Indian J. Phys. **43**, 76—82 (1969)

Impedance bridge network problem as solved by relaxation method

By S. N. Dutta

Department of Applied Physics, Calcutta University,

(Received 27 December 1968)

Here it is shown how the relaxation method can be advantageously used to solve the problems of A. C. networks containing complex circuit constants. This has been done in the solution of impedance bridge network problem, in which many useful information are obtained at a time. The results so obtained are compared with those calculated by the conventional method.

1. Introduction

Impedance bridges are commonly used to measure the circuit constants such as A. C. resistance, inductance etc. In this paper a network problem of unbalanced impedance bridge (figure 1) has been considered and the currents flowing in all the branches including the detector have been found out. This bridge circuit in a slightly unbalanced condition has great importance in having delicate sensing device. The methods

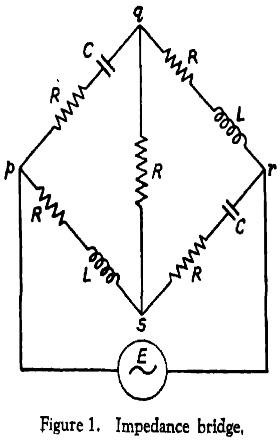

Figure 1. Impedance bridge,

which are generally used for analysis of this bridge configuration consisting of three meshes are a bit complicated due to the presence of complex circuit constants. It is described here how it can be solved easily and quickly by relaxation method when the equivalent circuit diagram (figure 2a) of the said network (figure 1) is considered.

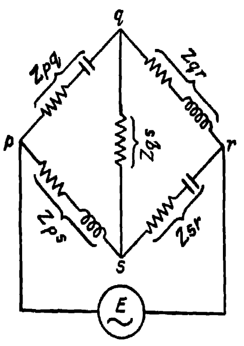

Figure 2a. Equivalent impedance network diagram of figure 1.

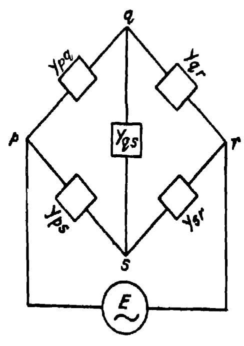

Figure 2b. Equivalent admittance network diagram of figure 1.

First of all Southwell & Black (1938) and later on Dutta (1966), and Basu & Dutta (1967) showed that the relaxation method can be suitably applied to solve the A. C. network problem without having much difficulty due to the presence of complex circuit constants. The method itself speaks of its advantage in getting many useful information simultaneously.

THE METHOD

Let Y_{pq}, Y_{qs} ....etc., be the reciprocals of the impedances Z_{pq}, Z_{qs} ....etc., of the branches '*pq*', '*qs*' .... etc., of the network shown in figure 2b.

Then the admittances of any branch for example '*qs*' can be written as,

$$Y_{qs} = g_{qs} + jb_{qs} \qquad ...(1)$$

Let the potentials at the nodal points '*q*' and '*s*' be

$$\left.\begin{aligned} V_q &= v_{x(q)} + jv_{y(q)} \\ V_s &= v_{x(s)} + jv_{y(s)} \end{aligned}\right\} \qquad ...(2)$$

So a current which flows from 'q' to 's' in branch 'qs' is given by,

$$I_{qs} = Y_{qs}\ (V_q - V_s)$$

$$= g_{qs}\{v_{x(q)} - v_{x(s)}\} - b_{qs}\{v_{y(q)} - v_{y(s)}\}$$

$$+ j\ [g_{qs}\{v_{y(q)} - v_{y(s)}\} + b_{qs}\{v_{x(q)} - v_{x(s)}\}]$$

If the total current flowing into 'q' from all the branches linked with it be,

$$-\sum_q I_{qs} = I_{q1} = \{i_{x(q)1} + j i_{y(q)1}\}$$

then,

$$\left.\begin{aligned} -i_{x(q)1} &= \sum_q [g_{qs}\{v_{x(q)} - v_{x(s)}\} - b_{qs}\{v_{y(q)} - v_{y(s)}\}] \\ \text{and} \quad -i_{y(q)1} &= \sum_q [g_{qs}\{v_{y(q)} - v_{y(s)}\} + b_{qs}\{v_{x(q)} - v_{x(s)}\}] \end{aligned}\right\} \quad \ldots(3)$$

Also, if the current supplied to 'q' from outside be,

$$I_{q2} = i_{x(q)2} + j i_{y(q)2}$$

then by Kirchhoff's law,

$$\left.\begin{aligned} i_{x(q)} &= i_{x(q)1} + i_{x(q)2} = 0 \\ \text{and} \quad i_{y(q)} &= i_{y(q)1} + i_{y(q)2} = 0 \end{aligned}\right\} \quad \ldots(4)$$

Let the vector potential of the point 'p' be unity and those of the points 'q' and 's' be zero. Then the currents flowing in the branches 'pq' and 'ps' are given by,

$$\left.\begin{aligned} I_{pq} &= Y_{qp} = g_{qp} + j b_{qp} \\ \text{and} \quad I_{ps} &= Y_{sp} = g_{sp} + j b_{sp}; \end{aligned}\right\} \quad \ldots(5)$$

and no current will flow in any other branch of the circuit. But in order to have the assumed potential correct, a current,

$$-I_{p2} = I_{pq} + I_{ps} = i_{x(p)2} + j i_{y(p)2},$$

is to be supplied to the point 'p' from outside. Under that condition the currents I_{pq} and I_{ps} leave the network at the points 'q' and 's' respectively. But actually no current enters into or leaves the network at the points 'q' and 's'. So on the assumed potentials those are to be superposed which would result if the currents $I_{q2} = I_{pq}$, and and $I_{s2} = I_{ps}$ were supplied at points 'q' and 's' and allowed to leave the network at the points 'p' and 'r', the latter points being maintained at zero potential.

Thereafter it is obtained initially as follows :

$$i_{x(q)} = i_{x(q)2} = g_{pq} \;\; ; \;\; i_{x(s)} = i_{x(s)2} = g_{sp} \;;$$

$$i_{y(q)} = i_{y(q)2} = b_{qp} \;\; ; \;\; i_{y(s)} = i_{y(s)2} = b_{sp} \;;$$

$$i_{x(p)} = i_{x(p)2} = -(g_{qp} + g_{sp}) \;;$$

$$i_{y(p)} = i_{y(p)2} = -(b_{qp} + b_{sp}) \;;$$

and

$$i_{x(r)} = i_{y(r)} = 0.$$

The standard operation table required for liquidation of the residuals that is, the initial values $i_{x(q)}$, $i_{y(q)}$, $i_{x(s)}$ and $i_{y(s)}$, by giving suitable vector potentials at the points 'q' and 's' only can be obtained using the following expression developed from the relations (3) :

$$\left.\begin{aligned} \frac{\delta i_{x(s)}}{\delta v_{x(q)}} &= g_{qs} = \frac{\delta i_{y(s)}}{\delta v_{y(q)}} \;; \\ -\frac{\delta i_{x(s)}}{\delta v_{y(q)}} &= b_{qs} = \frac{\delta i_{y(s)}}{\delta v_{x(q)}} \;; \\ \frac{\delta i_{x(q)}}{\delta v_{x(q)}} &= \frac{\delta i_{y(q)}}{\delta v_{y(q)}} = -\sum_{q}(g_{qs}) \;; \\ -\frac{\delta i_{x(q)}}{\delta v_{y(q)}} &= \frac{\delta i_{y(q)}}{\delta v_{x(q)}} = -\sum_{q}(b_{qs}) \;; \end{aligned}\right\} \quad \dots \quad (6)$$

The vector currents at the points 'p' and 'r' and the vector potentials at the points 'q' and 's' corresponding to unit potential difference between the points 'p' and and 'r' are obtained after liquidation of the residuals. The following illustration will show the advantage of the method.

ILLUSTRATION

This example (figure 1) worked out by Kerchner & Corcoran (1960) is taken up for illustration in which,

$V_{pr} = 100\angle 0°$ volts, $R_{ps} = 1$ ohm, $X_{ps} = 12$ ohms, $R_{pq} = 4$ ohms, $X_{pq} = 6$ ohms, $R_{sr} = 0.6$ ohms, $X_{sr} = 6.7$ ohms, $R_{qr} = 6.12$ ohms, $X_{qr} = 10.16$ ohms

The currents delivered by the source and those flowing in the four arms and the detector of the bridge network are to be found out.

Considering the different branches of the network shown in figure 2b, the corresponding admittances for the problem are calculated to be,

$$Y_{pq} = (\ 7.6923 + j\ 11.5385\ ) \times 10^{-2} \text{ mho},$$

$$Y_{qr} = (\ 4.3503 - j\ \ 7.2221\ ) \times 10^{-2} \quad ,, \ ,$$

$$Y_{ps} = (\ 0.6897 - j\ \ 8.2758\ ) \times 10^{-2} \quad ,, \ ,$$

$$Y_{sr} = (\ 1.3260 + j\ 14.8066\ ) \times 10^{-2} \quad ,, \ ,$$

$$Y_{qs} = (\ 33.3333 + \ 0 \qquad) \times 10^{-2} \quad ,, \ ,$$

The above admittances are multiplied by 10^2 to have a simplified calcultion and higher accuracy of results. This multiplying factor 10^2 and the potential difference V_{pr} have to be taken into account in calculating the currents.

Thus the currents which flow in the branches '*pq*' and '*ps*' are written as,

$$I_{pq} = 7.6923 + j\ 11.5385 \qquad \text{amps},$$

$$I_{ps} = 0.6897 - j\ \ 8.2758 \qquad ,,.$$

So the current to be flown from outside at the point '*p*' is found out to be,

$$-I_{p2} = 8.3820 + j\ 3.2627 \text{ amps}.$$

Thereafter the following initial values are obtained :

$$i_{x(q)} = \ 7.6923 \ ; \ i_{x(s)} = 0.6897 \ ; \ i_{x(p)} = -\ 8.3820 \ ;$$

$$i_{y(q)} = 11.5385 \ ; \ i_{y(s)} = -\ 8.2758 \ ; \ i_{y(p)} = -\ 3.2627 \ ;$$

and $i_{x(r)} = i_{y(r)} = 0.$

In liquidating the residuals the unit operation table (table 1) is obtained using the relations (6) in which the '*g*' and '*b*' values are multiplied by 10^2 for the above mentioned reason. Afterwards the group operations (Allen, 1954 ; Dutta, 1966), are performed as shown in table 2. Finally the residuals are liquidated in four steps only shown in the relaxation table (table 3) obtained by using tables 1 and 2.

The following values of the currents as wanted in the problem are found out by relaxation method and those obtained by the conventional one are shown side by side within the square brackets and they are seen to be in good agreement.

TABLE 1. UNIT OPERATION TABLE

Operation Steps	$\delta v_{x(q)}$	$\delta v_{x(s)}$	$\delta v_{y(q)}$	$\delta v_{y(s)}$	$\delta i_{x(p)}$	$\delta i_{x(q)}$	$\delta i_{x(s)}$	$\delta i_{x(r)}$	$\delta i_{y(p)}$	$\delta i_{(q)y}$	$\delta i_{y(s)}$	$\delta i_{y(r)}$
1	1	—	—	—	7·6923	−45·3759	33·3333	4·3503	11·5385	−4·3164	0	−7·2221
2	—	1	—	—	0·6897	33·3333	−35·3490	1·3260	−8·2758	0	−6·5308	14·8066
3	—	—	1	—	111·5385	4·3164	0	7·2221	7·6923	−45·3759	33.3333	4·3503
4	—	—	—	1	−8·2758	0	6·5308	−14·8066	0·6897	33·3333	−35·3490	1·3260

TABLE 2. GROUP OPERATION TABLE

Operation Steps	$\delta v_{x(q)}$	$\delta v_{x(s)}$	$\delta v_{y(q)}$	$\delta v_{y(s)}$	$\delta i_{x(p)}$	$\delta i_{x(q)}$	$\delta i_{x(s)}$	$\delta i_{x(r)}$	$\delta i_{y(p)}$	$\delta i_{y(q)}$	$\delta i_{y(s)}$	$\delta i_{y(r)}$
1	1	0	10·5124	−5·1040	−155·8447	0	0	155·8447	88·8828	−651·4592	530·8343	31·7421
2	0	1	−7·7225	5·4127	134·5902	0	0	−134·5903	−63·9464	530·8385	−448·7499	−11·6113
3	1	1·2272	1·0353	1·5385	10·3353	0	0	-9·3253	10·4074	0	−27·8891	17·4925

TABLE 3. RELAXATION TABLE

Liquidation Steps	$\delta v_{x(q)}$	$\delta v_{x(s)}$	$\delta v_{y(q)}$	$\delta v_{y(s)}$	$i_{x(p)}$	$i_{x(q)}$	$i_{x(s)}$	$i_{x(r)}$	$i_{y(p)}$	$i_{y(q)}$	$i_{y(s)}$	$i_{y(r)}$
		Initial Values			−8·3820	7·6923	0·6897	0	−3·2627	11·5385	−8·2758	0
1	0	0	-1·7821	0	12·1808	0	0·6897	−12·8705	−16·9711	92·4029	−67·6791	−7·7527
2	0	0	0	−0·1056	11.3069	0	0	−11·3069	−17·0439	88·8829	−63·9462	−7·8927
3	0·1364	0	1·4339	−0·6962	−9·9506	0	0	9·9506	−4·9202	0	8·4604	−3·5631
4	0·3034	0·3723	0·3141	0·4668	−7.1211	0	0	7.1209	−1·7624	0	0	1.7436
	0·4398	0·3723	−0·0341	−0·3350	−7·1211	0	0	7·1209	−1·7624	0	0	1·7436

The current delivered by the source is,

$$I_p = I_r = 7.121 + j1.753\ [7.140 + j1.760] \text{ amps.}$$

the currents in the four arms are,

$$I_{ps} = 3.205 - j4.963\ [3.190 - j4.960] \text{ amps,}$$

$$I_{pq} = 3.916 + j6.726\ [3.930 + j6.726] \quad ,, \ ,$$

$$I_{sr} = 5.454 + j5.068\ [5.444 + j5.090] \quad ,, \ ,$$

$$I_{qr} = 1.670 - j3.320\ [1.680 - j3.320] \quad ,, \ ,$$

and the detector current is,

$$I_{sq} = -2.250 - j10.030\ [-2.254 - j10.050] \text{ amps.}$$

Discussion

The solution of impedance bridge network, particularly, when it is unbalanced becomes laborious by the conventional method, whereas, the discussed one is seen to yield easily the values of the detector voltages (vector) viz., V_q and V_s, and the current I_r at a time. Thereafter, other required quantities can be calculated quickly. Also the number of residuals to be liquidated being four the labour involved in it is further reduced. Herein lies the advantage of the method over those followed normally.

The author is highly indebted to Prof. A. K. Sengupta, D. Sc., M. I. E. E. (London), Head of the Department of Applied Physics, Calcutta University, for his help and guidance throughout the progress of this work.

References

Allen, D. N. deG., 1954 *Relaxation Methods* McGraw-Hill Book Co., Inc., New York 17.

Basu, R. N. & Dutta, S. N. 1967 *Indian J. Phys* **41**, 382.

Black, A. N., & Southwell, R. V. 1938 *Proc. Roy. Soc.* (A), **164**, 447.

Dutta, S. N., 1966 *Indian J. Phys*, 40, 581.

Kerchner, R. M., & Corcoran, G. F. 1960 *Alternating Current Circuits*, John Wiley & Sons., Inc., New York 207.

Southwell, R. V., 1951 *Relaxation Methods in Engineering Science*, Oxford University Press, London 114.