Letters to the Editor

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17

MATRIX RELATIONS BETWEEN DIRECTION COSINES AND MILLERIAN INDICES

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For magnetic measurements in triclinic crystals, directions are usually specified by the Millerian indices of one or two planes, while for many calculations, the direction cosines with reference to some orthogonal axes x, y, z, are necessary. As an example, suppose the z axis coincides with the crystallographic c axis and the x axis lies in the $a-c$ plane close to the a axis. The cosines of the angles between the various axes are as follows.

	a	b	c
x	$\sin\beta$	$(\cos\gamma-\cos\alpha\cos\beta)/\sin\beta$	0
y	0	$M/\sin\beta$	0
z	$\cos\beta$	$\cos\alpha$	1

where $M = (1-\cos^2\alpha-\cos^2\beta-\cos^2\gamma+2\cos\alpha\cos\beta\cos\gamma)^{\frac{1}{2}}$ and is positive. The matrix $[J]$ of transformation from x, y, z to a, b, c, is the matrix formed by the trigonometrical quantities as arranged in the above table (Ghose, 1964). These coordinates x, y, z, have a slight advantage compared to those discussed by U. S. Ghosh and Mitra (1965), because the determinant of $|J|$ is positive. The values of $[J]$ and $[J]^{-}$ need be calculated only once for a particular variety of crystal. It may be verified that

$$[J]^{-1} = \begin{bmatrix} \dfrac{1}{\sin\beta} & -\dfrac{\cos\alpha\cos\beta-\cos\gamma}{M\sin\beta} & 0 \\[2ex] 0 & \dfrac{\sin\beta}{M} & 0 \\[2ex] -\dfrac{\cos\beta}{\sin\beta} & \dfrac{\cos\beta\cos\gamma-\cos\alpha}{M\sin\beta} & 1 \end{bmatrix}$$

As shown by Ghose (1964), a unit vector $[\xi\ \eta\ \zeta]$ in x, y, z coordinates, transforrms to a unit covariant vector $[V_p]$ and a unit contravariant vector $[V^p]$ in a, b, c coordinates such that

$$[\xi\ \eta\ \zeta] = [V_a\ V_b\ V_c][J]^{-1} = [J][V^a V^b V^c]' \qquad \ldots \quad (1)$$

It may be noted that V_a, V_b, V_c are cosines of the angles made by the unit vector with a, b, c axes while ξ, η, ζ are those made with the orthogonal x, y, z axes.

If $a:b:c$ be the axial length ratio and (hkl) be the Minerian indices of a plane, then any contravariant vector lying in this plane, is given by

$$[U^a\ U^b\ U^c] = \left[(m+n)\frac{a}{h} \quad -\frac{mb}{k} \quad -\frac{nc}{l}\right]$$

where m and n are any two numbers. For a covariant unit vector $[V_p]$ normal to this plane, $[V_p][U^p]$ is zero for all values of m and n. Hence we get

$$[V_a\ V_b\ V_c] = Q\left[\frac{h}{a} \quad \frac{k}{b} \quad \frac{l}{c}\right] \qquad \ldots \quad (2)$$

where Q is a suitable multiplier to make the magnitude of the vector equal to unity.

Again if two planes (hkl) and $(h'k'l')$ are both kept perpendicular to the plane of measurement, then the normal to the latter lies in both (hkl) and $(h'k'l')$ planes and is given by the contravariant vector

$$[V^a V^b V^c]' = Q_1[a(kl' - k'l)\ \ b(lh' - l'h)\ \ c(hk' - h'k)]' \qquad \ldots \quad (3)$$

where Q_1 is a suitable multiplier.

Values of ξ, η, ζ may be found by combining either eq. (2) or eq. (3) with eq.(1). If ξ_1, η_1, ζ_1 are the values obtained when Q or Q_1 is arbitrarily taken as 1, then the actual value of Q or Q_1 is $(\xi_1^2+\eta_1^2+\zeta_1^2)^{-\frac{1}{2}}$. When only the ratio of two direction cosines is required, Q need not be calculated.

This matrix method of calculating the direction cosines seems to be simpler and quicker than other methods.

REFERENCES

Ghose, J. K., 1964, *Ind. Jour. pure appl. Phys.*, **2**, 94.

Ghosh, U. S. and Mitra, S., 1964, *Ind. J. Phys.*, **38**, 19.