Letters to the Editor

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ON MEASUREMENT OF ELECTRON DENSITY OF PLASMA

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The usual methods for determining the electron density in a narrow plasma column are :

(a) Perturbation method, —to note the detuning of a cavity due to introduction of a coaxial plasma in it. The mode of the cavity, used most often, is TM_{010} mode. For a TM_{010} mode in a cylindrical cavity the change in the resonant frequency of the cavity is related to the electron density in Plasma as shown by Agdur and Enander (1962).

(b) Admittance method, —to note the admittance offered by the plasma column when placed in a waveguide parallel to the microwave electric field vector of TE_{10} mode. The relation between the admittance and electron density is given by Davidson and Farvis (1962).

However, both the above methods have certain limitations. The perturbation method is assumed to be valid if $N/N_c < 1+(\lambda/2\pi r_1)^2$ (Buchsbaum, Mower, and Brown, 1960), where N is the electron density in plasma, N_c is the critical electron density corresponding to the wavelength of microwave radiation λ, and r_1 is the radius of the plasma column. The admittance method is assumed to be applicable if $r_1/a << 1$ (Davidson and Farvis, 1962), where a is the width of the waveguide. Nevertheless the results given by the methods are somewhat approximate. Besides, in both the methods the effect of the plasma container, viz. a quartz or pyrex glass tube, is not properly taken into consideration.

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To assess the accuracy of the results and for comparison with the results as given by the perturbation and admittance methods for a typical plasma, a more accurate method termed, in this report, the exact method, has been developed. With the plasma placed coaxially in it, the cylindrical cavity operating in TM_{010} mode is tuned. Maxwell's equations for the different media in the cavity, viz. plasma, its container and air (air assumed equivalent to free space), are solved, and the following transcendental equations relating the operating frequency f and the electron density N are obtained by applying proper boundary conditions (Sen, Basu, and Ghoshal, 1964)

For $N < Nc$,

$$\left[J_0(\beta_2 r_2)-\sqrt{\frac{\epsilon_2}{\epsilon_0}}\,A J_1(\beta_2 r_2)\right]\left[Y_0(\beta_2 r_1)J_1(\beta_1 r_1)-\sqrt{\frac{\epsilon_2}{\epsilon_1}}\,J_0(\beta_1 r_1)Y_1(\beta_2 r_1)\right]$$

$$=\left[\sqrt{\frac{\epsilon_2}{\epsilon_0}}\,A Y_1(\beta_2 r_2)-Y_0(\beta_2 r_2)\right]\left[\sqrt{\frac{\epsilon_2}{\epsilon_1}}\,J_0(\beta_1 r_1)J_1(\beta_2 r_1)-J_1(\beta_2 r_1)J_1(\beta_1 r_1)\right] \quad \ldots \text{(1a)}$$

For $N > Nc$,

$$\left[J_0(\beta_2 r_2)-\sqrt{\frac{\epsilon_2}{\epsilon_0}}\,A J_1(\beta_2 r_2)\right]\left[Y_0(\beta_2 r_1)I_1(\beta_1 r_1)-\sqrt{\frac{\epsilon_2}{c_1}}\,I_0(\beta_1 r_1)Y_1(\beta_2 r_1)\right]$$

$$=\left[\sqrt{\frac{\epsilon_2}{\epsilon_0}}\,A Y_1(\beta_2 r_2)-Y_0(\beta_2 r_2)\right]\left[\sqrt{\frac{\epsilon_2}{\epsilon_1}}\,I_0(\beta_1 r_1)J_1(\beta_2 r_1)-J_0(\beta_2 r_1)I_1(\beta_1 r_1)\right] \quad \ldots \text{(1b)}$$

Where $$A=\frac{J_0(\beta_0 r_2)Y_0(\beta_0 r_3)-J_0(\beta_0 r_3)Y_0(\beta_0 r_2)}{J_1(\beta_0 r_2)Y_0(\beta_0 r_3)-J_0(\beta_0 r_3)Y_1(\beta_0 r_2)}$$

$\beta_0 =$ propagation constant corresponding to f

$\beta_1 =$ propagation constant in plasma $= \beta_0\sqrt{\frac{\epsilon_1}{\epsilon_0}} = \beta_0(1-N/Nc)^{\frac{1}{2}}$

$\beta_2 =$ propagation constant in the medium constituting the plasma container $= \beta_0\sqrt{\frac{\epsilon_2}{\epsilon_0}}$.

ϵ_0, ϵ_1, ϵ_2 are the permittivities of free space, plasma and its container respectively.

r_1, r_2 are the inner and outer radii of the container,

r_3 is the radius of the cavity.

J_0, J_1 are the Bessel functions of the first kind ; Y_0, Y_1 are the Bessel functions of the second kind,

I_0, I_1 are the modified Bessel functions of the first kind.

Experiments have been carried out, at S-band frequency, on a typical mercury discharge plasma column of radius $r_1 = 3.89$ mm, for discharge currents ranging from 0 to 500 mA. The plasma is contained in a pyrex glass tube of thickness 1.1 mm and dielectric constant 4. 58, the latter being determined by the method, the authors have described elsewhere (Sen, Basu, and Ghoshal, 1964). The cavity radius r_3 is 38.37mm., and the width of the waveguide a is 72 mm. Pressure in the plasma was of the order of 1μ.

The electron densities at various discharge currents have been determined by the perturbation, admittance method and by the exact method using equations (1a) and (1b). The electron density at the maximum current was of the order of 1.7×10^{11} per c.c., corresponding to $N/Nc \simeq 1.5$. Compared with the results given by the exact method, those by the perturbation and admittance methods were within 15% and 20% respectively. It appears that, of the perturbation and admittance methods, the former is somewhat more accurate.

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REFERENCES

Agdur, B., and Enander, B., 1962, *Journal of Applied Physics*, **33**, 575.

Buchsbaum, S. J., Mower, Lyman., and Brown, S. C., 1960, *Physics of Fluids*, **3**, 806.

Davidson, C. W., and Farvis, W. E. J., 1962, *Physical Review*, **127**, 1858.

Sen, S. K., Basu, J., and Ghoshal, A. K., 1964, *Ind. Jour. of Phys.* **38**, 601.