# VISCOSITY OF POLAR-NONPOLAR GAS MIXTURES EMPIRICAL METHOD 

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#### Abstract

The adequary of the familiar Sutherland exprossion for the viseosity of gas mixtures is tested suncesssfully for binary mixtures involving one component as polar. The limited calculations of this paper also reverl that the co-efficients of the Sutherland expression may be treated as approximately temperature independent. This interesting result may find great use in predicting values at high temperatures where no direct measurements are available.


## INTRODUCTION

Inspite of the fact that viscosity is the best theoretically understood transport property we still lack enough in our knowledge to be able to predict it with reliance for complicated systems such as polar gas mixtures. This is primarily because of the complicated nature of the interatomic forces of polar gas molecules, in general. Some calculations have recently been reported by Mason and Monchick (1962) for a somewhat over simplified potential model. This is about the best that has been achieved so far, unfortunately. Mathur and Saxena (1964a) have recently examined the Chapman-Enskog expression for the viscosity of a binary gas mixture, $\eta_{m i x}$, in a fashion analogous to that of Gambhir and Saxena (1964) for nonpolar gas mixtures. This study revealed that most of the simple polar gases are amenable to treatment on the general theory of Chapman-Enskog except the collision cross-sections may require still more sophisticated angular averaging than given by Monchick and Mason (1961). This oncouraged us to look into the procedures analogous to the approximate method of Wilke (1950) and semiempirical method of Saxena and Gambhir (1963a, 1963b). All these calculations, which are still to be published, indicated that the over-all accuracy of such calculations is the same as that obtained for nonpolar gases.

The purpose of this article is to look into the possibility of being able to correlate the experimental $\eta_{\text {mix }}$ data on the basis of the following equation, Sutherland (1895) :

$$
\begin{gathered}
\eta_{m i x}=\sum_{i=1}^{n} \frac{\eta_{i}}{1+\sum_{j=1}^{n} \phi_{i j} \frac{X_{j}}{X_{i}}}, \\
j \neq i
\end{gathered}
$$

where $\eta_{i}$ and $X_{i}$ are the coefficient of viscosity and mole fraction rospectively of the pure component of molecular weight $M_{i}$, and $\phi_{i j}$ is a constant which as shown by Mathur and Saxena (1964a) is a complicated function of $T$ (temperature), $M_{i}, X_{i}$, and collision integrals. They also showed that the dependence of $\phi_{i j}$ on temperature and composition is feeble. Here will treat $\phi_{i j}$ as disporable parameters to be determined from the knowledge of $\eta_{m i x}$. This endeavour holds a great promise in predicting the $\eta_{m i x}$ values hit high temperatures, as well as values of multicomponent gas mixtures.

## CALCULATION OF V SCOSITIES

If the two pure viscosities and the mixture viscosity values be known at two compositions of a hinary system, $\phi_{12}$ and $\phi_{21}$ canibe evaluated directly on the basis of Eq. (1). If the form of Eq. (1) is correct the it should be possible to correlate

TABLE I
Comparison of the calculated and experimental $\eta_{\text {mix }}$ values of $\mathrm{H}_{2}-\mathrm{NH}_{3}$ system and some other pertinent data

| Temp ${ }^{\text {c }}$. $K$ | $\times$ polar | $\times \eta_{\text {mix }}$ Exptl. | /(\%m ser ${ }^{\text {Calc. }}$ ) ( | \%dev. | $\phi_{12}$ | $\phi_{21}$ | $\begin{gathered} \eta_{N H S} \times 10^{7} \\ (\mathrm{gm} / \mathrm{cm} \\ \mathrm{Soc} .) \end{gathered}$ | $\begin{gathered} \eta_{H 2} \times 10^{7} \\ (\mathrm{gm} / \mathrm{cm} . \\ \text { Nec. } .) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 293.16 | 0.9005 | 1004 | 1005 | (++0.1) | 0.307 | 1.659 | $(\mathrm{a}, \mathrm{c}, \mathrm{~d})$ | (b) $881$ |
|  | 0.7087 | 1047 | 1047 | $(+0.0)$ |  |  |  |  |
|  | 0.5177 | 1080 | 1080 | $(++0.0)$ |  |  |  |  |
|  | 0.2975 | 1087 | 1087 | $(+0.0)$ |  |  |  |  |
|  | 0.2239 | 1072 | 1072 | $(+0.0)$ |  |  |  |  |
|  | 0.1082 | 1011 | 1012 | ( + 0.1) |  |  |  |  |
| 373.16 | 0.9005 | 1209 | 1302 | $(+0.2)$ |  |  | 1279 | 1035 |
|  | 0.7087 | 1333 | 1341 | $(+0.6)$ |  |  |  |  |
|  | 0.5177 | 1354 | 1367 | $(+1.0)$ |  |  |  |  |
|  | 0.2975 | 1329 | 1349 | $(+1.5)$ |  |  |  |  |
|  | 0.2239 | 1299 | 1320 | $(+1.6)$ |  |  |  |  |
|  | 0.1082 | 1204 | 1224 | $(+1.7)$ |  |  |  |  |
| 473.16 | 0.9005 | 1660 | 1668 | $(+0.5)$ |  |  | 1646 | 1217 |
|  | 0.7087 | 1680 | 1703 | $(+1.4)$ |  |  |  |  |
|  | 0.6177 | 1676 | 1718 | $(+2.5)$ |  |  |  |  |
|  | 0.2975 | 1610 | 1669 | $(+3.7)$ |  |  |  |  |
|  | 0.2239 | 1560 | 1620 | $(+3.8)$ |  |  |  |  |
|  | 0.1082 | 1432 | 1479 | $(+3.3)$ |  |  |  |  |
| 523.16 | 0.9005 | 1825 | 1836 | $(+0.6)$ |  |  | 1814 | 1303 |
|  | 0.7087 | 1837 | 1869 | $(+1.7)$ |  |  |  |  |
|  | 0.5177 | 1823 | 1879 | $(+3.1)$ |  |  |  |  |
|  | 0.2975 | 1737 | 1817 | $(+4.6)$ |  |  |  |  |
|  | 0.2239 | 1678 | 1760 | ( +4.9 ) |  |  |  |  |
| 523.16 | 0.9005 | 1825 | 1826(a) | $(+0.1)$ | 0.376 | 1.605 | 1814 | 1303 |
|  | 0.7087 | 1837 | 1837 | $(+0.0)$ |  |  |  |  |
|  | 0.5177 | 1823 | 1822 | $(-0.1)$ |  |  |  |  |
|  | 0.2975 | 1737 | 1737 | $(+0.0)$ |  |  |  |  |
|  | 0.2239 | 1678 | 1678 | $(+0.0)$ |  |  |  |  |

(a) Utilising $\varphi_{i j}$ calculated at this very temperature. $a, b, c$, and $d$ stand for the same references as given in the foot-note of Table 2,

## TABLE II

Comparison of the calculatod and experimental $\eta_{m i x}$ values of $\mathrm{N}_{2}-\mathrm{NH}_{3}$ system and some other pertinent data

| $\eta_{7 n_{i x} \times 10^{7}(\mathrm{gm} . / \mathrm{cm} \text {-sec }) ~}^{\text {c }}$ |  |  |  |  |  |  | $\eta_{N / 2} \times 107$ | $\eta_{\text {NII }} \times 107$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Temp. <br> ${ }^{\prime} K$ | $\times p$ olar | Exptl. | Culc. | \%dev | $\varphi_{12}$ | $\varphi_{21}$ <br> (gm | (gm/cm-sec) | (gm/cm-sec) |
| 293.16 |  |  |  |  | 0.954 | 0.839 | (b) | ${ }^{(a, c, c)}$ |
|  | 0.8883 | 1092 | 1093 | (-10.1) |  |  | $1758$ | 982 |
|  | 0.7147 | 1254 | 1254 | $(+0.0)$ |  |  |  |  |
|  | () 5638 | 1383 | 1383 | $(-10.0)$ |  |  |  |  |
|  | 0.2920 | 1885 | 1585 | $(+0.0)$ |  |  |  |  |
|  | 0.1111 | 1690 | 1698 | $(+0.5)$ |  |  |  |  |
| 37316 | 0.8883 | 1398 | 1407 | $(+0.6)$ |  |  | 2145 | 1279 |
|  | 0.7147 | 1569 | 1591 | $(+1.4)$ |  |  |  |  |
|  | 0.6638 | 1710 | 1736 | $(+1.5)$ |  |  |  |  |
|  | 0.2920 | 1920 | 1961 | $(+2.1)$ |  |  |  |  |
|  | 0 1111 | 2031 | 2082 | $(+2.5)$ |  |  |  |  |
| 473.16 | 0.8883 | 1768 | 1787 | $(+1.1)$ |  |  | 2580 | 1640 |
|  | 0. 7147 | 1946 | 1988 | $(+2.1)$ |  |  |  |  |
|  | 0.6638 | 2085 | 2145 | $(+2.9)$ |  |  |  |  |
|  | 0.2920 | 2296 | 2380 | $(+3.7)$ |  |  |  |  |
|  | 0.1111 | 2408 | 2501 | $(+3.9)$ |  |  |  |  |
| 523.16 | 0.8883 | 1939 | 1960 | $(+1.1)$ |  |  | 2740 | 1814 |
|  | $0.7147$ | 2112 | 2167 | $(+2.6)$ |  |  |  |  |
|  | 0.5038 | 2250 | 2327 | (-1 3.4) |  |  |  |  |
|  | 0.2920 | 2460 | 2564 | (-4.2) |  |  |  |  |
|  | 0.1111 | 2572 | 2683 | ( $-1-4.3$ ) |  |  |  |  |
|  |  |  |  |  |  | 2.401 |  | (a,c.d) |
| 293.16 | 0.8883 | 1092 | 1090 | (-0.2) | 0.533 |  | $1758$ | $985$ |
|  | 0.7147 | 1254 | 1254 | (10.0) |  |  |  |  |
|  | 0.5638 | 1383 | 1385 | (+0.1) |  |  |  |  |
|  | 0.2920 | 1585 | 1585 | (-0.0) |  |  |  |  |
|  | 0.1111 | 1690 | 1697 | (+0.4) |  |  |  |  |
| 373.16 | 0.8883 | 1398 | 1392 | $(-0.4)$ |  |  | 2145 | 1279 |
|  | 0.7147 | 1569 | 1572 | $(+0.2)$ |  |  |  |  |
|  | 0.5638 | 1710 | 1718 | $(+0.5)$ |  |  |  |  |
|  | 0.2920 | 1920 | 1946 | $(+1.4)$ |  |  |  |  |
|  | 0. 1111 | 2031 | 2074 | (+2.1) |  |  |  |  |
| 473.16 |  | 1788 | 1753 | (-0.8) |  |  | 2560 | 1646 |
|  | 0.7147 | 1946 | 1937 | $(-0.5)$ |  |  |  |  |
|  | 0.5638 | 2085 | 2092 | $(+0.3)$ |  |  |  |  |
|  | 0.2920 | 2296 | 2340 | $(+1.9)$ |  |  |  |  |
|  | 0.1111 | 2408 | 2482 | $(+3.1)$ |  |  |  |  |
| 523.16 | 0.8883 | 1939 | 1916 | (-1.2) |  |  | 2740 | 1814 |
|  | 0.7147 | 2112 | 2100 | (-0.6) |  |  |  |  |
|  | 0.5638 | 2250 | 2257 | (-0.3) |  |  |  |  |
|  | 0.2920 | 2460 | 2512 | $(+2.1)$ |  |  |  |  |
|  | 0.1111 | 2572 | 2659 | $(+3.4)$ |  |  |  |  |

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the entire composition range in this way. As this approach has proved very successful for the binary combinations of nonpolar gasos it will be interesting to see how far this procedure holds when polar gases are involved. To check this we consider the specific systems $\mathrm{H}_{2}-\mathrm{NH}_{3}$ and $\mathrm{N}_{2}-\mathrm{NH}_{3}$ for which claborate data are available as a function of composition at various temperatures. It may be pointed out that the procedure for $\phi_{i j}$ determination suggested above will. in general, lead to two sots of coupled $\phi_{i j}$ yalues. In certain cases it may be possible to discard one in favour of the othe but sonetimes it may not be. In Table I, we list the calculated values for $\mathrm{H}_{2}+\mathrm{NH}_{3}$ while in Table II for $\mathbf{N}_{\mathbf{2}}-\mathbf{N H}_{3}$. Only one set of appropriate $\phi_{i j}$ values wire found in the case, of $\mathrm{H}_{2}-\mathrm{NH}_{3}$ system but in the case of $\mathrm{N}_{2}-\mathrm{NH}_{3}$ system two sets wore found. It may be possible to decide about their relative appropriateness by the degree of agreement betweon the calculateif and experimental values. Thus, for the $\mathbf{N}_{2}-\mathrm{NH}_{3}$ system the first set of $\phi_{i j}{ }^{i}$ values, reproduce the $\eta_{\text {mix }}$ values within the average absolute deviation of $2.5 \%$, while for the latter set this number drops to $1.3 \%$ only. However, the other relatively rigorous procedures of evaluating $\phi_{i j}$ load to values closer to the first set (Mathur and Saxena, 1964a, 1964b) and therefore this particular set should be preferred over the second set. A little enhanced dis erepancy found in the values uring this sot when compared with the experimental values should not matter much for $\phi_{i j}$ are also dependent on $M_{i}$ and to some extent on the temperature, composition and nature of molecular interactions. The other set owes its origin to the fact that a large number of coupled $\phi_{i}$ 's are possible which will reproduce the data almo t equally well (Gray and Wright, 1961). Here, of course, we are looking for such values of $\phi_{i j}$, which are independent of temperature, composition and nature of potential energy functions.

Cowling (1961) gave a simple hut approximate interpretation of the physical significance of $\phi_{i j}$. Aceording to him $\phi_{i j}$ is the ratio of the efficiencies with which molecules $j$ and molecules $i$ separately impede the transport of momentum by molecules $i$. Thus, on this picture one should get only one set of $\phi_{i j}$ values. The fact that we get here two sets is not necessarily a contradiction. The reason for this anomaly lies in the assumptions made in arriving at this simple definition for $\phi_{i j}$. As pointed out by Cowling (1961) Eq. (1) with this meaning for $\phi_{i j}$ assumes the interaction between molecules of difforent gases purely as opposition to transport of momentum and neglects other interactions, for example, the one most important arises from the transfer of momentum transport from one gas to another at collisions. This has been discussed by Cowling, Gray and Wright (1963) and when this effect is also included much more complicated expressions result for $\phi_{i j}$ and its simple definition is found to be no more valid. Therefore, the only criterion of choosing one set of $\phi_{i j}$ values amongst many should be the concordance of their actual values with those obtained on the basis of rigorous theory (Mathur and Saxena, 1964a).

To investigate the adequacy of the temperature independent $\phi_{i j}$ for purposes of prediction at high temperatures we aloo compute $\eta_{\text {mix }}$ at high temperatures where direct measured values are available. The $\phi_{i j}$ were always evaluated at the lowest temperature and it is interesting that the reproduction even at the highest temperature is not bad. A somewhat better correlation is possible if $\phi_{i j}$ are calculated at the same temperature. Thus for $\mathrm{H}_{2}-\mathrm{NH}_{3}$, whore calculations were repeated at $T=523.16^{\wedge} \mathrm{K}$, it was found that the values get reproducex within the maximum deviation of $0.1 \%$

## CONCLUSIONS

As for nonpolar binary and higher order mixturos, the Sutherland form seems adequate to correlate the data even when polar gases are involved. Numerical calculations performed here for the two systems indicate that Sutherland's expression holds the promise of being extended to high temperatures with $\phi_{i j}$ values determined at a lower temperature. By this analogy one may expect the generalised Sutherland formula to be adequate also for multicomponent mixtures involving polar gases though no checks could be made becanse of the lack of experimental data.

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