11

ELECTRON VELOCITY DISTRIBUTION IN SLIGHTLY IONIZED ARGON WITH CROSSED ELECTRIC AND MAGNETIC FIELDS

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ABSTRACT. A method of solving the Boltzmann equation for the distribution function of electrons in slightly ionized argon with crossed electric and magnetic fields is outlined using Golant's analytical approximations of the experimental data on the cross-sections for elastic and inelastic collisions. The distribution function is obtained in the presence of an electric field of arbitrary frequency crossed with a constant magnetic field as well as in the presence of crossed d.c. electric and magnetic fields.

It has been found from the plots of the electron distribution function that (i) for a given E/p, the distribution with crossed d.c. electric and magnetic fields contains more low energy electrons than that with only d.c. electric field, and (ii) that in a given transverse magnetic field, the electron distribution for a higher E/p is richer in higher energy electrons than that for a lower value of E/p.

INTRODUCTION

The problem of electron velocity distribution in a gaseous discharge has been the subject of investigation for many years. The knowledge of the distribution function is important in the study of transport phenomena in ionized gases. Recently Golant (1957, 1959) has determined the electron distribution in argon under the influence of a high frequency field using an analytical approximation of the experimental data on cross-sections of elastic and inelastic collisions of electrons with argon atoms. By solving the Boltzmann equation numerically Engelhardt and Phelps (1964) have found that the electron distribution obtained by them agrees satisfactoritly with that of Golant. Thus the approximations used in the analysis of Golant have produced no appreciable distortion in the resulting electron distribution. In the present communication the method of Golant has been extended to obtain the electron distribution in argon in the presence of an electric field of arbitrary frequency crossed with a constaut magnetic field, The case of crossed d.c. electric and magnetic fields has also been treated.

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THE ELECTRON DISTRIBUTION FUNCTION

The electron velocity distribution function $f(\vec{v})$ can be obtained by solving the well-known Boltzmann transport equation. In the case where the electrons are acted on simultaneously by electric and magnetic fields and where the electron density gradient is negligible, the Boltzmann equation is given by

$$\frac{\partial f}{\partial t}+(\vec{a}+\vec{\omega}_b\times\vec{v})\cdot\vec{\nabla}_v f=\left(\frac{\delta f}{\delta t}\right)_{coll} \quad \ldots (1)$$

where $\vec{a}=-\frac{e\vec{E}}{m}$ (E is the electric field experienced by the particles),

$\vec{\omega}_b=\frac{e\vec{B}}{mc}$ the electron cyclotron frequency,

$\vec{\nabla}_v$ the gradient operator in the velocity space,

and $\left(\frac{\delta f}{\delta t}\right)_{coll}$ the change of the distribution function in time due to collision.

Expanding the distribution function $f(\vec{v})$ in spherical harmonics as

$$f(\vec{v})=f_0+\frac{\vec{v}\cdot\vec{f}_1}{v}+\ldots \quad \ldots (2)$$

and substituting the expansion, retaining only first two terms, into *Eq.* (1) we obtain the following equations for the components of the distribution function (Allis, 1956)

$$\frac{\partial f_0}{\partial t}-\frac{\vec{\gamma}\cos\omega t}{3v^2}\cdot\frac{\partial}{\partial v}(v^2\vec{f}_1)=\left(\frac{\delta f_0}{\delta t}\right)_{coll} \quad \ldots (3)$$

$$\frac{\partial \vec{f}_1}{\partial t}-\vec{\gamma}\cos\omega t\frac{\partial f_0}{\partial v}-\vec{\omega}_b\times\vec{f}_1=\left(\frac{\delta \vec{f}_1}{\delta t}\right)_{coll} \quad \ldots (4)$$

where

$$\vec{E}=\vec{E}_0\cos\omega t$$

$$\vec{\gamma}=\frac{e\vec{E}_0}{m}$$

Writing the collision terms for a slightly ionized gas following Golant (1957), Eqs. (3) and (4) become

$$\frac{\partial f_0}{\partial t}-\frac{\vec{\gamma}\cos\omega t}{3v^2}\cdot\frac{\partial}{\partial v}(v^2\vec{f}_1)=\frac{m}{M}\frac{1}{v^2}\frac{\partial}{\partial v}\left(\frac{v^4 f_0}{\lambda_e}\right)-\frac{vf_0}{\lambda_{ne}}, \quad \ldots (5)$$

$$\frac{\partial \vec{f_1}}{\partial t} - \vec{\gamma} \cos \omega t \frac{\partial f_0}{\partial v} - \vec{\omega_b} \times \vec{f_1} = -\frac{v \vec{f_0}}{\lambda_e} \quad \dots \quad (6)$$

Here λ_e is the diffusion mean freepath for elastic collisions and λ_{ne}, the corresponding quantity for inelastic processes. The last term in Eq. (5) is dropped out below the threshold for inelastic processes. The above form of collision terms have been derived on the assumptions that (i) the thermal energy of the neutral particles is negligible in comparison with the average electron energy, that (ii) an electron loses its energy completely during an inelastic collision and that (iii) the inelastic collisions do not affect the distribution symmetry.

In the case of crossed electric and magnetic fields, with the magnetic field along the z—axis, Eq. (6) can be written as

$$\frac{\partial f_{1x}}{\partial t} - \gamma_x \cos \omega t \frac{\partial f_0}{\partial v} + \omega_b f_{1y} = -\frac{v}{\lambda_e} f_{1x} \quad \dots \quad (6a)$$

$$\frac{\partial f_{1y}}{\partial t} - \gamma_y \cos \omega t \frac{\partial f_0}{\partial v} - \omega_b f_{1x} = -\frac{v}{\lambda_e} f_{1y} \quad \dots \quad (6b)$$

$$\frac{\partial f_{1z}}{\partial t} = -\frac{v}{\lambda_e} f_{1z} \quad \dots \quad (6c)$$

Multiplying Eq. (6*b*) by i, adding it to Eq. (6a), and putting

$$f_{1x} + i f_{1y} = f_1^1, \quad \gamma_x + i\gamma_y = \gamma^1$$

the following equation is obtained :

$$\frac{\partial f_1^1}{\partial t} - \gamma^1 \cos \omega t \frac{\partial f_0}{\partial v} = -\left(\frac{v}{\lambda_e} - i\omega_b\right) f_1^1 \quad \dots \quad (7)$$

Considering f_0 to be independent of time, the steady state solution of Eq. (7) is given by

$$f_1^1 = \gamma^1 \frac{\partial f_0}{\partial v} \left[\frac{\omega}{\omega^2 + (v/\lambda_e - i\omega_b)^2} \sin \omega t + \frac{v/\lambda_e - i\omega_b}{\omega^2 + (v/\lambda_e - i\omega_b)^2} \cos \omega t \right] \quad \dots \quad (8)$$

the steady state solution of Eq. (6c) yielding $f_{1z} = 0$. Substituting for $\vec{f_1}$ in Eq. (5) and taking the time average,

$$-\frac{\gamma^2}{6v^2} \frac{\partial}{\partial v} \left\{ \frac{v^3}{\lambda_e} \frac{\partial f_0}{\partial v} \frac{[\omega^2 + \omega_b^2 + v^2/\lambda_e^2]}{[(\omega - \omega_b)^2 + v^2/\lambda_e^2][(\omega + \omega_b)^2 + v^2/\lambda_e^2]} \right\}$$

$$= \frac{m}{M} \frac{1}{v^2} \frac{\partial}{\partial v} \left(\frac{v^4 f_0}{\lambda_e} \right) - \frac{v f_0}{\lambda_{ne}} \quad \dots \quad (9)$$

Substituting $v^2 = xu_0$, where $u_0 = 2eU_0/m$, U_0 being the threshold for inelastic processes, Eq. (9) becomes

$$-\frac{2}{3}\frac{\gamma^2}{\omega^2}x^{\frac{1}{2}}\frac{\partial}{\partial x}\left\{\frac{x^2}{\lambda_e}\frac{\partial f_0}{\partial x}\frac{[(1+(\omega_b/\omega)^2+xu_0/(\lambda_e^2\omega^2)]}{[(1-\omega_b/\omega)^2+xu_0/(\lambda_e^2\omega^2)][(1+\omega_b/\omega)^2+xu_0/(\lambda_e^2w^2)]}\right\}$$

$$= \frac{2mu_0x^{1/2}}{M}\frac{\partial}{\partial x}\left(\frac{x^2f_0}{\lambda_e}\right)+\frac{u_0x^{3/2}f_0}{\lambda_{ne}} \quad \dots \quad (10)$$

Upto this point the treatment is quite general and Eq. (10) can be solved for the isotropic part of the distribution function in any gas by using suitable values of collision cross sections. Lax, Allis and Brown (1950) have obtained a similar equation which they have eventually treated by assuming constant collision frequency. Following Golant (1957) we assume that the collision mean freepaths λ_e and λ_{ne} have energy dependences given by

$$\lambda_e = \begin{cases} \lambda_e^0 x^{\frac{1}{2}} & \text{for } x > 1 \\ \lambda_e^0/x & \text{for } 1 > x > 0.1 \\ 10\lambda_e^0 & \text{for } x < 0.1 \end{cases} \quad \dots \quad (11a)$$

and

$$\lambda_{ne} = \frac{\lambda_{ne}^0}{x^{\frac{1}{2}}(x-1)} \quad \text{for } x > 1 \quad \dots \quad (11b)$$

where $x = \frac{U}{U_0}$. v is the electron energy and $U_0 = 11.5$ ev is the first ionization potential of argon.

Thus for the region $x > 1$, Eq. (10) reduces to

$$x^{\frac{1}{2}}\frac{\partial}{\partial x}\left(x^{3/2}\frac{\partial f_0}{\partial x}\right)+\alpha x^{\frac{1}{2}}\frac{\partial}{\partial x}(x^{3/2}f_0)-\beta x^2(x-1)f_0 = 0, \quad \dots \quad (12)$$

where

$$\alpha = \frac{3m}{M}\frac{u_0^2}{\gamma^2\lambda_e^{0\,2}}[1+(1-\omega_b/\omega)^2\zeta]\left[1+\frac{2\omega_b}{\omega}\left\{1+\left(\frac{\omega_b}{\omega}\right)^2+\frac{1}{\zeta}\right\}^{-1}\right],$$

$$\beta = \frac{3}{2}\frac{u_0^2}{\gamma^2\lambda_e^0\lambda_{ne}^0}\left[1+\left((1-\frac{\omega_b}{\omega}\right)^2\zeta\right]\left[1+\frac{2\omega_b}{\omega}\left\{1+\left(\frac{\omega_b}{\omega}\right)^2+\frac{1}{\zeta}\right\}^{-1}\right],$$

$$\zeta = \frac{\omega^2\lambda_e^{0\,2}}{u_0}$$

As $\alpha << \beta$, the effect of elastic collision is negligible in the region $x > 1$ and Eq. (12) can be solved for the distribution function $f_0(x)$ in the region $x > 1$ (Golant, 1957) :

$$f_0(x) = x^{-3/4}(x-1)^{\frac{1}{2}}K_{1/3}\left[\frac{2}{3}\beta^{\frac{1}{2}}(x-1)^{3/2}\right] \quad \dots \quad (13)$$

where $K_{1/3}$ is the MacDonald function. An arbitrary multiplying constant is implied in the above solution which is obtained by normalization.

At $x = 1$ the distribution function $f_0(x)$ as given in Eq. (13) and its first derivative become

$$f_0(1) = 1.91\beta^{-1/6}; \quad f_0'(1)/f_0(1) = -[0.75+0.73\beta^{1/3}] \quad \dots \quad (14)$$

In the regions $0.1 < x < 1$ and $x < 0.1$ it has been shown by Golant (1957) that the term involving $\frac{m}{M}$ can be neglected. Thus substituting for λ_e in Eq.(10) from Eqs. (11) and integrating we obtain for the region $0.1 < x < 1$

$$\frac{\partial f_0}{\partial x} = \frac{A[(x^3+q^3)^2-k]}{x^3(x^3+q^3)}$$

and

$$f_0(x) = B+A\left[x-\left(1-\frac{k}{q^6}\right)\frac{q^3}{2x^2}+\frac{k}{6q^5}\left\{\ln\frac{(x+q)^2}{(x^2-qx+q^2)}+2\sqrt{3}\tan^{-1}\frac{2x-q}{q\sqrt{3}}\right\}\right], \quad \dots \quad (15)$$

where

$$q^3 = \zeta\left\{1+\left(\frac{\omega_b}{\omega}\right)^2\right\}$$

$$k = 4\zeta^2(\omega_b/\omega)^2$$

The constants of integration A and B determined by matching $\partial f_0/\partial x$ and $f_0(x)$ at $x = 1$, are given by

$$A = \frac{(1+q^3)}{[(1+q^3)^2-k]} \quad f'_0(1), \quad \dots \quad (16)$$

$$B = f_0(1)-A\left[1-\frac{q^3}{2}\left(1-\frac{k}{q^6}\right)+\frac{k}{6q^5}\left\{\ln\frac{(1+q)^2}{(1-q+q^2)}+2\sqrt{3}\tan^{-1}\frac{2-q}{q\sqrt{3}}\right\}\right] \quad (17)$$

Similarly for the region $x < 0.1$

$$\frac{\partial f_0}{\partial x} = \frac{c}{x^2}\frac{[(x+10^2q^3)^2-10^4k]}{[x+10^2q^3]}$$

and

$$f_0(x) = D + c[\ln x - \frac{10^2q^3}{x}\left(1-\frac{k}{q^6}\right) - \frac{k}{q}\ln\left(1+\frac{10^2q^3}{x}\right)] \quad \ldots \quad (18)$$

The constants of integration C and D determined by matching $\partial f_0/\partial x$ and $f_0(x)$ at $x = 0.1$, are given by

$$C = 0.1A \quad \ldots \quad (19)$$

$$D = f_0(1) - A[0.6697 - 50.5q^3\left(1-\frac{k}{q^6}\right) - \frac{k}{10q}\ln(1+10^3q^3)$$

$$+\frac{k}{6q^5}\left\{\ln\left[\left(\frac{1-10q+10^2q^2}{1-q+q^2}\right)\left(\frac{1+q}{1+10q}\right)^2\right] + 2\sqrt{3}\tan^{-1}\frac{9\sqrt{3}q}{(20q^2-11q+2)}\right\}] \quad (20)$$

The above expression [Eqs. (13), (15) and (18)] give the electron distribution function in slightly ionized argon in the presence of an *ac* electric field of frequency ω crossed with a constant magnetic field.

In the case of a d.c. electric field ($\omega = 0$) crossed with constant magnetic field, the following equation is obtained instead of Eq. (9) :

$$-\frac{\gamma^2}{3v^2}\frac{\partial}{\partial v}\left\{\frac{v^3}{\lambda_e}\frac{1}{(\omega_b^2+v^2/\lambda_e^2)}\frac{\partial f_0}{\partial v}\right\} = \frac{m}{M}\frac{1}{v^2}\frac{\partial}{\partial v}\left(\frac{v^4 f_0}{\lambda_e}\right) - \frac{vf_0}{\lambda_{ne}} \quad \ldots \quad (9a)$$

the factor of 1/2 in the left hand side of Eq. (9), obtained by taking the time average, being omitted. The solutions of Eq. (9a) in the three energy regions are :

$$f_0(x) = x^{-3/4}(x-1)^{\frac{1}{4}}K_{1/3}\left[\frac{2}{3}\beta^{\frac{1}{2}}_1(x-1)^{3/2}\right], \quad x > 1; \quad (21)$$

$$f_0(x) = A_1x - \tfrac{1}{2}A_1\zeta_1/x^2 + B_1, \quad 0.1 < x < 1; \quad \ldots \quad (22)$$

$$f_0(x) = -10^2C_1\zeta_1/x + C_1\ln x + D_1, \quad x < 0.1\ ; \quad \ldots \quad (23)$$

where

$$\beta_1 = \frac{3}{4}\frac{u_0^2}{\gamma^2\lambda_e^0\lambda^0_{ne}}[1+\zeta_1],$$

$$\zeta_1 = \frac{\omega_b^2\lambda_e^{02}}{u_0},$$

$$A_1 = \frac{1}{1+\zeta_1}f'_0(1)$$

$$B_1 = f_0(1) - f'_0(1)\frac{1-0.5\zeta_1}{1+\zeta_1}$$

$$C_1 = \frac{0.1}{1+\zeta_1} f'_0(1),$$

$$D_1 = f_0(1) - f'_0(1) \frac{0.6697 - 50.5\zeta_1}{1+\zeta_1}$$

These expressions [Eqs. (21), (22) and (23)] for the distribution function with crossed d.c. electric and magnetic fields become similar to those obtained by Golant (1957) for the case of a.c. electric field if ω_b is replaced by ω. The applicability of this substitution has been pointed out by Engelhardt and Phelps (1963) as well. Further it is to be noted that Eqs. (21), (22) and (23) can be obtained from Eqs. (13), (15) and (18) respectively, by putting $\omega = 0$ and $\beta/2 = \beta_1$.

From the electron velocity distribution function $f_0(x)$ one obtains then the electron energy distribution function $F(x)$ by means of the relationship

$$F(x) = \frac{x^{\frac{1}{2}} f_0(x)}{\int_0^\infty x^{\frac{1}{2}} f_0(x) dx} \quad \ldots \quad (24)$$

Plots of the electron energy distribution function are shown in Figs. (1) and (2) for several values of magnetic field and E/p in the case of crossed d.c. electric and magnetic fields. The corresponding curves for only d.c. electric field are also plotted in the same figures for comparison.

DISCUSSION

By using the cross-sections for elastic and inelastic collisions as given by Golant, we have determined the electron velocity distribution in argon under the influence of crossed electric and magnetic fields. Eqs. (13), (15) and (18) above give, respectively, the isotropic part of the electron distribution function $f_0(x)$ in the regions $x > 1$, $0.1 < x < 1$ and $x < 0.1$ for an a.c. electric field crossed with a constant magnetic field. For the case of crossed d.c. electric and magnetic fields the distribution function is given by the Eqs. (21), (22) and (23). It is evident from these equations as well as from the curves plotted in Figs. (1) and (2) that the presence of the magnetic field perpendicular to the electric field has altered the distribution considerably. In the low energy region curve 1 in Fig. 1 for zero magnetic field is lying below the other curves (2, 3, 4, 5 and 6) for successive values of the magnetic field, indicating fewer low energy electrons in the distribut-tion when the magnetic field is absent. Thus the application of the crossed d.c. electric and magnetic fields causes an excess of low energy electrons over the number when the electric field alone is present and the effect increases as the magnetic field is increased. Our plots of the distribution function further show (Fig. 2) that with the increase of E/p the distribution becomes richer in higher energy electrons.

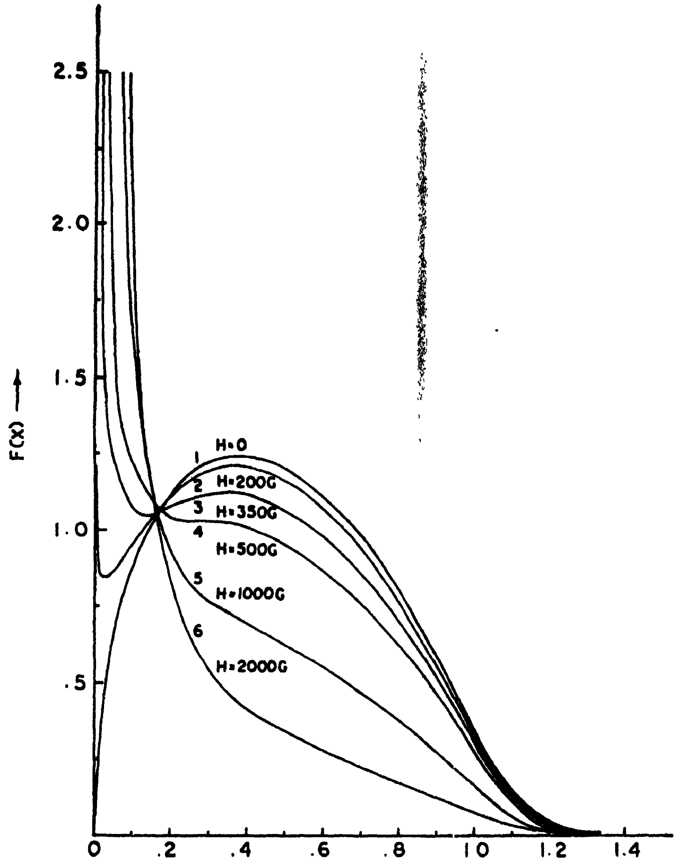

Fig. 1.—Electron energy distribution for d.c. electric field and crossed d.c. electric and magnetic fields with $E/p = 10$ V/cm. mm. Hg. and different values of the magnetic field. Curve 1 is for zero magnetic field and other curves (2, 3, 4, 5 and 6) for successively increasing value of the magnetic field.

It is interesting to note that the electron distribution obtained by us for crossed d.c. electric and magnetic fields is similar to that found by Golant (1957) for a.c. electric field without magnetic field. The similarity arises due to the fact that in the parameters β and ζ determining $f_0(x)$ the cyclotron frequency ω_b and the a.c. frequency ω occur in the same place. Perhaps this similarity between the two distributions indicates that the mechanism of energy transfer to the electrons is similar in both the a.c. fields and the crossed d.c. electric and magnetic fields.

Our derivation of the distribution function following the method of Golant has the limitation that the distribution function goes to infinity at $x = 0$. This infinity is spurious, since, unless there is a point source of electrons at the origin, the distribution function must be finite. According to Holstein (1946) this spurious infinity is due to break down of basic assumptions in the derivation of

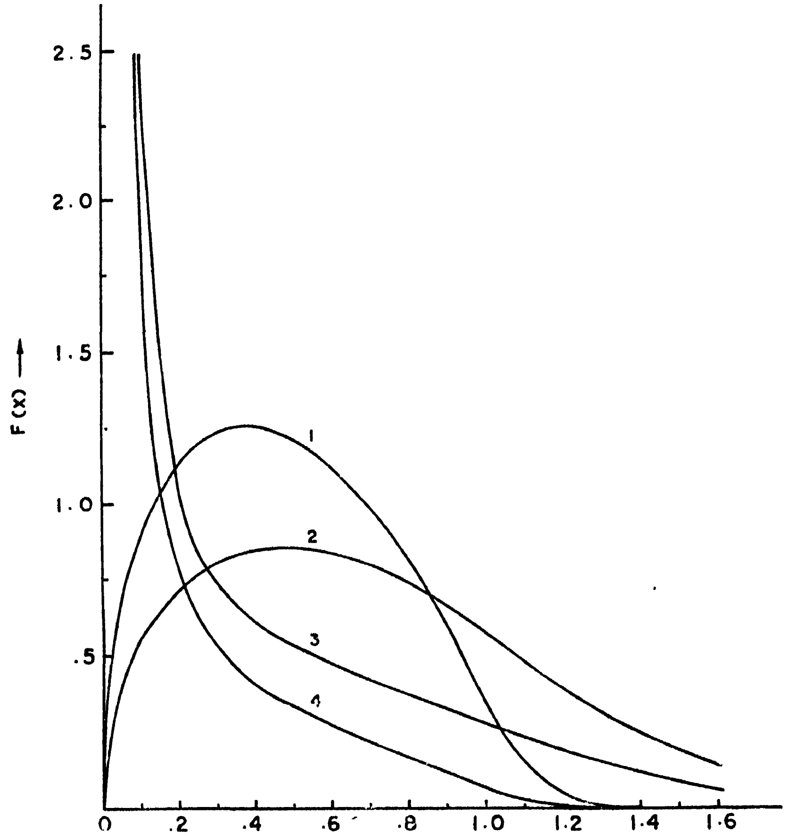

Fig. 2.—Electron energy distribution for different values of E/p and magnetic field. Curves : (1) E/p = 10 V/cm.mm.Hg H = 0; (2) E/p = 100 V/cm.mm.Hg., H = 0; (3) E/p = 100 V/cm.mm. Hg. H = 2000G; (4) E/p = 10 V/cm.mm.Hg., H = 2000G.

$F(x)$ when $x \rightarrow 0$. In particular the representation of the velocity distribution function $f(\vec{v})$ by the first two terms of the expansion in Eq.(2) is incorrect when $x << 1$. However, this is not a serious defect, since the total number of electrons having energy smaller than any limiting value remains finite and goes to zero as this limit tends to zero. Thus this would not influence derivation of the transport co-efficients by using the distribution function.

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