GROWTH OF HYDROMAGNETIC SHOCK WAVES

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ABSTRACT. In this technical note we consider the influence of a transverse magnetic field on the formation of a shocks wave in an electrically conducting field. We conclude that the presence of a transverse magnetic field is conducive to the growth of compression waves and the decay of the expansion waves.

It is well known that the ordinary hydrodynamic compression shock wave involves an increase in entropy and that the rarefaction shock wave decays immediately into a continuous expansion wave. This is so because in a compression wave, the waves nearer the source tend to overtake those further from it with the result that the wave profile becomes more and more steep until the pressure gradients become infinite. In this way a compression shock is formed which grows in strength as the process continues. In a rarefaction shock, on the other hand, the waves nearer the shock lag more and more behind those in front of it, the wave profile flattens till the pressure gradients vanish and ultimately no dis continuity effects are observed. In fact if a rarefaction shock is established, even momentarily, it would decay immediately into a continuous expansion wave. We will consider, in this note, the influence of a transverse magnetic field on the formation of a shock wave in an electrically conducting fluid. It will be seen that the presence of a transverse magnetic field is conducive to the formation of a shock wave.

For the sake of simplicity we will consider a plane one-dimensional shock wave propagating in a fluid of infinite electrical conductivity and specific volume τ with an external magnetic field H oriented in a direction normal to the direction in which the shock propagates. We define the quantities $p^{\times}$ and $c^{\times}$ according to Hoffmann *et al.*, 1950.

$$p^{\times} = p + \frac{H^2}{8\pi}, \quad \text{and} \quad c^{\times} = (c^2 + v^2_{ALF})^{\frac{1}{2}} \qquad \ldots (1)$$

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These quantities take account of the contributions of the hydromagnetic interaction to the pressure p and the velocity of sound c through the magnetic pressure $(H^2/8\pi)$ and the Alfven speed $v_{ALF} = H(\tau/4\pi)^{\frac{1}{2}}$. We will call $p^{\times}$ and $c^{\times}$ the total pressure and the modified velocity of sound respectively.

Let us now suppose that the properties of the fluid at two adjacent points differ in magnitude by $d\tau$, dH, dv, $dp^{\times}$ and $dc^{\times}$ where v denotes the gas speed. We also assume that the respective parts of the wave passing through these points differ in speed of propagation by dv_{ω}. For the sake of simplicity, let us further assume that the gradients of temperature and velocity are small so that the dissipative effects of viscosity and heat conduction are negligible. Therefore each elementary part of the wave travels with the local speed of sound with respect to the fluid. The velocity of propagation v_{ω} of this part of the wave with reference to a fixed coordinate system is

$$V_{\omega} = v + c^{\times} \quad \ldots \quad (2)$$

and the velocity of propagation of an adjacent part of the wave is

$$v_{\omega} + dv_{\omega} = v + dv + c^{\times} + dc^{\times} \quad \ldots \quad (3)$$

so that

$$\frac{dV_{\omega}}{dp^{\times}} = \frac{dv}{dp^{\times}} + \frac{dc^{\times}}{dp^{\times}} \quad \ldots \quad (4)$$

Let us assume that the entire fluid was initally at rest with uniform pressure and temperature, and that each particle of the fluid undergoes isentropic changes. Therefore, the increments in pressure and density between adjacent particles obey the relation

$$c^{\times 2} = -\tau^2 \frac{dp^{\times}}{d\tau} \quad \ldots \quad (5)$$

which yields on differentiation

$$2c^{\times} \frac{dc^{\times}}{dp} = -\frac{d\tau}{dp^{\times}} \frac{d}{d\tau} \left(\tau^2 \frac{dp^{\times}}{d\tau}\right) \quad \ldots \quad (6)$$

Again it can be shown from the equations of constant mass flux and constant momentum flux that

$$\frac{dv}{dp^{\times}} = \frac{\tau}{c^{\times}} \quad \ldots \quad (7)$$

On carrying out the substitutions from (6) and (7) in (4) we finally obtain

$$\frac{dV_\omega}{dp^\times} = -\frac{\tau}{2c^\times}\frac{(d^2p^\times/d\tau^2)}{(dp^\times/d\tau)} \qquad \dots \quad (8)$$

Now if $(dV_\omega/dp^\times)$ is positive, the high pressure parts of the wave overtake the low pressure parts and a wave of compression steepens as it progresses. Similarly a wave of rarefaction becomes less steep. On the other hand if $(dV_\omega/dp^\times)$ is negative, a wave of compression becomes less steep and a wave of rarefaction steepens into a compression shock.

A fluid is said to be thermodynamically stable if it does not collapse or expand catastrophically. For a fluid to be thermodynamically stable, $dV_\omega/dp^\times$ must be positive. It follows from (8) that the sign of $dV_\omega/dp^\times$ depends on the sign of

$$\frac{d^2p^\times}{d\tau^2} = \frac{d^2p}{d\tau^2} + \frac{3H^2}{4\pi\tau^2} \qquad \dots \quad (9)$$

From the considerations outlined above, we immediately arrive at the following conclusions :

(i) Compression waves steepen and rarefaction waves flatten when

$$\tau^2\frac{d^2p}{d\tau^2} + \frac{3H^2}{4\pi} > 0 \qquad \dots \quad (10)$$

This happens when

either (*a*) $d^2p/d\tau^2$ is positive

or (b) $d^2p/d\tau^2$ is negative, and

$$\left|\frac{d^2p}{d\tau^2}\right| < \frac{3H^2}{4\pi\tau^2}$$

Therefore, a flattening wave of compression will begin to steepen as soon as a magnetic field of suitable strength is switched on.

(ii) Compression waves flatten and rarefaction waves steepen when

$$\tau^2\frac{d^2p}{d\tau^2} + \frac{3H^2}{4\pi} < 0 \qquad (11)$$

This happens when $(d^2p/d\tau^2)$ is essentially notgative and

$$\frac{d^2p}{d\tau^2} > \frac{3H^2}{4\pi\tau^2} \qquad \dots \quad (12)$$

Hence it may be concluded that the magnetic field enhances the steepening of a compression wave and flattening of a rarefaction wave. Hence the presence of a transverse magnetic field is conducive to the growth of compression waves and the decay of the expansion waves.

REFERENCES

Hoffmann, F. de and Teller, E., 1950. *Phys. Rev.*, **80**, 692.