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OSCILLATIONS OF ROTATING COSMICAL BODIES IN THE PRESENCE OF MAGNETIC FIELD

J. N. TANDON

INDIAN NATIONAL COMMITTEE FOR THE IGY, NATIONAL PHYSICAL LABORATORY, DELHI

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ABSTRACT. The effect of rotation on the radial pulsations of cosmical fluid masses with special reference to spherical mass (magnetic variables) and cylindrical mass (spiral arm, solar-ion stream) has been investigated when the fluids are having volume electric currents. Two models of currents system are considered for cylindrical mass, viz., circular currents and line currents. It is found that for radial pulsations, rotation in general, helps in the dynamical stability of the cosmical bodies.

1. INTRODUCTION

Talwar and Tandon (1956) have earlier obtained an expression for the frequency of radial pulsations of spherical masses in the presence of megnetic field (magnetic variable stars). The magnetic field was assumed to be axially symmetric and derivable from volume currents flowing in the interior of the star. They also obtained an upper limit for the magnetic field above which the star will become dynamically unstable provided $\Gamma < 4/3$ where Γ is the ratio of the two specific heats Similar problem for radial pulsations of the infinitely long cylinder (spiral arm solar-ion streams etc.) having volume currents has also been investigated by Tandon and Talwar (1957). Two special cases, (1) circular currents and (2) line currents are investigated It is found that the cylinder remains dynamically stable for both the models

In this paper we have investigated the effect of rotation on the frequency of pulsations of the cosmical masses having volume currents. §2 deals with the radial pulsations of rotating spherical mass and is of great significance for magnetic variables. Ledoux (1945) has treated the similar problems for non-magnetic stars and has obtained the expression for frequency of radial pulsations. Our expression is similar to one obtained by Ledoux except that an additional term $\int \mathbf{r} \cdot (\mathbf{J} \times \mathbf{H})\, d\tau$ along with gravitational energy term Ω has been obtained It is also shown that rotation helps in the dynamical stability of the star provided $\frac{4}{3} < \Gamma < \frac{5}{3}$. In §3 we have considered the effects of the rotation on the radial pulsations of cylindrical fluid masses. The two special cases of the volume currents, viz., circular and line currents have been re-investigated. It is found that rotation helps in the dynamical stability of the cylinder also.

2 PULSATIONS OF ROTATING SPHERE WITH VOLUME CURRENTS

The equation of motion of a uniformly rotating fluid mass having an internal magnetic field arising from the volume currents can be written as

$$\frac{d\mathbf{u}}{dt}=-\frac{1}{\rho}\operatorname{grad}p-\operatorname{grad}V+\frac{1}{\rho}(\mathbf{J}\times\mathbf{H})$$

$$-\mathbf{w}\times(\mathbf{w}\times\mathbf{r})-2(\mathbf{w}\times\mathbf{u})-\left(\frac{d\mathbf{w}}{dt}\times\mathbf{r}\right)\quad\ldots\quad(1)$$

where ρ denotes the fluid density, V the gravitational potential, p the pressure and $\mathbf{w}$ the angular velocity at any point. The magnetic field $\mathbf{H}$ and the current density $\mathbf{j}$ satisfy the following relation inside

$$\operatorname{curl}\mathbf{H}=4\pi\mathbf{J}\quad\ldots\quad(2)$$

$$\operatorname{div}\mathbf{H}=0\quad\ldots\quad(3)$$

and the field outside is continuous at the boundary.

Assuming axial symmetry, $\mathbf{u}$ the fluid velocity vector will be in the meridian plane and the last two terms on the right hand side of eqn (1) are the only vector in this equation which are normal to this plane*. Thus we should have

$$2(\mathbf{w}\times\mathbf{u})+\left(\frac{d\mathbf{w}}{dt}\times\mathbf{r}\right)=0\quad\ldots\quad(4)$$

and

$$\frac{d\mathbf{u}}{dt}=-\frac{1}{\rho}\operatorname{grad}p-\operatorname{grad}V+\frac{1}{\rho}(\mathbf{J}\times\mathbf{H})-\mathbf{w}\times(\mathbf{w}\times\mathbf{r})\quad\ldots\quad(5)$$

We multiply equation (5) scalarly on a vector $\mathbf{r}$ and integrate over the entire mass of the configuration. The left hand side of the equation becomes

$$\int^{M}\mathbf{r}\,\frac{d\mathbf{u}}{dt}\,dm=\int^{M}\mathbf{r}\,.\,\frac{d^2r}{dt^2}\,dm=\frac{1}{2}\,\frac{d^2}{dt^2}\int^{M}r^2dm-\int^{M}|u|^2dm\quad\ldots\quad(6)$$

where $dm=\rho d\tau(\equiv\rho dx_1\,dx_2\,dx_3)$
and the integration is effected over the entire mass, M, of the configuration.

* It may be noted here that we are restricting ourselves to a case when the electromagnetic force $j\times H$ also lies only in the meridian plane.

Letting

$$I = \int^{M} r^2 dm$$

and

$$T = \frac{1}{2}\int^{M} |u|^2 d$$

denote the movement of inertia and kinetic energy of mass motion respectively, we have

$$\frac{1}{2}\frac{d^2I}{dt^2} - 2T = -\int_V \mathbf{r}\,.\,\text{grad}\; p d\tau + \int^{M} \frac{1}{\rho}\, r.(\mathbf{j}\times\mathbf{H})dm$$
$$-\int^{M} \mathbf{r}\,.\,(\text{grad V})dm - \int^{M} \mathbf{r}\,.\{\mathbf{w}\times(\mathbf{w}\times\mathbf{r})\}dm \quad \ldots \quad (7)$$

The third integral on the right hand side of this equation represents the gravitational potential energy Ω of the configuration. Now

$$\int_V \mathbf{r}\,.\,\text{grad}\, p\, d\tau = \int_S p\, \mathbf{r}.d\mathbf{S} - \int_V p\, div\, \mathbf{r}\, d\tau = -3\int_V p\, d\tau \quad \ldots \quad (8)$$

since the gas pressure vanishes at the boundary of the surface. Thus we should have

$$\int_V \mathbf{r}\,.\,\text{grad}\; p\, d\tau = -3(\Gamma-1)U \quad \ldots \quad (9)$$

where U is the internal energy of the system. Now, since $\mathbf{r}.\ \mathbf{w} = 0$, the last integral on the right hand side of equation (8) can be written as

$$\int^{M} \mathbf{r}\,.\{\mathbf{w}\times(\mathbf{w}\times\mathbf{r})\}dm = -\int^{M} w^2(x^2+y^2)dm$$

$$= -\int^{m} wdm \quad \ldots \quad (10)$$

$$= - wW$$

where W is the total angular momentum Further, putting

$$\int^{M} \frac{1}{\rho}\,\mathbf{r}\,.\,(\mathbf{j}\times\mathbf{H})dm = E \quad \ldots \quad (11)$$

the electromagnetic energy of the fluid, and substituting the values of various integrals in equation (8) we find

$$\frac{1}{2}\frac{d^2I}{dt^2} = 2T + 3(\Gamma-1)u + \Omega + E + wW \quad \ldots \quad (12)$$

This is the Virial theorem for a system of rotating fluid subjected to electromagnetic field. We shall now apply this equation to the adiabatic pulsations of a rotating fluid in which there are body currents. In analysing this problem we shall adopt the Lagrangian mode of description and follow each element of mass, dm, as it moves.

Considering periodic oscillations with angular frequency we shall let $\delta\mathbf{r}e^{i\sigma t}$ denote the displacement of an element of mass dm, from its equilibrium position r_0 Similarly, we shall denote by $\delta p e^{i\sigma t}$, $\delta\rho e^{i\sigma t}$, $\delta\mathbf{H}e^{i\sigma t}$, $\delta\mathbf{j}e^{i\sigma t}$ and $\delta\mathbf{w}e^{i\sigma t}$ the corresponding changes in the other physical variables as we follow the element, dm, during its motion. The assumption that oscillations take place adiabatically requires that the changes in pressure and density, as we follow the motion, should satisfy the relation

$$\delta p = \Gamma\frac{\delta\rho}{\rho}p \tag{13}$$

where Γ is the ratio of the specific heats (assumed to be constant in space and time) while the equation of continuity

$$\frac{\partial\rho}{\partial t}+\rho\,\operatorname{div}\mathbf{u}=0$$

requires that

$$\frac{\delta\rho}{\rho}=-\operatorname{div}\delta\mathbf{r} \tag{14}$$

Returning to equation (4) and assuming Z-axis as the axis of rotation we can write it in cylindrical coordinate system (ω, θ, z) as follows

$$2w\frac{\partial\omega}{\partial t}+\omega\frac{\partial\omega}{\partial t}=0 \quad \ldots \quad (15)$$

Upon integration this leads to the relation

$$w\omega^2 = \text{constant} \quad \ldots \quad (16)$$

which can also be expressed in cartesians as follows

$$w(x^2+y^2) = \text{constant} \quad \ldots \quad (17)$$

Equation (4) simply expresses the conservation of angular momentum w.

Letting $\delta Ie^{i\sigma t}$, $\delta\Omega e^{i\sigma t}$, $\delta ue^{i\sigma t}$, $\delta Ee^{i\sigma t}$ and $\delta(wW)e^{i\sigma t}$ denote the changes in I, Ω U, E and wW respectively we can write equation (12) as

$$-\tfrac{1}{2}\sigma^2\delta I = 3(\Gamma-1)\delta u+\delta\Omega+\delta E+\delta(wW). \quad \ldots \quad (18)$$

Since to the first order in the displacement, the terms involved in T do not make any significant contribution.

Now

$$\delta I = 2 \int^{M} \mathbf{r} \cdot \delta \mathbf{r} dm \qquad \ldots (19)$$

$$\begin{aligned} 3(\Gamma - 1)\delta U &= 3 \int^{M} \delta(p/\rho) dm \\ &= 3 \int^{M} \left(\frac{\delta p}{\rho} - \frac{p \delta \rho}{\rho^2} \right) dm \\ &= 3(\Gamma - 1) \int^{M} \frac{p}{\rho} \frac{\delta \rho}{\rho} dm \\ &= -3(\Gamma - 1) \int_V p \operatorname{div} \delta \mathbf{r} \, d\tau \\ &= -3(\Gamma - 1) \left[\int_S p \delta \mathbf{r} \cdot d\mathbf{s} - \int_V \delta \mathbf{r} \cdot \operatorname{grad} p \, d\tau \right] \\ &= 3(\Gamma - 1) \int_V \delta \mathbf{r} \cdot \operatorname{grad} p \, d\tau \end{aligned} \qquad \ldots (20)$$

In obtaining equation (20) we have made use of the equations (13) and (14) and of the fact that the fluid pressure vanishes on the bounding surface. Further for the equilibrium configuration equation (5) gives

$$\operatorname{grad} p = -\rho \operatorname{grad} V + (\mathbf{J} \times \mathbf{H}) - \rho \mathbf{w} \times (\mathbf{w} \times \mathbf{r}) \qquad (21)$$

Therefore

$$\begin{aligned} 3(\Gamma - 1)\delta U &= 3(\Gamma - 1) \int_V \delta \mathbf{r} \cdot \operatorname{grad} p d\tau \\ &= 3(\Gamma - 1) \Big[-\int^{M} \delta \mathbf{r} \cdot \operatorname{grad} V dm \\ &\quad + \int_V \delta r \cdot (\mathbf{J} \times \mathbf{H}) d\tau \\ &\quad - \int^{M} \delta \mathbf{r} \cdot \{\mathbf{w} \times (\mathbf{w} \times \mathbf{r})\} dm \Big] \end{aligned} \qquad \ldots (22)$$

Further we have

$$\delta \Omega = - \int^{M} \delta \mathbf{r} \cdot \operatorname{grad} V dm \qquad \ldots (23)$$

But

$$
\begin{aligned}
\mathfrak{T} &= \delta \int^{M} \frac{1}{\rho}\, \mathbf{r} \,.\, (\mathbf{J}\times\mathbf{H})dm \\
&= \int^{M} \Big[\Big\{ \frac{\delta \mathbf{r}}{\rho} - \frac{\delta\rho}{\rho^2}\, \mathbf{r} \Big\} (\mathbf{J}\times\mathbf{H}) \\
&\quad + \mathbf{r} \; \{(\delta\mathbf{j}\times\mathbf{H}) + (\mathbf{J}\times\delta\mathbf{H})\} \Big] dm \\
&= \int [\{\delta\mathbf{r} + (\mathrm{div}\;\delta\mathbf{r})\,.\,\mathbf{r}\}(\mathbf{J}\times\mathbf{H}) \qquad (24) \\
&\quad + \mathbf{r} \,.\, \{(\delta\mathbf{J}\times\mathbf{H}) + (\mathbf{J}\times\delta\mathbf{H})\}]d\tau
\end{aligned}
$$

and since the total angular momentum is preserved during pulsations, we have

$$\delta(wW) = W\delta w \qquad (25)$$

Substituting equations (19) to (25) in equation (18) we get

$$
\begin{aligned}
\int^{M} \mathbf{r} \,.\, \delta\mathbf{r}\; dm &= -(3\Gamma-4) \int^{M} \delta\mathbf{r} \,.\, \mathrm{grad}\; V dm \\
&\quad + (3\Gamma-2) \int_{V} \delta\mathbf{r} \,.\, (\mathbf{J}\times\mathbf{H})d\tau \\
&\quad + \int_{V} (\mathrm{div}\;\delta\,\mathbf{r})\, \mathbf{r} \,.\, (\mathbf{J}\times\mathbf{H})d\tau \qquad (26) \\
&\quad + \int_{V} \mathbf{r} \,.\, [(\delta\mathbf{J}\times\mathbf{H}) + (\mathbf{j}\times\delta\mathbf{H})]d\tau \\
&\quad -3(\Gamma-1) \int^{M} \delta\,\mathbf{r} \,.\, \{\mathbf{w}\times(\mathbf{w}\times\mathbf{r})\}dm \\
&\quad + W\;\delta w
\end{aligned}
$$

This is the required integral formula for σ^2. The change δH following the motion is given by (Chandrasekhar and Fermi, 1953)

$$\delta\mathbf{H} = \mathrm{curl}\;(\delta\mathbf{r}\times\mathbf{H}) + (\delta\mathbf{r}\,.\,\mathrm{grad})\,\mathbf{H}$$

while $\delta\mathbf{j}$ will be evaluated by substituting the value of this in equation (2) remembering that the independent variable is r_0 and not r while following the motion

To obtain the approximate relation for the frequency of pulsations we put,

$$\delta\mathbf{r} = \xi\mathbf{r} \qquad \ldots \quad (28)$$

where ξ is constant in space. Thus it can readily be seen that

$$\begin{aligned} \delta\mathbf{H} &= -2\xi\mathbf{H} \\ \delta\mathbf{j} &= -3\xi\,\mathbf{j} \\ \delta\mathbf{w} &= -2\xi\mathbf{w} \end{aligned} \qquad \dots \quad (29)$$

and

$$\int^{M} \delta\mathbf{r}\,.\,\{\mathbf{w}\times(\mathbf{w}\times\mathbf{r})\}dm = \xi\, w\, W$$

Substituting equations (28) and (29) in equation (27) we obtain after some reduction

$$\sigma^2 \int^{M} r^2 dm = -(3\Gamma-4)[E+\Omega]+(5-3\Gamma)wW$$

or

$$\sigma^2 = -(3\Gamma-4)\,\frac{E+\Omega}{I}+(5-3\Gamma)\,\frac{wW}{I} \qquad \dots \quad (30)$$

It is evident from equation (30) that rotation like gravitation helps in the dynamical stability of the sphere provided $\frac{4}{3} < \Gamma < \frac{5}{3}$. Also there exists an upper limit for the magnetic field set by the following equation, viz.,

$$E = |\Omega| - \frac{3\Gamma-5}{3\Gamma-4}\,wW \qquad \dots \quad (31)$$

3 RADIAL PULSATIONS OF A ROTATING CYLINDER WITH VOLUME CURRENTS

Let us now consider an infinitely long cylinder, rotating with a constant angular velocity $\mathbf{w}$ in which the currents are flowing. The equation of motion for the radial pulsation of such a configuration assuming axial symmetry can be written as follows*

$$\frac{du_w}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial \omega} - \frac{2Gm(\omega)}{\omega} + \frac{1}{\rho}(\mathbf{j}\times\mathbf{H})_{radial}$$

$$-\{\mathbf{w}\times(\mathbf{w}\times\omega)\}_{radial} \qquad \dots \quad (32)$$

and

$$2(\mathbf{w}\times\mathbf{u})+\left(\frac{d\mathbf{w}}{dt}\times\omega\right) = 0 \qquad \dots \quad (33)$$

Here $m(\omega)$ is the mass of the unit length of the cylinder interior to ω. Equation (33) with Z-axis of the cylindrical coordinate system (ω, θ, z) as the axis of the rotation can be written in the form (after integration)

$$w\omega^2 = \text{constant} \qquad \dots \quad (34)$$

* Here we assume that $j\times\mathbf{H}$ has only radial component.

This equation simply expresses the conservation of angular momentum. Multiplying equation (32) by ω and integrating over the entire mass of unit cylinder and proceeding exactly as in § 2, we find

$$\int^{M} \omega \frac{du_{\omega}}{dt} = \frac{1}{2}\frac{d^2}{dt^2}\int^{M} \omega^2 dm - 2T \qquad \ldots \quad (35)$$

where M is the mass of the unit cylinder and T is the kinetic energy of the mass motion. Also

$$\int^{M} \frac{\omega}{\rho}\frac{\partial p}{\partial \omega}\, dm = -2\int_{V} p\, d\tau = -2(\Gamma-1)U \qquad \ldots \quad (36)$$

since div $\omega = 2$, for a 2 dimensional case and U is the internal energy per unit length of the configuration For a homogeneous fluid mass we further have

$$\int^{M} \omega \cdot \frac{2Gm}{\omega}\, dm = GM^2 \qquad \ldots \quad (37)$$

$$\int^{M} \frac{1}{\rho}\, \omega \cdot (\mathbf{J}\times\mathbf{H})dm = E \qquad \ldots \quad (38)$$

and

$$\int^{M} \omega \cdot \{\mathbf{w}\times(\mathbf{w}\times\omega)\}dm = -\int^{W} w\, dW = -wW \qquad \ldots \quad (39)$$

where $dW = w\omega^2$ and m is the angular momentum per unit length of the cylinder Hence the Virial theorem for the study of radial pulsations of an rotating infinite cylinder having volume currents will be

$$\frac{1}{2}\frac{d^2}{dt^2}\int^{M} \omega^2 dm = 2T + 2(\Gamma-1)U - GM^2 + E + wW \qquad \ldots \quad (40)$$

To study the radial pulsations we adopt as before Lagrangian mode of description. Now consider periodic pulsations with the frequency σ and let $\delta\omega e^{i\sigma t}$ denote the displacement of an element of mass, dm, from its equilibrium configuration, ω_0 Similarly, denote the corresponding changes in other physical variables by $\delta p e^{i\sigma t}$ etc. Further, the change in the pressure δp for adiabatic pulsations and the equation of continuity are represented by equations (13) and (14) respectively.

Letting $\delta u e^{i\sigma t}$, $\delta m e^{i\sigma t}$ and $\delta(wW)e^{i\sigma t}$ denote the changes in quantities U, E and wW we have from the Virial theorem

$$\sigma^2\int^{M} \omega\delta\omega\, dm = 2(\Gamma-1)\delta U + \delta E + \delta(wW) \qquad \ldots \quad (41)$$

Since GM^2 is constant and to the first order in the displacement, the terms in T will not make any contribution. Further,

$$2(\Gamma-1)\delta U = 2(\Gamma-1)\int_V \delta\omega \frac{\partial p}{\partial \omega}\, d\tau$$

since the pressure vanishes at the bounding surface. Now for the equilibrium configuration

$$\frac{\partial p}{\partial \omega} = -\frac{2Gm}{\omega}\rho + (\mathbf{J}\times\mathbf{H})_{radial} + \rho\{\dot{\mathbf{w}}\times(\mathbf{w}\times\omega)\}_{radial}$$

Multiplying this equation by $\delta\omega$ and putting

$$\delta\omega = \xi\omega \tag{42}$$

we find

$$2(\Gamma-1)\delta U = 2(\Gamma-1)\Big[-\int^{M} \xi Gm\, dm$$

$$+\int \xi\omega\,.\,(\mathbf{J}\times\mathbf{H})d\tau + \int^{W} \xi w\, dW\Big] \tag{43}$$

Further

$$\delta E = \int \{\xi + \operatorname{div}(\xi\omega)\}\{\omega\,.\,(\mathbf{J}\times\mathbf{H})\}d\tau$$

$$+\int \omega\,.\,[(\delta\mathbf{j}\times\mathbf{H}) + (\mathbf{J}\times\delta\mathbf{H})]d\tau \tag{44}$$

and

$$\delta(wW) = W\,\delta w \tag{45}$$

since the angular momentum is constant. Now from equation (34) we get

$$\delta w = -2\xi W$$

Therefore

$$\delta(wW) = -2\xi w W$$

Hence equation (41) with the help of equations (43) to (46) reduces to

$$-\sigma^2 \int^{M} \xi\omega^2 dm = -2(\Gamma-1)\Big[\int^{M} \xi 2Gm\,dm - \int_V \xi\omega\,.\,(\mathbf{J}\times\mathbf{H})d\tau$$

$$- \int^{m} w\xi dm \Big] + \int_V [\xi + \operatorname{div}(\xi\omega)][\omega \,.\, (\mathbf{J} \times \mathbf{H})] d\tau$$

$$+ \int_V \omega \,.\, [(\delta\mathbf{j} \times \mathbf{H}) + (\mathbf{J} \times \delta\mathbf{H})] d\tau - 2\xi w W \qquad \ldots \quad (47)$$

This is the required integral formula for the frequency of radial pulsations of rotating infinitely long cylinder for all currents distribution having axial symmetry The changes $\delta\mathbf{H}$ in the magnetic field and $\delta\mathbf{j}$ in the current density can easily be evaluated with the help of equations (27) and (2).

Let us now obtain the approximate expression for the frequency of pulsation for two special cases of magnetic fields, viz., poloidal and toroidal Auluck and Kothari (1957) have discussed these two systems of fields in detail Let us make use of the usual assumption made in the theory of adiabatic pulsations of stars, viz.,

$$\xi = \text{constant in space.} \qquad \ldots \quad (48)$$

Case (i) The magnetic field is poloidal

This poloidal magnetic field is derived from circular currents of the form

$$\mathbf{j} = \left\{ 0, -\frac{k\omega}{4\pi}, 0 \right\} \qquad (49)$$

such that

$$\mathbf{H} = \left\{ 0, 0, \frac{k}{2}(\omega^2 - R^2) \right\} \qquad \ldots \quad (50)$$

where k is constant and R is the radius of the cylinder. For such a configuration, it was shown earlier by Tandon and Talwar (1957) that

$$\delta\mathbf{H} = -2\xi\mathbf{H}$$

and

$$\delta\mathbf{j} = -3\xi\mathbf{j} \qquad \ldots \quad (51)$$

The equation for the frequency of radial pulsations will then be

$$\sigma^2 \int^{M} \omega^2 dm = 2(\Gamma - 1)GM^2 - 2(\Gamma - 2) \int \omega . (\mathbf{J} \times \mathbf{H}) d\tau$$

$$- 2(\Gamma - 2) w W \qquad \ldots \quad (52)$$

or using the abbreviation

$$E' = \int_V \frac{H^2}{8\pi} d\tau \qquad \ldots \quad (53)$$

we get

$$\sigma^2 \int^{M} \omega^2 dm = 2(\Gamma-1)GM^2 + 2(2-\Gamma)[2E' + wW] \quad \ldots \quad (54)$$

Thus, in the case of rotation the term $2E'$ of equation (25) of Tandon and Talwar has been replaced by $2E'+wW$. This clearly indicates that the rotation is similar to magnetic field and helps in the dynamical stability of the cylinder for radial pulsations in the presence of circular currents.

Case (ii)—The magnetic field is toroidal

For this case we consider a system in which there are line currents of constant value such that

$$\mathbf{j} = \left(0, 0, \frac{k}{2\pi}\right) \quad . \quad (55)$$

and hence

$$\mathbf{H} = (0, k\omega, 0) \quad . \; . \quad (56)$$

where k is a constant. The change in the magnetic field $\delta\mathbf{H}$ and the change in the current density $\delta\mathbf{j}$ will then be given by

$$\delta\mathbf{H} = -\xi\mathbf{H}$$

and

$$\delta\mathbf{j} = -2\,\xi\,\mathbf{j} \quad \ldots \quad (57)$$

The equation for the frequency of pulsations thus becomes

$$\sigma^2 \int^{M} \omega^2 dm = 2(\Gamma-1)GM^2 - 2(\Gamma-1)\int_{V} \omega\,.\,(\mathbf{J}\times\mathbf{H})d\tau - 2(\Gamma-2)wW \quad . \quad (58)$$

Further using the abbreviation represented by equation (53) we obtain

$$\sigma^2 \int^{M} \omega^2 dm = 2(\Gamma-1)GM^2 + 8(\Gamma-1)E' + 2(2-\Gamma)wW \quad \ldots \quad (59)$$

This equation clearly indicates that the cylinder is stable for the radial pulsations in the presence of line currents as well.

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