60

THEORY OF CUMULUS CONVECTION.

By S. MULL and Y. P. RAO.

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ABSTRACT. A new theory of convection based on lateral flow of air into the cloud is discussed, as an alternative to the 'parcel and slice' theories

INTRODUCTION:

The fundamental assumption made in the derivation of equations representing "parcel and slice" method of convection is that the pressure gradient along the vertical is determined by the surrounding air. As this assumption would only be valid in the most initial stage of the growth of the convection clouds, *i.e.*, in the stage when the parcel is isolated and small in dimension and not at the stage when convection is represented by a rising column of cumulus or cumulonimbus, it was thought advisable to examine the problem of cumulus convection to see if a more realistic picture of the growth of the convection clouds could be given, at least qualitatively.

PRESENT IDEAS:

The present ideas about the growth of convection clouds (cumulus and cumulo-nimbus) are represented by the parcel and the slice theories of convection. Whereas, in the first process the parcel of air is assumed to ascend adiabatically with no lateral mixing through an environment at rest, in the second one the environment is allowed to subside in order to preserve mass continuity. The two processes, however, fundamentally depend upon the principle that a body floating or immersed in a fluid is subjected to an upward directed force equal to the weight of the amount of fluid that the body displaces. The body will rise or sink or remain at the same level depending on whether the force is greater than, less than, or equal to respectively the downward force on the body due to acceleration of gravity. Realising that we are dealing with a parcel of air instead of a fixed body, we may write, following Petterssen (1945), for a parcel of air embedded in an environment in hydrostatic equilibrium

$$\dot{\omega}' = -\alpha' \frac{\partial p'}{\partial z} - g$$

$$o = -\alpha \frac{\partial p}{\partial z} - g$$

where letters with indices refer to the parcel and letters without indices refer to the environment and where

$\dot{\omega}'$ = vertical acceleration

α = specific volume

p = pressure

g = acceleration of gravity

z = distance along a vertical axis.

If the pressure gradient along the vertical is determined by the surrounding air, then we have

$$\frac{\partial p'}{\partial z} = \frac{\partial p}{\partial z}$$

and therefore

$$\dot{\omega}' = g\,\frac{(\alpha' - \alpha)}{\alpha}$$

or

$$\dot{\omega}' = g\,\frac{(T' - T)}{T}$$

where T is the absolute temperature, which shows that the parcel will be accelerated upward if it is warmer and downwards if it is colder than the environment.

As it is clear, the above derivation is based on the fundamental assumption that the pressure gradient along the vertical is determined by the surrounding air. This assumption would certainly be valid in the most initial stage of cloud formation, since at that stage the parcel is isolated and small in dimension. It is, however, difficult to conceive how the same will hold good during the subsequent stages of cloud growth, where instead of a parcel we have to deal with a rising column, since under such circumstances the pressure difference between any two horizontal levels in the rising column and the surrounding environment cannot be the same.

The slice method as developed by Bjerknes (1938) and Petterssen (1939) computes the excess of heating of the rising column over the surrounding descending air after taking into consideration the modification in temperature due to vertical motion. No attempt is made to explain how the temperature difference results in acceleration or deceleration of the rising air. Presumably it is taken that the acceleration can be derived from the temperature difference as in the parcel method.

It is also assumed in the slice method that there is no net inflow or outflow along the horizontal into the slice under consideration. Temperature ascents inside convective clouds show that in a high percentage of cases, the lapse rate inside the cloud is in excess of the saturated adiabatic rate. This can only be explained on the assumption that there is appreciable lateral flow into the cloud. In situations favourable for convection, the environmental air has a lapse rate in excess of the saturated adiabatic lapse rate. If there is no immediate and thorough mixing between the air inflowing from the sides at any level and the air coming from below, it is very likely that the air which has newly come from the sides being unsaturated will first cool at dry adiabatic lapse rate. For the above two reasons the lapse rate inside the cloud will definitely be in excess of the saturated adiabatic. The simple picture of a convective cloud fed from below the base with no lateral inflow beyond turbulent mixing with the environment is insufficient to account for the observed lapse rates.

SUGGESTED PICTURE OF CONVECTION IN CUMULUS AND CUMULONIMBUS

(a) *Initial Stage*:

Let us consider a barotropic atmosphere with horizontal isobaric surfaces. A perturbation in the form of a vertical displacement is applied to the system, the magnitude of the displacement being maximum at C (Fig. 1) and zero at the ground as well as at a higher level *E*. The displacement also decreases in the horizontal with distance from the vertical *ABCDE*. The pressure surfaces, represented by dotted lines, will consequently be perturbed as shown in Fig. 1, so that at any horizontal level the pressure is higher along the vertical *ABCDE* than in the surrounding. At the levels *A* and *E*, the pressure surface is still horizontal. The above is merely a description of a vertical perturbation.

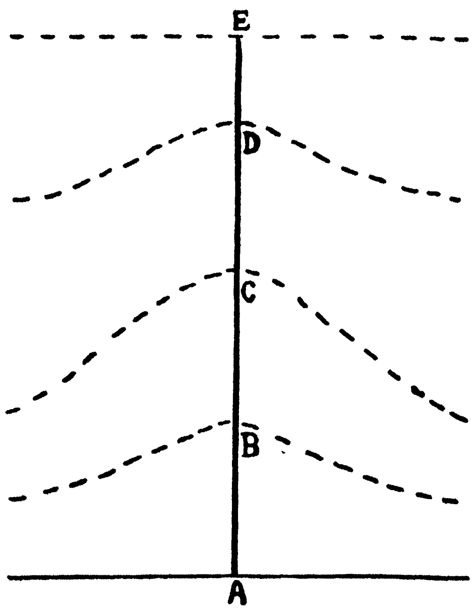

FIG. 1

Owing to the outward pressure gradient at all levels between A and E, air flows out radially and tends to decrease the pressure, its fall at any level being cumulative of the divergence at all levels aloft. This will be a maximum at the ground and as initially the pressure surface was horizontal at the ground, a low pressure will develop at the ground and progressively build upwards. This stage is represented in Fig. 2. Air will then flow

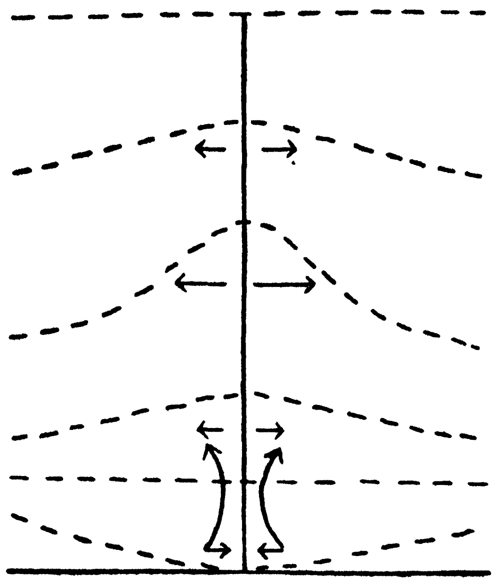

FIG. 2

radially inwards into the low pressure zone and outwards in the high pressure zone aloft. The inflowing air in the lower levels will then set up continuity with the outflowing air aloft in the form of a vertical current or in other words, the air in lower levels will tend to rise.

The rising air will cool either according to dry adiabatic or saturated adiabatic rate depending upon the humidity conditions. If the rising air is denser than the air it replaces (stable stratification), it will counteract the development of low by outflow of air aloft. The perturbation will be damped out in such a case. If on the other hand, the rising air is less dense than the air it replaces (unstable stratification), it will intensify the development of the low pressure below. The radial inflow into the low pressure will therefore increase and feed the rising column.

(*b*) *Large Cumulus Stage* :

Fig. 3 shows the outline of the cloud, pressure surfaces (broken lines) and air flow (arrow heads) at the stage of a large cumulus. The top of the growing cloud will push upwards the isobaric surfaces and consequently a high pressure will occur immediately above the top of the cloud. Owing to the lower density of the cloud air, the high pressure will change to a low one some distance below the top of the cloud. The low pressure will intensify from there up to the ground. Air will flow into the low pressure zone of

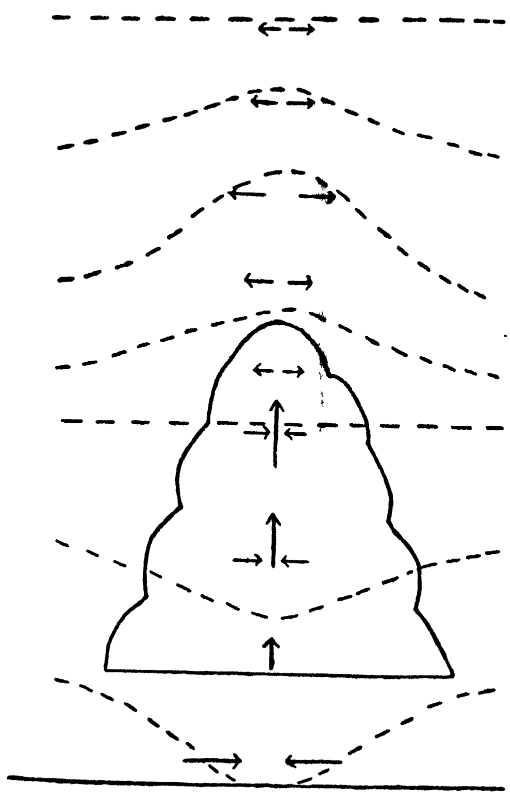

FIG. 3

the cloud from the sides while there will be some lateral outflow in the shallow high pressure zone in the top part of the cloud. The total convergence of the air into the low pressure zone of the cloud is appreciably greater than the divergence in the top portion of the cloud so that the cloud is still growing. It should be noted that though the top of the cloud forms a high pressure zone, its density is still less than that of the environment and as the cloud grows, the high pressure zone of the cloud is being changed into low pressure. Hence there is no persistent outflow from the top of the cloud and it does not spread out.

(c) *Beginning of Cumulo-nimbus Stage*:

The next stage in the growth of the cloud will be when with further ascent the top portion of the cloud commences to be more dense than the environment. This is shown in Fig. 4. As in the previous stage there is a high pressure zone just at the top of the cloud but due to the higher density of the cloud air (over the environment) the high pressure intensifies below the top up to the level where the density in the cloud is the same as that of the environment. As a result of this high pressure aloft, the high pressure

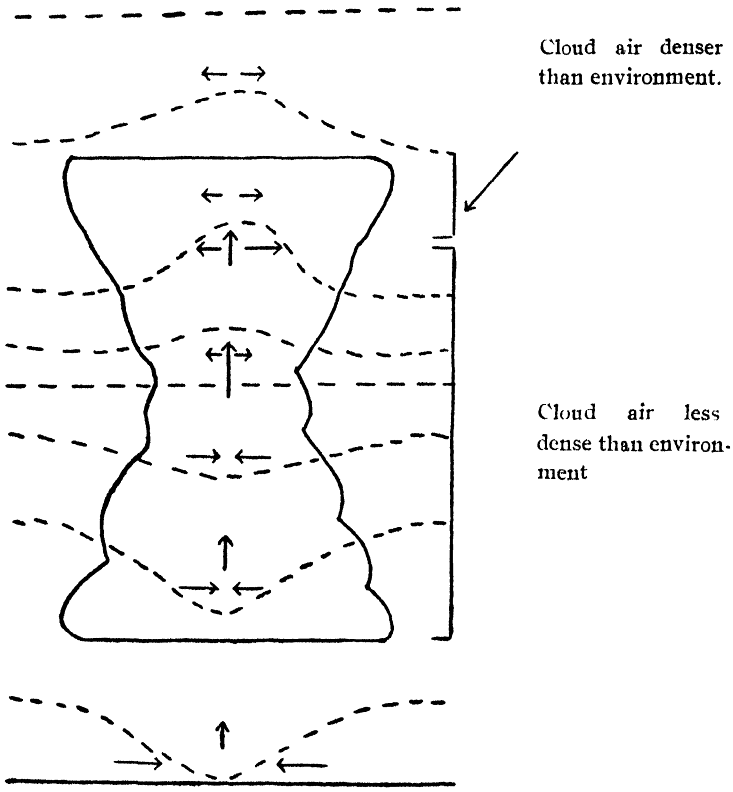

Fig. 4

zone extends down up to some level into that portion of the cloud which is less dense than the environment. Below that level, the low pressure zone builds up. The convergence in the low pressure zone is greater than the divergence in the high pressure portion of the cloud and hence the cloud is still growing. The cloud also spreads out in the high pressure zone unlike in the previous stage, as the high pressure is steadily maintained.

(*d*) *Fully Grown Cumulo-nimbus*:

At this stage represented in Fig. 5, the pressure right at the top of the cloud is the same as in the environment, as the cloud is no longer growing, and hence no more perturbing the upper pressure surfaces. A high pressure zone builds up below the top and is most intense at the level, *S*, where the cloud air comes to have the same density as the environment. Below that the cloud air is less dense than the environment and the high pressure zone first diminishes in intensity and finally changes into low pressure zone which attains the maximum intensity at the ground. The convergence in the low pressure zone below and the divergence in the high pressure zone aloft are in this stage equal so that there is no further growth of the cloud. But the lateral inflow or outflow and the vertical currents do not cease. Thus the

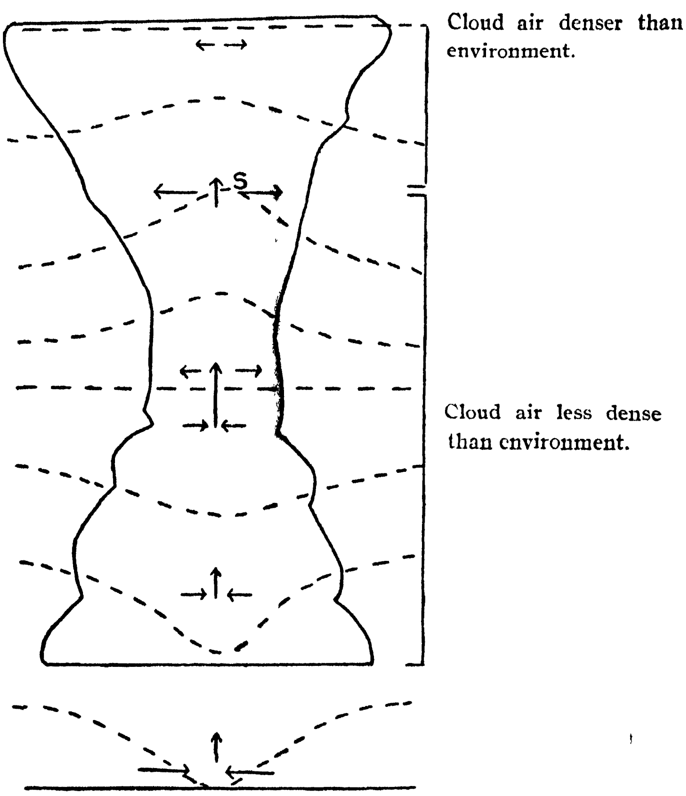

FIG. 5

fully grown cumulonimbus represents a state of dynamic equilibrium. On this basis it is possible to understand how a cumulonimbus cloud can maintain itself for considerable time without further growth or decay. The low pressure below the cloud is a necessity for causing inflow of air into the cloud. It is also to be noted that the high pressure zone extends from the top of the cloud into a portion of the cloud which is warmer than the environment and hence the outflow compensating the inflow lower down occurs also in a portion of the cloud which is rarer than the environment. Hence the vertical portion which is less dense than the environment, is generally greater than the portion which is denser.

Thus, according to the above discussion, it is possible to conceive purely on theoretical grounds, a stage in the development of the cumulonimbus cloud when the cloud, with its fully grown anvil, can maintain itself without any further growth or decay for hours together. As the cloud represents a system in dynamic equilibrium and as the up currents are stable, it may be expected that the cloud will not break into thunderstorm with the usual downdraft, squall and precipitation unless an additional mechanism comes into operation. Such an equilibrium stage in the life cycle of cumulonimbus is frequently observed in the tropics, where the cloud persists for a considerable time without any apparent change.

The discussion also clearly reveals how a low pressure occurs below convection clouds at all stages of its growth. This explains the anticlockwise circulation reported in cumulus and cumulonimbus clouds and the low pressure recorded with the passage of such clouds, though the fall of pressure is generally very small.

This paper is restricted to the stages prior to the breakdown of the cloud into a thunderstorm. The mechanism which brings out this change and the resulting circulation are being discussed in a separate paper.

REGIONAL METEOROLOGICAL CENTRE,
NAGPUR.

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