ON THE SCATTERING OF FAST PARTICLES OF SPIN 1 BY ATOM NUCLEI

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ABSTRACT. The wave-statistical theory of scattering due to spin-spin interaction discussed in the previous paper has been further extended for spin 1.

In a previous paper (Kar, 1947) the wave-statistical theory of scattering of fast particles of spin $\frac{1}{2}$ by atom nuclei was developed and the well-known Mott (1929) formula was derived. The formula derived for electron-electron scattering is, however, different from that of Möller (1932) in the general case although at the limiting cases for which the velocity is too low or too high, the two formulae completely agree.

The object of the present paper is to further extend the wave-statistical theory to the case of scattering of fast particles of spin 1.

It may be seen without difficulty (Kar, *loc. cit.*) that on neglecting the spinorbit interaction, we have for the differential equation satisfied by the first order scattering function

$$\Delta(\lambda_1\chi_1) + k^2(\lambda_1\chi_1) = \frac{4\pi^2}{h^2c^2}\chi_0[2EV + 2EV_{s-s} - V^2] \qquad \ldots \quad (1)$$

where V_{s-s} is the spin-spin interaction. It is apparent that the contributions of the terms $2EV$ and $-V^2$ are same as in the previous paper (Kar, *loc. cit.*). The contribution of the remaining term is, however, different as the interaction potential V_{s-s} is different in the present case. Let us suppose that the scattering nucleus has $\frac{1}{2}$ spin. Thus the interacting particles have unequal spins and so the exchange factor 2 should be dropped from the spin-spin interaction potential. We have then

$$V_{s-s} = \pm S_1S_2\frac{Ze^2}{r} \qquad \ldots \quad (2)$$

where the upper sign denotes that the coulomb force is repulsive

On using the above interaction potential and proceeding in the usual manner we have for the first order scattering function.

$$\lambda_1\chi_1 = \mp S_1S_2\frac{Ze^2}{2m_0v^2}(1-\beta^2)^{\frac{1}{2}}\operatorname{cosec}^2\tfrac{1}{2}\theta\cos k'r_0\Lambda\frac{e^{ikr}}{r} \qquad \ldots \quad (3)$$

As in the previous paper (*loc.cit.*) $\lambda_1 X_1$ should be multiplied by the spin and relativity factors, in order to get the complete scattering function due to spin-spin interaction. Now, it has been shown in the paper just referred to that the probability that there is no change in the sign of spin $\frac{1}{2}$, after scattering, is unity and is given by

$$D_s = P^{-\frac{1}{2}}_{+\frac{1}{2}}(\theta) P^{+\frac{1}{2}}_{-\frac{1}{2}}(\theta) e^{-i\frac{1}{2}\phi} e^{+i\frac{1}{2}\phi} = 1 \qquad \ldots \quad (4.1)$$

whereas, the probability that there is change of sign after scattering, is given by

$$D_s = P^{+\frac{1}{2}}_{+\frac{1}{2}}(\theta) P^{-\frac{1}{2}}_{+\frac{1}{2}}(\theta) e^{+i\frac{1}{2}\phi} e^{-i\frac{1}{2}\phi} = \cos\theta \qquad \ldots \quad (4.2)$$

where $P^{\frac{1}{2}}_{\frac{1}{2}}(\theta)$,...etc. are Legendre functions for $|m| = \frac{1}{2}$, $|n| = \frac{1}{2}$. The corresponding probabilities for the observer are obtained by putting $\pi-\theta$ for θ. Hence the total probability for the observer is evidently

$$\delta_s = 1 - \cos\theta \qquad \ldots \quad (4)$$

which is the spin factor by which the scattering function (3) should be multiplied in order to get the total scattering.

We have now to decide whether the corresponding spin factors (δ) for spin 1 should involve Legendre functions of the type $P^1_1(\theta)$, $P^{-1}_1(\theta)$,...etc. It is evident that these Legendre functions cannot represent spin 1, because in that case Legendre functions of the type $P^{\frac{1}{2}}_1(\theta)$ cannot be interpreted. The only other course left to us for representing spin 1 by Legendre functions would be to represent it by squares of $\frac{1}{2}-$ integral Legendre functions. Thus the probability in the present case corresponding to (4.1) should be

$$D_s = \left\{P^{-\frac{1}{2}}_{+\frac{1}{2}}(\theta)\right\}^2 \left\{P^{+\frac{1}{2}}_{-\frac{1}{2}}(\theta)\right\}^2 e^{-i1\phi} e^{+i1\phi} = 1 \qquad \ldots \quad (5.1)$$

while corresponding to (4.2) it should be

$$D_s = \left\{P^{+\frac{1}{2}}_{+\frac{1}{2}}(\theta)\right\}^2 \left\{P^{-\frac{1}{2}}_{+\frac{1}{2}}(\theta)\right\}^2 e^{i1\phi} e^{-i1\phi} = \cos^2\theta \qquad \ldots \quad (5.2)$$

The physical significance of taking squares is that the ultimate unit of spin is $\frac{1}{2}$. The spin 1 is developed due to the simultaneous existence of two component $\frac{1}{2}-$ spins. The probability of this simultaneous happening is obtained by taking squares according to the usual law of probability. Now, in the case of spin $\frac{1}{2}$ we took the spin factor with respect to the observer of the scattered wave (*vide* Eq. 4) by putting $\pi-\theta$ for θ. In the present case of spin 1 because we have to take squares we should take the geometric mean for the observers situated with the incident and scattered waves facing the scatterer. Consequently (5.2) should be $-\cos^2\theta$ being the product of $\cos(\pi-\theta)$ and $\cos\theta$. It should be noted that if one takes the geometric mean (5.1) remains unaffected. Thus the spin factor becomes

$$\delta_\kappa = 1-\cos^2\theta = \sin^2\theta \qquad \ldots \quad (6)$$

It should be noted that the results in (5.1) and (5.2) may also be obtained in the following way :

$$D'_\kappa = \left\{P^{-\frac{1}{2}}_{+\frac{1}{2}}(\theta)P^{+\frac{1}{2}}_{-\frac{1}{2}}(\theta)\right\}\left\{P^{-\frac{1}{2}}_{+\frac{1}{2}}(\theta)P^{+\frac{1}{2}}_{-\frac{1}{2}}(\theta)\right\}e^{-i\phi}e^{+i\phi}=1 \qquad \ldots \quad (7.1)$$

corresponding to (5.1) and

$$D'_\kappa = \left\{P^{+\frac{1}{2}}_{+\frac{1}{2}}(\theta)P^{-\frac{1}{2}}_{+\frac{1}{2}}(\theta)\right\}\left\{P^{+\frac{1}{2}}_{+\frac{1}{2}}(\theta)P^{-\frac{1}{2}}_{+\frac{1}{2}}(\theta)\right\}e^{+i\phi}e^{-i\phi} = \cos^2\theta \qquad \ldots \quad (7.2)$$

corresponding to (5.2). Taking into account these two different ways, it is evident that the spin factor should be normalised by dividing by 2. Accordingly the spin factor should be $\delta = \frac{1}{2}\sin^2\theta$ (*vide Eq.* (6)).

Next we consider the relativity factor. From its definition already given and remembering that in taking squares we should take the geometrical mean as in the case of the spin factor.

$$\delta_{rel} = \frac{1-\beta^2-1}{1}\cdot\frac{1-(1-\beta^2)}{1-\beta^2} = -\frac{\beta^4}{1-\beta^2} \qquad \ldots \quad (8)$$

Hence we have for the total scattering function, neglecting the effect of $-V^2$ term in (1).

$$\lambda_1 X_1 = \mp\left(\frac{Ze^2}{2m_0v^2}\right)(1-\beta^2)^{\frac{1}{2}}\operatorname{cosec}^2\tfrac{1}{2}\theta A\frac{e^{ikr}}{r}\cos k'r_0\left\{1-\tfrac{1}{2}S_1S_2\frac{\beta^4}{1-\beta^2}\sin^2\theta\right\} \quad (9)$$

Hence the relative intensity of scattering becomes

$$I = \left(\frac{Ze^2}{2m_0v^2}\right)^2(1-\beta^2)\operatorname{cosec}^4\tfrac{1}{2}\theta\cos^2k'r_0\left\{1-S_1S_2\frac{\beta^4}{1-\beta^2}\sin^2\theta\right\} \quad \ldots \quad (10)$$

Since the weights for anti-parallel to parallel spins are as 2:1 and since $S_1 = \frac{1}{2}$, $S_2 = 1$, we have for the total intensity of scattering

$$I = \left(\frac{Ze^2}{2m_0v^2}\right)^2(1-\beta^2)\operatorname{cosec}^4\tfrac{1}{2}\theta\cos^2k'r_0\left\{1+\frac{1}{6}\frac{\beta^4}{1-\beta^2}\sin^2\theta\right\} \quad \ldots \quad (11)$$

which is the formula obtained first by Massey and Corben (1939) in a different way.

It may be mentioned in conclusion that in the above formula we have considered the interaction between spin $\frac{1}{2}$ of the nucleus and spin 1 of the scattered particle. If, however, the nucleus has also spin 1, there is the exchange effect. And so the spin-spin interaction potential should be multiplied by the numerical factor 2. But because of the nuclear spin 1 there should be the additional weight factor $\frac{1}{2}$, which neutralises the exchange effect of 2. Thus it may be easily seen, remembering that $S_1 = 1$, $S_2 = 1$, that the intensity should be [*vide* Eq. (10)]

$$I = \left(\frac{Ze^2}{2m_0v^2} \right)^2 (1-\beta^2) \operatorname{cosec}^4 \tfrac{1}{2}\theta \cos^2 k' r_0 \left\{ 1 + \frac{1}{3} \frac{\beta^4}{1-\beta^2} \sin^2\theta \right\} \quad \ldots \quad (12)$$

which is slightly different from Massey and Corben's formula (11) in as much as the numerical factor in the second term is $\frac{1}{3}$ instead of $\frac{1}{6}$.

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