

## DIELECTRIC CONSTANTS OF SOME SOLID INSULATING MATERIALS AT ULTRA SHORT WAVES

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**ABSTRACT** Dielectric constants of several insulating materials in the form of thin slabs have been measured by the Lecher wire method over a wavelength range of 1.40 cms. to 57.7 cms. Dielectric constants in the case of materials under investigation have been found to vary with wavelengths. In the case of solid paraffin, dielectric constants have been found to increase with wavelengths becoming shorter. In other cases dielectric constants show a decreasing tendency with wavelength decreasing.

### INTRODUCTION

The behaviour of insulating materials under electric stress is a matter of fundamental importance to communication engineers. The uncertainties of dielectric behaviour of solids arise from the fact that under high frequency excitation various factors become operative within the body of the material depending on its physical and chemical constitution. Hartshorn (1926) stressed on the need of a thorough investigation of various causes responsible for the uncertain dielectric behaviour. The use of any material as a dielectric in the region of cm. waves presupposes a knowledge of the dielectric constant and loss factor of the material in the particular band of frequencies. Various methods have been evolved for the study of the dielectric behaviour of different materials. The frequency variation of an oscillator, due to the insertion of the material in the tank condenser, has been utilised by Niven (1911), Hyslop and Carman (1920), Kerr (1926) and in a modified form by Wyman (1930) etc. to determine the dielectric constants of various materials. Capacitance variation method has been employed by Hartshorn and Ward (1936) to determine the dielectric constant of various ceramic materials. The difference in the wavelength of standing waves produced by resonance on parallel wires with the wires being first wholly immersed in air and then in the dielectric has been utilised first by Cohen (1891) and later on more extensively by Drude (1895), Rudop (1913) etc. Instead of parallel wires, concentric tubes have been used by Drake, Pierce and Dow (1930). The standing wave method has been suitably modified by King (1937) to measure dielectric constants of a thin slab of materials. Theoretical and experimental development of resonator systems suitable for the determination of dielectric constant has been achieved by Horner and others (1946). The measurement of 'Q' of a resonator has been utilised by Dakin and Boggs (1944) and England (1944). Wave-guide properties have been used by Lamont (1940) for such measurements. A new technique has been developed by MacLean (1946) for dielectric

loss measurements. The object of the present investigation is to measure dielectric constants of thin slabs of insulating materials over a wide band of ultra short waves by employing a weak source of oscillation and following the method outlined by King.

T H E O R E T I C A L

The velocity  $v$  of a sinusoidal current in the steady state on two parallel wires, immersed wholly in air, is  $v = \omega/\beta$  where,  $\beta$  is the retardation angle per unit length of the line and  $\omega$  is the angular velocity of the impressed e. m. f. in radians per second. If the parallel wires are immersed in medium of dielectric constant  $\epsilon$ , the velocity  $v_\epsilon$ , is given by  $v_\epsilon = v/\sqrt{\epsilon}$ , the frequency  $\omega/2\pi$  of the wave being constant. Or, we can write  $\lambda_\epsilon = \lambda/\sqrt{\epsilon}$ . This amounts to, as if, the length of the wire has been shortened from  $\lambda$  to  $\lambda/\epsilon$  due to the insertion of the dielectric. If instead of immersing the parallel wire wholly in the dielectric, a thin slab of the dielectric is interposed, then calculating the input admittance (King, 1937) of the whole system from the telegraphist's line equation at ultra high frequencies, it can be shown that

$$\begin{aligned} & \tan \frac{1}{2} \beta (s_1 - s_2) = n \tan \frac{1}{2} n \beta s_1 \\ \text{or,} & \quad \tan \frac{1}{2} \beta (\text{max. shift} + s_1) = n \tan \frac{1}{2} n \beta s_1 \quad \dots (1) \\ \text{where,} & \quad s_1 - s_2 = \text{maximum shift due to the introduction of the slab,} \\ & \quad s_1 = \text{thickness of the slab,} \\ & \quad n = \text{refractive index of the material.} \end{aligned}$$

The retardation angle  $\beta$  is given by (Drake and Pierce, 1930)

$$\beta = \frac{\omega}{c^1} \cdot \sqrt{\epsilon} \cdot (1 - \eta s)^{\frac{1}{2}} \cdot f(h)$$

where,  $\eta = \frac{l}{l\omega}$  and  $s = \frac{g}{c\omega}$  and

$$f(h) = \left[ \frac{(1 + h^2)^{\frac{1}{2}} + 1}{2} \right]^{1/2}$$

where, as shown by Pierce (1922),

$$h = \frac{r\omega + g/l\omega}{lc^1\omega^2 - rg}$$

where,  $r$ ,  $c$  and  $l$  = resistance, capacity, and inductance respectively per loop unit length of a smooth line,

$c^1$  = velocity of electromagnetic waves in vacuum,

$g$  = leakage conductance per unit loop length of the line.

Then the velocity  $v_\epsilon = \frac{\omega}{\beta} = \frac{c^1}{\sqrt{\epsilon} \cdot (1 - \eta s)^{\frac{1}{2}} \cdot f(h)}$

or  $\frac{c^1}{v_\epsilon} = \sqrt{\epsilon} \cdot (1 - \eta s)^{\frac{1}{2}} \cdot f(h) = \frac{\lambda}{\lambda_\epsilon}$

But the ratio of wavelength in vacuum to that in the dielectric is the index of refraction  $n$ . Therefore

$$n = \sqrt{\epsilon} \cdot (1 - \eta s)^{\frac{1}{2}} \cdot f(h)$$

For a short and well insulated line,  $g$  may be considered to be negligibly small. Moreover, at cm. waves  $\omega$  is very high. So,

$$lc\omega^2 \gg rg \text{ and } rc\omega > gl\omega$$

Therefore,

$$h = \frac{rc\omega + gl\omega}{lc\omega^2 - rg} \doteq \frac{r}{l\omega}$$

and

$$(1 - \eta s)^{\frac{1}{2}} = \left(1 - \frac{rg}{lc\omega^2}\right)^{\frac{1}{2}} \doteq 1$$

The expression for the index of refraction, therefore, at ultra short waves for a short well insulated line, reduces to

$$n = \sqrt{\eta\epsilon} \quad \dots (2)$$

So by finding out the maximum bridge shift for a certain material, the refractive index and hence the dielectric constant of the material can be calculated with the help of the equations (1) and (2).

#### EXPERIMENTAL

The experimental technique mainly consists of the following :

1. The wavelength of the oscillator along the Lecher wire, when the latter is wholly immersed in air is determined by finding bridge positions for three consecutive maxima of the indicator reading. The bridge then being placed in its first position of maxima, the oscillator is moved in its truck parallel to the Lecher wire till the indicator shows its maximum. The oscillator is kept fixed in this position throughout the experiment.

2. The optimum position of the slab is determined so that the bridge shift ( $s^1 - s_2$ ) may be a maximum. This procedure minimises the error in the determination of  $\epsilon$ . The slab is provided with two holes at the centre, the diameters of the two holes being almost of the same size as that of the wires, so that the contact between slab and wires is properly ensured, while at the same time the slab can be moved smoothly and snugly along the wires. The slab is placed between the indicator and the bridge. The bridge is moved till the indicator reads maximum. Then moving the slab towards the bridge,

step by step, the bridge shift at each step is measured. Fig. 1 shows the relation between bridge shift and the slab position for one of the materials.

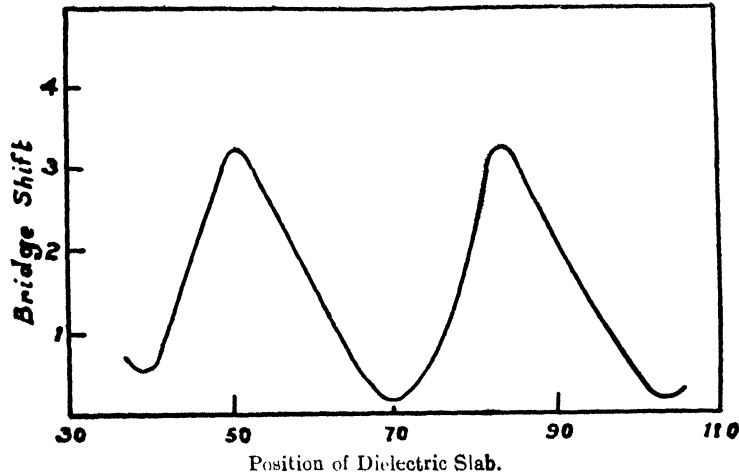


FIG. 1

3. The wavelength of excitation being varied from 140 cms to 57.7 cms maximum bridge shift ( $s_1 - s_2$ ) at each step corresponding to the optimum position of the slab is noted. The wavelength of excitation is checked before and after each observation.

4. The thickness of the material is measured.

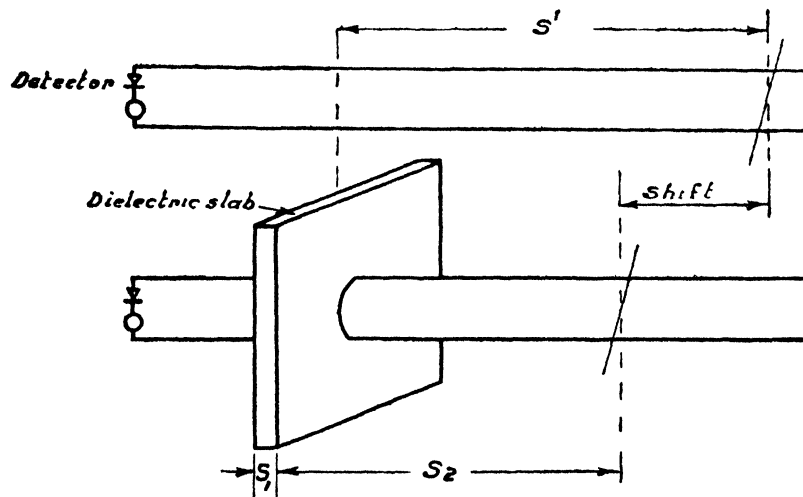


FIG. 2

The above procedure is repeated for ten different materials, in the form of thin slabs. The experimental arrangement is shown in figure 2.

The oscillator used is of the resonant type with two 955 tubes in parallel. Two oscillators have been used. One of them covers a range of 142 cms to 96 cms. The other covers a range of 98 to 56 cms. The resonant

lines of the first oscillator are 9 cms in length and 0.3 cms in diameter and spacing between lines is 0.43 cms. The filament chokes for the oscillator consists of air cored coils of 50 turns and diameter 0.4 cms made from

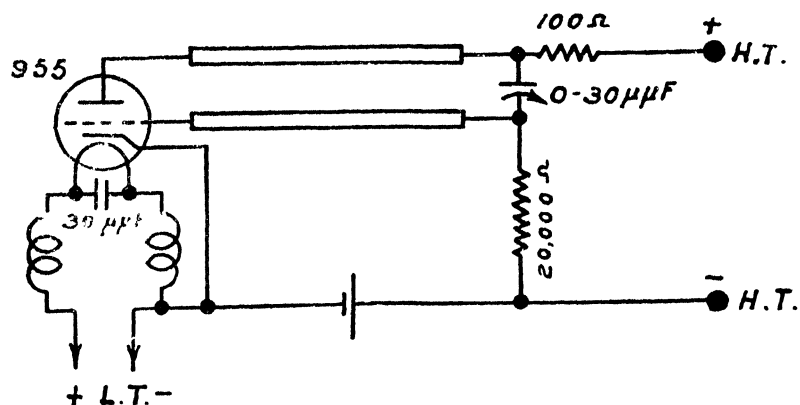


FIG. 3

26 S.W.G. enamelled copper wire. For the second oscillator, the resonant lines are of length 3.6 cms., diameter 0.3 cm. and spacing 0.5 cm. The filament chokes of diameter 0.3 cm. consists of 40 turns made from the same gauge of wire as above. The frequency of the oscillators is varied by changing the spacing between resonant lines and also by adjusting the trimmer shunting the end of the resonant lines. The oscillator circuit diagram is shown in Fig. 3.

The indicator used consists of a crystal detector-micro-ammeter combination fixed at one end of the Lecher wire line 1.

The diameter of the Lecher wires 0.326 cm. and the spacing 5 cms. from centre to centre have been chosen to satisfy the requirements of the parallel wire theory with regard to the ratio of the wire diameter to the wire separation viz.  $(d/a)^2 \ll 1$ .

The experimental results for the different materials are tabulated as follows. In order to calculate the refractive index from the equation 1 Newtonian approximation Sanden and Jahukes tables have been used.

TABLE I

| Material                                  | $\lambda$ (cms.) | Max. Shift (cms.) | $\epsilon$ | Material                                   | $\lambda$ (cms.) | Max. Shift (cms.) | $\epsilon$ |
|---|------------------|-------------------|------------|--|------------------|-------------------|------------|
| Celeron<br>(Thickness<br>$S_1 = 1.33$ cm) | 140.0            | 5.7               | 5.3        | Bakelite<br>(Thickness<br>$S_1 = 0.06$ cm) | 140.0            | 1.5               | 5.7        |
|   | 120.0            | 5.7               | 5.3        |  | 120.0            | 1.4               | 5.6        |
|   | 111.1            | 5.5               | 5.2        |  | 111.1            | 1.3               | 5.5        |
|   | 100.0            | 5.6               | 1.7        |  | 100.0            | 4.3               | 5.5        |
|   | 88.2             | 5.3               | 4.5        |  | 88.2             | 3.3               | 4.5        |
|   | 78.9             | 5.3               | 1.5        |  | 78.9             | 3.3               | 4.5        |
|   | 71.1             | 1.5               | 3.9        |  | 71.1             | 2.7               | 3.8        |
|   | 65.2             | 4.0               | 3.6        |  | 65.2             | 2.6               | 3.8        |
|   | 60.0             | 4.0               | 3.6        |  | 60.0             | 2.4               | 3.5        |
| 57.7                                      | 4.0              | 3.6               | 57.7       | 2.0  | 3.1              |                   |            |

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TABLE I—(contd.)

| Material                                | $\lambda$ (cms) | Max. Shift (cms) | $\epsilon$ | Material  | $\lambda$ (cms) | Max. Shift (cms) | $\epsilon$ |
|---|-----------------|------------------|------------|---|-----------------|------------------|------------|
| Mycalex<br>(Thickness<br>$S_1=0.97$ cm) | 140.0           | 4.1              | 5.2        | Mica<br>(Brown)<br>(Thickness<br>$S_1=0.63$ cm) | 140.0           | 2.6              | 5.0        |
|   | 120.0           | 3.9              | 4.9        |   | 120.0           | 2.2              | 4.3        |
|   | 111.1           | 4.0              | 5.1        |   | 111.1           | 2.5              | 4.9        |
|   | 100.0           | 4.0              | 4.5        |   | 100.0           | 2.5              | 4.9        |
|   | 88.2            | 4.3              | 5.5        |   | 88.2            | 2.1              | 4.8        |
|   | 78.9            | 4.4              | 5.6        |   | 78.9            | 2.4              | 4.7        |
|   | 71.1            | 4.5              | 5.7        |   | 71.1            | 2.4              | 4.8        |
|   | 65.2            | 4.5              | 5.7        |   | 65.2            | 2.5              | 4.9        |
|   | 60.0            | 4.0              | 5.2        |   | 60.0            | 2.2              | 4.5        |
| 57.7                                    | 4.3             | 5.0              | 57.7       | 2.2   | 4.5             |                  |            |

| Material                         | $\lambda$ (cms) | Max Shift (cms) | $\epsilon$ | Material                        | $\lambda$ (cms) | Max. Shift (cms) | $\epsilon$ |
|----------------------------------|-----------------|-----------------|------------|---------------------------------|-----------------|------------------|------------|
| Micanite<br>Thickness<br>0.92 cm | 140.0           | 3.7             | 4.9        | Ebonite<br>Thickness<br>1.25 cm | 140.0           | 3.6              | 3.8        |
|                                  | 120.0           | 3.7             | 5.0        |                                 | 120.0           | 3.5              | 3.8        |
|                                  | 111.1           | 3.7             | 5.0        |                                 | 111.1           | 3.5              | 3.8        |
|                                  | 100.0           | 3.7             | 5.0        |                                 | 100.0           | 3.0              | 3.4        |
|                                  | 88.2            | 3.5             | 4.8        |                                 | 88.2            | 2.9              | 3.3        |
|                                  | 78.9            | 3.2             | 4.5        |                                 | 78.9            | 3.0              | 3.4        |
|                                  | 71.1            | 3.3             | 4.4        |                                 | 71.1            | 3.0              | 3.4        |
|                                  | 65.2            | 3.4             | 4.7        |                                 | 65.2            | 3.0              | 3.4        |
|                                  | 60.0            | 3.2             | 4.5        |                                 | 60.0            | 3.0              | 3.4        |
| 57.7                             | 3.2             | 4.5             | 57.7       | 3.0                             | 3.4             |                  |            |

| Material                      | $\lambda$ (cms) | Max. Shift (cms) | $\epsilon$ | Material                          | $\lambda$ (cms) | Max. Shift (cms) | $\epsilon$ |
|-------------------------------|-----------------|------------------|------------|-----------------------------------|-----------------|------------------|------------|
| Fibre<br>Thickness<br>0.64 cm | 140.0           | 2.1              | 4.2        | Bees Wax<br>Thickness<br>2.02 cms | 140.0           | 2.6              | 2.0        |
|                               | 120.0           | 2.0              | 4.0        |                                   | 120.0           | 2.5              | 2.2        |
|                               | 111.1           | 2.0              | 4.0        |                                   | 111.1           | 2.3              | 2.1        |
|                               | 100.0           | 1.8              | 3.8        |                                   | 100.0           | 2.3              | 2.1        |
|                               | 88.2            | 2.2              | 4.4        |                                   | 88.2            | 2.2              | 2.2        |
|                               | 78.9            | 2.0              | 4.1        |                                   | 78.9            | 2.5              | 2.3        |
|                               | 71.1            | 1.8              | 3.7        |                                   | 71.1            | 2.6              | 2.3        |
|                               | 65.2            | 2.0              | 4.1        |                                   | 65.2            | 2.1              | 2.0        |
|                               | 60.0            | 1.8              | 3.6        |                                   | 60.0            | 1.8              | 1.9        |
| 57.7                          | 2.1             | 4.3              | 57.7       | 1.8                               | 1.9             |                  |            |

| Material                              | $\lambda$ (cms) | Max. Shift (cms) | $\epsilon$ | Material   | $\lambda$ (cms) | Max. Shift (cms) | $\epsilon$ |
|---------------------------------------|-----------------|------------------|------------|--|-----------------|------------------|------------|
| Paraffin Wax<br>Thickness<br>2.13 cms | 140.0           | 2.1              | 2.0        | Plexiglass<br>(Methyl-metha-<br>Crylate)<br>Thickness<br>0.64 cm | 140.0           | 1.35             | 3.0        |
|                                       | 120.0           | 2.2              | 2.0        |  | 120.0           | 1.20             | 2.8        |
|                                       | 111.1           | 2.2              | 2.0        |  | 111.1           | 1.30             | 2.9        |
|                                       | 100.0           | 2.0              | 2.1        |  | 100.0           | 1.10             | 2.9        |
|                                       | 88.2            | 2.1              | 2.3        |  | 88.2            | 1.10             | 2.7        |
|                                       | 78.9            | 2.1              | 2.0        |  | 78.9            | 1.30             | 2.9        |
|                                       | 71.1            | 2.3              | 2.1        |  | 71.1            | 1.00             | 2.9        |
|                                       | 65.2            | 2.4              | 2.1        |  | 65.2            | 1.00             | 2.5        |
|                                       | 60.0            | 2.5              | 2.2        |  | 60.0            | 1.00             | 2.5        |
| 57.7                                  | 2.5             | 2.2              | 57.7       | 1.00   | 2.5             |                  |            |

## DISCUSSION

The nature of variation of bridge shift with different position of dielectric slab, as shown in Fig. 1, can be shown to agree with that obtained theoretically from the equation (Lamont, 1940)

$$n \cot \beta_1 s_1 (\cot \beta s_1 - \cot \beta s_2) + \cot \beta s_1 \cot \beta s_2 + n^2 = 0$$

where,

$$s_2 = s_1 - (s_1 + \delta), \quad \delta = \text{shift}$$

This shows that the shift is a function of the absolute position of the slab. From the stand point of optical analogy, this result seems to be unexpected. In optical problems, such results are encountered due to the interference of two separate travelling waves. In the present case, interference occurs at the boundaries of the dielectric between the incident wave and a portion of it reflected. It has been pointed out by Lamont, that under resonant conditions reflections take place at the dielectric boundaries and at the bridge in such a manner that the resultant stationary waves show little change of intensity at the boundaries but exhibit a definite phase change, depending on the position of the dielectric slab. This variation of phase change is considered to be responsible for different amounts of bridge shifts with different dielectric slab positions with respect to the original standing wave system.

From the experimental results, dielectric constants for all the above cases of materials, except solid paraffin wax, have been found to decrease with the wavelength. As reported by Hormell (1902), the dielectric constant of paraffin of different densities varied from 2.13 to 2.22 and from 2.16 to 2.25 at wavelengths of 81.68 cms. and 61.32 cms. respectively. In the present investigation dielectric constants of paraffin (solid) at 78.9 cms. and 60 cms. have been found to be 2.0 and 2.19 respectively.

The frequency dependence of dielectric constants may be explained by molecular process or by a mechanism of interstitial conduction which requires the presence of adsorbed ions on the surface of structural units as suggested by Murphy and Lowry (1930). When an e.m.f. is applied to the dielectric, a polarised distribution of these ions takes place, due to a surface mobility, but after the source of excitation is removed the ions return to their normal distribution. The electrical effect of the adsorbed ions is, therefore, similar to that of a dipole and they produce a kind of dielectric polarisation. The structural unit with its adsorbed ions can be represented by an equivalent electrical circuit, where each layer of adsorbed ions is represented by a capacity in series with a resistance. Both the capacities and the resistances become smaller, the greater the distance from the interface of the layers of ions to which they refer. In figure the equivalent electrical circuit  $c_m$  represents the capacity due to the dielectric polarisation of molecules;  $c_1, c_2, \dots, c_{n-1}, c_n$  represent the capacities due to redistribution of adsorbed ions on the surface of the micelles;  $R_1, R_2, \dots, R_{n-1}, R_n$  represent the series resistances

equivalent to frictional resistances which hinder the process of formation of a polarised distribution of adsorbed ions and  $R_d$  is the D.C. resistance. By an

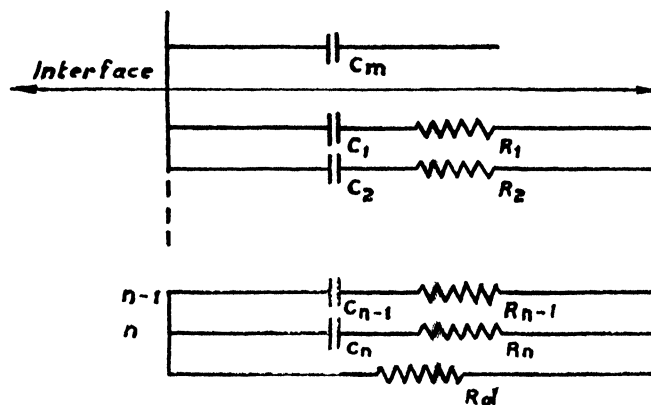


FIG. 1

analysis of the circuit, it can be shown that the equivalent parallel capacity decreases with decreasing wavelength. At very high frequency, the capacity term due to the adsorbed ions, or similar process involving long relaxation time, becomes zero, the only capacity that remains is that due to the polarisation of the charges within the molecules. In this connection, it is interesting to note that Debye (1912) suggested that if the system is compared with a pure capacity  $C$  and a resistance  $R$  in series, the following relations hold good.

$$e' = \frac{c/c_0}{1 + R^2\omega^2c^2}$$

$$e'' = \frac{c}{c_0} \left( \frac{R\omega c}{1 + R^2\omega^2c^2} \right)$$

where,  $e'$ ,  $e''$ , etc. have their usual significance.

Most of the solid dielectrics, for instance, ebonite, fibre, mycalex etc., investigated above, may be classed as either amorphous or of a crystalline type containing many lattice irregularities and impurities. Part of the dielectric constant of these materials may be regarded as due to the presence of dipoles and (or) ions in the dielectric. The electrical dipoles are either due to the polar groups existing in the molecules building up the amorphous substance or to the occluded or impurity molecules, which are more or less subjected to restricted rotation. The ions may be considered to originate from or belonging to the substance itself or formed by electrolytic impurities, which can both travel over certain distances through the structure of the amorphous solid, along crystal flaws or irregularities. The positions of a dipole, which is originally bound to certain position by the molecular field of the surroundings, can only be altered if a certain amount of energy is supplied. Similarly the displacement of the ions also needs a certain amount of energy. The amount



of activation energy needed for reorientation or rotation will be different for different dipoles or ions according as the solid is of amorphous or crystalline type.

On the assumption of a statistical scattering of the activation energies determining the progressive or rotational diffusion of ions or dipoles, Gevers and Pre' (1946) have worked out a theory which explains the dielectric properties of amorphous dielectrics. It has been concluded from this theory that, as a first approximation, contribution to the dielectric constant is made only by those ions and dipoles whose relaxation times ( $\tau$ ) exceed the value  $\tau = 1/\omega$ , where  $\omega$  is the angular frequency of the alternating field or the part of the dielectric constant which is dependent on the frequency is a function of the activation energy corresponding to  $\omega\tau = 1$ .

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