

A NOTE ON THE THERMODYNAMICS OF THE WET- AND DRY-BULB HYGROMETER*

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ABSTRACT. The paper deals with the thermodynamic study of the wet-and-dry bulb hygrometer as applied to other liquids besides water. Hygroscopic liquids are also included. The theory is found in reasonable accord with experiment.

INTRODUCTION

The present note is concerned with the thermodynamics of the wet-and-dry bulb hygrometer, generalised to apply to liquids other than water. Experiments were performed with different evaporating liquids and also hygroscopic liquids (particularly with strong sulphuric acid) which absorb moisture from the air. The standard theory of the wet- and dry-bulb can easily be extended to cover these cases. The experimental results were found to be in agreement with theory.

In the beginning the generalised formula, known as the psychrometer equation is derived. The symbols relevant for the purpose are the following:—

L = Latent heat of the liquid at the wet-bulb temperature.

T = Room temperature.

T' = Wet bulb temperature.

$x = e e / (p - e)$ = Humidity mixing ratio at temperature T .

$x' = e e' / (p - e')$ = Humidity mixing ratio at temperature T' .

e = Vapour pressure at temperature T

where e' = Vapour pressure at temperature T' .

ϵ = Ratio of densities of vapour and dry air at the same temperature

C_P = Specific heat of dry air at constant pressure.

$C_{P'}$ = Specific heat of vapour at constant pressure.

p = Total pressure.

When the wet-bulb has reached a constant temperature the heat lost by air in cooling from the dry-bulb to the wet-bulb temperature is utilised in the evaporation of the liquid, the vapour content of the air increasing from the

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initial value to a value corresponding to saturation at the wet-bulb temperature. Hence it follows that

$$(C_P + xC'_P)(T - T') = L'(x' - \lambda)$$

Substituting the values of λ and x' given above and assuming $xC'_P \ll C_P$ and $e', e \ll p$, we have finally

$$(e' - e) = \frac{C_P p}{L' \epsilon} (T - T') \quad \dots (1)$$

This is the general equation for all evaporating liquids, substituting $e = 0$, which in the present experiments is the case for all evaporating liquids other than water, equation (1) reduces to

$$e' = \frac{C_P p}{L' \epsilon} (T - T') \quad \dots (2)$$

The case of hygroscopic liquids is now considered. In this case the process is just reversed and the liquid absorbs practically all the water vapour from the air coming in contact with it. Consequently its temperature will rise due to heat of chemical combination and the heat of condensation. The rise in temperature will be proportional to the amount of water vapour present in the air and hence can provide a measure for humidity.

In order to derive, for hygroscopic liquids, a relation corresponding to the equation (1), it is assumed that the air after coming in contact with the hygroscopic liquid is completely robbed of its water vapour contents and acquires the temperature of the liquid. This temperature may be called the "anti-wet-bulb" temperature. In the steady state the heat taken away by the air is equal to the heat of reaction plus the heat of condensation. Heat taken away by one gram of air is $(T' - T)C_P$. If x grams of water vapour be associated with one gram of air, heat liberated is $(H + L')x$. Hence we have

$$(H + L')x = C_P(T' - T)$$

here H denotes the heat of reaction and T' the anti-wet-bulb temperature. Substituting for x the final expression is

$$e = \frac{C_P p (T' - T)}{(H + L') \epsilon} \quad \dots (3)$$

Thus the expression in the case of hygroscopic liquids comes out to be of the same form as for evaporating liquids. In fact equation (3) can be got directly from (1) by putting $e' = 0$ and replacing L' by $(H + L')$.

It is hardly necessary to mention that, the equations derived above hold when the air surrounding the wet-bulb is in a state of sufficient motion. When the air around the wet-bulb is not renewed fast enough, the wet-bulb depression will be less—its full value is obtained when the motion of the air exceeds a certain limiting value. The same applies to the "anti-wet-bulb" rise in temperature. In the experiments performed care was taken that the

full value of depression in the one case and the rise in temperature in the other case are obtained.*

The method of derivation of the psychrometric equation has been the subject of great controversy and much criticism has been levelled against it. It is not possible to justify, on purely theoretical grounds, what is implied in the above derivations—splitting of the air in two layers, having different properties. In the region just around the wet-bulb the stream lines are parallel to the surface. Beyond this there is the turbulent layer which affects the flow of heat and water-vapour. Alternative methods (Whipple, 1933) of derivation have been proposed, though not with much success. We should not however be appalled by the sweeping nature of an assumption, particularly when (as is the case here) we can compare theory with experiment.

EXPERIMENTAL VERIFICATION

The experimental verification equation (3) is first taken up. In this case it was found convenient to use the following arrangement instead of the usual "wick method". The liquid was taken in a thermos flask and its mouth closed by a stopper through which passed a thermometer and two tubes. One of the tubes was kept dipping in the liquid while the other much above its surface. Air from the surface was constantly sucked by connecting the second tube to a suction pump. Fresh air from the atmosphere entered through the first tube. If this process is allowed to run for some time (about half an hour) the liquid attains a steady temperature.

In order to check the working of this arrangement water was tried. The steady temperature in this case was found to nearly coincide with the wet-bulb temperature of the ventilated type of psychrometer.† A feature of this experiment is that it does not require the assumption of two distinct layers in the neighbourhood of the wet-bulb.

Coming back to the case of the hygroscopic liquids we notice that if 'e' is known, the value of H can be found from equation (3). If, however, this equation is used for finding e, H must be determined by a separate experiment. A graph, may be plotted between heat evolved in calories against the weight of water in grams added to a gram molecule of the liquid.

It will be noticed from such a plot that the heat emitted per gram of dissolved water is sensibly independent of the strength of the acid so long as it contains less than twelve grams of water per mol. of the acid. The following is a set of observations taken on a particular day using water and sulphuric acid in turn.

* In the case of still air the psychrometric equation does not retain the simple form (1) but involves some other factors. For a detailed discussion see Glazebrook's Dictionary of Applied Physics' P. 420.

† In this connection it is interesting to note that water in a 'Surahi' (earthen jug) placed in a room is cooled to within 2° of the wet-bulb temperature.

Room temperature = 30.75°C

(1) Depression in wet-bulb = 11.5

Vapour pressure = 10.1 mms.

(2) Rise in anti-wet-bulb = 35°C

H—(heat of solution) = 360 Calories.

Making use of equation (3) we get

Vapour Pressure = 10.8 mms.

The thermos flask method is a universal method—in the respect that any liquid can be used. The psychrometric method is very limited in its application. It fails in the case of hygroscopic liquids, especially strong sulphuric acid. In fact, the 'wick method' will always fail in these cases. It has been pointed out before that the hygroscopic substance behaves just like a sink for water-vapour present in the air. Consequently it will be diluted continuously and will soon reach the 'non-proportional' region—in which heat evolved is not directly proportional to the amount of water vapour present in the air. It will be seen presently that the psychrometer method has to be rejected in the case of most of the evaporating liquids.

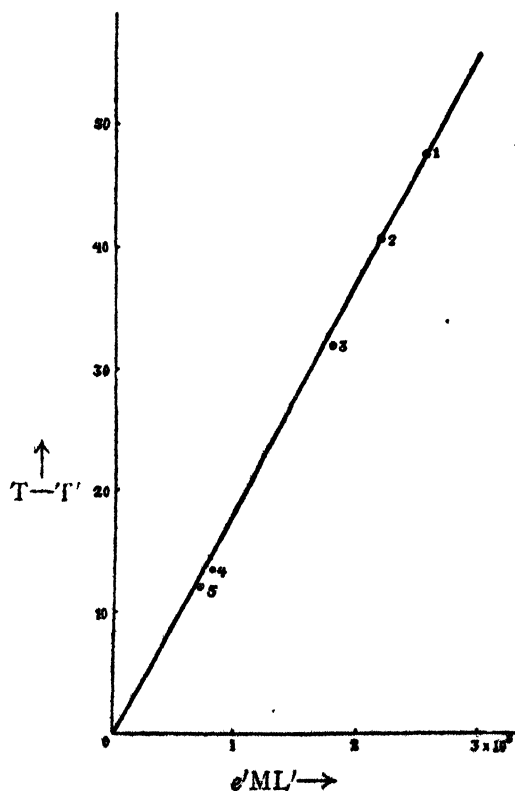


FIG. 1.

The experimental verification of equation (2), which is satisfied by all liquids for which $e=0$, is now considered. Substituting $\epsilon=M/M_0$ in (2) we have

$$e'ML' = A(T - T') \quad \dots (4)$$

where $A = C_p \rho M_0$

M = Molecular weight of the liquid.

M_0 = Molecular weight of dry air.

It is clear from equation (4) that a plot of $(T - T')$ against $e'ML'$ must be a straight line passing through the origin whose slope is $(1/A)$. The usual psychrometer arrangement was found to fail in the case of liquids with their wet-bulb temperature lower than that of water. The reasons are—

1. Assumption regarding the splitting of air in two distinct layers is no longer valid.
2. The accuracy of observations is considerably marred by the condensation of water vapour present in the air on the wet-bulb. The effect of condensation is to raise the wet-bulb temperature.

In order to get true wet-bulb temperature the thermos flask method, with a slight modification, was used. The air was not allowed to enter the thermos flask directly but was first made to pass through a calcium chloride U-tube and then a sulphuric acid bulb. In this process it was completely dehumidified. Since the absorption of water vapour is accompanied by rise in temperature of the hygroscopic substance, the U-tube and the bulb were cooled by circulating water. The temperature of the incoming air was noted by a thermometer whose bulb was sealed in the tube. The wet-bulb temperature on the other hand was measured by a thermocouple. With the above arrangement acetone, methyl alcohol, ethyl alcohol, acetic acid and butyl alcohol were tried in turn to get the depression in the wet-bulb temperature $(T - T')$. In the graph numerical values of e' and L' have been taken from the 'International Critical Tables' and have been substituted in millimeters and calories respectively. The various points lie approximately on a straight line whose slope is 1.85×10^{-4} . The slope of this line according to theory should be 1.94×10^{-4} . Since theory and experiment are in fair agreement, equation (2) can be taken as correct.

Before concluding it must be mentioned that the liquids used must be free from impurities. The presence of an impurity will change the true wet-bulb temperature. The specimen of ethyl alcohol used in the experiment contained about 4% of water. It will be noticed from the graph that the point representing ethyl alcohol is below the straight line—the depression in the wet-bulb temperature is less than the expected value. Now the wet-bulb temperature of water is known to be higher than for ethyl alcohol. The presence of water in alcohol will, therefore, tend to raise its wet-bulb temperature. This is connected with the depression in wet-bulb temperature of miscible liquids and will be discussed fully in a subsequent paper.

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REFERENCE

Whipple, 1933, *Prof. Phys. Soc. Lond.*, **45**, 307.