ANALYSIS OF RANDOM FADING RECORDS

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ABSTRACT. The analysis of the three-spaced-receiver fading records taken at Banaras from November, 1956 to March, 1958, with vertically-directed pulse transmission on 3.8 Mc/s has yielded the following results:

(*i*) The ratio of the drift velocity v_ω to the *r.m.s.* line-of-sight velocity v_0 of the ionospheric irregularities is not found to be constant, as is expected from theory. The ratio increases with the increasing drift velocity.

(*ii*) The ratio of the drift velocity v_ω to the product of the frequency of fading N and the wavelength λ is not found to be constant, as is expected from theory. The ratio decreases with the increasing drift velocity.

(*iii*) The angle of spread of the scattered components from the ionospheric irregularities obtained from $\theta_0 = \sin^{-1}(N.\lambda/2v_\omega)$ is found to increase with the increasing drift velocity.

THEORETICAL CONSIDERATIONS

Ratcliffe (1948) developed a theory of the randomly fading radio waves. According to the theory, the irregularities in the ionosphere are the scattering centres which may be regarded as gas molecules under thermal agitation. The distribution of velocity (in one dimension) can then be expressed as:

$$P(v) = A \,.\, \exp\left(-\frac{v^2}{v_0^2}\right) \qquad \ldots \quad (1)$$

where $\int_{-\infty}^{\infty} P(v)dv = 1$ and v_0 is the r.m.s. line-of sight velocity of the irregular scattering centres.

The frequency of the scattered components suffers a Döppler shift of frequency due to the line-of-sight velocity of the scattering centres. Considering the frequency-shift, $f-f_0$, the power-spectrum is given by

$$W(f) = B.\exp\left[-\frac{\lambda^2(f-f_0)^2}{8v_0^2}\right] \qquad \ldots \quad (2)$$

where λ is the wavelength of the wave.

The auto-correlation function of the fading pattern may be written as

$$P_R(\tau) = \frac{\overline{R(t) \,.\, R(t+\tau)} - [\overline{R(t)}]^2}{[\overline{R(t)}]^2 - [\overline{R(t)}]^2} \qquad \ldots \quad (3)$$

where $R(t)$ and $R(t+\tau)$ are the amplitudes of the fading signals at instants t and $t+\tau$. Since the auto-correlation function is proportional to the Fourier transform of the distribution of power in the power spectrum, it can be shown

$$P_R(\tau) = C.\ \exp.\left[-\frac{16\pi^2 v_0^2 \tau^2}{\lambda^2}\right] \qquad \ldots \quad (4)$$

The theory can be tested by finding whether the auto-correlation function of the fading pattern obeys this law, and if it does, the magnitude of v_0 can be obtained from the value of τ, where $P_R(\tau)$ falls to e^{-1} in the auto-correlogram Thus

$$v_0 = \frac{\lambda}{4\pi\tau} \qquad \ldots \quad (5)$$

If the spatial auto-correlation of the fading pattern is plotted as a function of distance in one direction from a fixed origin, it is possible that it would fall in a smooth manner. Ratcliffe and Pawsey (1933), Pawsey (1935) had shown that the spatial auto-correlation function falls to about 0.8 in a distance of one wave-length. If now the irregular ionosphere producing this pattern were to move with a velocity v_ω, the diffraction produced on the ground would move with velocity $2v_\omega$. Therefore the spatial auto-correlation function would fall to about 0.8 in time $\lambda/2v_\omega$.

If now we assume the space and time auto-correlation to be similar (say, Gaussian), then the space auto-correlation function will fall to e^{-1} in time

$$\tau = \frac{0.8\lambda}{2v_\omega \,.\, e^{-1}} = \frac{2.2\lambda}{2v_\omega} \qquad \ldots \quad (6)$$

Then comparing the relations (5) and (6) we obtain

$$v_0 = \frac{v_\omega}{4.4\pi}$$

or

$$v_\omega / v_0 \approx 14 \qquad \ldots \quad (7)$$

McNicol (1049) developed a quick method of determining the r.m.s. line-of-sight velocity v_0 by counting the number of maxima N of the fading pattern per sec. The approximate relation was given by

$$v_0 = \frac{N.\lambda}{5} \qquad \ldots \quad (8)$$

Considering (7) and (8) we have

$$v_\omega = 14v_0 = 14\,\frac{N.\lambda}{5} = 2.8N.\lambda \qquad \ldots \quad (9)$$

Further Briggs (1951) assuming that the horizontal movement of the reflecting layer to be the main cause of fading deduced the relation :

$$N = \frac{2v_\omega}{\lambda}\sin\,\theta_0 \qquad \ldots \quad (10)$$

where θ_0 is the semi-angle of the cone of the down-coming waves. It will be noticed that this relation is the same as the relation (9) given by McNicol except for the constants.

RESULTS OF THE ANALYSIS OF THE FADING RECORDS

A number of three-spaced-receiver fading records taken at Banaras from November 1956 to March 1958 with vertically-directed pulse-transmission on 3.8 Mc/s has been analysed. The cross-correlation method has given the drift velocity. The same sets of records have been used to find the r.m.s. line-of-sight velocity v_0 of the irregularities in the ionosphere by finding the time at which;the auto-correlation function falls to a value e^{-1} and using relation (5). Table I shows the results of the analysis. The various values of the drift velocities have been arranged in groups of 0–10, 10–20, 20–30 metres/sec. and the averages of these have been estimated.

TABLE I

Frequency 3.8 Mc/Sec.

Drift velocity v_ω metres/sec.	r.m.s. line-of-sight velocity of the ionospheric irregularities v_0 metres/sec.	v_ω/v_0
12	1.23	9.75
26	2.46	10.60
37	2.70	13.70
48	3.20	15.00
56	3.40	16.40
67	3.50	19.00
78	4.10	19.00
86	4.60	18.60
	mean	15.25

It is evident from the data given in Table I that the ratio of the drift velocity v_ω to the r.m.s. line-of-sight velocity v_0 of the ionospheric irregularities varies with the magnitude of the drift velocity, the ratio being smaller for the lower values of the drift velocity and larger for the higher values. Clearly the results show a departure from the theoretical relation (7).

The relation between the drift velocity v_ω and the frequency of fading N has also been found by analysing the random fading records. In Table II are given the values of the frequency of fading N and of the ratio of the drift velocity v_ω to $N.\lambda$ for the different groups of the drift velocity. It is clear that the ratio $v_\omega/N.\lambda$ decreases with the increase of the drift velocity. This variation is a departure from the theoretical relation (9). Using the relations (9) and (10), the angles of spread of the scattered components from the ionospheric irregularities for the various values of the drift velocity have been calculated. The calculated values of the spread angle are also entered in Table II. It is interesting to note that the spread angle increases with the increase of the drift velocity.

TABLE II

v_ω meters/sec.	60N in cycles/min	$v\omega/N\lambda$ =const.	$\theta_0 = \sin^{-1}\frac{N.\lambda}{2v_\omega}$
12	1 8	5 83	4° 50′
26	4 2	4.64	6° 19′
37	7 7	3.60	8° 2′
48	10.5	3.43	8° 20′
56	13.0	3 23	8° 55′
67	16.7	3 00	9° 35′
78	20.8	2 81	10° 23′
86	27.2	2.40	12° 7′
	mean	3.62	

It may be mentioned that the angle of spread of the scattered components is of the same order as that obtained by Briggs (1951) and Rao and Rao (1958). The values of the spread-angle are also in agreement with the values obtained by Briggs and Philips (1950) and by Khastgir and Singh (1960) from the three-spaced-receiver fading records.

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