ELASTIC SCATTERING AND POLARIZATION OF 300 MEV PROTONS

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ABSTRACT. Analysis of elastic scattering and polarization of 300 Mev energy protons is made in Born approximation for Woods-Saxon type of nucleon-nuclear potential with spin-orbit coupling.

It has been felt by Sternheimer (1955) and Bjorklund and others (1957) that to account for the polarization of elastically scattered beam of protons in the high energy region there should be a spin orbit coupling along with the central part of the nucleon-nuclear potential. In this paper we take the potential to be of the form :

$$V = [V_{CR}+iV_{CI}]\,\rho(r)+[V_{SR}+iV_{SI}]\left(\frac{\hbar}{\mu c}\right)^2 \frac{1}{r}\,\frac{d\rho(r)}{dr}\,\vec{\sigma}\cdot$$

$$\rho(r) = \frac{1}{1+e^{(r-R)/a}}\,;\quad R = r_0A^{1/3}$$

The Born approximation analysis gives the matrix element for the central part of the potential as

$$\langle f|V_C|i\rangle = -\frac{2M}{\hbar^2}\left[\frac{V_{CR}+iV_{CI}}{K}\left\{\frac{a^2\pi^2 \sin KR \cosh aK\pi}{\sin h^2\, aK\pi}\right.\right.$$

$$\left.\left.\frac{a\pi R\cos KR}{\sin h\; aK\pi} - 2Ka^3\sum_{n=1}^{\infty}\frac{(-)^n.\,e^{-\frac{nR}{a}}.\,n}{(n^2+K^2a^2)^2}\right\}\right] \tag{1}$$

where $$K = \frac{2p}{\hbar}\sin\theta/2$$

The matrix element of the spin-orbit part of the potential is given by

$$\langle f|V_S|i\rangle = -\frac{2M}{\hbar^2}(V_{SR}+iV_{SI})\left(\frac{\hbar}{\mu c}\right)^2 i\cos\theta/2\,\sin\phi\,\frac{p}{\hbar}\,.$$

$$\left[\frac{a^2\pi^2\sin KR\cosh aK\pi}{\sin h^2\, aK\pi} - \frac{a\pi R\cos KR}{\sin h\;aK\pi} - 2Ka^3\sum_{n=1}^{\infty}\frac{(-)^n.\,e^{-\frac{nR}{a}}.\,n}{(n^2+K^2a^2)^2}\right] \quad \ldots \tag{2}$$

Here we have considered the intial state a plane wave parallel to z axis with spin parallel to x axis. We are interested in the scattering in YZ plane. The differential scattering cross section $d\sigma/d\omega$ is given by

$$\frac{d\sigma}{d\omega} = \frac{4M^2}{\hbar^4}\left[\frac{a^2\pi^2 \sin KR \cosh a K\pi}{\sinh^2 aK\pi} - \frac{a\pi R \cos KR}{\sin \mathrm{h}\, aK\pi} - 2Ka^3 \sum_{n=1}^{\infty} \frac{(-)^n . e^{-\frac{nR}{a}} . n}{(n^2+K^2a^2)^2}\right]^2$$

$$\left[\left\{\frac{V_{CR}}{K} - V_{SI}\left(\frac{\hbar}{\mu c}\right)^2 \cos\theta/2 \sin\phi \frac{p}{\hbar}\right\}^2 + \left\{\frac{V_{CI}}{K} + V_{SR}\left(\frac{\hbar}{\mu c}\right)^2 . \cos\theta/2 \sin\phi \frac{p}{\hbar}\right\}^2\right] \quad \ldots \quad (3)$$

The square well limit for the scattering cross section as given by Fermi (1954) is obtained by making a tend to zero. Of course Fermi has taken $V_{SI} = 0$ and $V_{CR} = V_{SR}$. The intensity of polarization is as usual expressed by

$$e(\theta) = \frac{I\left(\phi = \frac{\pi}{2}\right) - I\left(\phi = \frac{3\pi}{2}\right)}{I\left(\phi = \frac{\pi}{2}\right) + I\left(\phi = \frac{3\pi}{2}\right)}$$

From formula (3) we obtain

$$e(\theta) = \frac{\frac{2}{K}\left(\frac{\hbar}{\mu c}\right)^2 \cos\theta/2 \frac{p}{\hbar}\ [V_{CI}V_{SR} - V_{CR}V_{SI}]}{\frac{1}{K^2}\ [V_{CR}^2 + V_{CI}^2] + \left[\left(\frac{\hbar}{\mu c}\right)^2 \cos\theta/z\ \frac{p}{\hbar}\right]^2 [V_{SI}^2 + V_{SR}^2]}$$

This expression is independent of the diffusivity parameter a and thus, as expected, we find that the polarization depends only upon the magnitudes of the potentials and not on their radial shapes. So, for the polarization, no better agreement with experiment is obtained from the diffuse well shape than from the square well potential as shown by Fermi (1954). For the scattering cross section we have taken average of $d\sigma/d\omega$ which is given by $\frac{\overline{d\sigma}}{d\omega} = \frac{1}{2}\left[I\left(\phi = \frac{\pi}{2}\right) + I\left(\phi = \frac{3\pi}{2}\right)\right]$. The real part of the central potential will be somewhat modified in forward scattering angles if the coulomb potential is taken into account. This we have neglected because in the 300 Mev region coulomb cross section drops off very rapidly with angle for the elements considered here.

We present here the result (figure 1) of our calculation of the differential scattering cross sections of 340 Mev protons scattered by lead and 313 Mev protons

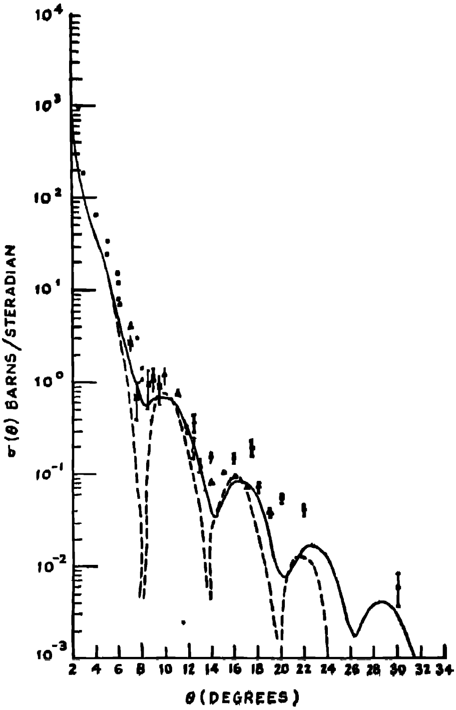

Fig. 1. —— Exact phase-shift analysis (1957).
- - - Born approximation analysis.

scattered by carbon (figure 2) along with the results of exact phase shift analysis of Bjorklund, Blandford and Fernbach (1957). As regards parameters, we have chosen $r_0 = 1.25 \times 10^{-13}$ cm, $a = 0.65 \times 10^{-13}$, $V_{CR} = 0$ Mev, $V_{CI} = 16$ Mev, $V_{SR} = 1.08$ Mev, and $V_{SI} = -2.28$ Mev. For small angles the scattering cross section is not affected by the spin orbit coupling term which predominates as the scattering angle increases. At 30° angle of scattering the contribution of spin-orbit coupling term to the scattering cross section is 10 times greater than that of the central potential. The effect of increasing the magnitude of the spin-orbit coupling term is to raise the scattering cross section at larger angles. It appears from the figures that the calculations in Born approximation for 300 Mev protons agree closely with those of the exact phase shift analysis.

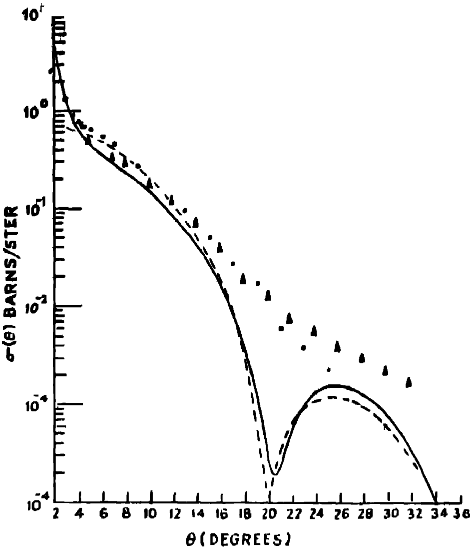

Fig. 2. —— Exact phase shift analysis (1957).
- - - Born approximation analysis.

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