DESIGN OF PULSE AMPLIFIER

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ABSTRACT. This paper describes a new approach to the design of pulse amplifier based on its transient response characteristics. A pulse amplifier is characterised by its gain, risetime and overshoot and its design is dependent upon the relation between these parameters and the circuit constants. Analytical relation between overshoot and risetime (defined as the time for 10% to 90% of the final value) cannot be obtained. Relation between gain, overshoot and the time to rise from zero to peak value has, however, been obtained, and the method of designing an actual pulse amplifier from a knowledge of these parameters shown. Results are compared with an actual pulse amplifier designed from the data obtained theoretically

INTRODUCTION

It is known that the unwanted shunt capacitance sets a limit to the high frequency response and hence the sharp risetime of R-C coupled pulse amplifiers. Since it is not possible to reduce its value indefinitely one aims at reducing its detrimental effects and thus improving the high frequency response with the help of some complicated circuitry. The simplest method is shunt compensation.

The degree of h.f. compensation may be ascertained fron the transient response characteristics. A short risetime for a step function input corresponds to a high upper 3 *db* point, and hence large bandwidth. In this paper a brief review is made of the effect of pole-zero location on the transient response characteristics and the process of designing parameters of a shunt compensated R–C coupled amplifier, which gives the best possible combination of gain, overshoot and risetime, is discussed. (Gupta Sharma, 1954 and Martin, 1955).

2. DESIGN ANALYSIS

If the mutual impedance function of a network is given by $g\,(p)$ the voltage output for a unit current step input is given by

$$v(t) = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} \frac{g(p)}{p}\,\epsilon^{pt}\,dp,$$

where the contour has to be chosen in such a way that all the singularities of $g(p)$ are to the left of the path of integration. Thus

$$v(t) = \frac{1}{2\pi j} \times 2\pi j \, \Sigma \text{ Residues } \frac{g(p)}{p} \epsilon^{pt} \text{ at the poles of } g(p)$$

$$= \Sigma \text{ Residues } \frac{g(p)}{p} \epsilon^{pt} \text{ at the poles of } g(p),$$

where $g(p)$ can be expressed as

$$H \cdot \frac{(p-\alpha_1)(p-\alpha_3)\ldots\ldots}{(p-\alpha_2)(p-\alpha_4)\ldots\ldots} \epsilon^{pt}$$

In the above expression H is a constant, $\alpha_1, \alpha_3, \ldots$ are zeros and $\alpha_2, \alpha_4 \ldots$ poles of the network function in the complex frequency plane.

Hence we can write

$$v(t) = H \; \Sigma \text{ Residue } \frac{1}{p} \frac{(p-\alpha_1)(p-\alpha_3)\ldots\ldots}{(p-\alpha_2)(p-\alpha_4)\ldots\ldots} \epsilon^{pt}$$

It may be mentioned here that pentode tubes are normally used for such amplifiers with relatively low values of plate load. So that equivalent generator for the output circuit is ideally a constant current generator and the output voltage is proportional to the output current which, in turn, is proportional to the input voltage. Hence, in calculating $v(t)$ we have taken a step current input function.

The conventional shunt compensated amplifier and its high frequency equivalent circuit are given in figures 1(a) and (b) respectively where C denotes the output capacitance of the first stage together with the input capacitance of the second stage.

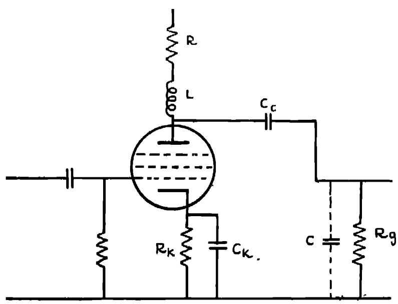

Fig. 1 (a)

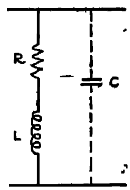

Fig. 1 (b) High frequency equivalent circuit.

For this equivalent circuit

$$g(p) = \frac{Z_1 . Z_2}{Z_1+Z_2+Z_3}$$

where

$$Z_1 = R+pL$$

$$Z_2 = \frac{1}{pC}$$

$$Z_3 = 0$$

$$\therefore \quad g(p) = \frac{1}{C} \cdot \frac{p+\frac{R}{L}}{p^2+p.\frac{R}{L}+\frac{1}{LC}}$$

$$= H. \frac{p+a}{(p-b)(p-c)}$$

Where

$$H = \frac{1}{C}; \quad a = R/L$$

$$b = -\frac{R}{2L} + j\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}} = -\alpha+j\beta \text{ (say)}$$

and

$$c = -\frac{R}{2L} - j\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}} = -\alpha-j\beta \quad \text{(say)}$$

Hence $v(t) = H \sum \text{Residue } \frac{1}{p} \frac{p+a}{(p-b)(p-c)} \epsilon^{pt}$

for unit step current input.

$$= H . \frac{a}{\alpha^2+\beta^2} \left[1+\frac{\epsilon^{\alpha t}}{\sin\phi} \sin(\beta t-\phi)\right]$$

$$= H. \frac{2\alpha}{\alpha^2+\beta^2} \left[1+\frac{c^{-\alpha t}}{\sin\phi} \sin(\beta t-\phi) \right]$$

where

$$\phi = \tan^{-1} \frac{a\beta}{\alpha^2+\beta^2-a\alpha}$$

The nature of the solution clearly indicates that the response is non-monotonic (oscillatory) and some overshoot is present which is always the case as long as poles of the network functions are complex conjugates. For real poles the response is monotonic (damped), but the risetime is large. In order to minimise the risetime the parameters have therefore to be so adjusted that the roots are complex conjugates and in such a case some overshoot has to be tolerated. To obtain such a condition the relation $\frac{L}{R^2} > \frac{C}{4}$ has to be satisfied.

Now locating the poles and zero's in the complex frequency plane (figure 2) we obtain

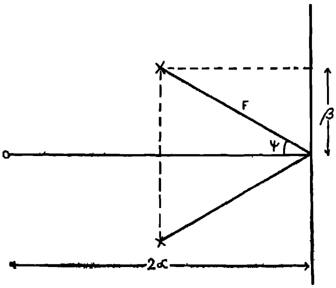

Fig. 2.

$$v(t) = H. \frac{2 \cos \psi}{F} \left[1 - \epsilon^{-\alpha t} \frac{\sin (\beta t + 2\psi)}{\sin 2\psi}\right]$$

where

$$\alpha = F \cos \psi$$

$$\beta = F \sin \psi$$

$$\psi = \tan^{-1} \frac{\beta}{\alpha} = \tan^{-1} \sqrt{\frac{4L}{CR^2} - 1}$$

$$F = \sqrt{\alpha^2 + \beta^2} = \sqrt{\frac{1}{LC}}$$

The plot of $v(t)$ against t as given in figure 3 reveals that $v(t)$ passes through alternate maxima and minima, before it reaches the steady state value given by $H \frac{2 \cos \psi}{F}$.

T_m is the risetime at which the first maximum is obtained and can be found out by satisfying the following two conditions simultaneously $\frac{d}{dt} v(t) = 0$

and $$\frac{d^2}{dt^2} v(t) = \text{negative}.$$

We thus have

$$\beta T_m = \pi - \psi$$

or $$T_m = \frac{\pi - \psi}{\beta} = \frac{\pi = \psi}{F \sin \psi}$$

for the first maximum.

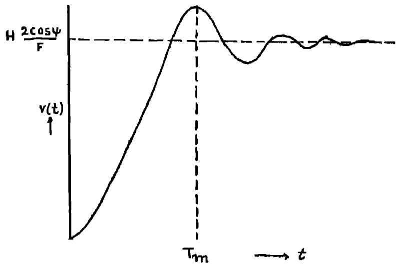

Fig. 3.

Substituting the value of T_m in the expression of $v(t)$ we get

$$v(t)_{max} = H.\frac{2\cos\psi}{F}\left(1+\frac{\epsilon^{-\alpha T_m}}{2\cos\psi}\right)$$

The percentage overshoot is therefore given by

$$\frac{v(t)_{max} - v(t)_\infty}{v(t)_\infty} \times 100$$

$$\frac{\epsilon^{-(\pi-\psi)\cot\psi}}{2\cos\psi} \times 100$$

From the expressions for overshoot, T_m and steady state value it is clear that overshoot is a function of ψ alone whereas the steady state value and T_m are functions of both ψ and F. It is therefore expected that by properly adjusting F and ψ, it is possible to obtain favourable combination of overshoot, risetime and gain to suit a particular purpose.

Now in any case C is fixed by the choice of the tube. So to vary ψ and F, we have to adjust L and R, but the limit of variation is decided by the relation $\frac{4L}{CR^2} > 1$.

Or, we can write $$\frac{L}{R^2} = K\frac{C}{4}$$

where $$K > 1$$

Let us now find for what value of K one can obtain an optimum combination of gain, overshoot and risetime. Expressing them in terms of C and K we get

$$\text{Overshoot } S = \frac{\sqrt{K}}{2}\,\epsilon^{-\frac{\pi-\tan^{-1}\sqrt{k-1}}{\sqrt{k-1}}}$$

$$T_m = \frac{CR}{2}\frac{K}{\sqrt{k-1}}(\pi - \tan^{-1}\sqrt{k-1})$$

Steady state gain $G = g_m R$. (midfrequency gain)

$$\therefore \quad \frac{T_m}{G} = \frac{C}{2g_m} \cdot \frac{K}{\sqrt{k-1}} (\pi - \tan^{-1}\sqrt{k-1})$$

The plot of S vs K and T_m/G vs K are shown respectively in figures 4 and 5.

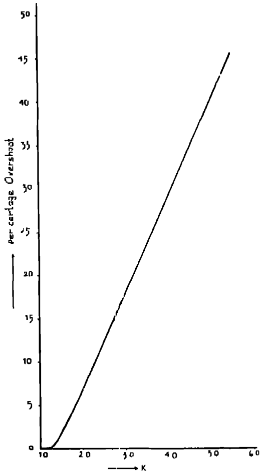

Fig. 4.

The above sets of curves can be taken as a guide to the design process. To start with one fixes up the amount of overshoot that can be tolerated. From the knowledge of tolerated overshoot, one knows the value of K which automatically fixes up T_m/g ratio. Then knowing the value of G, T_m is automatically fixed or from the knowledge of T_m, G is fixed up. In fact one has to make a compromise between T_m and G. For the same permissible overshoot, if a large gain is desired T_m will also have to be large and if, on the other hand, a small T_m is-required one has to sacrifice gain. G decides the value of R in the circuit. Thus knowing R and K, L can be known from a knowledge of C.

It will be noted from figure 5 showing the plot of T_m/G against K, that the curve passes through a minimum i.e., the gain-risetime ratio is maximum for a particular value of $K = 2.55$. The percentage overshoot for this value of K is 13%. If this can be tolerated, the combination of gain, overshoot and risetime as defined by this value of K should be taken as a guide to the design procedure.

It may further be noted that the minimum of the curve is not very sharp, it is rather flat for values of K greater than that for minimum. This means that beyond a certain value of K (≈ 2) although T_m/G does not vary appreciably, the overshoot rises rapidly with increase of K.

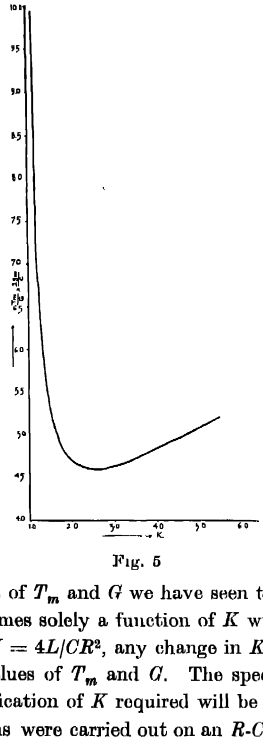

Fig. 5

From the expressions of T_m and G we have seen that both include R as the multiplier, but T_m/G becomes solely a function of K without having any multiplier R. Further, since $K = 4L/CR^2$, any change in K by varying R and L will modify the individual values of T_m and G. The specified gain will fix up the value of R and any modification of K required will be done only by changing L. The following observations were carried out on an R-C coupled amplifier with an inductance connected in series with the plate load resistance having the following circuit parameters, with a view to study its behaviour for different values of K involving changes in L and also R as discussed above.

Tube : $6AK5$
Plate load R : $2K\Omega$
Inductance : $20\mu H$
HT supply : $200V$.

The equivalent shunt capacity with the measuring aids connected to the corresponding points was approximately equal to $30p$F of which the input capacitance of the measuring probe was $8p$F. The input rectangular pulses were fed from Marconi Video Oscillator (TF885 A/1). The parameter $K\left(=\dfrac{4L}{CR^2}\right)$ was varied by varying L alone and the output waveform was examined on the calibrated pulse oscilloscope, Tektronix type 541. It was observed that with increase in the value of K from zero to unity (monotonic condition), effected by varying L from zero to $30\mu H (= CR^2/4)$ the risetime improved i.e. decreased uniformly, the steady state gain remaining constant; with further increase in the value of K risetime was still reduced but overshoot appeared.

When the inductance was further increased to $60\mu H$, keeping $R = 2K\ \Omega$ i. e. $K\left(= \frac{4L}{CR^2}\right) = 2$, the observed overshoot was approximately 6% whereas from the graph it was 6.7%. Now K was increased by decreasing R in steps. It was found that the risetime was reduced and the steady state gain began to fall. For large values of K with small values of R the output approximated a damped oscillation and the steady state gain approached zero. Under this condition the risetime, which is the time for a quarter cycle of oscillation is given by $\pi/2\sqrt{LC}$. Upto a value of K=2.5 there is a rapid improvement in the risetime with very little loss in steady state gain, thus resulting in a minimum value of T_m/G for this value of K. This value of K corresponds to about 13% overshoot and if that is tolerable (which is normally the case), pulse amplifiers can be conveniently designed with $K = 2.5$. With further increase in the value of K, loss of gain becomes more rapid than improvement in risetime, thus resulting in increased values of T_m/G (figure 5).

DESIGN EXAMPLE

Suppose we want to design a single stage pulse amplifier with a vacuum tube type 6Ak5 having gm = 5 mA/V. The unavoidable shunt capacity C at the under practical conditions is, say, $30pF$. Let the tolerable overshoot (S) in the amplifier be 12.5% . Reffering to figure 4, we find $K = 2.5$ for this value of S. From figure 5 we then have, for $K = 2.5$,

$$\frac{T_m}{G} \times \frac{2g_m}{C} = 4.6$$

Thus we see that the value of K automatically fixes up the T_m/G ratio, for a given value of g_m/C.

For the operating conditions specified above we get

$$\frac{T_m}{G} = 1.38 \times 10^{-8}$$

Now if we specify G, T_m will be fixed up and vice versa. If the desired gain is 10, the risetime becomes

$$T_m = 1.38 \times 10^{-7}$$

If permissible risetime is less than 0.138 μ sec., the value of G will be correspondingly reduced, keeping $T_m/G = 1.38 \times 10^{-8}$.

For $G = 10$ we have $R = 2000$ ohms. Substituting this value of R in the eqn. $\frac{L}{R^2} = K \cdot \frac{C}{4}$, we get $L = 75\mu H$. If, on the other hand, the permissible value of T_m is 0. 1μsec, G comes to be 7.24 and $R = 1488$ ohms, which in turn gives $L = 41.43\mu H$.

Experiments were carried out with a shunt compensated amplifier as discussed above, the input rectangular pulse being obtained from a pulse generator. The output was observed on a calibrated pulse oscilloscope.

Observations carried out with $R = 2000$ ohms, $L = 75\mu H$ and $C = 30\,pF$ gave $T_m = 0.15\mu$ sec, gain 9.2 and $S = 12\ \%$ approximately.

The experiment was further varified by connecting a 10pF condenser between anode and ground of the 6Ak5 tube in the amplifier and thereby making the total shunt capacity 40 pF. With the same overshoot and gain i.e. $K = 2.5$ and $G = 10$, since $\frac{T_m}{G} \times \frac{2g_m}{C} = 4.6$, we have for the operating conditions specified above $\frac{T_m}{G} = 1.84 \times 10^{-8}$ or $T_m = 1.84 \times 10^{-7}$ sec. with $G = 10$. Actual observations carried out with $R = 2000$ ohms, $L = 100\mu H$ and $C = 40pF$ gave $T_m = 0.2\ \mu$sec, gain 9.4 and $S = 12\%$ approximately.

The above design example refers to a single stage pulse amplifier. If the desired gain for a given overshoot is higher than that obtainable with a single tube the number of stages must have to be increased.

From the expression deduced above we have $\frac{T_m}{G} = \frac{C}{2g_m} \times$ some factor specified by the overshoot Hence for same overshoot using same tube, higher gain will automatically increase T_m to keep $\frac{T_m}{G}$ a constant. Now if T_m is to be reduced $\frac{g_m}{C}$ should be increased, which can be done by arranging the tubes in such a way that only the g_m of individual tubes are added and not their shunt capacitances. This again can be done by arranging the tubes to form a distributed amplifier.

APPENDIX: A

It is shown before that the mutual impedance of the high frequency equivalent circuit of the shunt compensated amplifier is given by

$$Z_m = \frac{(R + pL) \times \frac{1}{pC}}{R + pL + \frac{1}{pC}}$$

$$\text{or} \quad |Z_m| = R\sqrt{\frac{1+\frac{\omega^2L^2}{R^2}}{(1-\omega^2LC)^2+\omega^2C^2R^2}}$$

A_H (high frequency amplification)

$$= g_m \cdot |Z_m| = g_m R\sqrt{\frac{1+\frac{\omega^2L^2}{R^2}}{(1-\omega^2LC)^2+\omega^2C^2R^2}}$$

$$= A_r\sqrt{\frac{1+\frac{\omega^2L^2}{R^2}}{(1-\omega^2LC)^2+\omega^2C^2R^2}}$$

$$= A_r \cdot f(\omega)$$

Where $A_r =$ the mid. frequency gain.

Now to get the peak in the frequency response curve i.e the maximum value of $|Z_m|$ we are to satisfy the conditions :

$$\frac{d}{d\omega}f(\omega) = 0$$

and

$$\frac{d^2}{d^2\omega^2}f(\omega) = \text{negative}$$

Satisfying the above two conditions we get

$$\omega^2 = \frac{\sqrt{2LCR^2+L^2}}{L^2C} - \frac{R^2}{L^2}$$

Expressing ω^2 in terms of K, we have

$$\omega^2 = \frac{2}{LC}\left[\sqrt{\frac{2}{K}+\frac{1}{4}} - \frac{2}{K}\right]$$

From this expression we get for ω to be real and positive

Or $$K^2 + 8K - 16 > 0$$

Or $$K > 4 \cdot (\sqrt{2} - 1)$$

That is, the frequency response characteristic will show a peak only for values of $K > 4(\sqrt{2}-1)$. Thus, it is seen that although the transient response shows a peak (overshoot) as soon as the amplifier load circuit becomes underdamped ($K > 1$), the steady state response (frequency response) shows a peak only when $|Z_m|$ exceeds R. i.e. $K > 1.656$.

In order that $|Z_m| > R$ at some values of f, K must be greater than $[4(\sqrt{2}-1)]$. For value of K lying between unity and $[4(\sqrt{2}-1]$, there will be no peak in the frequency response characteristic although overshoots will occur for pulse amplification. Thus we see that to construct a pulse amplifier, a knowledge of transient response is more important than its steady state response. As such a design procedure directly based on the transient response, as done here, will be of much use.

APPENDIX : B

The above discussions are all valid if the Power Supply impedance is zero. But practical power supplies are not perfectly regulated and hence, the resulting internal impedance will modify the load impedance of the amplifier. The shift in the pole-zero location depends upon the nature and magnitude of the impedance. The equivalent circuit of the amplifier with the power supply impedance is shown and its effects on the response are discussed. It may be mentioned that if the power supply has a resistive impedance then its effect can be minimised by connecting a large condenser across the power supply. In any case the power supply source can be made to behave as a source of practically zero impedance by using an *R-C* filter consisting of a resistance in series with and a condenser across the supply. The power supply with its internal impedance Z_{bb} is shown in figure 6. If the source impedance is reactive the shunt condenser across the source has some peculiar effects on the load impedance characteristics and hence on pulse response, as discussed bellow :

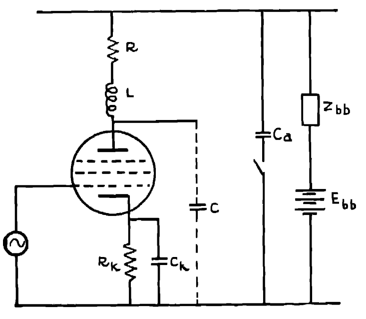

Fig. 6.

(1) *Internal impedance is capacitive.*

When the condenser is connected across the power supply, the effective inductance is reduced,

$\therefore$ K is reduced, and hence overshoot is decreased and risetime is increased.

(2) *Internal impedance is inductive*:

We know if the power supply impedance is zero, the load impedance approaches $\frac{1}{\omega C}$ (C = the total shunt capacity), but when the power supply impedance is inductive, it can be shown that as the power supply is shorted by a condenser, magnitude of the load impedance passes alternately through a maximum followed by a minimum and a maximum before the impedance approaches $\frac{1}{\omega C}$ at the high frequency. This is obvious from the equivalent circuit of the load impedance together with the inductive internal impedance of the power supply,

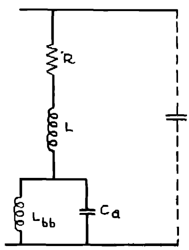

Fig. 7 The equivalent circuit with inductive internal impedance of the power supply.

figure 7. Since it is a minimum phase-shift type of network, the imaginary part of the complex impedance also changes from inductive to capacitive nature along with the impedance variation. Thus the zero-pole location changes and hence overshoot, risetime etc., are also modified accordingly.

Experimental observations revealed the fact that the power supply had inductive impedance, and hence the response was modified when a condenser was connected across the power supply, as explained above. Now, since, in the case of inductive internal impedance if the capacitance across the power supply

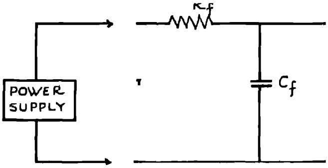

Fig. 8. R – C filter

is not very large, the internal impedance of power supply instead of being zero becomes more complicated. It is found that an *R-C* filter as shown in figure 8 will give better performance.

CONCLUSION

In this article we have defined risetime to be equal to the time required to reach the peak and not as time required to reach 10% to 90% of the final value. Hence in actual practice the risetime according to the conventional definition will be less than the calculated value.

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REFERENCES

Gupta Sarma, D., 1954, *Wireless Engineer*, **31**, 327.
Martin, T. L. Jr., 1955, Electronic Circuits, Prentice Hall, Inc.