

# RANGE-ENERGY RELATION FOR PROTONS IN VARIOUS SUBSTANCES

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**ABSTRACT.** A simple relation between the range ( $R$ ) and energy ( $E$ ) of heavy charged particles has been proposed:  $R = a(E+c)^n$ , where,  $a$ ,  $c$  and  $n$  are constants. The relation has been shown to be quite successful for proton ranges in a number of substances, up to 100 Mev.

## INTRODUCTION

All charged particles in their passage through matter lose energy by a number of processes. For not too high energies, the chief causes are the excitation and ionization of atoms and molecules of the medium. The practical problem of estimating the energy of a nuclear particle from its observed range in a known medium is of great importance and many experimental and theoretical studies have been made with the object of establishing standard range-energy relations. The subject has been reviewed in recent years by Taylor (1952), Bethe and Ashkin (1953), Allison and Warshaw (1953) and Uehling (1954).

It is convenient to distinguish the charged particles in two types: electrons and positrons on one hand and all charged particles heavier than electrons on the other. In the present paper we would confine ourselves to heavier particles. In the following  $E$  is the energy of the particle in Mev,  $v$  is its velocity and  $R$  is its range. The different units of range are indicated at appropriate places.

Semi-theoretical or empirical relations are frequently employed for representing the relation between the energy and the range of a particle.

The best known of these relations is due to Geiger (1910):

$$R = kv^3 = aE^{3/2} \quad \dots (1)$$

Originally Geiger had suggested this for alpha particles having ranges between 3 and 7 cm.

For low energy alpha particles having ranges less than 4 mm., Blackott and Loes (1932) found the following equation to be satisfactory:

$$R = Cv^{1.43} \quad \dots (2)$$

On the other hand, Briggs (1933) showed that for alpha particles having ranges greater than 5 cm., Geiger's relation should be modified to

$$R = kv^{3.26} \quad \dots (3)$$

Rutherford, Wynn-Williams, Lewis and Bowden (1933) noticed deviations from the Geiger relation and gave a correction curve for the same.

Meyer (1935) put forward the following relation for alpha particles :

$$R = -r + Av + Bv^2 + Cv^3 \quad \dots (4)$$

where  $r$ ,  $A$ ,  $B$  and  $C$  are constants.

It becomes apparent that power relations have only limited applicability. Livingston and Bethe (1937) have given a graph showing the variation of the exponent  $n$  in Geiger law with  $E$  (figure 35, p. 278 of their paper).

Rogers and Rogers (1938) have proposed a number of relations for different energy intervals. For example, for protons in air, with  $R$  in cm. :

for  $R \leq 6$

$$\log_{10} E = -0.3845 + [0.595 \pm 0.45e^{-4R} + 0.0025 \sin^2(2\pi R/9) \cos(\pi R/3)] \log_{10} 2R \quad \dots (5a)$$

$6 \leq R \leq 30$

$$E = 1.819 + 0.16758(R-6) - 0.003(R-6)^2 + 0.00004(R-6)^3 \quad \dots (5b)$$

$30 \leq R \leq 90$

$$E = 4.661 + 0.08715(R-30) - 0.000454(R-30)^2 + 0.00000217(R-30)^3 \quad \dots (5c)$$

Wilson and Brobeck (Wilson 1947) found for protons in air :

$$R \text{ (in metres)} = (E/9.3)^{1.8} \quad \dots (6)$$

Rogozinski (1951) advocates :

$$R = a^{-1.8} E^{1.8} \quad \dots (7)$$

If  $R$  is in gm/cm<sup>2</sup>,  $a = 27.5$  for Al and  $a = 29.0$  for air.

Cook, Jones and Jorgensen (1953) have employed a modification of Geiger relation for low energy protons (50—250 kev) :

$$R = C(E + E_1)^{3/2} \quad \dots (8)$$

where  $E_1$  and  $C$  are constants.

Recently Kaila (1955) has deduced from semi-theoretical considerations an equation for the ranges of alphas in air. But the values of the constants are fitted empirically :

$$E = 0.4359R^{1/3} + 5.676R^{1/3} \log R + 1.477R^{-2/3} + 0.9712R^{-2/3} \log R \quad \dots (9)$$

The equation is useful below 2.5 Mev.

Another recent equation is due to Gobeli (1956). For ranges of alpha particles in three semiconductors, he finds that his results can be represented by

$$R = aE^n + b \quad \dots (10)$$

where  $a$ ,  $b$  and  $n$  are constants. A similar equation has been used by Livesey (1956).

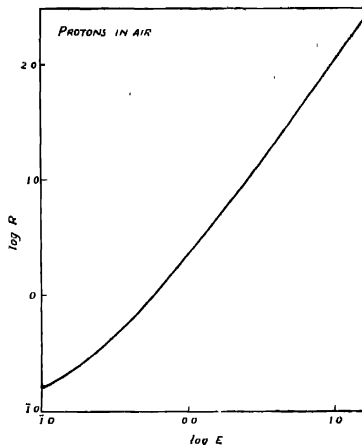


Fig. 1

## NEW RELATION

From the above it will be evident that simple power relations hold only for small energy intervals. The point has been illustrated in figure 1, in which  $\log R$  has been plotted against  $\log E$  for protons in air for  $E = 0.1$  to 15 Mev. For the applicability of the power relation, the curve should have been a straight line. Some of the relations proposed by earlier workers are rather complicated.

The present paper suggests a simple range-energy relation for heavy particles. The relation is

$$R = a(E+c)^n \quad \dots (11a)$$

where  $a$ ,  $c$  and  $n$  are constants.

The relation may be put as

$$\log_{10} R = n \log_{10}(E+c) + B \quad \dots (11b)$$

Presently we would show its applicability to protons for wide energy intervals.

Range-energy data for protons in a number of substances are available from experimental determinations and/or theoretical calculations. Individual substances have been treated below separately.

## AIR

Experimental determination of range-energy relation for protons in air have been made by a number of workers. These have been reviewed by Clarke and Bartholomew (1949), Bethe (1950), Jesse and Sadauskis (1950) and Bethe and Ashkin (1953). Theoretical calculations have been carried out by Smith

(1947) by numerical integration of the relativistic expression for  $dE/dx$ . The data for the region .05 Mev to 100 Mev can be represented by

$$\log R(\text{cm}) = 1.795 \log (E+0.13) + 0.2598$$

Calculated values have been compared with experimental and theoretical values in Table I.

TABLE I

Air			
$E$ (Mev)	$R$ (cm) Calc.	$R$ (cm) Expt. & Theor.*	% error
0.05	0.0838	0.085	- 1.41
0.1	0.1301	0.15	-13.3
0.2	0.2486	0.285	-12.8
0.4	0.582	0.61	- 4.6
0.6	1.034	1.04	- 0.58
0.8	1.597	1.6	- 0.19
1.0	2.265	2.27	- 0.22
1.5	4.372	4.4	- 0.64
2.0	7.068	7.1	- 0.45
4.0	23.2	23	+ 0.87
6.0	47.14	46.7	+ 0.94
8.0	78.23	77.3	+ 1.20
10.0	116.1	114.8	+ 1.13
15.0	238.6	238.5	+ 0.04
19.0	363.4	362.3	+ 0.30
30	821.5	810.4	+ 0.26
40	1374	1374	0
50	2049	2053	- 0.19
60	2840	2849	- 0.32
70	3743	3756	- 0.34
80	4754	4769	- 0.31
90	5872	—	—
100	7095	7095	0

\* Sources of data :

.05 - 1.5 Mev—Read from the curve given by Bethe (1950).

1.5 - 15 Mev—Read from curve given by Bethe and Ashkin (1953).

15 - 100 Mev—Theoretical calculations of Smith (1947).

#### BERYLLIUM

Aron (1951) has theoretically calculated the ranges, but as the value of the effective ionization potential used by him was found to be rather low, Bichsel, Mozley and Aron (1957) have recalculated ranges for energies between 1 to 21 Mev. From these the values of the constants in eq. (11b) were determined:

$$\log R (\text{mg/cm}^2) = 1.816 \log (E+0.18) + 0.3443$$

Table II shows the results.

TABLE II

## Beryllium

$E$ (Mev)	$R$ (mg/cm <sup>2</sup> ) Calc.	$R$ (mg/cm <sup>2</sup> ) Theor.*	% error
1	2.99	3.0	-0.33
2	9.10	9.1	0
3	18.06	18.0	+0.33
4	29.67	29.6	+0.24
5	43.8	43.6	+0.46
6	60.37	60.1	+0.45
7	79.25	78.9	+0.44
8	100.4	100.0	+0.4
9	123.8	123.3	+0.4
10	149.4	148.8	+0.4
11	177.1	176.5	+0.34
12	206.8	206.3	+0.24
13	238.7	238.2	+0.21
14	272.7	272.1	+0.22
15	308.6	308.1	+0.16
16	346.5	346.1	+0.11
17	386.4	386.1	+0.08
18	428.3	428.1	+0.05
19	472.0	472.0	0
20	517.6	517.6	0
21	565.1	565.9	-0.14

\* Theoretical values from Bichsel *et al.* (1957).

## ALUMINUM

Aluminum is one of the most useful substances for range measurements at both low and high energy because it can be obtained both in thin foils and in bulk, with high purity and satisfactory uniformity. It is an element, hence preferable to mica whose chemical composition varies and it has at the same time sufficiently low atomic weight to permit the application of theory at relatively low energy.

Experimental results have been summarised by Bethe and Ashkin (1953). Smith (1947) has tabulated theoretical results for  $E > 1$  Mev.

The data can be expressed by

$$\log R(\text{mg/cm}^2) = 1.777 \log (E+0.15) + 0.4384$$

The calculated values by this relation together with Smith's values are recorded in Table III.

TABLE V

Silver

$E$ (Mev)	$R$ (mg/cm <sup>2</sup> ) Calc.	$R$ (mg/cm <sup>2</sup> ) Theor.*
4	62.61	62.60
6	114.7	112.63
8	179.8	175.97
10	257.2	251.89
14	447.2	439.35
20	813.2	803.56
26	1272	1261.0
30	1624	1615.0
34	2014	2006.7
38	2439	2435.0
42	2901	2898.9
46	3396	3397.4
50	3923	3929.6
60	5389	5402.8
70	7050	7071.1
80	8902	8924.8
90	10940	10955
100	13150	13155

\* Theoretical values from Aron (1951).

## GOLD

For low energy region Wilcox (1948) gives a range-energy curve obtained by integrating the  $dE/dx$  curve. Theoretical values have been provided by Bichsel, Mozley and Aron (1957). Their data can be expressed by

$$\log R \text{ (mg/cm}^2\text{)} = 1.74 \log (E+1.15) + 0.7329$$

Table VI shows the agreement between the two sets of values.

TABLE VI  
Gold

$E$ (Mev)	$R$ (mg/cm <sup>2</sup> ) Calc.	$R$ (mg/cm <sup>2</sup> ) Theor.*
2	39.8	39.7
3	64.3	64.3
4	93.62	93.6
5	127.5	127.3
6	165.7	165.5
7	208.1	207.8
8	254.5	254.2
9	304.8	304.5
10	359.1	358.6
11	417.0	416.4
12	478.4	477.8
13	543.4	542.9
14	612.0	611.5
15	684.2	683.5
16	759.7	759.0
17	838.3	837.8
18	920.0	920.0
19	1005.7	1005.5
20	1093.7	1094.2
21	1185.6	1186.2

\* Theoretical values from Bichsel *et al* (1957).GLYCEROL TRISTEARATE  $C_{57}H_{110}O_6$ 

In many neutron experiments hydrogen is used in the form of an organic compound. Glycerol tristearate is one of the important ones. Hirschfelder and Magee (1948) have calculated ranges by a semi-theoretical method. The equation is

$$\log R \text{ (mg/cm}^2\text{)} = 1.81 \log (E+0.12) + 0.2247$$

Table VII shows some of the theoretical values of Hirschfelder and Magee together with the calculated values from the above formula.

TABLE VII

Glycerol tristearate

<i>E</i> (Mev)	<i>R</i> (mg/cm <sup>2</sup> ) Calc.	<i>R</i> (mg/cm <sup>2</sup> ) Theor.*
0.02	0.0478	0.046
0.05	0.0679	0.068
0.1	0.1083	0.106
0.2	0.2132	0.205
0.4	0.5137	0.500
0.6	0.9256	0.911
0.8	1.442	1.430
1.0	2.060	2.050
1.2	2.773	2.767
1.4	3.579	3.576
1.6	4.477	4.476
1.8	5.464	5.464
2.0	6.536	6.539
2.2	7.697	7.698
2.4	8.937	8.941
2.6	10.26	10.267
2.8	11.67	11.673
3.0	13.16	13.160

\* Theoretical values from Hirschfelder and Magee (1948).

PARAFFIN (CH<sub>2</sub>)<sub>n</sub>

Hirschfelder and Magee (1948) have tabulated theoretical results for paraffin also. The equation is

$$\log R \text{ (mg/cm}^2\text{)} = 1.834 \log (E+0.12) + 0.1905$$

Results are given in Table VIII.



TABLE VIII

## Paraffin

$E$ (Mev)	$R$ (mg/cm <sup>2</sup> )	
	Calc.	Theor.*
0.05	0.06013	0.06096
0.1	0.0965	0.09613
0.2	0.1918	0.18908
0.4	0.4673	0.46833
0.6	0.8487	0.86101
0.8	1.331	1.35750
1.0	1.909	1.95202
1.5	3.756	3.83030
2.0	6.150	6.2683
3.0	12.50	12.652
4.0	20.81	20.960
5.0	31.00	31.118
6.0	43.00	43.070
7.0	56.75	56.771
8.0	72.23	72.182
9.0	89.34	89.275
10	108.1	108.022
11	128.5	128.40
12	150.5	150.40
13	174.1	173.99
14	199.2	199.17
15	225.9	225.91

\* Theoretical values from Hirschfölder and Magee (1948).

## NUCLEAR EMULSION

Due to the great importance of nuclear emulsion technique, extensive experimental and theoretical investigations have been carried out for establishing accurately the range-energy relation for the nuclear emulsions.

Simple power relations have been employed by some workers (Lattes, Occhialini and Powell, 1948; Bradner *et al.*, 1950; Lees *et al.*, 1953) for representing their data.

Recently Daniel, George and Peters (1955) have examined some of the experimental and theoretical data and given a 'best value' table of range and energy for Ilford C2 emulsion (density 3.94 gm/cm<sup>3</sup>). We have utilized their data for finding the values of the constants in the formula

$$\log R = 1.749 \log (E+0.25) - 2.0301$$

where  $R$  is in mm.

The calculated values by the above expression have been compared with the values given by Daniel *et al.* in Table IX.

TABLE IX  
Ilford C2 nuclear emulsion

$E$ (Mev)	$R$ (mm) Calc.	$R$ (mm) Expt. & Theor.*	% error
1	0.01379	0.0138	-0.07
1.5	0.02482	0.0255	-2.67
2	0.03854	0.0399	-3.4
3	0.07332	0.0746	-1.71
4	0.1172	0.1185	-1.1
5	0.1696	0.1725	-1.68
6	0.2301	0.2350	-1.96
7	0.2983	0.3030	-1.55
8	0.3740	0.3780	-1.06
10	0.5465	0.5540	-1.35
15	1.095	1.100	-0.45
20	1.799	1.815	-0.88
30	3.627	3.615	+0.33
40	5.977	5.980	+0.28
50	8.812	8.750	+0.71
60	12.11	11.85	+2.19
80	20.0	19.80	+1.01
100	29.50	29.70	-0.67

\* Data from Daniel, George and Peters (1955).

#### DISCUSSION

The values of the constants in equation (11) for the various substances have been summarised in Table X.

TABLE X  
Values of the constants ( $E$  in Mev)

Substance	Unit of $R$	$n$	$c$	$B$	$a$
Air	cm	1.795	0.13	0.2598	1.819
Beryllium	mg/cm <sup>2</sup>	1.816	0.18	0.3443	2.210
Aluminum	mg/cm <sup>2</sup>	1.777	0.15	0.4384	2.745
Copper	mg/cm <sup>2</sup>	1.778	0.5	0.5076	3.218
Silver	,,	1.768	0.9	0.5763	3.770
Gold	,,	1.74	1.15	0.7329	5.406
Glycerol tristearate	,,	1.81	0.12	0.2247	1.678
Paraffin	,,	1.834	0.12	0.1905	1.551
Nuclear emulsion	mm	1.749	0.25	-2.0301	0.009330

Though there is no regular variation in the values of the constants, yet a rough observation may be made that  $n$  has a tendency to decrease and  $C$  tends to increase as we go to substances of higher effective atomic numbers. Value of  $a$  is dependent on the unit of  $R$ . In those cases where  $R$  has been expressed in  $\text{mg}/\text{cm}^2$  it may be noticed that  $a$  increases with increase in effective atomic number.

It will be observed from Tables I to IX that the agreement between the calculated values from eq. (11b) and the theoretical and/or experimental values is quite satisfactory up to 100 Mev. Errors are usually of the order of 1%. It may be added that the accuracy of theoretical and experimental figures is usually of the order of 2%. Further, the constants were not determined by the method of least squares; if they are determined by the method of least squares the results can be expected to be even better.

Below 0.1 Mev protons begin to capture electrons. The theory of the electron capture has not yet been fully worked out, though approximate studies have been made. In a number of cases (glycerol tristearate, paraffin etc.) theoretically calculated values are available for  $E < 0.1$  Mev; but as these values do not include the effect of electron capture, they may be in error.

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