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QUADRUPOLE VIBRATIONS AND THE ELECTRIC QUADRUPOLE MOMENT OF SOME CLOSED SHELL PLUS (OR MINUS) A SINGLE NUCLEON NUCLEI

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ABSTRACT. A critical survey has been made in this paper on the present position of the quadrupole moment of closed shell plus (or minus) a single nucleon nuclei. It is found that collective model formalism of the quadrupole moment is essentially correct if one calculates the rigidity of the core from the data on the vibrational spectra of even-even nuclei. Using the available data on vibrational levels the strength of surface coupling has been calculated and it is found that in the region of closed shell, an intermediate coupling picture holds true. The values of the quadrupole moment are also improved in this scheme.

1. INTRODUCTION

It is well known that although spin and energy levels of nuclei can sometimes be satisfactorily explained by the shell model, it is very difficult to account for the magnitude of their magnetic moments and electric quadrupole moments. Specially the latter is very sensitive to the choice of the ground state wave function. In the extreme single particle model the ground state quadrupole moment for a closed shell plus a single proton nucleus is given by

$$Q_j = \langle r^2(3\cos^2\theta - 1)\rangle_{j=m}$$

$$= -\frac{2j-1}{2(j+1)}\langle r^2\rangle \qquad \ldots \quad (1)$$

where $\mathbf{j}$ is the total angular momentum of the satellite proton, m its Z-component and $\mathbf{r}$ its position vector.

On the average $\langle r^2\rangle$ can be taken as $3/5\, R_0^2$ (Mayer and Jensen, 1955), where R_0 is the nuclear charge radius. It is well known that

$$R_0 = KA^{1/3} \qquad \ldots \quad (2)$$

where $K = 1.19\times 10^{-13}$cm (as given by electron scattering experiments). If there is a hole instead of a proton then the sign of (1) is to be interchanged. This explains the fact that just before a magic number the quadrupole moment is positive and after a magic number it is negative. But apart from this qualitative agree-

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ment, the single particle model is not adequate to account for the magnitude of the quadrupole moments. In fact, the ratio Q_{obs}/Q_j is sometimes as large as forty.

In 1950, Rainwater showed that if instead of a central field $V(r)$, a spheroidal field of the form $V(r, \theta)$ is taken then the value of Q_j is somewhat increased. This was further developed in the collective model of nuclei by A. Bohr in 1952. In this model the closed shell part of the nucleus is treated as a liquid drop, capable of collective oscillations and around this core one or more particles are moving in their shell orbits. As a result of the surface coupling the core will be distorted and its equilibrium shape will be a spheroid.

As a consequence of this there will be a good amount of contribution to the quadrupole moment from the core.

It can be easily proved (Bohr and Mottelson, 1953) that for core plus a single proton

$$Q = Q_c + Q_j$$

$$= -\left[1-3 \cdot \frac{2I+1}{(I+1)(2I+3)} \cdot \frac{x^2}{\sqrt{x^4+4/9}}\right] \frac{3}{4\pi} \cdot \frac{2I-1}{2(I+1)} \cdot \frac{k}{C} ZR_o^2$$

$$-\frac{3}{5} \cdot \frac{2I-1}{2(I+1)} \cdot R_0^2 \quad \ldots \quad (3)$$

In (3) k is a term appearing in the interaction hamiltonian of the particle and surface. Its sign is reversed if the particle is replaced by a hole. On the average $k \simeq 40$ Mev. The factor C which appears in (3) is known as the nuclear rigidity. For a uniformly charged nucleus of constant surface tension S

$$C = 4R_0^2 S - \frac{3}{10\pi} \cdot \frac{Z^2 e^2}{R_0} \text{ [A. Bohr, 1952]} \quad \ldots \quad (4)$$

The dimensionless parameter

$$x = \sqrt{\frac{5}{16\pi}} \cdot \frac{k}{\sqrt{\hbar\omega j C}} \quad \ldots \quad (5)$$

It is a measure of the strength of coupling between the particle and the surface. In (5) ω is the frequency of collective oscillations of the core. It can be shown (A. Bohr, 1952) that

$$\omega = \sqrt{\frac{C}{B}} \quad \ldots \quad (6)$$

where B is the nuclear inertial parameter.

For a uniform liquid in irrotational flow,

$$B = \frac{3}{8\pi} \cdot AMR^2_0 \qquad \dots \quad (7)$$

Using (4) and (7), Bohr and Mottelson (1953) have calculated Q for various nuclei and found that in general Q is many times larger than its actual value (see Table 1. column 15). This is hardly surprising since both (4) and (7) are very approximate equations. It is obvious that both B and C should depend strongly on the shell structure of the core.

Marumori *et al.* (1956) showed that C, the surface rigidity depends strongly on the shell structure configuration of the core. Using a simple form of interaction they have calculated the values of C and found that these values of C give excellent values of the quadrupole moment. But here again the agreement has little meaning since these authors have used the same irrotational value of B as given by equation (7).

The purpose of the present paper is to show that the collective model formalism of the quadrupole moment, as given in equation (3), is essentially correct.

2. CALCULATION OF QUADRUPOLE MOMENT FROM THE DATA OF THE VIBRATIONAL LEVEL OF EVEN-EVEN NUCLEI

The core of the nuclei under our consideration are even-even nuclei and hence they are capable of oscillation about a spherical equilibrium shape when the extra particle is absent (Alder *et al.* 1956). The lowest mode will be obviously of quadrupole type if the nucleus is to remain symmetric. The energy of the first excited level is given by the well known oscillator equation.

$$E_2 = \hbar\omega$$

$$= \hbar\sqrt{\frac{C}{B}} \qquad \dots \quad (8)$$

It is well established that the spin of the first excited level of even-even nuclei is almost always 2 and the dominant mode of decay is by E_2 radiation. That the observed excitation is of collective origin is exhibited by the fact that the reduced transition probability $B(E_2)$ is many times larger than the single particle estimate.

If one assumes that these vibrations correspond to one photon excitation then it is easy to show that

$$B(E_2, 0 \rightarrow 2) = \Sigma \mid < 0 \mid m(E_2\mu) \mid 1 > \mid^2$$

TABLE I

The values of quadrupole moment

Isotope	Configuration Proton	Configuration Neutron	Core	Spin	E_2 Mev	$B(E_2)$ $\times 10^{-48}$ cm^4.e^2	$\frac{B(E_2)}{B(E_2)_{sp}}$	C_2 Mev	λ	$P_q(\pi)$	In Barn Q_o	Q_s	Q_j	$Q = Q_s+Q_j$	Q BM	Q obs
$_5B^{11}$	$(1p_{3/2})^{-1}$	—	$_6C^{12}$	3/2	4.40	0.009	11.42	14	1.31	0.258	0.121	0.031	0.016	.047	0.093	0.036
$_{27}Co^{59}$	$(1f_{7/2})^{-1}$	—	$_{28}Ni^{60}$	7/2	1.33	0.12	17.80	59	0.76	0.650	0.656	0.426	0.085	0.511	0.90	0.5
$_{31}Ga^{69}$	$(2p_{3/2})^{-1}$	—	$_{32}Ge^{70}$	3/2	1.08	0.077	9.62	120	0.93	0.363	0.245	0.089	.058	.147	0.52	0.23
$_{31}Ga^{71}$	$(2p_{3/2})^{-1}$	—	$_{32}Ge^{72}$	3/2	0.83	0.23	26.77	32	1.999	0.211	0.937	0.197	0.058	.255	0.52	0.14
$_{79}Au^{197}$	$(2d_{3/2})^{-1}$	—	$_{80}Hg^{198}$	3/2	0.41	1.0	30.3	88	1.714	0.22	1.688	0.371	0.115	0.486	0.967	0.48
$_{32}Ge^{73}$	—	$1g_{9/2}$	$_{32}Ge^{72}$	9/2	0.83	0.23	26.77	32	1.154	0.593	-1.703	-1.01	0	-1.01	-1.1	-0.2

$$= \left(\frac{3}{4\pi} ZeR_0^2 \right)^2 \sum_{\mu} | < 0 | \alpha_{2\mu}^* | | 1 > |^2$$

where $\alpha_{2\mu}$ is the familiar deformation parameter.

$$= 5. \left(\frac{3ZeR_0^2}{4\pi} \right)^2 . \frac{\hbar}{2(BC)^{1/2}} \quad \ldots \quad (9)$$

From (8) and (9)

$$C = \frac{45}{32\pi^2} \cdot \frac{Z^2e^2R_0^4E_2}{B(E_2, 0 \rightarrow 2)} \quad \ldots \quad (10)$$

Thus here is a nice way to determine the value of C as well as x from the data of the first excited state of even-even nuclei. From (5) it is obvious that

$$x = \sqrt{\frac{100 J}{2\pi j E}} \quad (11)$$

where we have taken $k = 40$Mev and E_2 and C are expressed in Mev.

To be sure that the levels we have used to calculate C and, x are of collective origin, we have compared the experimental $B(E_2)$ values with that of single particle estimate. (Blatt and Weisskoff, 1952).

$$B_{sp}(E_2, 0 \rightarrow 2) = \frac{9}{20\pi} e^2R_0^4 \quad \ldots \quad (12)$$

This is presented in Table I column 8, where one can see that the ratio $\frac{B_{exp}(E_2)}{B_{sp}(E_2)}$ is always much greater than one.

The coupling strength x as calculated from (11) is presented in column 10 of the same table. It is seen that x is almost always of the order of one, indicating an intermediate coupling. It is of interest to note that x depends sensitively on the orbital angular momentum of the satellite particle. Thus for $_{32}Ge^{72}$ core $x \sim 2$ when the outside particle is in $2p$ orbit, while $x \sim 1.1$ when the particle is in $1g$.

The last three columns of Table I give our calculated values of Q, the previous hydrodynamical values Q_{BM}, and observed values Q_{obs}. It will be seen that our values agree better with the observed values. One exception is that of $_{32}Ge^{73}$. This can be accounted for by the fact that the core $_{32}Ge^{72}$ has or more rigid neutron core ($N = 40$) than proton core, and the values of C as determined from the vibrational spectra gives essentially the rigidity of the proton core.

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