ANALOGUE MULTIPLIER AND FUNCTION GENERATOR WITH CATHODE RAY TUBE

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ABSTRACT. A multiplier and a function generator using a cathode ray tube with a capacitative pick up device are described. The device simplifies the construction of the multiplier and function generator without impairing the speed of response or the accuracy of the instrument.

INTRODUCTION

Function generators and multipliers using cathode ray tubes have been described by many authors (MacNee, 1949, Mackay, 1947; Deely and Mackay, 1949; Sunstein, 1949). Use of the electron beam of the C R. tube as the dynamic element in these function generators and multipliers results in high speed and makes them specially suited for applications in high speed analogue computers. However, in all such function generators and multipliers, the voltage giving the function or the product is obtained with the help of photocells placed in front of the C. R. tube. Instead of the photocells as the pick up device, MacNee suggested the use of metallic pick up plates mounted inside the C. R tube. Evidently, this is only suited for the multiplier If it is to be used for the function generator a separate C. R. tube is required for each new function. In this paper a multiplier and function generator with a new pick up device is described.The device is simple and can be easily replaced so that the same C. R. tube can be used for the multiplier or for generating different types of functions.

PRINCIPLE OF THE MULTIPLIER

The electron beam in a cathode ray tube, when subjected to an electric field $\mathbf{E}_x$ in the X-direction and a magnetic field $\mathbf{H}_z$ in the Z-direction, is deflected in the Y-direction and the deflection is given by y, where

$$y = K_1 \mathbf{E}_x \times \mathbf{H}_z$$

Since, velocity of the spot in the Y-direction is proportional to $\mathbf{E}_x \times \mathbf{H}_z$.

The deflection in the Y-direction is proportional to the product of the electric and magnetic field intensities. One of the voltage to be multiplied (V_1) is applied to the X-plates (figure 1). Thus $\mathbf{E}_x$ is made proportional to V_1. The other voltage V_2 is applied to an amplifier which sends a current proportional to V_2 through a coil mounted coaxially on the neck of the tube between X and Y plates. The resulting Y-deflection is proportional to the product of V_1 and V_2.

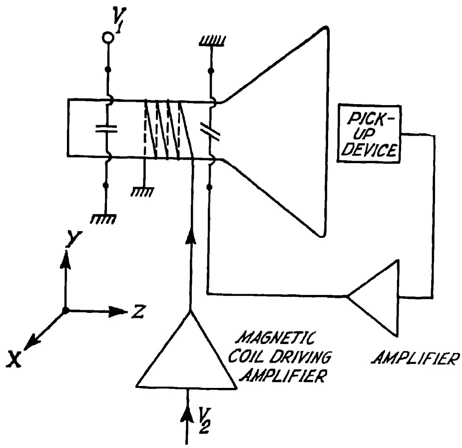

Fig. 1 · Block diagram of the multiplier.

On the face of the cathode ray tube is mounted a pick up device. It gives an output voltage which varies as the spot is deflected from the zero-Y line. If the output voltage of the pick up device is proportional to the spot deflection, it is also proportional to the product of V_1 and V_2 Usually this is not so, and to obtain a voltage proportional to the product, the output of the pick up device is amplified and applied to the Y-plates, in such a phase that it tries to bring the spot back to its initial position, whenever it is deflected due to the applications of V_1 and V_2. Let the pick up device give an output voltage v per unit Y deflection and δ_y be the Y-plate deflection sensitivity of the tube. Then, evidently the voltage appearing at the output of the amplifier, when voltages V_1 and V_2 are applied, is V_{out}, where

$$V_{out} = \frac{K_2 V_1 V_2}{\delta_y - \frac{1}{vA}} = \frac{K_2/\delta_y}{1 - \frac{1}{\delta_y vA}} V_1 V_2. \quad \ldots (1)$$

A is the gain of the amplifier and K_2 is a constant related to K_1, the X-plate sensitivity and the constant relating V_2 with V_1. If the gain of the amplifier is made so large, that $\frac{1}{\delta_y vA}$ is very small compared to 1, V_{out} is proportional to the product of V_1 and V_2.

PICK UP DEVICE

The pick up device consists of metallic plates mounted on the face of the cathode ray tube as shown in the figure 2. One of the plates is directly grounded and the other is connected to ground through a high impedance element. The electron beam of the C. R. tube is intensity modulated. As a result the charge appearing before the plates varies with time and thereby induces charges on the

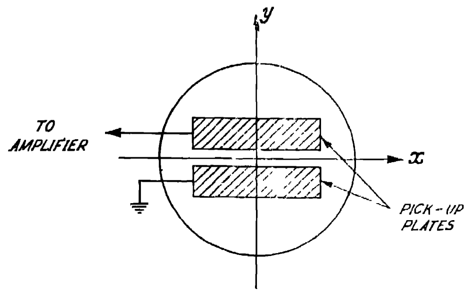

Fig. 2. Shape of the pickup plates for the multiplier.

plates which also vary with time. The varying charge on the metallic plate connected to ground through high impedance causes a voltage to be developed across the impedance. Evidently the magnitude of this voltage depends on the position of the cathode ray spot relative to this pick up plate and decreases as the spot recedes from its edge. The presence of the grounded plate enhances this rate of variation.

CIRCUITRY OF THE MULTIPLIER

The amplifier circuit following the pick up device is shown in figure 3. The high impedance element connected to the pick up plate consists of a heavily damped parallel tuned circuit. It is tuned to the frequency at which the electron beam is intensity modulated, which is 400 kc/s. The oscillator supplying the intensity modulation voltage is crystal controlled. The voltage developed across the pick up device is amplified by R-C coupled amplifiers in two stages, and then rectified by a thermionic diode. The rectified voltage is applied to the input of a d.c. amplifier. The output of the d.c. amplifier is adjusted to be zero when the spot is on the zero-Y line by the zero adjuster. Also the phase of the output voltage is so adjusted that as the spot tries to leave the zero-Y line, the amplifier output voltage, when applied to the Y-plate, brings it back. The gain of the amplifier is such that $v\delta_y A$ is equal to 1000. Thus error introduced from this source is less than 1 in 1000.

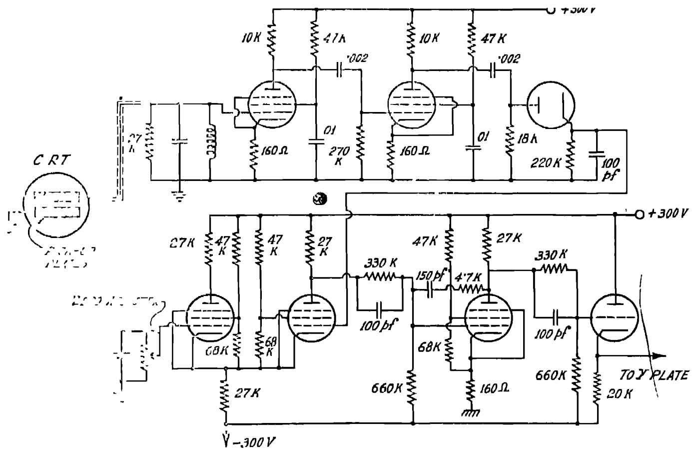

Fig. : Circuit arrangement of the amplifier.

The circuit arrangement of the amplifier supplying the current to the magnetic deflection coil is shown in figure 4. The frequency response of this amplifier is flat up to 3 Kc/s. The magnetic deflection coil consists of 5000 turns of 46 gauge copper wire and has a resistance of 1 K Ω. The C-R tube used for the multiplier is of the type 5BP1.

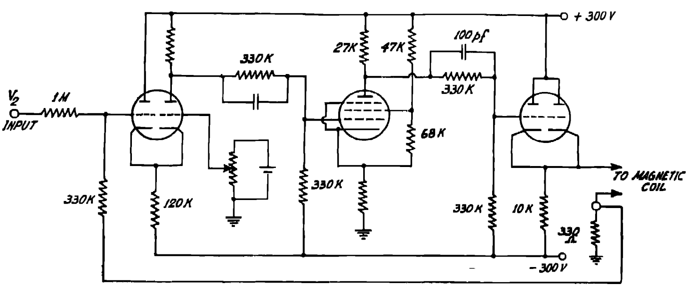

Fig. 4. : Magnetic-deflection amplifier circuit.

MULTIPLIER CHARACTERISTICS

The measured d.c. characteristics of the multiplier is shown in figure 5. It shows an accuracy of 2% of full output. The frequency response of the multiplier is flat up to 5 Kc/s for voltages applied to the X-plates and up to 3Kc/s for voltages applied to the magnetic-deflection coil.

The accuracy of the multiplier is limited by limitations of mechanical alignments. The edges of the pick up plates are not perfectly parallel to the zero-Y deflection line and also the axis of the magnetic deflection coil cannot be accurately aligned with the tube axis. With better mechanical arrangements the accuracy figure can be further improved. In this connection, it may be noted that as the

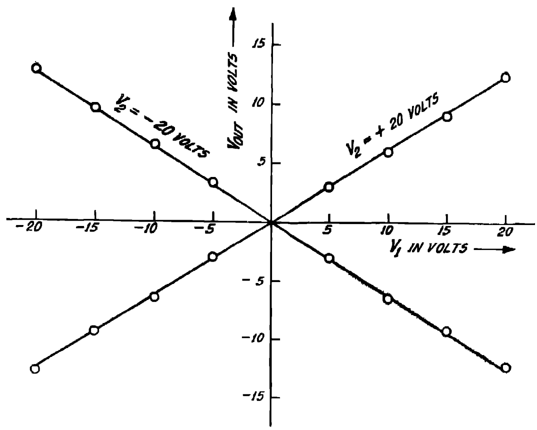

Fig 5 : Multiplier D. C. characteristics.

spot is deflected from its centre position it gets defocussed. This defocussing is further increased when the magnetic field is applied. However, defocussing only reduces the output of the pick-up plate and thereby affects the overall gain. The reduction in the gain does not affect the accuracy much since even the reduced gain is well over 700 and this may contribute an error of only 1 in 700.

The frequency response of the multiplier for voltages applied at the Y-plate may be further improved by using a higher frequency for beam intensity modulation and increasing the bandwidth of the parallel-tuned circuit. The frequency response for voltage applied at the magnetic deflection coil also may be improved by increasing the bandwidth of the magnetic deflection amplifier. This requires the use of a magnetic-deflection coil with smaller number of turns and therefore involves greater power consumption. Since, however, the differential analyser of which the multiplier forms a part has a fixed value of CR equal to 1/4, further improvement of the frequency response was thought unnecessary.

Solution of an equation with the multiplier

The performance of the multiplier in its actual use in the computer is illustrated by solutions of Mathieu equation obtained by the differential analyser using the multiplier.

Mathieu equation in its general form is written as

$$\frac{d^2y}{dz^2} + (a - 2q \cos 2z)y = 0 \qquad \ldots \quad (2)$$

The set-up of the computer for solving the equation is shown in figure 6. Cos $\omega_1 t$ is generated in a different part of the analyser. The computer set-up solves the equation

$$\frac{d^2y}{dt^2} + \omega_0^2(\alpha_1 - \alpha_2 \cos \omega_1 t)y = 0 \qquad \ldots \quad (3)$$

where $\omega_0^2 = 1/(CR)^2$.

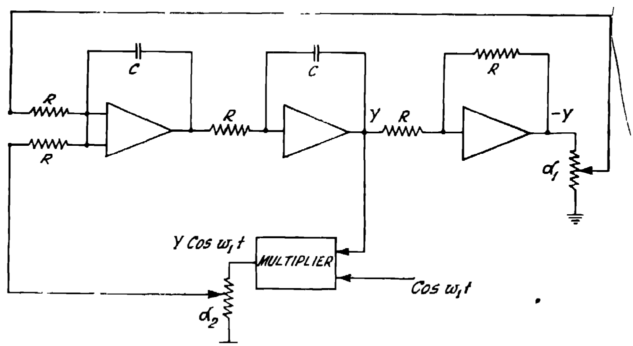

Fig. 6 · Computer set up for solving the equation,

$$\frac{d^2y}{dz^2} + (a - 2q \cos 2z)y = 0.$$

On substituting $2z$ for $\omega_1 t$ Eqn. (3) reduces to

$$\frac{d^2y}{dz^2} + \left[\frac{4\omega_0^2}{\omega_1^2} \alpha_1 - 2 \, . \, \frac{2\omega_0^2}{\omega_1^2} \alpha_2 \cos 2z \right] y = 0.$$

Hence $a = \frac{4\omega_0^2}{\omega_1^2} \alpha_1$, and $q = \frac{2\omega_0^2}{\omega_1^2} \alpha_2$.

In figure 7 is given the solution of the equation as obtained by the analyser for $a = 3$, $q = 2$ and $\dot{y} = 0$.

FUNCTION GENERATOR

Let the function to be generated be given by $f(V_1)$. The pick up plate mounted on the face of the C.R. tube is replaced by two other plates with their edges shaped to the form given by $f(V_1)$. The form of the pick up plates for $f(V_1) = V_1^2$ is shown

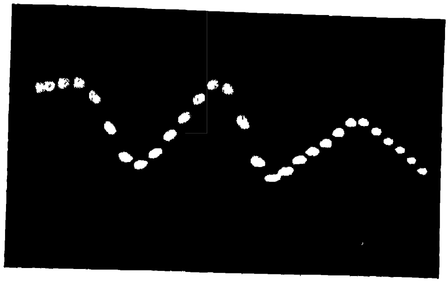

Fig. 7 : Computer solution of Mathiou equation for

$a = 3, \quad q = 2, \quad \dot{y}_o = 0.$

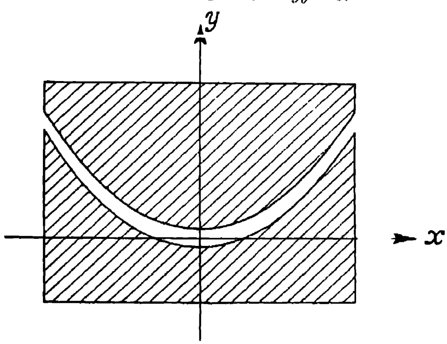

Fig. 8 : Shapo of tho pick up plates for generating V_1^2.

Fig. 9 : Plot of V_1^2 generated by tho function generator against V_1.

in figure 8. Now for generating V_1^2, V_2 is made zero and V_1 applied to the X-plates. As V_1 is applied, the spot moves along the X-axis, but the feed back device keeps it in between the edges of the two plates. As in the case of the multiplier if the gain of the amplifier is large, the spot moves along the centre line between the edges, and since the plate edges are shaped to V_1^2, the voltage appearing at the output of the amplifier is proportional to V_1^2.

V_1^2 plotted against V_1 as generated by the function generator is shown in figure 9. The frequency response of the function generator is flat up to 5 Kc/s, which, however, can be improved further as indicated before by increasing the beam intensity modulation frequency.

CONCLUSION

The new pick up device described in this paper can be used successfully for constructing a multiplier or a function generator employing a C. R. tube. The use of this device does not affect the speed of response. The accuracy is mainly limited by mechanical mis-alignments.

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