Leptonic CP Violation and Leptogenesis

M. N. Rebelo

Departamento de Física and Centro de Física Teorica de Partículas (CFTP)
Instituto Superior Tecnico (IST), Av. Rovisco Pais, 1049-001 Lisboa, Portugal
CERN, Department of Physics, Theory Division, CH-1211 Geneve 23, Switzerland
Presently at CERN on sabbatical leave from IST.
E-mail: margarida.rebelo@cern.ch and rebelo@ist.utl.pt

We review some recent results on the connection between CP violation at low energies and Leptogenesis in the framework of specific flavour structures for the fundamental leptonic mass matrices with zero textures.

Keywords: Leptonic CP violation; Neutrino masses; Leptogenesis

1. Introduction

Neutrinos have masses which are much smaller than the other fermionic masses and there is large mixing in the leptonic sector. The Standard Model (SM) of electroweak interactions cannot accommodate the observed neutrino masses and leptonic mixing since in the Standard Model neutrinos are strictly massless: the absence of righthanded components for the neutrino fields does not allow one to write a Dirac mass term; the fact that the lefthanded components of the neutrino fields are part of a doublet of $SU(2)$ rules out the possibility of introducing Majorana mass terms since these would violate gauge symmetry; finally, in the SM, $B - L$ is exactly conserved, therefore Majorana mass terms cannot be generated neither radiatively in higher orders nor nonperturbatively. Therefore, neutrino masses require physics beyond the SM. At present, this is the only direct evidence for physics beyond the SM. The origin of neutrino masses remains an open question. It is part of a wider puzzle, the flavour puzzle, with questions such as whether or not there is a connection between quarks and leptons explaining the different patterns of flavour mixing in each sector and the different mass hierarchies. In the seesaw framework[1–5] the explanation of the observed smallness of neutrino masses is related to the existence of heavy neutrinos with masses that can be of the order of the unification scale and have profound implications for cosmology. Mixing in the leptonic sector leads to the possibility of leptonic CP violation both at low and at high energies. CP violation in the decay of heavy neutrinos may allow for the explanation of the observed baryon asymmetry of the Universe (BAU) through leptogenesis.[6] Neutrino physics may also be relevant to the understanding of dark matter and dark energy as well as galaxy-cluster formation. Recent detailed analyses of the present theoretical and experimental situation in neutrino physics and its future, can be found in Refs. 7 and 8.

In this work the possibility that BAU may be generated via leptogenesis through the decay of heavy neutrinos is discussed. Leptogenesis requires CP violation in the decays of heavy neutrinos. However, in general it is not possible to establish a connection between CP violation required for leptogenesis and low energy CP violation.[9,10] This connection can only be established in speci c avour models. The fact that in this framework the masses of the heavy neutrinos are so large that they cannot be produced at present colliders and would have decayed in the early Universe shows the relevance of avour models in order to prove leptogenesis. In what follows it will be shown how the imposition of texture zeros in the neutrino Yukawa couplings may at the same time constrain physics at low energies and lead to predictions for leptogenesis.

2. Framework and Notation

The work described here is done in the seesaw framework, which provides an elegent way to explain the smallness of neutrino masses, when compared to the masses of the other fermions.

In the minimal seesaw framework, the SM is extended only through the inclusion of righthanded components for the neutrinos which are singlets of SU(2) U(1). Frequently, one righthanded neutrino component per generation is introduced. This will be the case in what follows, unless otherwise stated. In fact, neutrino masses can be generated without requiring the number of righthanded and lefthanded neutrinos to be equal. Present observations are consistent with the introdution of two righthanded components only. In this case one of the three light neutrinos would be massless.

With one righthanded neutrino component per generation the number of fermionic degrees of freedom for neutrinos equals those of all other fermions in the theory. However neutrinos are the only known fermions which have zero electrical charge and this allows one to write Majorana mass terms for the singlet fermion elds. After spontaneous symmetry breakdown (SSB) the leptonic mass term is of the form:

$$
\begin{aligned}
\mathcal{L}_m &= -[\overline{\nu_L^0} m_D \nu_R^0 + \frac{1}{2} \nu_R^{0T} C M_R \nu_R^0 + \overline{l_L^0} m_l l_R^0] + h.c. = \\
&= -[\frac{1}{2} n_L^T C \mathcal{M}^* n_L + \overline{l_L^0} m_l l_R^0] + h.c.
\end{aligned}
\tag{1}
$$

with the 6×6 matrix $\mathcal{M}$ given by:

$$
\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}
\tag{2}
$$

the upperscript 0 in the neutrino (ν) and charged lepton fields (l) is used to indicate that we are still in a weak basis (WB), i.e., the gauge currents are still diagonal. The charged current is given by:

$$
\mathcal{L}_W = -\frac{g}{\sqrt{2}} W_\mu^+ \overline{l_L^0} \gamma^\mu \nu_L^0 + h.c.
\tag{3}
$$

Since the Majorana mass term is gauge invariant there are no constraints on the scale of M_R. The seesaw limit consists of taking this scale to be much larger than the scale of the Dirac mass matrices m_D and m_l. The Dirac mass matrices are generated from Yukawa couplings after SSB and are therefore at most of the electroweak scale. As a result the spectrum of the neutrino masses splits into two sets, one consisting of very heavy neutrinos with masses of the order of that of the matrix M_R and the other set with masses obtained, to a very good approximation, from the diagonalisation of an effective Majorana mass matrix given by:

$$
m_{eff} = -m_D \frac{1}{M_R} m_D^T
\tag{4}
$$

This expression shows that the light neutrino masses are strongly suppressed with respect to the electroweak scale. There is no loss of generality in choosing a WB where m_l is real diagonal and positive. The diagonalization of $\mathcal{M}$ is performed via the unitary transformation:

$$
V^T \mathcal{M}^* V = \mathcal{D}
\tag{5}
$$

where $\mathcal{D} = \mathrm{diag}(m_1, m_2, m_3, M_1, M_2, M_3)$, with m_i and M_i denoting the physical masses of the light and heavy Majorana neutrinos, respectively. It is convenient to write V and $\mathcal{D}$ in the following block form:

$$
V = \begin{pmatrix} K & G \\ S & T \end{pmatrix} ; \qquad \mathcal{D} = \begin{pmatrix} d & 0 \\ 0 & D \end{pmatrix} .
\tag{6}
$$

The neutrino weak-eigenstates are then related to the mass eigenstates by:

$$
\nu_{iL}^0 = V_{i\alpha} \nu_{\alpha L} = (K, G) \begin{pmatrix} \nu_{iL} \\ N_{iL} \end{pmatrix} \qquad \begin{pmatrix} i = 1, 2, 3 \\ \alpha = 1, 2, \ldots 6 \end{pmatrix}
\tag{7}
$$

and the leptonic charged current interactions are given by:

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}}\left(\overline{l_{iL}}\gamma_\mu K_{ij}\nu_{jL} + \overline{l_{iL}}\gamma_\mu G_{ij}N_{jL}\right)W^\mu + h.c. \tag{8}$$

with K and G being the charged current couplings of charged leptons to the light neutrinos ν_j and to the heavy neutrinos N_j, respectively.

In the seesaw limit the matrix K coincides to an excellent approximation with the unitary matrix U_ν that diagonalises m_{eff} of Eq. (4):

$$-U_\nu^\dagger m_D \frac{1}{M_R} m_D^T U_\nu^* = d \tag{9}$$

and the matrix G verifies the exact relation:

$$G = m_D T^* D^{-1} \tag{10}$$

and is therefore very suppressed.

In a general framework, with M symmetric, without the zero block present in Eq. (2) the 3×6 physical matrix (K, G) of the 6×6 unitary matrix V would depend on six independent mixing angles and twelve independent CP violating phases.[11] This would be possible with a further extention of the SM including a Higgs triplet. The presence of the zero block reduces the number of independent CP violating phases to six.[12] In the seesaw framework massive neutrinos lead to the possibility of CP violation in the leptonic sector both at low and at high energies. CP violation at high energies manifests itself in the decays of heavy neutrinos and is sensitive to phases appearing in the matrix G.

3. Low Energy Leptonic Physics

The light neutrino masses are obtained from the diagonalisation of m_{eff} defined by Eq. (4) which is an effective Majorana mass matrix. The unitary matrix U_ν that diagonalises m_{eff} in the WB where the charged lepton masses are already diagonal real and positive is known as the Pontecorvo, Maki, Nakagawa, Sakata (PMNS) matrix,[13] and can be parametrised as:[14]

$$U_\nu = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \cdot P \tag{11}$$

with $P = \mathrm{diag}(1, e^{i\alpha}, e^{i\beta})$, α and β are phases associated to the Majorana character of neutrinos. There are three CP violating phases in U_ν.

Experimentally it is not yet known whether any of the three CP violating phases of the leptonic sector is different from zero. The current experimental bounds on neutrino masses and leptonic mixing are:[14]

$$\Delta m^2_{21} = 8.0^{+0.4}_{-0.3} \times 10^{-5}\ \mathrm{eV}^2 \tag{12}$$
$$\sin^2(2\theta_{12}) = 0.86^{+0.03}_{-0.04} \tag{13}$$
$$|\Delta m^2_{32}| = (1.9 \text{ to } 3.0) \times 10^{-3}\ \mathrm{eV}^2 \tag{14}$$
$$\sin^2(2\theta_{23}) > 0.92 \tag{15}$$
$$\sin^2\theta_{13} < 0.05 \tag{16}$$

with $\Delta m^2_{ij} \equiv m^2_j - m^2_i$. The angle θ_{23} may be maximal, meaning 45°, whilst θ_{12} is already known to deviate from this value. At the moment, there is only an experimental upper bound on the angle θ_{13}.

It is also not yet known whether the ordering of the light neutrino masses is normal, i.e., $m_1 < m_2 < m_3$ or inverted $m_3 < m_1 < m_2$. The scale of the neutrino masses is also not yet established. Direct kinematical limits from Mainz[15] and Troitsk[16] place an upper bound on m_β defined as:

$$m_\beta \equiv \sqrt{\sum_i |U_{ei}|^2 m_i^2} \tag{17}$$

given by $m_\beta \leq 2.3$ eV (Mainz), $m_\beta \leq 2.2$ eV (Troitsk). The forthcoming KATRIN experiment[17] is expected to be sensitive to $m_\beta > 0.2$ eV and to start taking data in 2010.[18]

It is possible to obtain information on the absolute scale of neutrino masses from the study of the cosmic microwave radiation spectrum together with the study of the large scale structure of the universe. For a flat universe, WMAP combined with other astronomical data leads to[19] $\sum_i m_i \leq 0.66$ eV (95% CL).

Neutrinoless double beta decay can also provide information on the absolute scale of the neutrino masses. In the present framework, in the absence of additional lepton number violating interactions, it provides a measurement of the effective Majorana mass given by:

$$m_{ee} = \left| m_1 U^2_{e1} + m_2 U^2_{e2} + m_3 U^2_{e3} \right| \tag{18}$$

The present upper limit is $m_{ee} \leq 0.9$ eV[20] from the Heidelberg-Moskow[21] and the IGEX[22] experiments. There is a claim of discovery of neutrinoless double beta decay by the Heidelberg-Moscow collaboration.[23] Interpreted in terms of a Majorana mass of the neutrino, this implies m_{ee} between 0.12 eV to 0.90 eV. This result awaits confirmation from other experiments and would constitute a major discovery.

It was shown that the strength of CP violation at low energies, observable for example through neutrino oscillations can be obtained from the following low energy WB invariant:[24]

$$Tr[h_{eff}, h_l]^3 = -6i\,\Delta_{21}\Delta_{32}\Delta_{31}\,\mathrm{Im}\{(h_{eff})_{12}(h_{eff})_{23}(h_{eff})_{31}\} \quad (19)$$

where $h_{eff} = m_{eff} m_{eff}^{\dagger}$, $h_l = m_l m_l^{\dagger}$, and $\Delta_{21} = (m_{\mu}^2 - m_e^2)$ with analogous expressions for Δ_{31}, Δ_{32}. The righthand side of this equation is the computation of this invariant in the special WB where the charged lepton masses are real and diagonal. In the case of no CP violation of Dirac type in the leptonic sector this WB invariant vanishes; on the other hand, it is not sensitive to the presence of Majorana phases. This quantity can be computed in any WB and therefore is extremely useful for model building since it enables one to investigate whether a specific ansatz leads to Dirac type CP violation or not, without the need to go to the physical basis. It is also possible to write WB invariant conditions sensitive to the Majorana phases. The general procedure was outlined in Ref. 25 where it was applied to the quark sector. For three generations it was shown that the following four conditions are sufficient [24] to guarantee CP invariance:

$$\mathrm{Im}\,\mathrm{tr}\left[h_l (m_{eff}^* m_{eff})(m_{eff}^* h_l^* m_{eff})\right] = 0 \quad (20)$$

$$\mathrm{Im}\,\mathrm{tr}\left[h_l (m_{eff}^* m_{eff})^2 (m_{eff}^* h_l^* m_{eff})\right] = 0 \quad (21)$$

$$\mathrm{Im}\,\mathrm{tr}\left[h_l (m_{eff}^* m_{eff})^2 (m_{eff}^* h_l^* m_{eff})(m_{eff}^* m_{eff})\right] = 0 \quad (22)$$

$$\mathrm{Im}\,\det\left[(m_{eff}^* h_l m_{eff}) + (h_l^* m_{eff}^* m_{eff})\right] = 0 \quad (23)$$

provided that neutrino masses are nonzero and nondegenerate (see also Ref. 26). In Ref. 27 alternative WB invariant conditions necessary to guarantee CP invariance in the leptonic sector under less general circumstances are given.

4. Leptogenesis

The observed baryon asymmetry of the universe (BAU) is given by:[28]

$$\frac{n_B - n_{\overline{B}}}{n_\gamma} = (6.1^{+0.3}_{-0.2}) \times 10^{-10} \, . \quad (24)$$

It is already established that this observation requires physics beyond the SM in order to be explained. One of the most plausibe explanations is Leptogenesis[6] where out-of-equilibrium L-violating decays of heavy Majorana neutrinos generate a lepton asymmetry which is partially converted through sphaleron processes[29] into a baryon asymmetry. The lepton number asymmetry ε_{N_j}, thus produced was computed by several authors.[30–34] Summing over all charged leptons one obtains for the asymmetry produced by the decay of the heavy Majorana neutrino N_j into the charged leptons $l_i^{\pm}$ ($i = e, \mu, \tau$):

$$
\begin{aligned}
\varepsilon_{N_j} &= \\
&= \frac{g^2}{M_W^2} \sum_{k \neq j} \left[\mathrm{Im} \left((m_D^{\dagger} m_D)_{jk} (m_D^{\dagger} m_D)_{jk} \right) \frac{1}{16\pi} \left(I(x_k) + \frac{\sqrt{x_k}}{1 - x_k} \right) \right] \frac{1}{(m_D^{\dagger} m_D)_{jj}} = \\
&= \frac{g^2}{M_W^2} \sum_{k \neq j} \left[(M_k)^2 \, \mathrm{Im} \left((G^{\dagger} G)_{jk} (G^{\dagger} G)_{jk} \right) \frac{1}{16\pi} \left(I(x_k) + \frac{\sqrt{x_k}}{1 - x_k} \right) \right] \frac{1}{(G^{\dagger} G)_{jj}}
\end{aligned}
\tag{25}
$$

where M_k denote the heavy neutrino masses, the variable x_k is defined as $x_k = \frac{M_k^2}{M_j^2}$ and $I(x_k) = \sqrt{x_k} \left(1 + (1 + x_k) \log\left(\frac{x_k}{1 + x_k}\right) \right)$. From Equation (25) it can be seen that, when one sums over all charged leptons, the lepton-number asymmetry is only sensitive to the CP-violating phases appearing in $m_D^{\dagger} m_D$ in the WB, where M_R is diagonal. Weak basis invariants relevant for leptogenesis were derived in:[9]

$$I_1 \equiv \mathrm{Im\,Tr}[h_D H_R M_R^{*} h_D^{*} M_R] = 0 \tag{26}$$

$$I_2 \equiv \mathrm{Im\,Tr}[h_D H_R^2 M_R^{*} h_D^{*} M_R] = 0 \tag{27}$$

$$I_3 \equiv \mathrm{Im\,Tr}[h_D H_R^2 M_R^{*} h_D^{*} M_R H_R] = 0 \tag{28}$$

with $h_D = m_D^{\dagger} m_D$ and $H_R = M_R^{\dagger} M_R$. These constitute a set of necessary and sufficient conditions in the case of three heavy neutrinos. See also.[33]

The simplest realisation of thermal leptogenesis consists of having hierarchical heavy neutrinos. In this case there is a lower bound for the mass of the lightest of the heavy neutrinos.[35,36] Depending on the cosmological scenario, the range for minimal M_1 varies from order 10^7 Gev to 10^9 Gev.[37,38] Furthermore, an upper bound on the light neutrino masses is obtained in order for leptogenesis to be viable. With the assumption that washout effects are not sensitive to the different flavours of charged leptons into which the heavy neutrino decays this bound is approximately 0.1 ev.[39–42] However, it was recently pointed out[43–51] that there are cases where flavour matters and the commonly used expressions for the lepton asymmetry, which depend on the total CP asymmetry and one single efficiency factor, may fail to reproduce the correct lepton asymmetry. In this cases, the calculation of the baryon asymmetry with hierarchical righthanded neutrinos must take into consideration flavour dependent washout processes. As a result, in this case, the previous upper limit on the light neutrino masses does not survive and leptogenesis can be made viable with neutrino masses reaching the cosmological bound of $\sum_i m_i \leq 0.66$ eV. The lower bound on M_1 does not move much with the inclusion of flavour effects. Flavour effects bring new sources of CP violation to leptogenesis and the possibility of having a common origin for CP violation at low energies and for leptogenesis.[52–55]

There are very interesting alternative scenarios to the minimal leptogenesis scenario briefly mentioned here. It was pointed out at this conference that an SU(2)-singlet neutrino with a keV mass is a viable dark matter candidate.[56] Some leptogenesis scenarios are compatible with much lower heavy neutrino masses than the values required for minimal leptogenesis.

5. Implications from Zero neutrino Yukawa Textures

The general seesaw framework contains a large number of free parameters. The introduction of zero textures and/or the reduction of the number of righthanded neutrinos to two, allows to reduce the number of parameters. In this work only zero textures imposed in the fundamental leptonic mass matrices are considered and, in particular, zero textures of the Dirac neutrino mass matrix, m_D in the WB where M_R and m_l are real and diagonal. Zero textures of the low energy effective neutrino mass matrix are also very interesting phenomenologically.[57] The physical meaning of the zero textures that appear in most of the leptonic mass ansatze was analysed in a recent work[58] where it is shown that some leptonic zero texture ansatze can be obtained from WB transformations and therefore have no physical meaning.

In general, zero textures reduce the number of CP violating phases, as a result some sets of zero textures imply the vanishing of certain CP-odd WB invariants.[59] This is an important fact since clearly zero textures are not WB invariant, therefore in a different WB the zeros may not be present making it difficult to recognise the ansatz. Furthermore, it was also shown[59] that starting from arbitrary leptonic mass matrices, the vanishing of certain CP-odd WB invariants, together with the assumption of no conspiracy among the parameters of the Dirac and Majorana mass terms, one is automatically lead to given sets of zero textures in a particular WB.

Frampton, Glashow and Yanagida have shown[60] that it is possible to uniquely relate the sign of the baryon number of the Universe to CP violation in neutrino oscillation experiments by imposing two zeros in m_D, in the seesaw framework with only two righthanded neutrino components. Two examples were given by these authors:

$$m_D = \begin{pmatrix} a & 0 \\ a' & b \\ 0 & b' \end{pmatrix} \quad \text{or} \quad m_D = \begin{pmatrix} a & 0 \\ 0 & b \\ a' & b' \end{pmatrix} \tag{29}$$

The two zeros in m_D eliminate two CP violating phases, so that only one CP violating phase remains. This is the most economical extension of the standard model leading to leptogenesis and at the same time allowing for low energy CP violation. Imposing that the model accommodates the experimental facts at low energy strongly constrains its parameters.

In Ref. 61 minimal scenarios for leptogenesis and CP violation at low energies were analysed in some specific realisations of seesaw models with three righthanded neutrinos and four zero textures in m_D, where three of the zeros are in the upper triangular part of the matrix. This latter particular feature was motivated by the fact that there is no loss of generality in parametrising m_D as:

$$m_D = U Y_4 \; ; \tag{30}$$

with U a unitary matrix and Y_4 a lower triangular matrix, i.e.:

$$Y_4 = \begin{pmatrix} y_{11} & 0 & 0 \\ y_{21} e^{i\phi_{21}} & y_{22} & 0 \\ y_{31} e^{i\phi_{31}} & y_{32} e^{i\phi_{32}} & y_{33} \end{pmatrix} ; \tag{31}$$

where y_{ij} are real positive numbers. Choosing $U = 1$ reduces the number of parameters in m_D. Moreover, U cancels out in the combination $m_D^\dagger m_D$ relevant in the case of unflavoured leptogenesis, whilst it does not cancel in m_{eff}. Therefore choosing $U = 1$ allows for a connection between low energy CP violation and leptogenesis to be established since in this case the same phases affect both phenomena. The nonzero entries of m_D were written in terms of powers of a small parameter a la Frogatt Nielsen[62] and chosen in such a way as to accommodate the experimental data. Viable leptogenesis was found requiring the existence of low energy CP violating effects within the range of sensitivity of the future long baseline neutrino oscillation experiments under consideration.

In order to understand how the connection between CP violation required for leptogenesis and low energy physics is established in the presence of zeros in the matrix m_D, the following relation derived from Eq. (9) in the WB where M_R and m_l are real positive and diagonal is important:

$$m_D = iU\sqrt{d}R\sqrt{D_R} \tag{32}$$

with R an orthogonal complex matrix, $\sqrt{D_R}$ a diagonal real matrix verifying the relation $\sqrt{D_R}\sqrt{D_R} = D_R$ and $\sqrt{d}$ a real matrix with a maximum number of zeros such that $\sqrt{d}\sqrt{d}^T = d$. This is the well known Casas and Ibarra parametrisation.[63] From this equation it follows that:

$$m_D^\dagger m_D = \sqrt{D_R}R^\dagger\sqrt{d}^T\sqrt{d}R\sqrt{D_R} \tag{33}$$

Since the CP violating phases relevant for leptogenesis in the unflavoured case are those contained in $m_D^\dagger m_D$, it is clear that leptogenesis can occur even if there is no CP violation at low energies i.e. no Majorana- or Dirac-type CP phases at low energies.[10] Unflavoured leptogenesis requires the matrix R to be complex. In flavoured leptogenesis the separate lepton i family asymmetry generated from the decay of the kth heavy Majorana neutrino depends on the combination[45] $\text{Im}\left[(m_D^\dagger m_D)_{kk'}(m_D^*)_{ik}(m_D)_{ik'}\right]$ as well as on $\text{Im}\left[(m_D^\dagger m_D)_{k'k}(m_D^*)_{ik}(m_D)_{ik'}\right]$. The matrix U does not cancel in each of these terms and it was shown that it is possible to have viable leptogenesis even in the case of real R, with CP violation in the PMNS matrix as the source of CP violation required for leptogenesis.

From Eq. (32) it is clear that one zero in $(m_D)_{ij}$ corresponds to having an orthogonality relation between the ith row of the matrix $U\sqrt{d}$ and the jth column of the matrix R:

$$(m_D)_{ij} = 0 \;:\quad (U)_{ik}\sqrt{d}_{kl} R_{lj} = 0 \tag{34}$$

Ibarra and Ross[64] showed that, in the seesaw case with only two righthanded neutrinos, a single zero texture, has the special feature of fixing the matrix R, up to a reflection, without imposing any further restriction on light neutrino masses and mixing. The predictions from models with two zero textures in m_D were also analysed in detail in their work, including the constraints on leptogenesis and lepton flavour violating processes. The number of all different two texture zeros is fifteen. Two zeros imply two simultaneous conditions of the type given by Eq. (34). Compatibility of these two conditions implies restrictions on U and $\sqrt{m_i}$. Only five of these cases turned out to be allowed experimentally, including the two cases of Eq. (29) in this reference.

All of these two zero texture ansatze satify the following WB invariant condition:[59]

$$I_1 \equiv \text{tr}\left[m_D M_R^\dagger M_R m_D^\dagger, h_l\right]^3 = 0 \tag{35}$$

with $h_l = m_l m_l^\dagger$, as before. It was also shown[59] that for arbitrary complex leptonic mass matrices, assuming that there are no special relations among the entries of M_R and those of m_D this condition automatically leads to one of the two zero anzatze classified in Ref. 64. The assumption that M_R and m_D are not related to each other is quite natural, since m_D and M_R originate from different terms of the Lagrangian.

There are other CP-odd WB invariants which vanish for all of the two zero textures just mentioned, even if they arise in a basis where M_R is not diagonal. An example is the following WB invariant condition:[59]

$$I^0 \equiv \mathrm{tr}\left[m_D m_D^\dagger ; h_1\right]^3 = 0 \qquad (36)$$

which is verified for any texture with two zeros in m_D in a WB where m_1 is diagonal, while M_R is arbitrary.

The case of zero textures with three righthanded neutrinos was also considered in Ref 59. In this case the WB invariant I_1 always vanishes for three zero textures in m_D with two orthogonal rows, which implies that one row has no zeros. The case of three zeros corresponding to two orthogonal columns of m_D, which in this case implies that one column has no zeros leads to the vanishing of a new invariant I_2, defined by:

$$I_2 \equiv \mathrm{tr}\left[M_R^\dagger M_R ; m_D^\dagger m_D\right]^3 \qquad (37)$$

Four zero textures in the context of seesaw with three righthanded neutrinos are studied in detail in Ref. 65. It is shown that four is the maximum number of zeros in textures compatible with the observed leptonic mixing and with the additional requirement that none of the neutrino masses vanishes. It is also shown that such textures lead to important constraints both at low and high energies, and allow for a tight connection between leptogenesis and low energy parameters. It is possible in all cases to completely specify the matrix R in terms of light neutrino masses and the PMNS matrix. These relations are explicitly given in Ref. 65.

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