K_{e4} decays and Wigner cusp

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1   Introduction

The single-flavour quark condensate $\langle 0 \,|\bar{q}q|\, 0\rangle$ is a fundamental parameter of χPT, determining the relative size of mass and momentum terms in the expansion. Since it can not be predicted theoretically, its value must be determined experimentally, e.g. by measuring the $\pi\pi$ scattering lengths, whose values are predicted very precisely within the framework of χPT, assuming a big quark condensate [1], or of generalised χPT, where the quark condensate is a free parameter [2].

The K_{e4}^{+-} decay is a very clean environment for the measurement of $\pi\pi$ scattering lengths, since the two pions are the only hadrons and they are produced close to threshold. The only theoretical uncertainty enters through the constraint [3] between the scattering lengths a_0^2 and a_0^0. In the $K^\pm \to \pi^0\pi^0\pi^\pm$ decay a cusp-like structure can be observed at $M_{00}^2 = 4m_{\pi^+}^2$, due to re-scattering from $K^\pm \to \pi^+\pi^-\pi^\pm$. The scattering lengths can be extracted from a fit of the M_{00}^2 distribution around the discontinuity.

2   Experimental setup

Simultaneous K^+ and K^- beams were produced by 400 GeV energy protons from the CERN SPS, impinging on a beryllium target. The kaons were deflected in a front-end achromat in order to select the momentum band of (60 ± 3) GeV/c and focused at the beginning of the detector, about 200 m downstream. For the measurements presented here, the most important detector components are the magnet spectrometer, consisting of two drift chambers before and two after a dipole magnet and the quasi-homogeneous liquid krypton electromagnetic calorimeter. The momentum of

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the charged particles and the energy of the photons are measured with a relative uncertainty of 1% at 20 GeV. A detailed description of the NA48/2 detector can be found in Ref. [4].

3 $K^\pm \to \pi^+\pi^- e^\pm \nu_e$

The K_{e4}^{+-} selection consisted of geometrical criteria, like the requirement of having three tracks within the detector acceptance and building a good vertex; particle identification requirements, based mainly on the different fraction of energy deposited by pions and electrons in the electromagnetic calorimeter; kinematical cuts for background rejection, like an elliptical cut in the $(p_T, M_{3\pi})$ plane centered at $(0, M_K)$. In order to improve the pion rejection, the electron identification also included a Linear Discriminant Analysis combining the three quantities with the highest discriminating power. Two reconstruction strategies can be applied to the K_{e4}^{+-} events: either imposing the kaon mass and extracting the kaon momentum from a quadratic equation, or imposing the kaon momentum to be the mean beam momentum (60 GeV/c along the beam axis) and extracting the kaon mass from a linear equation (see Fig. 1).
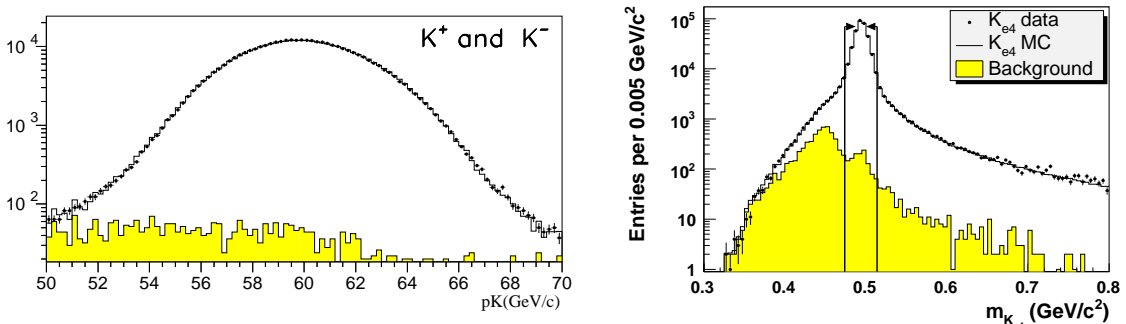

Figure 1: Kaon momentum (left) and mass (right) of the K_{e4}^{+-} events reconstructed with a quadratic or a linear equation, respectively. The data (crosses) are compared to signal MC (open histogram) plus background (yellow).

Analysing part of the 2003 data, 3.7×10^5 K_{e4}^{+-} events were selected with a background contamination below 1%. The background level was estimated from data, using the so-called "wrong sign" events, i.e. with the signature $\pi^\pm\pi^\pm e^\mp \nu_e$, that, at the present statistical level, can only be background, since the corresponding kaon decay violates the $\Delta S = \Delta Q$ rule and is therefore strongly suppressed [5]. The main background contributions are due to $K^\pm \to \pi^+\pi^-\pi^\pm$ events with $\pi \to e\nu$ or a pion mis-identified as an electron. The background estimate from data was cross-checked using Monte Carlo simulation (MC).
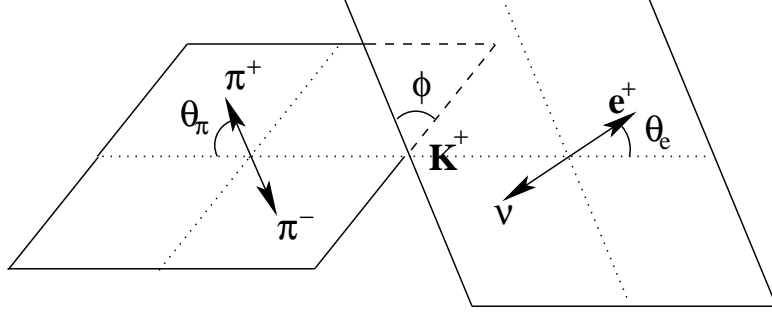
3.1 Form factors

Figure 2: Topology of the K_{e4} decay.

The form factors of the K_{e4}^{+-} decay are parametrised as a function of five kinematic variables [6] (see Fig. 2): the invariant masses $M_{\pi\pi}$ and $M_{e\nu}$ and the angles θ_π, θ_e and ϕ. The matrix element

$$T = \frac{G_F}{\sqrt{2}} V_{us}^* \bar{u}(p_\nu)\gamma_\mu(1 - \gamma_5)v(p_e)(V^\mu - A^\mu)$$

contains a hadronic part, that can be described using two axial (F and G) and one vector (H) form factors [7]. After expanding them into partial waves and into a Taylor series in $q^2 = M_{\pi\pi}^2/4m_{\pi+}^2 - 1$, the following parametrisation was used to determine the form factors from the experimental data [8, 9]:

$$
\begin{aligned}
F &= (f_s + f_s'q^2 + f_s''q^4)e^{i\delta_0^0(q^2)} + f_p\cos\theta_\pi e^{i\delta_1^1(q^2)} \\
G &= (g_p + g_p'q^2)e^{i\delta_1^1(q^2)} \\
H &= h_p e^{i\delta_1^1(q^2)}.
\end{aligned}
$$

In a first step, ten independent five-parameter fits were performed for each bin in $M_{\pi\pi}$, comparing data and MC in four-dimensional histograms in $M_{e\nu}$, $\cos\theta_\pi$, $\cos\theta_e$ and ϕ, with 1500 equal population bins each. The second step consisted in a fit of the distributions in $M_{\pi\pi}$ (see Figs. 3,4), to extract the (constant) form factor parameters.

The polynomial expansion in q^2 was truncated according to the experimental sensitivity. The dependence on $M_{e\nu}$ and the D-wave were found to be negligible within the total uncertainty and the corresponding parameters were therefore set to zero. The $\delta = \delta_0^0 - \delta_1^1$ distribution was fitted with a one-parameter function given by the numerical solution of the Roy equations [3], in order to determine a_0^0, while a_0^2 was constrained to lie on the centre of the universal band. The following preliminary result was obtained:

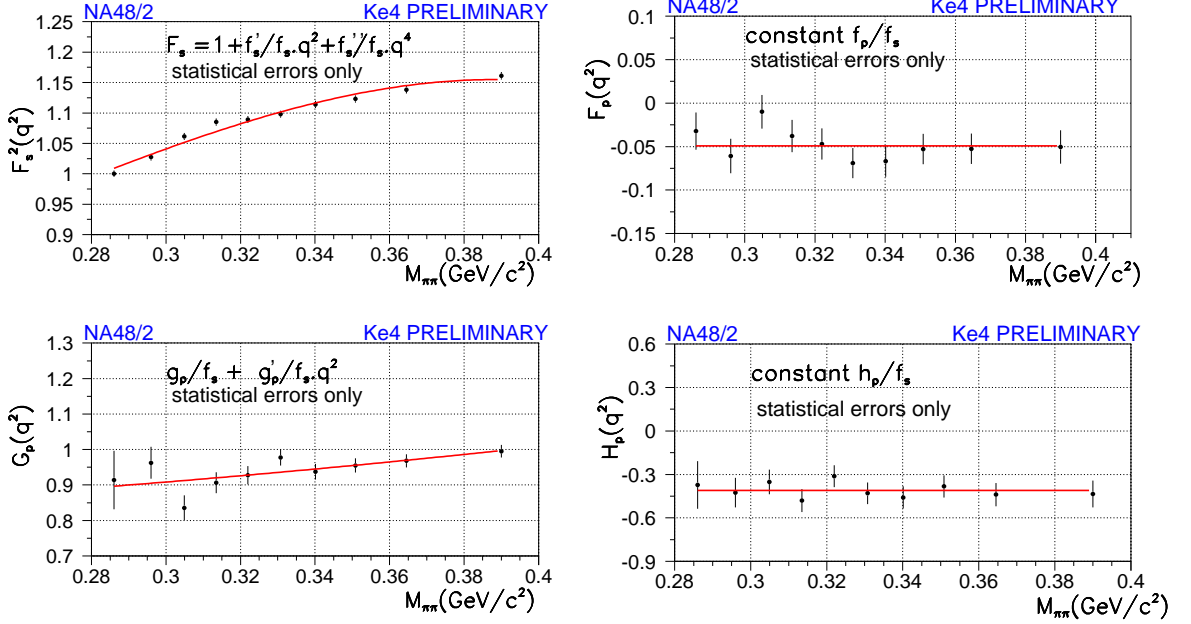$$f_s'/f_s = 0.169 \pm 0.009_{stat} \pm 0.034_{syst}$$

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Figure 3: F, G and H dependence on $M_{\pi\pi}$. The points represent the results of the first-step fits, the lines are fitted in the second step.

$$
\begin{aligned}
f_s''/f_s &= -0.091 \pm 0.009_{stat} \pm 0.031_{syst} \\
f_p/f_s &= -0.047 \pm 0.006_{stat} \pm 0.008_{syst} \\
g_p/f_s &= 0.891 \pm 0.019_{stat} \pm 0.020_{syst} \\
g_p'/f_s &= 0.111 \pm 0.031_{stat} \pm 0.032_{syst} \\
h_p/f_s &= -0.411 \pm 0.027_{stat} \pm 0.038_{syst} \\
a_0^0 &= 0.256 \pm 0.008_{stat} \pm 0.007_{syst} \pm 0.018_{theor},
\end{aligned}
$$

where the systematic uncertainty was determined by comparing two independent analyses and taking into account the effect of reconstruction method, acceptance, fit method, uncertainty on background estimate, electron-ID efficiency, radiative corrections and bias due to the neglected $M_{e\nu}$ dependence. The form factors are measured relative to f_s, which is related to the decay rate. The obtained value for a_0^0 is compatible with the χPT prediction $a_0^0 = 0.220 \pm 0.005$ [10] and with previous measurements [11, 12].

4 $K^{\pm} \rightarrow \pi^0\pi^0 e^{\pm}\nu_e$

About 10,000 K_{e4}^{00} events were selected from the 2003 data and about 30,000 from the 2004 data with a background contamination of 3% and 2%, respectively. The
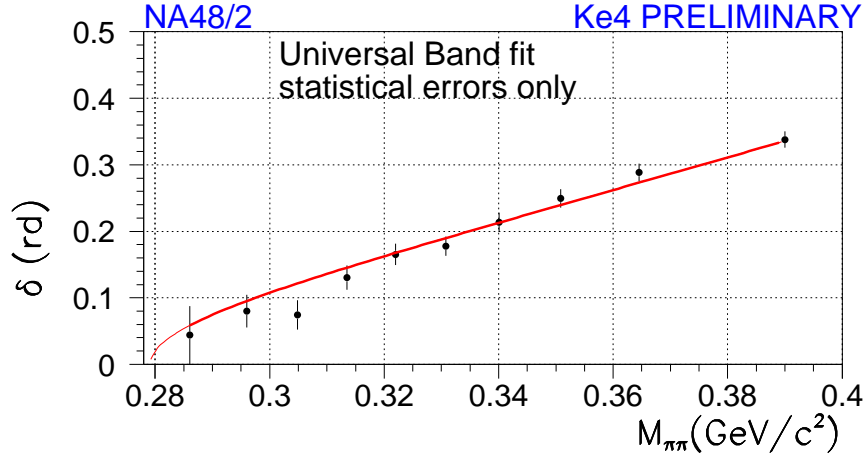
Figure 4: $\delta = \delta_0^0 - \delta_1^1$ distribution as a function of $M_{\pi\pi}$. The points represent the results of the first-step fits, the line is fitted in the second step.

selection criteria were similar to the ones used for the K_{e4}^{+-} events, apart from the requirement of containing one track and 4 photons compatible with two π^0s at the same vertex. The electron identification was based on the fraction of energy deposited in the electromagnetic calorimeter and on the width of the corresponding shower. The background level was estimated from data by reversing some of the selection criteria and was found to be mainly due to $K^\pm \to \pi^0\pi^0\pi^\pm$ events with a pion mis-identified as an electron (see Fig. 5).
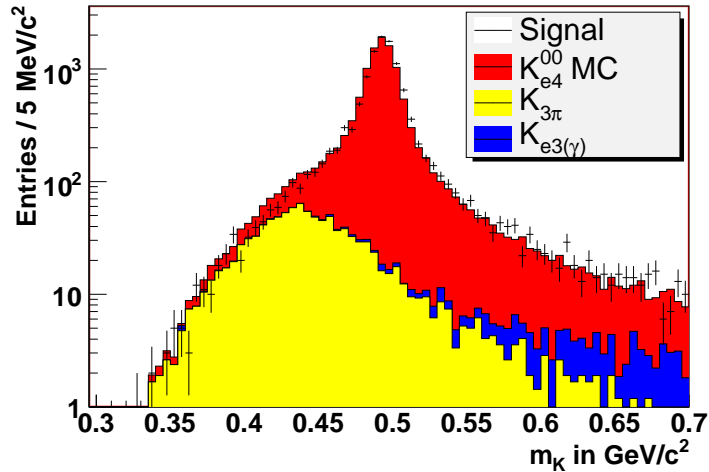

Figure 5: Invariant mass distribution in logarithmic scale of the K_{e4}^{00} events selected from the 2003 data (crosses) compared to the signal MC (red) plus physical (yellow) and accidental (blue) background.

The branching fraction was measured, as a preliminary result from the 2003 data

5

only, normalised to $K^\pm \to \pi^0\pi^0\pi^\pm$:

$$BR(K_{e4}^{00}) = (2.587 \pm 0.026_{stat} \pm 0.019_{syst} \pm 0.029_{ext}) \times 10^{-5},$$

where the systematic uncertainty takes into account the effect of acceptance, trigger efficiency and energy measurement of the calorimeter, while the external uncertainty is due to the uncertainty on the $K^\pm \to \pi^0\pi^0\pi^\pm$ branching fraction. This result is about eight times more precise than the best previous measurement [13].

For the form factors the same formalism is used as in K_{e4}^{+-}, but, due to the symmetry of the $\pi^0\pi^0$ system, the P-wave is missing and only two parameters are left: f_s'/f_s and f_s''/f_s. Using the full data sample, the following preliminary result was obtained:

$$f_s'/f_s = 0.129 \pm 0.036_{stat} \pm 0.020_{syst}$$
$$f_s''/f_s = -0.040 \pm 0.034_{stat} \pm 0.020_{syst},$$

which is compatible with the K_{e4}^{+-} result (see Fig. 6).
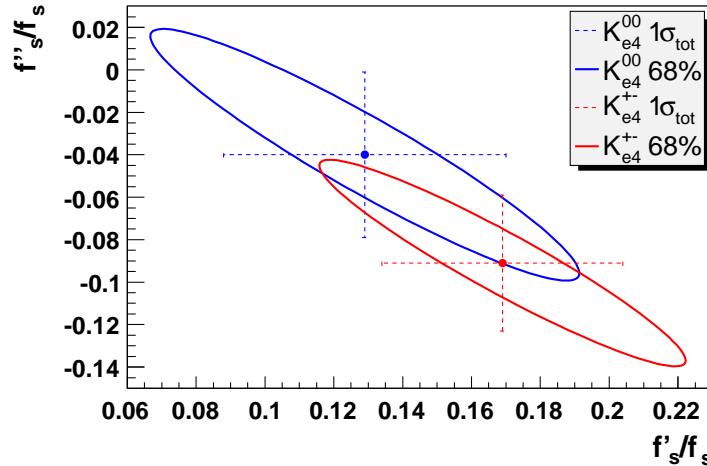

Figure 6: Comparison of the f_s'/f_s and f_s''/f_s measurements in K_{e4}^{+-} and K_{e4}^{00}.

5 $K^\pm \to \pi^0\pi^0\pi^\pm$

From 2003 data, about 23 million $K^\pm \to \pi^0\pi^0\pi^\pm$ events were selected, with negligible background. The squared invariant mass of the $\pi^0\pi^0$ system (M_{00}^2) was computed imposing the mean vertex of the π^0s, in order to improve its resolution close to threshold. At $M_{00}^2 = 4m_{\pi^+}^2$, the distribution shows evidence for a cusp-like structure (see Fig. 7, left) due to $\pi\pi$ re-scattering.
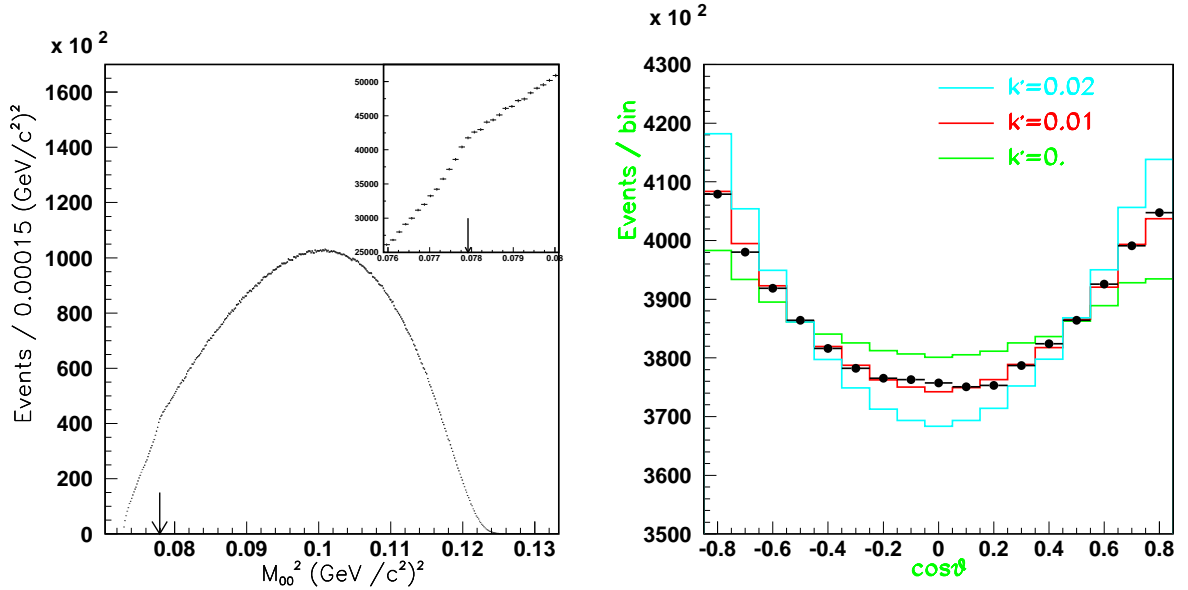
Figure 7: Left: M_{00}^2 of the selection $K^\pm \to \pi^0\pi^0\pi^\pm$ data events. The arrow indicates the position of the cusp. Right: angle between the $\pi^\pm$ and the π^0 in the $\pi^0\pi^0$ centre of mass system. The points represent the data, the three curves, the MC distribution for different values of k'

Fitting the distribution with the theoretical model presented in Ref. [14] and using the unperturbed matrix element

$$M_0 = A_0(1 + \tfrac{1}{2}g_0 u + \tfrac{1}{2}h'u^2 + \tfrac{1}{2}k'v^2),$$

the following result was obtained [15], assuming $k' = 0$ [16]:

$$
\begin{aligned}
g_0 &= 0.645 \pm 0.004_{stat} \pm 0.009_{syst} \\
h' &= -0.047 \pm 0.012_{stat} \pm 0.011_{syst} \\
a_2 &= -0.041 \pm 0.022_{stat} \pm 0.014_{syst} \\
a_0 - a_2 &= 0.268 \pm 0.010_{stat} \pm 0.004_{syst} \pm 0.013_{theor},
\end{aligned}
$$

where the $a_0 - a_2$ measurement is dominated by the uncertainty on the theoretical model.

In a further analysis, the value of k' was obtained from a fit above the cusp in the plane $\cos\theta$ vs M_{00}^2, where θ is the angle between the π^+ and the π^0 in the $\pi^0\pi^0$ centre of mass system. Evidence was found for a non-zero value of k' (see Fig. 7, right):

$$k' = 0.0097 \pm 0.0003_{stat} \pm 0.0008_{syst},$$

where the systematic uncertainty takes into account the effect of acceptance and trigger efficiency. Reweighting the MC with the obtained value of k', the standard fit of the M_{00}^2 distribution with the Cabibbo-Isidori model was performed to obtain the cusp parameters, that were found to be compatible with the published values.

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