

Signals of non-extensive statistical mechanics in high-energy nuclear collisions

W.M. Alberico^{1,2}, P. Czerski³, A. Lavagno^{4,2}, M. Nardi^{5,2}, V. Somá¹

¹*Dipartimento di Fisica Teorica, Università di Torino,*

Via P. Giuria 1, I-10126 Torino, Italy

²*Istituto Nazionale di Fisica Nucleare, Sezione di Torino*

³*The Henryk Niewodniczanski Institute of Nuclear Physics,*

Polish Academy of Sciences, Krakow, Poland

⁴*Dipartimento di Fisica, Politecnico di Torino,*

C.so Duca degli Abruzzi 24, I-10129 Torino, Italy

⁵*Centro Studi e Ricerche E.Fermi, Compendio Viminale 00184 Roma*

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Abstract

Starting from the presence of non-ideal plasma effects due to strongly coupled plasma in the early stage of relativistic heavy-ion collisions, we investigate, from a phenomenological point of view, the relevance of non-conventional statistical mechanics effects on the rapidity spectra of net proton yield at AGS, SPS and RHIC. We show that the broad rapidity shape measured at RHIC can be very well reproduced in the framework of a non-linear relativistic Fokker-Planck equation which incorporates non-extensive statistics and anomalous diffusion.

1 Introduction

It is expected that hadrons dissociate into a plasma of their elementary constituents, quarks and gluons (QGP), at a density several times the nuclear

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matter density and/or at temperatures of the order of $T_c = 170$ MeV, which is the critical temperature of the transition from the hadronic gas phase to the QGP phase, as expected from lattice QCD calculations. In central Au+Au collisions at RHIC energy densities are reached that are far above the critical energy density (of the order of $1 \text{ GeV}/\text{fm}^3$) for a transition to the QGP phase.

Since interactions among quarks and gluons become weak at small distance or high energy, one usually expects that QGP is a weakly interacting, ideal plasma, which can be described by perturbative QCD. However, this is rigorously true only at very high temperature, while non-perturbative phenomena prevail up to temperatures of several times T_c [1]. For example, as it is known from several approaches, partons in a hot plasma acquire a temperature dependent Debye mass, which deeply modifies the QCD dynamics, yet inducing collective effects. Hence the final states in an ultrarelativistic nucleus-nucleus collision can be very sensibly affected by non-ideal plasma effects, including long-range interactions and memory effects. Many properties concerning the formation of the expected new phase and the consequent hadronization process are still under debate [2, 3, 4, 5]. Pre-thermalization or metastability conditions can be realized and the standard equilibrium statistical mechanics assumptions can not be taken for granted in the description of the system toward equilibrium [6].

In this paper, we want to investigate the relevance of such non-ideal and non-equilibrium plasma effects within a phenomenological study of the net-baryon rapidity distribution in various relativistic heavy ion collisions, up to the central Au+Au collisions at the highest RHIC energy of $\sqrt{s_{NN}}=200$ GeV. The interest of this observable lies in the fact that in these processes nuclear matter reaches high energy densities and nuclei undergoing central collisions strongly reduce their original longitudinal momentum. This loss of rapidity, usually referred to as baryon stopping, is an important characteristic to understand the reaction mechanism at high energy density and the net rapidity distribution is very sensitive to the dynamical and statistical properties of nucleus-nucleus at high energy.

We study the evolution of the rapidity distribution from a macroscopic point of view by using a non-linear relativistic Fokker-Planck equation and we will show that the observed broad rapidity shape could be a signal of non-equilibrium properties of the system in which non-extensive Tsallis statistical mechanics [7, 8] emerges in a natural way. Similar approaches have been considered in the past to analyze transverse momentum distributions [2, 9, 10,

11, 12] and power law spectra at large p_{\perp} in terms of various non conventional extensions of the Boltzmann-Gibbs thermostatics, including the Tsallis one.

The net-proton rapidity distribution appears to be a more elusive observable with respect to equilibrium considerations, since the main component is expected to be related to the initial beam protons; however the results obtained at RHIC do not appear easily reproduced within standard approaches. Encouraging results have been obtained by Wolschin [13, 14, 15] within a three-components Relativistic Diffusion Model (RDM). Here again a possible link with Tsallis statistics is suggested [14].

The paper is structured as follows. In Sec. 2, we qualitatively review some basic features of a non-ideal QGP like the one hypothetically generated in the relativistic heavy-ion collisions at AGS, SPS and RHIC energies. We also show that the main assumptions contained in the derivation of the standard dynamical kinetic equations, describing the evolution of the system toward the equilibrium, are no longer valid. In Sec. 3, we introduce a non-linear relativistic kinetic equation containing anomalous diffusion effects in the framework of the non-extensive Tsallis thermostatics. In Sec. 4, we study the net proton rapidity spectra comparing the AGS, SPS and RHIC data. Conclusions are reported in Sec. 5.

2 Non-ideal QGP plasmas and kinetic assumptions towards the equilibrium

In the literature, an ordinary plasma is usually characterized by the value of the plasma parameter Γ [16]

$$\Gamma = \frac{\langle U \rangle}{\langle T \rangle}, \quad (1)$$

defined as the ratio between potential energy $\langle U \rangle$ versus kinetic energy $\langle T \rangle$. When $\Gamma \ll 1$, one has a dilute weakly interacting gas; the Debye screening length λ_D is much greater than the average interparticle distance r_0 and a large number of particles is contained in the Debye sphere. Binary collisions induced by screened forces produce, in the classical case, the standard Maxwell-Boltzmann velocity distribution. If $\Gamma \approx 0.1 \div 1$, then $\lambda_D \approx r_0$, and it is not possible to clearly separate individual and collective degrees

of freedom: this situation refers to a weakly interacting, non-ideal plasma. Finally, if $\Gamma \geq 1$, the plasma is strongly interacting, Coulomb interaction and quantum effects dominate and determine the structure of the system.

The quark-gluon plasma close to the critical temperature is a strongly interacting system. In fact, following Ref.[2, 3, 4], the color-Coulomb coupling parameter of the QGP is defined, in analogy with the one of the classical plasma, as

$$\Gamma \approx C \frac{g^2}{4\pi r_0 T}, \quad (2)$$

where $C = 4/3$ or 3 is the Casimir invariant for the quarks or gluons, respectively; for typical temperatures attained in relativistic heavy ion collisions, $T \simeq 200$ MeV, $\alpha_s = g^2/(4\pi) = 0.2 \div 0.5$, and $r_0 \simeq n^{-1/3} \simeq 0.5$ fm (n being the particle density for an ideal gas of 2 quark flavors in QGP). Consequently, one obtains $\Gamma \simeq 1.5 - 5$ and the plasma can be considered to be in a non-ideal liquid phase [3, 4].

In these conditions, the generated QGP does not satisfy anymore the basic assumptions (BBGKY hierarchy) of a kinetic equation (Boltzmann or Fokker-Planck equation) which describes a system toward the equilibrium. In fact, near the phase transition the interaction range is much larger than the Debye screening length and a small number of partons is contained in the Debye sphere [2, 4]. Therefore, the collision time is not much smaller than the mean time between collisions and the interaction is not local. The binary collisions approximation is not satisfied, memory effects and long-range color interactions give rise to the presence of non-Markovian processes in the kinetic equation, thus affecting the thermalization process toward equilibrium as well as the standard equilibrium distribution. In the next section we will see how such effects can be taken into account, from a macroscopic point of view, by means of an appropriate generalization of the standard statistical mechanics.

3 Relativistic non-extensive kinetic equations

In many-body long-range-interacting systems, it has been recently observed the emergence of long-living quasi stationary (metastable) states characterized by non-Gaussian power law velocity distributions, before the Boltzmann-Gibbs equilibrium is attained. In this case, the standard statistical mechanics

is no longer appropriate to describe such a behavior [17, 18, 19]. Indeed, the basic assumption of the Boltzmann-Gibbs statistical mechanics is that the system can be subdivided into a set of non-overlapping subsystems, the total entropy of which is the sum of the entropies of the independent subsystems. In the presence of memory effects and long range forces the entropy, which is a measure of the information about the particle distribution in the states available to the system, does not satisfy the extensivity property.

Recently, there is an increasing evidence that the generalized non-extensive statistical mechanics, proposed by Tsallis [7, 8], can be considered as an appropriate basis for a theoretical framework to describe physical phenomena where long-range interactions, long-range microscopic memories and/or fractal space-time constraints are present. A considerable variety of physical issues show a quantitative agreement between experimental data and theoretical models based on Tsallis' thermostatistics. In particular, there is a growing interest to high energy physics applications of non-extensive statistics. Several authors outline the possibility that experimental observations in relativistic heavy-ion collisions can reflect non-extensive statistical mechanics effects during the early stage of the collisions and the thermalization evolution of the system [2, 10, 12, 20, 21, 22, 23].

In order to study from a phenomenological point of view experimental observables in relativistic heavy-ion collisions, we can introduce the basic macroscopic variables in the language of relativistic kinetic theory following the Tsallis' prescriptions for the non-extensive statistical mechanics. In this framework the particle (four-vector) flow can be generalized as [24]

$$N^\mu(x) = \frac{1}{Z_q} \int \frac{d^3p}{p^0} p^\mu f(x, p), \quad (3)$$

and the energy-momentum flow as

$$T^{\mu\nu}(x) = \frac{1}{Z_q} \int \frac{d^3p}{p^0} p^\mu p^\nu [f(x, p)]^q, \quad (4)$$

where we have set $\hbar = c = 1$, $x \equiv x^\mu = (t, \mathbf{x})$, $p \equiv p^\mu = (p^0, \mathbf{p})$ and $p^0 = \sqrt{\mathbf{p}^2 + m^2}$ is the relativistic energy. In the above $Z_q = \int d\Omega [f(x, p)]^q$ is the non-extensive partition function, $d\Omega$ stands for the corresponding phase space volume element and q is the deformation parameter. The limit $q \rightarrow 1$ corresponds to ordinary statistics. The four-vector $N^\mu = (n, \mathbf{j})$ contains the probability density $n = n(x)$ (which is normalized to unity) and the probability flow $\mathbf{j} = \mathbf{j}(x)$. The energy-momentum tensor contains the normalized

q -mean expectation value¹ of the energy density, as well as the energy flow, the momentum and the momentum flow per particle.

On the basis of the above definitions, one can show that it is possible to obtain a generalized non-linear relativistic Boltzmann equation [24]

$$p^\mu \partial_\mu [f(x, p)]^q = C_q(x, p) , \quad (6)$$

where the function $C_q(x, p)$ implicitly defines a generalized non-extensive collision term

$$C_q(x, p) = \frac{1}{2} \int \frac{d^3 p_1}{p_1^0} \frac{d^3 p'}{p'^0} \frac{d^3 p'_1}{p'^0_1} \left\{ h_q[f', f'_1] W(p', p'_1 | p, p_1) - h_q[f, f_1] W(p, p_1 | p', p'_1) \right\} . \quad (7)$$

Here $W(p, p_1 | p', p'_1)$ is the transition rate between a two-particle state with initial four-momenta p and p_1 and a final state with four-momenta p' and p'_1 ; $h_q[f, f_1]$ is the q -correlation function relative to two particles in the same space-time position but with different four-momenta p and p_1 , respectively. Such a transport equation conserves the probability normalization (number of particles) and is consistent with the energy-momentum conservation laws. The collision term contains a generalized expression of the molecular chaos and for $q > 0$ implies the validity of a generalized H -theorem, if the following, non-extensive, local four-density entropy is assumed

$$S_q^\mu(x) = -k_B \int \frac{d^3 p}{p^0} p^\mu f[(x, p)]^q [\ln_q f(x, p) - 1] , \quad (8)$$

where we have used the definition $\ln_q x = (x^{1-q} - 1)/(1 - q)$. At equilibrium, the solution of the above Boltzmann equation is a relativistic Tsallis-like (power law) distribution and can be written as

$$f^{eq}(p) = \frac{1}{Z_q} \left[1 - (1 - q) \frac{p^\mu U_\mu}{k_B T} \right]^{1/(1-q)} . \quad (9)$$

In the limit $q \rightarrow 1$, Eq.(9) reduces to the standard relativistic equilibrium Jüttner distribution.

The above relations represent the basic framework in which, in the next section, will be studied the net-baryon rapidity distribution near equilibrium.

¹The q -mean expectation value is defined as [8, 24]:

$$\langle O(x) \rangle_q = \int d^3 p O(x, p) [f(x, p)]^q / \int d\Omega [f(x, p)]^q . \quad (5)$$

4 Net-baryon rapidity distribution

The energy loss of colliding nuclei is a fundamental quantity in order to determine the energy available for particle production in heavy-ion collisions. Since the baryon number is conserved and rapidity distributions are only affected by rescattering in the late stages of the collision, the measured net-baryon ($B - \bar{B}$) distribution retains information about the energy loss and allows one to obtain the degree of nuclear stopping.

Recent results for net-proton rapidity spectra in central Au+Au collisions at RHIC [25] show an unexpectedly large rapidity density at midrapidity in comparison with analogous spectra at lower energy at SPS [26] and AGS [27]. As outlined from different authors, such spectra can reflect non-equilibrium effects even if the energy dependence of the rapidity spectra is not very well understood [25, 28].

We are going to study the evolution of the rapidity distribution from a macroscopic point of view by means of a non-linear relativistic Fokker-Planck equation which can be viewed as the near-equilibrium approximation of the generalized non-linear Boltzmann equation (6). Therefore, the use of the Fokker-Planck equation appears to be correct only not too far from equilibrium [13, 14, 15, 22, 30, 31, 32, 33, 34].

In order to study the rapidity spectra, it is convenient to separate the kinetic variables into their transverse and longitudinal components, the latter being related to the rapidity y . If we assume that the particle distribution function $f(y, m_{\perp}, t)$, at fixed transverse mass $m_{\perp} = \sqrt{m^2 + p_{\perp}^2}$, is not appreciably influenced by the transverse dynamics (which is considered in thermal equilibrium), the non-linear Fokker-Planck equation in the rapidity space y can be written as

$$\frac{\partial}{\partial t}[f(y, m_{\perp}, t)] = \frac{\partial}{\partial y} \left[J(y, m_{\perp})[f(y, m_{\perp}, t)] + D \frac{\partial}{\partial y}[f(y, m_{\perp}, t)]^{\mu} \right], \quad (10)$$

where D and J are the diffusion and drift coefficients, respectively.

Tsallis and Bukman [29] have shown that, for linear drift, the time dependent solution of the above equation is a Tsallis (non-relativistic) distribution with $\mu = 2 - q$ and that a value of $q \neq 1$ implies anomalous diffusion, i.e., $[y(t) - y_M(t)]^2$ scales like t^{α} , with $\alpha = 2/(3 - q)$. For $q < 1$, the above equation implies anomalous subdiffusion, while for $q > 1$, we have a superdiffusion process in the rapidity space.

Let us note that a similar approach, within a linear and non-linear Fokker-Planck equation, has been previously studied in Ref.[13, 14, 15] involving directly the time evolution of the rapidity distribution instead of the particle distribution function $f(y, m_{\perp}, t)$, as in the Eq.(10). The two approaches are equivalent only if the particle distribution is completely decoupled in the transverse and in the longitudinal coordinates and the drift coefficient J does not depend on transverse momentum. This is the case, for example, when the drift coefficient is assumed to be linear in the rapidity space. In Ref.[13, 14, 15], the author first study a linear Fokker-Planck equation with a linear drift coefficient and a free parameter diffusion coefficient (connected to a collective longitudinal expansion) which implies a strong violation of the fluctuation-dissipation theorem. This approach fails to reproduce the rapidity spectra at RHIC energy unless it is assumed that a fraction of net protons, near midrapidity, undergoes a fast transition to local thermal equilibrium. The non-linear Fokker-Planck equation for the rapidity distribution function is also considered in Ref.[13, 14, 15], even if only approximate solutions are considered by means of a linear superposition of power-law distribution functions.

We claim that the choice of the diffusion and the drift coefficients plays a crucial rôle in the solution of the above non-linear Fokker-Planck equation (10). Such a choice influences the time evolution of the system and its equilibrium distribution. This important point has been extensively studied in Ref.[9] where the authors obtained a generalized fluctuation-dissipation theorem, which implies an implicit relation between the drift and diffusion coefficient in the framework of a linear non-relativistic Fokker-Planck equation.

We are going to show that by generalizing the Brownian motion in a relativistic framework, the standard Einstein relation is satisfied and Tsallis non-extensive statistics emerges in a natural way from the non-linearity of the Fokker-Planck equation. In fact, by imposing the validity of the Einstein relation for Brownian particles and setting henceforward the Boltzmann constant k_B to unity, we can generalize to the relativistic case the standard expressions of diffusion and drift coefficients as follows

$$D = \gamma T , \quad J(y, m_{\perp}) = \gamma m_{\perp} \sinh(y) \equiv \gamma p_{\parallel} , \quad (11)$$

where p_{\parallel} is the longitudinal momentum, T is the temperature and γ is a common constant. Let us remark that the above definition of the diffusion

and drift coefficients, previously introduced by us in Ref.[20], appears as the natural generalization to the relativistic Brownian case in the rapidity space. The drift coefficient which is, as usual, *linear* in the longitudinal momentum p_{\parallel} becomes *non-linear* in the rapidity coordinate.

It is easy to see that the above coefficients give us the Boltzmann stationary distribution in the linear case ($q = \mu = 1$), while the equilibrium solution $f^{eq}(y, m_{\perp})$ of Eq.(10), with $\mu = 2 - q$, is a Tsallis-like distribution (introduced in Eq.(9)) with the relativistic energy $E = m_{\perp} \cosh(y)$

$$f^{eq}(y, m_{\perp}) = \left[1 - (1 - q) m_{\perp} \cosh(y)/T \right]^{1/(1-q)}. \quad (12)$$

Out of equilibrium the rapidity distribution at fixed time can be obtained by means of numerical integration of Eq.(10) with delta function initial conditions depending upon the value of the experimental projectile rapidities. The rapidity distribution at fixed time is then obtained by numerical integration over the transverse mass m_{\perp} as follows

$$\frac{dN}{dy}(y, t) = c \int_m^{\infty} m_{\perp}^2 \cosh(y) f(y, m_{\perp}, t) dm_{\perp}, \quad (13)$$

where m is the mass of the considered particles and c is the normalization constant, fixed by the experimental data. The rapidity spectra calculated from Eq.(13) will ultimately depend on two parameters: the “interaction” time $\tau_{int} = \gamma t$ and the non-extensive parameter q .

Let us observe that in the numerical solution of Eq.(13) we have not explicitly assumed the presence of longitudinal flow. This fact does not exclude the dynamical effects of a collective (mainly due to rescattering) flow but rather incorporates a description of it in the adopted non-extensive statistical mechanics. In fact, as explicitly shown in Ref. [2] by studying the transverse mass spectrum, dynamical collective interactions are intrinsically involved in the generalized statistical mechanics and, in a purely thermal source, a generalized q -blue shift factor (depending on q and on m_{\perp}) appears. We expect a similar behavior also in the longitudinal degrees of freedom of the system. Let us also notice that collective transverse flow effects in the framework of a non-extensive statistical mechanics have been investigated in Ref. [9] as well.

In Figs. 1 and 2 we plot the numerical solution of Eq.s (10) and (13) at different interaction times τ_{int} in the linear case ($q = 1$) and in the non-linear case, with $q = 1.25$, keeping the same initial conditions. In comparing Fig. 1

with Fig. 2, we can observe that the rapidity spectra appear to be broader in the non-linear case. This is a consequence of the anomalous (super)diffusion process implied in the time evolution of Eq.(10) and in the nature of the power law distribution function which, for $q > 1$, enhances the probability to find particles at high rapidity values.

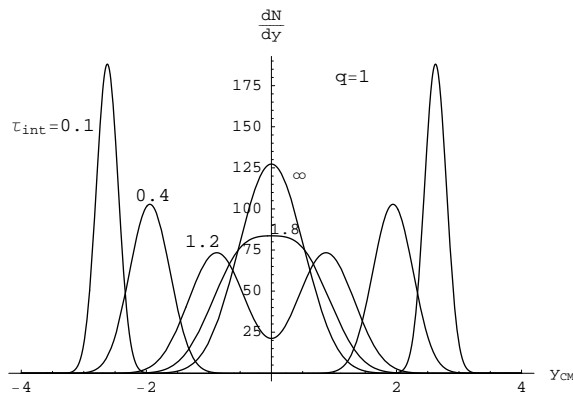


Figure 1: Numerical solution of Eq.s(10) and (13) at different interaction times τ_{int} in the linear case ($q = 1$).

In Fig. 3, we report the obtained rapidity distribution (full line) for the net proton production ($p - \bar{p}$) compared with the experimental data of RHIC (Au+Au at $\sqrt{s_{NN}} = 200$ GeV, [25]), SPS (Pb+Pb at $\sqrt{s_{NN}} = 17.3$ GeV, [26]) and AGS (Au+Au at $\sqrt{s_{NN}} = 5$ GeV, [27]). The parameters employed for the three curves are: $q = 1.485$ with $\tau_{int} = 0.47$ for RHIC, $q = 1.235$ with $\tau_{int} = 0.84$ for SPS and $q = 1.09$ with $\tau_{int} = 0.95$ for AGS, respectively. We notice that, although q and τ_{int} appear, in principle, as independent parameters, in fitting the data they are not. Indeed, we can see that only in the non-linear case ($q \neq 1$) there exists one and only one (finite) time τ_{int} for which the obtained rapidity spectrum well reproduces the broad experimental shape. On the contrary, for $q = 1$, no value of τ_{int} can be found, which allows to reproduce the data.

We obtain a remarkable agreement with the experimental data by increasing the value of the non-linear deformation parameter q as the beam energy

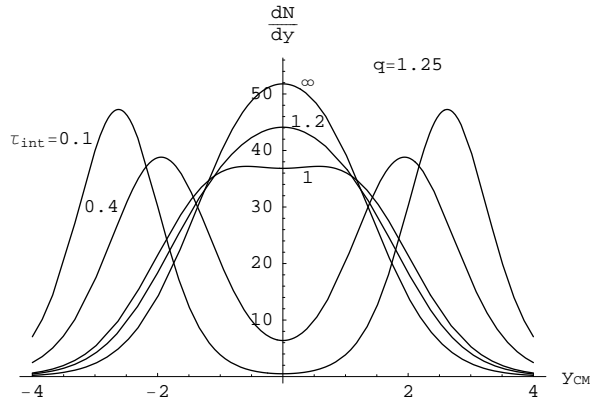


Figure 2: The same of Fig.1 but in the non-linear case with $q = 1.25$.

increases. At AGS energy, the non-extensive statistical effects are negligible and the spectrum is well reproduced within the standard quasi-equilibrium linear approach. At SPS energy, non-equilibrium effects and non-linear evolution become remarkable ($q = 1.235$) and such effects are even more evident for the very broad RHIC spectra ($q = 1.485$). Let us observe that such an excellent agreement with the RHIC experimental data has not been reached in the similar (but different, as pointed out in the introduction) approach of Ref.[13, 14, 15], especially in reproducing the experimental points far from the midrapidity region.

From a phenomenological point of view, we can read the larger value of the parameter q for the RHIC data, corresponding to non-linear anomalous (super)diffusion, as a signal of the non-ideal nature of the plasma formed at a temperature larger than the critical one. As confirmed by recent microscopic calculations [3, 4], strongly coupled non-ideal plasma is generated at energy densities corresponding to the order of the critical phase transition temperature and in such a regime we find, in our macroscopic approach, strong deviations from the standard thermostatics. At much higher energy, such as LHC, we can expect a minor relevance of such non-ideal effects since the considerable energy density reached is far above the critical one. However, an anomalous diffusion behavior should still affect the time evolution process toward the equilibrium due to long range color magnetic forces which remain

unscreened (in leading order) at all temperatures.

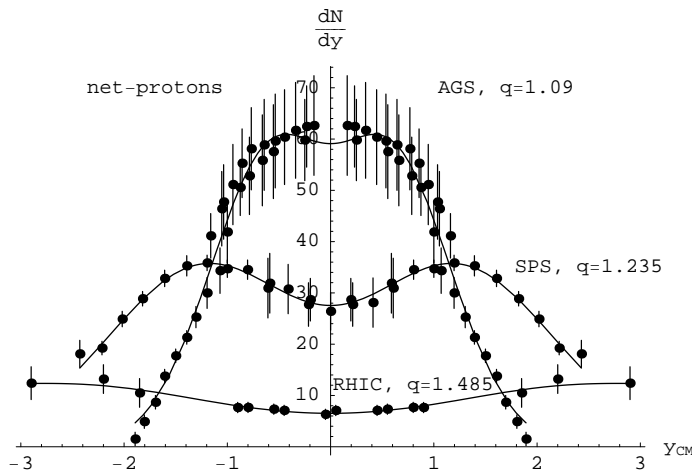


Figure 3: Rapidity spectra for net proton production ($p-\bar{p}$) at RHIC (Au+Au at $\sqrt{s_{NN}} = 200$ GeV, BRAHMS data), SPS (Pb+Pb at $\sqrt{s_{NN}} = 17.3$ GeV, NA49 data) and AGS (Au+Au at $\sqrt{s_{NN}} = 5$ GeV, E802, E877, E917).

5 Conclusions

We have studied the rapidity spectra from a macroscopic point of view by means of a non-linear kinetic equation which preserves the fluctuation-dissipation theorem by introducing an appropriate relativistic generalization of the drift and diffusion coefficients. Such a generalized evolution equation lies inside the framework of Tsallis' non-extensive thermostatistics and contains anomalous diffusion effects which are strongly related to the presence of non-Markovian memory interactions and long-range color forces. Multi-particle rescattering, very large at SPS and at RHIC, could be a signal of this effects. We recall that non-extensive features in relativistic collisions are suggested in several works, even if a microscopic justification of these effects is still lacking and lies out of the scope of this paper. The relevant point which was found in the present work is that the larger the beam energy of

the heavy ions, the greater are the values of the deformation parameter q , which are required to reproduce the experimental rapidity spectra. This behavior of the q -parameter can be viewed as a phenomenological indication of the onset of a strongly coupled QGP plasma phase in the early stages of the collision.

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