

CP violation in top pair production
at an e^+e^- collider

Darwin Chang^(1,2), Wai-Yee Keung^(3,4)

Ivan Phillips⁽¹⁾

⁽¹⁾*Department of Physics and Astronomy,*

Northwestern University, Evanston, IL 60208, USA.

⁽²⁾*Institute of Physics, Academia Sinica, Taipei, Taiwan, R.O.C.*

⁽³⁾*Physics Department, University of Illinois at Chicago, IL 60680, USA*

⁽⁴⁾*Theory Division, CERN, CH-1211, Geneva 23, Switzerland*

Abstract

We investigate a possible CP violating effect in e^+e^- annihilation into $t\bar{t}$ top quark pairs. As an illustrative example, we assume the source of the CP nonconservation is in the Yukawa couplings of a neutral Higgs boson which contain both scalar and pseudoscalar pieces. One of the interesting observable effects is the difference in production rates between the two CP conjugate polarized $t\bar{t}$ states.

Since the top quark is widely believed to be within the reach of the present collider machines, it is not unreasonable for theorists to imagine what we can learn from the top quark. The best place to study the top quark in detail is in an e^+e^- collider. One of the facts one would like to learn from the discovery of the top quark is the origin of the still mysterious CP violation. In this paper we investigate a way CP violation can manifest itself in the top pair production of an e^+e^- collider.

Among the various mechanisms of CP violation the one that may manifest itself most easily is the neutral Higgs mediated CP violation. Since the neutral Higgs couplings are typically proportional to the quark mass, the large mass of the top quark naturally gives large couplings to the neutral Higgs bosons. In addition, the top quark, due to its short lifetime, is widely believed to decay before it hadronizes. Therefore the information about its polarization may be preserved in its decay products. If that is the case, then one can investigate the source of CP nonconservation by measuring the CP violating observable involving a polarized top pair in the final state. This idea of detecting the rate asymmetry between different polarized states was recently pursued in Refs. [1,2]. For $t\bar{t}$ production through the virtual photonic or Z intermediate states, to the lowest order in the final state quark mass, the polarizations of the quarks are either $t_L\bar{t}_R$ or $t_R\bar{t}_L$. (Note that we have adopted the notation that \bar{t}_L is the antiparticle of t_R and should be left handed.) These two modes are CP self-conjugate. However since the top quark is heavy, there will also be large percentage of $t_L\bar{t}_L$ and $t_R\bar{t}_R$ modes which are CP conjugates of each other. Therefore one can consider the CP asymmetry in the event rate difference, $N(t_L\bar{t}_L) - N(t_R\bar{t}_R)$.

For detection of the asymmetry $N(t_L\bar{t}_L) - N(t_R\bar{t}_R)$ [1,2], one assumes that the t quark decays semileptonically through the usual $V - A$ weak interaction. Assuming that the hadronization time is much longer than the decay time [3], one can analyze polarization dependence of its decay at the quark level. The top quark first decays into a b quark and a W^+ boson, which subsequently becomes $l^+\nu$. For heavy top quark, the W^+ boson produced in top decay is predominantly longitudinal. Due to the $V - A$ interaction, the b quark is preferentially produced with left-handed helicity. So the longitudinal W^+ boson

is preferentially produced along the direction of the top quark polarization. Therefore the anti-lepton l^+ produced in the W^+ decay is also preferentially in that direction. In the rest frame of the t , the angular distribution [4] of the produced l^+ has the form $1 + \cos \psi$, with ψ as the angle between l^+ and the helicity axis of the t . Above the $t\bar{t}$ threshold, the top quark is produced with nonzero momentum. As a result of the Lorentz boost, the anti-lepton l^+ produced in the decay of the right handed top quark t_R has a higher energy than that produced in the decay of the left handed top quark t_L . Similarly, the lepton l produced in the decay of \bar{t}_L has a higher energy than that produced in the decay of \bar{t}_R . Consequently, in the decay of the pair $t_L\bar{t}_L$ the lepton from \bar{t}_L has a higher energy than the anti-lepton from t_L ; while in the decay of $t_R\bar{t}_R$ the anti-lepton has a higher energy. Therefore one can observe $N(t_L\bar{t}_L) - N(t_R\bar{t}_R)$ by measuring the energy asymmetry in the resulting leptons [5].

In order to generate the asymmetries $N(t_L\bar{t}_L) - N(t_R\bar{t}_R)$ it is necessary to include effects of the final state interactions in order to escape from the hermiticity constraint at the tree-level due to the CPT theorem. In our case both CP violation and the final state effect are produced by the same one-loop graphs.

CP non-conservation occurs in the complex Yukawa coupling,

$$\mathcal{L}_{CPX} = -(m_t/v)\bar{t}(AP_L + A^*P_R)tH + BH(m_Z^2/v)Z^\nu Z_\nu. \quad (1)$$

Here $v = (\sqrt{2}G_F)^{-\frac{1}{2}} \simeq 246$ GeV. The complex coefficient A is a combination of model-dependent mixing angles. Simultaneous presence of both the real part $A_R = \text{Re}A$ and the imaginary part $A_I = \text{Im}A$ guarantees CP asymmetry. For example, in the low energy regime, it can give rise to the electric dipole moment of elementary particles [6,7]. Here we will show that CP nonconservation manifests itself in the event rate difference in collider experiments.

The vertex amplitude $ie\Gamma^j$ for the virtual γ^* or Z^* turning into $t(p)$ and $\bar{t}(p')$ is parametrized in the following expression:

$$\Gamma_\mu^j = c_v^j \gamma_\mu + c_a^j \gamma_\mu \gamma_5 + c_d^j i \gamma_5 \frac{p_\mu - p'_\mu}{2m_t} + \dots, \quad j = \gamma, Z. \quad (2)$$

We use the tree-level values for c_v and c_a . They are

$$\begin{aligned} c_v^\gamma &= \frac{2}{3}, & c_a^\gamma &= 0, \\ c_v^Z &= (\frac{1}{4} - \frac{2}{3}x_W)/\sqrt{x_W(1-x_W)}, \\ c_a^Z &= -\frac{1}{4}/\sqrt{x_W(1-x_W)}. \end{aligned} \quad (3)$$

The c_d term is the the electric dipole moment factor. The spinor structure can be recast into another familiar form $\sigma_{\mu\nu}(p+p')^\nu\gamma_5/2m_t$. It is induced at the one-loop level as shown in the Fig. 1. We are interested in the absorptive parts which can be easily calculated according to the Cutkosky rules. It can also be easily shown that, in the limit that the electron mass is ignored, $\text{Im}c_d^j$ are the only relevant one loop form factors for the CP violating quantities we are interested in.

The leading contribution to $\text{Im}c_d^\gamma$ comes from the rescattering of the top quark pair through the Higgs-boson exchange.

$$\text{Im}c_d^\gamma = c_v^\gamma \left(\frac{m_t}{v}\right)^2 \frac{A_R A_I t^2}{2\pi\beta} \left(1 - \frac{h^2}{\beta^2} \log\left(1 + \frac{\beta^2}{h^2}\right)\right). \quad (4)$$

The dimensionless variables are defined by, $t = m_t/\sqrt{s}$, $z = m_Z/\sqrt{s}$, and $h = m_H/\sqrt{s}$. For $\text{Im}c_d^Z$, there is a similar contribution. In addition, there could be a contribution due to the ZH intermediate state, Fig 1b, provided the kinematics is allowed.

$$\text{Im}c_d^Z = \frac{c_v^Z}{c_v^\gamma} \text{Im}c_d^\gamma - \frac{\alpha A_I B c_v^Z t^2}{2(1-x_W)x_W\beta^3} [\beta\beta_Z + (2t^2 + 2t^2h^2 - 2t^2z^2 - h^2)L]. \quad (5)$$

Here $\beta_Z^2 = 1 + h^4 + z^4 - 2z^2 - 2h^2 - 2h^2z^2$, $\beta^2 = 1 - 4t^2$, and the logarithmic factor $L = \log(L_-/L_+)$ with $L_\pm = 1 - z^2 - h^2 \pm \beta\beta_Z$. Other irrelevant terms, like the magnetic moments, are not listed in Eq.(2). Note that there is no CP violating contribution due to c_a^Z coupling in the limit that the electron mass is ignored. Our expression in Eq.(5) agrees with that in Ref. [8,9]. The amplitudes for the process $e^+e^- \rightarrow t\bar{t}$ of different helicities have been given in the literature [10]. Now, we can obtain the explicit CP asymmetry in the difference of the production rates,

$$\delta \equiv [N(t_L\bar{t}_L) - N(t_R\bar{t}_R)]/N(t\bar{t}; \text{all})$$

$$= \frac{\sum_{i=L,R} 2\beta[(1-z^2)c_v^\gamma + r_i c_v^Z][(1-z^2)\text{Im}c_d^\gamma + r_i \text{Im}c_d^Z]}{\sum_{i=L,R} (3-\beta^2)[(1-z^2)c_v^\gamma + r_i c_v^Z]^2 + 2\beta^2 r_i^2 c_a^{Z^2}}. \quad (6)$$

Here $r_i = (\frac{1}{2}\delta_{i,L} - x_W)/\sqrt{x_W(1-x_W)}$, which is the Z -coupling of the electron of different chiralities. Typical values of δ are shown in Fig. 2.

We can make use of this asymmetry parameter δ to illustrate the the difference in the energy distributions of l^+ or l^- from the t or \bar{t} decays. The energy $E_0(l^+)$ distribution of a static t quark decay $t \rightarrow l^+ \nu b$ is very simple [4] in the narrow width Γ_W approximation when m_b is negligible.

$$f(x_0) = \begin{cases} x_0(1-x_0)/D & \text{if } m_W^2/m_t^2 < x < 1, \\ 0 & \text{otherwise.} \end{cases} \quad (7)$$

Here we denote the scaling variable $x_0 = 2E_0(l^+)/m_t$ and the normalization factor $D = \frac{1}{6} - \frac{1}{2}(m_W/m_t)^4 + \frac{1}{3}(m_W/m_t)^6$. When the t quark is not static, but moves at a speed β with helicity L or R , the distribution expression becomes a convolution,

$$f_{R,L}(x, \beta) = \int_{x/(1+\beta)}^{x/(1-\beta)} f(x_0) \frac{\beta x_0 \pm (x-x_0)}{2x_0^2 \beta^2} dx_0. \quad (8)$$

Here $x = 2E(l^+)/E_t$. Similar distributions for the \bar{t} decay is related by CP conjugation at the tree-level. Using the polarization asymmetry formula in Eq.(6), we can derive an expression for the difference in the energy distributions of l^- and l^+ :

$$\frac{1}{N} \left[\frac{dN}{dx(l^+)} - \frac{dN}{dx(l^-)} \right] = \delta [f_L(x, \beta) - f_R(x, \beta)]. \quad (9)$$

Here distributions are compared at the same energy for the lepton and the anti-lepton, $x(l^-) = x(l^+) = x = 4E(l^\pm)/\sqrt{s}$. The count N includes events with prompt leptons or anti-leptons from the top pair production. It is useful to compare Eq.(9) with that of the overall energy distribution,

$$\begin{aligned} \frac{1}{N} \left[\frac{dN}{dx(l^-)} + \frac{dN}{dx(l^+)} \right] &= \frac{\sum_{i=L,R} 4\beta r_i c_a^Z [(1-z^2)c_v^\gamma + r_i c_v^Z] [f_R(x, \beta) - f_L(x, \beta)]}{\sum_{i=L,R} (3-\beta^2)[(1-z^2)c_v^\gamma + r_i c_v^Z]^2 + 2\beta^2 r_i^2 c_a^{Z^2}} \\ &+ f_L(x, \beta) + f_R(x, \beta). \end{aligned} \quad (10)$$

Here we only keep the dominant tree-level contribution. Fig. 3 shows the overall prompt lepton energy distribution of Eq.(10), and the ratio of the expressions in Eq.(9) and Eq.(10).

In conclusion, we have shown that the CP violation in top pair production in e^+e^- annihilation can be at the level of 10^{-3} . Experiments can measure the asymmetry in the energy distributions of prompt leptons and anti-leptons. An e^+e^- machine of high luminosity is needed to accumulate several million prompt lepton events from the $t\bar{t}$ production in order to search for the CP nonconservation.

D.C. wishes to thank the Theory Group at the Institute of Physics at Academia Sinica in Taipei, Taiwan and Department of Physics and Astronomy at University of Hawaii at Manoa for hospitality while this work was in progress. This work is supported by grants from Department of Energy and from National Science Council of Republic of China.

REFERENCES

- [1] C.R. Schmidt and M. E. Peskin, Phys. Rev. Lett. **69**, 410 (1992).
- [2] D. Chang and W.-Y. Keung, Northwestern preprint, NUHEP-TH-92-14 (1992).
- [3] I. Bigi and H. Krasemann, Z. Phys. **C7**, 127 (1981); J. Kühn, Acta Phys. Austr. Suppl. **XXIV**, 203 (1982); I. Bigi, Y. Dokshitzer, V. Khoze, J. Kühn, and P. Zerwas, Phys. Lett. **181B**, 157 (1986).
- [4] M. Jezabek and J.H. Kühn, Nucl. Phys. **B320**, 20, (1989).
- [5] Additional CP-odd observables in collider physics have been studied by G. Kane, G.A. Ladinsky, and C.-P. Yuan, Phys. Rev. **D45**, 124 (1991); J.F. Donoghue and G. Valencia, Phys. Rev. Lett. **58**, 451 (1987); W. Bernreuther and O. Nachtmann, Phys. Rev. Lett. **63**, 2787 (1989); G. Valencia and A. Soni, Phys. Lett. **B263**, 517 (1991); A. Bilal, E. Massó, and A. De Rújula, Nucl. Phys. **355**, 549 (1991); D. Atwood and A. Soni, Brookhaven Report BNL-46892 (1991).
- [6] S. Weinberg, Phys. Rev. Lett. **63**, 2333 (1989); Phys. Rev. **D42**, 860 (1990).
- [7] S. Barr and A. Zee, Phys. Rev. Lett. **65**, 21 (1990); See also J. Gunion and R. Vega, Phys. Lett. **251B**, 21 (1990); D. Chang, W.-Y. Keung, and T.C. Yuan, Phys. Lett. **251B**, 608 (1990); D. Chang, W.-Y. Keung, and T.C. Yuan, Phys. Rev. **D43**, R14 (1991); R. Leigh, S. Paban, and R. Xu, Nucl. Phys. **B352**, 45 (1991).
- [8] W. Bernreuther, T. Schröder, and T.N. Pham, Phys. Lett. **B279**, 389 (1992).
- [9] W. Bernreuther, J.P. Ma, and T. Schröder, Heidelberg preprint, HD-THEP-92-30 (1992).
- [10] G. Kane, G.A. Ladinsky, and C.-P. Yuan, Phys. Rev. **D45**, 124 (1991). G

FIGURES

FIG. 1. Feynman diagrams for the process $e^+e^- \rightarrow t\bar{t}$. The tree amplitude interferes with those one-loop amplitudes with (a) the final state interactions due to the exchange of a Higgs boson, or (b) the intermediate state production of the ZH bosons.

FIG. 2. $[N(t_L\bar{t}_L) - N(t_R\bar{t}_R)]/N(\text{all } t\bar{t})$ versus \sqrt{s} in the solid (dotted, dashed) curve for the case that $m_H = 50$ (150,200) GeV and $m_t = 100$ GeV. The parameters are chosen to be $A_I = A_R = B = 1$.

FIG. 3. The energy distributions of prompt leptons, for the case that $m_t = 120$ GeV, $m_H = 50$ GeV, $\sqrt{s} = 300$ GeV, and $A_I = A_R = B = 1$. Case (a) for $N^{-1}[dN/dx(l^+) + dN/dx(l^-)]$, and case (b) for $[dN/dx(l^+) - dN/dx(l^-)]/[dN/dx(l^+) + dN/dx(l^-)]$.