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# MODELLING OPERATIONAL RISK MEASUREMENT IN ISLAMIC BANKING: A THEORETICAL AND EMPIRICAL INVESTIGATION

## **HYLMUN IZHAR**

A Doctoral Thesis submitted in fulfilment of the requirement for the Degree of Doctor of Philosophy at School of Government and International Affairs, Durham University, United Kingdom

# I dedicate this PhD Thesis to my late Mum and Dad, Suhartiningsih and Imam Mahdi

For having established a comprehensive set of essential foundations for my life

(May Allah SWT bless their souls and grant them paradise)

## Modelling Operational Risk Measurement in Islamic Banking: A Theoretical and Empirical Investigation

## Hylmun Izhar

#### **Abstract:**

With the emergence and development of Islamic banking industry, the need to cater operational risks issues has attracted the attention of academics in recent years. Such studies commonly agree that operational risk is relatively higher and serious than credit risk and market risk for Islamic banks. However, there is not any single research in the context of Islamic banking which thoroughly tackles the issue of operational risks by tackling it in three main aspects: theoretical, methodological, and empirical. This may be due to the fact that operational risk is relatively new area, which requires further research to understand the complexities it carries. This is the sources of motivation for the research, which aims to fill this observed gap in the literature by responding to the mentioned three aspects.

This research, hence, aims to develop a new measurement model of operational risk exposures in Islamic banking with the objective of theoretically determining the underlying features of operational risk exposures and its measurement particularly for Islamic banks.

In its attempt to develop a theoretical framework of the proposed model, the research provides a classification of operational risks in major Islamic financial contracts. In addition, rather than adopting the existing operational risk measurement methods, this research develops a proposed measurement model attributed as *Delta Gamma Sensitivity Analysis- Extreme Value Theory (DGSA-EVT)* model. *DGSA-EVT* is a model to measure high frequency-low severity (*HF-LS*) and low frequency-high severity (*LF-HS*) type of operational risks. This is the core of this research's methodological contribution.

As regards to the empirical contributions, in analysing operational value at risk (opVaR), this research carefully analyses the behaviour of the data by taking into account volatility, skewness and kurtosis of the variables. In the modelling, volatility analysis employs two models: constant-variance model and exponential weighted moving average (EWMA) model. Results of the empirical tests show that the operational risk variables in this research are non-normal; thus, non-normality involving skewness and kurtosis as well as volatility has to be taken into account in the estimation of VaR. In doing so, this research employs Cornish-Fisher expansion upon which the confidence interval of operational variables is an explicit function of the skewness and kurtosis as well as the volatility.

Empirical findings by deploying a set of econometrics tests reveal that for financing activities, the role of maintaining operational efficiency as part of an Islamic bank's fiduciary responsibilities is immensely high. However, people risk is enormous and plays a dominant role in affecting the level of operational risk exposures in Islamic banks in investment activities.

## **ACKNOWLEDGEMENT**

In the name of Allah; the Most Gracious, Most Benevolent, and Most Merciful.

For sure, this work would have never been completed without His Mercy and Guidance. Hours, days and months I have spent to grasp, comprehend the concepts and develop ideas on my PhD work whose substance, in many respects, became clear in unexpected moments. I really am most grateful to you my Lord.

I am thankful to my family and relatives in Indonesia for their constant prayers and supports, especially for my Mum Endang Noerlaela, my twin brother Hylman Fadly, my youngest brother Syahmi Manistato'a, my late grandma Soeminem (May Allah SWT bless her soul), uncle Wahid and aunty Tutik, and uncle Tuhron. To Baba and Anne in Istanbul, thank you for your trust, continuous support and prayers; you both have always made me feel like the *golden boy* in the family.

I sincerely extend my gratitude to Dr. Mehmet Asutay, my PhD supervisor, mentor, and *Baba* who has, not only, groomed me over the past few years, but has also taught me that PhD is not solely about obtaining an academic degree; it entails a spirit and process through which an individual can nurture his/her intellectual as well as personal development in order to make a positive and significant impact wherever he/she might be in. Indeed, his guidance, supervision and more importantly his trust have enabled me to improve and develop ideas during my PhD research. I also thank you for the innumerable helps and inspirations you have provided for my professional as well as for my personal development. May Allah (SWT) grant you His best reward as you deserve it.

To the Graduate School of Durham University; thank you so much for honouring me with the prestigious Doctoral Fellowship award. A PhD, without any doubt, signifies the pinnacle of one's formal education. In this respect, I would like to thank all of my teachers and lecturers at the kindergarten up to the university. At last, I could pull it off.

My appreciation also goes to the Markfield Institute of Higher Education (MIHE) and Islamic Foundation (IF) in Leicestershire United Kingdom for the exceptional student's life I had during my master degree and also for giving me an opportunity to be part of the academic staff and sponsoring me to deliver my research paper at Harvard Law School, Harvard University, USA in 2010. Surely, Markfield is a unique place I will never forget where I had my acquaintance with special people and a unique community who, one way or the other, gave a positive influence to my life.

Indeed, words cannot express my deepest gratitude to my immense source of support; my wife Humeyra Ceylan Izhar. In her capacity as a professional chartered librarian at MIHE-Islamic Foundation Library, she tirelessly provided me with up-to-date materials related to my PhD research. More importantly, her patience, sacrifice, love and understanding are second to none; for which I highly appreciate and value. My dear, marrying you is truly a great blessing from Allah (SWT).

## **DECLARATION**

I hereby confirm that this thesis is a result of my original work.. None of the materials in this thesis has previously been submitted for any other degrees in this or any other university.

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## LIST OF ABBREVIATIONS

AAOIFI : Accounting and Auditing Organization for Islamic Financial Institutions

ADF : Augmented Dickey Fuller

AMA : Advanced Measurement Approach

BCBS : Basel Committee on Banking Supervision

BIA : Basic Indicator Approach

BIS : Bank for International Settlements

BLUE : Best Linear Unbiased Estimator

BOPO : Operating Expenses to Operating Income

CAMEL : Capital Assets Management Earning Liquidity

CAPM : Capital Asset Pricing Model

CAR : Capital Adequacy Ratio

DGSA : Delta Gamma Sensitivity Analysis

DGSA-EVT : Delta Gamma Sensitivity Analysis-Extreme Value Theory

DSN : Dewan Syariah Nasional (Shariah National Board)

DW : Durbin Watson

E : Elasticity

EL : Expected Loss

EVT : Extreme Value Theory

EWMA : Exponential Weighted Moving Average

F : Financing

FR : Fiduciary Risk

FS : Financial securities

GARCH : General Autoregressive Conditional Heteroskedasticity

GPD : Generalized Pareto Distribution

HS-LS : High Severity and Low Severity

IAH : Investment Account Holders

IDR : Indonesian Rupiah

IFSB : Islamic Financial Services Board

IGC : Income Generating Channel

IGC<sub>F</sub>: Income Generating Channel for Financing

IGC<sub>I</sub>: Income Generating Channel for Investment

IGCs : Income Generating Channel for Services

IIFS : Institutions offering Islamic Financial Services

JB : Jarque Bera

LF-HS : Low Frequency and High Severity

LOB : Lines of Business

LR : Legal Risk

OLD : Operating Loss Distribution

OLS : Ordinary Least Squares

OpVaR : Operational Value at Risk

PDF : Probability Density Function

PDs : Partial Derivatives

POT : Peak Over Threshold

PR : People Risk

RoF : Rate of Return on Financing

RoSD : Return on Saving Deposits

RoTD : Return on 1-Month Time Deposits

SA : Sensitivity Analysis

SPV : Special Purpose Vehicle

SR : Shariah non-compliance Risk

TR : Technology Risk

Tr : Training expenses

TSA : The Standardized Approach

UL : Unexpected Loss

VaR : Value at Risk

WLS : Weighted Least Squares

## **CHAPTER 1**

## INTRODUCTION

#### 1.1 BACKGROUND

Of all the different types of risks that affect financial institutions, operational risks can be among the most destructive and most difficult to foresee. Operational risks, therefore, continue to receive keen attention among market participants and regulators, triggering dialogues and debates on the best ways to identify, measure, and manage this important risk. This recognition has led to an increased prominence of the importance of sound operational risk management in financial institutions and, to a greater degree, operational risk in banks' internal capital assessment and allocation processes.

In fact, the banking industry is currently undergoing a surge of innovation and development in these areas. The extraordinary demands of setting up a robust yet sensible and practical operational risk management system are puzzling risk professionals in every industry, and even more in financial institutions, where the regulators set out very detailed requirements.

The Basel 2 Accord focuses on bringing together the world's financial institutions under a common regulatory framework, although the way to manage operational risk is different for each financial institution. In addition, it was also expected that Basel 2 would enable banks to align regulatory requirements more closely with their internal risk measurement and to improve operational processes. This initiative has led banks around the world to collect data, which would allow them to model the occurrence and severity of losses and to use these models to estimate their economic capital requirement. Interestingly, in a survey conducted by the Risk Management Association, the main reasons given for engaging in operational risk management were to improve governance, and protect against the loss of reputation rather than meeting Basel 2 regulatory requirements (Moosa, 2007).

Nevertheless, forward-thinking institutions recognise that the accord also provides a unique opportunity to modernise and upgrade their overall risk practices and risk infrastructure, especially for credit and operational risk. For these banks, Basel 2 means more than compliance; rather, it denotes the opportunity to achieve distinct competitive advantage in a tight global market. Some banking institutions have begun developing processes required by Basel 2, but only a few of them made the operational risk framework as a practical tool to drive bottom-line results by enhancing operational and performance effectiveness.

Amid all such developments, the emergence of Islamic banks and financial institutions provides an alternative financing and banking method. While they are considered better protected against various risks due to the internal Shari'ah screening process, the real life realities demonstrates that 'the resilience of Islamic bank and financial institutions' may not necessarily an established fact, as they seem to be subjected to the same market conditions and risks.

Considering the increased importance of the sector and its robust development, in January 2006 the Islamic Financial Services Board (IFSB) issued its first two standards on risk management in Islamic financial institutions and capital adequacy for Islamic financial institutions. Operational risk, its management, and its use in calculating risk-based capital form part of these standards. The IFSB Standard No. 1 covers guiding principles for risk management in institutions offering Islamic financial services. While referring to the definition of operational risk used by the Basel Committee, namely the risk of loss resulting from inadequate or failed internal processes, people, and systems, or from external events, it also highlights additional operational risks faced by Islamic financial institutions. These additional operational risks for Islamic financial institutions are described as 'Shariah non-compliance risk' and those associated with the institutions' fiduciary responsibilities toward different fund providers.

The IFSB standard No. 2 covers capital adequacy for institutions offering Islamic financial services. It states that the measurement of capital to cater for operational risk may be based on either the Basic Indicator Approach or the Standardised Approach as set out in Basel 2. Under the Basic Indicator Approach, a fixed percentage of 15% of annual gross income, averaged over the previous three years, is set aside. Under the

Standardised Approach, an amount of capital to cater operational risks is set aside according to a defined beta percentage, ranging from 12% to 18%, in eight lines of business (LOB); corporate finance, trading and sales, and payment and settlement, commercial banking, agency services, retail banking, asset management, and retail brokerage. In this regard, IFSB proposed that the Basic Indicator Approach could be used by Islamic banks at the current state. However, subject to the supervisory authority defining the applicable business lines, the supervisory authority may allow Islamic banks in its jurisdiction to apply the Standardised Approach in which a percentage (12%, 15%, or 18%) of gross income is set aside according to the business lines.

As the efforts by IFSB and similar institutions evidence operational risk management carries an important weight for a healthy running of Islamic financial institutions. It is, thus, the aim of this research to explore the measurement of operational risk of Islamic banks and financial institutions with the objective of developing a sound and effective operational risk management process.

#### 1.2. STATEMENT OF THE RESEARCH

The previous section indicates that capital calculation for operational risk, as suggested by IFSB, may be based on the Basic Indicator Approach or the Standardised Approach. This statement, however, raises a number of issues as highlighted below:

- (i) An adaptation of capital calculation methods, namely the Basic Indicator Approach (BIA), the Standardised Approach (TSA) and Advanced Measurement Approach (AMA) to cater for operational risk based on Basel is deemed to be inappropriate due to different contractual features of an Islamic bank as compared to its conventional counterpart;
- (ii) The determination of beta percentage in eight lines of business may be arbitrary as it is extracted from a conventional bank. Moreover, it may not reflect the characteristics and nature of financial transactions in an Islamic bank;
- (iii) An Islamic bank may have different line of business due to different nature of transactional forms;

This research, hence, argues that to understand how much capital is needed to be set aside is contingent upon the knowledge of exposure level of operational risk, which should be known *ex-ante*. This suggests that there is an initial step, which is overlooked in the process, namely the measurement of the operational risk exposure itself. It is the aim of this research to contribute to this topic by developing an efficient operational risk measurement method for Islamic banks and financial institutions.

#### 1.3 RESEARCH AIM AND OBJECTIVES

The aim of this research is to explore measurement of operational risk management methods in the case of Islamic banks with the objective of developing a new model to measure the exposure of operational risks in Islamic banking. In order to reach the aim, following objectives are constructed:

- to identify the extents to which operational risks are similar or dissimilar in an Islamic banks as compared to its conventional counterparts;
- b. to identify the spectrums of operational risks in an Islamic bank;
- to identify the dimensions of operational risks in major Islamic financial contracts;
- d. to develop models for the measurement of operational risk in an Islamic bank;
- e. to test the proposed models based on available data.

## 1.4 RESEARCH QUESTIONS

Research questions, which are clearly formulated, help to respond to the aim and objectives of the research. Hence, the research questions of this research in line with the aims and objectives are as follows:

a. Does the definition of operational risk in an Islamic bank embody the same dimensions and framework as conventional banks?

- b. What are the dimensions and framework of operational risks in major Islamic financial contracts, such as *mudharabah*, *musharakah*, *murabahah*, *ijara*, *salam* and *istisna*'?
- c. What are the underlying features of operational risk model in an Islamic bank?
- d. How to empirically estimate the proposed model?
- e. Among the identified risk factors, which one is the most dominant? Why?

The following chapters aim to respond to these research questions within the defined aims and objectives.

#### 1.5 RESEARCH METHODOLOGY

This research deploys mainly quantitative methodology encompassing from mathematics, statistics and econometrics. However, this research also adopts qualitative approach in the sense of developing a clear understanding on the subject matter through the survey of the available material in its attempt to explicate the dimensions of operational risks in an Islamic bank (Chapter 3).

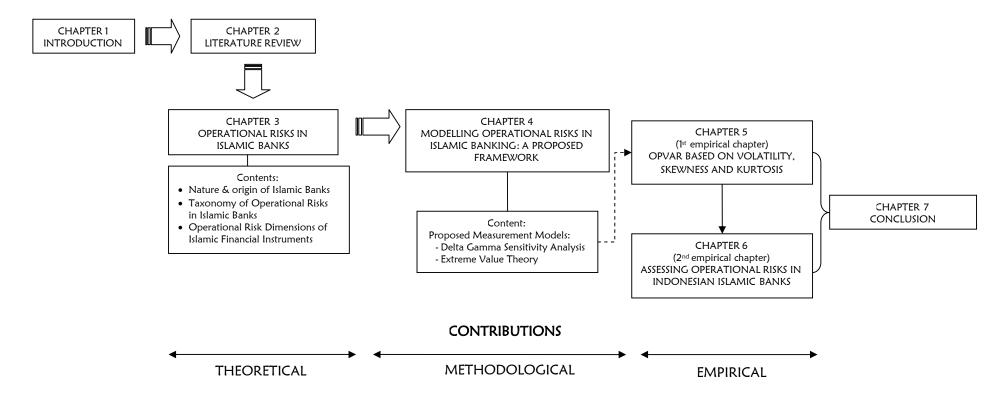
In terms of research method, different quantitative approaches are employed according to the objective of the research question. In developing the proposed measurement model for instance, this research utilises theoretical mathematics and statistics; which can be found in Chapter 4. In addition, statistics and econometrics are deployed in empirically estimating the proposed model, as exemplified in Chapter 5 and 6.

## 1.6. SIGNIFICANCE AND CONTRIBUTIONS OF THE RESEARCH

In Islamic banking industry, the need to cater operational risks issues has been highlighted by a number of recent study including Akkizidis and Kumar (2008), Archer and Haron (2007), Hossain (2005), Iqbal and Mirakhor (2007), Khan and Ahmed (2001), and Sundararajan and Errico (2002). All of these studies identified operational risk management as an essential area of risk exposure to be managed for the successful operations and functioning of Islamic banks due to the fact that Islamic banks operate in a similar, if not the same, business environment. Khan and Ahmed

(2001), for example, show that operational risk is relatively higher and serious than credit risk and market risk for Islamic banks. Unfortunately, to the best of the researcher's knowledge, there is not any single research in the context of Islamic banking which thoroughly tackles the issue of operational risks in three respects; (i) being theoretical, (ii) being methodological, and (iii) being empirical. This may be due to the fact that operational risk is relatively new area which requires further research to tackle the complexities it carries. This is the reason from which this research is developed; which attempts to fill this observed gap in the literature. Thus, the contributions of this research emerge in three aspects: theoretical, methodological and empirical, which is depicted in figure 1.1.

Figure 1.1: Taxonomy of the Research's Contributions



#### 1.7 STRUCTURE OF THE RESEARCH

This research is composed of seven chapters that are assembled into three main parts: Chapters 2 and 3 are theoretical; Chapter 4 is methodological; Chapters 5 and 6 are empirical. Chapter 7 is the last chapter, which provides an integrated discussion and concludes the research.

This research, hence, starts with an Introduction (Chapter 1), being this very chapter, consisting of background, aim and objectives, issues of the research, research questions, significance and contribution of the research and lastly, structure of the research. An outline of the remaining chapters is as follows:

Chapter 2 (Operational Risk Management: Literature Survey on Theory and Empirical Studies) discusses the historical background, definition, classification, and capital allocation for operational risks. A review on the empirical studies in operational risks is also presented in the last section of the chapter. Chapter 2 is mainly a survey of literature with an objective to gain a basic idea on the concept of operational risk in general. In addition, highlights of relevant literature are also presented in each chapter depending upon the context of the discussion.

Chapter 3 (*Operational Risks in Islamic Banks*) starts with a discussion on the nature and origin of Islamic banks and analyses why an Islamic bank has distinct operational aspects, as compared to the conventional one. It continues by examining the operational risk exposures in Islamic banks. The subsequent section discusses how to identify and conduct a mapping of operational risk in Islamic banks. Islamic banks are also different from conventional ones on the structure of their financial contracts; thus they bring different features of operational risks in different contracts. These are discussed in the later section. The analysis of issues in having adequate capital in order to cover operational losses is also presented.

Chapter 4 (Modelling Operational Risks in Islamic Banking: A Proposed Framework) commences with a review of the existing models in operational risk measurement and its classifications, which is followed by a theoretical background of the proposed model and

its features. In the subsequent section, attention is focused on the empirical aspect of the proposed model.

Chapter 5 (Operational Value at Risk (OpVaR) Based on Volatility, Skewness and Kurtosis) revisits the theoretical background of value at risk (VaR); and provides detailed explanation on the methodology used in the study. It also renders a discussion on the empirical findings. As the detailed discussion in the chapter identifies, in analysing VaR, this study does not simply use the data and follow a prescribed assumption to produce VaR. Rather, it carefully analyses the behaviour of the data by taking into account volatility, skewness and kurtosis of the variables. As shown in the chapter, volatility analysis employs two models: constant-variance model and exponential weighted moving average (EWMA) model. This approach has been adopted by Li (1999), Hull and White (1998), and RiskMetrics (1996).

Chapter 6 (Applying Econometrics to Operational Risks Analysis) is a continuation of empirical examination conducted in chapter 5. While the objective of the previous chapter is to figure out the level of scaled-standard deviation of volatility of earnings, which is represented by the value of operational value at risk in  $IGC_I$  (Income Generating Channel for Investment) and  $IGC_F$  (Income Generating Channel for Financing); the focus of this chapter is to examine the relationship between identified risk factors with return on securities (RoS) and return on financing (RoF). As Chapter 5 explains, RoS is the earnings which is defined in terms of a series of risk factors in  $IGC_I$  and RoF is earnings in  $IGC_F$ . By employing regression techniques and running a set of econometric tests, it is expected that such techniques would provide a cause-effect framework; hence significant determinants of operational risk could be identified. Thus, Chapter 6 presents a highlight of empirical studies in operational risk, and describes the data and methodology, and examines and discusses the empirical results

Chapter 7 being the *Conclusion* chapter discusses the finding of this research in light of the aim, objectives and research questions as suggested in chapter 1.

## **CHAPTER 2**

# OPERATIONAL RISK MANAGEMENT:A LITERATURE SURVEY ON THEORETICAL FRAMEWORK AND EMPIRICAL STUDIES

## 2.1 INTRODUCTION

Until very recently, it has been believed that banks are exposed to two main risks, namely credit risk and market risk. Operational risk has been regarded as a mere part of 'other' risk. However, operational risk is not a new concept for banks, as it has been reflected in banks' balance sheets for many decades, as it occurs in the banking industry everyday and affects the soundness and operating efficiency of all banking activities and all business units.

The collapse of Barings in 1995, Britain's oldest merchant bank, and the \$1.8 billion losses suffered by Sumitomo Corporation has shown the need for managing operational risk into corporate consciousness. It is also worth to mention, as Chernobai *et al.* (2007) observe, that the fall-down taking place at Orange County and Daiwa Bank in 1990s and at Allied Irish Banks and Enron in early 2000s provides examples of what is so called high-magnitude operational losses.

As Nash contends operational risk is the most striking and has been a "fundamental part" of doing business, and hence, cannot be fully eliminated (as cited in Alexander, 2003: 3). It has also become a dominant risk and has contributed an even larger share of total risk (Ferguson, 2003; as cited in Chernobai et al., 2007). HSBC Group shares the same view in its report, stating that:

...regulators are increasingly focusing on operational risk...This extends to operational risk the principle of supporting credit and market risk with capital, since arguably it is operational risk that potentially poses the greatest risk.<sup>1</sup>

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<sup>&</sup>lt;sup>1</sup> HSBC Operational Risk Consultancy group was founded in 1990, and is a division of HSBC Insurance Brokers.

It is, therefore, argued that a sound operational risk management is essential to the prudent operation of a financial system as a whole.

While risk management can be understood as the use of financial engineering technology of various sorts to manage the risks associated with financial positions and exposures (Marshall, 2001: 150), operational risk is often defined as the entire process of policies, procedures, expertise and systems that an institution needs in order to manage all the risks resulting from its financial transactions (Hussain, 2000: 91).

Nonetheless, there has not been unanimity in defining what and how operational risk occurs in the business as well as explicating the importance of managing operational risk, as it is not just 'other' risk. For this reason, in the following section, the chapter discusses the historical background, definition, classification, and capital allocation for operational risks. A review on the empirical studies in operational risks is also presented in the last section of the chapter.

## 2.2. THE BASEL CAPITAL ACCORD: A HISTORICAL BACKGROUND

Prior to 1988, bank capital was regulated by setting minimum levels for the ratio of capital to total assets. However, definitions of capital and ratios considered acceptable varied from country to country, but also some countries enforced their regulations more diligently than others. Banks were competing globally and a bank operating in a country where capital regulations were relaxed was considered to have a competitive edge over one operating in a country with tighter more strictly enforced capital regulations. The increased international competition among banks during 1980s emphasized how inconsistently banks were regulated with regard to capital. As Crouhy *et al.* (2001) observe that Japanese bank regulations contained no formal capital adequacy requirements, while in the United States and the United Kingdom banks were required to finance more than 5 percent of their risky assets by means of equity.

Another problem was that the types of transactions entered into by banks were becoming more complicated. The over the counter derivatives market for products such as interest rate swaps, currency swaps, and foreign exchange options was growing fast. These

contracts increase the credit risks being taken by a bank. Many of these newer transactions were *off-balance sheet*<sup>2</sup> activities meaning that they had no effect on the level of assets reported by a bank, nor on the amount of capital the bank was required to keep, as a consequence.

The major increase in off-balance sheet activity by banks that took place in the 1980s, surely, altered the risk profile of banks, while the regulatory requirements concerning equity ratios remained the same. It, then, became apparent to regulators that total assets were no longer a reliable indicator of the total risks being taken. A more sophisticated approach than that of setting minimum levels for the ratio of capital to total balance sheet assets was needed. The 1988 Basel Accord (known also as the 1988 BIS Accord, or the 'Accord') established international minimum capital guidelines that linked banks' capital requirements to the financial assets in their portfolio. It was also intended to harmonize minimum capital ratios.

The BIS Accord defined two minimum standards for meeting acceptable capital adequacy requirement; namely, an asset to capital multiple and a risk based capital ratio. The first standard is an overall measure of the bank's capital adequacy, which was similar to that existing prior to 1988. It required banks to have an assets-to-capital multiple of at most 20. The second standard, known as *Cooke ratio*, focuses on the credit risk associated with specific on- and off-balance sheet asset categories. It takes the form of solvency ratio, and is defined as the ratio of capital to risk weighted on-balance sheet plus off-balance sheet exposures, where the weights are assigned on the basis of counterparty credit risk.

There have been numerous criticisms of the 1988 Basel Accord. One of the primary criticisms has focused on the fact that there is a much greater variation in the quality of assets than those defined by the four categories named under the Accord. Krainer (2002: 425) points out that the categories are based on legal classifications of assets, which are only remotely related to the investment quality of the assets. In addition, the Accord is only advisory; its capital requirements are actually determined by local bank regulators.

<sup>&</sup>lt;sup>2</sup> Off-balance sheet is a position with potential financial consequences that does not appear on either the asset side or the liabilities side of a balance sheet.

For example, Krainer (2002: 426) reports that if an important economically powerful country, such as Japan or Germany, relaxes its capital adequacy requirements, the BIS tends to revise the Accord to accommodate that country.

The scope of the Accord is also limited since it does not address issues related to capital adequacy such as *portfolio effects*<sup>3</sup> and *netting*<sup>4</sup>. Another limitation is that the requirements do not distinguish between loans to firms with differing risk levels, which can lead to adverse incentive effects (Kirstein, 2002:394).

In a normal economic condition, when banks shift the composition of their portfolio into more risky assets, they must increase their reserves and therefore decrease their financial leverage. Moreover, Krainer (2002:425) argues that banks generally tend to decrease the risk of their portfolios in recessions (the so-called *flight to quality*); resulting in increased financial leverage of banks, these capital requirements will be countercyclical. However, the regulators focused primarily on credit risk, and ignored market risk and operational risk.

In response to the several problems with the 1988 Basel Accord, in April 1995 the Basle Committee issued a consultative proposal to amend the Accord which became known as the '1996 Amendment', or after it was implemented, 'BIS 98'.

The introduction of market risk to the scope of risks has made BIS 98 more comprehensive than the Accord. Moreover, it also requires financial institutions to measure and hold capital to cover their exposure to the market risk associated with debt and equity positions in their trading book, foreign exchange and commodity positions in both trading and banking book.

The amendment started to distinguish between a bank's trading book and its banking book (Hull, 2007: 176). The banking book consists mainly of loans and is not usually

<sup>4</sup> Netting is the ability to offset contracts or claims against each other, with positive and negative values, recognizing only the net amount. It takes place in the event of a default by counterparty.

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<sup>&</sup>lt;sup>3</sup> Portfolio effects is the term used to describe various benefits that arise when a portfolio is well diversified across issuers, industries, and geographical locations; naturally, a well-diversified portfolio is much less likely to suffer from credit losses than is a portfolio of deals concentrated with one party, one industry, and one geographical area.

marked to market for managerial and accounting purposes. The trading book consists of numerous financial instruments, whether on or off-balance sheet, which are intentionally held for short-term trading, and which are taken on by the bank with the intention of making profit from short-term changes in prices, rates, and volatilities. All trading book positions must be marked to market daily.

After reflecting the developments in the financial industry, the Basel Committee decided to undertake a comprehensive amendment of the Accord, which is now commonly referred to *Basel I*, and account for the diversity of risks taken by the banks. The new capital accord of 1998 is now known as *Basel II*.

The document of 'Operational Risk Management' was released in 1998, which discussed the importance of operational risk as a substantial financial risk factor.<sup>5</sup> This is actually one of the main objectives of Basel II; introducing operational risk as a new risk class.

Under Basel II, operational risk is subject to a regulatory capital charge. The Accord defines and sets detailed instructions on the capital assessment of operational risk and proposes several approaches that banks may consider to estimate the operational capital charge, as well as outlines necessary managerial and disclosure requirements.

It seems that regulators are introducing a capital charge for operational risk for three reasons (Hull: 2007). The first is that in an increasingly complex environment, banks face many risks arising from the possibilities of human and computer error. The second is that regulators want banks to pay more attention to their internal systems to avoid *catastrophic losses* like at Barings Bank. The third is that the effect of the Basel II credit risk calculation will be to reduce the capital requirements for most banks, and regulators want another capital charge to bring the total capital back to roughly where it was before.

<sup>&</sup>lt;sup>5</sup> See BIS (1998)

<sup>&</sup>lt;sup>6</sup> All errors are ultimately human errors. In the case of a "computer error", someone at a certain stage made a mistake programming the computer

<sup>&</sup>lt;sup>7</sup> Catastrophic loss is the most extreme and the rarest forms of operational risk event which can destroy the bank entirely, involving insider fraud, bad lending, poorly understood derivatives, counterparty failures, natural disasters, and snowballing reputational losses.

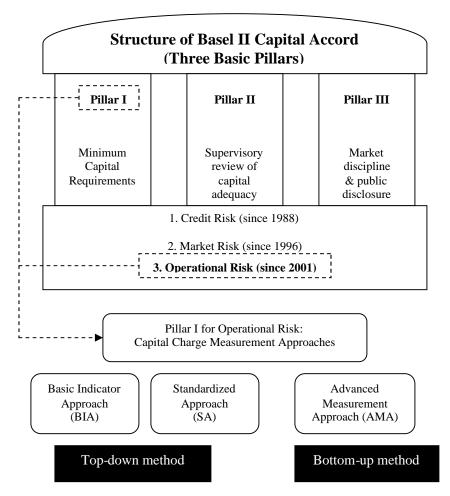
The Basel II Capital Accord underwent a number of amendments and was finalized in June 2006.<sup>8</sup>.

In sum, the proposals for the new Basel Accord makes changes to areas that were already included the Accord and add another important dimension to regulatory capital requirements – operational risk. Thus, there are now three areas of risk that are related to the minimum capital requirement; (i) credit risk (which was the focus of the original 1988 Accord); (ii) market risk of trading activities (which was introduced in a 1996 amendment to the Accord), and (iii) operational risk.

It should be noted that a key conceptual change within Basel II is the introduction of the comprehensive framework for capital regulation based on three pillars—*minimum capital requirements*, *supervisory review and market discipline*; as depicted in Figure 2.1.

 $<sup>^{8}</sup>$  see Basel Committee (2001a, 2001b, 2002a, 2002b) for more discussions of the issues related to the proposed Accord

Figure 2.1 Structure of Basel II Capital Accord and Pillar I for Operational Risk



Source: Chernobai et al., (2007: 38)

Under *Pillar 1*, banks are required to calculate minimum capital charge, referred to as regulatory capital, with the aim of bringing the quantification of this minimal capital more in line with the banks' economic loss potential. Under the Basel II framework there will be a capital charge for credit risk, market risk and, for the first time, operational risk. Whereas the treatment of market risk is unchanged relative to the 1996 Amendment of the Basel 1 Capital Accord, the capital charge for credit risk has been revised substantially.

It is further recognized that any quantitative approach to risk management should be embedded in a well-functioning corporate governance structure. Thus best practice risk management imposes clear constraints on the organization of the institution, *i.e.* the board

of directors, management, employees, internal and external audit processes. In particular, the board of directors assumes the ultimate responsibility for oversight of the risk landscape and the formulation of the company's risk appetite. This is where *Pillar 2* enters. Through this important pillar, also referred to as the *supervisory review process*, local regulators review the various checks and balances put in place. This pillar recognizes the necessity of an effective overview of the banks' internal assessment of their overall risk and ensures that management is exercising sound judgement and has set aside adequate capital for the various risks.

Finally, in order to fulfil its promise that increased regulation will also diminish *systemic risks*<sup>9</sup>; clear reporting guidelines on risks carried by financial institutions are called for. Pillar 3 seeks to establish *market discipline* through a better public disclosure of risk measures and other information relevant to risk management. In particular, banks will have to offer greater insight into the adequacy of their capitalization.

As Figure 2.1 shows, Basel II (under Pillar I), the regulators also offer three approaches (BIS:2001a); namely (a) the basic indicator approach, (b) the standardized approach, and (c) the advanced measurement approach in order to measure operational risk regulatory capital 10. The observation indicates that which of these is used depends on the sophistication of the bank. The simplest approach is the basic indicator approach. This sets the operational risk capital equal to the bank's average annual gross income over the last three years multiplied by 0.15. 11 The standardized approach is similar to the basic indicator approach except that a different factor is applied to the gross income from different business lines. In the advanced measurement approach the bank uses its own internal models to calculate the operational risk loss that it is 99.9% certain will not be exceeded in one year. Details of how these three approaches operate are presented in the section 2.5.

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<sup>&</sup>lt;sup>9</sup> Systemic risk is the risk that a default by one financial institution will lead to defaults by other financial institutions.

<sup>&</sup>lt;sup>10</sup> Regulatory capital is the capital that a bank is required to keep by regulators. It is designed to ensure that there is enough capital in the banking system.

<sup>&</sup>lt;sup>11</sup> Gross income is defined as net interest income plus non-interest income. Net interest income is the excess of income earned on loans over interest paid on deposits and other instruments that are used to fund the loans. Years where gross income is negative are not included in the calculations.

The first two approaches are based on *top-down* method, while the last approach is following *bottom-up* method (see figure 2.1). *Top-down* method takes aggregate targets, such as net income or net asset value, to analyze the operational risk factors and loss events that cause fluctuations in the target. Meanwhile, bottom-up method disaggregates the targets into many sub-targets and evaluates the impact that factors and events have on these sub-targets.

#### 2.3 OPERATIONAL RISK: A THEORETICAL FRAMEWORK

Risk is the fundamental element in finance which will inevitably affect financial behaviour. For such fundamental concept, risk has a wide range of definitions.

The Oxford English Dictionary (2001), for instance, defines risk as "hazard, a chance of bad consequences, loss or exposure to mischance". McNeil *et. al.* (2005: 1) defines risk as "any event or action that may adversely affect an organization's ability to achieve its objectives and execute its strategies" or, alternatively, "the quantifiable likelihood of loss or less than expected returns".

Certainly, risk has been associated with negative consequences, in other word, risk is perceived as the probability of a negative deviation or sustaining a loss. As Vaughan and Vaughan (1995: 8) notes that risk is "a condition in which there is a possibility of an adverse deviation from a desired outcome that is expected or hoped for". This negative approach is also suggested by Geiger (as cited in Chernobai *et al.*, 2007: 16), when he defines risk as "an expression of the danger that the effective future outcome will deviate from the expected or planned outcome in a negative way".

Interestingly, Chernobai *et al.* (2007) raise a very important and valid point regarding risk, stating that risk is not necessarily perceived as a negative concept. For example, in the perspective of investment, risk is the volatility of expected future cash flows (measured, for example, by the standard deviation). Because of this uncertainty and because fluctuations in the underlying value may occur in either negative or positive direction, risk defined in this way does not exclude the possibility of positive outcomes. Hence, risk is not necessarily associated with a negative concept.

In banking, the best known type of risk is *credit risk*, the risk of not receiving promised repayments on outstanding investments such as loans and bonds, because of the default of the borrower (Crouhy et al, 2001). The next important category is probably *market risk*, which is described as the change in the value of a financial position due to changes in the value of the underlying components on which that position depends, such as stock and bond prices, exchange rates, commodity prices, *etc.* There are other two categories of risks which have also become important part of banking operations: *liquidity risk* and *model risk*. Liquidity risk can be defined as the risk stemming from the lack of marketability of an investment that can not be bought or sold quickly enough to prevent or minimize loss (McNeil et al., 2005:3). Model risk, on the other hand, is associated with using a mis-specified (inappropriate) model for measuring risks (Dowd, 1997: 193). The most recent risk category which has received a lot of attention, and also the focus of this study, is *operational risk*, the risk of losses resulting from inadequate or failed internal processes, people and systems, or from external events. Figure 2.2 depicts the spectrum of financial risks.

FINANCIAL RISKS IN BANKS Market Risk Credit Risk Operational Risk Other risks • Currency risk • Counterparty Internal Liquidity risk operational risks: • Interest rate default People, Process, risk • Settlement risk Systems Model risk • Equity price Counterparty credit rating change Political risk External operational risks: Business risk People Natural disasters Legal risk

Figure 2.2: Spectrum of Financial Risks

Source: Modified version of Moosa (2008: 7)

Referring to the definition of risk discussed before, in the context of operational risk, a negative approach towards risk is considered to be more relevant (Chernobai *et al*, 2007), since it deals with mitigating the risk in the events of having the possibility of loss.

# 2.3.1. A Conceptual Definition

The collapse of Barings Bank in 1995 is without doubt has challenged the concerned stakeholders in financial industry to define risk which gave a great impact on the industry and yet was not clearly classified. However, the financial industry, later on, started to recognise the type of risk, which can not be classified as either credit risk or market risk, as operational risk.

Many different ways of defining operational risk have been gradually proposed. Albeit the difficulties of describing operational risk, due to its diversity (Buchelt and Untregger, 2004) and complexity (Milligan, 2004), nevertheless attempts to settle the definition and the coverage of operational risk have been made by Alexander (2003), Crouhy *et al.*, (2001), Cagan (2001), Tripe (2000), Lopez (2002), Jarrow (2007), Moosa (2007) and Jobst (2007), amongst others.

Attributed as "Risk X" (Metcalfe, 2003), operational risk is not only a lot easier to be said than done (Allen and Bali, 2004), but it is also considered as a "fuzzy" concept (Crouhy *et al.*,2001), because of the difficulties in distinguishing between operational risk and the normal uncertainties faced by the organisation in its daily operations.

There are two general approaches in defining operational risk. The first approach is 'residual approach' which states that operational risk is everything other than credit risk or market risk (Rao and Dev, 2006; Hull, 2007). More specifically, operational risk can be drawn by looking at the bank's financial statements and remove from the income statement (i) the impact of credit losses and (ii) the profit or losses from market risk exposure, thus the variation in the resulting income would the attributed to operational risk (Hull, 2007). However, most people agree that this definition is too broad. Also, this

definition does not include, as Marshall (2001: 45), Dowd (in Alexander, 2003: 45), and Hull (2007: 323) point out, what is so called *rogue trading*<sup>12</sup> risk.

Despite the fact that this view is deemed to be hardly suitable for identifying its scope precisely (Buchelt and Unteregger, 2004), however, Medova and Kyriacou (2001) and Jameson (1998) argue that the understanding of operational risk is everything that is not exposed to credit and market risk, in other word, as a *residual* of credit and market risk, remains prevalent among practitioners. Responding to this approach, Moosa (2007) asserts that this sort of definition is probably a reflection of the lack of understanding and the diversity of operational risk.

The second approach, which is 'non-residual approach', suggests that operational risk is, in fact, the risk arising from operations (Crouhy *et al.*, 2001). This includes the risk of mistakes in processing transactions, making payment, *etc*. This definition of operational risk, is unfortunately, too narrow and has amounted to the confusion between operational risk and operations risk, given that the former is broader term and is only associated with value-driving operations such as foreign exchange trading and settlement (Moosa, 2007: 170).

Early on, The Group of Thirty (1993: 4) defined operational risk as "uncertainty related to losses resulting from inadequate systems or controls, human error or management". Furthermore, the Commonwealth Bank of Australia (1999: 17) came up with the broad definition that operational risk is "all risks other than credit risk and market risk, which could cause volatility of revenues, expenses and the value of the Bank's business". An early definition of operational risk came up in a seminar at the Federal Reserve Bank of New York when Shepheard-Walwyn and Litterman (1998: 176) characterised operational risk as "a general term that applies to all the risk failures that influence the volatility of the firm's cost structure as opposed to its revenue structure".

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<sup>&</sup>lt;sup>12</sup> Rogue trading is an activity conducted by so-called *rogue trader* who acts independently of others - and, typically, recklessly - usually to the detriment of both the clients and the institution that employs him or her. In most cases this type of trading is high risk and can create huge losses. The most famous rogue trader is Nick Leeson, who was a derivatives trader at the Singapore office of Britain's Barings Bank

It is important to note that in the definition of The Commonwealth Bank of Australia (1999), operational risk impinges upon both the revenue and cost sides of the business, but in the definition of Shepherd-Walwyn and Litterman (1998) it covers the cost side only. The difference between the two approach gives rise to the question if operational risk is one sided.

In addition to the discussion above, the definition that identifies internal and external sources of operational risk has been highlighted by Crouhy *et al.*, (2006: 332), who assert that operational risk is "the risk that external events, or deficiencies in internal controls or information systems, will result in a loss-whether the loss is anticipated to some extent or entirely unexpected".

The diversity of defining operational risk in the early stage shows that there was no consensus in the industry on the precise definition of operational risk (Webb: 1999). However, some other vague concepts have been put forward, such as Tripe (2000) who attributes operational risk as operational loss, without elaborating further. This of course, does not reflect the diversity of the scope of operational risk. Likewise, Lopez (2002) argues that operational risk is every unquantifiable risk, which seems to be the antithesis of measuring regulatory capital against operational risk as required by the Basel II Accord.

The first official publication of the Bank of International Settlements (BIS: 1998) defines operational risk as (i) other risks, (ii) any risk not categorized as credit and market risk, and (iii) the risk of loss arising from various types of human or technical errors.

Defining in a broader aspects, Barclay Bank (2004) suggests that the major sources of operational risk include operational process reliability, IT security, outsourcing of operations, dependence on key suppliers, implementation of strategic change, integration of acquisitions, fraud, error, customer service quality, regulatory compliance, recruitment, training and retention of staff, and social and environmental impact.

Large banks and financial institutions sometimes prefer to use their own definition of operational risk. For example, Deutsche Bank (2005: 45) defines operational risk as

"potential for incurring losses in relation to employees, contractual specifications and documentation, technology, infrastructure failure and disasters, external influences and customer relationships".

The formal definition that is widely accepted was initially proposed by the British Bankers Association (1997: 4) and adopted by BIS in January 2001, in which, operational risk was defined as "the risk of direct or indirect loss resulting from inadequate or failed internal processes, people or system or from external events".

The second issue is regarding the wide scope of operational risk encountering human risk (e.g. incompetence, fraud), process risk (e.g. model, transaction and operational risk control) and technology risk (e.g. system failure, programming error). Whereas everyone agrees that such risks are very important, much disagreement exists on how far one is able to quantify such risks. This becomes particularly difficult when financially more important risks like fraud and litigation are taken into account. None doubts the importance of operational risk for the financial sector, but much less agreement exists on how to measure this risk.

A refined definition of operational risk dropped the two terms, hence finalising the definition of operational risk as "operational risk is the risk of loss resulting from inadequate or failed internal processes, people or system, or from external events" (BIS, 2003b: 2).

Chernobai *et al.*(2007) argues that the definition is "causal based", providing a breakdown of operational risk into four categories based on its sources: (i) people; (ii) processes; (iii) systems; and (iv) external factors.

In addition, the BIS paper also clarifies that the Pillar I capital charge is not meant to capture or reflect systemic risks. It is important to note that this definition is based on the underlying causes of operational risk, as it seeks to delineate operational risks from other risks by referring to key internal and external aspects of the business operation that, alone or in combination, can cause operational losses.

# 2.3.2. Indicators of Operational Risk Exposures

The probability of operational risk events will increase with a large number of personnel (due to increased probability of committing an error) and with a greater transaction volume. Following BIS (2001a, Annex 4), Haunbenstock (2003), and Allen *et al.*, (2004), the operational risk exposure indicators include gross income, volume of trades or new deals, value of assets under management, value of transactions, number of transactions, number of employees, employees' years of experience, capital structure (debt-to-equity-ratio), historical operational losses, and historical insurance claims for operational losses.

In a study gauging the dependence between a bank size and operational loss amounts, Shih *et al.*, (2000) find that on average, for every unit increase in a bank size, operational losses are predicted to increase roughly a fourth root of that. This means that when they regressed log-losses on a bank's log size, the estimated coefficient was approximately 0.25. However, in a similar study, Chapelle *et al.*, (2005) estimated the coefficient to be 0.15.

It can therefore be argued that the exposures of operational risk in a financial institution are likely to increase in proportion with the size and complexity of financial transactions it engages with. However, in order to find out the sources of operational risk, the subsequent section attempts to classify the different classes of operational risks.

# 2.3.3. Classification of Operational Risks

A number of technical concepts have been introduced since the acknowledgement of operational risk as a new class of risk. Therefore, it is noteworthy to get familiarised with some basic concepts as well as a classification of operational risk (see Figure 2.3).

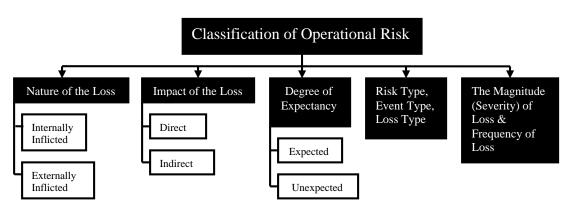


Figure 2.3: Operational Risk Classification

Source: Modified version of Chorafas (2004: 32) and Chernobai et al., (2007: 19)

The above figure shows that the classification of operational risk are based on five classes: (i) nature of the loss, (ii) impact of the loss, (iii) degree of expectancy, (iv) risk type, event type, loss type, (v) the magnitude (severity) of loss and frequency of loss. These are discussed in the following sections:

# 2.3.3.1. Nature of the Loss

Operational losses can be internally inflicted or can result from external sources, or, in the words of Crouhy, Galai and Mark (2001), operational loss is the cause of *operational failure risk*<sup>13</sup> and *operational strategic risk*<sup>14</sup>.

Internally inflicted sources include most of the losses caused by human, process, and technology failures, such as those due to human errors, internal fraud, unauthorized trading, injuries, and business delays due to computer failures or telecommunication problems. External sources include man made incidents such as external fraud, theft,

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<sup>&</sup>lt;sup>13</sup> Operational failure risk refers to internal event.

<sup>&</sup>lt;sup>14</sup> Operational strategic risk refers to external event.

computer hacking, terrorist activities, and natural disasters such as damage to physical assets due to hurricanes, floods, and fires.

Many of the internal operational failures can be prevented with appropriate internal management practices; for example, tightened controls and management of the personnel can help prevent some employee errors and internal fraud, and improved telecommunication networks can help prevent some technological failures.

External losses are very difficult to prevent. However, Marshall (2001), Young and Ashby (2001), van den Brink (2002) and Hoffman (2002) contend that it is still possible to design insurance or other hedging strategies to reduce or possibly eliminate externally inflicted losses.

## 2.3.3.2. Direct vs Indirect Operational Losses

Direct losses are the losses that directly arise from the associated events. For example, in incompetent currency trader can result in a loss for the bank due to adverse exchange rate movements. Another example might be by mistakenly charging the amount of £10,000 instead of £15,000 resulting in the loss for the bank in the amount of £5,000. The Basel II sets guidelines regarding the estimation of the regulatory capital charge by the banks based only on direct losses. Table 2.1 identifies the Basel II categories and definitions of direct operational losses.

Table 2.1: Direct Loss Type in Operational Risk

Loss Type	Contents
Write-downs	Direct reduction in the value of assets due to theft, fraud, unauthorized activity, or market and credit losses arising as a result of operational events.
Loss of recourse	Payments or disbursements made to incorrect parties and not covered.
Restitution	Payments to clients of principal and/or interest by way of restitution, or the cost of any other form of compensation paid to clients.
Legal liability	Judgements, settlements, and other legal cots.
Regulatory and	Taxation penalties, fines, or the direct cost of any other
Compliance	penalties, such as license revocations.
Loss of or damage to	Direct reductions in the value of physical assets, including
assets	certificates, due to an accident, such as neglect, accident, fire,
	and earthquake.

Source: BIS (2001a: 3)

Indirect losses are generally opportunity costs and the losses associated with the costs of fixing an operational risk problem such as near-miss losses.

In addition, near-miss losses have been mentioned in the regulatory proposals (BIS: 2001a), and there are hints that they might be used to augment internal loss data in the calibration of the capital calculation models. Near-miss losses are actually the estimated losses from those events that could potentially occur but were successfully prevented.

## 2.3.3.3. Expected and Unexpected Operational Losses

Some operational losses are expected; some are not. The expected losses (*EL*) are generally those that occur on regular (such as everyday) basis, such as minor employee errors and minor credit card fraud. In other word, expected loss is anticipated for the next time period. For infrequent events, i.e. those which are extremely unlikely to occur more than once in a given time period, expected losses are:

$$EL = \sum_{event_i} Loss_i \times LikelihoodofLoss_i$$
(2.1)

For more frequent events, expected losses E(L) depend on the form of the probability distribution p(L) for the event frequencies and impacts; and in the continuous limit can be written as:

$$E(L) = \int_{-\infty}^{\infty} Lp(L)dL$$
(2.2)

Unexpected losses (*UL*), on the other hand, are those losses that generally cannot be easily foreseen, such as natural disasters and large scale internal fraud.

For infrequent events, the following formula can be used to estimate the unexpected loss over a number of possible outcomes denoted by i:

$$UL = \sqrt{\sum_{event_i} LikelihoodofLoss_i \times (Loss_i - EL)^2}$$
 (2.3)

Or its continuous equivalent:

$$UL = \sqrt{\int_{-\infty}^{\infty} [L - E(L)^2 p(L) dL}$$
(2.4)

As with expected losses, it is assumed that the number of occurrences (N) of the event in a time period, and the individual events impacts (I) are independent and identically

BIS (2001a) suggested that the capital charge for operational risk should cover unexpected losses (UL) due to operational risk, and that provisions should cover expected losses (EL). This is due to the fact that many banking activities with a highly likely incidence of expected regular operational risk losses (such as fraud losses in credit card), EL are deducted from reported income in the particular year. Therefore, in 2001 BIS proposed to calibrate the capital charge for operational risk based on both EL and UL, but to deduct the amount due to provisioning and loss deduction (rather than EL) from the minimum capital requirement.

However, accounting rules in many countries do not provide a robust and clear approach to setting provisions, from example allowing provisions set only for future obligations related to events that have already occurred. In this sense, they may not accurately reflect the true scope of *EL*. Therefore, in the 2004 version of Accord, it was proposed to estimate the capital charge as sum of *EL* and *UL* first and then subtract the *EL* portion in those cases when the bank is able to demonstrate its ability to capture the *EL* by its internal business practices. BIS (2006a: 56) further clarify the idea:

For operational risk EL to be measured to the satisfaction of national supervisors, the bank's measure of EL must be consistent with the EL plus UL capital charge calculated using the AMA model approved by supervisors. ...Allowable offsets for operational risk EL must be clear capital substitutes or otherwise available to cover EL with a high degree of certainty over a one year time horizon. Where the offset is something other than provisions, its availability should be limited to those business lines and event types with highly predictable, routine losses. Because exceptional operational risk losses do not fall within EL, specific reserves for any such events that have already occurred will not qualify as allowable EL offsets.

Figure 2.4 portrays the dimensions of operational risk, showing the *catastrophic loss/stress loss* which is the loss in the excess of the upper boundary of the estimated UL such as 99.9% *value at risk*<sup>15</sup>. It requires no capital coverage; however, Mori and Harada (2001), van den Brink (2002), and Chorafas (2004) suggest that insurance coverage may be considered.

<sup>&</sup>lt;sup>15</sup> Value at risk is the worst loss that may occur with a given confidence level and for a given period.

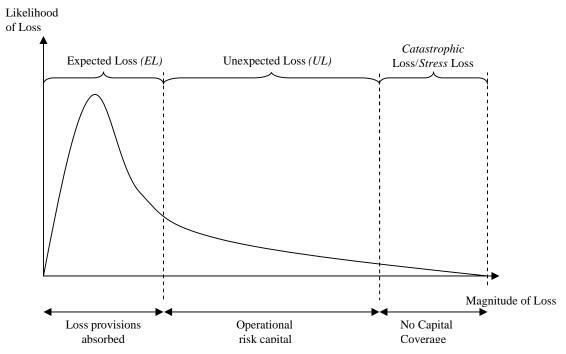


Figure 2.4: Coverage of Operational Risk

Source: Marshall (2001: 80)

# 2.3.3.4. Risk Type, Event Type and Loss Type

Unlike market risk management, in which the risks largely result from continuous changes in market prices and rates (risk factors), operational risk management is often about preventing, controlling, and mitigating loss events.

What is loss event? What is the relation between loss events and risks? Confusion arises in the operational risk because of the distinction between risk type (or hazard type), event type, and loss type. When banks record their operational loss data, it is very essential to record it separately according to event type and loss type, and precisely identify the risk type as well. Mori and Harada (2001), Alvarez (2002), and Dowd (2003) suggest that the distinction between the three is comparable to cause and the effect:

- *Hazard* constitutes one or more factors that increase the probability of occurrence of an event.
- Event is a single incident that leads directly to one or more effects (e.g. losses).
- Loss constitutes the amount of financial damage resulting from an event

**HAZAR** LOSS **EVENT** cause of event effect of hazard cause of loss effect of event **Examples of hazard Examples of event types:** Loss types categories: types: • Internal fraud (e.g. • Write downs • Inadequate employee Loss of recourse unauthorized trading, management theft) Restitution • Obsolete computer • External fraud (e.g. Legal liability systems credit card fraud) Regulatory and Inexperienced • Diversity/discriminatio compliance (e.g., personnel n events fines, penalties and • Large transaction · Improper business and taxation) volumes market practices Loss of or damage to • Diversity and cultural • Failed/inaccurate physical assets differences reporting Other • Unfavourable climate • System failure conditions or Natural disasters geographical

Figure 2.5
The Process of Loss Occurrence

Source: Modified version of Mori and Harada (2001: 3), Dowd (in Alexander, 2003: 37), and Chernobai et al. (2007: 23)

Figure 2.5 shows how operational losses would occur in a process called "cause-effect" (Mori and Harada, 2001, 3) relationship between *hazard*, *event*, and *loss*. As illustrated in figure 2.4, *loss* is effect of *event* while *event* is cause of *loss*. Yet, *event* is effect of *hazard* while *hazard* is cause of *event*. In other words, every *loss* must be associated with an *event* that caused the *loss*, while every event must be associated with one or multiple *hazards* that caused the *event*. Note that in the literature related to operational risk, *hazard* is also termed as *risk* (Marshall, 2001), or *cause* (Dowd, in Alexander: 2003), while *loss* might be named as *effect* (Dowd, in Alexander: 2003).

By giving the following examples, Mori and Harada (2001: 4) further contend how critical the correct identification of *event type* is, in order to decide whether a loss of a particular *loss type* is attributed to credit, market, or operational risk:

- (i) A reduction in the value of a bond due to change in the market price;
- (ii) A reduction in the value of bond due to the bankruptcy of the issuer;
- (iii) A reduction in the value of a bond due to a delivery failure.

In this case, the write-down of the bond (the loss type) belongs to the scope of market risk, credit risk, and operational risk, respectively.

Accurate documentation of operational risk by the type of hazard, event, and loss is also essential for understanding of operational risk. Marshall (2001) points out that event usually involves a subject and an active verb (e.g. counterparty defaults, incorrect counterparty entered, and fax machine fails), and has to be well defined and be clear whether it has occurred or not. Unrecorded and accumulative events might be dependently correlated (Powojowski et al., 2002), hence, it is necessary to be alert in order to avoid its huge impact on losses.

The Basel II classifies operational risk into seven event types groups, as follows<sup>16</sup>:

- 1. *Internal fraud*: Acts of a type intended to defraud, misappropriate property or circumvent regulations, the law, or company policy. Examples includes intentional misreporting of positions, employee theft, and insider trading on an employee's own account.
- 2. *External fraud*: Acts by third party of a type intended to defraud, misappropriate property or circumvent the law. Examples include robbery, forgery, check kiting, and damage from computer hacking.

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<sup>&</sup>lt;sup>16</sup> This categorisation is established by Basel Committee on Bank Supervision. For details, see "Sound Practices for the Management and Supervision of Operational Risk", Bank International Settlements, July 2002.

- 3. Employment practices & workplace safety: Acts inconsistent with employment, health or safety laws or agreements, or which result in payment of personal injury claims, or claims relating to diversity or discrimination issues. Examples include workers compensation claims, violation of employee health and safety rules, organised labour activities, discrimination claims, and general liability (e.g., a customer slipping and falling at a branch office)
- 4. Clients, products & business practices: Unintentional or negligent failure to meet a professional obligation to specific clients (including fiduciary and suitability requirements), or from the nature or design of a product. Examples include fiduciary breaches, misuse of confidential customer information, improper trading activities on the bank's account, money laundering, and the sale of unauthorised products.
- 5. *Damages to physical assets*: Loss or damage to physical assets from natural disasters or other events. Examples include terrorism, vandalism, earthquakes, fires, and floods.
- 6. *Business disruption and system failures*: Disruption of business or system failures. Examples include hardware and software failures, telecommunication problems, and utility outages.
- 7. Execution, delivery and process management: Failed transaction processing or process management, and relations with trade counterparties and vendors. Examples include data entry errors, collateral management failures, incomplete legal documentation, unapproved access given to clients' accounts, non-client counterparty misperformance, and vendor disputes.

# 2.3.3.5. Operational Loss Severity and Frequency

Expected losses generally refer to the losses of low severity (or magnitude) and high frequency. Generalizing this idea, operational losses can be broadly classified into four main groups:

- (i) Low frequency/low severity;
- (ii) High frequency/low severity;
- (iii) High frequency/high severity;
- (iv) Low frequency/high severity

The 'severity-frequency quadrant' shown in Figure 2.6 gives an idea on the assessment of the likelihood (frequency) of operational risk and the magnitude (severity) of loss. It also provides information on operational risk exposures across the bank.

As clearly seen in top half of figure 2.6 that if a business unit falls in the upper right hand quadrant (high frequency/high severity), the business has a high likelihood of operational risk and a high severity of loss, if failure occurs. However, Samad-Khan (2006) argues that this is unlikely to happen; therefore it is not very useful for operational risk modelling (Scandizzo: 2005). In addition, Chernobai *et al.*, (2007) contend that the first group (low frequency/low severity) is not feasible as well. Consequently, the two remaining categories of operational losses that the financial industry needs to focus on are 'high frequency/low severity' and 'low severity/high frequency' losses. The two areas are described in the bottom half of Figure 2.6.

Loss Severity Low Frequency/ **High Frequency/ High Severity High Severity** Low Frequency/ **High Frequency/** Low Severity **Low Severity** Loss Frequency Loss Severity Low Frequency/ NA **High Severity** NA High Frequency/ **Low Severity** Loss **▶**Frequency

Figure 2.6: Classification of operational risk by frequency and severity

Source: Chernobai et al. (2007: 25)

The losses of high frequency/low severity are relatively unimportant for an institution and often can be prevented. What cause the greatest damage are the low frequency/high severity losses. Banks must be particularly attentive to these losses, because these cause the greatest financial consequences to the institution, including potential bankruptcy.<sup>17</sup> Just a few of such events may result in bankruptcy or a significant decline in the value of the bank.

## 2.4 CAPITAL ATTRIBUTION FOR OPERATIONAL RISKS

One of the most critical resources in any business is its capital. Capital can be narrowly defined as equity plus reserves, or more broadly to also include some types of long-term subordinated debt (Marshall, 2001: 495).

<sup>&</sup>lt;sup>17</sup> The events that incur such losses are often called the *tail events*.

The primary role of capital charge imposed by Basel II is to serve as a buffer to protect against the damage resulting from risk. It can also be seen as a form of self-insurance tool. A precise definition should be addressed in order to distinguish between two types of risk-based capital, economic capital and regulatory capital. According to Chorafas (2004), economic capital is the amount of capital market forces dictate for risk in the bank. It is the cushion that provides protection against the various risks inherent in the business institution and is designed to absorb unexpected losses up to some level of insolvency risk. It does not include expected losses because these should already be captured in loss provisions <sup>18</sup>.

Regulatory capital is the amount of capital necessary to provide adequate coverage of banks' exposures to financial risk. It is designed to ensure that there is enough capital in the banking system.

Whereas, according to Marshall (2001), economic capital is what really matter and its role becomes supreme because it is based on the objective estimates of business risk. Moreover, the calculation of economic capital is on a firmer basis of quantifiable internal and external events (Chorafas: 2004).

In a study on the allocation of operational risk capital, Jorion (2000), Crouhy *et al.*,(2001), and Cruz (2002) suggest 25%, 20%, and 35% of risk-weighted assets respectively. According to BIS (2001a), the portion of economic capital allocated to operational risk ranges from 15% to 25% of risk-weighted assets. A 2000 Loss Data Collection Exercise carried out by the Risk Management Group of the BIS revealed the overall allocation of minimum regulatory/economic capital to operational risk across 41 banks, shown in Table 2.2.

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<sup>&</sup>lt;sup>18</sup> See figure 2.4

Table 2.2: Ratio of Operational Risk Economic Capital to Overall Economic Capital and to Minimum Regulatory Capital

	Median	Mean	Minimum	25 <sup>th</sup>	75 <sup>th</sup>	Maximum
				percentile	percentile	
Operational risk	0.150	0.149	0.009	0.086	0.197	0.351
capital/Overall economic capital						
Operational risk capital/Minimum regulatory capital	0.128	0.153	0.009	0.074	0.170	0.876

Source: BIS (2001b: 26)

The results shown in table 2.2 suggests that, on average for banks in the sample, operational risk capital represents about 15 percent of overall economic capital, although there is some amount of dispersion around this figure. Operational risk capital, however, appears to represent a rather smaller share of minimum regulatory capital, somewhat over 12 percent for the median bank in the sample. The results also imply that a reasonable level of the overall operational risk capital charge would be about 14 to 15 percent of minimum regulatory capital.

## 2.5. DETERMINATION OF REGULATORY CAPITAL

Banks have three alternatives for determining operational risk regulatory capital, basic indicator approach (BA), standardized approach (SA), and advanced measurement approach (AMA). In all three, the amount of regulatory operational risk capital is proportional to some exposure indicator, which is an accounting measure of bank activity.

# 2.5.1. Basic Indicator Approach (BIA)

The simplest approach is the *basic indicator* approach (*BIA*). Under this approach, operational risk capital is set equal to 15% (denoted by  $\alpha$ ) of annual gross income over the previous three years. Gross income is defined as net interest income plus non-interest income. Hence the risk capital under the BI approach for operational risk in year t is given by

$$RC_{BI}^{t}(OR) = \frac{1}{Z_{t}} \sum_{i=1}^{3} \alpha \max(GI^{t-i}, 0),$$
 (2.5)

where  $Z_t = \sum_{i=1}^3 I_{\{GI^{t-i} > 0\}}$  and  $GI^{t-1}$  stands for gross income in year t-i. It is important to note that an operational risk-capital charge is calculated on a yearly basis.

No quantitative and qualitative requirements are specified by Basel II regarding the use of (*BIA*). However, banks are encouraged to comply with the guidelines as described in BIS (2003) and BIS (2006b). The approach is particularly convenient for small and medium-size banks in the early stage of their implementation of the capital requirements.

# 2.5.2. Standardised Approach (SA)

Under the *standardised approach* (SA), banks' activities are divided into eight *business lines*: corporate finance; trading and sales; retail banking; commercial banking; payment & settlement; agency services; asset management; and retail brokerage. Within each business line, gross income is a broad indicator that serves as a proxy for the scale of business operations and thus the likely scale of operational risk exposure. The capital charge for each business line is calculated by multiplying gross income by a factor (denoted by  $\beta$ ) assigned to that business line. As in 2.1, the total capital charge is calculated as a three year average over positive GIs, resulting in the following capital charge formula:

$$RC_S^t(OR) = \frac{1}{3} \sum_{i=1}^{3} \max \left[ \sum_{j=1}^{8} \beta_j GI_j^{t-1}, 0 \right]$$
 (2.6)

Note that in formula (2.2), in any given year t-i, negative capital charges (resulting from negative gross income) in some business line j may offset positive capital charges in other business lines.

The June 2004 Basel II guidelines suggested an option to adopt an alternative version of the standardized approach, which remain valid under the recent June 2006 Basel II guidelines. Once a bank has been allowed to use this alternative, it will not be allowed to

revert to use the standardized approach without the permission of its supervisor. Under *alternative standardized approach*, the capital charge for the retail banking and the commercial banking business lines is calculated by taking total loans and advances as the operational risk exposure indicator, instead of gross income.

As set by the Basel Committee, the beta coefficient for each business line is shown in the following table.

Table 2.3: Beta Factors for the Standardised Approach

Business line ( <i>j</i> )	Beta factors $(\beta_j)$		
j = 1, corporate finance	18%		
j = 2, trading & sales	18%		
j = 3, retail banking	12%		
j = 4, commercial banking	15%		
j = 5, payment & settlement	18%		
j = 6, agency services	15%		
j = 7, asset management	12%		
j = 8, retail brokerage	12%		

Source: BIS (2006b)

Hull (2007) contends that several conditions must be satisfied by a bank in order to use the standardised approach:

- 1. The bank must have an operational risk management function that is responsible for identifying, assessing, monitoring, and controlling operational risk;
- 2. The bank must keep track of relevant losses by business line and must create incentives for the improvement of operational risk;
- 3. There must be regular reporting of operational risk losses throughout the bank;
- 4. The bank's operational risk management system must be well documented;
- 5. The bank's operational risk management processes and assessment system must be subject to regular independent reviews by internal auditors. It must also be subject to regular review by external auditors or supervisors or both.

In sum, in order to qualify for the standardized approach, banks must be able to map their business activities into the business lines.

# 2.5.3. Advanced Measurement Approach (AMA)

Advanced measurement approach (AMA) are the most complex and advanced, as the resulting capital charge is the risk measure directly derived from the bank's internal loss data history and employs quantitative and qualitative aspects of the bank's risk measurement system for the assessment of the regulatory capital charge. To ensure reliability of the assessment methodology, apart from the internal data, banks may supplement their databases with external data as well as utilizes technique such as factor analysis, stress tests, and Bayesian methods.

While the *BIA* and *SA* approaches prescribe the explicit formulas (2.1) and (2.2), *AMA* approach lays down general guidelines. In the words of the Basel Committee (BIS: 2003):

Given the continuing evolution of analytical approaches for operational risk, the Committee is not specifying the approach or distributional assumptions used to generate the operational risk measure for regulatory capital purposes. However, a bank must be able to demonstrate that its approach captures potentially severe "tail" loss events. Whatever approach is used, a bank must demonstrate that its operational risk measure meets a soundness standard comparable to that of the internal ratings-based approach for credit risk (comparable to a one year holding period and the 99.9 percent confidence interval).

Banks are expected to gather internal data on repetitive, high frequency losses (three to five years of data), as well as relevant external data on non-repetitive low-frequency losses. Moreover, they must add stress scenarios both at the level of loss severity (parameter shocks to model parameters) and correlation between loss types. In the absence of detailed joint models for different loss types, risk measures for the aggregate loss should be calculated by summing across the different loss categories. In general, both so-called *expected* and *unexpected* losses should be taken into account (*i.e.* risk measure estimates cannot be reduced by subtraction of an expected loss amount).

$$\left\{X_{k}^{t-i,b,\ell}: i=1\right\}, \dots, T; b=1, \dots, 8; \ell=1, \dots, 7; k=1, \dots, N^{t-i,b,\ell}\right\}$$
(2.7)

where  $X_k^{t-i,b,\ell}$  stands for the k th loss of type for business line b in year t-i;  $N^{t-i,b,\ell}$  is the number for such losses and  $T \ge 5$  years, say. Note that thresholds may be imposed for each  $(i,b,\ell)$  category and small losses less than the threshold may be neglected. Consequently, the total historical loss amount for business line b in year t-I is as follows:

$$L^{t-i,b} = \sum_{\ell=1}^{7} \sum_{k=1}^{N^{t-i,b,\ell}} X_k^{t-i,b,\ell}$$
(2.8)

And the total loss amount for year t - i is

$$L^{t-i} = \sum_{b=1}^{8} L^{t-i,b} \tag{2.9}$$

The problem in the *advanced measurement* approach is to use the loss data to estimate the distribution of  $L_t$  for year t and to calculate risk measures such as VaR. Denoting  $\wp_\alpha$  for the risk measure at a confidence level  $\alpha$ , the regulatory capital is determined by

$$RC_{M}^{t}(OR) = \wp_{\alpha}(L^{t}), \tag{2.10}$$

where  $\alpha$  would typically take a value in the range 0.99-0.999 imposed by the local regulator. Because the joint distributional structure of the losses in (2.8) and (2.9) for any given year is generally unknown, it would then typically resort to simple aggregation of risk measures across loss categories to obtain a formula of the form

$$RC_{AM}^{t}(OR) = \sum_{b=1}^{8} \wp_{\alpha}(L^{t,b})$$
 (2.11)

A crucial requirement that the banks must satisfy in order to qualify for the *AMA* is to have the availability of a minimum of five years of internal data. In the event of the lack of data and the bank is exposed to infrequent which leads to potentially severe losses, internal data may be complemented by external data, such as publicly available data or pooled industry data.

Nonetheless, with respect to quantification of regulatory capital, it is still a nascent subject, particularly in the implementation of *AMA*. As reported in *Quantitative Impact Study* by Bank for International Settlement (BIS), out of 357 banks surveyed, only 22 banks gave an estimate of operational risk capital using *AMA* approach, all of which belong to the G10 countries (BIS, 2006). Interestingly, the study also reveals that operational risk is the main driver for increasing minimum required capital. It is the single largest positive contributor to increase in risk capital among all other exposures, including credit risk exposure and market risk exposure. In light of this, the subsequent section discusses some empirical aspects of operational risks and also highlights some of the data issues in conducting empirical measurement for operational risk exposures.

#### 2.6 A REVIEW OF EMPIRICAL STUDIES

As mentioned above, losses due to operational risk can be direct and indirect. Previous studies have addressed the indirect impact of operational risk on the market value of a bank's equity. Particularly, several studies explored reputational risk caused by operational loss events, and its impact on the market value of a bank. Reputational damage inflicted on a bank as a result of an operational loss can be considered as an indirect operational loss. Perry and de Fontnouvelle (2005) argue that from the point of view of shareholders, operational loss announcements convey negative information about the bank's internal activities and signal direct adverse effect on the future cash flows generated by the bank. Therefore, the bank's stock price will be directly affected by such an announcement. Similar studies have also been conducted by Palmrose *et al.* (2004), and Murphy *et al.* (2004).

Having the similar framework of analysis, Cummins *et al.* (2004) examine the effect of the announcements of large operational loss events on the market value of financial firm. They argue that announcements of large operational losses events have statistically significant adverse effect on the stock price. They found statistically significant evidence that the material damage to the firm value exceeded the amount of actual operational loss, which in turn implies that there is reputational damage to the firm due to an announcement of a large operational loss. Furthermore, they showed that firms with

stronger growth opportunities suffer greater losses in the market value due to operational loss announcements.

Another important empirical finding of operational risk is generated by de Fontnouvelle et al. (2004), who used loss data covering six large internationally active banks as part of the BIS' (2003a) operational risk loss data exercise to find out if the regularities in the loss data make consistent modeling of operational losses possible. Their results turned out to be consistent with the publicly reported operational risk capital estimates produced by banks' internal economic capital models. Using the similar data from the BIS's exercise, Moscadelli (2005) performs a thorough comparison of traditional full-data analyses and extreme value methods for estimating loss severity distributions. He found that extreme value theory outperformed the traditional methods in all of the eight business lines proposed by the BIS. He also found the severity distribution to be very heavy-tailed and that a substantial difference exists in loss severity across business lines. In a similar study, Wei (2003) examined operational risk in the insurance industry. By using data from the OpVaR operational loss database, his results indicate that operational loss events have a significantly negative effect on the market value 182 of the affected firms and that the effect of operational losses goes beyond the firm experiencing the loss event. The conclusion derived from this study is that the significant damage of market values of both the insurers and the insurance industry caused by operational losses should provide an incentive for operational risk management in the U.S. insurance industry. In addition, Wei (2007) expanded the analysis by estimating the aggregate tail operational risk exposure, implementing a Bayesian approach to estimate the frequency distribution, while estimating the severity distribution by introducing a covariate. He concluded that the main driving force of the capital requirement is the tail distribution and the size of a bank.

In another study, Wei (2006) examined the impact of operational loss events on the market value of announcing and non-announcing U.S. financial institutions using data from the *OpVaR* database. The study evidences significantly negative impact of the announcement of operational losses on stock prices, which also shows declines in market value to be of a larger magnitude than the operational losses causing them, which

supports the conjecture put forward by Cummins *et al.* (2006). By using data from the same source, Cummins *et al.* (2006) conducted an event study of the impact of operational loss events on the market values of U.S. banks and insurance companies, obtaining similar results to those obtained by Wei (2006). They found losses to be proportionately larger for institutions with higher Tobin's Q ratios, which implies that operational losses are more serious for firms with strong growth prospects.

With regard to the data issues, it has been commonly viewed that operational risk data can be a hindrance to conduct empirical studies on the measurement, causes and consequences of operational risk. This point is made explicit by Wei (2007), who suggested that quantification of operational risk has been hindered by the lack of internal and external data on operational losses.

To deal with the data problem, Allen and Bali (2004) estimate an operational risk model for individual financial institutions using monthly time series of stock returns over the period 1973–2003. In another study, de Fontnouvelle *et al.* (2006) address the problem of sample selection bias using an econometric model in which the truncation point for each loss is modeled as an unobserved random variable. By using two external operational loss databases to estimate the loss distribution and estimate the capital charge, they conclude that the regulatory capital held against operational risk often exceeds that held against market risk. They also conclude that supplementing internal data with external data on extremely large events could result in a significant improvement in operational risk models.

# 2.7. CONCLUDING REMARKS

The Basel standard, with the proposed variety of increasingly sophisticated and risk sensitive methodologies for quantifying the capital charge, conceives measurement of operational risk in the wider and more comprehensive context. It happens as a response of an increasing number of business institutions suffering from huge operational losses during 1980s and 1990s.

Assessing operational risk in numerical terms is crucial, and yet, very challenging, since the aspects of operational risks encompassing quantifiable and non-quantifiable variables. However, one important factor has to be maintained; the availability and the right calculation of risk based capital. While regulatory capital is essential to ensure that there is a minimum amount of capital held in the banks, economic capital is very crucial for the bank to protect the banks against insolvency due to various risks inherently attached in the banking business. Moreover, quantifying economic capital is basically the expression for unexpected losses.

Three major standards; basic indicator approach (BIA), standardized approach (SA), and advanced measurement approach (AMA) have also been introduced by the regulators in order to help banks determine the amount of regulatory capital that they should hold, depending on the scale of business as well as their capitalization.

# **CHAPTER 3**

# OPERATIONAL RISKS IN ISLAMIC BANKS: A CONCEPTUAL INQUIRY

#### 3.1 INTRODUCTION

Operational risk management in financial institutions has undoubtedly attracted more attention from the regulators, practitioners, and also academia over the last decade. This is partly attributed to the huge losses incurred by a number of financial institutions such as Barings, Daiwa and Merril Lynch, resulting from malfunctioning of their operational risk management (Hoffman, 2002; Hull, 2007; and Hussain, 2000). Learning from such incidents, the regulators and practitioners have seriously taken the issue. In spite of the wide range of areas and issues in operational risks that need to be catered, attempts to define and classify operational risk have been made by several institutions, most notably by Basel Committee on Banking Supervision or Bank for International Settlements (BIS). For this purpose, BCBS proposed a definition of operational risk through its consultative document on operational risk (BIS, 2001a).

The industry has a wide range of responses to the definition proposed by BIS. Despite the raised criticisms, a positive side of the proposal is that banks started to realise the importance of managing operational risk, and therefore they began to put aside a certain percentage of capital for operational risk, in addition to credit and market risk.

As an emerging and growing industry, Islamic banking industry also need to develop strategies and instruments to cater the same issue as highlighted by Akkizidis and Kumar (2008), Archer and Haron (2007), Hossain (2005), Iqbal and Mirakhor (2007), Khan and Ahmed (2001), and Sundararajan and Errico (2002), among others. This is not surprising since Islamic banks operate in a similar, if not the same, business environment.

Unfortunately, there have not been many studies tackling the operational risk related issues thoroughly in relation to Islamic banking. This may be due to the fact that operational risk is relatively new area which needs more research, but also the complexities it carries should be considered as another reason for less available material. This is the main reason motivated this chapter.

The chapter starts with a discussion on the nature and origin of Islamic banks and analyse why an Islamic bank has a distinct operational aspect, as compared to the conventional one. It goes on with an examination of operational risk exposures in Islamic banks. The following section discusses how to identify and conduct a mapping of operational risk in Islamic banks, as Islamic banks are also different from conventional ones on the structure of their financial contracts; thus they bring different features of operational risks in different contracts. This is the issue which will be discussed in the subsequent section.

It should be noted that the analysis would not have been complete without tackling the issue of having adequate capital in order to cover operational losses. Finally, the last section is allocated for concluding remarks.

## 3.2. NATURE AND ORIGIN OF ISLAMIC BANKS

The way financial system is constructed can be very central for efficient resource allocation, as the history has shown that the financial system is determined by the nature of financial intermediation. The rapid development in financial system has made financial intermediary more important in the economy. The acquisition and processing of information about economic agents, the packaging and repackaging of financial claims, a financial contracting are among the activities that differentiate financial intermediation from other economic activities (Mishkin, 2004). Consequently, the nature of intermediation has changed drastically over the last three decades due to the changes in macroeconomic policies, liberalisation of capital accounts, deregulation, and advances in financial theory as well as breakthroughs in technology. For example, lending based operations which characterise traditional banking activity has been replaced by more feebased services that bring investors and borrowers directly in contact with each other. In addition, financial intermediation in the form of traditional banking—mainly based on the

operations of lending—has declined considerably in developed countries where marketbased intermediation has become dominant.

In Islamic history, financial intermediation has established a historical record and has made significant contributions to economic development over time. The simplest manifestation of financial services within the early Muslim states took the form of money-changers (*sayarifah*; sing., *sarraf*) who were also partially engaged in the holding of deposits and the short-term financing of trade (Chapra and Khan, 2000). However, a more sophisticated form of banking finance for trade and government was represented by the *jahabidhah* (sing., *jahbadh*) who practiced much of the modern financing activities under the supervision of the Muslim state (Chachi, 2005; Heck, 2006).

In the highly developed market economy of the Abbasid State, *jahabidhah* bankers proliferated throughout the state, even though they were mostly of Jews who enjoyed the status of Ahl al-Kitab origin (People of the Book) within the early Muslim state. The *jahabidhah* were basically trade vendors who concurrently practiced business of financing and commercial transactions to others. Banking operations were therefore ancillary to primary mercantile operations, yet they seemed to have grown to sizeable banking functions particularly when the *jahabidhah* accepted deposits in efforts to augment their own businesses. Moreover, the high streets of Basra were so much supplied with money-changers and *jahabidhah* that the banking network in Basra was rightly called by a Western historian 'the Wall-Street of the Middle Ages' (Heck 2006). In this regard, the famous Persian historian, Nasir-i Khusraw, was reported to have estimated the number of *jahabidhah* bankers in the state of Isfahan alone at 200 banks (Heck 2006).

It was such a complex network of banking activities that the call for appropriate government supervision and regulation was acknowledged by the Islamic state. To this effect, the Abbasid State established a central banking agency in year 316 H/ 929 A.D called *Diwan al-Jahabidhah* to foresee the performance and growth of banks within the empire. Dar al-Mal, on the other hand, was also established as a Treasury institution whose function was to supervise an equally intense *jahabidhah* activities in Fatimid

Egypt. Among the most commonly practiced banking instruments were the *sakk* (the Arabic root of 'cheque') and the *suftajah* (which combined features of traveller cheques and letters of credit), the *hawalah* (which is a means credit transfer), *wadi'ah* (i.e. deposit), *ruq'ah* (which was a sort of promissory note). The use of cheque (*sakk*) was particularly known since the time of the Rightly-Guided Caliphs. A renowned historian, Ibn Abdel-Hakam, reported that Umar ibn al-Khattab paid for the grains delivered to the state warehouses by cheque and that he used to pay government wages by cheques signed by his treasurer Zaid ibn Thabit (Heck, 2006).

Having been regarded as an alternative financial intermediary with profit and loss sharing contract (in *mudarabah* and *musharakah* contract) as its cornerstone, an Islamic bank is, theoretically, expected to bring more stabilisation and efficiency in resource allocation. In addition, an Islamic bank is also equipped with contracts which may, slightly, look similar to what a conventional bank has been commonly practising; *i.e.* debt financing (in *murabahah* contract). Nevertheless, the nature of debt in an Islamic bank is qualitatively different from that of conventional bank since debt contract in an Islamic bank requires to be tied to some underlying assets (Ahmed, 2005 and Khan, 1995). Consequently, the distinctive contractual structure that an Islamic bank embodies necessitates different treatment on the management of the operational system of an Islamic bank.

## 3.3 OPERATIONAL RISK EXPOSURES OF ISLAMIC BANKS

As a modern form of *jahbadh*, an Islamic bank is an institution offering financial services conforming with *Shariah*. A set of *Shariah* principles governing the operations of Islamic banks are:

- (i) prohibition of dealing with interest (*riba*);
- (ii) financial contracts must be cleared from contractual uncertainty (gharar);
- (iii) exclusion of gambling (maysir) in any financial activity;
- (iv) profit must not be originated from *haram* industries (prohibited industries such as those related to pork products, pornography, or alcoholic beverages);
- (v) each financial transaction must refer to a tangible, identifiable underlying asset, and
- (vi) parties to a financial transaction must share in the risks and rewards attached to it.

The principles mentioned above must be, conceptually, inherent in Islamic banks, in order to distinguish them from conventional banks.

With regard to operational risk, Islamic banks face the same challenge as conventional ones, to the extent that it exists in the ordinary course of various banking activities (Archer and Haron, 2007; and Hossain, 2005). At this phase, the challenge is fairly similar for all financial intermediaries, whether *Shariah*-compliant or not. Nevertheless, the challenges are more complex for Islamic banks owing to their activities and unique features of financial contracts. Islamic Financial Services Board (IFSB) clearly mentions in its publication that Islamic banks are exposed to "a range of operational risks that could materially affect their operations" (IFSB, 2007a: 22). Further, it is argued that operational risks are likely to be more significant for Islamic banks due to their specific contractual features (Fiennes, 2007; Greuning and Iqbal, 2008; Iqbal and Mirakhor, 2007; Khan and Ahmed, 2001; Kumar, 2008; Sundararajan and Errico, 2002; and Sundararajan, 2005).

In Islamic banks, operational risk is associated with the loss resulting from "inadequate or failed internal processes, people and system, or from external events, including losses resulting from *Shariah* non compliance and the failure in fiduciary responsibilities" (IFSB, 2005a: 26). It is understood that the definition of operational risk in Islamic banks includes legal risk (Archer and Haron, 2007; Cihak and Hesse, 2008; Djojosugito, 2008, Fiennes, 2007; Khan and Ahmed, 2001; and Sundararajan, 2005), and also reputational risk (Fiennes, 2007; Akkizidis and Kumar, 2008; Standard & Poor's, 2008). The foremost distinctive feature of this definition, as compared to the definition by Basel II, is the inclusion of *Shariah* non-compliance risk and fiduciary risk. As a matter of fact, *Shariah* non-compliance risk is considered to have a significant portion in operational risk (IFSB, 2007b: 6).

*Shariah* non-compliance risk is the risk arising from Islamic banks' failure to comply with the *Shariah* rules and principles determined by the *Shariah* Board or the relevant body in the jurisdiction in which the Islamic bank operates (IFSB, 2005a). The failure to comply with such principle will result in the transaction being cancelled, and hence the

income or loss cannot be recognised. Moreover, fiduciary risk is the risk that arises from Islamic banks' failure to perform in accordance with explicit and implicit standards applicable to their fiduciary responsibilities (IFSB, 2005a).

Another distinctive aspect from the definition is the recognition of reputational risk. Maintaining good reputation is crucial for Islamic banks (Hamidi, 2006) since a failure to do such thing could trigger an exodus of funds, which would result in a liquidity crisis. Reputational damage could also make retail customers stop requesting financing from Islamic banks, triggering a downturn in profitability. Therefore, in order to keep good reputation, it is suggested that Islamic banks need to do two things; firstly, to ensure that their financial products are *Shariah* compliant (Greuning and Iqbal, 2008; and Iqbal and Mirakhor, 2007), secondly, to effectively maintain their fiduciary roles (Muljawan, 2005).

The spotlight above explains why operational risk management in Islamic banks is not similar to that in conventional banks. There are a number of dimensions need to be added in the analysis. Although it is argued earlier that the challenge is somewhat similar, however, it is only to the extent that Islamic banks and conventional banks are dealing with various banking activities. To a greater extent, operational risk management in Islamic banks requires more thorough understanding of the sources of operational risk from which the loss could occur. Operational risks in Islamic banks could, therefore, appear based on the following major sources: (i) *Shariah* non-compliance risk; (ii) fiduciary risk; (iii) people risk; and (iv) legal risk

# 3.3.1 Shariah Non-Compliance Risk

IFSB guiding principles of risk management for institutions offering Islamic financial services—other than insurance institutions, clearly mentions the definition of *Shariah* non-compliance risk. It is the risk which arises from "IIFSs" failure to comply with the *Shariah* rules and principles determined by the *Shariah* board of the IIFS or the relevant

<sup>&</sup>lt;sup>19</sup> IIFS stands for institutions (other than insurance companies) which offer only Islamic financial services. In many literatures, the term "Islamic banks", "IIFS" or "Islamic financial institutions" are used interchangeably. IFSB opts to use IIFS in its publication.

body in the jurisdiction in which the IIFS operate" (IFSB, 2005a: 26). For Islamic banks, to be *Shariah* compliant is paramount. According to IFSB principle 7.1, Islamic banks shall have in place adequate system and controls, including Shariah Board/Advisor, to ensure compliance with *Shariah* rules and principles (IFSB, 2005a: 27). Such compliance requirements must be pervasively infused throughout the organisation as well as in their products and activities. *Shariah* compliance is considered by IFSB as a higher priority category in relation to the other identified risks, since violation of *Shariah* principles will result in the transactions being cancelled or income generated from them shall be considered as illegitimate.

The need to ensure compliance with *Shariah* in operational risk management is vital (Aziz, 2006) and it must encompass the products, activities, and contract documentation — with regard to formation, termination and elements which might possibly affect contract performance such as fraud, misrepresentation, or duress. Furthermore, the degree of *Shariah* compliance, as IFSB (2005a) suggests has to be reviewed, at least, annually which can be performed by a credible party, either from a separate *Shariah* control department or as part of the existing internal and external audit. The main objective is to ensure that (a) the nature of Islamic banks' financing and equity investment and (b) their operations are executed in adherence to the *Shariah* principles.

In the event that *Shariah* non compliance occuring, either in the products or activities, Islamic banks need to keep record of the profits out of it. The record will help Islamic banks assess the probability of similar cases arising in the future. Further, historical reviews and data of potential areas of *Shariah* non-compliance will enable Islamic banks to make an assessment on the potential profits which can not be recognised as legitimate profits. In order word, potential loss could be managed, hence, reduced to a minimum level.

With respect to *Shariah* requirements in financing contracts, albeit the diversity of interpretations prevalent in the industry, Accounting and Auditing Organisation for Islamic Financial Institutions (AAOIFI) has already issued its latest *Shariah* standard (AAOIFI, 2005) that could be referred to by Islamic banks. In sum, *Shariah* compliant

financing in the main contractual forms needs to fulfil the following *shariah* requirements:

# (i) Murabahah and ijarah contracts:

- The asset is in existence at the time of sale or lease or, in *Ijarah*, the lease contract should be preceded by acquisition of the usufruct of the leased asset.
- The asset is legally owned by Islamic banks when it is sold.
- The asset is intended to be used by the buyer/lessee for activities or business permissible by *Shariah*; if the asset is leased back to its owner in the first lease period, it should not lead to contract of 'inah, by varying the rent or the duration.
- In the event of late payment, there is no penalty fee or increase in price in exchange for extending or rescheduling the date of payment of accounts receivable or lease receivable, irrespective of whether the debtor is solvent or insolvent.

# (b) Salam and istisna' contracts:

- A sale and purchase contract cannot be inter-dependent and inter-conditional on each other. This is for the case of *salam* and parallel *salam* or *istisna* and parallel *istisna*.
- It is not allowed to stipulate a penalty clause in respect of delay in delivery of a commodity that is purchased under *salam* contract. However, it is allowed under *istisna*' or parallel *istisna*'.
- The subject matter of an *istisna*' contract may not physically exist upon entering into the contract.

# (c) Musharakah and mudarabah contracts:

- The capital of the Islamic banks is to be invested in *Shariah* compliant investments or business activities.
- A partner in *musharakah* cannot guarantee the capital of another partner or a *mudarib* guarantees the capital of the *mudarabah*.
- The purchase price of other partner's share in a *musharakah* with a binding promise to purchase can only be set as per the market value or as per the agreement at the date of buying. It is not permissible to stipulate that the share be acquired at its face value.

## 3.3.2 Fiduciary Risk

Islamic banks are liable for losses arising from their negligence, misconduct or breach of their investment mandate; the risk of losses which arises from such events is characterised as a fiduciary risk. In other word, fiduciary risk is an indication of failure to "perform in accordance with explicit and implicit standards applicable to their fiduciary responsibilities" (IFSB, 2005a: 26). The indication of such failure can be seen from the high degree of their earnings volatility. As a result of losses, Islamic banks may become insolvent and as a consequence unable to (a) meet the demands of current account holders for repayment of their funds, or (b) protect the interests of its investment account holders.

In performing their fiduciary role, Islamic banks are enforced to preserve the interests of all fund providers, as prescribed by IFSB standard on risk management principle 7.2 (IFSB, 2005a: 2). In doing so, Islamic banks must ensure that the bases for "asset, revenue, expense and profit allocations are established, applied and reported in a manner consistent with Islamic banks' fiduciary responsibilities" (IFSB, 2005a: 27).

Islamic banks' fiduciary duty is all about preserving the trust from all fund providers. Two important aspects that seriously need to be taken into consideration in safeguarding the trust are; (a) *Shariah* aspect; Islamic banks must ensure that the activities and the products are *Shariah*-compliance, (b) *Performance* aspect; Islamic banks are required to have sound financial performance, without which, fund providers might indicate that there is mismanagement or misconduct.

In the *Shariah* aspect, Islamic banks may follow the guidance set by their own or independent *Shariah* supervisory board, while in the *performance* aspect Islamic banks may create policy which includes the following:

- Identification of investing activities that contribute to investment returns and taking reasonable steps to carry on those activities in accordance with the Islamic banks' fiduciary and agency duties and to treat all their fund providers appropriately in conformity with the terms and conditions of their investment agreements;
- Allocation of assets and profits between the IIFS and their investment account holders (IAH) will be managed and applied appropriately to IAH having funds invested over different investment periods;
- Determination of appropriate reserves at levels that do not discriminate against the right for better returns of existing IAH;
- Limitation the risk transmission between current and investment accounts.
- Timely provision of information disclosure to IAH and the market as a reliable basis for assessing their risk profiles and investment performance.

In terms of fiduciary risk, the element of trust is very important in the relationship between Islamic banks and the fund providers. This relationship, as Iqbal and Mirakhor (2007) argue, distinguishes Islamic banks from conventional ones and is the sole justification for the existence of the Islamic banks. Thus, Islamic banks are always expected to act in the best interests of their fund providers, *i.e.* investors/depositors and shareholders. In respect with fiduciary role, Islamic banks are exposed to fiduciary risk if they are unable to align the objectives of the investors and shareholders with the actions that they are supposed to carry out.

In terms of consequences, the fiduciary risk can be enormous, particularly if Islamic banks start to lose their reputation from their customers. Iqbal and Mirakhor (2007) show that fiduciary risk can give a huge impact on the bank's cost and access to liquidity. If the

banks are declared to be insolvent, which is the worst case, the banks are unlikely able to meet the demands of the current and investment account holders. Hence, a sound level of solvability help Islamic banks enhances their credibility in sights of the fund provider. For this reason, Muljawan (2005) suggests three numerical indicators which can possibly be used to indicate the level of a bank's solvency; first, *capital adequacy ratio* (CAR) based on IFSB directives; second, *equity coverage ratio* which reflects the capability of the own capital to effectively cover the potential loss emanated from bank's financial exposures; and third, *leverage ratio* that indicates the estimate of the residual claims of the bank.

Examples of fiduciary risk exposures are as follows (Greuning and Iqbal, 2008; Iqbal and Mirakhor, 2007):

- In the case of a partnership based investment in the form of *mudarabah* and *musharakah* on the assets side, the bank is expected to perform adequate screening and monitoring of projects and any deliberate or even non-deliberate negligence in evaluating and monitoring the project can lead to fiduciary risk. It becomes incumbent upon management to perform due diligence before committing the investors/depositors' funds.
- Mismanagement of funds of current account holders, which are accepted on trust (amanah) basis, can expose the bank to fiduciary risk as well. It is a common practice of Islamic banks to utilise current account holders' funds without any obligation to share the profit with them. However, in a case of heavy losses on the investment financed by current account holders' funds, the depositors can lose confidence in the bank and this can lead to their taking legal recourse.
- Mismanagement in governing the business, incurring unnecessary expenses or allocation of excessive expenses to investment account holders is a breach of the implicit contract to act in a transparent fashion.

# 3.3.3 People Risk

People risk is another type of operational risk arising from incompetence or fraud, which exposes Islamic banks to potential losses. This includes human errors, lack of expertise, compliance and fraud (Akkizidis and Kumar: 2008). The risk of a loss intentionally or unintentionally caused by an employee, such as employee error and employee misdeeds, or involving employees such as in the area of employment disputes, is the risk class that covers internal organisations problems, fraud and losses. Unfortunately, as Akkizidis and Kumar (2008) contend, the largest amount of losses comes from intentional activities such as fraud and unauthorised trading. For instance, an internal control problem cost the Dubai Islamic Bank US\$50 million in 1998 when a bank official did not conform to the bank's credit terms. This also resulted in a run on its deposits of US\$138 million, representing 7% of the bank's total deposits, in just one day (Warde, 2000: 155). Another case involving a large unauthorised loan, around US\$242 million, was also caused by bank official of the Dubai Islamic Bank and West African tycoon Foutanga Dit Babani Sissoko (Warde, 2000: 156).

Although there has not been any single research assessing the exposure of people risk in Islamic banks, it is understood that the challenge is considerably high. The thriving development of Islamic banking industry, unfortunately, has not been matched up with the number of people who have credentials in running and directing the business, which as an issues has been highlighted by Aziz (2006), Edwardes (2002), Jackson-Moore (2007), Khan (2004), Khan and Ahmed (2001), and Kumar (2008), and Nienhaus (2007).

The dimension of people risk in Islamic banks is understandably wider than in conventional ones since the personnel of Islamic banks' personnel are required to be well-versed in both conventional banking products and their status in relation to Islamic requirements (Aziz, 2006; Ebrahim, 2007; Nienhaus, 2007). There is a need, hence, that Islamic banking industry must be equipped with a new breed of innovators, risk managers, regulators and supervisors who have the right blend of knowledge of finance and the understanding of the *Shariah* (Aziz, 2006).

Furthermore, they should be aware of the existing Islamic alternatives and their commercial advantages and disadvantages compared to the conventional products (Nienhaus, 2007). A shortage in skilled bankers, who are, at the same time, well versed in *Shariah* or *Shariah* scholars who are familiar with conventional banking products, as Jackson-Moore (2007) contend, will lead to higher people risk. In other word, inadequately trained staff or incapable personnel will expose Islamic banks unnecessarily to operational risk. In response to a very demanding industry, staffs of Islamic banks must be able to design *Shariah* compliant financial innovations in order to meet the diversified needs of the clients and to match the ever-increasing scope of conventional techniques, procedures, and products. More importantly, despite the fact of such challenges, staffs of Islamic banks should be able to create financial contracts which are more than just legally interest free. In sum, skilled staffs of Islamic banks will ensure that the products are efficient as well as *Shariah*-compliant. Unskilled staffs can cause the product to be, either illegitimate according to *Shariah* or inefficient.

The above mentioned fraud mentioned above affecting the the Dubai Islamic Bank shows that Islamic banking institutions is not free from fraud, whether intentional or unintentional. According to Akkizidis and Kumar (2008), it is intentional activities such as fraud and unauthorised trading which has caused largest amount of losses. Fraud invades every area of businesses and is committed when a motive coincides with an opportunity. Moreover, Akkizidis and Kumar (2008) suggest that financial institutions should establish appropriate system and thorough control for the management of operational risks that may arise from employee. Hence, the following direction can be established (Akkizidis and Kumar, 2008: 194-195):

- A selection of employees that respect and follow the *Shariah* principles
- A separation of the employees' duties
- An internal supervision of the employees' performances
- A monitoring of the employees' behaviour
- Well established policies that are complying with the *Shariah* principles and are well known by all employees
- Training process to direct the employees in the process of the risk management
- Well defined employment termination policies and procedures.

### 3.3.4 Technology Risk

In an advanced financial industry, an Islamic bank's operations are very much dependent on its technological system. Its success depends, in great part, on its ability to assemble increasingly rich databases and make timely decisions in anticipation of client demands and industry changes. The advanced use of information technology (IT) has also brought a new facet in the current competition of Islamic banking industry, as it is often that a success of an Islamic bank's business is determined by the ability to capitalise the use of an information technology in different ways. An inability to keep up with the advanced use of an information technology could cause an Islamic bank fall behind its competitors. Therefore, every Islamic bank must be committed to an ongoing process of upgrading, enhancing, and testing its technology, to effectively meet (Chorafas, 2004: 91): (a) sophisticated client requirements, (b) market and regulatory changes, and (c) evolving internal needs for information and knowledge management.

Chorafas (2004) argues that a failure to respond to the above prerequisites could increase an exposure to operational risk related to IT. In addition, the use of software and telecommunications systems that are not tailored to the need of Islamic banks could also contribute to technology risk, as well as many other internal such as such as human error, internal fraud through software manipulation (Chorafas, 2004: 91), programming errors, IT crash caused by new applications, incompatibility with the existing systems, failures of system to meet the business requirements (Akkizidis and Kumar, 2008: 191), external fraud by intruders; obsolescence in applications and machines, reliability issues, mismanagement, and the effect of natural disasters.

It is clear, hence, that the extensive use of an information technology could increase IT related operational risk in number and severity originating from internal as well as external events. However, high technology allows a visualisation which turns numbers into graphs and images. Unfortunately, only few financial institutions have the ability to capitalise the best that the technology can offer (Chorafas, 2004). Spending big sums of money on technology without the corresponding return on investment (ROI) is also an indication of an IT-related operational risk.

### 3.3.5 Legal Risk

The inclusion of legal risk as part of broader notion of operational risk, however, has been a subject of debate among academician and practitioners (Hadjiemmanuil, 2003; and Scott, 2001). One of the reasons might be due to the difficulties in defining its nature (Scott, 2001). Furthermore, as Scott (2001) argues, legal risk has an unpredictable effect although it determines the amount of losses that banks have to incur. Integrating legal risk as a subset of operational risk is also criticised because of being neither self-evident nor universally accepted (Hadjiemmanuil, 2003). For instance, in May 2000 the IFCI Financial Risk Institute, a non-profit foundation established by derivatives exchanges, market participants and regulators issued descriptions of principal sources of risks which concern regulators' in derivatives and commodities markets. The documents specifically include market, credit, settlement and 'other' risk. On this account, the residual 'other' category covers, in particular, liquidity, legal and operational risks. With regard to legal risk, the document defines it as "the risk that a transaction proves unenforceable in law" (Jorion, 2009: 590). Typical examples of legal risk are also given. These include legal uncertainties surrounding the legal capacity of banks' contractual counterparties to enter into binding transactions, the legality of derivatives transactions and/or the recognition and effectiveness of netting arrangements in bankruptcy (IFCI Financial Risk Institute, 2000; as cited in Hadjiemmanuil, 2003).

As a matter of fact, the meaning of legal risk varies, depending on the specific context and the practical concerns of the persons employing it (Hadjiemmanuil, 2003). In relation to litigation or liability insurance, the term may refer mainly to civil liabilities, including duties to compensate the victims of torts and to make contractual payments or provide indemnities in certain contingencies. In derivatives market, much emphasis is placed on uncertainties regarding the legal recognition of novel contractual arrangements, which have not been tested in the courts. In international lending or project finance, a major concern is the relative risk of doing business in different countries; to a significant extent, this depends on differences between their legal and judicial systems - in particular, their effectiveness in enforcing creditors' rights.

Furthermore, Hadjiemmanuil (2003: 77) suggests that there are different ways in which loss may arise, all of which are often classified under the domain of legal risk. Thus, the loss may be attributable to:

- (i) Legally flawed actions of the bank or its employees and agents, as a result of which the bank either incurs direct liabilities or becomes unable to ascertain in law certain rights in order to protect its interests;
- (ii) Legal uncertainty; which does not depend on any fault of the bank itself, since this is an external parameter, it affects even the most diligently and prudently run institutions. Sometimes, the law is intentionally expressed in general and abstract terms. Because of informational constraints, it is impossible to draft complete rules which make special provision for each and every eventuality;
- (iii) Legal uncertainties and financial innovation. Innovation, however, is a significant contributor to legal risk as well. The adoption of new and complex transactional techniques, in particular, is often surrounded by significant legal uncertainty and can expose banks to potentially catastrophic risk;
- (iv) Country specific legal perils and costs. The term legal risk can also refer to the relative risk of doing business in different countries, as a function of the quality of their legal system. Jurisdictions can be compared by reference to the effects of their laws and judicial systems in terms of increasing or attenuating the risk. From this perspective, legal risk is primarily an attribute of legal system, not of the banking institutions or of their activities. This approach may be useful in relation to international lending or project-finance activities, where the evaluation of a country's relative legal risk can have significant pricing and risk management implications.

Despite his critics, Hadjiemmanuil (2003) shows the reasons why legal risk is associated with operational risk, it is because fraud which occurs in financial institutions is considered to be both (a) the most significant category of operational loss event, and (b) a legal issue.

In Islamic banking context, although the term 'legal risk' is not clearly mentioned in the IFSB standard when specifying the aspect of operational risk, nevertheless, from the contributions by Cihak and Hesse (2008), Djojosugito (2008), Hassan and Dicle (2005), Iqbal (2005), Kahf (2005), Kumar (2008), Nienhaus (2005), and Sundararajan (2005), the impacts of legal risk on Islamic bank, with regard to the spectrum of operational risk management, are substantial and cannot be neglected. In Islamic banks, legal risk may arise from uncertainty in laws (Kumar, 2008), lack of reliable legal system to enforce financial contracts (Djojosugito, 2008; Iqbal, 2005; Sundararajan and Errico, 2002; and Sundararajan, 2005), legal uncertainty in the interpretations of contracts (Cihak and Hesse, 2008), the legality of financial instruments (Djojosugito, 2008), lack of availability of legal experts (Kumar 2008), and exposure to unanticipated changes in laws and regulations (Djojosugito, 2008). In addition, it is argued that some operational aspects of Islamic banking activities are not sufficiently covered by laws, which in turn, results in the exposure of legal risk to Islamic banks (Djojosugito, 2008). It comes from the fact that most of Islamic banks, at the current stage, operate within similar legal and business environments (Hassan and Dicle, 2005; and Kahf, 2005).

Although the profile of legal risk in Islamic banks seemingly to be similar to the conventional ones, however, the fact can be substantially different if the *Shariah* aspect is taken into account in the operation of laws. For instance, there is a requirement that the Court refer to the question of *Shariah* to *Shariah* people. However, the legal risk is still present since the final decision will still be decided by the Court.

The other problem is regarding the jurisdiction of *Shariah* board. In Indonesia, for example, the *fatwas* of *National Shariah Board* (DSN<sup>20</sup>) are only binding upon the *Shariah* board of the Islamic banks, but not to the financing recipient of the Islamic banks. Consequently, the party who receives financing can freely invest in any way he likes. While it seems that it will not affect the Islamic bank directly, it will certainly affect the income of the Islamic banks which in turn result in the income considered illegitimate as it does not comply with *Shariah* principles. Although *Shariah* principles

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<sup>&</sup>lt;sup>20</sup> DSN stands for Dewan Syariah Nasional, which means Shariah National Board

which must be adhered to, can be included in the covenants, but such practice will not be effective unless there is a consistent *Shariah* interpretation and legal enforcement.

Another common legal risk faced by Islamic banks is the issue of legality of Islamic financial instruments (Djojosugito, 2008). The absence of recognisable laws pertaining to Islamic financial instruments will also cause some transactions considered as illegal by law even though they meet the requirements of *Shariah* principles. This is the case in Indonesia where the law views some of *mudarabah* bonds issued as debt which in effect is guaranteed by the patrimony of *mubarib*. While *Shariah* prohibits such recourse, the law will not uphold the *Shariah* prohibition.

Legal capacity is another element of legal risk that affects the operations of Islamic banks. The legal capacity is defined as the legal authority to enter into a contract. The consequence of non-existence of any legal capacity is that the contract is deemed *ultra vires*, and is therefore unenforceable. One example can be cited in this regard. It is related to the use of Special Purpose Vehicle (SPV) in project finance transactions. While the *Shariah* will view the SPV as either a *mudarib* or a *wakil*, the law will view them as a trustee (Djojosugito, 2008: 117). At the outset, the difference seems trivial as all of them share many things in common. However, when it comes to the issue of legal capacity of the SPV to enter into a contract, the legal consequences depend on whether the SPV is regarded as *mudarib*, *wakil* or simply a trustee. A transaction entered into by a SPV which is perfectly acceptable under *Shariah* may be declared unlawful or illegal by a court (Djojosugito, 2008: 118).

Compared to jurisprudences in Common and Civil law regimes, the *Shariah* jurisprudence has different phase of development. While the Common Law jurisprudence was developed by way of precedent and the Civil law by way of codification, the Islamic jurisprudence was developed by way of scholarship. These facts make the conventional law sometimes cannot accommodate fully the principles of *Shariah*. One example may be cited in relation to IDB *Sukuk* (Djojosugito, 2008: 118). It is stated that the decrease in the proportion of the pool to below 25 % will trigger a dissolution event. While the decrease in the proportion itself is another type of risk, the treatment arises from such

decrease will trigger a legal risk. While such a condition is not acceptable from *Shariah* point of view and must lead to dissolution of the *Sukuk* because the prohibition in trading of debts is absolute, the liquidation may not be authorised from legal point of view because the law will see the reason as to why the proportion decreases to such level taking into account the interest of the *Sukuk*'s holders. The other relevant issue is on governing law, the fact that all Islamic banks' financing is governed by laws other than *Shari'ah*. The Shamil Bank case is a good representation on what a court may interpret *Shari'ah*. The judgment of the case clearly state that *Shariah* cannot invalidate English law even though the governing law of the contract is subject to the Glorious Principles of *Shariah*. This represents legal risk as a contract, which is supposed to be governed by *Shari'ah*, will be interpreted not according to *Shariah*.

#### 3.4 IDENTIFICATION OF OPERATIONAL RISKS

A main part in designing an effective operational risk management system is the identification of both internal and external operational risks (Akkizidis and Kumar, 2008). Internal operational risk attributes loss exposure to the potential for failure of people, processes and technology in the course of regular business operations, such as breaches in internal controls and monitoring, internal and external fraud, legal claims or business disruptions and improper business practices (Zamorski, 2003). These risks are more specifically defined as

- (i) *process risk* associated with operational failures stemming from the breakdown in established processes, failure to follow processes or inadequate process mapping within business lines;
- (ii) *people risk* from management failure, organisational structure or other human failures, which may be exacerbated by poor training, inadequate controls, poor staffing resources, or other factors, and
- (iii) *system* risk, which reflects the operational exposure to disruptions and outright system failure in both internal and outsourced operations. *External* operational risk (or external dependency risk) arises from environmental factors, such as new

competitor that changes the business paradigm, a major political and regulatory regime change, unforeseen (natural) disasters, terrorism, vandalism, and other such factors that are outside the control of the firm (Mark, 2002, as cited in Jobst, 2007).

In Islamic banks, such identification should also refer to the operational risks for the insufficient compliance with *Shariah* rules and principles as clearly exemplified in the section 3.3.1. Financial institutions should identify and assess the operational risk inherently in all products, activities, processes, and systems. Moreover, based on Basel II and IFSB directives, further states that risk identification is essential for the consequent development of a practical operational risk monitoring and control system. However, the key factors that negatively affect the financial institution in terms of reaching their business objectives should be identified first.

Effective risk identification considers both internal and external factors that could negatively affect the process of reaching the financial institution's objectives. Some internal factors are:

- (i) the structures of the institution's accounts;
- (ii) the corresponding contracts;
- (iii) the nature of the institution's activities;
- (iv) the quality of the institution's human resources; and
- (v) the organisational changes and employee turnover.

Moreover, some external factors are:

- (i) the changes in the industry; and
- (ii) the technological advances.

It should be a standard practice for a financial institution's management to implement policies and procedures to manage risks arising from their operational activities. The institution, hence, should maintain written policies and procedures that identify the risk tolerances approved by the board of directors and should clearly define lines of authority and responsibility for managing the risks. The institution's employees should be fully aware of all policies and procedure that relate to their specific duties.

The above factors should also be considered in the process for mapping the business operations and the risks that influence them. The mapping of the operations' processes is used to define the key business operations, the various business units, the organisational functions, and process flows as well as their direct or indirect links to business targets and objectives. Note that operations used in the Islamic financial contracts must also be linked to *Shari'ah* compliance. For instance, the commodities, assets, or constructions agreed in *istisna* and *salam* contracts should always be linked to the *Shari'ah* principles. Moreover, the operations that refer to the process of producing and delivering products and services should be well defined and monitored in regards to risk of not complying with *Shariah* principles. In addition, when the financial institution agrees on a partnership type of agreement, such as the *musharakah* and *mudarabah*, additional mappings of the operational processes that are linked to these contracts should also be designed.

The operational process mapping exercise is used to identify key operations and design a roadmap of the combined key operations by defining inputs and outputs and linkage between them. In the risk-mapping process, all possible risks that might affect the operational processes are identified and linked to the operations process map. The operational risk mapping is used as the basis to identify the types of operational risks' causes and their existence in Islamic financial contracts.

#### 3.4.1 Identification of Hazard, Events, and Losses

Having performed the operational risk mapping, an Islamic bank should be able to identify what are the causes of the risks, what are the events, and what are the downstream effects and consequences. However, it is sometimes difficult to identify the differences between causes, events, and consequences. In general, operational risk analysts and managers should have in their minds that:

- A 'cause' or 'hazard' should result in one or more events;
- An 'event' should have at least one cause and it must result in one or more consequences;
- A 'consequence' or 'loss' must result from one or more events and may result in new cause.

Confusion usually arises in the operational risk because of the distinction between risk type (or *hazard* type), event type, and *consequence* (or *loss* type). When banks record their operational loss data, it is very essential to record it separately according to event type and loss type, and precisely identify the risk type as well. Mori and Harada (2001), Alvarez (2002) and Dowd (2003) suggest that the distinction between the three is comparable to cause and the effect. While *hazard* constitutes one or more factors that increase the probability of occurrence of an event; *event* is a single incident that leads directly to one or more effects (*e.g.* losses); and loss constitutes the amount of financial damage resulting from an event.

Mori and Harada (2001) shows how operational losses would occur in a process called 'cause-effect'relationship between *hazard*, *event*, and *loss*. *loss* is effect of *event* while *event* is cause of *loss*. However, *event* is the effect of *hazard*, while *hazard* is cause of *event*. In other words, every *loss* must be associated with an *event* that caused the *loss*, while every event must be associated with one or multiple *hazards* that caused the *event*. It should, therefore, be noted that in the operational risk literature, *hazard* is also termed as *risk* (Marshall, 2001), or *cause* (Dowd, in Alexander, 2003). While *loss* and *effects*, are often used interchangeably (Dowd, in Alexander, 2003).

Operational risk causes, events, and losses are usually associated with internal control weaknesses or lack of compliance with existing internal procedures as well as with the *Shariah* principles. In explaining, examples of causes, events and losses are shown in table 3.1.

Table 3.1: Examples of Causes, Events, and Losses

Source: Akkizidis and Kumar (2008: 188)

A variety of causes and events can be found in all areas of an institution and are mainly caused by the combined actions of people, technological systems, processes, and some unpredictable events. People, for example, as human resources are the area of greatest variability and, as a result, the sources of the majority of operational risks. It is, therefore, recommended that the organisation look for root causes as opposed to effect. When a risk event is evaluated, its causes or originating source must be identified as well as what consequences or resulting effect it will have on other risks. The resulting consequences will then give an indication to the risk manager whether a certain risk is to be 'accepted',

'avoided', or 'mitigated'. Therefore, an identification of root causes combined with linking causation to relevant business activities is important, as it can help identify the most significant risk which can have a detrimental effect to the financial institution.

The real life experience, however, indicates that, realistically, some operational risks must be accepted. How much is accepted, or not accepted, mainly depends on the operational risk impact and internal policies of the organisation. Operational risks with a high degree of impact should not be accepted, even if their probability is low. The decision to accept operational risk is affected by many inputs and policies. When a manager decides to accept operational risks, the decision should be coordinated whenever practical with the affected personnel and organisations, and then documented so that in the future everyone will know and understand the elements of the decision and why it was made.

## 3.4.2. Sources of Operational Risk

Mapping the operational risk during the identification process allows Islamic banks to define and measure the risks within the business and better understand their operational risk loss profile. Each financial institution has its own, individual and unique operational settings. Thus, to be able to manage operational risk might require tailoring its definition to the institution's specific settings. In operational risk identification analysis, hence, all major business disruptions that result in operational risk losses initiated from people, system, and technology, policies, processes and delivery failures, transactions, and/or internal and external events should be taken into account (Akkizidis and Kumar, 2008) as follows:

- 1. People: humans are one of the main sources of operational risks and play a major role in Islamic financial contracts.
- 2. Transactions: failures in financial transactions.
- 3. Systems and technology: this refers to systems and technology that are initiated by internal and external events.

- 4. Process and delivery failures: such disruption may refer to process execution and delivery and present in most Islamic financial contracts.
- 5. Internal and external events: these are events that cause losses to Islamic banks due to external events, political uncertainties, natural disasters, and the actual implementation of the Islamic contracts.
- 6. Policies: which refer to incomplete or missing legal documentations which affect the compliance to the *Shariah* principles. Furthermore, it includes unapproved access given to client accounts, or even to employment practices and workplace safety.

In Islamic bank, nevertheless, operational losses may also arise due to different types of Islamic financial contracts, which include *murabaha*, *ijara*, *salam*, and *istisna*. Operational losses also appear in *musharakah* and *mudarabah* contracts, where the institution has a close business relation with the counterparties. In such agreements, the institution can be exposed to a great degree of operational risk since it has the full responsibility for covering the entire amount of associated losses.

For this reason, it is important to understand how different aspects of operational risks arise in various Islamic financial contracts, which is discussed in the following section.

# 3.5 OPERATIONAL RISKS IN ISLAMIC FINANCIAL CONTRACTS

This section discusses different dimension of operational risk in different type of Islamic financial contracts. As can be seen in Table 3.2, five dimensions of operational risk are *Shari'ah* compliance risk (SR), fiduciary risk (FR), people risk (PR), legal risk (LR), and technology risk (TR). The first three dimensions are, by nature, internally inflicted; while the fourth one is naturally from external source. As for technology risk (TR); it can originate from either internal or external operational failures. These are discussed in the following sections:

**Table 3.2: The Dimensions of Operational Risk in Islamic Financial Contracts** 

Contracts	Internal Risks				External Risks	
	Shariah Compliance Risk (SR)	Fiduciary Risk (FR)	People Risk (PR)	Technology Risk (TR)	Legal Risk (LR)	Technology Risk (TR)
Murabaha	Exchange of money and commodity needs to be ensured     In the event of late payment, penalty must be avoided as it will tantamount to riba.	Inability to meet the specified product stipulated in the contract	Fail to deliver the product	Incompatibilit y of the new accounting software	Products to be sold must be legally owned by the bank	System failures and external security breaches
Salam	Final payment of monetary rewards must be concluded in advance Penalty clause is illegitimate in the event of seller's default in delivering the goods In parallel salam, execution of second salam contract is not contingent on the settlement of the first salam contract	Inability to meet the specified product stipulated in the contract. Delivery of inferior goods can not be accepted	Mismatch in the commodity's specification due to inability of seller to provide the exact product mentioned in the contract.	Incompatibilit y of the new accounting software	Goods must be delivered when it is due, as agreed in the contract	Specification mismatching in commodities productions agreed in the contract
Istisna	Should not be used as a legal device; e.g. the party ordering the product to be produced is the manufacturer himself     In parallel istisna', contracts should be separated to avoid two sales in one deal	Need to ensure the quality standards of the products	Inability to deliver the product on time	Incompatibilit y of the new accounting software	Disagreement with the sub- contractor or the customer in the event of remedying the defects	Specification mismatching in commodities productions agreed in the contract
Ijara	Need to ensure that leased asset is used in a Shariah compliant manner In ijarah muntahia bittamleek, an option to purchase can not be enforced.	Major maintenance of the leased asset is the responsibility of the banks or any party acting as lessor.	Lessor needs to understand that in the event of payment delay, rental due can not be increased as clearly exemplified by AAOIFI	Incompatibilit y of the new accounting software	Enforcement of contractual right to repossess the asset in case of default or misconduct by the lessee	Losses of information on the leased assets specified in the contract due to external security breaches
Musharakah	Profit allocation is based on actual profit, not expected profit	Inadequate monitoring of the financial performance of the venture	Lack of technical expertise in assessing the project	Incompatibilit y of the new accounting software	A mixture of shares in one entity may lead to legal risk if the regulation does not facilitate such action	Losses of information on the projects specified in the contract due to external security breaches
Mudarabah	Profit allocation is based on actual profit, not expected profit	Inadequate monitoring of the business	Inability to provide regular and transparent financial	Incompatibiliy of the new accounting software	Misinterpretati on of civil law upon implementatio n of Shariah	Losses of information on the projects specified in the contract due to

	performance	compliant	external security
	of the project	mudaraba	breaches

Source: Author's own

#### 3.5.1 Murabahah

Murabahah is "selling a commodity as per the purchasing price with a defined and agreed profit mark-up" (AAOIFI, 2005: 127). This mark-up may be a percentage of the selling price or a lump sum. Moreover, according to AAOIFI standard (2005), this transaction may be concluded either without a prior promise to buy, in which case it is called ordinary murabahah, or with a prior promise to buy submitted by a person interested in acquiring goods through the institution, in which it is called a "financial murabahah", i.e. murabahah to the purchase orderer. This transaction is one of the trust-based contracts that depends on transparency as to the actual purchasing price or cost price in addition to common expenses.

*Murabahah* is the most popular contract in terms of its use, since most of Islamic commercial banks operating worldwide rely on this contract in generating income. Different dimensions of operational risk, which can arise in *murabahah* transaction are as follows:

- Shariah compliance risk (SR) may arise if the Islamic banks give money, instead of commodity, which will then result in the exchange of money and money. This is prohibited in Shari'ah, since the exchange of money with money, plus additional amount above the principal and paid in different time will tantamount to riba. AAOIFI Shari'ah standard (2005) also requires Islamic banks to own, legally, the commodity before they sell it to the customers. It is important to note that the sequence of the contract is very central in murabahah transaction. Inability or failure to conform with the sequence and Shari'ah requirement will result in the transaction to be deemed illegitimate.
- *Fiduciary risk* (FR) arises due to the inability to meet the specified commodity stipulated in the contract.

- Legal risk (LR): profit originated from murabahah can not be equated with interest, although it looks similar. The main difference is because the resulting profit is tied with the underlying commodity. This might create legal problem as in certain countries, the regulators only give limitation on interest rate, not profit rate. Hence, the absence of so called 'profit rate cap' has the potential to crate legal problems if there is any dispute. Another potential problem can occur at the contract signing stage, since the contract requires the Islamic bank to purchase the asset first before selling it to the customer, the bank needs to ensure that the legal implications of the contract properly match the commercial intent of the transactions
- *People Risk* (PR) can result from two sides, seller as well as buyer. *PR* from the seller side occurs if Islamic banks fail to deliver the specified product agreed in the contract on due date, while *PR* from the buyer side takes place when the buyers does not keep their promise to buy the commodity. This can happen in the binding *murabahah* contract.
- *Technology risk (TR)* may result from an incompatibility of the new accounting software or an external system failure.

#### 3.5.2 Salam and Parallel Salam

AAOIFI *Shari'ah* standards (2005: 174) define *salam* as a transaction of the purchase of a commodity for the deferred delivery in exchange for immediate payment. It is a type of sale in which the price, known as the *salam* capital, is paid at the time of contracting, while the delivery of the item to be sold, know as *al-muslam fihi* (the subject matter of a *salam* contract), is deferred. The seller and the buyer are known as *al-muslam ilaihi* and *al-muslam* or *rabb al-salam* respectively.

It should be noted that *salam* is also known as *salaf*, and a modification of *salam* is called parallel *salam* whereby a seller enters into another separate *salam* contract with a third party to acquire goods, the specification of which corresponds to that of the commodity specified in the first *salam* contract (AAOIFI, 2005).

- Shariah compliance risk (SR): One of the very central conditions in salam contract is that the payment of salam capital must be paid full in advance. If payment is delayed, the transaction is not called salam (AAOIFI, 2005: 172). Any delay in payment of the capital and dispersal of the parties renders the transaction a sale of debt for debt, which is prohibited, and the scholars agreed on its prohibition (AAOIFI, 2005: 172). Another aspect which might lead to SR may also occur in parallel salam; this will take place if the execution of the second salam contract is contingent on the execution of the first salam contract. Penalty clause is also not allowed, in the event of a seller's default in delivering the good. The basis for not allowing penalty in salam is because al-muslam fihi (the subject matter of a salam contract) is considered to be a debt; hence it is not permitted to stipulate payment in excess of the principal amounts of debt (AAOIFI, 2005: 173).
- Fiduciary risk (FR): Salam is generally associated with the agricultural sector. The buyer must either rejects goods of an inferior quality to that specified in the contract, or accept them at the original price. In the latter case, the goods would have to be sold at a discount (unless the customer under a parallel salam agreed to accept the goods at the originally agreed price)
- Legal risk (LR): Islamic banks may face legal risk if the goods can not be delivered at the specified time (unless the customer under parallel salam agrees to modify the delivery date).
- People risk (PR) can arise due to a seller's default in delivering the commodity or due to the commodity's specification mismatching. Financial institutions may minimise such type of operational risks by asking from the seller guarantees that they are following a quality management system or following any standard system, or by asking for references on past promises on *salam* contract or by collateralising their losses via insurance policies.
- *Technology risk (TR)* may result from an incompatibility of the new accounting software or the system fails to specify precisely the commodities agreed in the contract.

#### 3.5.3 Istisna' and Parallel Istisna'

Istisna' is another type of forward contract, but the role of an Islamic bank as a financial intermediary differs from that in a salam contract. In this case, the bank contracts to supply a constructed asset (such as a building or a ship) for a customer. In turn, the bank enters into a parallel istisna' with a sub-contractor in order to have the asset constructed. Its reliance on the parallel istisna counterparty (the sub-contractor) exposes it to various operational risks, which need to be managed by a combination of legal precautions, due diligence in choosing sub-contractors, and technical management by appropriately qualified staff or consultants of the execution of the contract by the sub-contractor. Islamic banks that specialise in istisna' financing may have an engineering department. Risks may include the following:

- Shariah compliance risk (SR) could arise if istisna is being used as a legal device for mere interest based financing. For instance, an institution buys items from the contractor on a cash payment basis and sells them back to the manufacturer on a deferred payment basis at a higher price; or where the party ordering the subject matter to be produced is the manufacturer himself; or where one third or more of the facility in which the subject matter will be produced belongs to the customer. All the circumstances mentioned above would make the deal an interest based financing deal in which the subject matter never genuinely changes hands, even if the deal won through competitive bidding. This rule is intended to avoid sale and buy back transactions (bay al-inah). In parallel istisna', the separation of contracts is a must, hence this is not an instance of two sales in one deal, which is prohibited.
- *Fiduciary risk* (*FR*) may arise when the sub-contractor may fail to meet quality standards or other requirements of the specification, as agreed with the costumer under the *istisna*' contract.
- Legal risk (LR): Islamic banks may face legal risk if no agreement is reached with the sub-contractor and the customer either for remedying the defects or for reducing the contract price.

- People Risk (PR) may arise if the Islamic bank may be unable to deliver the asset on time, owing to time overruns by the sub-contractor under the parallel istisna', and may thus face penalties for late completion.
- *Technology risk (TR)* may result from an incompatibility of the new accounting software or the system fails to specify precisely the commodities that would be produced in the contract

## 3.5.4. *Ijarah* and *Ijarah Muntahia Bittamleek*

In simple terms, an *ijarah* contract is an operating lease, whereas *ijarah muntahia bittamleek* is a lease to purchase, while operational risk exposures during the purchase and holding of the assets may be similar to those in case of *murabahah*. Other operational risk aspects include the following:

- Shariah compliance risk (SR): The Islamic banks need to ensure that the asset will be used in a Shariah compliant manner. Otherwise, it is exposed to non-recognition of the lease income as non-permissible.
- *Fiduciary Risk (FR)*: Major maintenance is the responsibility of an Islamic bank as a lessor, as directed by AAOIFI Shariah standards (2005: 154). In addition, it is the duty of the lessor to ensure that the usufruct is intact, and this is not possible unless the asset is maintained and kept safe so that the lessor may be entitled to the rentals in consideration for the usufruct. Thus, deficiencies in maintaining such responsibility can be deemed to be sources of *FR* in *ijarah* contract.
- Legal risk (LR): The Islamic bank may be exposed to legal risk in respect of the enforcement of its contractual right to repossess the asset in case of default or misconduct by the lessee. This may be the case particularly when the asset is a house or apartment that is the lessee's home, and the lessee enjoys protection as a tenant.
- People Risk (PR) may arise when lessor is not allowed to increase the rental due, in case of delay of payment by the lessee, as identified by AAOIFI (2005).
   Misunderstanding of this principle by the staff is a source of losses caused by PR,

because the income generated from this, is not permissible from *Shariah* point of view.

• *Technology risk (TR)* may occur due to an incompatibility of the new accounting software or losses of information on the leased assets due to external security breaches.

#### 3.5.5 Musharakah

As known *musharakah* is a profit and loss sharing partnership contract. The Islamic bank may enter into a *musharakah* with a customer for the purpose of providing a *Shari'ah* compliant financing facility to the customer on a profit and loss sharing basis. The customer will normally be the managing partner in the venture, but the bank may participate in the management and thus be able to monitor the use of the funds more closely. Typically, a diminishing *musharakah* is used for this purpose, and the customer will progressively purchase the bank's share of the venture.

Operational risks that may be associated with *musharakah* investments are as follows:

- Shariah compliance risk (SR): The source of SR may arise due to the final allocation of profit taking place based on expected profit. AAOIFI (2005: 205) commands that it is necessary that the allocation of profit is done on the basis of actual profit earned through actual or constructive valuation of the sold assets.
- Fiduciary Risk (FR): Any misconduct or negligence of the partners are the sources of FR. This can happen in the absence of adequate monitoring of the financial performance of the venture.
- Legal Risk (LR): An Islamic bank which enters into musharakah contract needs to acquire some shares from separate legal entity that undertake Shariah compliant activities. A mixture of shares in one entity may lead to legal risk if the regulation does not allow doing such action.

- *People Risk (PR)*: Lack of appropriate technical expertise can be a cause of failure in a new business activity.
- Technology risk (TR) may occur due to an incompatibility of the new accounting software or losses of the precise information on projects undertaken due to external security breaches.

#### 3.5.6 Mudarabah

As equity based financial contract, *mudarabah* is defined as a profit sharing and loss bearing contract under which the financier (*rab al mal*) entrusts his funds to an entrepreneur (*mudarib*). The exposure of operational risk in *mudarabah* is somewhat similar to that of *musharakah* due to the similarities in the substance and form nature.

Since this type of contract may be used on the assets side of the balance sheet, as well as being used on the funding side for mobilising investment accounts, the operational risk is first analysed from the assets-side perspective and then from the funding side perspective (which is related to fiduciary risk)

#### 3.5.6.1 Asset-side mudarabah

Contractually, an Islamic bank has no control over the management of the business financed through this mode, the entrepreneur having complete freedom to run the enterprise according to his best judge judgement. The bank is contractually entitled only to share with the entrepreneur the profits generated by the venture according to the contractually agreed profit sharing ratio. The entrepreneur as *mudarib* does not share in any losses which are borne entirely by the *rab al mal*. The *mudarib* has an obligation to act in a fiduciary capacity as the manager of the bank's funds, but the situation gives rise to moral hazard especially if there is information asymmetry - that is, the bank does not receive regular and reliable financial reports on the performance of the *mudarib*. Hence, in addition to due diligence before advancing the funds, the bank needs to take precautions against problems of information asymmetry during the period of investment.

### 3.5.6.2 Funding-side mudarabah

Profit-sharing (and loss bearing) investment accounts are a *Shari'ah* compliant alternative to conventional interest-bearing deposit account. Since a *mudarabah* contract is employed between the Islamic bank and its investment account holders, the investment account holders (IAHs) share the profits and bear all losses without having any control or rights of governance over the Islamic bank. In return, the Islamic bank has fiduciary responsibilities in managing the IAHs' funds. The IAHs typically expect returns on their funds that are comparable to the returns paid by competitors (both other Islamic banks and conventional institutions). However, they also expect the Islamic bank to comply with *Shariah* rules and principles at all times. If the Islamic bank is seen to be deficient in its *Shariah* compliance, it is exposed to the risk of IAHs withdrawing their funds and, in serious cases, of being accused of misconduct and negligence. In the latter case, the funds of the IAHs may be considered to be a liability of the Islamic bank, thus jeopardising its solvency.

## 3.6 CAPITAL REQUIREMENT FOR OPERATIONAL RISKS

Prior to discussing the measurement of capital requirement for operational risks in Islamic banks, it is important to understand why banks should have adequate capital. For this reason, the first part of this section attempts to elucidate the rational behind capital adequacy requirement. This also explains, briefly, the relationship between bank capitalisation and risk taking behaviour. Following to the discussion in the first part, the subsequent parts discuss the measurement of capital attribution for operational risks and operational risk capital charge in Islamic banks respectively.

# 3.6.1 Why Do Banks Need to Hold Capital?

Traditionally, capital adequacy requirements have been imposed to ensure solvency. Following Maisel (1979, 1981) and Merton (1979), a bank can be declared 'insolvent' or 'bankrupt' when the market value of the bank liabilities to depositors, computed by assuming that the bank's obligations to depositors would be fully met, exceeds the market value of the bank assets reduced by the costs of liquidation. In other words, negative net

worth (based on market values) implies insolvency. For this reason banks generally attempt to boost their risk-based capital ratios by means of (i) increasing the measures of regulatory capital appearing in the numerators of leverage ratio, or (ii) decreasing the regulatory measures of total risk appearing in the denominators (*e.g.*, total risk-weighted assets). Jones (2000) suggests that in the short run, most banks have tended to react to capital pressures in the ways broadly envisioned by the framers of the Accord. That is, by increasing their capacity to absorb unexpected losses through increased earnings retentions or new capital issues, and by lowering their assumed risks through reductions in loans and other footings.

The relationship between banks' capitalisations and risk taking behaviours is one of the central issues in the banking literature, because of the potential implications for regulatory policies. The minimum capital requirement which currently constitutes the core regulatory instrument for the banking industry is based on the premise that increased capital enhances bank safety (Jeitschko and Jeung, 2007). As also discussed by Jeitschko and Jeung (2005), however, this premise may not hold under some relevant circumstances. Indeed, if increased capital induces a bank to increase asset risk (asset substitution effect of capital), and this effect supersede the buffer effect of capital (larger capital absorbs more risk), then it is possible that a more highly capitalised bank has a higher probability of failure. This risk taking behaviour of banks related to capitalisation explains why banks often experience rapid, large declines in their capital to asset ratio (CAR), and are classified by regulators from well capitalised to troubled banks in as little as a single reporting period. The implication of this positive relationship between risk taking and capitalisation is that capital regulation alone may not be adequate to guarantee the soundness of the banking business.

## 3.6.2 Measurement of Operational Risk based Capital

Basel II implemented an additional add-on to capital for operational risk. Prior to this proposal, the Basel Committee on banking Supervision (BCBS) had argued that operational risk exposures of banks were adequately taken care of by 8% credit risk-adjusted ratio. But increased visibility of operational risks in recent years (as discussed in

chapter 2) has induced regulators to propose a separate capital requirement for credit and operational risks. BCBS now believes that operational risks are sufficiently important for banks to devote resources to quantify such risks and to incorporate them separately into their assessment of their capital adequacy. In the 2001 and 2003 Consultative Documents the Basel Committee outlined three specific methods by which banks can calculate capital to protect against operational risk: the Basic Indicator Approach (*BIA*), the Standardised Approach (*SA*), and the Advanced Measurement Approach (*AMA*).

The Basic Indicator Approach is structured so that banks, on average, will hold 12% of their total regulatory capital for operational risk. This 12% target was based on a widespread survey conducted internationally of current practices by large banks.<sup>21</sup> To achieve this target, the Basic Indicator Approach focuses on the gross income of the bank, that is, its net profits. This equals a bank's net interest income plus net non-interest income:

#### GrossIncome = netInterestIncome + netNonInterestIncome

According to BCBS calculations, a bank that holds a fraction ( $\alpha$ ) of its gross income for operational risk capital, where alpha ( $\alpha$ ) is set at 15%, will generate enough capital for operational risk such that this amount will be 12% of its regulatory capital holdings against all risks (*i.e.*: credit, market, and operational risks). For example, under the Basic Indicator Approach:

Operationa lCapital =  $\alpha \times GrossIncom\ e$ = .15×GrossIncome

The problem with the Basic Indicator Approach is that it is too aggregative, or 'top-down', and does not differentiate at all among different areas in which operational risks may differ (e.g., Payment and Settlement may have a very different operational risk profile from Retail Brokerage). A second issue is that  $\alpha$  implies operational risk that is proportional to gross income. This ignores, according to Saunders and Cornett (2008),

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<sup>&</sup>lt;sup>21</sup> Research has found that the amount of capital held for operational risk according to these models will often exceed capital held for market risk and that the largest banks could choose to allocate several billion dollars in capital to operational risk. See: Defontnouvelle et al., (2006).

possible economies of scale effects that would make this relationship nonlinear (non-proportional); that is,  $\alpha$  might fall as bank profits and/or size grows.

In an attempt to provide a finer differentiation of operational risks in a bank across different activity lines while still retaining a basically *top-down* approach, the BCBS offers a second method for operational capital calculation. The second method, the Standardised Approach, divides activities into eight major business units and lines. Within each business line, there is a specified broad indicator (defined as beta,  $\beta$ ) that reflects the scale or volume of a bank's activities in that area. The indicator relates to the gross income reported for a particular line of business. It serves as a rough proxy for the amount of operational risk within each of these lines. A capital charge is calculated by multiplying the  $\beta$  for each line by the indicator assigned to the line and then summing these components. The  $\beta$  reflects the importance of each activity in the average bank. The  $\beta$  is set by regulators and is calculated from average industry figures from a selected sample of banks.

Suppose the industry  $\beta$  for Corporate Finance is 18% and gross income from the Corporate Finance line of business (the activity indicator) is £30 million for the bank. Then, the regulatory capital charge for this line for this line for this year is:

Capital<sub>Corporate Finance</sub> =  $\beta$  x Gross Income from the Corporate Finance line business for the bank

= 18% x £30 million = £5,400,000

The total capital charge is calculated as the three-year average of the simple summation of the regulatory capital charge across each of the eight business lines.<sup>22</sup>

The third method, the Advanced Measurement Approach, allows individual banks to rely on internal data for regulatory capital purposes subject to supervisory approval. Under the Advanced Measurement Approach, supervisors require the bank to calculate its

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The Basel's Committee's Loss Data Collection Exercise for Operational Risk (March 2003), based on data provided by 89 banks from 19 countries, revealed that about 61 percent of operational loss events occurred in the retail area, with an average loss of \$79,300. Also, only 0.9 percent of operational loss events occurred in the corporate finance area, but with an average loss of \$646,600.

regulatory capital requirement as the sum of the expected loss (EL) and unexpected loss (UL) for each event type.

Internally generated operational risk measures used for regulatory capital purposes must be based on a minimum three year observation period of internal loss data, whether the internal loss data are used directly to build the loss measure or to validate it.

As mentioned in the previous chapter, there are some requirements that need to be fulfilled for the bank to apply *AMA*, one of which is the robust data requirement. This condition also applies to an Islamic bank which operates in a similar financial environment as its conventional counterpart. The next sub-section, thus, briefly discusses to what extent the Islamic finance industry adopts the calculation of risk capital requirement as set out by Basel 2.

# 3.6.3 Operational Risk Capital Charge in Islamic Banks

The proposed measurement of capital to cater for operational risk in Islamic banks is also adopting the methods set by BIS. As IFSB (2005b: 17) mentions in its standards that the calculation of operational risk based capital in Islamic banks "may be based on either the Basic Indicator Approach or the Standardised Approach as set out in Basel II". However, there is dissimilarity as regard with the use of the Standardised Approach (SA), since IFSB (2005b) views that Islamic banks have different structure of business lines. Hence, at the present stage, the Basic Indicator Approach (BIA) can be adopted by Islamic banks. BIA requires the setting aside of a fixed percentage of average annual gross income over the previous three years.

Problems of measurement is likely to arise due to lack of data, hence the extent of losses arising from non-compliance with *Shariah* rules can not be ascertained. Therefore, IFSB (2005b: 18) does not require Islamic banks to set aside any additional amount over and above the 15% of average annual gross income over the preceding three years for operational risk. Furthermore, in determining risk weights for operational risk, IFSB (2005b: 18) recommends the exclusion of the share of profit sharing investment account

holders from gross income. It is necessary to adjust this, since Islamic banks share profits with their depositor-investors (Greuning and Iqbal, 2008).

#### 3.7 CONCLUDING REMARKS

Operational risk is a recent addition to the list of risks faced by financial institutions, which has received an increased attention in recent times. It has implications for Islamic banks as well, as the management of operational risk in Islamic banks is similar to that in conventional banks but includes several additional elements. In addition, due to the unique features of their financial contracts, operational risk in Islamic banks can be substantially different from what is exposed to the conventional ones. The relative complexity of contracts, combined with the fiduciary obligations of Islamic banks, imply that for Islamic banks, operational risk is a very important consideration. More importantly, *Shariah* compliance risk as part of operational risk is paramount to Islamic banks, which means Islamic banks must ensure, at all times, that all activities and products are in conformity with *Shariah* principles. It is, then, apparent that the dimension of operational risk exposure in Islamic banks is more sophisticated than in conventional banks.

Operational risk is now recognised as a type of risk which can contribute to significant losses in all financial institutions. For this reason, various techniques being applied in banks today in order to measure and manage operational risk. The methods set out by BIS help the Islamic banks determine their capital in order to absorb operational losses. However, due to the small size of Islamic banks compared to the overall financial industry, the more advanced methods in the calculation of operational risk based capital is still not feasible to be implemented. The absence of significant amount of loss data is also one of the problems that hinder Islamic banks to implement more sophisticated methods. Given the rapid growth of Islamic financial industry, it is expected that lack of data will not be the main issue in the near future.

# **CHAPTER 4**

# MODELLING OPERATIONAL RISKS IN ISLAMIC BANKING: A PROPOSED FRAMEWORK

#### 4.1 INTRODUCTION

The previous chapters identified the complexity of the multidimensionality and the complexity of operational risk management and its measurement. The complexity of operational risk measurement has been exacerbated by two dimensions of operational risk data, namely high frequency-low severity (*HF-LS*) and low frequency-high severity (*LF-HS*). Due to their nature, consequently, each type requires different approach to cater for operational risk.

The current literature on operational risk almost exclusively focuses on two issues: firstly, the estimation of operational risk loss processes using extreme value theory or Cox processes, (Chavez-Demoulin et al., 2006; Coleman, 2003; de Fontnouvelle *et al.*, 2004; de Fontnouvelle *et al.*, 2006; Ebnother *et al.*, 2001; Jang, 2004; Moscadelli, 2004; Lindskog and MecNeil, 2003), and secondly, the application of these estimates to the determination of economic capital (de Fontnouvelle *et al.*, 2004; de Fontnouvelle *et al.*, 2006, Moscadelli, 2004).

It should be noted that in the estimation of economic capital for operational risk, the estimates appear to be quite large, in fact, at least as large as that necessary to cover market risk. As evidenced by the references mentioned earlier that the modelling and estimation of operational risk is treated identically to market and credit risk, *i.e.* a loss process is modelled and estimated. However, this is where the similarity ends, as unlike market and credit risk, which are external to the bank in their origin, operational risk is internal to the bank.

There is also an intensive use of Value at Risk (*VaR*) in the measurement of risk exposure in financial institutions. For a long time, financial economists have considered empirical behaviour models of banks where these institutions maximise some utility criteria under a solvency constraint of *VaR* type (Gollier *et al.*, 1996; Santomero and Babbel, 1996). Similarly, other researchers have studies optimal portfolio selection under limited downside risk as an alternative to traditional mean-variance efficiency frontiers (Roy, 1952; Levy and Sarnat, 1972; Arzac and Bawa, 1977). Finally, internal use of *VaR* by financial institutions has been addressed in a delegated risk management framework in order to mitigate agency problems (Kimball, 1997; Froot and Stein, 1998; Stoughton and Zechner, 1999).

Despite a growing interest in *VaR*, there is, unfortunately, a very limited study in the theoretical properties of risk measures and their consequences on operational risk management in Islamic banking. Although IFSB Draft No. 2 on Capital Adequacy Standard (2005) mentions the definition of operational risk and proposes Basic Indicator Approach (*BIA*) and the Standardised Approach (*TSA*) as methods to calculate operational risk capital. However, the standard excluded a very essential step, *i.e.* a method to measure the magnitude of operational risk exposures. This shortcoming is also identified in the studies by Khan and Ahmed (2001), Hassan and Dicle (2005), Ismail and Sulaiman (2005), Kahf (2005), Muljawan (2005), and Sundararajan (2005). This study, therefore, attempts to fill this gap by developing a new modelling for the measurement of operational risk in Islamic banking, as presented in the following section.

The proposed model in this study, namely *Delta-Gamma Sensitivity Analysis-Extreme Value Theory* (*DGSA-EVT*), is a model to measure *HF-LS* and *LF-HS* type of operational risks. The first leg of the proposed model, namely *DGSA*, is a methodology that deals with propagation of errors in the value adding activities which works by using measures of fluctuations in the activities. The sensitivities of the output, hence, are deployed to estimate the performance volatility. Through operating loss distribution that is the result of the entire quantification process, the *DGSA* would help us in generating the level of operational value at risk (*OpVaR*) of the analysed Islamic banks. Furthermore, the second leg of the proposed model, *Extreme Value Theory* (*EVT*), is a technique to cater for an

excess operational loss over a defined threshold, which is normally characterised by low frequency and high severity (*LF-HS*) type of loss.

The second section of this chapter reviews in some more detail the existing models in operational risk measurement and its classifications, while the third section explains the theoretical background of the proposed model and its features. In the fourth section, the chapter focuses on the empirical aspect of the proposed model. Conclusion is reached in the fifth section.

#### 4.2 A REVIEW OF OPERATIONAL RISK MODELLING

Operational risk models encompass a variety of statistical and econometric models designed to measure the regulatory and economic capital to be held against operational risk. Different models are also designed to study causes and consequences of operational risk. This is partly due to the constantly changing financial environment, which has made modelling of operational risk imperative (Peccia, 2003), Furthermore, operational risk modelling is also needed to provide bank management with a tool to make a better decision in carrying out a desirable level of operational risk. Bocker and Kluppelberg (2005) suggest that the only feasible way to effectively manage operational risk is by identifying and minimising it, which requires the development of adequate quantification techniques. This position is substantiated by Fujii (2005), who points out that quantifying operational risk is a prerequisite for the formulation of an effective economic capital framework. Furthermore, Consiglio and Zenois (2003) emphasise the importance of operational risk models by attributing some widely-publicised loss events to the use of inadequate models rather than anything else. Actually, Giraud (2005) attributes the collapse of Long-Term Capital Management in large part to "model bias in the risk management process". Holmes (2003) argues that even if operational risk modelling is not scientific or reliable, it may force banks to carry more capital and encourage better behaviour.

# 4.2.1 Taxonomy of Operational Risk Modelling

The operational risk can broadly be classified into three categories: (i) process approach, (ii) factor approach, and (iii) actuarial approach. Additional techniques have taken place under process approach, which is shown in italics in Table 4.1.

**Table 4.1 Approaches in Operational Risk Modelling** 

Process Approach	Factor Approach	Actuarial Approach
<ul> <li>Causal models</li> <li>Bayesian belief networks</li> <li>Fuzzy logic</li> <li>Statistical quality control and reliability analysis</li> <li>Connectivity</li> <li>System Dynamics</li> </ul>	<ul> <li>Risk Indicators</li> <li>CAPM like models</li> <li>Predictive models</li> </ul>	<ul> <li>Empirical loss distributions</li> <li>Explicit distributions parameterised using historical data</li> <li>Extreme value theory</li> </ul>

Source: Smithson and Song (2004: 58)

The following sections discuss the each of the identified categories of operational risk in detail.

## 4.2.1.1 Process approach

The process approach focuses on the chain of activities that comprise an operation or transaction in much the same way that an industrial engineer examines a manufacturing process by looking at the individual work stations. Examples of this approach include:

• Causal models attempts to look at a specific outcome (for example, a settlement payment) in terms of the probabilistic impact of the activities that are in the chain (for example, recognition that a payment date has occurred, calculation of the settlement amount, notification of the counterparty, and paying or receiving) (Chernobai *et al.*, 2007: 72). The success of each activity in the chain might be modelled as a function of inputs. Each of the inputs and implied outcomes is given a probabilities description, and, using conditional probabilities, the probability of a failure further down the chain can be estimated.

- Statistical quality control and reliability analysis; and
- *Connectivity* which requires the modelling process to develop a 'connectivity matrix' that can then be used to estimate the likelihood of failure (or potential losses) for the process as a whole (Moosa, 2008: 144).

Three additional techniques that could be considered 'process' approaches are:

- Bayesian belief network, which extends the 'causal model' technique by treating the initial model as the null hypothesis, so, as data is collected, the model can be tested to provide a more accurate picture of the process (Chernobai *et al.*, 2007: 72).
- *Fuzzy logic* is a branch of mathematics that facilitates decision-making when some of the inputs are vague, or if they are subjective judgements (Cruz, 2002: 169). In a 'causal model', fuzzy logic could provide a way to aggregate the subjective drivers of a process.
- *System dynamics*, which extends 'connectivity', is carried out by making the connections between activities dynamic (stochastic) (Chernobai *et al.*, 2007: 75). This technique requires a development of the model to simulate the cause-effect interactions among activities that make up the processes within the firm.

## 4.2.1.2 Factor approach

A factor approach was initiated as an attempt to identify the significant determinants of operational risk – either at the institution level or at the level of an individual business or individual process. The objective is to obtain an equation that relates the level of operational risk for institution i (or business i or process i) to a set of factors, as expressed in the following model:

$$(OperationalRisk)_i = \alpha + \beta(Factor1) + \gamma(Factor2)$$
(4.1)

The key element of factor approach is the identification of appropriate factors in order to obtain the measures parameters  $\alpha$ ,  $\beta$ , and  $\gamma$ . As a result, an estimation of the level of operational risk that will exist in future periods can be materialised. In the analysis of

operational risk quantification, Smith and Song (2004) describe three applications of the factor approach;

- *Risk indicators* for which regression techniques are utilised to identify the significant operational risk factors.
- *CAPM-like model* relates the volatility in share returns (and earnings and other components of the institution's valuation) to operational risk factors.
- *Predictive models*, which use discriminated analysis and similar techniques to identify factors that 'lead' operational losses.

## 4.2.1.3 Actuarial approach

An actuarial approach attempts to identify the loss distribution associated with operational risk – either at the level of institution or at the level of a business or process.

- *Empirical loss distribution* is the most straightforward way to estimate the loss distribution, using the institution's own data on losses or both internal data and (properly scaled) external data. However, empirical loss distributions will probably suffer from limited data points (especially in the tail of the distribution).
- Explicit distributions parameterized using historical data is one way to get around the sparse data problem. The analyst specifies a distributional form for the loss distribution or a distribution for the frequency of occurrence of losses and a different distribution for the severity of the losses.
- Extreme value theory provides another way of getting around the data sparseness problem. This theory is an area of statistics concerned with modelling the limiting behaviour of sample extremes, which indicates that, for a large class of distributions, losses in excess of a high enough threshold all follow the same distribution (a generalised Pareto distribution).

# 4.2.2 Operational Risk in Islamic Banking: Empirical Research

In the IFSB Draft No. 2 on Capital Adequacy Standard, operational risk is defined as the risk of losses resulting from inadequate or failed internal risk and Shariah compliance risk (IFSB, 2005: 22). This definition is rather different from Basel 2 on operational risk. However, IFSB adopts Basel 2's methodology in the calculation of minimum capital requirement for operational risk exposure. Three methods have been proposed; namely the Basic Indicator approach (BIA), the Standardised approach (TSA) and the Advanced Measurement approach (AMA). BIA takes the moving average of gross income as a proxy of the size of operational risk exposure and suggests a parameter of 15% to calculate the minimum capital required to stand for this kind of risk. TSA is a little more refined as it takes average gross income at the activity level, after dividing a bank's activities into 8 categories and suggests a parameter for each of them. Finally, AMA allows using internal measurement methodologies to calculate the minimum capital requirement for operational risk exposure provided the bank satisfies certain qualification criteria to assure the supervisory authority of the existence of efficient and independent operational risk management system and of its ability to fairly estimate operational risk and the capital needed to face it, including the expected losses as well as the unexpected losses.

The IFSB standards provide fairly detailed guidance on adaptation of Basel 2 to the specific risk characteristics of Islamic banks. In particular, the IFSB draft proposes an adaptation of standardised approach to risk measurement—based on externally provided rating categories—and within this framework allows supervisory discretion to recognise the extent of risks assumed by the PSIA's<sup>23</sup> in computing capital adequacy for Islamic banks. Kahf (2005) opposes the use of gross income as a proxy of operational risk exposure as set out by IFSB. In this respect, his argument is in line with Sundararajan (2005) who argued that the use of gross income as the basic indicator for operational risk measurement could be misleading in Islamic banks, as large volume of transactions in commodities, and the use of structure finance raise operational exposures that will not be captured by gross income. However, Sundarajan is still supporting the standardised

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<sup>&</sup>lt;sup>23</sup> PSIA refers to profit sharing investment account

approach that allows for different business lines to be better suited, but would still need adaptation to the needs of Islamic banks.

Some empirical aspects of the operational soundness in Islamic banks are conducted by Ismail and Suleiman (2005), Hassan and Dicle (2005) and Muljawan (2005). Using the Cavello and Majnoni model, Ismail and Suleiman (2005) discuss the interaction between the capital requirement as stated in the New Basel Capital Accord and the cyclical pattern of profit. In addition, CAMEL framework is deployed by Muljawan (2005) as an alternative tool to assess the operational soundness of Islamic banks. The analysis of Hassan and Dicle (2005) is somewhat broader than other papers in the sense that it also discusses the nature of operational risk. However, it does not make any suggestion on how to handle capital requirements with respect to Islamic banks.

There are two issues that can be highlighted from the survey presented above: first, it is clear that the empirical research that have been conducted are on the aspect of capital attribution for operational risk; second, there has not been unanimity upon the standard of operational risk measurement method. The most recent research on this issue is conducted by Jackson-Moore (2007); nevertheless he could not come up with a conclusive suggestion on the refined measurement method.

The following section discusses the framework proposed by this study in quantifying operational risk exposures in Islamic banks.

# 4.3. DELTA-GAMMA SENSITIVITY ANALYSIS (DGSA)

One of the main objectives of measuring operational risk exposures is to decide which type of operational risk indicators are significant so a mitigation technique can be established accordingly. Therefore, a refined measurement method is essentially required to provide a measure that has a defined relationship to a risk factor that can be assigned as controllable or uncontrollable. This would result in the determination of an appropriate intervention for controllable risks by focusing on their causes.

Given the foregoing discussion, the impact on operations can be separated into controllable risk and uncontrollable risk. In this study, 'controllable risk' is defined as the

risks, which have assignable causes that can be influenced. Generally, process-related risks will have assignable causes and therefore, it is controllable. For instance, classifying loan customers into the wrong credit categories can result in substantial differences in the default rates and loan provision requirements, and is an example of a risk that is controllable because the cause is known.

'Uncontrollable risk', on the other hand, is defined as any risks that do not have causal factors that can be influenced. Their impact is determined through loss models that analyse extreme values (losses), and use classification instead of causes. Ideally, extreme loss models will be used with scenarios that provide stress points for the analysis. Uncontrollable does not mean that there is nothing that can be done about it, as there are many mitigation strategies that can be implemented in order to reduce the effects of loss. Also uncontrollable risk may become controllable if an assignable cause can be found and which would enable the management to carry out a corrective action.

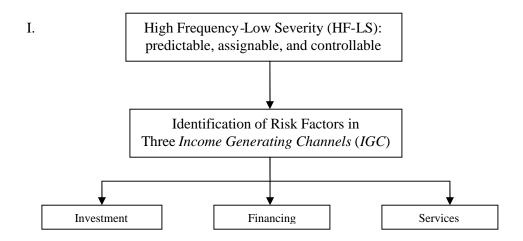
The proposed *DGSA* deals with controllable risk; in other word *DGSA* is designed to be linked with the causality process in *HF-LS* type of operational risks.

# 4.3.1. Building Blocks of DGSA

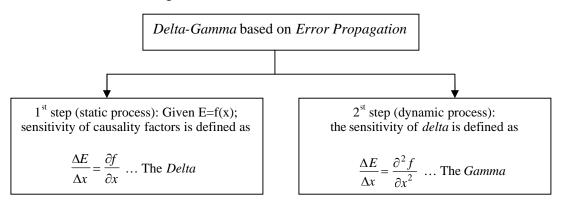
The analysis of *DGSA* begins by developing a function for a value adding process and then examining the key factors that contribute to the performance and their associated errors (uncertainties). This can be done by partitioning the business unit into different income generating channels, which would then function as the unit of analysis for measuring operational risk, as shown in Figure 4.1.

Figure 4.1: Delta-Gamma Sensitivity Analysis Framework for Operational Risk

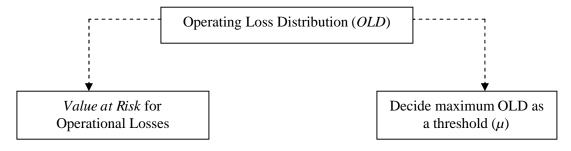
Measurement in Islamic Banks



II. Establish earnings functions related to risk factors in each *IGC*. How?



## **Output:**



Source: Author's own

Income generating channels can be defined as the production unit by which a bank creates a product valuable to its customers. An activity in the income generating channels employ purchased inputs, human resources, capital, and some form of technology to perform its function (Porter, 1985). Since a business unit has profit and loss reporting (by definition), its income generating processes are the key components that make up the profit and loss for the business unit. In this proposed model, Islamic banks business model can be partitioned into three income-generating channels, namely; (i) investment channel; (ii) financing channel, and (iii) service channel.

- a. Investment channel comprises any investment in the form of a partnership. There are two types of investing instruments: fund management (*mudarabah*) and equity partnership (*musharakah*). *Mudarabah*, which can be short, medium, or long term, is a trust-based financing agreement whereby an investor entrusts capital to an agent to undertake a project. Profits are based on a pre-agreed, agreed ratio. *Musharakah*, which can be either medium or long term, is a hybrid of *shirka* (partnership) and *mudarabah*, combining the act of investment and management. In the absence of debt security, the *Shariah* encourages this form of financing.
- b. Financing channel contains any financing instruments that are used primarily to finance obligations arising from the trade and sale of commodities or property. Financing instruments also include instruments generating rental cash flows against exchange of rights to use the assets such as *ijarah* and *istisna*'. Financing instruments are closely linked to a sale contract and therefore are collateralised by the product being financed. These instruments are the basis of short-term assets for the Islamic banks. *Murabahah*, a cost-plus sales contract, is one of the most popular contracts for purchasing commodities and other products on credit.
- c. Service channel consists of any financial transactions that create earnings by charging fees. The example of which is *wakala*.

For each income generating channel, an earning figure can be located and linked up with causal factors for the business. Causal factors can be defined as factors that have impacts on earnings. In other words, *DGSA* uses risk factors resulting from causal factors that create losses with a random uncertainty to measure the variability of earnings.

The next step in *DGSA* is to divide losses into assignable and un-assignable losses. It is important to note that the cut-off point of assignable and un-assignable losses is the threshold determined from the maximum operating loss distribution. Assignable loss, by definition is *HF-LS* type of operational risk which can be linked to a risk factor that could contribute to the loss. In contrast, un-assignable loss cannot be tied to a risk factor, since the cause is normally unknown as it can be from unprecedented external events. Based on the causality between risk factors contributing to assignable losses, an earning function can be produced in each income generating channel. The *DGSA* methods use factors, which lead to loss and their sensitivities to generate loss distributions in different business unit.

It is worth noting here that losses within business unit are not normally accounted for in a systematic way that would allow their direct assignment to risk factors. Since there are a large number of small losses, many banks simply aggregate operational losses in general accounts along with other entries. They may be included as a cost of doing business or simply mixed up in the profit and loss accounting. Without having a loss figure that can be linked to risk factors, therefore, it is almost impossible to produce a direct measurement of operational risk caused by assignable loss. Hence *DGSA* method can overcome this problem.

In summary, the steps of building *DGSA* frameworks are as follows:

- 1) Establish the business model with income generating channel;
- 2) Determine the risk factors for the major activities in the income generating channel;
- Determine the relations between risk factors and earning through setting up earnings function in different income generating channel;
- 4) Estimate operational losses using uncertainty of the risk factors propagated to the risk in earnings (*Delta-Gamma* method);

- 5) Set the threshold of operating losses from the processes using the risk factor uncertainties and operating losses from *Delta-Gamma* method;
- 6) Filter the large losses using the threshold.

The following section discusses the key features of the proposed model.

# 4.3.2. Key Features of DGSA

The *DGSA* methodology is the calculation technique to determine the value of the assignable losses based on the sensitivity of the causality between the risk factors. *DGSA* is produced through error propagation of the risk factors to measure operational risk. The uncertainty of the risk factors is utilised to calculate the uncertainty in earnings using sensitivities; with which the relation of the change in earnings to a change in the risk factors can be located.

In *DGSA*, operational risk is measured as the uncertainty in earnings due to two parts. First using the uncertainty in causal factors for losses up to a threshold and second using a large loss model for un-assignable loss above a threshold. Causality model, hence, plays a critical role in determining the risk factors establishing the model. Hence, the combination of the two constitutes *DGSA* and is described by the operational risk formula as follows:

$$u(E) = fu(\Delta X_1)...u(\Delta X_n) + \Phi(L_{unassignable} | L_{unassignable} > \mu)$$
(4.2)

Uncertainty in earnings due to operational risk is a function of the uncertainties in a set of risk factors plus a function of the distribution of un-assignable losses larger than a given threshold ( $\mu$ ). *DGSA* method is used to calculate the first term in the model. This model expresses the uncertainty in earnings as a function of the uncertainty in a set of risk factors:

$$u(E) = f(u(\Delta X_1)...u(\Delta X_n))$$
(4.3)

DGSA method for measuring operational risk is based on five following key concepts:

(i) Earnings as a function of causal factors.

In *DGSA* method, it is assumed that earnings are described by a series of causal factors. For a given earnings level, there is a set of causal factors whose values are used to estimate earnings:

$$earnings = f(causal factors)$$
 (4.4)

Earnings described as a function of a set of causal factors. For example, earnings may be calculated as 20% of sales revenue minus an adjustment for rejects. By separating the causal factors into constants and volatilities, earnings can be described by a set of performance drivers that create the expected level of earnings and a set of risk factors that create volatility in the level of earnings (risk):

$$earnings = f(performance drivers) \pm f(risk factors)$$
 (4.5)

Earnings described as a function of performance drivers for level and risk factors for volatility. Therefore in the model earnings are calculated as 20% of sales revenue minus the variance to target cost for rejects. 'Sales revenue' is the performance driver and 'rejects' is the risk.

(ii) The risk in earnings is a random fluctuation in value caused by uncertainty in the risk factors. Given

$$E = f(x) \tag{4.6}$$

Therefore

$$u(E) \approx f(u(x)) \tag{4.7}$$

(iii) The basic measure of uncertainty for operational risk is the standard deviation of the mean, or standard error.

In general, for any measure the standard deviation of the mean of the measured values is referred to as the standard error or simply error (Damodaran, 2003). It is calculated from a sample of n measures using the following formula:

$$\sigma_{\bar{x}} = \sqrt{\frac{1}{n(n-1)} \sum_{k=1}^{n} (x_k - \bar{x})^2}$$
(4.8)

where  $\bar{x}$  is the mean of the measures. Note that this measure is different from the standard deviation of the measures. In fact it is related to it by the simple formula:

$$\sigma_{\bar{x}} = \frac{\sigma_{x}}{\sqrt{n}} \tag{4.9}$$

(iv) Uncertainties are combined using the formula for the expected value of the sum of variances.

This formula is given for the simple case of correlation values of only 0 or 1, corresponding to independent measures and measures that are perfectly correlated. Normally this should be sufficient for operational risk measures.

$$\sigma_z^2 = \sum_i \sigma_i^2 + \left(\sum_j \sigma_j\right)^2 \tag{4.10}$$

Formula for combining uncertainties using standard errors where the i's are uncorrelated and the j's are correlated (perfectly) measures.

(v) Uncertainties for functions of uncertainty measures are calculated using *the law of error propagation*. For each risk factor the sensitivity of the earnings with respect to the factor is needed. The sensitivity is the amount of change in earnings given a single unit change in the factor with everything else remaining unchanged, or the partial derivative of the earnings function with respect to the factor. Given the earnings function that expresses earnings as a function of a factor

$$E = f(x)$$

Then sensitivity is defined as

$$\frac{\Delta E}{\Delta x} = \frac{\partial f}{\partial x} \tag{4.11}$$

The method of combining measurement uncertainties from various factors and accounting for their correlation is known as the *propagation of uncertainty*. The basic formula uses the sensitivities (partial derivatives) of the factors to calculate the standard deviation of the estimate. It is based on a Taylor approximation for the uncertainty in terms of factors such as:

$$R = f(X_{1,} X_{2} ... X_{n}) (4.12)$$

Using the Taylor approximation's first term, the uncertainty for the measure can be figured out using the following formula:

$$u^{2}(r) = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_{i}}\right)^{2} u^{2}(x_{i}) + 2\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\partial f}{\partial w_{i}} \frac{\partial f}{\partial w_{j}} \sigma_{i} \sigma_{j} \rho_{ij}$$

$$(4.13)$$

The formula in 4.13 is the formula for the calculation of combined uncertainty from many factors, also known as the 'general law of error propagation', where  $u(\cdot)$  denotes the uncertainty in the value, r is the risk measurement, x is the factor, and f is the functional relationship between x and r. The partial derivative term is known as the 'sensitivity to the factor'. This formula also explicitly considers correlation between factors  $\rho_{ii}$ .

(vi) The gamma ( $\Gamma$ ) of a portfolio on an underlying assets is the rate of change of the portfolio's delta with respect to the price of the underlying asset, while the delta is the first derivative of the model, the gamma is the second partial derivative of the portfolio with respect to different risk factors:

$$Gamma = \frac{\partial^2 \pi}{\partial S^2} \tag{4.14}$$

If the value of *gamma* is small, the *delta* changes slowly and adjustments to keep a portfolio delta *neutral* only need to be made relatively infrequently. However, if *gamma* is large in absolute terms, then *delta* is highly sensitive to the price of the underlying asset. It is then quite risky to leave a *delta* neutral portfolio unchanged for any length of time. In our analysis, *gamma* is an important factor in determining which risk factors are more influential to income generating channels.

It is expected that partnership type of financing, such as *mudarabah* and *musharaka* would give higher value since they are likely to increase the level of operational risk exposures.

7) Threshold is used to separate losses to be analysed using DGSA from those that are not assignable. As highlighted in the earlier paragraph, DGSA deals with small losses (HF-LS type of operational risks); hence, the threshold is the transition point from small loss (HF-LS) to large loss (LF-HS). However, to ensure that there will not be any overlap; meticulous calculations must be carried out to set the threshold precisely since losses assigned to risk factors using DGSA method are assumed to have random error properties; and DGSA is used to estimate the central tendency of this uncertainty.

It is important to reiterate that the first stage of the proposed model, namely *Delta Gamma* will result in the figure showing the level of operational risk exposures from any controllable risks, reflected by the level of its *value at risk*. As shown in figure 4.1, operational value at risk (*opvar*) of the delta gamma is generated from the operating loss distribution of the earnings functions in each income generating function. However, with regard to the sensitivity analysis showing the causality relationship between operational risk factors, a more thorough analysis on how it will be operationalised in the proposed model is discussed in the following section.

## 4.4 WHY SENSITIVITY ANALYSIS?

The activity in the field of sensitivity analysis (SA) has been steadily growing, due to the increasing complexity of numerical models, whereby SA has acquired a key role in testing the correctness and corroborating the robustness of models in several discipline.

This has led to the development and application of several new *SA* techniques (Borgonovo and Apostolakis, 2001a; Saltelli, 1997; Saltelli, 1999; Saltelli, Tarantolla and Chan, 1999; Turany and Rabitz, 2000). It should be noted that most of the recent literature in portfolio management has proposed *SA* approaches based on partial derivatives (*PD*s) (Drudi, Generale and Majnoni, 1997; Gourieroux, Laurent and Scaillet, 2000; Manganelli, 2004; McNeil and Frey, 2000).

Nevertheless, recent studies in the *SA* literature have highlighted that *PDs*-based *SA* suffers from several limitations when used for parameter impact evaluation and risk management purposes (Borgonovo and Apostolakis, 2001a; Borgonovo and Apostolakis, 2001b; Borgonovo and Peccati, 2004; Cheok, Parry and Sherry, 1998). It is shown that when a *PDs*-based *SA* is employed in the model to evaluate the impact of parameter changes with respect to the generic model output, it will suffer from the two limitations (Borgonovo and Apostolakis, 2001a; Borgonovo and Apostolakis, 2001b; Borgonovo and Peccati, 2004):

- 1) It is equivalent to neglecting the relative parameter changes, or, equivalently, to impose that all the parameters are varied in the same way;
- 2) It does not allow the appreciation of the model sensitivity to changes in groups of parameters

Therefore, using Elasticity (E) is considered to be a better alternative as compared to PDs (Simon and Blume, 1994) since the model does not neglect the relative parameter changes in the model. Therefore, using elasticity based sensitivity analysis will theoretically overcome limitiation 1 as mentioned above. However, limitation 2 might still be in place, as E is not additive (Borgonovo and Apostolakis, 2001a; Borgonovo and Apostolakis, 2001b; Borgonovo and Peccati, 2004).

This study aims to explore and demonstrate that the use of Differential Importance Measure (*D*) would overcome the two above mentioned limitations.

Let us consider the generic model output:

$$Y = f(x) \tag{4.15}$$

where  $x = \{x_i, i = 1, 2, ..., n\}$  is the set of the input parameters. Suppose:

$$dx = [dx_1, dx_2,...,dx_n]^{\mathrm{T}}$$
 (4.16)

which denote the vector of change; if f(x) is is differentiable, then the differential importance of  $x_s$  at  $x^0$  is defined as (Borgonovo and Peccati, 2004)

$$D_{s}(x^{0}, dx) = \frac{df_{s}(x^{0})}{df(x^{0})} = \frac{f'_{x_{s}}(x^{0})dx_{s}}{\sum_{i=1}^{n} f_{i}(x^{0})dx_{i}}$$
(4.17)

D can be interpreted as the ratio of the (infinitesimal) change in Y caused by a change in  $x_s$  and the total change in Y caused by a change in all the parameters. Thus, D is the normalised change in Y provoked by a change in parameter  $x_s$ . It can be shown that (Borgonovo and Apostolakis, 2001a; Borgonovo and Apostolakis, 2001b; Borgonovo and Peccati, 2004; Borgonovo and Peccati, 2005):

(i) D shares the additivity property with respect to the various inputs, for example, the impact of the change in some set of parameters coincides with the sum of the individual parameter impacts. More formally, let  $S \subseteq \{1,2,...,n\}$  identify some subset of interest of the input set; hence it would give:

$$D_{s}(x^{0}, d_{x}) = \frac{\sum_{s \in S} f_{s}(x^{0}) dx_{s}}{\sum_{j=1}^{n} f_{j}(x^{0}) dx_{j}} = \sum_{s \in S} D_{s}(x^{0}, dx)$$
(4.18)

As a consequence,

$$\sum_{s=1}^{n} D_{s}(x^{0}, dx) = 1$$
 (4.19)

for example, the sum of the  $D_i$  (i=1,...,n) of all parameters is equal to unity.

(ii) Equation (4.17) shows that D accounts for the relative parameters changes through the dependence on dx. In fact, equation (4.17) can be rewritten as:

$$D_{s}(x^{0}, dx) = \frac{f_{s}(x^{0})}{\sum_{j=1}^{n} f_{j}(x^{0}) \frac{dx_{j}}{dx_{s}}}$$
(4.20)

In the hypothesis of uniform parameter changes (*H1*) ( $dx_j=dx_s \ \forall j,s$ ), the following can be produced:

$$D1_{s}(x^{0}) = \frac{f_{s}(x^{0})}{\sum_{j=1}^{n} fj(x^{0})}$$
(4.21)

Hypothesis of proportional changes (H2)  $\left(\frac{dx_j}{x_j^0} = \omega \forall j\right)$ , would result in:

$$D2_{s}(x^{0}) = \frac{f_{s}(x^{0}) \cdot x_{s}^{0}}{\sum_{j=1}^{n} fj(x^{0}) \cdot x_{j}^{0}}$$
(4.22)

It can be shown that *D* generalises other local *SA* techniques as the *Fussel-Vesely importance* measure and *Local Importance Measure* based on normalised partial derivatives. More specifically, in case *H2* it holds that (Borgonovo and Peccatti, 2004):

$$D2_{s}(x^{0}) = \frac{E_{s}(x^{0})}{\sum_{j=1}^{n} E_{j}(x^{0})}$$
(4.23)

where  $Es(x^0)$  is the elasticity of Y with respect to  $x_s$  at  $x^0$ . Equation (4.23) shows that E produces the importance of parameters for proportional changes.

This section highlights the benefit of rectifying the partial derivatives based sensitivity analysis with the elasticity based one. The main reason is mainly due to the different variation that different variables might have. To put it differently, assuming that any observed parameters will invariably move in a similar fashion may not be correct, in particular when there are some derivatives products involved in the process. The next challenge, hence, is to determine which risk factors will contribute significantly to the whole process of determining the operational risk exposures, which is discussed in the subsequent section.

#### 4.5 DETERMINATION OF RISK FACTOR CONTRIBUTION

From the practitioners' point of view (Ebnother *et al.*, 2001), a relevant question for them is how much each of the process contributes to the risk exposure. If it turns out that only a fraction of all processes significantly contribute to the risk exposure, then the risk manager should only focus on this particular process. It is, therefore, important to analyse how much each single process contributes to the total risk. This study considers operational value at risk (OpVaR) resulting from operating loss distribution as a risk measure. To split up the risk into its process components, this study considers comparing the incremental risk (IR) of the processes.

Let  $IR_{\alpha}(i)$  be the risk contribution of process i to OpVaR at the confidence level  $\alpha$ .

$$IR_{\alpha}(i) = OpVaR_{\alpha}(P) - OpVaR_{\alpha}(P\backslash\{i\}), \tag{4.24}$$

where  $P\setminus\{i\}$  is the whole set of workflows without process i. Since the sum over all  $IR_{\alpha}$ 's is generally to equal to the OpVaR, the relative incremental risk  $(RIC_{\alpha}(i))$  of process i is defined as the  $IR_{\alpha}(i)$  normalised by the sum over all  $IR_{\alpha}$ , i.e.

$$RIC_{\alpha}(i) = \frac{IR_{\alpha}(i)}{\sum_{j} IR_{\alpha}(j)} = \frac{OpVaR_{\alpha}(P) - OpVaR\alpha(P \setminus \{i\})}{\sum_{j} IR_{\alpha}(j)}$$
(4.25)

as a further step, for each  $\alpha$ , this paper counts the number of processes that exceed a relative incremental risk of 1%. Equation 4.25 also suggests that in determining the risk factor contribution, apart from the level of risk exposure, which sometimes is related to the normality assumption of the model, the other key factor is the determination of the confidence level. It is, therefore, essential to take the two elements into account in generating a more accurate operational value at risk. Following the DGSA, which is the first stage of the proposed model, the next step is to determine the maximum threshold. One it is achieved, the subsequent stage is to analyse any operational risk variables

beyond the threshold, which will be dealt with by extreme value theory technique discussed in the following section.

# 4.6. EXTREME VALUE THEORY (EVT)

Extreme value theory (*EVT*) is a field of study in statistics that focuses on the properties and behaviour of extreme events. In general, there are two main kinds of model for extreme values. The most traditional models are the *block maxima* models; these are models for largest observations collected from large samples of identically distributed observations (McNeil *et al.*, 2005: 271). The second type of model, which is more comprehensive is the peak over threshold (*POT*) model; this is a model for all large observations that exceed some high level, and is generally considered to be the most useful for practical applications, due to their more efficient use of the data (often limited) on extreme outcomes (McNeil *et al.*, 2005: 301).

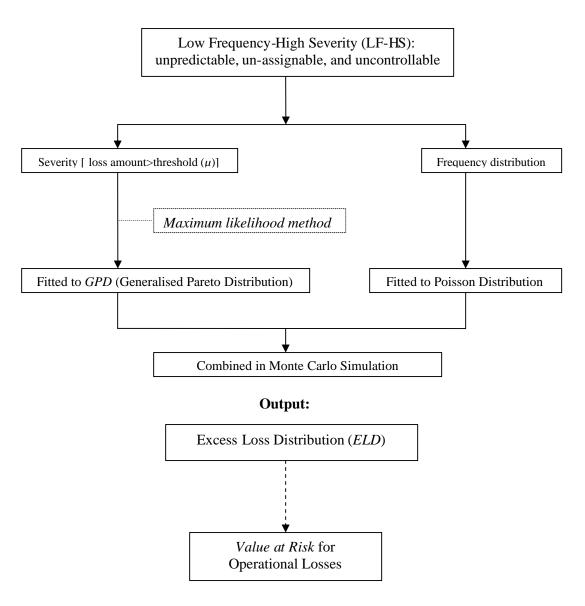
In our analysis, the application of *EVT* as the second leg of the proposed model starts after the determination of a transition point resulting from *DGSA*. It is important to note that the transition point is typically classified as the maximum threshold. *EVT* offers a parametric statistical approach for the extreme values of data. Its roots are in the physical sciences and it has recently been applied to insurance. Since traditional statistical techniques focus on measures of central tendency (*e.g.* mean), they are not an accurate estimators, when estimating values very far from the centre of the data. *EVT*, on the other hand, deals only with the extreme values and ignores the majority of the underlying data and its measures in order to provide better estimates of the 'tails'.

The EVT methodology for operational risk is basically a loss model for large losses using a GPD for the severity. The technique for fitting the GPD to data is the peaks over threshold method (POT), where large values over a specific threshold are fitted to the GPD. Following McNeil  $et\ al.(2005)$ , the POT method deployed in the analysis uses the following basic assumptions: i) the excesses of an independent identically distributed (or stationary) sequence over a high threshold u occur at the times of a Poisson process; ii) the corresponding excesses over u are independent and have a GPD; iii) excesses and exceedance times are independent of each other.

# **4.6.1 Operating Framework for** *EVT*

As depicted in Figure 4.2, the steps for operating *EVT* in the analysis of the proposed model start with the separation of loss amount into its severity and frequency.

Figure 4.2: The Application of Extreme Value Theory for Operational Risk Measurement in Islamic Banks



Source: Author's own

Furthermore, excess losses are fit to a *GPD* to determine the severity of a loss given that it exceeds the threshold. This is a conditional severity distribution for large losses. Since the number of exceedances follows a Poisson distribution, it is fitted and used to estimate the frequency of exceedances. Combining the severity and frequency distributions in a Monte Carlo simulation gives the excess loss distribution. The resulting excess loss distribution is a multi-period loss distribution for only those losses that exceed the threshold.

# 4.6.2 Theoretical Building Blocks of EVT: Fisher-Tippet-Gnedenko Theorem

The Fisher-Tippet-Gnedenko theorem states that given a sample of independent identically distributed loss data  $\{x_1, x_2, ..., x_n\}$ , as the number of observations n becomes increasingly large, the maximum of the sequence of observations, under vert general conditions, is approximately distributed as the 'generalised extreme value' (*GEV*) distribution with cumulative probability distribution function (Lewis, 2004: 204):

$$F(x) = \begin{cases} exp \left\{ -\left[1 + \xi \left(\frac{x - \mu}{\sigma}\right)\right]^{-\frac{1}{\xi}} \right\} for \xi \neq 0 \\ exp - \left\{ exp \left[ -\left(\frac{x - \mu}{\sigma}\right)\right] \right\} for \xi = 0 \end{cases}$$

$$(4.26)$$

where  $\mu$  is the location parameter,  $\sigma > 0$  is a scale parameter,  $1 + \xi z > 0$ ,  $-\infty \le \xi \le \infty, \sigma > 0$ , and  $\xi$  is the tail index parameter. The *GEV* has three forms; if  $\xi > 0$ , then the distribution takes the form of a type II (Frechet) heavy-tailed distribution. For  $\xi < 0$ , the distribution is takes the type III (Weibull) distribution. When  $\xi = 0$ , the distribution is the type I (Gumbel) light-tailed distribution. In fact, the larger the tail index parameter, the fatter is the tail.

#### 4.6.3 Parameter Estimation

The parameter  $\mu$  and  $\sigma$  can be estimated from the sample mean and sample standard deviation, respectively. If we rank the data in order size so that  $x_1>x_2>...>x_n$ , the tail index parameter  $\xi$  can be estimated using the Hill estimator (Cruz, 2002: 78):

Method I: 
$$\hat{\xi}_k = \left(\frac{1}{k-1} \sum_{j=1}^{k-1} ln(x_j)\right) - ln(x_k)$$
 (4.27)

Method II: 
$$\hat{\xi}_k = \left(\frac{1}{k} \sum_{j=1}^k ln(x_j)\right) - ln(x_k)$$
(4.28)

The problem now is how to choose k values, for which theory gives little susbtance as to what value to choose. Furthermore, the actual estimate will be sensitive to the value of k chosen. In practice, the average estimator, using either of the following two formulas, often works well as identified in the following methods (Lewis, 2004: 204):

Method 1: 
$$\hat{\xi} = \frac{1}{n} \sum_{i=1}^{n} \theta_i$$
 where  $\theta_k = \left(\frac{1}{k-1} \sum_{j=1}^{k-1} ln(x_j)\right) - ln(x_k)$  for  $k = 1, 2, ..., n$  (4.29)

Method 2: 
$$\hat{\xi} = \frac{1}{n} \sum_{i=1}^{n} \theta_i$$
 where  $\theta_k = \left(\frac{1}{k} \sum_{j=1}^{k} ln(x_j)\right) - ln(x_k)$  for  $k = 1, 2, ..., n$  (4.30)

Equation 4.29 and 4.30 attempt to substantiate what is mentioned in the earlier paragraph as to the choice of k in the determination of the tail index parameter ( $\xi$ ) can be estimated using the Hill estimator. Although the only difference is only on (k-l) and (k) as shown in both equations; the impact on  $\xi$ , however, can be really substantial as the result would significantly affect the distribution fitting within the area of extreme values above the threshold. How big or small the tail index parameter ( $\xi$ ), would subsequently impinge upon the modelling of the severity and frequency of extreme value distribution is discussed in the following sub-section.

# 4.6.4 Severity Model

An alternative *EVT* approach to calculate *OpVaR* is to use peaks over threshold (*POT*) modelling. The underlying principle of the operating framework is to use peaks over threshold. Although the method of block maxima utilises the Fisher-Tippet-Gnedenko theorem to inform us what the distribution of the maximum loss is, *POT* uses the

Picklands-Dalkema-de Hann to inform us what is the probability distribution of all events greater than some large present threshold. The Picklands-Dalkema-de Hann theorem states that if  $F_u$  is the conditional excess distribution function of values of the ordered losses X above some threshold,  $\mu$  is given by

$$F_{\mu} = Pr \ ob(X - \mu \le y | X > \mu), 0 \le y \le x_F - \mu.$$
 (4.31)

Then for a suitably high threshold the limiting distribution of  $F_u$  is a generalised Pareto distribution (GPD) with cumulative distribution function (Jorion, 2007: 59):

$$GPD_{\xi\sigma} = \begin{cases} 1 - \left(1 + \xi \frac{x}{\sigma}\right)^{-\frac{1}{\xi}} i, \xi \neq 0 \\ 1 - exp\left(-\frac{x}{\sigma}\right), \xi = 0 \end{cases}$$

$$(4.32)$$

where  $\sigma > 0$ , and  $x \ge 0$  when  $\xi \ge 0$  and  $0 \le x \le -\beta/\xi$  when  $\xi < 0$ . The parameters  $\xi$  and  $\beta$  are referred to, respectively, as the *shape* and *scale* parameters. In other words, ys are called *excesses* whereas xs are called *exceedances*.

It is possible to determine the conditional distribution function of the excesses (i.e., ys) as a function of x:

$$F_{u}(y) = P(X - u \le y | X > u) = \frac{F_{x}(x) - F_{x}(u)}{1 - F_{x}(u)}$$
(4.33)

In this representations the parameters  $\xi$  is crucial, when  $\xi = 0$ , we have an exponential distribution; when  $\xi < 0$ , we have a Pareto distribution—II Type and when  $\xi > 0$ , we have Pareto distribution—I Type. Moreover, this parameter has a direct connection with the existence of finite moments of the losses distributions. We have the following equations:

$$E(x^k) = \infty \text{ if } k \ge 1/\xi \tag{4.34}$$

Hence in the case of a *GPD* as a Pareto—I Type, when  $\xi \ge 1$ , we have infinite mean models. As also shown by Moscadelli (2004) and Neslehova *et. al* (2006).

Following Di Clemente-Romano (2004), we suggest to model the loss severity using the lognormal for the body of the distribution and *EVT* for the tail in the following way:

$$F_{i}(x) = \begin{cases} \Phi\left(\frac{\ln x - \mu(i)}{\sigma(i)}\right) & 0 < x < u(i) \\ 1 - \frac{N_{u}(i)}{N_{i}} \left(1 + \xi(i)\frac{x - u(i)}{\beta(i)}\right)^{-1/\xi(i)} & u(i) \le x \end{cases}$$

$$(4.35)$$

where:

 $\Phi$  = standardised normal cumulative distribution functions;

 $N_u(i)$  = number of losses exceeding the threshold u(i);

N(i) = number of loss data observed in the *i*th ET;

 $\beta(i)$  = scale parameters of a *GPD*;

 $\xi(i)$  = shape parameters of a *GPD*;

An important issue to consider is the estimation of the severity distribution parameters. While the estimation maximum likelihood (ML) in the lognormal case is straightforward, in the EVT case, it is extremely important to consider whether ML or the alternative probability weighted moment (PWM) routines are able to capture the dynamics underlying losses severities.

With respect to ML, the log-likelihood function equals

$$l((\xi,\beta;X) = -n \ln \beta - \left(\frac{1}{\xi} + 1\right) \sum_{i=1}^{n} \ln \left(1 + \frac{\xi}{\beta} X_i\right)$$
(4.36)

This method works well if  $\xi > -1/2$ . In this case, it is possible to show that

$$n^{1/2} \left( \hat{\xi}_n - \xi, \frac{\hat{\beta}_n}{\beta} - 1 \right) \xrightarrow{d} N(0, M^{-1}), n \longrightarrow \infty$$

$$(4.37)$$

where

$$M^{-1} = (1 + \xi) \begin{pmatrix} 1 + \xi & -1 \\ -1 & 2 \end{pmatrix}$$
 (4.38)

Instead, the *PWM* consists of equating model moments based on a certain parametric distribution function to the corresponding empirical moments based on the data. Estimations based on *PWM* are often considered to be superior to standard moment-based estimates. In our case, this approach is based on these quantities (Di Clemente-Romano ,2004):

$$w_r = E[Z(G\overline{P}D_{\xi,\beta}(Z))^r] = \frac{\beta}{(r+1)(r+1-\xi)}, \quad r = 0,1,...$$
 (4.39)

where  $G\overline{P}D_{\xi,\beta} = 1 - GPD_{\xi,\beta}$ , Z follows a  $GPD_{\xi\beta}$ 

From the above equations, it is possible to show that

$$\beta = \frac{2w_0 w_1}{w_0 - 2w_1} \text{ and } \xi = 2 - \frac{w_0}{w_0 - 2w_1}$$
(4.40)

Hosking and Wallis (1987) show that *PWM* is a viable alternative to *ML*, when  $\xi \geq 0$ . Analysis in the proposed model, however, intends to estimate the *GPD* parameters using *mean excess plot* together with the standard Hill estimator. The reason is simply to anticipate the small number of observations beyond the threshold. The small numbers of observations in the tail area will not only have high variance, it will also create difficulties in the process of modelling the frequency distribution as discussed in the following section.

# 4.6.5 Frequency Model

Having fitted a *GPD* to the amount of loss for a set of excess losses, the next step is to determine the frequency of losses using a Poisson distribution, which is well known as a single parameter distribution for the number of occurrences of an event with relatively small probabilities given a relatively large sample. The formula for the Poisson distribution is (Jorion, 2007: 57):

$$Pr(x) = \frac{\lambda e^{-x}}{x!} \tag{4.41}$$

where  $x \ge 0$ , and the parameter  $\lambda > 0$  can be interpreted as arithmetic mean. The fitting of the Poisson to a set of occurrences proceeds using the inter-arrival times for the loss events. That is, the average time between events can be used to determine the arrival rate or *lambda* for the Poisson formula. Note that the arrival rate is simply the inverse of the inter-arrival time. For the Poisson distribution, it can be shown that the maximum likelihood estimator for  $\lambda$  is given by the mean arrival rate formula as below (Johnson *et al.*, 1994)

$$\lambda = \frac{\sum_{k} k n_k}{n} \tag{4.42}$$

Formula for estimating  $\lambda$  for the Poisson distribution; where

k is the number of events in a period;

 $n_k$  is the number of periods with k events;

n is the total number of periods.

A goodness of fit statistic for the Poisson distribution can be found using a simple  $\chi^2$ squared test. The test statistic is (McNeil *et al.*, 2005: 486):

$$\chi^{2} = \sum_{k} \frac{(n_{k} - n \operatorname{Pr}(k; \lambda))^{2}}{n \operatorname{Pr}(k; \lambda)}$$
(4.43)

Chi-squared test statistic for the goodness of fit of the Poisson distribution to a set of data, where  $Pr(k; \lambda)$  is the probability of k events for the Poisson distribution with parameter  $\lambda$ . The degrees of freedom are n-2.

Although there are other types of modelling frequency distribution, such as binomial distribution and negative binomial distribution; being the most practical model, the analysis in this study intends to apply poisson distribution in the frequency distribution.

Once the analysis completes the modelling of severity and frequency, the next step is to combine these two in order to generate excess loss distribution as shown in figure 4.2. The following sub-section discusses the techniques in combining severity and frequency modelling.

# 4.6.6. Compounding via Monte Carlo Methods

Once the severity and frequency distributions have been estimated, it is necessary to compound them via Monte Carlo methods to get a new data series of aggregate losses, so that we can then compute the desired risk measures, such as the *VaR* and expected shortfall.

The random sum  $L=X_1+...+X_n$  (where N follows a Poisson distribution) have distribution function (McNeil et al., 2005: 367):

$$F_{L}(x) = Pr (L \le x)$$

$$= \sum_{n=0}^{\infty} p_{n} Pr(L \le x | N = n)$$

$$= \sum_{n=0}^{\infty} p_{n} F_{x}^{*n}(x)$$

$$(4.44)$$

where  $F_x(x) = Pr(X \le x) = \text{distribution function of the severities } X_i$ 

 $F_x^{*n} = n$ -fold convolution of the cumulative distribution function of X.

Hence, the aggregation of frequencies and severities is performed as a sum of severities distribution function convolutions, thus determining a compound distribution, whose density function can be obtained by:

$$f_{L}(x) = \sum_{n=0}^{\infty} p_{n} F_{x}^{*n}(x)$$
 (4.45)

The analysis in this study computes this aggregation via convolution using Monte Carlo methods. It should be noted that that the convolution is a bit more complex due to the split of severity distribution in two parts: the body of the distribution, which follows a lognormal distribution, and the tail, which follows a GPD. As a consequence, the study will have two different severity levels. Hence, the probability associated at each severity (i.e., the number of observations obtained by the Poisson distribution) has to be congruent with the fact that losses may belong to the body or to the tail. The analysis will, however, emphasize the severity level that belongs to the tail. Therefore, it is crucial to consider F(u), where u is the GPD threshold and F is the distribution function associated to this point. After having sampled from the two severity distributions, every single loss  $X_i$  whose F(Xi) > F(u) will be modeled as a GPD random draw; the process of which will be used to estimate operational value at risk in the tail area. The next step is to create proximity of operational risk exposures by summing up the VaR resulted in the EVT process, being the second stage of the proposed model and the DGSA process, being the first stage of the proposed model.

## 4.7 CONCLUDING REMARKS

The scope of this chapter was to show that measuring operational risk, adapted to the structure of business unit or value adding process, is feasible if the causality taking place in the banking operations is considered. The proposed *DGSA-EVT* model would give a number advantages to the operational risk managers. First, it is a reflection of potential loss that is not merely based on actual loss figure which is rarely available. This aspect is very crucial since in most cases, operational losses are not recorded. This advantage is

achieved by combining operational value at risk in the first stage (DGSA) and second stage (EVT) of the proposed model.

Second, the models will show which risk factors that dominantly contribute to the level of exposure of operational risk. This is done through the analysis of cause-effect relationship that takes place in the first stage of the proposed model. Moreover, running the *delta* or *gamma* within the earnings function in each income generating channel will not only show the dominant risk factor, but it will also possibly show the pattern of the relationship among the analysed operational risk variables, whether they are linear or nonlinear relationship.

In addition, through EVT process, the proposed model will also help operational risk manager in the determination of risk which has a detrimental impact on the whole system. However, in running the EVT, an analyst or operational risk manager, as mentioned in the earlier paragraph, has to be careful in choosing the right method in determining the tail index parameter ( $\xi$ ), so the chosen threshold does not result in the high variance of the analysed variables.

Third, the proposed model, if implemented, is not costly, since it can be operationalised by using a variety of key financial indicators in the financial statements of the bank. Being too costly is actually one of the main reasons why financial institutions are reluctant in setting up the infrastructure for operational risk management, especially for its measurement.

# **CHAPTER 5**

# OPERATIONAL VALUE AT RISK (OpVaR) BASED ON **VOLATILITY, SKEWNESS AND KURTOSIS**

## **5.1 INTRODUCTION**

As discussed in chapter 4, the proposed model in this study has two main stages consisting of (i) an estimation of Delta-Gamma Sensitivity Analysis (DGSA) which will result in value at risk (VaR) for operating loss distribution<sup>24</sup>, and (ii) an estimation of extreme values resulting in VaR for excess loss distribution<sup>25</sup>. Thus, summing up both VaRs would provide an approximation of the operational risk level in the analysed bank. In furthering the details of the proposed measurement model, while the previous chapter is theoretical construction in nature, this chapter aims at empirically testing the proposed model by using empirical data.

Due to its computational and numerical intensity coupled with limited availability of data, a few adjustments is made in testing the proposed model. Firstly, the process through which VaR is generated needs a slight modification since the variables in the analysis do not reflect non-linearity relationships. It should be noted that non-linearity might arise from derivative products such as options. As shown in the later section, none of the variables in the model are categorised as one, hence the issue of non-linearity is irrelevant since a quadratic relationship among variables does not exist. A quadratic relationship between variables occurs when a change in one variable affects a change in another variable in a non-linear manner. As a result, the issue of first and second derivative relationship, namely delta-gamma cannot be analysed. The analysis, therefore, focuses on the issue of normality vis a vis non-normality of the distribution density functions of the analysed variables. Secondly, through a range of statistical tests as well

 <sup>&</sup>lt;sup>24</sup> See figure 4.1 in chapter 4
 <sup>25</sup> See figure 4.2 in chapter 4

as simulation of histograms and graphs of probability density functions, the industrial level data drawn from published monthly financial reports demonstrate that only two variables under analysis constitute extreme value distribution reflecting extreme events taking place within certain period of time. Consequently, the second leg of the proposed model, namely extreme value theory model (*EVT*) could not be carried out.

In analysing VaR, this study does not simply use the data and follow a prescribed assumption to produce VaR. One of the very strong assumptions in VaR is normal distribution, which according to the report by Financial Times (2012), is no longer the norm and an over reliance to it was perceived to be one of the causes leading to the 2008 global financial crisis.

Therefore, the analysis in this study will carefully examine the behaviour of the data by taking into account volatility, skewness and kurtosis of the variables. As is shown in the later section, volatility resulting from the variance employs two models: constant-variance model and exponential weighted moving average (*EWMA*) model, which as an approach is adopted by Li (1999), Hull and White (1998), and RiskMetrics (1996). While the analysis in skewness and kurtosis of the data is applied to examine the level of non-normality of the analysed variables.

In order to investigate the normality or non-normality of the analysed variables, this study also employs a generation of probability density function through a simulation and deploys two statistical tests, namely *Kolmogorov-Smirnov* and *Anderson-Darling* tests.

The results of all the tests mentioned above will help in determining the technique to be applied to estimate the *operational value at risk*.

The organisation of this chapter is as follows; after this introduction, the second section revisits the theoretical background of value at risk (*VaR*), while the third section provides an explanation on the methodology used in the study. Section four, provides a discussion on the empirical findings before it is concluded in the fifth section.

## 5.2 WHAT IS VALUE AT RISK (VAR)?

Formally the Value at Risk (VaR) can be defined as the maximum loss that could occur in a given confidence level within a certain period of time (Jorion, 2005 :252). If the investor's temporal horizon is denoted by  $\tau$ , with  $R_t(\tau)$  the series data in the interval (t,  $t+\tau$ ) and with  $\theta$  the level of confidence, the VaR given by the loss such that,

$$P(R_t(\tau) \le -VaR) = 1-\theta \tag{5.1}$$

Thus, the VaR is the percentile at the  $(1-\theta)\%$  of the variables under analysis in the interval  $(t, t+\tau)$ . In the model, the temporal horizon  $\tau$  and the level of confidence  $\theta$  are parameter chosen by the investor. The choice of  $\tau$  depends on the frequency with which the investor wishes to control his/her investment.

As mentioned earlier that the *RiskMetrics* (1996) model assumes that the conditional distribution  $R_t(\tau)$  is a standard Gaussian distribution. In particular, *RiskMetrics* assumes that the financial returns, conditioned to the forecast volatility level, are distributed like a standardised normal distribution:

$$R_t(\tau)/\sigma_t \sim N(0,1) \tag{5.2}$$

The explicit modelling of the volatility series captures the time varying persistent volatility observed in the real financial market. Given the hypothesis of normality, the VaR at  $(1-\theta)\%$  is given by simply multiplying the tabulated value of the corresponding percentile,  $k_{1-\theta}$ , times the volatility forecast in the period  $(t, t+\tau)$ , therefore,

$$VaR_t = k_{1-\theta} \int_t^{\sigma} (5.3)$$

The *RiskMetrics* hypothesis simplifies the VaR calculation for those portfolios with many assets<sup>26</sup>. If it is denoted with  $[w_1, w_2, ..., w_n]'$  the vector of the positions taken in n assets forming the portfolio, its return at time t is given by,

$$R_{p,t} = \sum (5.4)$$

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<sup>&</sup>lt;sup>26</sup> See RiskMetrics Technical Document (1996)

$$, , , \dots,$$
 (5.5)

which follows a joint Gaussian distribution, and the linear combination of the return is normally distributed, ( , ) where is the mean and is the standard deviation of portfolio . Assuming that within a short period of time the expected return is null, the VaR is determined by the portfolio standard deviation,  $VaR_t = k_{1-\theta}^{\sigma}_{p,t}$ .

More importantly, VaR is defined contingent on two arbitrarily chosen parameters; (i) a confidence level  $(\alpha)$ , which indicates the likelihood that an investor will get an outcome no worse than predicted VaR, and which might be any value between 0 and 1; and (ii) a holding or horizon period, during which a certain portfolio is measured, and it could be a day, a month or any time period.

Some VaRs are illustrated in figure 5.1, which shows a common probability density function (pdf) of profit/loss over a chosen holding period.<sup>27</sup> Positive values correspond to profits, and negative observations to losses. If  $\alpha$ =0.95, the VaR is given by the negative point on the x-axis that cuts off the top 95% of profit/loss observations from the bottom 5% of tail observations. In this case, the relevant x-axis value (or quantile) is -1.645, so the VaR is 1.645. The negative profit/loss value corresponds to a positive VaR, indicating that the worst outcome at this level of confidence is a loss of 1.645<sup>28</sup>. Alternatively.  $\alpha$ =0.99 could be set and in this case the VaR would be negative of the cut-off between the bottom 1% tail and everything else. The 99% *VaR* is 2.326.

 $<sup>^{27}</sup>$  The figure is constructed on the assumption that profit/loss is normally distributed with mean 0 and standard deviation 1 over a holding period of 1 day.

<sup>&</sup>lt;sup>28</sup> In practice, the point on the x-axis corresponding to VaR will usually be negative and, where it is, will correspond to a (positive) loss and a positive VaR. However, this x-point can sometimes be positive, in which case it indicates a profit rather than a loss and, hence, a negative VaR. This also makes sense; if the worst outcome at this confidence level is a particular profit rather than a loss, then the VaR, the likely loss, must be negative.

0.4 - 95% VaR = 1.645 | - 99% VaR = 2.326 | 0.25 - 99% VaR = 2.326 | 0.15 - 0.15 - 0.0

Figure 5.1: Value at Risk

Source: Dowd (2005: 28)

Since *VaR* is contingent on the choice of confidence level, Figure 5.1 suggests that it will usually increase when the confidence level changes<sup>29</sup>. This point is further illustrated in Figure 5.2, which shows how the *VaR* varies, as we change the confidence level. In this particular case, which is also quite common in practice, the *VaR* not only rises with the confidence level, but also rises at an increasing rate.

Profit (+)/loss (-)

Since the VaR is also contingent on the holding period, it should be taken into consideration as to how the VaR would vary as the holding period changes. This behaviour is illustrated in Figure 5.3, which plots 95% VaRs based on two alternative  $\mu$  values against a holding period that varies from 1 day to 100 days.

<sup>29</sup> Strictly speaking, the VaR is non-decreasing with the confidence level, which means that the VaR can sometimes remain the same as the confidence level rises. However, the VaR cannot fall as the confidence level rises.

-

Figure 5.2: VaR and Confidence Level

Source: Dowd (2005: 29)

0.93

0.92

0.94

Confidence level

0.95

0.97

0.98

0.9

0.91

With  $\mu=0$ , the VaR rises with the square root of the holding period, but with  $\mu>0$ , the VaR rises at a lower rate and would in fact eventually turn down. Thus, the VaR varies with the holding period, and the way it varies with the holding period depends significantly on the  $\mu$  parameter.

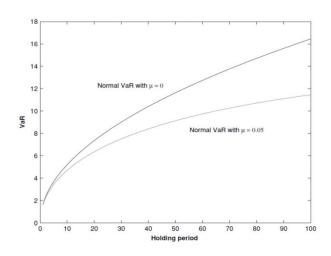


Figure 5.3: VaR and Holding Period

Source: Dowd (2005: 30)

VaR method earned its prominence among academics and practitioners since it was introduced by JP Morgan in the mid-nineties. Many researchers have also given their positive responses to the idea ever since. Risk managers, particularly, are very keen to utilise VaR because of its analytical tractability. Although numerical methods to calculate VaR have been developed leading to more accurate results depending on less restrictive assumptions, many institutions seems to still rely on analytical methodologies. The most important advantage of analytical methods over their numerical counterparts is the saving of computing time that makes real time calculations possible. This seems to outweigh their disadvantages for many practical applications. Nevertheless, the methods are not without any flaws. One of the shortcomings is the normality assumption which has become the main subject of criticism indicated by the subsistence of high skewness and excess kurtosis of the distribution functions (Hull and White, 1998; Kupiec, 1995; Li and Turtle, 1997). This study, for that matter, attempts to resound similar criticisms in light of the proposed theoretical framework presented in chapter 4.

Duffie and Pan (1997) identify jumps and stochastic volatility as possible causes of kurtosis. They point out that under a jump-diffusion model, kurtosis is a declining function of the time horizon, whereas under a stochastic volatility model, it is an increasing function of the time horizon (at least for the time horizons normally considered in *VaR* calculations).

Nevertheless, many alternatives exist for the statistical and computational decisions to be made for the computation of *VaR*, which is a quantile of a portfolio's loss distribution over a given time horizon. Several methods have been proposed to compute a quantile of the distribution, among them are Monte-Carlo simulation (Pritsker, 1996), Johnson transformations (Zangari, 1996a; Longerstaey, 1996), Cornish-Fisher expansions (Zangari, 1996b; Fallon, 1996), moment-based approximations utilising the theory of estimating functions (Li, 1999), and Fourier-inversion (Rouvinez, 1997; Albanese, 2000).

It should be noted that with respect to time and calculations efficiency, Pichler and Selitsch (1999), Mina and Ulmer (1999) and Jaschke (2001) demonstrate that *Cornish*-

*Fisher* expansions are preferable to other methods, such as Johnson transformations, Delta-Normal, and Fourier inversion.

However, before further analysing the technicalities of *VaR* based on *Cornish Fisher* expansion, the following section is to discuss the potential risk factors within three income generating channels indicated in chapter 4 followed by a discussion on the behaviour of data in three aspects; volatility, skewness and kurtosis.

## 5.3 CONSTRUCTING THE PROPOSED DGSA METHODOLOGY

The proposed *DGSA* methodology is an estimation technique to determine the value of assignable (controllable) losses based on the causality between risk factors. The model commences by partitioning the business unit into different income generating channel (*IGC*), which would then function as the unit of analysis for measuring operational risk<sup>30</sup>. Each *IGC* is partitioned into three different units: (i) investment channel, (ii) financing channel, and (iii) service channel.<sup>31</sup> The level of operational risk exposures is measured as the uncertainty of earnings in each *IGC* in terms of a series of risk factors. This study, however, defines uncertainty as volatility.<sup>32</sup>

An investment channel consists of any investment in the form of a partnership, while financing channel contains any financing instruments that are used primarily to finance obligations arising from the trade and sale of commodities or property. Service channel, however, consists of any financial transactions that create earnings by charging fees.

In *DGSA*, it is assumed that earnings are described by a series of risk factors. For a given earnings level, there is a set of risk factors whose values are used to estimate earnings:

$$= (5.6)$$

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<sup>&</sup>lt;sup>30</sup> Income generating channel is defined as the production unit by which an Islamic bank creates a product to its customers.

<sup>&</sup>lt;sup>31</sup> Hence, DGSA has three IGCs, namely  $IGC_I$  (income generating channel for investment channel),  $IGC_F$  (income generating channel for financing channel), and  $IGC_S$  (income generating channel for service channel)

<sup>&</sup>lt;sup>32</sup> From this point onward, the term volatility is used to explain uncertainty

The equation above shows that earnings are defined as a function of a set of risk factors, which can be transformed into:

$$() \approx (( ... )) \tag{5.7}$$

Suggesting that volatility of earnings, v(E) is determined by a set of volatility of risk factors, ( ( ... )).

Based on the *IGC* partitions, equation (5.7) can be expanded as follows:

$$(\ )\approx\ (\ ...\ )$$
 (5.9)

$$() \approx ( \dots )$$
 (5.10)

where

( ) = volatility of earnings in investment channel

( ) = volatility of earnings in financing channel

( ) = volatility of earnings in service channel

... = volatility of a set of risk factors in investment channel

... = volatility of a set of risk factors in financing channel

... = volatility of a set of risk factors in service channel

In  $IGC_I$ , earnings are represented by the rate of return on financial securities (RoS), which is based on profit sharing. The financial securities represent ownership of profit sharing based investments carried out by Islamic banks.

A proposed set of risk categories that have been highlighted in chapter 3 in relation to operational risks are *shariah* non-compliance risk, fiduciary risk, people risk, legal risk

and technology risk.<sup>33</sup> Due to the quantitative nature of the data, *shariah* non-compliance risk, legal risk and technology risk, which are qualitative in nature, is not be included in the model constructed in thus study, as their relevant data cannot be captured by the model. This, as a result, leaves the model with fiduciary risk and people risk.

Fiduciary risk can be characterised as a condition whereby Islamic banks are liable for losses arising from their negligence, misconduct or breach of their investment mandate. In other words, fiduciary risk is an indication of failure to "perform in accordance with explicit and implicit standards applicable to their fiduciary responsibilities" (IFSB, 2005:26).

In the model, a set of fiduciary risk is decomposed into a series of risk factors, namely volatilities of (i) volume of investment in financial securities (FS), (ii) return on saving deposits (RoSD), (iii) return on 1-month time deposits (RoTD), and (iv) ratio of operating expenses over operating income ( $BOPO_I$ ).

People risk, nonetheless, is another type of operational risk arising from incompetence or fraud, which exposes Islamic banks to potential losses. This includes human errors, lack of expertise, compliance and fraud (Akkizidis and Kumar: 2008). It should be noted that the volatility of training expenses  $(Tr_I)$  are used as a proxy for people risk.

In  $IGC_F$ , on the other hand, rate of return on financing (RoF) signifies earnings. RoFcontains a bulk of returns on financing, which unsurprisingly, is still predominated by murabaha mode of finance.  $^{34}$  Fiduciary risk in  $IGC_F$  is symbolised by the volatilities of two elements, namely volume of financing (F) and the ratio of operating expenses over operating income  $(BOPO_F)$ . As for people risk,  $IGC_F$  employs a similar variable as  $IGC_I$ , that is the volatility of training expenses  $(Tr_F)$ .

 $IGC_S$  is deliberately taken out of the analysis, since the volume of service fee does not contribute significantly to the value adding process.<sup>35</sup> Hence, IGC<sub>I</sub> has the following the earnings function:

See table 3.2 in Chapter 3
 As of June 2010, *murabaha* makes up over 60 percent of the total financing.
 As of June 2010, service fee only contributes less than 2% to the value adding process

$$( ) \approx ( , , , , )$$
 (5.11)

Volatility of return on securities is a function of a series of risk factors, *i.e.* volatilities of investment in securities ( ), return on saving deposits ( ), return on 1-month time deposits ( ), ratio of operating expenses over operating income in investment channel ( ), and training expenses in investment channel ( ).

Consequently,  $IGC_F$  has the following earnings function:

$$( ) \approx ( , , ) \qquad (5.12)$$

It should be noted that the volatility of return on financing is a function of a set of risk factors: volatilities of volumes of financing ( ), ratio of operating expenses over operating income in financing channel ( ), and training expenses in financing channel ( ).

It is important to note that the focus of this chapter is to find out the level of scaled-standard deviation of volatility of earnings in  $IGC_I$  and  $IGC_F$  with the objective of developing an alternative measurement model of operational risk in Islamic banking. To put it differently, this chapter centres around operational risk VaR based on volatility analysis, skewness and kurtosis of A and A.

Surveying the available body of knowledge indicates that examining *VaR* based on the volatility of earnings is widely accepted in the sphere of theoretical and empirical studies and has become the backbone of risk measurement analysis (Alexander, 2008; Li, 1999; and Marshall, 2001); without which, measuring risk exposures can be futile. Such VaR examination is what Sundararajan (2005), Allen *et al.* (2004), and Hiwatashi and Ashida (2002), describe as profit at risk models, earnings at risk models and volatility approach respectively.

Based on this logic, empirical tests and discussion of the relationships between identified risk factors with and are to be elaborated in chapter 6 as they are beyond the

scope of this chapter. The following section, hence, attempts to examine the *Gaussian* features of the data in the analysis. Such an examination is essential as an underlying foundation to establish an accurate estimation of value at risk.

# 5.3.1 Empirical Analysis: Data, Skewness and Kurtosis

The data set for the empirical analysis in testing the proposed model of this study is extracted from published monthly financial reports (balance sheet and income statement) of Islamic banking industry in Indonesia. Such monthly data set, which comprises of 10 full-fledge Islamic commercial banks and 23 Islamic business units spanning from January 2001 to June 2010, is chosen simply because of its accessibility.<sup>36</sup> Following Hull and White (1998) and Li (1999), this study forecasts the volatility of operational risk variables in two ways: (i) a constant-variance model, and (ii) an exponential weighted moving average (*EWMA*) model.<sup>37</sup>

In addition, skewness and kurtosis are also taken into account since omitting these components could lead to a detrimental impact on the valuation of VaR especially when it is found out that normality (Gaussian) assumption of the probability density function is violated. Skewness is a measure of asymmetry and a third moment of a distribution.<sup>38</sup> Symmetry of a distribution is viewed around its mean. Therefore, if the skewness of a distribution is zero, the shape of the distribution on the left side of the mean will be a mirror image of the shape on the right side of the mean. Thus, skewness of a normal distribution will always be zero, as this distribution is perfectly symmetrical. Positively skewed distributions are characterised by the existence of a few very large positive values, *i.e.* large distributions right tail. Negatively skewed distributions, on the contrary, have a few large negative values, *i.e.* large left tail. As an example, Elton *et al.* (2003) shed some lights on why positive skewness matters for investors. It is argued that positive skewness of a distribution would demonstrate higher probability for large gains and

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This figure is accurate until June 2010. Up-to-date monthly financial reports can be accessed at http://www.bi.go.id/web/en/Statistik/Statistik+Perbankan/Statistik+Perbankan+Syariah/

<sup>&</sup>lt;sup>37</sup> The term estimate and forecast should not be confused. A forecast is the value that an analyst expects a certain parameter or other quantity to take at the end of a defined future horizon period, so all forecasts are estimates but not all estimates are forecasts. In this study, the terms 'forecast' and 'estimate' can be used interchangeably. This position, however, is also adopted by Dowd (2005) and RiskMetrics (1996).

<sup>&</sup>lt;sup>38</sup> Pearson (1895) is claimed to be the first person who introduced measures of skewness.

limited loss. This assertion is also in line with Fogler and Radcliffe (1974) and Alderfer and Bierman (1970) stating that negative skewness should be avoided; hence, it explains why people find the need to buy insurance premium. To provide a better understanding, three different shapes of skewness are illustrated in Figure 5.4.

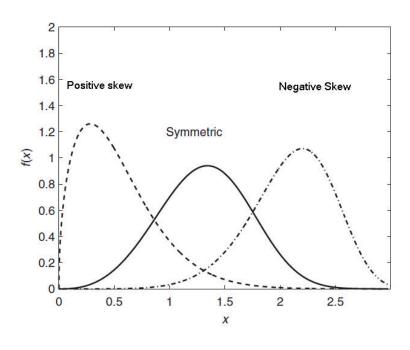


Figure 5.4. Shapes of Skewness

Source: Chernobai, Rachev and Fabozzi (2007: 139)

Furthermore, being a fourth moment of a distribution, kurtosis indicates the *peakedness* of the data (Johnson and Kotz, 1985), which is often thought as a measure of non-normality (Hample's 1968 as cited in Ruppert, 1987). By employing influence function, Darlington (1970), Hample (1974), Johnson and Kotz's (1985) and Ruppert (1987) prove how an existence of kurtosis could deviate a distribution from being normal (Gaussian). A higher kurtosis means that there is a high peak in the centre of the data, which implies that the data embody heavy tails. The data that are severely *kurtotic* and also contain heavy tails are often called *leptokurtic* (Allen, Boudoukh, and Saunders, 2004; Chernobai, Rachev and Fabozzi, 2007; Moosa, 2007; and Alexander, 2008). The degree of kurtosis in a distribution is measured by the sample kurtosis coefficient. Figure 5.5 depicts different shapes of kurtosis.

0.8 0.7 0.6 Kurtosis > 3 0.5 € 0.4 0.3 Kurtosis = 3 0.2 Kurtosis < 3 0.1 0 3 0 2 5 6

Figure 5.5: Various Shapes of Kurtosis

Source: Chernobai et al., (2007: 139)

# 5.3.2 Volatility in Constant-Variance Model

The first step in the volatility analysis is to calculate monthly logarithm change,  $= \ln(--)$  for all variables in the analysis.  $S_t$  is the rate of the variables at time t, and  $S_{t-1}$  is the rate of the variables at time t-1 (previous month). Subsequently, a new time series data is obtained by dividing the return series by their respective standard deviation; this method is attributed as constant-variance model (Hull and White, 1998).

Although the focus of this empirical attempt is on the volatility of *RoS and RoF*, other variables are also included in the descriptive statistics to develop a better understanding. The main reason is to observe their likely changing behaviour in two different variance models, namely constant-variance model and exponential weighted moving average model (*EWMA*).<sup>39</sup> The latter is discussed in the subsequent subsection.

<sup>&</sup>lt;sup>39</sup> Full results of descriptive statistics are presented in Appendix 5A

Table 5.1. presents the characteristics of all variables in constant-variance model based on the data described above.

Table 5.1: Summary Statistics of Operational Risk Variables in Constant-Variance Model

Variables	Mean	Standard Deviation	Skewness	Kurtosis
RoS	-0.000347	0.06045	-1.186	12.623
RoF	0.004009	0.03724	1.5672	4.924
FS	0.02415	0.0373	1.3769	4.212
F	0.03362	0.0251	1.3807	3.531
$T_r$	0.04825	0.4754	-0.0419	0.851
RoSD	-0.00181	0.0907	0.0569	3.41
RoTD	-0.0023	0.0826	0.4721	2.389
ВОРО	0.00134	0.1551	0.1727	1.863

As shown in Table 5.1.,all variables have a non-zero skewness; RoF, FS, F, RoSD, RoTD, and BOPO have skewness more than zero (>0) or non-symmetrical, which means they do not constitute normal distribution. A zero skewness is required for a distribution to be qualified as normal (Gaussian) distribution (Cruz, 2002: 42). Meanwhile, skewness for RoS is -1.186 and Tr is -0.0419. The negative value of skewness implies that both RoS and Tr make up for negatively skewed distribution, since their skewness is below zero (<0). Moreover, all variables exhibit a very significant excess kurtosis, ranging from 3.5 for variable F up to 12.6 for RoSD, except for Tr, RoTD and BOPO, which are 0.851, 2.389, and 1.863 respectively.<sup>40</sup> As the theory suggests, a value of kurtosis is 3 for a standard normal distribution (Bollerslev, 1986; Cruz, 2002). It is very obvious that all variables display non-normality.

can be decomposed into and , while can be broken down into and .

A question might arise as to whether the variables would demonstrate a changing behaviour in an exponential weighted moving average (*EWMA*). Before answering this question, the subsequent sub-section attempts to theoretically analyse why *EWMA* can be a better forecast than constant-variance model.

## **5.3.3. Rationalising Exponentially Weighted Moving Average (EWMA)**

The problems encountered when applying constant variance model stem from the small jumps that are often encountered in financial asset prices but from the large jumps that are only rarely encountered. When a long averaging period is used, the importance of a single extreme event is averaged out within a large sample of returns. Hence, a very long term constant-variance volatility estimation will not respond very much to a short, sharp shock in the market (Alexander, 2008: 117).

Moreover, the main reason why this study utilises *EWMA* based volatility is due to its superiority over other methods, such as equal weighted moving average and GARCH. A number of studies have empirically proven that *VaR* forecasts based on the *EWMA* estimator are superior to those based on the GARCH model (Alexander and Leigh, 1997; Boudoukh, Ricardson and Whitelaw, 1997; Guermat and Harris, 2002).

One way to capture such sharp shocks or the dynamic features of volatility is to use an exponential moving average of historical observations where the latest observations carry the highest weight in the volatility estimate (RiskMetrics, 1996; Alexander, 2008). This approach has two important advantages over constant-variance model and equally weighted model. First, volatility reacts faster to shocks in the market as recent data carry more weight than data in the distant past (Hull and White, 1998). Second, following a shock, the volatility declines exponentially as the weight of the shock observation falls (Li, 1999). In contrast, the use of a simple constant-variance leads to relatively abrupt changes in the standard deviation once the shock falls out of the measurement sample, which, in most cases, can be several months after it occurs.

An essential component underpinning EWMA is its moving average with declining weights that gives greater weight to more recent observations and less weight to more

distant ones. This type of weighting scheme might arguably be justified by claiming that volatility tends to change over time in a stable way, which is certainly more reasonable than assuming it to be constant. The volatility forecasting model proposed by this study follows the form expressed in equation 5.13.:

=

(5.13)

where the weights, the terms, decline as i gets larger, and sum to 1. In *EWMA* model, the weights would decline exponentially over time. This means that — = , where  $\lambda$  is a constant between 0 and 1. This assumption leads to the following volatility forecasting equations (5.14):

$$\approx (1 - ) \tag{5.14}$$

The *EWMA* model has the intuitively appealing property that the influence of any observation declines over time at a stable rate, and it is easy to apply as it relies on one parameter only, namely  $\lambda^{41}$ . The *EWMA* also leads to a very straightforward volatility formula. If equation 5.13 is lagged by one period and is multiplied throughout by , the following equation could be obtained:

$$\approx (1-) \qquad = (1-) \tag{5.15}$$

Rearranging equation (5.15) and (5.14), the formula in 5.16 is obtained:

-

 $<sup>^{41}</sup>$   $\lambda$  is also called decay factor

$$\approx +(1-) \tag{5.16}$$

Equation 5.16 shows that the estimate of volatility at time t, , made at the end of the time t-1, is calculated from the previous month's volatility,  $\sigma_{t$ -1, and the previous month's return,  $x_{t$ -1. The *EWMA* rule equation 5.16 can, therefore, be interpreted as a simple updating rule that allows an update monthly volatility estimate each month based on the most recent monthly return. A high  $\lambda$  means that the weight declines slowly, and a low  $\lambda$  means it declines quickly.

Equation 5.16 also exhibits that there are two terms; the first term is (1-) and the second one is . The first term determines the intensity of reaction of volatility to market events (Alexander, 2008: 139); the smaller is  $\lambda$  the more the volatility reacts to the market information in yesterday's return. The second term, nonetheless determines the persistence in volatility irrespective of what happens in the market, if volatility was high in the previous month, it will be still be high in this month. The closer  $\lambda$  is to 1, the more persistent is volatility following a market shock.<sup>42</sup>

Thus, a high  $\lambda$  gives little reaction to actual market events, but great persistence in volatility; and a low  $\lambda$  gives highly reactive volatilities that quickly shy away. An unfortunate restriction of *EWMA* models is that they assume that the reaction and persistence parameters are not independent: the strength of reaction to market events is determined by I-  $\lambda$  and the persistence of shocks is determined by  $\lambda$ .

Values of mean, standard deviation, skewness and kurtosis of all variables resulting from *EWMA* method based on the date collected from Indonesian Islamic banking sector for the period of January 2001 up to June 2010 are presented in table 5.2. It is expected that there will be slight changes taking place in the value of mean, standard deviation, skewness and kurtosis after the transformation of operational risk variables from constant-variance *to EWMA*. Changes are expected as the transformed data might be able

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<sup>&</sup>lt;sup>42</sup> Since the data set is monthly, this study deploys  $\lambda$ =0.97 as suggested by RiskMetrics (1996).

to capture the time-varying and persistent volatility in the data series. If this happens, it is hoped that the transformed data will exhibit normality (Gaussian) features.

Table 5.2: Summary Statistics of Operational Risk Variables in Exponential Weighted Moving Average (EWMA)

Variables	Mean	Standard Deviation	Skewness	Kurtosis
RoS	0.007021	0.00408	1.0389	1.549
RoF	0.001045	0.00039	-0.7226	0.8176
FS	0.000466	0.000475	1.3959	0.90716
F	0.000619	0.000209	-0.0863	1.421
$T_r$	0.25291	0.0617	0.7135	9.3704
RoSD	0.00737	0.00197	-0.4074	1.0276
RoTD	0.00491	0.00249	-0.6797	-0.9218
ВОРО	0.0181	0.00675	-0.5093	-0.8881

Table 5.2 shows that all operational risk variables still have a non-zero skewness and kurtosis. As clearly exhibited by the summary of the statistics in table 5.2, skewness are still non-zero and the kurtosis of the variables are either below 3 or above 3. In other words, the transformed series are still skewed and leptokurtic. In general, standard deviation, skewness and kurtosis display a decreasing trend after transformation using the EWMA method although there is a drastic change on the kurtosis of RoS and  $T_r$ . RoS's kurtosis slumps from 12.623 in constant-variance to 1.549 in EWMA; and  $T_r$ 's kurtosis shoots up from 0.851 to 9.37. The average of standard deviation, skewness, and kurtosis,

however, is reduced to 0.01, 0.092 and 1.66 from 0.12, 3.798, and 33.8 respectively<sup>43</sup>. This evidence suggests that the transformed data is less disperse than the previous model. More importantly, both models strongly suggest that the operational risk variables chosen in this study exhibit non-normality which implies that the use of a modified version of a standard VaR is inevitable in the estimation of operational value at risk.

Nevertheless, the findings above also prove that conditional normality assumption set by RiskMetrics is not consistent, which is in line with empirical findings of Li (1999), Hull and White (1998).

# 5.3.4 Fitting Probability Distributions

The analysis of volatility, skewness and kurtosis in the previous section has evidenced the non-normality of operational risk variables. In order to examine the behaviour of the data leading to VaR calculation, it is also very important to construct their histogram and observe what might stand out. It has also been indicated that the data series are still skewed and leptokurtic, even after the transformation, but the exact type of distribution for each variable is still not known. Thus, it is important to understand what sort of distribution which might fit the series data in the analysis.

To help with this process, a flow of analysis is illustrated in figure 5.6 to show how this study utilises different techniques to ensure that the outcome of this chapter, namely operational risk VaR; is robust and is theoretically justified.

 $<sup>^{43}</sup>$  The figures are generated after comparing the average of mean, standard deviation, skewness and kurtosis in Table 5.1 and Table 5.2

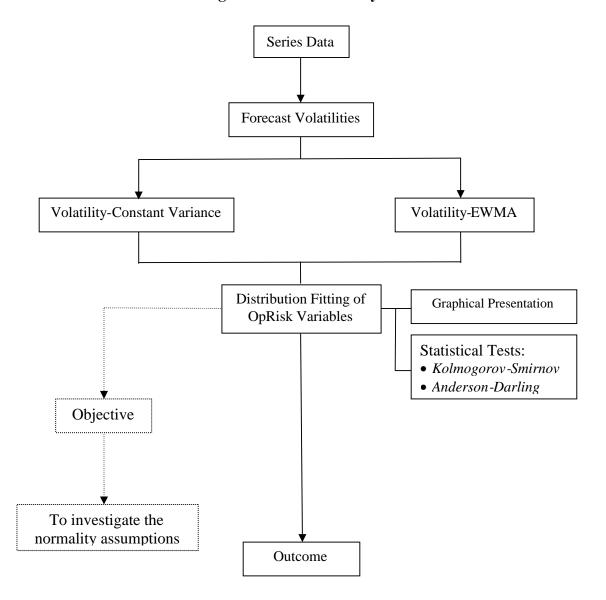


Figure 5.6: Flow of Analysis

Source: Author's own

As reflected in figure 5.6, the analysis of this study has now reached a stage called 'distribution fitting of operational risk variables'. The purpose of this stage is to examine the non-normality of operational risk through non-parametric approach by utilising two primary techniques: (i) graphical presentation, and (ii) statistical tests consisting of two main tests; *Kolmogorov-Smirnov* test and *Anderson-Darling* test.

It should be noted for the purpose of analysis, graphs are powerful tools for analysing trends and structures. They facilitate comparison of performance and structures over time, and show trend lines and changes in significant aspects of bank operations and performance (Nagafuji *et al.*, 2011). In addition, they provide the bank management with a high-level overview of risk trends in a bank.

Graphical presentation of operational risk variables in this study is carried out through simulation in a manner that produces histogram and probability density function that fits to the variables. In doing so, all results were produced with the help of two important statistical software, namely *easy fit 5.5* and *XL Stat*.

Easy fit 5.5 is a statistical package tailored to help this study deal with volatility and make informed decisions by analysing probability data and then selecting the best fitting distribution. It also allows the data to easily fit to a large number of distributions in seconds. Furthermore, XL Stat is add-in statistical software which offers a wide variety of functions to enhance the analytical capabilities of Excel, making it the ideal tool for data analysis and statistics requirements. An important advantage of utilising XL Stat is that it helps eliminate the complicated and risky data transfers between applications that had been a requisite for data analysis.

In addition to generating histogram and probability density function of the data, the simulation process will also produce QQ plot to confirm the specified distribution shown simulated histogram and probability density function. In QQ plot, any reference distributions that produce non-linear QQ plots can then be dismissed, and any distribution that produces a linear QQ plot is a good candidate distribution for the data in the analysis.

Statistical tests are carried out to investigate the *goodness of fit* of the data. To put it simply, the tests are employed to evaluate the quality of fit between the data and the distribution. The first one is *Kolmogorov-Smirnov* test statistic, defined as

$$= | ( ) - ( ) | (5.16)$$

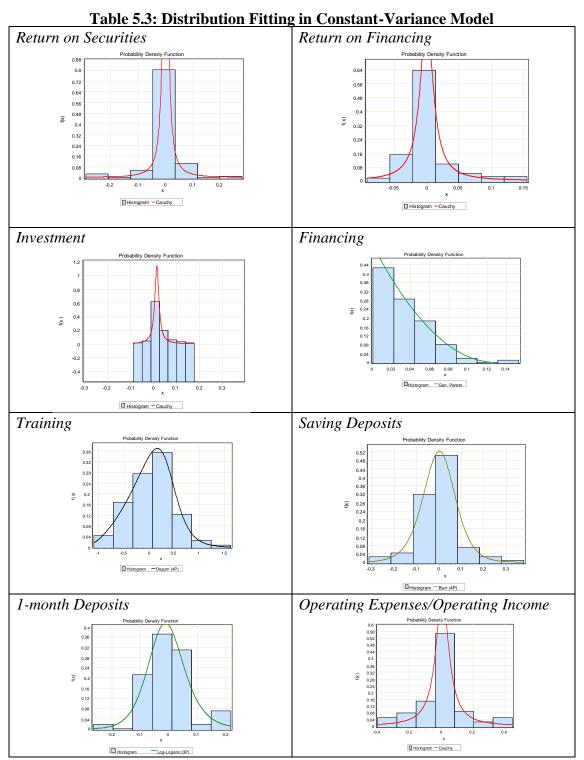
It should be noted that *KS* is the maximum of the vertical differences between two cumulative distribution functions: ( ) and ( ). In the *Kolmogorov-Smirnov* test, the *KS* statistic focuses on the fit between two distributions around their means. Furthermore, this study also employs the modified version of the *KS* test, called *Anderson-Darling*. The reason is mainly to observe the fit between two distributions in their tails.<sup>44</sup>

The followings are a brief description of table 5.3 and table 5.4. The aim is to see the trend and structures of the data, particularly in the aspect of normality or non-normality of the data. This will also help in determining as to whether the *VaR* analysis can go straight to the standard analysis or a modification may be needed as a result.

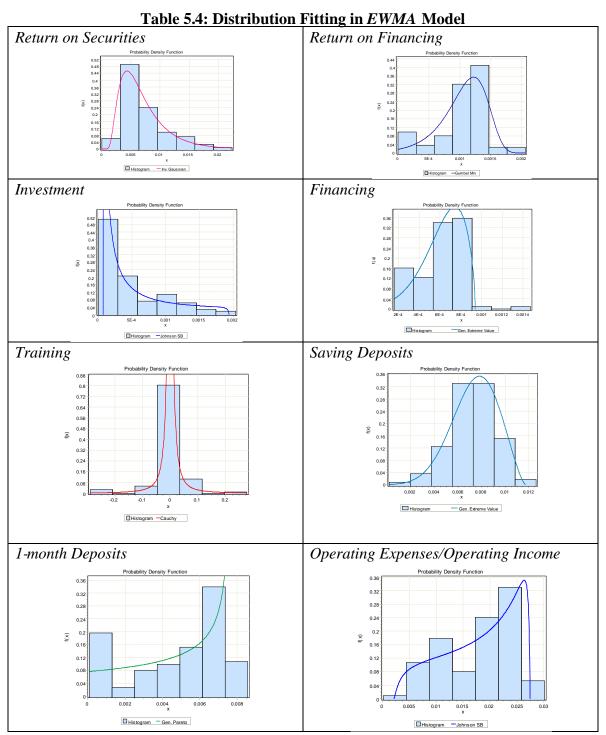
- The x-axis stands for probability density function;
- The y-axis stands for the value of assigned variables;
- Axes in table 5.4 are originated from constant variance model;
- Axes in table 5.5 are originated from exponential weighted moving average model (EWMA);
- Both tables present a combination of histograms and probability density functions of operational risk variables.

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<sup>&</sup>lt;sup>44</sup> Due to the extensive results of simulation, results of *Kolmogorov-Smirnov* and *Anderson-Darling* tests are located in the appendix of this chapter.

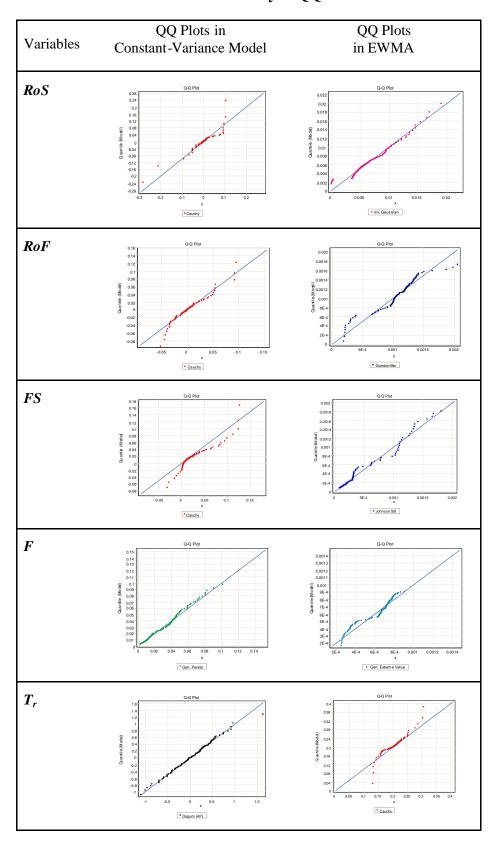


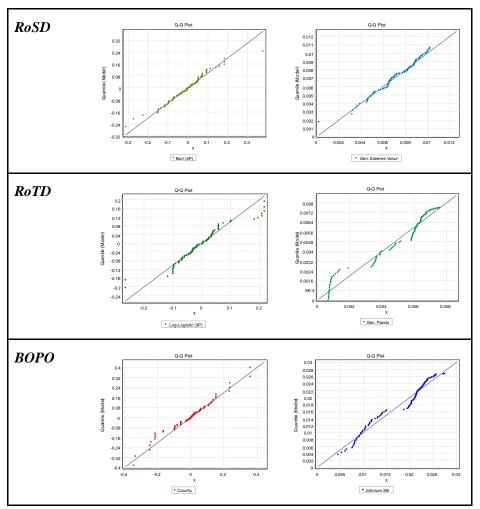
Source: Author's simulation



Source: Author's simulation

**Table 5.5: Summary of QQ Plots** 





Source: Author's simulation

Confirming the non-parametric tests conducted in sub-section 5.3.2 and 5.3.3, the simulation results as presented in table 5.3 and 5.4 demonstrate that none of the operational risk variables in this study exhibit normality, which implies that the use of conventional standard *VaR* will not be valid. Therefore, the cause for non-normality has to be taken into consideration in the estimation of *VaR*.

Moreover, the results of histograms and simulated probability density functions are substantiated by QQ plots as demonstrated in table 5.5. and a summary of the simulation results are illustrated in table 5.6 below.

**Table 5.6: Summary of Distribution Fitting** 

Variables	Distribution Functions in Constant-Variance Model	Distribution Function in EWMA
RoS	Cauchy	Inv. Gaussian
RoF	Cauchy	Gumbel min.
FS	Cauchy	Johnson SB
F	General Pareto	Gen. Extreme Value
$T_r$	Dagum (4P)	Cauchy
RoSD	Burr (4P)	Gen. Extreme Value
RoTD	Log Logistic	Gen. Pareto
ВОРО	Cauchy	Johnson

Under constant-variance model, distribution functions are predominantly made up of *Cauchy* distribution. Meanwhile, a transformation of the series data results in a mixture of distribution functions, spanning from *inverse Gaussian*, *Gumbel min.*, *Johnson*, *Cauchy, Gen. Extreme Value*, to *Gen. Pareto distribution*. Table 5.6 above also proves the plausibility of changing behaviour of data series after the transformation.

The simulation results also verify a remark stated in the introduction that it is unlikely to employ extreme value theory test since there are only two variables, i.e. *F* and *RoSD* after the transformation that constitute extreme value distribution. Furthermore, the results of simulation are in coherence with the statistical findings of *Kolmogorov-Smirnov* tests and *Anderson-Darling* tests.<sup>45</sup>

A range of non-parametric tests conducted above suggests that incorporating skewness and kurtosis into the VaR calculation is imminent, which is discussed in the following section.

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<sup>&</sup>lt;sup>45</sup> Details of the findings are presented in the appendix

# 5.4 ESTIMATING VAR USING CORNISH-FISHER EXPANSION: EMPIRICAL EVIDENCE

Non-parametric approach seeks to estimate risk measures without making strong assumptions about the distribution. The essence of this approach is to let the data speak for themselves as much as possible and use the empirical or simulated distribution to estimate risk measures. It estimates risks by fitting probability curves to the data and then inferring the risk measure (Dowd, 1998; 2002).

Parametric approach, on the contrary, use additional information contained in the assumed density or distribution function to estimate risk (Alexander, 2003; 2008). The objective of any parametric approach is basically to make assumptions that are consistent with the features of the empirical process, which is to be modelled.

By using non parametric approach in the previous section, it has been empirically shown the volatility forecast and simulation that data in this study are *skewed* and *leptokurtic*. Based on this finding and by considering the time and cost efficiency as pointed out in section 5.2, the analysis now moves on to parametric approach using *Cornish-Fisher* expansion in the calculation of *VaR*.

*VaR* of a portfolio, as commonly understood, is defined as the maximum loss that will occur over a given period of time at a given probability level. The calculation of *VaR* requires some assumptions about the distributional properties of the returns of the portfolio components. One of the popular *VaR* models, *delta-normal* approach is based on the assumptions of normally distributed returns of specified risk factors. <sup>46</sup> Under this model, it is viewed that there is a strictly linear relationship between the returns of the risk factors and the market value of the portfolio. Consequently, non-linearity which might result from nonlinear portfolio such as options is not taken into account in the model, which eventually exposes a shortcoming of delta-normal approach.

A first step to solve this problem is to include the quadratic term of a Taylor series expansion of the option pricing relation, *i.e.* the *gamma* matrix, in the *VaR* calculation

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<sup>&</sup>lt;sup>46</sup> Delta normal uwas originally promoted by JP Morgan's RiskMetrics

framework. The inclusion of quadratic terms implies that non-linearity is being dealt with by Taylor approximation. Other than nonlinearity, non normality due to the existence of skewness and excess kurtosis can also be a hindrance to the accuracy of *VaR* calculation (Mathai and Provost, 1992). There are several attempts to incorporate higher moments of any distribution in approximation procedures to calculate the required quantile of the distribution.

In a first attempt, Zangari (1996a) suggested to use the Cornish-Fisher expansion to directly calculate the quantile of a distribution with known skewness and kurtosis. Other approaches attempt to find a moment matching distribution for which the quantiles can be calculated. This class of approaches contain Zangari (1996b) who suggested to use the Johnson family of distributions to match the first four moments, Britten-Jones and Schaefe (1997) who suggested to use a central  $\chi^2$ -distribution to match the first three moments, and a simplifying approach that uses the normal distribution to match the first two moments (El-Jahel *et al.*, 1999).

It should be noted that there is a range of parametric approaches in the literature, such as Levy approach (Mittnik et al., 1998, 2000; Mantegna and Stanley, 2000), Elliptical and Hyperbolic approach (Eberlein et al., 1998; Eberlein, 1999; Bauer, 2000; Breckling et al., 2000), normal mixture approach (Zangari, 1996a; Venkataraman, 1997), Jump Diffusion (Merton, 1976; Duffie and Pan, 1997; Zangari, 1997; and Gibson, 2001), Gram-Charlier expansion (Polanski and Stoja, 2008), and stochastic volatility approach (Duffie and Pan, 1997; Billio and Pelizzon, 1997; Eberlein et al., 2001). This study, however, employs Cornish-Fisher expansion, since it is proved to be superior and preferable to other approaches, particularly in terms of time and calculations efficiency (Cornish and Fisher, 1937 as cited in Mina and Ulmer, 1999; Dowd, 2002; Alexander, 2003, 2008, Mina and Ulmer, 1999; Jaschke, 2001; and Pichler and Selitsch, 1999).

In essence, Cornish-Fisher expansion is used to determine the percentiles of distributions, which are non-normal. The actual expansion provides an adjustment factor that can be used to adjust estimated percentiles for non-normality, and the adjustment is reliable provided departures from normality are 'small' (Dowd, 2005: 171).

Since the non-normality dominates the portfolios return distribution, hence a standard *VaR* methodology is no longer appropriate. This study, therefore employs the Cornish Fisher expansion, which is a normal analytical approximation, in finding the percentiles of this portfolio's distribution with which *VaR* can be estimated. This technique is also attributed as partial simulation technique (Zangari, 1996a: 9).

With 95% level of confidence, calculation of *VaR* in normal distributions would typically rely on critical values +/- 1.65. In the presence of non-normality which is characterised by significant values of skewness and kurtosis, using the same critical values would give misleading risk estimates. The reason for this is very straight forward: +/-1.65 come from the normal distribution. As seen in figure 5.4 and 5.5, the presence of the third and fourth moments causes the distribution to be positively skewed or negatively skewed.

Basically, the discrepancy between normal and non-normal *VaR* leads this study to search for methods that augment the standard *VaR* methodology to account for the skewed return distribution. In particular, this study seeks the counterparts to the quantiles +/-1.65 that capture the skewness and kurtosis of the distribution.

After taking into account skewness and kurtosis, the new distribution is, suppose . Subsequently, the higher moments (skewness and kurtosis) of the distribution can be used to estimate the percentiles of . The critical points of distribution (counterparts to +/-1.65), hence, are estimated by applying the Cornish-Fisher expansion.

It should be noted that the applications of normal analytical approximations are motivated by the understanding that any distribution can be viewed as a function of any other one (Zangari, 1996a: 9). Suppose, the  $5^{th}$  and  $95^{th}$  percentiles of 's distributions are denoted by and which can be calculated as a function of the standard normal percentile  $z_{.05}$ =1.65,  $z_{.95}$ =1.65, and 's estimated moments. Say, a critical value, CV, of the normal 95% confidence interval around the mean portfolio return E[R] is defined as

$$= \{-1.65 + (), () + 1.65 \}$$
 (5.17)

Under the maintained assumptions, when R is no longer normal, that is, when R becomes non-normal , the approximate confidence interval for  $E[R^{SK}]$  can be written (Jaschke, 2002: 40):

$$= \{ [ ] + (-1.65 + [ ] , [ ] + (1.65 + [ ] ) \}$$

$$= \{ [ ] + ( [ ] ) , [ ] + ( [ ] ) \}$$
 (5.18)

The main purpose of the correction terms  $s_{\alpha}$  is to adjust for skewness. To a lesser extent it corrects for higher order departures from normality. In the case of the normal approximation interval,  $s_{.05}$ = $s_{.95}$ =0. In practice, the Cornish-Fisher expansion allows us to compute the adjusted critical values and as a function of the normal critical values  $z_{.05}$  and  $z_{.95}$  directly, as identified by the following (Jaschke, 2002: 42; Javanainen, 2004: 8; Monteiro, 2004: 23):

$$= +\frac{1}{6}(-1) + \frac{1}{24}(-3) - \frac{1}{36}(2-5)$$
(5.19)

where

= critical value of a normal distribution depending on the confidence level;

= 
$$[( - [ ]) / measures R^m$$
's skewness; and

$$= [( - ]) / - 3$$
 measures  $R^m$ 's kurtosis.

The following tables compare the critical values resulting from Cornish-Fisher expansion for two variables in the analysis, namely return on securities (RoS) and return on financing (RoF).

Table 5.7: Percentiles for Normal and Cornish-Fisher Approximation of Return on Securities

Percentile	$1^{ ext{st}}$	5 <sup>th</sup>	95 <sup>th</sup>	99 <sup>th</sup>
Normal	-2.33	-1.645	1.645	2.33
Cornish Fisher Expansion	-3.036	-1.298	2.003	3.053
Relative Difference	30.5%	31.3%	21.7%	31.03%

**Table 5.8: Percentiles for Normal and Cornish-Fisher Approximation of Return on Financing** 

Percentile	$1^{\mathrm{st}}$	5 <sup>th</sup>	95 <sup>th</sup>	99 <sup>th</sup>
Normal	-2.33	-1.645	1.645	2.33
Cornish Fisher Expansion	-3.592	-1.824	1.473	1.79
Relative Difference	54.2%	10.9%	10.45%	23.2%

As depicted in table 5.7 and 5.8, there are significant differences between critical values resulting from Cornish-Fisher expansion and commonly perceived normal distributions. As for *RoS*, the use of Cornish-Fisher expansion generate new critical values which are less than the normal one for its 1<sup>st</sup> and 5<sup>th</sup> percentiles, -3.036 and -1.298 as compared to the normal ones, -2.33 and -1.645 respectively. As for 95<sup>th</sup> and 99<sup>th</sup> percentiles, Cornish-Fisher expansion gives quite higher critical values, around 21.7% and 31.03% difference from the normal distribution.

Meanwhile, the use of Cornish-Fisher expansion in the calculation of *RoF*'s critical values demonstrate a similar pattern for the 1<sup>st</sup> and the 5<sup>th</sup> percentiles, except for the 95<sup>th</sup>

and 99<sup>th</sup> percentiles, which result in critical values that are less than the normal distribution.

In general, nonetheless, the discrepancy of critical values is higher for *RoF* than *RoS*; ranging from 10.45% to 54.2% for *RoF* and only 21.7% up to 31.3% for the latter. The results also suggest that incorporating higher moments in the confidence interval valuation is essential, without which the whole process of *VaR* calculation can be misleading.

Based on the findings presented above, the next step is to calculate *VaR* of *RoS* and *RoF*, which is calculated by multiplying the market value of a portfolio with its volatility and critical value.

$$VaR ext{ for } RoS =$$
 (5.20)

$$VaR ext{ for } RoF =$$
 . (5.21)

## where

= market value of return on securities;

= market value of return on financing;

= volatility of return on securities;

= volatility of return on financing;

= critical value of return on securities based on Cornish-Fisher expansion;

= critical value of return on financing based on Cornish-Fisher expansion.

Consequently, *RoS* and *RoF* have the following features:

= 16.57

= 19.68

= 0.0041

= 0.0004

= 2.003

= 1.473

= 3.053

= 1.79

Based on the features above, the table 5.9. presents a comparison of *VaR* calculation based on normal and Cornish-Fisher expansion at two levels of confidence, 95% and 99%.<sup>47</sup>

Table 5.9: VaR under normality and Cornish-Fisher Approximation

Confidence	Normal		Cornish-Fisher		
Level (%)	Return on	Return on	Return on	Return on	
	Securities	Financing	Securities	Financing	
95%	0.111318106	0.012611879	0.135556176	0.011297024	
99%	0.157672454	0.017863634	0.206605051	0.013733838	

Table 5.9 illustrates risk measures for  $IGC_I$  and  $IGC_F$  represented by RoS and RoF respectively. Under Cornish-Fisher expansion, the table can be read as follows:

- Given the volatility of 0.0041 for *RoS*, this study is 95% confident that the worst loss in *IGC*<sub>I</sub> within a month will not exceed 0.13.
- Given the volatility of 0.0041 for RoS, this study is 99% confident that the worst loss in  $IGC_I$  within a month will not exceed 0.21.

Moreover, in the context of  $IGC_F$ , the results of VaR can be interpreted as follows:

- Given the volatility of 0.0004 for RoF, this study has 95% confidence that the worst loss in  $IGC_F$  within a month will not exceed 0.0113
- Given the volatility of 0.0004 for *RoF*, this study has 99% confidence that the worst loss in *IGCF* within a month will not exceed 0.014

The empirical evidence presented in table 5.9 show that incorporating skewness and kurtosis will produce higher *VaRs* for *RoS* as compared to normal *VaR*. As clearly shown in the table, normal *VaR* produces 0.111 and 0.157, lower than 0.13 and 0.206 at 95% and

 $<sup>^{47}</sup>$  Results of VaR are generated from EWMA model.

99% confidence level. The results suggest that omitting higher moments would underestimate the exposure of operational risk in  $IGC_I$ . A similar picture, however, is not found in RoF, since Cornish-Fisher expansion produces lower VaRs for  $IGC_F$ . In  $IGC_F$ , normal VaR produces around 0.012 and 0.017 in comparison to slightly lower 0.011 and 0.013 at the level of confidence 95% and 99% respectively. The different VaR behaviour between RoS and RoF might be contributed by the different level of volatility between two variables leading to different patterns of confidence level.

The estimation exercise conducted above shows that incorporating volatility, skewness and kurtosis does certainly make a significant difference to the estimation of operational value at risk. Comparing the critical values produced by Cornish-Fisher expansion technique as compared to the normal one, the results in table 5.7 and 5.8 suggest that for *RoS*, the critical values of Cornish-Fisher tends to be higher than the normal one. But this is not the case for *RoF* as the normal critical values are higher than the ones produced by Cornish-Fisher expansion. It indicates that the *VaR* value for *RoS* is likely to be higher than the *VaR* for *RoF*, which, interestingly is verified by the empirical results of *VaR* estimation as depicted in table 5.9.

Based on the results shown in table 5.9, the analyst or operational risk manager in the bank can come to the decision that the operational risk exposures in  $IGC_I$  is higher than the one in  $IGC_F$ . Moreover, with respect to the comparison with the normal VaR; the operational risk manager may give a suggestion to the management that for  $IGC_I$ , the bank needs to set aside a higher amount of capital, as compared to the one allocated for operational risk in IGCF. In other words, the results also give an indication that investment activities in Islamic banking industry in Indonesia have higher exposure to operational risk than financing activities.

The other message from the empirical results is that if volatility, skewness and kurtosis of the operational risk variable are not taken into account, the bank will under estimate the amount of capital setting aside for the investment activities ( $IGC_I$ ), which could give a detrimental financial impact on the bank. As for financing activities ( $IGC_I$ ), however, had the volatility, skewness and kurtosis were not incorporated in the VaR estimation, the

bank could have overestimated the amount of capital that will be put aside to cater operational risks. Overestimation of capital will, obviously, not be favourable for the bank as it will adversely affect the profitability level for financing activities.

## 5.5 CONCLUDING REMARKS

The aim of this chapter is mainly to empirically test the proposed *DGSA-EVT* model presented in chapter 4 with the financial data. After deconstructing the proposed model as well as identifying the characteristics of the data, it is found out that there is a need to establish some adjustments in testing the model. This is needed due to the computational and numerical intensity of the model. Moreover, the adjustment are also made due to the nature of the financial instruments reflected in the collected data which do not show nonlinearity relationships among variables nor any extreme events taking place during the period of the data, without the latter, an examination of extreme value theory (*EVT*) model becomes implausible. Regarding the absence of nonlinearity, this might make sense in the context of Islamic banking since non-linear financial instruments, such as options are not largely flourishing. This study, for that reason, shifts its focus to normality *vis a vis* non-normality instead. This has left the model with testing *DGSA* as a result.

This chapter focuses on the assessment of operational risk exposures represented by the value of VaR in two income generating channels, namely  $IGC_I$  and  $IGC_F$ ; while an examination of the causality of some identified risk factors is discussed in the chapter 6. It should be noted that VaR in service fee channel is not taken into consideration since its contribution to value adding process in the bank is not significant.

The presence of non-normality is addressed by using non parametric approach which has been proven to be somewhat overwhelming in the data. Another attempt to smoothen the non-normality is by transforming operational risk variables from constant-variance model to exponential weighted moving average (*EWMA*).

Results of empirical tests have shown that the operational risk variables in this study are non-normal; thus, non-normality involving skewness and kurtosis as well as volatility

has to be taken into account in the calculation of *VaR*. In doing so, this study employs a parametric approach called Cornish-Fisher expansion upon which the confidence interval of operational variables is an explicit function of the skewness and kurtosis as well as the volatility. This is also an indication that length of the confidence interval is related to skewness and kurtosis of the variables. More importantly, the empirical findings also suggest that incorporating higher moments might likely prevent calculation of *VaR* to be underestimated or overestimated. As shown by the empirical results that if the volatility, skewness and kurtosis were not taken into account in the estimation of *VaR*; in other words, if the probability density function of the operational risk variables were assumed to follow Gaussian features, the amount of capital to be set aside for investment activities (*RoS*) would be underestimated. On the other hand, the amount of capital setting aside for financing activities would be higher than necessary had the volatility, skewness and kurtosis were not taken into consideration.

# **CHAPTER 6**

# ASSESSING OPERATIONAL RISK IN INDONESIAN ISLAMIC BANKS: AN EMPIRICAL ANALYSIS

#### 6.1 INTRODUCTION

This chapter is a continuation of the empirical tests conducted in Chapter 5 on testing the developed model of operational risk in Indonesian Islamic banking sector. While the objective of the previous chapter is to figure out the level of scaled-standard deviation of volatility of earnings with the objective of examining the exposures level of operational risk, which is represented by the value of operational value at risk in  $IGC_I$  and  $IGC_F$ ; the focus of this chapter, however, is to examine the relationship between identified risk factors with return on securities (RoS) and return on financing (RoF). As explained in the previous chapter, RoS is the earnings which is defined in terms of a series of risk factors in  $IGC_I$  and RoF is earnings in  $IGC_F$ .

By employing regression techniques and running a set of econometric tests, it is expected that such techniques would provide a cause-effect framework through which significant determinants of operational risk could be identified.

The use of regression technique is not alien in operational risk analysis. A number of studies in this field have benefited from this technique, such as Allen and Bali (2004), Cruz (2002), Chernobai *et al.* (2007), and Moosa (2007). The application of this technique is often intertwined with risk factor approach since its main objective is to identify which risk factor is dominant in the model.

This chapter is organised as follows; section 2 presents a highlight of empirical studies in operational risk, section 3 describes the data and methodology, section 4 is devoted to the examination of empirical results, and section 5 concludes the chapter.

#### 6.2 A SURVEY OF THE RELATED LITERATURE

The objective of this section is to present a snapshot on the role of regression techniques and financial ratio in operational risk analysis. As might have been noticed that in the operational risk literature, empirical studies with simulated data are arguably very dominant, this might be due to the shortage of comprehensive data. This is contrary to the studies that use actual loss data. An alternative to the estimation of operational risk is to measure it as the residual of an econometric model that accounts explicitly for market and credit risk, which is deployed by Allen and Bali (2004). Another alternative is to use the factor approach, which is what Chernobai *et al.* (2007) utilised in identifying the determinants of operational losses.

To deal with the data problem, Allen and Bali (2004) estimate an operational risk measure for individual financial institutions using a monthly time series of stock returns over the period 1973-2003. The model is represented by ordinary least squares (OLS) regression of the monthly rate of return on a large number of explanatory variables, which include the first difference of twenty-two variables representing credit risk, interest rate risk, exchange rate risk and market risk. The three Fama-French (1993) factors are also used as explanatory variables. An important finding resulting from Allen and Bali's research is that operational risk exposures often exceed market risk resulting in higher capital charge for operational risk.

Nevertheless, Tripe (2000) sees some benefit in using ratios derived from financial statement to determine capital requirements for a number of New Zealand banks based on the volatility of non-interest expenses. He also considers two ratios: the ratio of operating expenses to total assets and the ratio of operating expenses to income. The figures suggest that bank capital levels should embody a significant element of operational risk. Ford and Sundmacher (2007) also advocate the use of financial ratio such as cost to income ratio as a leading indicator of operational risk. The underlying argument is that a reduction of this ratio, which is essentially a measure of efficiency, is favourable since it implies lower cost per dollar of income. The volatility of this ratio is also a leading indicator as it results from factors associated with operational risk such as asset write-

downs, unstable or unpredictable cost structures and volatile income sources. They also suggest a ratio of training expenditure to total expenses and the proportion of incentive based remuneration.

In a recent study, Chernobai *et al.* (2007) examine the microeconomic and macroeconomic determinants of potential losses in financial institutions. On the basis of twenty-four years of US public operational loss data covering the period 1980-2003, they demonstrate that the firm-specific characteristics (such as size, leverage, volatility, profitability and the number of employees) turned out to be highly significant in their models. They also found that the overall macroeconomic environment is less important, although operational losses tend to be more frequent and more severe during economic downturns. The evidence they obtained indicates that contrary to the traditional view that operational risk is unsystematic, operational loss events cluster at the industry level in excess of what is predicted by the stochastic frequency estimates.

In the context of Islamic banking, there has not been any single study on both, methodological and empirical studies in operational risk. Nonetheless, some empirical studies on the aspect of operational soundness in Islamic banks are conducted by Ismail and Suleiman (2005), Hassan and Dicle (2005) and Muljawan (2005).

Using the Cavello and Majnoni model, Ismail and Suleiman (2005) discuss the interaction between the capital requirement as stated in the New Basel Capital Accord and the cyclical pattern of profit. In addition, CAMEL framework is deployed by Muljawan (2005) as an alternative tool to assess the operational soundness of Islamic banks. The analysis of Hassan and Dicle (2005) is somewhat broader than other papers in the sense that it also discusses the nature of operational risk.

In light of the brief available review above, this study fills an important gap, since it is the first attempt in empirically analysing the exposure of operational risk in Islamic banking. The following section describes the data and methodology utilised in this chapter.

# 6.3 A BRIEF BACKGROUND OF ISLAMIC BANKING INDUSTRY IN INDONESIA

The history of Islamic banks in Indonesia can be traced back from the establishment of the first Islamic bank, Bank Muamalat Indonesia November 1, 1991 in Jakarta. The bank started its operation on 1 Nov, 1991 and subsequently was inaugurated by the Vice-President on November 15th, 1991. Even though the regulation on Islamic banks did not exist at the time, due to political support from the political elites, Bank Muamalat became the only Islamic Commercial Bank in Indonesia over the period of 1991 up to 1998.

The Indonesian banking industry was regulated by Act No 7/1993, which was modified by Banking Act 10/1998 which allows the banking industry to apply dual banking system. Being the regulator of the banking industry in Indonesia, Bank Indonesia has outlined the blue-print of Islamic development banks in Indonesia; a strategic plan with the following objectives: (i) to formulate and improve regulation on Islamic banks, (ii) to develop the Islamic bank network, (iii) to increase the understanding of people of the Islamic banks system, (iv) to prepare infra-structures and supporting institutions to support the development of Islamic banks, (v) to increase the efficiency, service quality and competitiveness of Islamic banks, (vi) to develop profit-sharing scheme, and (vii) to ensure Islamic banks to comply with professional and international standards.

As a result of the strategies outlined in the blue print, currently Islamic commercial banks in Indonesia are composed of 10 full-fledge Islamic commercial banks and 23 Islamic business units <sup>48</sup>. Islamic business units within conventional banks in 2010 have grown up to nearly 47 % as compared to 2009 (Indonesian Islamic Banking Outlook, 2011: 31). The first Islamic unit was initiated by Bank IFI, followed by Bank Jabar, Bank BNI, Bukopin, Bank BRI, Bank Danamon, and lastly Bank BII which are currently are spread across 20 provinces.

In all aspects, the growth of Islamic banking has been very fast. In terms of total assets in 2010, the sum was around IDR 90 trillion, an increase of IDR 67 trillion or grows around

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<sup>&</sup>lt;sup>48</sup> In Indonesia, Islamic business unit is commonly termed as Shariah unit.

36 %. In terms of assets, the share of Islamic banks in Indonesian banking is almost 3.7 %. The share of financing is more remarkable <sup>49</sup>. In 2010 total financing increased from IDR 46.8 trillion to IDR 68.1 trillion or a growth of 68.7%. This figure shows that Islamic banks are responsive to the development of the real economic sector.

FDR of Islamic banks in 2010 is 89.67% upon which, the financing scheme is still dominated by murabahah i.e. 70.9%, followed by mudharabah, 15.2%, and mudharabah, 1.8%, and the rest is a small portion of salam, istishna, ijarah, rahn, and hawalah (Indonesian Islamic Banking Outlook, 2011: 26). With regard to non-performing financing, Islamic banking industry in Indonesia marked a very impressive figure, 3.02 only in 2010. As compared to the non performing financing (loan) in conventional banks, which was around 12.1%.

Moreover, a better operational efficiency of the industry coupled was demonstrated by a declining figure of the ratio of operating expense over operating income, which was 79.17% in 2010 as compared to 83.91 in the previous year. This promising figure is coupled with an increasing trend of operating income, which grows up to 22% than the year before.

#### 6.4 DATA AND METHODOLOGY

The dataset utilised in this study is times series monthly data spanning from January 2001 up to June 2010. The data is extracted from published monthly financial reports (balance sheet and income statement) of Islamic banking industry in Indonesia consisting of 10 full-fledge Islamic commercial banks and 23 Islamic business units. 50 In the area of social research, research methodology can be classified into two categories: qualitative and quantitative. This study, for that matter, utilises the latter due to its attempts to operate a

 $<sup>^{49}</sup>$  *IDR* stands for Indonesian Rupiah, a national currency for Indonesia  $^{50}$  An up-to-date monthly financial reports can be retrieved from http://www.bi.go.id/web/en/Statistik/Statistik+Perbankan/Statistik+Perbankan+Syariah/

range of econometric tests in analysing the relationship between a set proposed of operational risk variables.

## **6.4.1 Estimation Methods**

Methods are actually the specific technique of research. As Nachmias and Nachmias assert, method is described as "a systematic procedure for attaining an object or doing something" (1981: 15). In other words, method refers to data collection, data analysis and the definition of analytical methods to test the hypothesis. In line with this, Figure 6.1 clearly describes the research process through which the final empirical outcome is achieved.

First of all, all the raw statistical data are transformed into log form. In econometrics, a logarithmic transformation is very popular for several reasons: first, many economic time series exhibit a strong trend, *i.e.* a consistent upward or downward movement in the values. When this is caused by some underlying growth process, a plot of the series will reveal an exponential curve. In such cases, the exponential/growth component dominates other features of the series (*e.g.* cyclical and irregular components of time series) and may thus obscure the more interesting relationship between this variable and another growing variable.

The next step is to run unit root tests. Unit root tests are carried out to examine the stationarity of the data. As Koop (2005: 145) points out that the distinction between stationary and non-stationary time series is an extremely important one. Financial economists usually focus on non-stationary data that seems to be present in many times series data, namely *unit root non-stationarity* (Koop, 2005: 146). In this respect, Stock and Watson (1993) and Harris (1995) assert that estimated equations must not comprise non-stationary variables to avoid the 'spurious-regression' problem, which may be indicated by inflated *R*-squares and incorrect test statistics. A variable is said to be stationary or having no unit roots if its stochastic properties (mean, variance, and covariance with other variables) are time invariant (Koop, 2000:133). Therefore, testing for non-stationarity has become a necessary prelude to any robust empirical analysis. A given non-stationarity variable can be converted to a stationary one by differencing it

appropriately. Statistically speaking, the following are few different ways of thinking about whether a time series variable, suppose *Y*, is stationary or has a unit root (Koop, 2005: 145):

- 1. If *Y* has a unit root then its autocorrelations will be near one and will not drop much as lag length increases;
- 2. If Y has a unit root then the series will exhibit trend behaviour;
- 3. If Y has a unit root, then  $\Delta Y$  will be stationary. For this reason, series with unit roots are often referred to as difference stationary series.

Once unit root tests are conducted, the next step is to perform regression based on OLS. Following regressions based on OLS, misspecification tests are performed to ensure that the models do not violate the assumptions of classical linear regression model (CLRM) comprising the following (Gujarati, 1999: 153-154):

- (i) The explanatory variable X is uncorrelated with the disturbance term u;
- (ii) The expected, or mean, value of the disturbance term u is zero. That is,  $E(u_i) = 0$ ;
- (iii) The variance of each  $u_i$ , is constant, or homoskedastic (homo means equal and scedastic means variance). That is var  $(u_i)=\sigma^2$ ;
- (iv) There is no correlation between two error terms. This is the assumption of no autocorrelation. Algebraically, this assumption can be written as

$$=0 \neq$$

It is very essential to meet all the assumptions mentioned above so the estimation based on regression technique can satisfy a condition called 'the best linear unbiased estimator', or *BLUE* (Gujarati, 1999: 103). If an estimator is linear, unbiased, and has a minimum variance in the class of all linear unbiased estimators of a parameter, it is called a best linear unbiased estimator (Gujarati, 1999: 103-104). With respect to this, there are three elements that will be examined, normality of the model, autocorrelation and heteroskedasticity. In doing so, this study will utilise Jarque-Bera, Lagrange-Multiplier and White Heteroskedasticity tests.

Data Log Transformation Unit Root Tests Ordinary Least Squares (OLS) Jarque-Bera Misspecification Tests Lagrange Multiplier White Heteroskedasticity Reject Accept Но Но Estimated OLS is Rectified OLS sufficient

Figure 6.1: Flow of Analysis

Source: Author's own

As figure 6.1 demonstrates, the first OLS model can be considered as sufficiently robust if the null hypothesis is accepted in all these three tests. However, the models need to be rectified if the misspecification tests state otherwise.

## **6.4.2 Specification of the Models**

In  $IGC_I$ , earnings are represented by the rate of return on financial securities (RoS), which is based on profit sharing. The financial securities represent ownership of profit sharing based investments carried out by Islamic banks.

A proposed set of risk categories that was highlighted in chapter 3 and chapter 5 are *Shariah* non-compliance risk, fiduciary risk, people risk, legal risk and technology risk. <sup>51</sup> As explained in Chapter 5, *shariah* non-compliance risk, legal risk and technology risk, which are qualitative in nature, are not included in the model, as they cannot be captured by the data. As a result, that leaves the model with fiduciary risk and people risk.

Fiduciary risk can be characterised as a condition whereby Islamic banks are liable for losses arising from their negligence, misconduct or breach of their investment mandate. In other words, fiduciary risk is an indication of failure to "perform in accordance with explicit and implicit standards applicable to their fiduciary responsibilities" (IFSB, 2005:26).

As set out by *IFSB Guiding Principles of Risk Management 2005* Paragraph No. 123 under the section of operational risk, fiduciary risk is defined as "the risk that arises from IIFS's failure to perform in accordance with explicit and implicit standards applicable to their fiduciary responsibilities. As a result of losses in investments, IIFS may become insolvent and therefore unable to (a) meet the demands of current account holders for repayment of their funds; and (b) safeguard the interests of their IAH. IIFS may fail to act with due care when managing investment resulting in the risk of possible forgone profits to IAH."

<sup>&</sup>lt;sup>51</sup> Details of proposed operational risk categories are available in table 3.2 in Chapter 3.

Furthermore, *IFSB Guiding Principles of Risk Management 2005* Principles No 7.2 on page 2 states that "IIFS shall have in place appropriate mechanism to safeguard the interests of all fund providers. Where IAH funds are commingled with the IIFS's own funds, the IIFS shall ensure that the bases for asset, revenue, expense and profit allocations are established, applied and reported in a manner consistent with the IIFS's fiduciary responsibilities".

In the model, a set of proxies for fiduciary risks is decomposed into a series of risk factors, namely volatilities of (i) volume of investment in financial securities (FS), (ii) return on saving deposits (RoSD), (iii) return on 1-month time deposits (RoTD), and (iv) ratio of operating expenses over operating income ( $BOPO_I$ ).

The choice of a proposed set of fiduciary risks mentioned in the previous paragraph results from an inference drawn from the definition of fiduciary risk set out by IFSB. It can also be read as follows: an unstable volume of investment in securities, return on saving deposits, return on 1-month time deposits and operating expenses; reflected by their respective high (low) volatility would demonstrate an inability (ability) of Islamic banks to sustain their fiduciary responsibilities. In other words, a highly steady performance of investment, return on saving deposits and time deposits demonstrates an ability of an Islamic bank to safeguard the interest of their fund providers. Moreover, a high volatility of operating expense might attribute to the operational errors that a business may incur during its operations (Makridakis, 1998).

It is important to reiterate here that this study examines the relationship of the variables in the form of their respective volatility rather than their absolute value. The reason is because volatility is often used to describe the degree of risk exposure (Damodaran, 2001; Hull and White, 1998; Kupiec, 2001; Li, 1999)

People risk, nonetheless, is another type of operational risk arising from incompetence or fraud, which exposes Islamic banks to potential losses. This includes human errors, lack of expertise, compliance and fraud (Akkizidis and Kumar: 2008).

A proxy for people risk, on the other hand, is represented by the volatility of training expenses ( $Tr_I$ ). It is argued that training expenditure has a reverse effect on the number of employee errors and customer complaints (Taylor and Hoffman, 1999 and Shih, Samad-Khan and Medapa, 2000). The more skilled the bankers, which is a result from intensive training, the less people risk the bank would incur (Jackson-Moore, 2007). Hence, the higher the volatility of training expenditure the higher it would affect the volatility of operational risk indicator. Therefore, it is expected that the two variables have positive relationship.  $IGC_I$ , therefore, has the following earnings function:

$$( ) \approx ( , , , , , )$$
 (6.1)

Volatility of return on securities is a function of a series of risk factors, including volatilities of investment in securities ( ), return on saving deposits ( ), return on 1-month time deposits ( ), ratio of operating expenses over operating income in investment channel ( ), and training expenses in investment channel ( ).

In equation 6.1, proxies for fiduciary risk, namely FS, RoSD, RoTD, and BOPO are also expected to have a positive relationship with the target variable, RoS. The argument is that the more unsteady or volatile the explanatory variables the more volatile RoS can be. To put it differently, a higher fiduciary risk will likely to positively impact a greater exposure of operational risk, represented by the volatility of RoS.

A summary of the expected sign of each variable used in investment function model is depicted in Table 6.1.

**Table 6.1: The Expected Signs of Variables in Investment Function Model Dependent Variable:** v(RoS)

Variables	Expected Sign
v(FS)	+
v(RoSD)	+
v(RoTD	+
v(BOPO)	+
$v(Tr_I)$	+

It should be noted that equation 6.1. is a combination of income-based model and expense-based model. Income-based model of operational risk is basically a model to analyse the volatility of historical income as a proxy of operational risk exposure or operational losses in terms of some specific underlying risk factors (Marshall, 2001 and Matten, 1996). On the other hand, expense-based model associates operational risk with fluctuations in historical expenses, such as operating expense (Marshall, 2001).

In  $IGC_F$ , rate of return on financing (RoF) signifies earnings. RoF contains a bulk of returns on financing, which unsurprisingly, is still predominated by murabahah mode of finance. Fiduciary risk in  $IGC_F$  is symbolised by the volatilities of two elements: volume of financing (F) and the ratio of operating expenses over operating income  $(BOPO_F)$ . As for people risk,  $IGC_F$  employs a similar variable as  $IGC_I$ , that is the volatility of training expenses  $(Tr_F)$ . Consequently,  $IGC_F$  has the following earnings function:

$$( ) \approx ( , , ) \qquad (6.2)$$

In equation, 6.2., volatility of return on financing is a function of a set of risk factors: volatilities of volumes of financing (), ratio of operating expenses over operating income in financing channel (), and training expenses in financing channel (). All these explanatory variables are expected to have a positive relationship with the proxy for operational risk exposures in  $IGC_E$ , namely (), as depicted in table 6.2.

<sup>&</sup>lt;sup>52</sup> As of June 2010, *murabaha* makes up over 60 percent of the total financing.

Table 6.2: The Expected Signs of Variables in Financing Function Model Dependent Variable: v(RoF)

Variables	Expected Sign
v(F)	+
v(BOPO)	+
$v(Tr_F)$	+

It should be noted that  $IGC_S$  is deliberately taken out of the analysis since the volume of service fee does not contribute significantly to the value adding process.<sup>53</sup>

#### 6.5 EMPIRICAL ANALYSIS AND THE FINDINGS

This section presents the empirical analysis process by referring to each of the part and the relevant test before presenting the concluding findings.

#### **6.5.1 Unit Root Tests**

The unit root tests, Dickey-Fuller and Philip Perron, used in this study are to ensure that the time series data are stationary. Stationary is an important concept in an econometric analysis, because if the series is non-stationary then all the typical results of the classical regression analysis are not valid. Regressions with non-stationary series, hence, may have no meaning and are therefore called 'spurious'.

The specification for the Dickey Fuller unit root test is:

$$Y_{t} = \alpha_{0} + \alpha_{1} Y_{t-1} + \alpha_{2} \Delta Y_{t-1} + \mu_{t}$$
(6.3)

in the ADF test, the unit root test can be presented as:

$$\Delta Y_t = \alpha_i Y_{t-1} + \sum_{j=1}^k \alpha_j \Delta_{t-j} + \mu_t$$
, where j= 1,2, .... k. (6.4)

 $<sup>^{53}</sup>$  As of June 2010, service fee only contributes less than 2% to the value adding process

If  $\alpha_1$  from the above equation is significantly different from zero then it can be said that  $Y_t$  is stationary or does not have unit roots.

The distribution theory supporting the Dickey-Fuller tests is based on the assumption that the error terms are statistically independent and have a constant variance. In using the ADF methodology it is important to make sure that the error terms are uncorrelated and that they really have a constant variance. Phillips and Perron (1988) developed a generalisation of the ADF test procedure that allows for fairly mild assumptions concerning the distribution of errors. It is a nonparametric method of controlling for higher-order serial correlation in a series. The test regression for the Phillips-Perron (PP) test is the AR(1) process:

$$\Delta Y_{t} = \alpha + \beta Y_{t-1} + \varepsilon_{t} \tag{6.5}$$

While the ADF test corrects for higher order serial correlation by adding lagged differenced terms on the right-hand side, the PP test makes a correction to the t-statistic of the  $\gamma$  coefficient from the AR(1) regression to account for the serial correlation in  $\varepsilon$ .

The PP statistics, hence, are just modifications of the ADF t statistics which take into account the less restrictive nature of the error process. The asymptotic distribution of the PP t statistics is the same as the ADF t statistics, and therefore the MacKinnon (1991) critical values are still applicable. The summary of unit root tests is presented in table 6.3 and 6.4 below.

Table 6.3: Summary of Augmented Dickey-Fuller (ADF) and Philip-Perron (PP)
Statistics at Level

Variables	ADF	PP
RoS	4.22*	-5.53*
RoF	0.58	1.14
FS	-3.59**	-4.6*
F	-3.67**	-3.46**
$T_r$	-3.68**	-1.58
RoSD	-3.24***	-3.71**
RoTD	2.12**	1.98**
BOPO	1.44	-1.67

Note: (1) The ADF and PP statistics were generated by model with constant and trend. k is the lag length and was determined by Akaike info criterion and Schwarz criterion for the ADF test. The PP test use the automatic lag length that suggested by Newey-West. All variables were tested in log form. (2) '\*' denotes rejection of the null at 1% level; '\*\*' denotes rejection of the null at 5% level; '\*\*\*' denotes rejection of the null at 10%

Table 6.4: Summary of Augmented Dickey-Fuller (ADF) and Philip-Perron (PP)
Statistics at 1<sup>st</sup> Difference

Variables	ADF	PP
RoS	2.05*	-1.53*
RoF	-4.38*	-3.21**
FS	-4.18*	-5.67*
F	-5.62*	-15.43*
$T_r$	-10.54*	-10.54*
RoSD	-5.87*	-7.32*
RoTD	-4.73*	-8.98*
BOPO	-5.27*	-6.75*

*Note*: (1) The ADF and PP statistics were generated by model with constant and trend. k is the lag length and was determined by Akaike info criterion and Schwarz criterion for the ADF test. The PP test use the automatic lag length that suggested by Newey-West. All variables were tested in log form. (2) '\*' denotes rejection of the null at 1% level; '\*\*' denotes rejection of the null at 5% level; '\*\*\*' denotes rejection of the null at 10%

Using 2-lag difference for ADF and 4 truncation lag for PP, both tests were run with the inclusion of a trend and intercept. As depicted in table 6.3 and 6.4, most variables are already stationary at level with a different degree of significance level, except for *RoF* and *BOPO*, which are stationary at first difference. *RoS* is already stationary at level with 1% level of significance. Meanwhile, *FS*, *F*, *Tr*, and *RoTD* are stationary at 5% level of significance respectively; whereas *RoSD* is stationary at level with 10% significance level.

# **6.5.2 Discussing the Regression Results**

Marshall (2001: 268) argues that a line of best fit between inputs and outputs is built by minimising the sum of the squared deviations between y variables (operational risk exposures) and a linear combination of risk factors (explanatory variables). The sensitivity of losses to risk factor changes is then the beta coefficient of the factor. The t score of the parameter can be used to test the significance of the factor. The extent to which the regression captures the variation in the losses is captured by its R-squared and the significance of the model as a whole can be analysed using its F-statistics. The analysis under this study, two primary models are examined by using regression technique: the first model results from  $IGC_I$  (investment function) and the second one is generated from  $IGC_F$  (financing function). The results of the OLS-based regression as a result of time series analysis are shown in table 6.5 and table 6.6 below.

Table 6.5: Regression Results for Investment Function
Dependent Variable: *LROS* 

Dependent variable: LKOS			
Specification	<b>Estimated Values</b>		
Constant	1.28		
	(1.07)		
LFS	-0.78		
	(-7.80)*		
LRoSD	2.13		
	(6.84)*		
LRoTD	0.21		
	(2.14)**		
D(LBOPO)	0.79		
	(2.94)*		
LTR	0.69		
	(4.54)*		
R-squared	0.74		
F-stat	59.62		
DW	0.42		

Note: '\*' denotes rejection of the null hypothesis at 1% level; '\*\*' denotes rejection of the null hypothesis at 5% level

In line with the results of unit root tests presented in the previous section, all variables in investment function are regressed at level, except for *BOPO* which is stationary at first difference.

The regression results for the investment function in table 6.5 shows that most of the operational risk variables are statistically significant at 99% level of confidence with the *p*-value less than 0.001, except *LRTOD* which is significant at 95% level of confidence. Furthermore, all variables also produced expected signs as highlighted in the previous section, except for LFS which has negative coefficient. *R*-squared value shows that around 74% variation of *RoS* is explained by *FS*, *RoSD*, *RoTD*, *BOPO*, and *TR*. This finding is also strengthen by the high result of *F*-test and also significant at 1% level of confidence. However, due to a very low value of Durbin-Watson, which is only 0.42; the investment function model is indicated a severe serial correlation problem.

Table 6.6: Regression Results for Financing Function Dependent Variable: D(LROF)

Dependent variable. D(LKO1)			
Specification	Estimated Values		
Constant	0.56		
	(1.26)		
$L\!F$	0.03		
	(0.51)		
D(LBOPO)	0.75		
	(10.95)*		
LTR	0.20		
	(3.86)*		
R-squared	0.73		
F-stat	96.58		
DW	2.22		

<sup>\*</sup> denotes rejection of the null hypothesis at 1% level

As shown in table 6.6, most of the explanatory variables in financing function model are also statistically significant: BOPO and TR are significant at 1% level of confidence, while F is not significant at all. In financing function model, RoF is regressed at first difference as vindicated by the results of unit root tests presented in table 6.4. The R-squared value also shows that the 73% of variation in RoF can be explained by the F,

BOPO and TR. As compared to investment model, financing model might not suffer from serial correlation problem, since the value of DW is relatively close to 2.54 Nonetheless, in order to reach a conclusive result as to whether the models presented above are robust and can explain the relationship between explanatory variables and target variables, there is a need to conduct misspecification tests, the process and the result of which presented in the following section.

## **6.5.3 Misspecification Tests**

In an econometric analysis, it is necessary to observe the regression residuals to detect the potential misspecification problems. One of the assumptions of classical linear regression model for instance, is that the residuals are normally distributed with a zero mean and a constant variance. Violation of this assumption, therefore, leads to the inferential statistics of a regression model, *i.e. t*-stats, *F*-stats, not being valid. Consequently, it is very essential to test for normality of residuals. This section primarily examines the misspecification of the model in the three elements; namely normality, autocorrelation, and heteroskedasticity.

## **6.5.3.1 Normality Test**

Jarque-Bera (JB) test of normality that has now become very popular and is included in several statistical packages.<sup>55</sup> This is an *asymptotic*, or large sample, *test* and is based on OLS residuals. This test first computes the coefficients of skewness, *S* (a measure of asymmetry of a probability density function [PDF]), and *kurtosis*, *K* (a measure of how tall or flat a PDF is in relation to the normal distribution), of a random variable; for a normally distributed variable, skewness is zero and kurtosis is 3.

Jarque and Bera (JB) have developed the following test statistic:

$$= - + \frac{( )}{} \tag{6.6}$$

\_

<sup>&</sup>lt;sup>54</sup> A perfect condition of no serial correlation is normally indicated by the value of DW equal to 2 (Asteriou and Hall, 2007)

<sup>&</sup>lt;sup>55</sup> The test is based on the work by C.M. Jarque and A.K. Bera, "A Test for Normality of Observations and Regression Residuals", published in *International Statistical Review*, Vol. 55, 1987, p. 163-172.

where *n* is the sample size, *S* represents skewness, and *K* represents kurtosis. They have shown that under the normality assumption the JB statistic given in the equation above asymptotically (*i.e.*, in large samples) follows the chi-square distribution with 2 degree of freedom. Symbolically,

$$\sim$$
 (6.7)

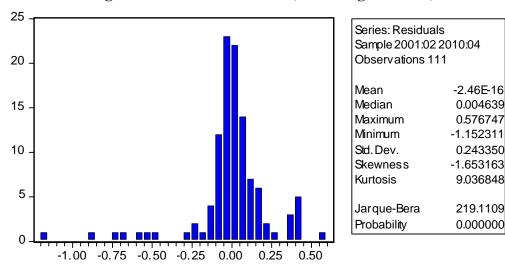
where asy means asymptotically.

As can be seen from equation 6.6., if a variable is normally distributed, *S* is zero and (*K*-3) is also zero, and therefore the value of the JB statistic is zero. But if a variable is not normally distributed, the JB statistic will assume increasingly larger values. What constitutes a large or small value of the JB statistic can be easily learned from the chi-square table. If the computed chi-square value from equation 6.6. exceeds the critical Chi-square value for 2 *d.f.* at the chosen level of significance, that would suggest a rejection of the null hypothesis of normal distribution; but if it does not exceed the critical Chi-square value, the null hypothesis cannot be rejected. In this study, JB test is conducted in EViews as presented for each of the function in Figure 6.2. and Figure 6.3.

14 Series: Residuals Sample 2001:02 2010:04 12 Observations 111 10 Mean -3.00E-15 Median 0.108071 8 Maximum 1.384728 Minimum -2.419147 6 Std. Dev. 0.711388 Skewness -1.022425 4 Kurtosis 4.273612 2 Jarque-Bera 26.84120 Probability 0.000001 -1.5 -1.0 -0.5 0.0

**Figure 6.2: JB Test for** *IGC*<sub>I</sub> **(Investment Function)** 

Figure 6.3: JB Test for  $IGC_F$  (Financing Function)



As shown in figures 6.2 and 6.3, the number of observations, which is 111, is qualified for JB test<sup>56</sup>. Secondly, it is rather obvious from the shape of histograms produced in *EViews* that the models do not seem to be normally distributed. Thirdly, the figures vividly demonstrate the non-normality of both the investment and financing function. This is demonstrated by the high value of computed JB value, 26.86 and 219.11 for investment and financing function, respectively. The results of JB tests are also significant at 1% level of significance. Moreover, non-normality of both functions are also confirmed by the value of skewness and kurtosis which are less than zero and greater than 3.

#### **6.5.3.2** Testing for Serial Correlation

Lagrange-Multiplier test is an alternative to the Durbin-Watson statistics for testing serial correlation on any errors, which may exhibit autocorrelation. This is considered very crucial, since the results of regression tests, as shown in table 6.3 and 6.4, indicate that the models might suffer from a severe serial correlation problem, particularly for investment function whose DW value is only 0.42. In this respect, the study employs Breusch-Godfrey Serial Correlation LM Test. The Breusch-Godfrey LM test overcomes the DW test, which has several drawbacks that make its use inappropriate in various cases. As highlighted by Asteriou and Hall (2007), a few drawback embedded in DW

<sup>56</sup> 111 is quite large number to qualify for JarqueBera test, as indicated by Gujarati (1999: 178)

test are (i) it may give inconclusive results, (ii) it is not applicable when a lagged dependent variable is used, and (iii) it cannot take into account higher orders of serial correlation. For these reasons, Breusch (1978) and Godfrey (1978) developed an LM test which can accommodate all the above cases. Consider the model:

$$= + + + + \cdots + + (6.8)$$

where

$$= + + \cdots + + \qquad (6.9)$$

The Breusch-Godfrey LM test combines these two equations:

$$=$$
 + + +  $\cdots$  + + +  $\cdots$  +  $\cdots$ 

And therefore the null hypothesis and the alternative hypotheses are defined as:

$$H_0$$
:  $\rho_1 = \rho_2 = \dots = \rho_p = 0$  no autocorrelation

 $H_a$ : at least one of the  $\rho_s$  is not zero, thus, serial correlation.

**Table 6.7: LM Test for Investment Function** 

Breusch-Godfrey Serial Correlation LM Test:

-			
F-statistic	110.4335	Probability	0.000000
Obs*R-squared	75.69846	Probability	0.000000

Test Equation:

Dependent Variable: RESID Method: Least Squares Date: 08/23/11 Time: 15:29

Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
С	0.304949	0.682094	0.447077	0.6558
LFS	0.044421	0.057524	0.772221	0.4418
LROSD	-0.025021	0.177452	-0.141003	0.8881
LROTD	0.069109	0.058954	1.172249	0.2438
D(LBOPO)	-0.174140	0.155022	-1.123327	0.2639
LTR	-0.182586	0.089848	-2.032167	0.0447
RESID(-1)	0.563473	0.091484	6.159255	0.0000
RESID(-2)	0.344589	0.095472	3.609319	0.0005
R-squared	0.681968	Mean depende	ent var	-3.00E-15
Adjusted R-squared	0.660354	S.D. depender	nt var	0.711388

S.E. of regression	0.414591	Akaike info criterion	1.146292
Sum squared resid	17.70419	Schwarz criterion	1.341574
Log likelihood	-55.61923	F-statistic	31.55242
Durbin-Watson stat	1.945681	Prob(F-statistic)	_ 0.000000

The result of LM test for investment function suggests that it suffers from autocorrelation problem, as the value of *observation\*R*-squared being 75.7 is really high, and strongly significant at 1% confidence level. Hence, the result rejects the null hypothesis of no serial correlation. The value of calculated F-stat also suggests that the model suffers from serial correlation problem (reject  $H_0$ ).

**Table 6.8: LM Test for Financing Function** 

Breusch-Godfrey Serial Correlation LM Test:				
F-statistic	1.751800	Probability	0.161042	
Obs*R-squared	5.339319	Probability	0.148571	

**Test Equation:** 

Dependent Variable: RESID Method: Least Squares Date: 08/23/11 Time: 15:41

Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
С	0.036560	0.443530	0.082429	0.9345
LF	0.012140	0.063462	0.191294	0.8487
D(LBOPO)	0.004501	0.068730	0.065495	0.9479
LTR	-0.033401	0.054794	-0.609584	0.5435
RESID(-1)	-0.185397	0.104107	-1.780836	0.0779
RESID(-2)	-0.169930	0.103320	-1.644690	0.1031
RESID(-3)	0.004197	0.103067	0.040725	0.9676
R-squared	0.048102	Mean depende	ent var	-2.46E-16
Adjusted R-squared	-0.006815	S.D. depender	nt var	0.243350
S.E. of regression	0.244178	Akaike info c	riterion	0.079150
Sum squared resid	6.200792	Schwarz crite	rion	0.250021
Log likelihood	2.607174	F-statistic		0.875900
Durbin-Watson stat	1.933922	Prob(F-statist	ic)	0.515359

On the other hand, financing function is statistically immune from autocorrelation since the result of LM test cannot reject the null hypothesis. As shown in the table 6.8, the values Observation\*R-squared coupled with F-statistic are low and insignificant. The financing function is therefore free from autocorrelation problem. F-stat is insignificant

suggesting the acceptance of null hypothesis. In other words, the model is statistically immune from serial correlation problem.

# **6.5.3.3** Testing for Heteroskedasticity

One of the assumptions of the classical linear regression is that the disturbances should have a constant (equal) variance independent of t, given in mathematical form by the following equation:

$$( ) =$$
  $(6.11)$ 

Therefore, having an equal variance means that the disturbances are homoskedastic. However, it is quite common in regression analysis to have cases where this assumption is violated. In such cases, it is said that the homoskedasticity assumption is violated.

Suppose a classical linear regression is modelled as follows:

$$= + + + + \cdots + + (6.12)$$

If the error term in the equation 6.12 is known to be heteroskedastic, then the consequences on the OLS estimators or , can be summarised as follows (Asteriou and Hull, 2007: 104):

- The OLS estimation for the are still unbiased and consistent. This is because none of the explanatory variables are correlated with the error term. So, a correctly specified equation that suffers only from the presence of heteroskedasticity will give us values of which are relatively good.
- Heteroskedasticity affects the distribution of the increasing the variances of the distributions and therefore making the estimators of the OLS method inefficient (because it violates the minimum variance property). It is viewed that heteroskedasticity does not cause bias, because is centred around  $\beta$  (so
  - = ), but widening the distribution makes it no longer efficient implying that OLS is no longer the most efficient estimator.

Heteroskedasticity also affects the variances (and therefore the standard errors as well) of the estimated . In fact, the presence of heteroskedasticity causes the OLS method to underestimate the variances (and standard errors), hence leading to higher than expected values of t statistics and F statistics. Therefore, heteroskedasticity has a wide impact on hypothesis testing: neither the t statistics nor the F statistics are reliable any more for hypothesis testing, because they will lead us to reject the null hypothesis too often.

One way to detect the presence of heteroskedasticity is by applying appropriate tests; such as Breusch-Pagan LM test, Glesjer test, Goldfeld-Quandt test, and White test. This study employs heteroskedasticity test developed by White (1980).

Table 6.9: White Heteroskedasticity Test for Investment Function

White Heteroskedasticity Test:

F-statistic	34.85517	Probability	0.000000
Obs*R-squared	86.25369	Probability	0.000000

**Test Equation:** 

Dependent Variable: RESID^2 Method: Least Squares Date: 08/23/11 Time: 15:30 Sample: 2001:02 2010:04

Included observations: 111

Variable Coefficient Std. Error t-Statistic Prob. C 106.0048 11.34988 9.339733 0.0000 **LFS** -4.525468 1.171285 -3.863677 0.0002 -3.902649 LFS^2 -0.298688 0.076535 0.0002 **LROSD** 50.85265 4.377724 11.61623 0.0000 LROSD^2 5.248876 0.436647 12.02087 0.0000 **LROTD** -0.220079 1.658346 -0.132710 0.8947 LROTD^2 0.013378 0.134129 0.099743 0.9207 -5.819194 D(LBOPO) -2.016736 0.346566 0.0000 (D(LBOPO))^2 -4.878183 0.433383 -11.25605 0.0000 LTR 1.016713 0.648089 1.568785 0.1199 2.019543 LTR<sup>2</sup> 0.208988 0.103483 0.0461 R-squared 0.777060 Mean dependent var 0.501513 Adjusted R-squared S.D. dependent var 0.911509 0.754766 S.E. of regression 0.451389 Akaike info criterion 1.340865 Sum squared resid Schwarz criterion 1.609377 20.37523 Log likelihood -63.41801 F-statistic 34.85517 **Durbin-Watson stat** 1.292891 Prob(F-statistic) 0.000000

Table 6.10: White Heteroskedasticity Test for Financing Function

White Heteroskedasticity Test:

F-statistic	2.485150	Probability	0.027505
Obs*R-squared	13.91891	Probability	0.030555

Test Equation:

Dependent Variable: RESID^2 Method: Least Squares Date: 08/23/11 Time: 15:41 Sample: 2001:02 2010:04 Included observations: 111

Variable	Coefficient	Std. Error	t-Statistic	Prob.
С	1.494102	5.242167	0.285016	0.7762
LF	0.419422	1.382614	0.303354	0.7622
LF^2	0.035630	0.090363	0.394298	0.6942
D(LBOPO)	-0.097779	0.104115	-0.939137	0.3498
$(D(LBOPO))^2$	-0.017172	0.036019	-0.476750	0.6345
LTR	0.238074	0.165158	1.441492	0.1525
LTR^2	0.031636	0.025365	1.247239	0.2151
R-squared	0.125396	Mean dependent var		0.058686
Adjusted R-squared	0.074938	S.D. dependent var		0.167125
S.E. of regression	0.160741	Akaike info criterion		-0.757055
Sum squared resid	2.687125	Schwarz criterion		-0.586184
Log likelihood	49.01656	F-statistic		2.485150
Durbin-Watson stat	2.191003	Prob(F-statisti	c)	0.027505

The results of White test for both the investment and financing function show that computed White's hetersokedasticity statistics in investment and financing function are bigger than the critical value, 86.25 and 13.92 respectively; both suggest the presence of heteroskedasticity in the models. In addition, the findings are also confirmed by the p-values as well as F-stats of both functions, 0.00, 0.030 and 0.027 for investment and financing function respectively suggesting the evidence of heteroskedasticity.

## **6.5.4 Modifying the Models**

The previous section presents that the OLS-based regressions conducted in section 6.5.2 have violated the basic assumption of classical linear regression model in three respects; normality, presence of serial correlation and heteroskedasticity. Thus, the regression results cannot be used as the basis of drawing the conclusion in the analysis. This section, hence, attempts to address the issue by modifying the regression models accordingly.

A technique that is adopted in this study is weighted least squares (WLS) which, according to Asteriou and Hall (2007), is one of the effective ways in overcoming heteroskedasticity. WLS is conducted by re-estimating the model in a way which will fully recognizes the presence of heteroskedasticity problem. WLS would then produce (i) a new set of parameter estimates which would be more efficient than the OLS ones, and (ii) a correct set of covariances and *t*-statistics. The advantage of using this method is that it can also simultaneously correct the problem of non-normality.

In addition, this study also utilises White Heteroskedasticity test developed by White (1980). White's test is basically an LM test, but it has the following advantages: (i) it does not assume any prior knowledge of heteroskedasticity, and (ii) it does not depend on the normality assumption.

Another problem that is addressed in this section is the presence of serial correlation or autocorrelation, which prevail in the investment function model. The reasons why there is a need of making a correction on autocorrelation are as follows (Asteriou and Hall, 2007: 137):

- (1) The OLS estimators of the  $\beta s$  are still unbiased and consistent;
- (2) The OLS estimators will be inefficient and therefore no longer BLUE; and
- (3) The estimated variances of the regression coefficients will be biased and inconsistent, and therefore hypothesis testing is no longer valid. In most cases, *R*-squared will be overestimated which may indicate a better fit than the one that truly exist and the *t*-statistics will tend to be higher indicating higher significance of the estimates than the correct one.

A popular remedy, among others, for autocorrelation problem is the Cochrane-Orcutt iterative procedure (Asteriou and Hall, 2007; and Gujarati, 1999). In *EViews*, the estimates from this iterative method can be obtained by simply adding the AR(1) error terms to the end of the equation specification list. The implication behind the AR(1) model is that the time series behaviour of, suppose  $Y_t$ , is largely determined by its own value in the preceding period. Thus, what will happen in the present time, suppose t, is

largely dependent on what happened in t-1, or alternatively what will happen in t+1 is to be determined by the behaviour of the series in the current time t.

The table 6.11 and table 6.12 summarise the results of rectified regression analysis based on two methods: weighted least squares coupled with white heteroskedasticity, and the Cochrane–Orcutt iterative procedure for investment function model.

Table 6.11: Regression Results for Investment Function Dependent Variable: *LROS* 

Specification	<b>Estimated Values</b>
Constant	4.00
	(2.42)**
LFS	0.13
	(0.46)
LRoSD	1.32
	(3.07)**
LRoTD	0.22
	(1.24)
D(LBOPO)	0.18
	(0.85)
LTR	0.43
	(3.12)*
AR(1)	0.97
	(26.67)*
R-squared	0.95
F-stat	342.8
DW	1.75

Note: '\*' denotes rejection of the null hypothesis at 1% level; '\*\*' denotes rejection of the null hypothesis at 5% level

The results in table 6.11 depict that all the variables produce coefficients according to the expected signs. Due to the presence of autocorrelation problem, investment function model is transformed into autoregressive model through Cochrane-Orcutt iterative procedure, which proves to make a positive difference in its DW value from 0.42 to 1.75. More importantly, the model transformation is also very significant at 1% confidence level, shown by variable AR(1).

Table 6.11 also shows that there are two main determinants of operational risk exposures in  $IGC_I$ ; the volatility of training expenses and the volatility of return on saving deposits. Volatility of training expenses is significant at 1% confidence level, whereas volatility of return on saving deposits are positively related to the target variable and significant at 5% level of confidence. The dominance of training in investment activities might show an indication of the role of an intensive training which will produce highly specialised human resources who are well versed in both *Shariah* and financial economics. In other words, the result shows that the performance of investment activities in Islamic banking is significantly affected by highly-skilled personnel who run the banking activities, which unfortunately quite scarce at the present stage. The finding also confirms the concerns raised by Archer and Haroon (2007) and Jackson-Moore (2007).

It is also apparent that volatility of return on saving deposits, which represents a fiduciary role an Islamic bank, is also essential as demonstrated by the significance of its *t*-statistics. However, it can be argued that investment activities in Indonesian Islamic banks is highly affected by people risk as compared to other type of operational risks.

The rectified model, nonetheless, is also proved to be the best model, since the variability of explanatory variables can be explained by the model at around 95% as shown by the value of *R*-squared.

Table 6.12: Regression Results for Financing Function Dependent Variable: D(LRoF)

Specification	<b>Estimated Values</b>
Constant	0.56
	(0.91)
LF	0.03
	(0.39)
D(LBOPO)	0.75
	(5.4)*
LTR	0.20
	(2.16)**
R-squared	0.73
F-stat	96.58
DW	2.22

Note: '\*' denotes rejection of the null hypothesis at 1% level; '\*\*' denotes rejection of the null hypothesis at 5% level

As for financing function model, it is only re-estimated by using WLS, OR weighted least squares combined with white heteroskedasticity to address the issue of non-normality and the presence of heteroskedasticity. An autocorrelation problem, however, is not an issue for financing function model; this has been statistically proven by using Breusch-Godfrey Serial Correlation LM tests in section 6.5.3.2 and depicted in table 6.8.

The results of the rectified regression model in Table 6.12 show that all variables produce positive coefficient as expected. Meanwhile, the values of *R*-squared, *F*-stat and *DW*, which are 73%, 96.58 and 2.2 respectively; shows that the model now fits in explaining the variation of the explanatory variables with respect to the target variable, namely volatility of return on financing.

Although people risk, represented by *TR* and is significant at 5% level of confidence, still plays an important role in affecting the magnitude of operational risk exposures; it is no longer dominant in financing activities as compared to its impact in investment activities. Ratio of operating expenses to operating income (*BOPO*), on the other hand, plays a major role in this regard.

The findings indicate that for financing activities, the role of maintaining operational efficiency as part of an Islamic bank's fiduciary responsibilities is immensely high in the case of Indonesian Islamic banks. In the model, *BOPO* is significant at 1% level of confidence.

#### 6.6 CONCLUDING REMARKS

The focus of this chapter is to examine the relationship between the operational risk variables in two income generating channels, namely investment ( $IGC_I$ ) and financing ( $IGC_F$ ). By employing a range of econometrics tests, this study attempts to come up with the determinants affecting operational risk exposures in Islamic banking activities. Due to some misspecifications of the initial model, rectified models are then generated to respond to such issues identified in the section 6.5.3.

Following some corrections on the initial models, the empirical finding in this study shows that fiduciary risk and people risk are significant in both channels,  $IGC_I$  and  $IGC_F$ . However, people risk is immense and plays a major role in affecting the magnitude of operational risk exposures in Islamic banks, particularly in investment activities. The finding is in line with the concern raised by Archer and Haroon (2007), Jackson-Moore (2007) and Nienhaus (2007) over the high degree of people risk in Islamic banking activities. This finding also validates the priority of Bank Indonesia, being the regulator of Indonesian Islamic Banking industry, to enhance the capacity of human resource that run the Islamic banking business, particularly in investment activities which shows a growing trend over the last three years (Indonesian Islamic Banking Outlook, 2011).

As for financing activities, although people risk is also significant, its impact on the exposure of operational risk is outweighed by the role of operational efficiency ratio, represented by *BOPO*. The results suggest that an expansion of financing activities should retain the prudential principle combined with a principle of costs minimisation; as it would otherwise increase the level of un-repayment. Such finding is in line with the projection of central bank of Indonesia as reported in Indonesian Islamic Banking Outlook (2011).

The important implications of the findings is that the right policy prescription, by means of customising it to the nature of activities within the Islamic banking, should be applied in order to achieve an effective operational risk management. Reflecting to the empirical findings, an orientation towards enhancing human development and increasing the *know-how* on investment activities should be the priority so as to make investments activities in Islamic banking more attractive; whereas a cost reduction strategy can be emphasised in financing activities in Islamic banking industry in Indonesia.

# **CHAPTER 7**

# AN INTERPRETATIVE DISCUSSION AND CONCLUSION

#### 7.1 INTRODUCTION

Prior to presenting the conclusion, this chapter provides a discussion of the research's findings upon which an elucidation based on the nature of this study's contribution in theoretical, methodological, and empirical areas are presented.

As depicted in figure 1.1, theoretical contribution of this study is presented in Chapter 2 and Chapter 3. While the analysis in Chapter 2 is on general-theoretical concept of operational risks, the analysis in Chapter 3 is substantiated in the context of Islamic banking. Chapter 4, further attempts to contribute to the methodological aspect of operational risk measurement by developing a new model to measure operational risk exposures in Islamic banks. A contribution of this kind, however, is still lacking in the area of risk management in Islamic banking, particularly in operational risk management. Furthermore, an empirical contribution of the research is presented in Chapter 5 as well as 6.

The primary goal of the discussion in this chapter is mainly to relate the research's findings to the aim and objectives mentioned in the first chapter. As mentioned in chapter 1, the aim of this research is to develop and propose a new model to measure the exposure of operational risks in Islamic banking. In explicating the discussion of research's findings, this chapter follows a sequential order of the research questions formulated in section 1.4.

#### 7.2 REFLECTING ON THE THEORETICAL CONTRIBUTION

# 7.2.1 Does the Definition of Operational Risk in Islamic Banks Embody the Same Dimensions as Conventional Banks?

Albeit the difficulties of describing operational risk, due to its diversity (Buchelt and Untregger: 2004) and complexity (Milligan: 2004); attempts to settle the definition and the coverage of operational risk have been made by a number of contenders including Alexander (2003), Crouhy *et al.* (2001), Cagan (2001), Tripe (2000), Lopez (2002), Jarrow (2007), Moosa (2007) and Jobst (2007).

There are two general approaches in defining operational risk. The first approach is 'residual approach' which states that operational risk is everything other than credit risk or market risk (Rao and Dev, 2006; Hull (2007). More specifically, operational risk can be drawn by looking at the bank's financial statements and remove from the income statement (i) the impact of credit losses and (ii) the profit or losses from market risk exposure, thus the variation in the resulting income would the attributed to operational risk (Hull: 2007).

Despite the fact that this view is deemed to be hardly suitable for identifying its scope precisely (Buchelt and Unteregger: 2004), Medova and Kyriacou (2001) and Jameson (1998) argue that the understanding of operational risk is everything that is not exposed to credit and market risk. In response to this, Moosa (2007) asserts that a definition of this sort is probably a reflection of the lack of understanding on the diversity of operational risk.

The second approach, which is 'non-residual approach', suggests that operational risk is, in fact, the risk arising from operations (Crouhy *et al.*, 2001). This includes the risk of mistakes in processing transactions, making payment, etc.

The diversity of defining operational risk in the initial stage shows that there was no consensus in the industry on the precise definition of operational risk. In addition, some other vague concepts have been put forward, such as Tripe (2000) who contends that operational risk is simply an operational loss, without elaborating further. This of course,

does not reflect the diversity of the scope of operational risk. Similarly, Lopez (2002) argues that operational risk is every unquantifiable risk, which seems to be the antithesis of measuring regulatory capital against operational risk as required by the Basel 2 Accord.

A refined definition of operational risk that is currently widely accepted, and as mentioned in Chapter 2, is the risk of loss resulting from inadequate or failed internal processes, people or system, or from external events. It is clear that such definition entails four main sources of operational risk; (i) people, (ii) processes, (iii) systems, and (iv) external factors.

An Islamic bank, on the other hand, is a financial institution that runs its business by following a set of *Shari'ah* principles as set out in Chapter 3. With respect to operational risk, it is viewed that an Islamic bank encounters very similar financial challenges as its conventional counterpart since they are operating in a similar, if not the same business environment. One of the very differences in the operation between the two is the *shariah* restrictions that are attached to the running of business of an Islamic bank.

Nevertheless, it is also understood that the challenges are more complex for Islamic banks due to their activities and unique features of Islamic financial contracts that are not only consisting of debt financing, but also hybrid financing as well as equity financing; each of which would bring different implications to the dimensions of operational risk.

In an attempt to define operational risk, one of the very influential governing bodies in Islamic finance industry, namely *Islamic Financial Services Board*, defines operational risk as the risk of loss resulting from inadequate or failed internal processes, people and system, or from external events, including losses from *shariah* non-compliance and the failure in fiduciary responsibilities. Although it is also claimed that the definition of operational risk in Islamic banks includes legal risk and reputational risk, its distinctive features, however, is the inclusion of *shari'ah* non-compliance risk and fiduciary risk.

Chapter 3 clearly defines shariah non-compliance risk as the risk arising from Islamic banks' failure to comply with the shariah rules and principles determined by the shari'ah

board or the relevant body in the jurisdiction in which the Islamic bank operates; whereas fiduciary risk arises in the event that Islamic banks fail to perform in accordance with explicit and implicit standards applicable to their fiduciary responsibilities.

The spotlight above explains why operational risk management in Islamic banks is not similar to that in conventional banks. As indicated in Chapter 3, operational risk management in Islamic banks requires more rigorous understanding as to the sources of operational risk due to a range of financial contracts that should comply with certain *shariah* restrictions, which bring about somewhat different fiduciary responsibilities to Islamic banks. In brief, operational risks in Islamic banks could appear based on the following major sources: (i) *shari'ah* non-compliance risk, (ii) fiduciary risk, (iii) people risk, (iv) technology risk, and (v) legal risk

# 7.2.2. Operational Risks Dimensions in Major Islamic Financial Contracts

With regard to the dimension of operational risks in Islamic financial contracts, it is understood that different contracts carry different complexity of operational risks. According to the discussion in Chapter 3, five identified sources of operational risk that are spread across major financial contracts are as follows; *shariah* non-compliance risk, fiduciary risk, people risk, technology risk, and legal risk. Out of five, the first four are categorised as internal sources, while the last one, namely legal risk is considered as an external source. Moreover, Chapter 3 also attempts to systematically analyse the proposed sources of operational risk (see Table 3.2).

# 7.3. REFLECTING ON THE METHODOLOGICAL CONTRIBUTION OF THE STUDY

After discussing the theoretical elements of operational risk in Chapter 2 and 3, this study moves on with the methodological analysis of the subject. In this respect, an attempt has been made to develop an entirely new model to measure operational risks exposure in Islamic bank, which constitutes the methodological and empirical contribution of this study.

As widely observed by academics as well as market players, and is clearly pointed out in Chapter 4, one of the challenges in the analysis of operational risk is not only about its multi-dimensionalities, but also due to the scarcity of operational risk data which makes operational risk even more difficult to be measured quantitatively. In this respect, this study attempts to contribute to the modelling of operational risk measurement in Islamic banking by developing a model attributed as *Delta-Gamma Sensitivity Analysis Extreme Value Theory (DGSA-EVT)* model. It is expected that the constructed model, as discussed in Chapter 4, is able to capture the operational risk exposures taking place in two types of areas, namely high frequency-low severity (*HF-LS*) and low frequency-high severity (*LF-HS*).

In dealing with the two types of aforementioned data, DGSA being the first leg of the proposed model is designed to cater HF-LS data, while EVT being the second leg of the model is tailored towards dealing with LF-HS type of data. In doing so, DGSA starts with a separation of different value adding process, namely income generating channels (IGC) in Islamic banking. As discussed in Chapter 4, income generating channel is defined as the production unit by which a bank creates a product valuable to its customers. In this regard, the bank is then partitioned into three income generating channels comprising of investment ( $IGC_I$ ), financing ( $IGC_F$ ) and services ( $IGC_S$ ) activities. Investment channel is composed of any investment in the form of a partnership, while financing channel consists of any financing instruments that are used primarily to finance obligations arising from the trade and sale of commodities. In addition, service channel contains any financial transactions that create earnings by charging fees.

Following the partition in three income generating channels, the next step, as the proposed model suggests, is to establish the earnings function in each *IGC*. Fluctuation or volatility of the target variable is considered as the proximity of operational risk exposures. As the name of the model indicates, *Delta-Gamma* attributes to the non-linear relationship taking place between variables in the earnings function, while *sensitivity analysis*, as thoroughly discussed in Chapter 4, refers to the technique used to determine the causality between analysed variables within earnings function.

A theoretical examination is also addressed to the key factors that contribute to the performance or the fluctuations of target variable in this regard. In other words, an establishment of key risk indicators influencing the target variable is necessary. It should be noted that in the proposed model, an *HF-LS* area constitutes operating loss distribution leading to the estimation of operational value at risk, within which causality between risk factors in the earning function can be established.

Furthermore, the second leg of the proposed model, *EVT*, is a technique to cater for an excess operational loss over a defined threshold which is normally characterised by low frequency and high severity (*LF-HS*) type of loss.

A subsequent attempt to develop a new measurement model of operational risk is by deploying *EVT* in catering *LF-HS* type of data. Prior to deploying this technique, as depicted in Figure 4.1, the proposed model asserts the necessity of determining maximum operating loss distribution of *DGSA* which would then function as a threshold or cut-off point between the *DGSA* and *EVT*.

EVT, being the second leg of the proposed model, attempts to fit the distribution of maximum losses beyond DGSA, which is characterised by an extremely skewed density area.

In determining tail index parameter within *EVT*, the proposed model utilises *mean excess plot* together with the standard *Hill* estimator. The reason is mainly to anticipate a small number of observations beyond the threshold which may not only cause a high variance; but it may also create difficulties in the process of modeling the frequency distribution. Following an examination of severity and frequency of the data, the proposed model combines the two in order to to generate excess loss distribution as shown in figure 4.2 (see Chapter 4).

In the final stage of the model, proximity of operational risk exposures is established by summing up the *VaR* resulted in the *EVT* process, being the second stage of the proposed model and the *DGSA* process, being the first stage of the proposed model.

Any modeling exercise, how robust it is, may be subject to empirical testing; in this respect, an empirical test of the proposed model is presented in Chapter 5 and 6, and is discussed in the subsequent section.

#### 7.4. REFLECTING ON THE EMPIRICAL CONTRIBUTIONS OF THE STUDY

# 7.4.1. Empirical Estimation of the Proposed Model

As discussed in chapter 4, the proposed model in this study has two main stages comprising (i) an estimation of Delta-Gamma Sensitivity Analysis (*DGSA*) resulting in value at risk (*VaR*) for operating loss distribution (see Figure 4.1 in Chapter 4), and (ii) an estimation of extreme values resulting in *VaR* for excess loss distribution (see Figure 4.2 in Chapter 4). Both *VaR*s, thus, would provide an approximation of the operational risk level in the analysed bank. While the Chapter 4 is aimed at contributing to the methodological aspect of operational risk measurement, Chapter 5 and Chapter 6's primary goal is to empirically test the proposed model by using some empirical data.

In empirically testing the proposed model, the study deploys monthly data ranging from January 2001 until June 2010, which was extracted from the published financial reports of Islamic banking Industry in Indonesia. However, due to the computational and numerical intensity of the proposed model coupled with the scarcity, or rather the unavailability of data, it is argued that there is a need to conduct a number of alterations in testing the model. Another factor contributing to the adjustment of empirical tests of the proposed model is the non-linearity relationship among the analysed variables that is non-existent.

An absence of non-linear relationship among variables implies that a deployment of *delta-gamma* technique is prevented. It has also been discovered from the simulation of the density functions of analysed variables conducted in Chapter 5 that none of the variables demonstrates an extreme density function. An application of *EVT*, thus, becomes unlikely.

It is argued in Chapter 5 that nonexistence of sophisticated derivatives products is the main factor as to why there is no non-linear relationship between analysed variables.

However, as discussed in Chapter 5, it is still possible to operate *DGSA* by shifting its focus to the normality *vis a vis* non-normality instead.

In DGSA, the level of operational risk exposures in each income generating channel is represented by VaR, which in this study is analysed by incorporating the volatility, skewness and kurtosis of the analysed variables. However, Due to the insignificant contribution to the value adding process,  $IGC_S$  is taken out of the analysis.

In estimating operational VaR in  $IGC_I$  and  $IGC_F$ , this study meticulously examines the behaviour of the volatility, skewness and kurtosis of the analysed variables. As discussed in Chapter 5, volatility analysis in this study adopts two methods, namely constant-variance model and exponential weighted moving average (EWMA) model. The rationale behind the use of both techniques is to compare the level of volatility of the analysed variables. It turns out that a transformation of data from constant-variance to EWMA tends to display a declining trend of standard deviation, skewness and kurtosis of data. In other words, the transformed data to EWMA shows a less disperse pattern. Another important message from the transformation is that the data still shows non-normality which suggests that a modification of standard VaR estimation is very likely. Furthermore, non-normality of the data are also proven by two statistical tests: (i) Kolmogorov-Smirnov, and (ii) Anderson-Darling whose thorough results are presented in the appendix.

In response to the non-normality of the variables, this study, therefore, deploys a technique called *Cornish-Fisher* expansion. Such a technique is a method of estimating *VaR* whereby a confidence interval of operational variables is an explicit function of the skewness, kurtosis, and volatility.

A substantial difference in the value of *VaR* has been discovered when *Cornish-Fisher* expansion is deployed as compared to the normal *VaR*. For investment activities, *VaR* based-*Cornish Fisher* expansion is higher than the normal one; whereas for financing activities, a deployment of *Cornish-Fisher* expansion gives a lower *VaR* as compared to the normal *VaR*.

An inference that can be withdrawn from the results is that an underestimation or overestimation of the capital catering for operational risk for investment and financing activities respectively, could have occurred, should the volatility, skewness and kurtosis are not incorporated in the estimation of *VaR*.

A continuation of testing the proposed model is discussed in Chapter 6 with an objective to examine the relationship between identified risk factors with their respective earnings functions, namely return on securities (RoS) in  $IGC_I$  and return on financing (RoF) in  $IGC_F$ . In doing so, as systematically depicted in Figure 6.1 the research mainly utilises regression techniques combined with misspecification tests which comprise (i) *jarque-bera*, (ii) *lagrange multiplier*, and (iii) *white heteroskedasticity*.

The proposed model by this study, thus, can be empirically tested by the identified tests and methods as presented above and illustrated in the relevant chapters.

#### 7.4.2. The Most Dominant Identified Risk Factors

With an objective of examining the relationship between operational risk variables, Chapter 6 attempts to determine the most significant factors influencing operational risk exposures in two income generating channels, namely investment ( $IGC_I$ ) and financing ( $IGC_F$ ). In order to reach accurate empirical results, a rectification of the initial model is deemed necessary due to some misspecifications of the initial regression model in three respects: normality, presence of serial correlation and heteroskedasticity.

In overcoming heteroskedasticity, a technique that is adopted in this study is weighted least squares (*WLS*) which, according to Asteriou and Hall (2007), is quite effective remedy for such problem.

In addition, this study also utilises White heteroskedasticity test, since it has two advantages; (i) it does not assume any prior knowledge of heteroskedasticity, and (ii) it does not depend on the normality assumption.

The presence of serial correlation or autocorrelation in investment function model, however, is overcome by the use of Cochrane-Orcutt iterative procedure which results in

a change of DW value from 0.42 to 1.75. More importantly, the model transformation is also significant at 1% confidence level, shown by variable AR(I). Chapter 6 also shows that there are two main determinants of operational risk exposures in  $IGC_I$ ; the volatility of training expenses and the volatility of return on saving deposits. Volatility of training expenses is significant at 1% confident level; whereas volatility of return on saving deposits are positively related to the target variable and significant at 5% level of confidence. The dominance of training in investment activities might show an indication of the role of an intensive training which will produce highly specialised human resources who are well versed in both *Shariah* and financial economics. In other words, the result shows that the performance of investment activities in Islamic banking is significantly affected by highly-skilled personnel who run the banking activities, which unfortunately quite scarce at the present stage.

It is also apparent that volatility of return on saving deposits, which represents a fiduciary role an Islamic bank, is also essential as demonstrated by the significance of its t-statistics. However, it can be argued that investment activities in Islamic bank is highly affected by people risk as compared to other type of operational risks.

The rectified model, nonetheless, is also proved to be the best model since the variability of explanatory variables can be explained by the model at around 95% as shown by the value of R-squared.

As for financing function model, since an autocorrelation problem does not exist in the initial model, a re-estimation is only carried out to address the issue of non-normality and the presence of heteroskedasticity.

The results of the rectified regression show that all variables produce positive coefficient as expected. Meanwhile, the values of *R*-squared, *F*-stat and *DW* show that the model is already fit in explaining the variability of the explanatory variables with respect to the target variable, namely volatility of return on financing.

In financing activities, people risk is significant at 5% confident level which shows that it plays an important role in affecting the level of operational risk exposures.

Nonetheless, it is no longer dominant in financing activities as compared to its impact in investment activities. On the other hand, ratio of operating expenses to operating income (*BOPO*), plays a significant role in this regard, shown by its significance at 1% confidence level. The finding indicate that for financing activities, the role of maintaining operational efficiency as part of an Islamic bank's fiduciary responsibilities is immensely high.

A bottom line that can be drawn from the rectified models in both,  $IGC_I$  and  $IGC_F$ , is that fiduciary responsibilities and people risk are immense in investment as well as financing activities.

#### 7.5 RECOMMENDATIONS OF THE RESEARCH

Theoretical and methodological contribution of this study as well as the empirical analysis conducted by this research points out a number of recommendations which can be summarised as follows:

- There is a dire need in developing human capacity in Islamic banking. The greatest challenge is to produce individuals who are well versed in both financial economics and *shari'ah*, especially in *fiqh muamalat*. A systematic education programme and training will hopefully help in overcoming the dearth of qualified individuals which contribute to the high level of people risk especially in investment activities as suggested by the empirical finding of this study. Moreover, an integrated programme involving universities, *Shari'ah* scholars and market players to address such needs of the industry may be considered;
- With regard to financing activities in Islamic banking, this research suggests that an
  operational efficiency, as part of an Islamic bank's fiduciary responsibilities, needs
  to be enhanced;
- A standard setting body such as IFSB may consider conducting a survey and
  research with an objective to examine the extent to which an implementation of risk
  measurement methods outlined in IFSB's standards on risk management have been
  implemented by Islamic banks around the world. The results of such surveys would

provide crucial information on whether some Islamic banks come up with their own operational risk measurement methods or are in compliance with the standards;

- IFSB may also consider carrying out a research to generate beta percentage which would suit better to Islamic banks. It is expected that the findings of such research would produce a more refined version of the standardised approach;
- The industry should start building a comprehensive operational risk data which can help stakeholders in assessing the degree of operational risk more accurately. In the absence of such data, the proposed model developed in this study can be used as alternative measurement methods since it can be operated based on the standard published financial reports.

#### 7.6 LIMITATIONS AND SUGGESTIONS FOR FUTURE RESEARCH

As repetitively mentioned and highlighted in the previous section, this research's contributions lie in three aspects: theoretical, methodological, and empirical. It should be reiterated that a research of this kind is the first attempt which thoroughly tackles three aspects aforementioned in the area of operational risk measurement in Islamic banking.

Nonetheless, some limitations can also be highlighted from this research. One of which is related to the empirical aspects, namely the data which are extracted from financial statements. The research would have given more accurate results, if it utilised internal operational loss data from each individual bank (micro-data), which unfortunately is difficult to generate or is not available at all.

Another limitation is that the research has not been able to capture qualitative components of operational risk, such as *Shariah* non-compliance risk, technology risk, and legal risk. This would have again enhanced the predictability power of the model presented in this study by providing a comprehensive perspective.

Reflecting from the limitations pointed out above, some recommendations for future research can be sketched out as follows:

- An expansion of data can be considered to cover more quantitative as well as qualitative aspects of operational risks;
- An expansion of data covering more regions such as South East Asian, GCC, and Europe may also be considered. It would be important to help interested parties in understanding the behaviour of market players in managing operational risks across the regions;
- A market survey may also be crucial in capturing the qualitative aspect of the data, hence the results would be more accurate.

#### 7.7 EPILOGUE

This research is designed to address the theoretical, methodological, and empirical aspects of operational risks in Islamic banking, which is still lacking in the literature, by proposing a new model to measure operational risk in Islamic banking. Through its comprehensive elucidation as demonstrated in chapter 2 until chapter 6, it can be stated that the research has achieved its aim and objectives. It is hoped that the findings and recommendations of this research would give benefit and are taken into account by academics as well as market players. It is hoped in particular that the model and methodology developed and proposed by this study can be considered as an important contribution to the available body of knowledge, and can be considered to be implemented in the industry. Furthermore, it is also expected that this research would trigger further studies, especially in the area of operational risk management in Islamic banking.

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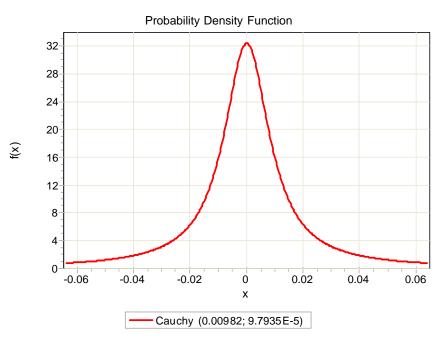
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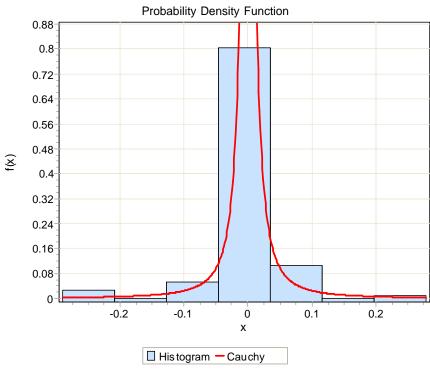
 ${\bf APPENDIX~5.1}$  Descriptive Statistics for  ${\it RoS}$  and  ${\it RoF}$  in Constant Variance and EWMA

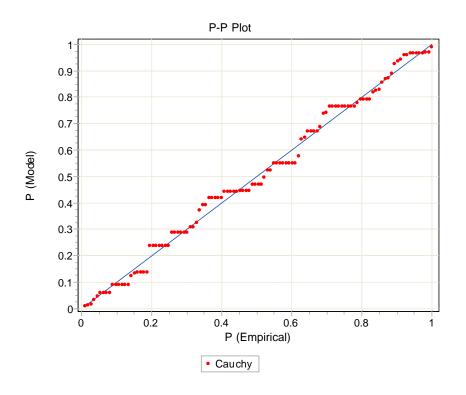
Return on Se	ecurities	Return on Secu	rities EWMA
Mean	-0.000346803	Mean	0.007021176
Standard Error	0.005686561	Standard Error	0.000385894
Median	-0.000832986	Median	0.005878347
Mode	-0.010446057	Mode	#N/A
Standard		Standard	·
Deviation	0.06044897	Deviation	0.004083922
Sample Variance	0.003654078	Sample Variance	1.66784E-05
Kurtosis	12.62335777	Kurtosis	1.549670716
Skewness	-1.186232734	Skewness	1.038922483
Range	0.568468408	Range	0.022355536
Minimum	-0.289929953	Minimum	7.68E-08
Maximum	0.278538455	Maximum	0.022355613
Sum	-0.039188688	Sum	0.786371766
Count	113	Count	
Return on Fi	inancing	RETURN ON FINA	ANCING ewma
Mean	0.004009509	Mean	0.001045753
Standard Error	0.003503083	Standard Error	3.68112E-05
Median	0	Median	0.001105029
Mode	-0.010446057	Mode	#N/A
Standard		Standard	
Deviation	0.037238288	Deviation	0.000389573
Sample Variance	0.00138669	Sample Variance	1.51767E-07
Kurtosis	4.924353855	Kurtosis	0.817625143
Skewness	1.56722886	Skewness	-0.722683906
Range	0.247260619	Range	0.00205196
Minimum	-0.09220188	Minimum	3.2736E-06
Maximum	0.155058739	Maximum	0.002055234
Sum	0.453074567	Sum	0.117124292
Count	113	Count	112

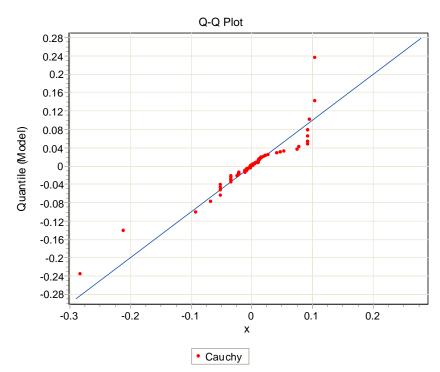
# **APPENDIX 5.2** Simulation Results

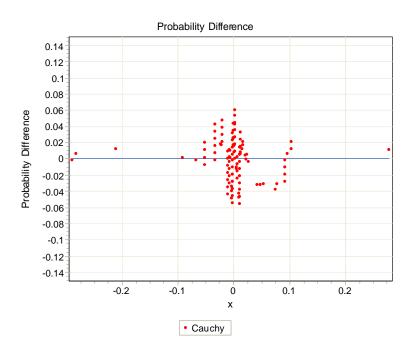
### RoS (constant-variance)











# **Fitting Results**

#	Distribution	Parameters
1	Beta	$\alpha_1$ =4825.6 $\alpha_2$ =150.22 $\alpha$ =-24.124 $\alpha$ =0.7508
2	Burr (4P)	k=0.92181 α=3.6158E+8 β=8.3733E+6 γ=-8.3733E+6
3	Cauchy	σ=0.00982 μ=9.7935Ε-5
4	Dagum (4P)	k=0.69323 α=1.0534E+8 β=2.0603E+6 γ=-2.0603E+6
5	Erlang (3P)	m=369 β=0.00331 γ=-1.2214
6	Error	k=1.0 σ=0.06045 μ=-3.4680E-4
7	Error Function	h=11.698
8	Exponential (2P)	λ=3.4532 γ=-0.28993
9	Fatigue Life (3P)	α=0.00148 β=40.317 γ=-40.317
10	Frechet (3P)	α=2.3456E+8 β=2.1619E+7 γ=-2.1619E+7
11	Gamma (3P)	α=396.19 β=0.00318 γ=-1.2578
12	Gen. Extreme Value	k=-0.37 σ=0.04538 μ=-0.01392
13	Gen. Gamma (4P)	k=2.2189 α=240.65 β=0.17684 γ=-2.0922
14	Gen. Pareto	k=-1.1971 σ=0.17331 μ=-0.07923

15	Gumbel Max	σ=0.04713 μ=-0.02755
16	Gumbel Min	σ=0.04713 μ=0.02686
17	Hypersecant	σ=0.06045 μ=-3.4680Ε-4
18	Inv. Gaussian (3P)	λ=2.2779E+6 μ=20.302 γ=-20.302
19	Johnson SU	$\gamma$ =0.28737 $\delta$ =1.205 $\lambda$ =0.04784 $\xi$ =0.0159
20	Kumaraswamy	α <sub>1</sub> =6.078 α <sub>2</sub> =2541.7 a=-0.38174 b=1.0879
21	Laplace	λ=23.395 μ=-3.4680Ε-4
22	Levy (2P)	σ=0.24857 γ=-0.3089
23	Log-Logistic (3P)	α=3.5984E+8 β=8.2822E+6 γ=-8.2822E+6
24	Logistic	σ=0.03333 μ=-3.4680Ε-4
25	Lognormal (3P)	σ=0.01822 μ=1.2066 γ=-3.3441
26	Normal	σ=0.06045 μ=-3.4680Ε-4
27	Pearson 5 (3P)	α=920.93 β=1755.5 γ=-1.9126
28	Pearson 6 (4P)	$\alpha_1$ =1.5878E+6 $\alpha_2$ =3.4093E+5 $\beta$ =6.8449 $\gamma$ =-31.879
29	Pert	m=8.6977E-4 a=-0.30856 b=0.28901
30	Power Function	α=1.367 a=-0.29376 b=0.27854
31	Rayleigh (2P)	σ=0.21174 γ=-0.29371
32	Triangular	m=-8.2902E-4 a=-0.29841 b=0.28426
33	Uniform	a=-0.10505 b=0.10435
34	Weibull (3P)	α=6.0971 β=0.40531 γ=-0.38271
35	Burr	No fit (data min < 0)
36	Chi-Squared	No fit (data min < 0)
37	Chi-Squared (2P)	No fit
38	Dagum	No fit (data min < 0)
39	Erlang	No fit (data min < 0)
40	Exponential	No fit (data min < 0)
41	Fatigue Life	No fit (data min < 0)
42	Frechet	No fit (data min < 0)
43	Gamma	No fit (data min < 0)
44	Gen. Gamma	No fit (data min < 0)
45	Inv. Gaussian	No fit (data min < 0)

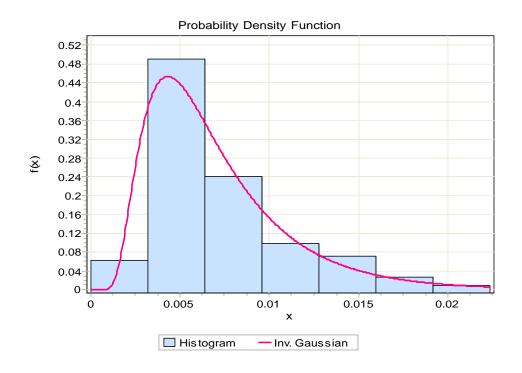
46	Johnson SB	No fit
47	Levy	No fit (data min < 0)
48	Log-Gamma	No fit
49	Log-Logistic	No fit (data min < 0)
50	Log-Pearson 3	No fit
51	Lognormal	No fit (data min < 0)
52	Nakagami	No fit
53	Pareto	No fit
54	Pareto 2	No fit
55	Pearson 5	No fit (data min < 0)
56	Pearson 6	No fit (data min < 0)
57	Rayleigh	No fit (data min < 0)
58	Reciprocal	No fit
59	Rice	No fit
60	Student's t	No fit
61	Weibull	No fit (data min < 0)

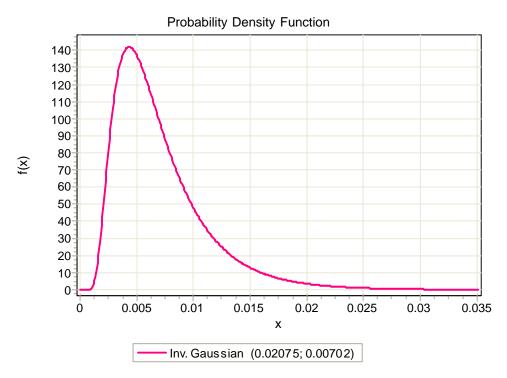
## **Goodness of Fit - Summary**

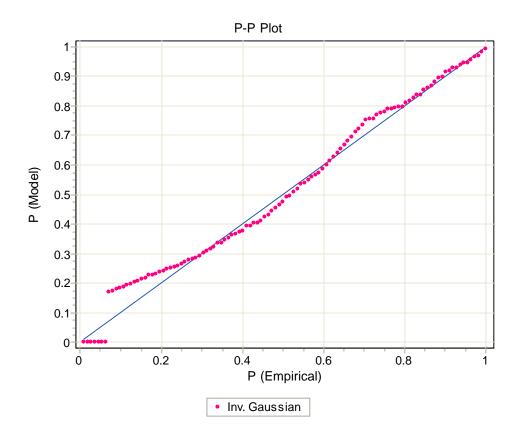
#	<u>Distribution</u>	Kolmogorov Smirnov		Anderson Darling		Chi-Squared	
		Statistic	Rank	Statistic	Rank	Statistic	Rank
1	Beta	0.24994	18	11.562	10	148.45	21
2	Burr (4P)	0.20247	3	6.6271	4	51.967	4
3	Cauchy	0.06438	1	0.5631	1	11.649	2
4	Dagum (4P)	0.20492	4	6.5057	3	56.893	5

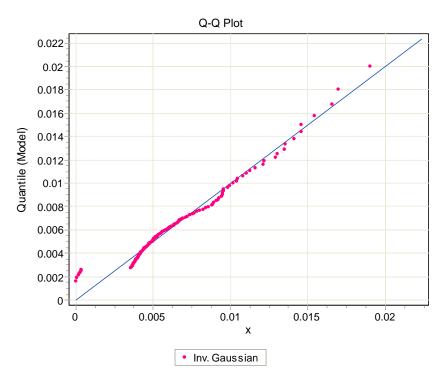
5	Erlang (3P)	0.24896	16	12.381	19	188.72	24
6	Error	0.20894	6	7.0046	6	65.432	6
7	Error Function	0.24556	12	11.696	13	152.29	22
8	Exponential (2P)	0.51693	33	38.392	32	87.394	9
9	Fatigue Life (3P)	0.2498	17	11.627	11	137.42	16
10	Frechet (3P)	0.29334	26	19.836	25	284.55	27
11	Gamma (3P)	0.25432	19	12.35	18	137.42	19
12	Gen. Extreme Value	0.22547	7	13.933	23	N/A	
13	Gen. Gamma (4P)	0.24795	15	11.799	16	137.42	18
14	Gen. Pareto	0.25448	20	47.902	34	N/A	
15	Gumbel Max	0.31292	28	22.686	28	91.764	10
16	Gumbel Min	0.29926	27	14.028	24	106.7	13
17	Hypersecant	0.23157	8	9.0568	8	92.103	11
18	Inv. Gaussian (3P)	0.2459	13	11.734	15	188.71	23
19	Johnson SU	0.24032	10	7.7903	7	80.727	8
20	Kumaraswamy	0.26191	22	13.786	22	197.64	26
21	Laplace	0.20894	5	7.0046	5	65.432	7
22	Levy (2P)	0.55337	34	43.27	33	0.4377	1
23	Log-Logistic (3P)	0.19575	2	6.4343	2	48.621	3
24	Logistic	0.23898	9	10.056	9	95.374	12
25	Lognormal (3P)	0.25889	21	11.983	17	128.75	15
26	Normal	0.24782	14	11.714	14	137.42	17
27	Pearson 5 (3P)	0.28426	25	13.032	20	113.36	14
28	Pearson 6 (4P)	0.24419	11	11.633	12	142.38	20
29	<u>Pert</u>	0.32136	29	21.419	27	313.63	29
30	Power Function	0.43165	31	31.313	31	378.25	31
31	Rayleigh (2P)	0.44988	32	29.869	30	344.8	30
32	<u>Triangular</u>	0.32454	30	21.208	26	304.46	28
33	<u>Uniform</u>	0.26914	24	26.77	29	N/A	
34	Weibull (3P)	0.2628	23	13.783	21	190.7	25
35	Burr	No fit (d	ata min	(0)			
36	Chi-Squared	No fit (data min < 0)					
37	Chi-Squared (2P)	No fit					
38	Dagum	No fit (d	ata min	(0)			
	- '						

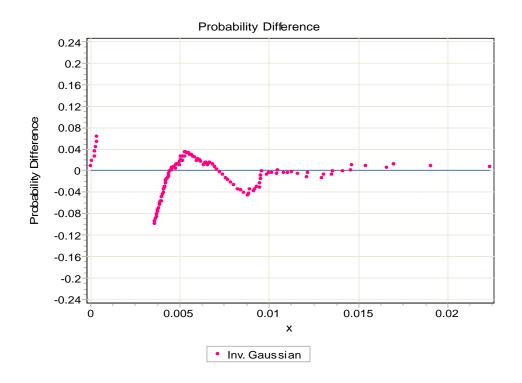
39	Erlang	No fit (data min < 0)
40	Exponential	No fit (data min < 0)
41	Fatigue Life	No fit (data min < 0)
42	Frechet	No fit (data min < 0)
43	Gamma	No fit (data min < 0)
44	Gen. Gamma	No fit (data min < 0)
45	Inv. Gaussian	No fit (data min < 0)
46	Johnson SB	No fit
47	Levy	No fit (data min < 0)
48	Log-Gamma	No fit
49	Log-Logistic	No fit (data min < 0)
50	Log-Pearson 3	No fit
51	Lognormal	No fit (data min < 0)
52	Nakagami	No fit
53	Pareto	No fit
54	Pareto 2	No fit
55	Pearson 5	No fit (data min < 0)
56	Pearson 6	No fit (data min < 0)
57	Rayleigh	No fit (data min < 0)
58	Reciprocal	No fit
59	Rice	No fit
60	Student's t	No fit
61	Weibull	No fit (data min < 0)











## **Fitting Results**

#	Distribution	Parameters
1	Beta	α <sub>1</sub> =6.1572 α <sub>2</sub> =1.4626E+5 a=-0.00294 b=236.76
2	Burr	k=2193.6 α=1.5695 β=1.0305
3	Burr (4P)	k=46274.0 α=1.9054 β=2.3436 γ=-3.0055E-4
4	Cauchy	σ=0.00189 μ=0.00556
5	Dagum	k=0.23592 α=5.0101 β=0.011
6	Dagum (4P)	k=0.21642 α=0.80105 β=0.76348 γ=7.6800E-8
7	Erlang	m=2 β=0.00238
8	Erlang (3P)	m=6 β=0.00157 γ=-0.00312
9	Error	k=1.2445 σ=0.00408 μ=0.00702
10	Error Function	h=173.14
11	Exponential	λ=142.43
12	Exponential (2P)	λ=142.43 γ=7.6800Ε-8
13	Fatigue Life	α=12.777 β=4.3847Ε-5
14	Fatigue Life (3P)	α=1173.3 β=5.3860Ε-9 γ=7.6800Ε-8

15	Frechet	α=0.52833 β=0.00172
16	Frechet (3P)	α=0.71995 β=0.00325 γ=-2.8081Ε-7
17	Gamma	α=2.9557 β=0.00238
18	Gamma (3P)	α=6.452 β=0.00157 γ=-0.00312
19	Gen. Extreme Value	k=0.06035 σ=0.00298 μ=0.00511
20	Gen. Gamma	k=0.84573 α=2.3683 β=0.00238
21	Gen. Gamma (4P)	k=1.1286 α=4.493 β=0.00256 γ=-0.00259
22	Gen. Pareto	k=-0.3077 σ=0.00661 μ=0.00196
23	Gumbel Max	σ=0.00318 μ=0.00518
24	Gumbel Min	σ=0.00318 μ=0.00886
25	Hypersecant	σ=0.00408 μ=0.00702
26	Inv. Gaussian	λ=0.02075 μ=0.00702
27	Inv. Gaussian (3P)	λ=0.00327 μ=0.00708 γ=6.9990Ε-8
28	Johnson SB	γ=3.6165 δ=1.925 $ λ=0.06417 ξ=-0.00219$
29	Kumaraswamy	α <sub>1</sub> =1.9544 α <sub>2</sub> =431.51 a=-6.6282E-4 b=0.19197
30	Laplace	λ=346.29 μ=0.00702
31	Levy	σ=0.00223
32	Levy (2P)	σ=0.00223 γ=6.9783Ε-8
33	Log-Logistic	α=0.89205 β=0.00497
34	Log-Logistic (3P)	α=5.2621 β=0.01094 γ=-0.00461
35	Log-Pearson 3	α=0.14311 β=-3.6689 γ=-4.7649
36	Logistic	σ=0.00225 μ=0.00702
37	Lognormal	σ=1.3817 μ=-5.29
38	Lognormal (3P)	σ=0.29015 μ=-4.3439 γ=-0.00653
39	Nakagami	m=0.70049 Ω=6.5826E-5
40	Normal	σ=0.00408 μ=0.00702
41	Pareto	α=0.09015 β=7.6800Ε-8
42	Pareto 2	α=533.25 β=3.7407
43	Pearson 5	α=0.66664 β=0.00149
44	Pearson 5 (3P)	α=0.66651 β=0.00149 γ=7.6800Ε-8

45	Pearson 6	$\alpha_1$ =1.7792 $\alpha_2$ =2.4876E+8 $\beta$ =9.5788E+5
46	Pearson 6 (4P)	$\alpha_1$ =6.4521 $\alpha_2$ =4.6033E+5 $\beta$ =723.16 $\gamma$ =-0.00311
47	Pert	m=0.00449 a=-3.6765E-4 b=0.02537
48	Power Function	α=0.54942 a=7.6800E-8 b=0.02236
49	Rayleigh	σ=0.0056
50	Rayleigh (2P)	σ=0.00621 γ=-7.5158Ε-4
51	Reciprocal	a=7.6762E-8 b=0.02237
52	Rice	v=4.8417E-5 σ=0.00574
53	Triangular	m=6.9886E-8 a=5.5302E-8 b=0.02282
54	Uniform	a=-5.2383E-5 b=0.01409
55	Weibull	α=0.69206 β=0.01119
56	Weibull (3P)	α=1.9595 β=0.00866 γ=-6.6614Ε-4
57	Chi-Squared	No fit
58	Chi-Squared (2P)	No fit
59	Johnson SU	No fit
60	Log-Gamma	No fit
61	Student's t	No fit

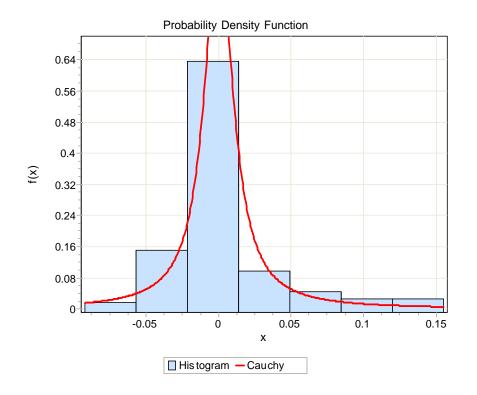
## **Goodness of Fit - Summary**

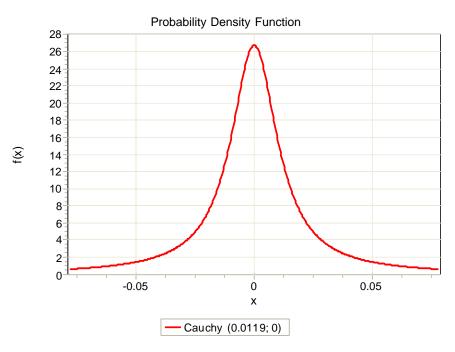
#	<u>Distribution</u>	Kolmogorov Smirnov		Anderson Darling		Chi-Squared	
		Statistic	Rank	Statistic	Rank	Statistic	Rank
1	Beta	0.13619	11	1.7021	7	18.027	12
2	<u>Burr</u>	0.19857	30	3.3979	25	36.052	24
3	Burr (4P)	0.14459	17	2.2436	13	16.227	9
4	Cauchy	0.1803	25	5.0117	27	41.019	27
5	<u>Dagum</u>	0.2023	31	3.0583	18	37.276	25
6	Dagum (4P)	0.47048	51	33.444	49	227.48	48
7	Erlang	0.3815	46	26.62	47	44.751	29
8	Erlang (3P)	0.19407	28	3.2937	24	20.774	19
9	<u>Error</u>	0.14986	18	3.2659	22	43.08	28

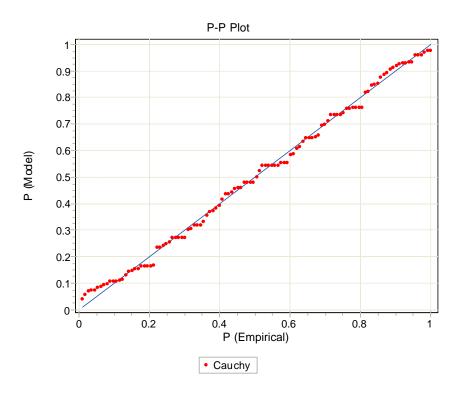
10	Error Function	0.74689	55	151.99	56	374.55	50
11	<u>Exponential</u>	0.3366	40	10.783	32	77.474	41
12	Exponential (2P)	0.3366	39	10.924	34	77.472	40
13	Fatigue Life	0.69496	54	76.252	53	224.16	47
14	Fatigue Life (3P)	0.69381	53	75.815	52	223.13	46
15	Frechet	0.44456	49	25.113	45	116.41	43
16	Frechet (3P)	0.33037	38	20.486	40	N/A	
17	<u>Gamma</u>	0.13831	14	3.2909	23	12.313	3
18	Gamma (3P)	0.1349	10	1.6947	6	19.286	16
19	Gen. Extreme Value	0.12238	5	1.6614	4	14.769	5
20	Gen. Gamma	0.24415	34	5.776	29	10.444	1
21	Gen. Gamma (4P)	0.13915	15	1.7223	8	18.048	13
22	Gen. Pareto	0.16132	22	27.907	48	N/A	
23	Gumbel Max	0.12831	6	1.5185	2	16.371	10
24	Gumbel Min	0.19247	26	10.48	31	45.046	30
25	Hypersecant	0.14416	16	3.1158	20	39.983	26
26	Inv. Gaussian	0.10724	1	19.624	39	13.298	4
27	Inv. Gaussian (3P)	0.44473	50	26.408	46	98.936	42
28	Johnson SB	0.13827	13	1.8586	9	15.195	6
29	Kumaraswamy	0.1577	21	2.1072	11	15.99	8
30	Laplace	0.17257	23	4.1023	26	30.894	23
31	Levy	0.36682	44	21.591	41	70.13	38
32	Levy (2P)	0.3672	45	21.601	42	70.087	37
33	Log-Logistic	0.36442	43	17.247	38	145.23	45
34	Log-Logistic (3P)	0.11579	3	1.3423	1	21.917	21
35	Log-Pearson 3	0.30957	37	138.38	55	N/A	
36	Logistic	0.1312	8	2.8657	14	55.192	36
37	Lognormal	0.33935	42	13.865	36	47.505	33
38	Lognormal (3P)	0.13093	7	1.5879	3	19.798	17
39	<u>Nakagami</u>	0.19503	29	3.1315	21	29.624	22
40	Normal	0.13696	12	2.9265	17	45.776	32
41	<u>Pareto</u>	0.55805	52	46.007	51	123.93	44
42	Pareto 2	0.33673	41	10.797	33	75.164	39
43	Pearson 5	0.41021	47	22.87	43	50.586	34

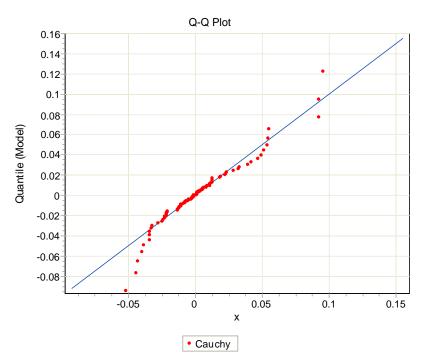
44	Pearson 5 (3P)	0.41029	48	24.698	44	N/A	
45	Pearson 6	0.23922	33	5.3076	28	11.616	2
46	Pearson 6 (4P)	0.13489	9	1.6947	5	19.286	15
47	<u>Pert</u>	0.17967	24	2.9223	16	21.096	20
48	Power Function	0.30281	36	11.654	35	45.347	31
49	Rayleigh	0.12183	4	2.8839	15	19.007	14
50	Rayleigh (2P)	0.15338	19	2.0907	10	16.821	11
51	Reciprocal	0.79177	56	124.03	54	233.1	49
52	Rice	0.11407	2	3.1001	19	N/A	
53	<u>Triangular</u>	0.22634	32	6.002	30	20.265	18
54	<u>Uniform</u>	0.19398	27	33.674	50	N/A	
55	Weibull Weibull	0.30239	35	15.604	37	52.031	35
56	Weibull (3P)	0.15653	20	2.1077	12	15.986	7
57	Chi-Squared	No fit					
58	Chi-Squared (2P)	No fit					
59	Johnson SU	No fit					
60	Log-Gamma	No fit					
61	Student's t	No fit					

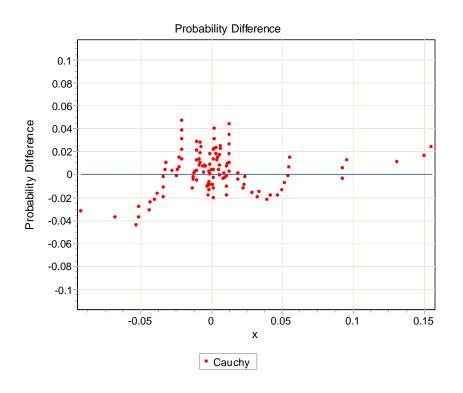
### RoF (constant variance)











## **Fitting Results**

#	Distribution	Parameters
1	Beta	α <sub>1</sub> =12.502 α <sub>2</sub> =1.2174E+7 a=-0.12515 b=1.2603E+5
2	Burr (4P)	k=0.57999 α=25.318 β=0.32672 γ=-0.33777
3	Cauchy	σ=0.0119 μ=0
4	Dagum (4P)	k=0.95291 α=9.2154 β=0.15418 γ=-0.15349
5	Erlang (3P)	m=16 β=0.00871 γ=-0.13922
6	Error	k=1.0 σ=0.03724 μ=0.00401
7	Error Function	h=18.989
8	Exponential (2P)	λ=10.394 γ=-0.0922
9	Fatigue Life (3P)	α=0.18589 β=0.18579 γ=-0.18499
10	Frechet (3P)	α=3.0351 β=0.08688 γ=-0.1022
11	Gamma (3P)	α=16.437 β=0.00871 γ=-0.13922
12	Gen. Extreme Value	k=0.02033 σ=0.02594 μ=-0.0115
13	Gen. Gamma (4P)	k=0.81113 α=25.323 β=0.00267 γ=-0.14032

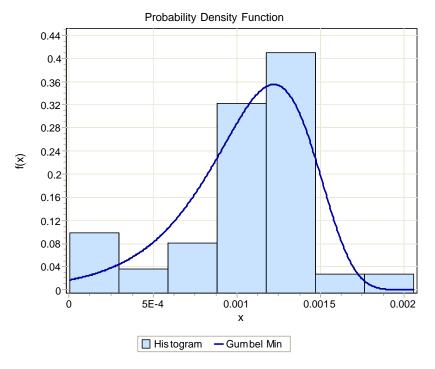
14	Gen. Pareto	k=-0.38107 σ=0.06027 μ=-0.03963
15	Gumbel Max	$\sigma$ =0.02903 $\mu$ =-0.01275
16	Gumbel Min	σ=0.02903 μ=0.02077
17	Hypersecant	σ=0.03724 μ=0.00401
18	Inv. Gaussian (3P)	λ=5.4272 μ=0.18905 γ=-0.18504
10	IIIv. Gaussian (31)	
19	Johnson SU	$\gamma$ =-3.0373 $\delta$ =2.0391 $\lambda$ =0.02745 $\xi$ =-0.06114
20	Kumaraswamy	α <sub>1</sub> =2.7138 α <sub>2</sub> =2254.1 a=-0.09753 b=1.8515
21	Laplace	λ=37.977 μ=0.00401
22	Levy (2P)	σ=0.07963 γ=-0.09731
23	Log-Logistic (3P)	$\alpha = 9.4135 \beta = 0.159 \gamma = -0.15941$
24	Logistic	σ=0.02053 μ=0.00401
25	Lognormal (3P)	σ=0.19572 μ=-1.7438 γ=-0.1743
26	Normal	σ=0.03724 μ=0.00401
27	Pearson 5 (3P)	α=41.435 β=8.9399 γ=-0.21718
28	Pearson 6 (4P)	$\alpha_1$ =138.15 $\alpha_2$ =42.697 $\beta$ =0.05948 $\gamma$ =-0.19317
29	Pert	m=-0.00839 a=-0.09489 b=0.17383
30	Power Function	α=0.76433 a=-0.0922 b=0.15506
31	Rayleigh (2P)	σ=0.07381 γ=-0.09356
32	Triangular	m=-0.00703 a=-0.09542 b=0.15931
33	Uniform	a=-0.06049 b=0.06851
34	Weibull (3P)	α=2.7198 β=0.11335 γ=-0.09751
35	Burr	No fit (data min < 0)
36	Chi-Squared	No fit (data min < 0)
37	Chi-Squared (2P)	No fit
38	Dagum	No fit (data min < 0)
39	Erlang	No fit (data min < 0)
40	Exponential	No fit (data min < 0)
41	Fatigue Life	No fit (data min < 0)
42	Frechet	No fit (data min < 0)
43	Gamma	No fit (data min < 0)
44	Gen. Gamma	No fit (data min < 0)

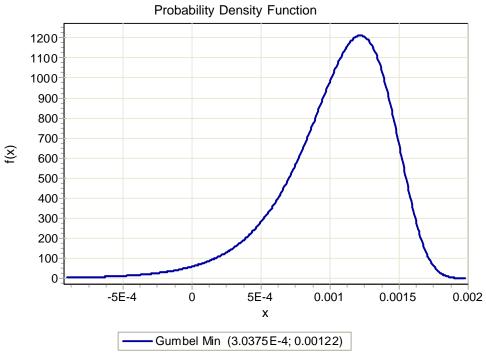
45	Inv. Gaussian	No fit (data min < 0)
46	Johnson SB	No fit
47	Levy	No fit (data min < 0)
48	Log-Gamma	No fit
49	Log-Logistic	No fit (data min < 0)
50	Log-Pearson 3	No fit
51	Lognormal	No fit (data min < 0)
52	Nakagami	No fit
53	Pareto	No fit
54	Pareto 2	No fit
55	Pearson 5	No fit (data min < 0)
56	Pearson 6	No fit (data min < 0)
57	Rayleigh	No fit (data min < 0)
58	Reciprocal	No fit
59	Rice	No fit
60	Student's t	No fit
61	Weibull	No fit (data min < 0)

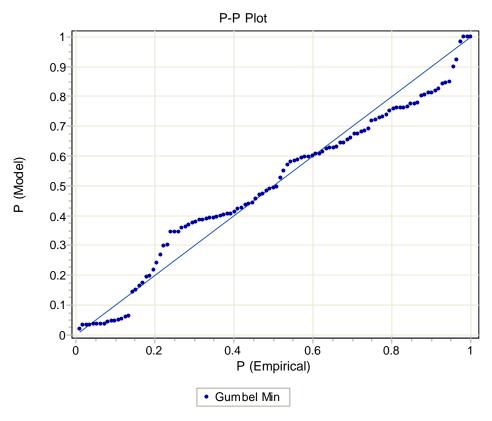
#	<u>Distribution</u>	Kolmogorov Smirnov		Anderson Darling		Chi-Squared	
		Statistic	Rank	Statistic	Rank	Statistic	Rank
1	<u>Beta</u>	0.17932	19	3.6901	17	32.945	19
2	Burr (4P)	0.13024	4	1.5143	2	17.896	4
3	Cauchy	0.05277	1	0.44318	1	4.1846	1
4	Dagum (4P)	0.1253	2	1.641	3	20.817	5

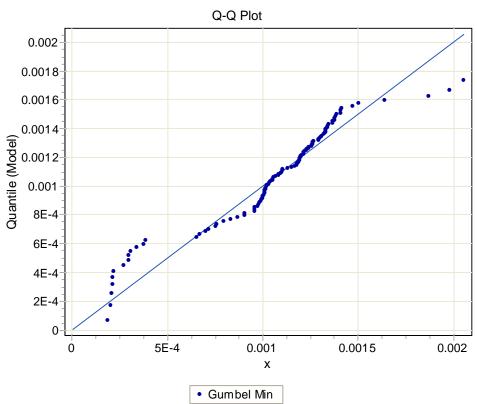
5	Erlang (3P)	0.16407	10	3.8315	18	34.157	20
6	Error	0.16363	9	2.6807	7	11.205	2
7	Error Function	0.17101	15	4.7394	21	37.674	21
8	Exponential (2P)	0.37607	33	24.543	31	193.14	29
9	Fatigue Life (3P)	0.17365	16	3.3312	13	32.553	15
10	Frechet (3P)	0.1818	20	9.3232	26	N/A	
11	Gamma (3P)	0.17776	18	3.4808	16	29.575	11
12	Gen. Extreme Value	0.1321	5	2.66	5	21.236	7
13	Gen. Gamma (4P)	0.177	17	3.4247	15	29.704	12
14	Gen. Pareto	0.16819	13	33.441	33	N/A	
15	Gumbel Max	0.14921	6	3.1552	11	28.764	10
16	Gumbel Min	0.2732	30	15.189	29	45.68	23
17	Hypersecant	0.19012	21	3.3661	14	24.031	8
18	Inv. Gaussian (3P)	0.20929	25	4.1076	20	30.839	13
19	Johnson SU	0.15503	7	3.0957	8	31.071	14
20	Kumaraswamy	0.19995	22	5.6266	24	48.175	25
21	<u>Laplace</u>	0.16363	8	2.6807	6	11.205	3
22	Levy (2P)	0.47724	34	35.118	34	449.58	31
23	Log-Logistic (3P)	0.12615	3	1.6423	4	20.819	6
24	Logistic	0.20008	23	3.903	19	27.903	9
25	Lognormal (3P)	0.17016	14	3.2227	12	32.561	16
26	Normal	0.21219	26	5.0263	22	41.902	22
27	Pearson 5 (3P)	0.16694	11	3.1171	9	32.573	18
28	Pearson 6 (4P)	0.16732	12	3.1349	10	32.572	17
29	<u>Pert</u>	0.24127	29	9.1724	25	104.86	28
30	Power Function	0.28924	31	19.157	30	210.15	30
31	Rayleigh (2P)	0.23439	27	9.4484	27	71.81	26
32	<u>Triangular</u>	0.31201	32	12.46	28	83.589	27
33	<u>Uniform</u>	0.2373	28	30.243	32	N/A	
34	Weibull (3P)	0.20056	24	5.6137	23	48.174	24
35	Burr	No fit (d	ata min	< 0)			
36	Chi-Squared	No fit (data min < 0)					
37	Chi-Squared (2P)	No fit					
38	Dagum	No fit (d	ata min	< 0)			
	ı						

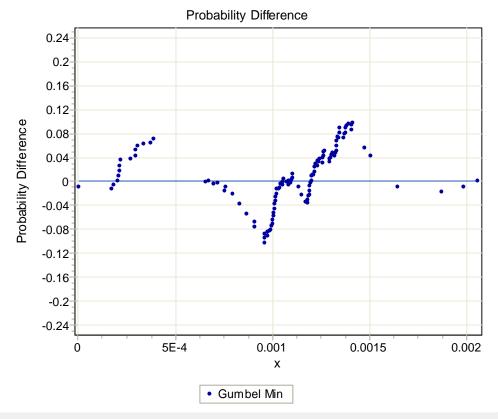
39	Erlang	No fit (data min < 0)
40	Exponential	No fit (data min < 0)
41	Fatigue Life	No fit (data min < 0)
42	Frechet	No fit (data min < 0)
43	Gamma	No fit (data min < 0)
44	Gen. Gamma	No fit (data min < 0)
45	Inv. Gaussian	No fit (data min < 0)
46	Johnson SB	No fit
47	Levy	No fit (data min < 0)
48	Log-Gamma	No fit
49	Log-Logistic	No fit (data min < 0)
50	Log-Pearson 3	No fit
51	Lognormal	No fit (data min < 0)
52	Nakagami	No fit
53	Pareto	No fit
54	Pareto 2	No fit
55	Pearson 5	No fit (data min < 0)
56	Pearson 6	No fit (data min < 0)
57	Rayleigh	No fit (data min < 0)
58	Reciprocal	No fit
59	Rice	No fit
60	Student's t	No fit
61	Weibull	No fit (data min < 0)











#	Distribution	Parameters
1	Beta	α <sub>1</sub> =3.1835E+6 α <sub>2</sub> =26.902 a=-234.15 b=0.00302
2	Burr	k=826.91 α=2.8147 β=0.01257
3	Burr (4P)	k=4763.3 α=7.0124 β=0.00819 γ=-0.00125
4	Cauchy	σ=1.7254Ε-4 μ=0.00115
5	Dagum	k=0.14447 α=14.233 β=0.00144
6	Dagum (4P)	k=0.20062 α=0.71406 β=0.76732 γ=3.2736E-6
7	Erlang	m=7 β=1.4513E-4
8	Erlang (3P)	m=137 β=3.4487E-5 γ=-0.00367
9	Error	k=1.4764 σ=3.8957E-4 μ=0.00105
10	Error Function	h=1815.1
11	Exponential	λ=956.25
12	Exponential (2P)	λ=959.25 γ=3.2736E-6

Fatigue Life (3P)   α=0.00746   β=0.05257   γ=-0.05152	13	Fatigue Life	α=1.4715 β=4.2378Ε-4
15   Frechet	_		<u> </u>
Gamma   α=1.4374 β=6.9818E-4 γ=-3.9867E-6     Gamma   α=7.2058 β=1.4513E-4     Gamma   α=10.75 β=3.8408E-5 γ=-0.00317     Gen. Extreme Value   k=-0.65708 σ=4.1118E-4 μ=9.8393E-4     Gen. Gamma   k=0.83419 α=5.0766 β=1.4513E-4     Gen. Gamma   k=0.83419 α=5.0766 β=1.4513E-4     Gen. Gamma   (4P)   k=16.713 α=2.5695 β=0.00901 γ=-0.00838     Gen. Pareto   k=-1.9331 σ=0.00238 μ=2.3429E-4     Gumbel Max   σ=3.0375E-4 μ=8.7042E-4     Gumbel Min   σ=3.0375E-4 μ=0.00102     Hypersecant   σ=3.8957E-4 μ=0.00105     Inv. Gaussian   λ=0.00754 μ=0.00105 γ=2.9065E-6     Inv. Gaussian   λ=0.00286 μ=0.00105 γ=2.9065E-6     Johnson SB   γ=6.0898 δ=3.0675 λ=0.0109 ξ=0.00849     Kumaraswamy   α <sub>1</sub> =7.0013 α <sub>2</sub> =412.97 a=-0.00124 b=0.00453     Levy   σ=0.00477     Levy   σ=0.00477     Levy   σ=7.7568E-4 γ=3.0014E-6     33			
17 Gamma       α=7.2058 β=1.4513E-4         18 Gamma (3P)       α=109.75 β=3.8408E-5 γ=0.00317         19 Gen. Extreme Value       k=-0.65708 σ=4.1118E-4 μ=9.8393E-4         20 Gen. Gamma       k=0.83419 α=5.0766 β=1.4513E-4         21 Gen. Gamma (4P)       k=16.713 α=2.5695 β=0.00901 γ=-0.00838         22 Gen. Pareto       k=-1.9331 σ=0.00238 μ=2.3429E-4         23 Gumbel Max       σ=3.0375E-4 μ=8.7042E-4         24 Gumbel Min       σ=3.0375E-4 μ=0.00122         25 Hypersecant       σ=3.8957E-4 μ=0.00105         26 Inv. Gaussian       λ=0.00754 μ=0.00105         27 Inv. Gaussian (3P)       λ=0.00286 μ=0.00105 γ=2.9065E-6         28 Johnson SB       γ=-6.0898 δ=3.0675 λ=0.0109 ξ=-0.00849         29 Kumaraswamy       α <sub>1</sub> =7.0013 α <sub>2</sub> =412.97 a=-0.00124 b=0.00453         30 Laplace       λ=3630.2 μ=0.00105         31 Levy       σ=0.00477         32 Levy (2P)       σ=7.7568E-4 γ=3.0014E-6         33 Log-Logistic       α=1.7208 β=8.9410E-4         34 Log-Pearson 3       α=0.2222 β=-1.6166 γ=-6.6531         35 Log-Pearson 3       α=0.2222 β=-1.6166 γ=-6.6531         36 Logistic       σ=2.1478E-4 μ=0.00105         37 Lognormal       σ=0.75862 μ=-7.0123         38 Lognormal (3P)       σ=0.02919 μ=-4.3075 γ=-0.01243         39 Nakag	-		<u> </u>
18   Gamma (3P)   α=109.75   β=3.8408E-5   γ=-0.00317     19   Gen. Extreme Value   k=-0.65708   σ=4.1118E-4   μ=9.8393E-4     20   Gen. Gamma   k=0.83419   α=5.0766   β=1.4513E-4     21   Gen. Gamma (4P)   k=16.713   α=2.5695   β=0.00901   γ=-0.00838     22   Gen. Pareto   k=-1.9331   σ=0.00238   μ=2.3429E-4     23   Gumbel Max   σ=3.0375E-4   μ=8.7042E-4     24   Gumbel Min   σ=3.0375E-4   μ=0.00122     25   Hypersecant   σ=3.8957E-4   μ=0.00105     26   Inv. Gaussian   λ=0.00754   μ=0.00105     27   Inv. Gaussian (3P)   λ=0.00286   μ=0.00105   γ=2.9065E-6     28   Johnson SB   γ=-6.0898   δ=3.0675   λ=0.0109   ξ=-0.00849     29   Kumaraswamy   α <sub>1</sub> =7.0013   α <sub>2</sub> =412.97   α=-0.00124   b=0.00453     30   Laplace   λ=3630.2   μ=0.00105     31   Levy   σ=0.00477     32   Levy (2P)   σ=7.7568E-4   γ=3.0014E-6     33   Log-Logistic   α=1.7208   β=8.9410E-4     44   Log-Logistic (3P)   α=2.7712E+8   β=57773.0   γ=-57773.0     35   Log-Pearson 3   α=0.2222   β=-1.6166   γ=-6.6531     36   Logistic   σ=2.1478E-4   μ=0.00105     37   Lognormal   σ=0.75862   μ=7.0123     38   Lognormal   σ=0.75862   μ=7.0123     39   Nakagami   m=2.8105   Ω=1.2440E-6     40   Normal   σ=3.8957E-4   μ=0.00105     41   Pareto   α=0.17802   β=3.2736E-6	_	<u> </u>	
Gen. Extreme Value   k=-0.65708 σ=4.1118E-4 μ=9.8393E-4     Gen. Gamma   k=0.83419 α=5.0766 β=1.4513E-4     Gen. Gamma (4P)   k=16.713 α=2.5695 β=0.00901 γ=-0.00838     Gen. Pareto   k=-1.9331 σ=0.00238 μ=2.3429E-4     Gumbel Max   σ=3.0375E-4 μ=8.7042E-4     Gumbel Min   σ=3.0375E-4 μ=0.00105     Inv. Gaussian   λ=0.00754 μ=0.00105     Inv. Gaussian (3P)   λ=0.00286 μ=0.00105 γ=2.9065E-6     Inv. Gaussian (3P)   λ=0.00286 μ=0.00105 γ=2.9065E-6     Johnson SB   γ=-6.0898 δ=3.0675 λ=0.0109 ξ=-0.00849     Kumaraswamy   α <sub>1</sub> =7.0013 α <sub>2</sub> =412.97 a=-0.00124 b=0.00453     Laplace   λ=3630.2 μ=0.00105     Levy   σ=0.00477     Levy   σ=0.00477     Log-Logistic (3P)   α=2.7712E+8 β=57773.0 γ=-57773.0     Log-Pearson 3   α=0.2222 β=-1.6166 γ=-6.6531     Logistic   σ=2.1478E-4 μ=0.00105     Lognormal   σ=0.75862 μ=-7.0123     Rognormal (3P)   σ=0.02919 μ=-4.3075 γ=0.01243     Nakagami   m=2.8105 Ω=1.2440E-6     O Normal   σ=3.8957E-4 μ=0.00105     α=0.17802 β=3.2736E-6	17		α=7.2058 β=1.4513Ε-4
20 Gen. Gamma	18	Gamma (3P)	α=109.75 β=3.8408Ε-5 γ=-0.00317
Gen. Gamma (4P)	19	Gen. Extreme Value	k=-0.65708 σ=4.1118E-4 μ=9.8393E-4
β=0.00901 γ=-0.00838     β=0.00901 γ=-0.00838     β=0.00238 μ=2.3429E-4     β=0.00238 μ=2.3429E-4     β=0.00238 μ=2.3429E-4     β=0.00128     β=0.00105     β=0.00105     β=0.00105     β=0.00105     β=0.00105     β=0.00105     β=0.00849     β=0.00108     β=0.00105     β=0.00105     β=0.00849     β=0.00105     β=0.00105     β=0.00124     β=0.00105     β=0.00105	20	Gen. Gamma	k=0.83419 α=5.0766 β=1.4513E-4
Gumbel Max   G=3.0375E-4   µ=8.7042E-4	21	Gen. Gamma (4P)	
24 Gumbel Min	22	Gen. Pareto	k=-1.9331 σ=0.00238 μ=2.3429E-4
Second	23	Gumbel Max	σ=3.0375Ε-4 μ=8.7042Ε-4
26 Inv. Gaussian $\lambda$ =0.00754 $\mu$ =0.00105 27 Inv. Gaussian (3P) $\lambda$ =0.00286 $\mu$ =0.00105 $\gamma$ =2.9065E-6 28 Johnson SB $\gamma$ =-6.0898 $\delta$ =3.0675 $\lambda$ =0.0109 $\xi$ =-0.00849 29 Kumaraswamy $\alpha_1$ =7.0013 $\alpha_2$ =412.97 $\alpha$ =-0.00124 $\alpha$ =-0.00125 30 Laplace $\alpha$ =3630.2 $\alpha$ =0.00477 31 Levy $\alpha$ =0.00477 32 Levy (2P) $\alpha$ =7.7568E-4 $\alpha$ =3.0014E-6 33 Log-Logistic $\alpha$ =1.7208 $\alpha$ =8.9410E-4 34 Log-Logistic (3P) $\alpha$ =2.7712E+8 $\alpha$ =57773.0 $\alpha$ =-57773.0 35 Log-Pearson 3 $\alpha$ =0.2222 $\alpha$ =-1.6166 $\alpha$ =-6.6531 36 Logistic $\alpha$ =2.1478E-4 $\alpha$ =0.00105 37 Lognormal $\alpha$ =0.75862 $\alpha$ =-7.0123 38 Lognormal (3P) $\alpha$ =0.02919 $\alpha$ =-4.3075 $\alpha$ =-0.01243 39 Nakagami $\alpha$ =2.8105 $\alpha$ =1.2440E-6 40 Normal $\alpha$ =3.8957E-4 $\alpha$ =0.00105 41 Pareto $\alpha$ =0.17802 $\alpha$ =3.2736E-6	24	Gumbel Min	σ=3.0375Ε-4 μ=0.00122
27 Inv. Gaussian (3P) $\lambda$ =0.00286 $\mu$ =0.00105 $\gamma$ =2.9065E-6  28 Johnson SB $\gamma$ =-6.0898 $\gamma$ =-0.00849  29 Kumaraswamy $\gamma$ =-0.00124 $\gamma$ =-0.00124 $\gamma$ =-0.00125  30 Laplace $\gamma$ =-0.00124 $\gamma$ =-0.00105  31 Levy $\gamma$ =-0.00477  32 Levy (2P) $\gamma$ =-7.7568E-4 $\gamma$ =-3.0014E-6  33 Log-Logistic $\gamma$ =-5.7773.0 $\gamma$ =-5.7773.0  35 Log-Pearson 3 $\gamma$ =-2.712E+8 $\gamma$ =-7.66531  36 Logistic $\gamma$ =-2.1478E-4 $\gamma$ =-0.00105  37 Lognormal $\gamma$ =-7.75862 $\gamma$ =-7.0123  38 Lognormal $\gamma$ =-0.02919 $\gamma$ =-4.3075 $\gamma$ =-0.01243  39 Nakagami $\gamma$ =-2.8105 $\gamma$ =-1.2440E-6  40 Normal $\gamma$ =-3.8957E-4 $\gamma$ =-0.00105  41 Pareto $\gamma$ =-0.17802 $\gamma$ =-3.2736E-6	25	Hypersecant	σ=3.8957Ε-4 μ=0.00105
28 Johnson SB $\gamma=-6.0898$ δ=3.0675 $\lambda=0.0109$ ξ=-0.00849  29 Kumaraswamy $\alpha_1=7.0013$ $\alpha_2=412.97$ $\alpha=-0.00124$ b=0.00453  30 Laplace $\lambda=3630.2$ $\mu=0.00105$ 31 Levy $\sigma=0.00477$ 32 Levy (2P) $\sigma=7.7568E-4$ $\gamma=3.0014E-6$ 33 Log-Logistic $\alpha=1.7208$ β=8.9410E-4  34 Log-Logistic (3P) $\alpha=2.7712E+8$ β=57773.0 $\gamma=-57773.0$ 35 Log-Pearson 3 $\alpha=0.2222$ β=-1.6166 $\gamma=-6.6531$ 36 Logistic $\sigma=2.1478E-4$ $\mu=0.00105$ 37 Lognormal $\sigma=0.75862$ $\mu=-7.0123$ 38 Lognormal (3P) $\sigma=0.02919$ $\mu=-4.3075$ $\gamma=-0.01243$ 39 Nakagami $m=2.8105$ $\Omega=1.2440E-6$ 40 Normal $\sigma=3.8957E-4$ $\mu=0.00105$	26	Inv. Gaussian	λ=0.00754 μ=0.00105
28 Johnson SB $\lambda$ =0.0109 ξ=-0.00849  29 Kumaraswamy $\alpha_1$ =7.0013 $\alpha_2$ =412.97 $\alpha$ =-0.00124 b=0.00453  30 Laplace $\lambda$ =3630.2 $\mu$ =0.00105  31 Levy $\sigma$ =0.00477  32 Levy (2P) $\sigma$ =7.7568E-4 $\gamma$ =3.0014E-6  33 Log-Logistic $\alpha$ =1.7208 $\beta$ =8.9410E-4  34 Log-Logistic (3P) $\alpha$ =2.7712E+8 $\beta$ =57773.0 $\gamma$ =-57773.0  35 Log-Pearson 3 $\alpha$ =0.2222 $\beta$ =-1.6166 $\gamma$ =-6.6531  36 Logistic $\sigma$ =2.1478E-4 $\mu$ =0.00105  37 Lognormal $\sigma$ =0.75862 $\mu$ =-7.0123  38 Lognormal (3P) $\sigma$ =0.02919 $\mu$ =-4.3075 $\gamma$ =-0.01243  39 Nakagami $\sigma$ =2.8105 $\Omega$ =1.2440E-6  40 Normal $\sigma$ =3.8957E-4 $\mu$ =0.00105  41 Pareto $\alpha$ =0.17802 $\beta$ =3.2736E-6	27	Inv. Gaussian (3P)	λ=0.00286 μ=0.00105 γ=2.9065Ε-6
Rumaraswamy   $a=-0.00124$   $b=0.00453$   $b=0.00453$   $b=0.00105$   $b=0.00477$   $b=0.00477$   $b=0.00477$   $b=0.00477$   $b=0.00486$   $b=0.00486$   $b=0.00486$   $b=0.00487$   $b=0.00486$   $b=0.00487$   $b=0.00486$   $b=0.00487$   $b=0.00486$   $b=0.00487$   $b=0.00487$	28	Johnson SB	·
31 Levy $\sigma=0.00477$ 32 Levy (2P) $\sigma=7.7568E-4 \ \gamma=3.0014E-6$ 33 Log-Logistic $\alpha=1.7208 \ \beta=8.9410E-4$ 34 Log-Logistic (3P) $\alpha=2.7712E+8 \ \beta=57773.0 \ \gamma=-57773.0$ 35 Log-Pearson 3 $\alpha=0.2222 \ \beta=-1.6166 \ \gamma=-6.6531$ 36 Logistic $\sigma=2.1478E-4 \ \mu=0.00105$ 37 Lognormal $\sigma=0.75862 \ \mu=-7.0123$ 38 Lognormal (3P) $\sigma=0.02919 \ \mu=-4.3075 \ \gamma=-0.01243$ 39 Nakagami $m=2.8105 \ \Omega=1.2440E-6$ 40 Normal $\sigma=3.8957E-4 \ \mu=0.00105$ 41 Pareto $\alpha=0.17802 \ \beta=3.2736E-6$	29	Kumaraswamy	
32 Levy (2P) $\sigma=7.7568E-4 \ \gamma=3.0014E-6$ 33 Log-Logistic $\alpha=1.7208 \ \beta=8.9410E-4$ 34 Log-Logistic (3P) $\alpha=2.7712E+8 \ \beta=57773.0 \ \gamma=-57773.0$ 35 Log-Pearson 3 $\alpha=0.2222 \ \beta=-1.6166 \ \gamma=-6.6531$ 36 Logistic $\sigma=2.1478E-4 \ \mu=0.00105$ 37 Lognormal $\sigma=0.75862 \ \mu=-7.0123$ 38 Lognormal (3P) $\sigma=0.02919 \ \mu=-4.3075 \ \gamma=-0.01243$ 39 Nakagami $m=2.8105 \ \Omega=1.2440E-6$ 40 Normal $\sigma=3.8957E-4 \ \mu=0.00105$ 41 Pareto $\alpha=0.17802 \ \beta=3.2736E-6$	30	Laplace	λ=3630.2 μ=0.00105
33 Log-Logistic $\alpha$ =1.7208 $\beta$ =8.9410E-4  34 Log-Logistic (3P) $\alpha$ =2.7712E+8 $\beta$ =57773.0 $\gamma$ =-57773.0  35 Log-Pearson 3 $\alpha$ =0.2222 $\beta$ =-1.6166 $\gamma$ =-6.6531  36 Logistic $\sigma$ =2.1478E-4 $\mu$ =0.00105  37 Lognormal $\sigma$ =0.75862 $\mu$ =-7.0123  38 Lognormal (3P) $\sigma$ =0.02919 $\mu$ =-4.3075 $\gamma$ =-0.01243  39 Nakagami $\sigma$ =2.8105 $\Omega$ =1.2440E-6  40 Normal $\sigma$ =3.8957E-4 $\mu$ =0.00105  41 Pareto $\sigma$ =0.17802 $\sigma$ =3.2736E-6	31	Levy	σ=0.00477
34 Log-Logistic (3P) $\alpha$ =2.7712E+8 $\beta$ =57773.0 $\gamma$ =-57773.0 35 Log-Pearson 3 $\alpha$ =0.2222 $\beta$ =-1.6166 $\gamma$ =-6.6531 36 Logistic $\sigma$ =2.1478E-4 $\mu$ =0.00105 37 Lognormal $\sigma$ =0.75862 $\mu$ =-7.0123 38 Lognormal (3P) $\sigma$ =0.02919 $\mu$ =-4.3075 $\gamma$ =-0.01243 39 Nakagami $\sigma$ =2.8105 $\Omega$ =1.2440E-6 40 Normal $\sigma$ =3.8957E-4 $\mu$ =0.00105 41 Pareto $\alpha$ =0.17802 $\beta$ =3.2736E-6	32	Levy (2P)	σ=7.7568Ε-4 γ=3.0014Ε-6
35 Log-Pearson 3 $\alpha$ =0.2222 β=-1.6166 γ=-6.6531 36 Logistic $\sigma$ =2.1478E-4 $\mu$ =0.00105 37 Lognormal $\sigma$ =0.75862 $\mu$ =-7.0123 38 Lognormal (3P) $\sigma$ =0.02919 $\mu$ =-4.3075 γ=-0.01243 39 Nakagami $\sigma$ =2.8105 $\Omega$ =1.2440E-6 40 Normal $\sigma$ =3.8957E-4 $\mu$ =0.00105 41 Pareto $\sigma$ =0.17802 $\sigma$ =3.2736E-6	33	Log-Logistic	α=1.7208 β=8.9410Ε-4
36 Logistic σ=2.1478E-4 μ=0.00105  37 Lognormal σ=0.75862 μ=-7.0123  38 Lognormal (3P) σ=0.02919 μ=-4.3075 γ=-0.01243  39 Nakagami m=2.8105 Ω=1.2440E-6  40 Normal σ=3.8957E-4 μ=0.00105  41 Pareto α=0.17802 β=3.2736E-6	34	Log-Logistic (3P)	α=2.7712E+8 β=57773.0 γ=-57773.0
37 Lognormal σ=0.75862 μ=-7.0123 38 Lognormal (3P) σ=0.02919 μ=-4.3075 γ=-0.01243 39 Nakagami m=2.8105 Ω=1.2440E-6 40 Normal σ=3.8957E-4 μ=0.00105 41 Pareto α=0.17802 β=3.2736E-6	35	Log-Pearson 3	$\alpha$ =0.2222 $\beta$ =-1.6166 $\gamma$ =-6.6531
38 Lognormal (3P) σ=0.02919 μ=-4.3075 γ=-0.01243 39 Nakagami m=2.8105 Ω=1.2440E-6 40 Normal σ=3.8957E-4 μ=0.00105 41 Pareto α=0.17802 β=3.2736E-6	36	Logistic	σ=2.1478Ε-4 μ=0.00105
39 Nakagami m=2.8105 Ω=1.2440E-6 40 Normal σ=3.8957E-4 μ=0.00105 41 Pareto α=0.17802 β=3.2736E-6	37	Lognormal	σ=0.75862 μ=-7.0123
40 Normal σ=3.8957E-4 μ=0.00105 41 Pareto α=0.17802 β=3.2736E-6	38	Lognormal (3P)	$\sigma$ =0.02919 μ=-4.3075 γ=-0.01243
41 Pareto α=0.17802 β=3.2736E-6	39	Nakagami	m=2.8105 Ω=1.2440E-6
	40	Normal	σ=3.8957Ε-4 μ=0.00105
42 Pareto 2 α=1518.9 β=1.588	41	Pareto	α=0.17802 β=3.2736Ε-6
	42	Pareto 2	α=1518.9 β=1.588

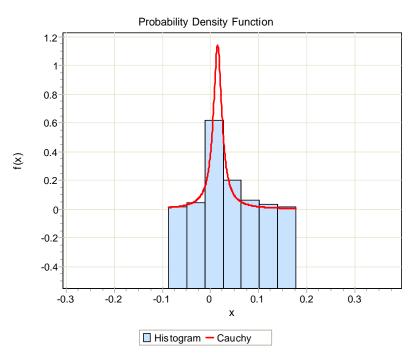
43	Pearson 5	α=3.0402 β=0.00242
44	Pearson 5 (3P)	α=2.6532 β=0.00206 γ=4.8790Ε-7
45	Pearson 6	$\alpha_1$ =3.371 $\alpha_2$ =1.2966E+8 $\beta$ =40478.0
46	Pearson 6 (4P)	$\alpha_1$ =116.41 $\alpha_2$ =11995.0 $\beta$ =0.45593 $\gamma$ =-0.0034
47	Pert	m=0.00107 a=-1.7454E-4 b=0.00213
48	Power Function	α=1.2254 a=-3.2308E-6 b=0.00206
49	Rayleigh	σ=8.3439E-4
50	Rayleigh (2P)	σ=8.1059Ε-4 γ=2.2666Ε-6
51	Reciprocal	a=3.2559E-6 b=0.00207
52	Rice	v=9.5791E-4 σ=4.0440E-4
53	Triangular	m=0.0012 a=2.6598E-6 b=0.00209
54	Uniform	a=3.7099E-4 b=0.00172
55	Weibull	α=1.3492 β=0.00136
56	Weibull (3P)	α=7.0179 β=0.00245 γ=-0.00125
57	Chi-Squared	No fit
58	Chi-Squared (2P)	No fit
59	Johnson SU	No fit
60	Log-Gamma	No fit
61	Student's t	No fit

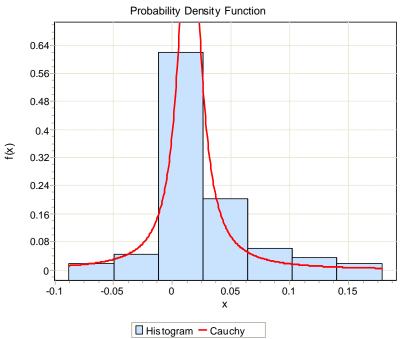
#	<u>Distribution</u>	Kolmogorov Smirnov		Anderson Darling		Chi-Squared	
		Statistic	Rank	Statistic	Rank	Statistic	Rank
1	Beta	0.15355	10	3.2575	9	21.624	6
2	<u>Burr</u>	0.21303	27	6.3768	23	42.658	23

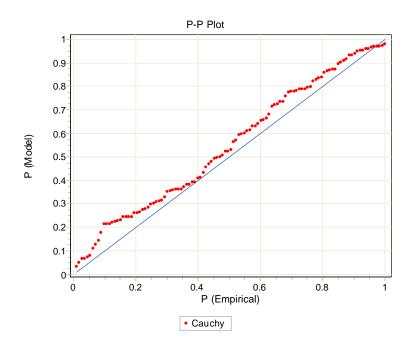
3	Burr (4P)	0.14988	9	3.0351	7	19.375	3
4	Cauchy	0.1315	4	2.8612	3	26.385	10
5	<u>Dagum</u>	0.20059	25	4.5297	16	29.562	13
6	Dagum (4P)	0.57324	53	45.779	51	N/A	
7	Erlang	0.2573	34	12.006	31	52.92	28
8	Erlang (3P)	0.17884	19	4.8618	18	38.481	22
9	Error	0.16333	14	3.8579	12	32.601	16
10	Error Function	0.82013	55	403.38	56	483.91	47
11	<u>Exponential</u>	0.36781	49	21.426	43	262.89	43
12	Exponential (2P)	0.36771	47	21.524	45	261.4	42
13	Fatigue Life	0.48743	50	41.268	49	66.357	31
14	Fatigue Life (3P)	0.17362	17	4.3841	14	30.871	15
15	Frechet	0.35551	46	21.673	46	199.21	40
16	Frechet (3P)	0.30006	40	19.506	41	N/A	
17	<u>Gamma</u>	0.22558	29	10.597	29	50.529	27
18	Gamma (3P)	0.19736	23	5.2926	19	25.561	9
19	Gen. Extreme Value	0.11311	2	17.365	38	N/A	
20	Gen. Gamma	0.28258	37	10.693	30	96.725	34
21	Gen. Gamma (4P)	0.14476	6	2.9053	4	20.058	5
22	Gen. Pareto	0.16727	16	34.371	48	N/A	
23	Gumbel Max	0.24057	31	13.504	33	53.653	29
24	Gumbel Min	0.11129	1	2.247	1	24.655	7
25	Hypersecant	0.15763	13	3.7177	10	27.963	12
26	Inv. Gaussian	0.24488	32	20.132	42	57.749	30
27	Inv. Gaussian (3P)	0.31571	44	16.351	37	132.18	36
28	Johnson SB	0.13463	5	2.7768	2	18.986	1
29	Kumaraswamy	0.14745	7	3.0107	5	19.111	2
30	<u>Laplace</u>	0.15548	11	3.8244	11	27.712	11
31	Levy	0.88928	56	247.49	55	1318.8	48
32	Levy (2P)	0.49182	51	31.598	47	420.15	46
33	Log-Logistic	0.29756	39	15.312	35	203.52	41
34	Log-Logistic (3P)	0.1187	3	3.1711	8	24.811	8
35	Log-Pearson 3	0.17546	18	98.818	53	N/A	
36	Logistic	0.16723	15	3.8733	13	35.118	19

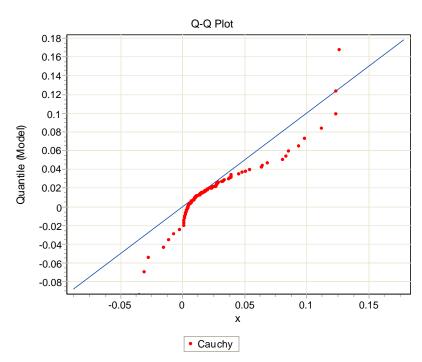
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37	Lognormal	0.30027	41	13.58	34	157.59	38
38	Lognormal (3P)	0.18852	21	4.8441	17	35.609	20
39	<u>Nakagami</u>	0.1575	12	8.9043	26	33.471	18
40	<u>Normal</u>	0.17885	20	4.4261	15	32.798	17
41	<u>Pareto</u>	0.49686	52	44.572	50	93.809	33
42	Pareto 2	0.36781	48	21.429	44	262.93	44
43	Pearson 5	0.31396	43	17.811	40	152.33	37
44	Pearson 5 (3P)	0.31684	45	17.567	39	161.11	39
45	Pearson 6	0.27356	36	10.102	28	89.46	32
46	Pearson 6 (4P)	0.21502	28	5.7657	22	43.515	25
47	<u>Pert</u>	0.19848	24	5.4825	21	44.207	26
48	Power Function	0.31324	42	15.89	36	N/A	
49	Rayleigh	0.25062	33	8.5839	25	42.763	24
50	Rayleigh (2P)	0.2689	35	9.1746	27	36.943	21
51	Reciprocal	0.68796	54	116.03	54	398.54	45
52	Rice	0.22791	30	6.794	24	N/A	
53	Triangular	0.19375	22	5.3189	20	30.068	14
54	<u>Uniform</u>	0.20291	26	52.268	52	N/A	
55	Weibull	0.29316	38	13.373	32	111.01	35
56	Weibull (3P)	0.14984	8	3.0343	6	19.376	4
57	Chi-Squared	No fit					
58	Chi-Squared (2P)	No fit					
59	Johnson SU	No fit					
60	Log-Gamma	No fit					
61	Student's t	No fit					

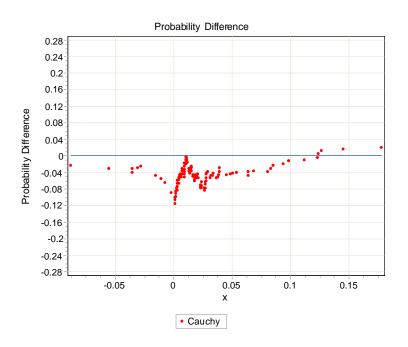
#### FS (constant-variance)











#	Distribution	Parameters
1	Beta	α <sub>1</sub> =24.359 α <sub>2</sub> =6.0720E+6 a=-0.15453 b=44556.0
2	Burr (4P)	k=0.50763 α=4.1916E+6 $β$ =54415.0 $γ$ =-54415.0
3	Cauchy	σ=0.01062 μ=0.0147
4	Dagum (4P)	k=1.1974 α=11.795 β=0.20023 γ=-0.18521
5	Erlang (3P)	m=28 β=0.00681 γ=-0.16447
6	Error	k=1.0 σ=0.03733 μ=0.02415
7	Error Function	h=18.945
8	Exponential (2P)	λ=8.9191 γ=-0.08797
9	Fatigue Life (3P)	α=0.14362 β=0.24591 γ=-0.2243
10	Frechet (3P)	α=4.115 β=0.11875 γ=-0.11207
11	Gamma (3P)	α=27.701 β=0.00681 γ=-0.16447
12	Gen. Extreme Value	k=0.12158 σ=0.02289 μ=0.00783
13	Gen. Gamma (4P)	k=0.89365 α=33.778 β=0.00361 γ=-0.16163

14         Gen. Pareto         k=-0.19877 σ=0.04742 μ=-0.0154           15         Gumbel Max         σ=0.0291 μ=0.00735           16         Gumbel Min         σ=0.0291 μ=0.04095           17         Hypersecant         σ=0.03733 μ=0.02415           18         Inv. Gaussian (3P)         λ=11.982 μ=0.24843 γ=-0.22428           19         Johnson SU         γ=-1.9533 δ=1.9963 λ=0.04162 ξ=-0.02975           20         Kumaraswamy         α <sub>1</sub> =3.148 α <sub>2</sub> =1064.5 a=-0.09626 b=1.1262           21         Laplace         λ=37.889 μ=0.02415           22         Levy (2P)         σ=0.09587 γ=-0.09407           23         Log-Logistic (3P)         α=10.82 β=0.17746 γ=-0.1585           24         Logistic         σ=0.09258 μ=0.02415           25         Lognormal (3P)         σ=0.15257 μ=-1.4695 γ=-0.20863           26         Normal         σ=0.03733 μ=0.02415           27         Pearson 5 (3P)         α=66.604 β=18.815 γ=-0.26271           28         Pearson 6 (4P)         α <sub>1</sub> =312.64 α <sub>2</sub> =67.633 β=0.05593 γ=-0.23833           29         Pert         m=0.01606 a=-0.09165 b=0.18934           30         Power Function         α=0.85816 a=-0.08797 b=0.17835           31         Rayleigh (2P)         σ=0.08444 γ=-0.08933			I
16         Gumbel Min $\sigma$ =0.0291 μ=0.04095           17         Hypersecant $\sigma$ =0.03733 μ=0.02415           18         Inv. Gaussian (3P) $\lambda$ =11.982 μ=0.24843 γ=-0.22428           19         Johnson SU $\gamma$ =-1.9533 δ=1.9963 $\lambda$ =0.04162 $\xi$ =-0.02975           20         Kumaraswamy $\alpha_1$ =3.148 $\alpha_2$ =1064.5 a=-0.09626 b=1.1262           21         Laplace $\lambda$ =37.889 μ=0.02415           22         Levy (2P) $\sigma$ =0.09587 γ=-0.09407           23         Log-Logistic (3P) $\alpha$ =10.82 β=0.17746 γ=-0.1585           24         Logistic $\sigma$ =0.02058 μ=0.02415           25         Lognormal (3P) $\sigma$ =0.15257 μ=-1.4695 γ=-0.20863           26         Normal $\sigma$ =0.03733 μ=0.02415           27         Pearson 5 (3P) $\alpha$ =66.604 β=18.815 γ=-0.26271           28         Pearson 6 (4P) $\alpha$ 1=312.64 $\alpha$ 2=67.633 β=0.05593 γ=-0.23833           29         Pert $m$ =0.01606 a=-0.09165 b=0.18934           30         Power Function $\alpha$ =0.85816 a=-0.08797 b=0.17835           31         Rayleigh (2P) $\sigma$ =0.08444 γ=-0.08933           32         Triangular $m$ =0.01092 a=-0.09082 b=0.18177           33         Uniform $\alpha$ =-3.1576 β	14	Gen. Pareto	k=-0.19877 σ=0.04742 μ=-0.0154
To Hypersecant	15	Gumbel Max	σ=0.0291 μ=0.00735
18 Inv. Gaussian (3P)	16	Gumbel Min	σ=0.0291 μ=0.04095
Johnson SU   $\gamma=-1.9533$ δ=1.9963  \( \lambda=0.04162 \) \( \xi=-0.02975 \)	17	Hypersecant	σ=0.03733 μ=0.02415
λ=0.04162 ξ=-0.02975     λ=0.04162 ξ=-0.02975     λ=0.09626 b=1.1262     Laplace   λ=37.889 μ=0.02415     22 Levy (2P)   σ=0.09587 γ=-0.09407     23 Log-Logistic (3P)   α=10.82 β=0.17746 γ=-0.1585     24 Logistic   σ=0.02058 μ=0.02415     25 Lognormal (3P)   σ=0.15257 μ=-1.4695 γ=-0.20863     26 Normal   σ=0.03733 μ=0.02415     27 Pearson 5 (3P)   α=66.604 β=18.815 γ=-0.26271     28 Pearson 6 (4P)   α <sub>1</sub> =312.64 α <sub>2</sub> =67.633     β=0.05593 γ=-0.23833     29 Pert   m=0.01606 a=-0.09165 b=0.18934     30 Power Function   α=0.85816 a=-0.08797 b=0.17835     31 Rayleigh (2P)   σ=0.08444 γ=-0.08933     32 Triangular   m=0.01092 a=-0.09082 b=0.18177     33 Uniform   a=-0.0405 b=0.0888     34 Weibull (3P)   α=3.1576 β=0.13327 γ=-0.09618     35 Burr   No fit (data min < 0)     36 Chi-Squared   No fit (data min < 0)     37 Chi-Squared (2P)   No fit     38 Dagum   No fit (data min < 0)     40 Exponential   No fit (data min < 0)     41 Fatigue Life   No fit (data min < 0)     42 Frechet   No fit (data min < 0)     43 Gamma   No fit (data min < 0)     44 Fatigue Life   No fit (data min < 0)     45 Gamma   No fit (data min < 0)     46 Fatigue Life   No fit (data min < 0)     47 Fatigue Life   No fit (data min < 0)     48 Gamma   No fit (data min < 0)     49 Frechet   No fit (data min < 0)     40 Frechet   No fit (data min < 0)     41 Fatigue Life   No fit (data min < 0)     42 Frechet   No fit (data min < 0)     43 Gamma   No fit (data min < 0)	18	Inv. Gaussian (3P)	λ=11.982 μ=0.24843 γ=-0.22428
20         Rumaswamy         a=-0.09626 b=1.1262           21         Laplace         λ=37.889 μ=0.02415           22         Levy (2P)         σ=0.09587 γ=-0.09407           23         Log-Logistic (3P)         α=10.82 β=0.17746 γ=-0.1585           24         Logistic         σ=0.02058 μ=0.02415           25         Lognormal (3P)         σ=0.15257 μ=-1.4695 γ=-0.20863           26         Normal         σ=0.03733 μ=0.02415           27         Pearson 5 (3P)         α=66.604 β=18.815 γ=-0.26271           28         Pearson 6 (4P)         α <sub>1</sub> =312.64 α <sub>2</sub> =67.633 β=0.05593 γ=-0.23833           29         Pert         m=0.01606 a=-0.09165 b=0.18934           30         Power Function         α=0.85816 a=-0.08797 b=0.17835           31         Rayleigh (2P)         σ=0.08444 γ=-0.08933           32         Triangular         m=0.01092 a=-0.09082 b=0.18177           33         Uniform         a=-0.0405 b=0.0888           34         Weibull (3P)         α=3.1576 β=0.13327 γ=-0.09618           35         Burr         No fit (data min < 0)	19	Johnson SU	1 -
22 Levy (2P)	20	Kumaraswamy	
23 Log-Logistic (3P) $\alpha$ =10.82 $\beta$ =0.17746 $\gamma$ =-0.1585 24 Logistic $\sigma$ =0.02058 $\mu$ =0.02415 25 Lognormal (3P) $\sigma$ =0.15257 $\mu$ =-1.4695 $\gamma$ =-0.20863 26 Normal $\sigma$ =0.03733 $\mu$ =0.02415 27 Pearson 5 (3P) $\sigma$ =66.604 $\sigma$ =18.815 $\sigma$ =-0.26271 28 Pearson 6 (4P) $\sigma$ =12.64 $\sigma$ =67.633 $\sigma$ =0.05593 $\sigma$ =-0.23833 29 Pert $\sigma$ =0.01606 $\sigma$ =-0.09165 $\sigma$ =0.18934 30 Power Function $\sigma$ =0.85816 $\sigma$ =-0.08797 $\sigma$ =0.17835 31 Rayleigh (2P) $\sigma$ =0.08444 $\sigma$ =-0.08933 32 Triangular $\sigma$ =0.01092 $\sigma$ =-0.09082 $\sigma$ =0.18177 33 Uniform $\sigma$ =0.0405 $\sigma$ =0.13327 $\sigma$ =-0.09618 35 Burr No fit (data min < 0) 36 Chi-Squared No fit (data min < 0) 37 Chi-Squared (2P) No fit 38 Dagum No fit (data min < 0) 40 Exponential No fit (data min < 0) 41 Fatigue Life No fit (data min < 0) 43 Gamma No fit (data min < 0)	21	Laplace	λ=37.889 μ=0.02415
24 Logistic $\sigma$ =0.02058 μ=0.02415  25 Lognormal (3P) $\sigma$ =0.15257 μ=-1.4695 γ=-0.20863  26 Normal $\sigma$ =0.03733 μ=0.02415  27 Pearson 5 (3P) $\sigma$ =66.604 $\sigma$ =18.815 γ=-0.26271  28 Pearson 6 (4P) $\sigma$ =12.64 $\sigma$ =67.633 $\sigma$ =0.05593 γ=-0.23833  29 Pert $\sigma$ =0.01606 $\sigma$ =-0.09165 $\sigma$ =0.18934  30 Power Function $\sigma$ =0.85816 $\sigma$ =-0.08797 $\sigma$ =0.17835  31 Rayleigh (2P) $\sigma$ =0.08444 γ=-0.08933  32 Triangular $\sigma$ =0.01092 $\sigma$ =-0.09082 $\sigma$ =0.18177  33 Uniform $\sigma$ =0.0405 $\sigma$ =0.13327 γ=-0.09618  35 Burr No fit (data min < 0)  36 Chi-Squared No fit (data min < 0)  37 Chi-Squared (2P) No fit  38 Dagum No fit (data min < 0)  39 Erlang No fit (data min < 0)  40 Exponential No fit (data min < 0)  41 Fatigue Life No fit (data min < 0)  42 Frechet No fit (data min < 0)  43 Gamma No fit (data min < 0)	22	Levy (2P)	σ=0.09587 γ=-0.09407
25 Lognormal (3P) $\sigma$ =0.15257 $\mu$ =-1.4695 $\gamma$ =-0.20863 26 Normal $\sigma$ =0.03733 $\mu$ =0.02415 27 Pearson 5 (3P) $\sigma$ =66.604 $\sigma$ =18.815 $\sigma$ =-0.26271 28 Pearson 6 (4P) $\sigma$ =12.64 $\sigma$ =67.633 $\sigma$ =0.05593 $\sigma$ =-0.23833 29 Pert $\sigma$ =0.01606 $\sigma$ =-0.09165 $\sigma$ =0.18934 30 Power Function $\sigma$ =0.85816 $\sigma$ =-0.08797 $\sigma$ =0.17835 31 Rayleigh (2P) $\sigma$ =0.08444 $\sigma$ =-0.08933 32 Triangular $\sigma$ =0.01092 $\sigma$ =-0.09082 $\sigma$ =0.18177 33 Uniform $\sigma$ =-0.0405 $\sigma$ =0.13327 $\sigma$ =-0.09618 35 Burr No fit (data min < 0) 36 Chi-Squared No fit (data min < 0) 37 Chi-Squared (2P) No fit 38 Dagum No fit (data min < 0) 39 Erlang No fit (data min < 0) 40 Exponential No fit (data min < 0) 41 Fatigue Life No fit (data min < 0) 42 Frechet No fit (data min < 0) 43 Gamma No fit (data min < 0)	23	Log-Logistic (3P)	α=10.82 β=0.17746 γ=-0.1585
26         Normal         σ=0.03733 μ=0.02415           27         Pearson 5 (3P)         α=66.604 β=18.815 γ=-0.26271           28         Pearson 6 (4P)         α <sub>1</sub> =312.64 α <sub>2</sub> =67.633 β=0.05593 γ=-0.23833           29         Pert         m=0.01606 a=-0.09165 b=0.18934           30         Power Function         α=0.85816 a=-0.08797 b=0.17835           31         Rayleigh (2P)         σ=0.08444 γ=-0.08933           32         Triangular         m=0.01092 a=-0.09082 b=0.18177           33         Uniform         a=-0.0405 b=0.0888           34         Weibull (3P)         α=3.1576 β=0.13327 γ=-0.09618           35         Burr         No fit (data min < 0)	24	Logistic	σ=0.02058 μ=0.02415
27       Pearson 5 (3P)       α=66.604 β=18.815 γ=-0.26271         28       Pearson 6 (4P)       α <sub>1</sub> =312.64 α <sub>2</sub> =67.633 β=0.05593 γ=-0.23833         29       Pert       m=0.01606 a=-0.09165 b=0.18934         30       Power Function       α=0.85816 a=-0.08797 b=0.17835         31       Rayleigh (2P)       σ=0.08444 γ=-0.08933         32       Triangular       m=0.01092 a=-0.09082 b=0.18177         33       Uniform       a=-0.0405 b=0.0888         34       Weibull (3P)       α=3.1576 β=0.13327 γ=-0.09618         35       Burr       No fit (data min < 0)	25	Lognormal (3P)	σ=0.15257 μ=-1.4695 γ=-0.20863
28       Pearson 6 (4P) $α_1 = 312.64$ $α_2 = 67.633$ $β = 0.05593$ $γ = -0.23833$ 29       Pert $m = 0.01606$ $a = -0.09165$ $b = 0.18934$ 30       Power Function $α = 0.85816$ $a = -0.08797$ $b = 0.17835$ 31       Rayleigh (2P) $σ = 0.08444$ $γ = -0.08933$ 32       Triangular $m = 0.01092$ $a = -0.09082$ $b = 0.18177$ 33       Uniform $a = -0.0405$ $b = 0.0888$ 34       Weibull (3P) $α = 3.1576$ $β = 0.13327$ $γ = -0.09618$ 35       Burr       No fit (data min < 0)	26	Normal	σ=0.03733 μ=0.02415
28       Pearson 6 (4P) $\beta$ =0.05593 $\gamma$ =-0.23833         29       Pert       m=0.01606 a=-0.09165 b=0.18934         30       Power Function $\alpha$ =0.85816 a=-0.08797 b=0.17835         31       Rayleigh (2P) $\sigma$ =0.08444 $\gamma$ =-0.08933         32       Triangular       m=0.01092 a=-0.09082 b=0.18177         33       Uniform       a=-0.0405 b=0.0888         34       Weibull (3P) $\alpha$ =3.1576 $\beta$ =0.13327 $\gamma$ =-0.09618         35       Burr       No fit (data min < 0)	27	Pearson 5 (3P)	α=66.604 β=18.815 γ=-0.26271
30         Power Function         α=0.85816 a=-0.08797 b=0.17835           31         Rayleigh (2P)         σ=0.08444 γ=-0.08933           32         Triangular         m=0.01092 a=-0.09082 b=0.18177           33         Uniform         a=-0.0405 b=0.0888           34         Weibull (3P)         α=3.1576 β=0.13327 γ=-0.09618           35         Burr         No fit (data min < 0)	28	Pearson 6 (4P)	
31       Rayleigh (2P)       σ=0.08444 γ=-0.08933         32       Triangular       m=0.01092 a=-0.09082 b=0.18177         33       Uniform       a=-0.0405 b=0.0888         34       Weibull (3P)       α=3.1576 β=0.13327 γ=-0.09618         35       Burr       No fit (data min < 0)	29	Pert	m=0.01606 a=-0.09165 b=0.18934
32 Triangular m=0.01092 a=-0.09082 b=0.18177 33 Uniform a=-0.0405 b=0.0888 34 Weibull (3P) α=3.1576 β=0.13327 γ=-0.09618 35 Burr No fit (data min < 0) 36 Chi-Squared No fit (data min < 0) 37 Chi-Squared (2P) No fit 38 Dagum No fit (data min < 0) 39 Erlang No fit (data min < 0) 40 Exponential No fit (data min < 0) 41 Fatigue Life No fit (data min < 0) 42 Frechet No fit (data min < 0) 43 Gamma No fit (data min < 0)	30	Power Function	α=0.85816 a=-0.08797 b=0.17835
33 Uniform a=-0.0405 b=0.0888  34 Weibull (3P) α=3.1576 β=0.13327 γ=-0.09618  35 Burr No fit (data min < 0)  36 Chi-Squared No fit (data min < 0)  37 Chi-Squared (2P) No fit  38 Dagum No fit (data min < 0)  39 Erlang No fit (data min < 0)  40 Exponential No fit (data min < 0)  41 Fatigue Life No fit (data min < 0)  42 Frechet No fit (data min < 0)  43 Gamma No fit (data min < 0)	31	Rayleigh (2P)	σ=0.08444 γ=-0.08933
34       Weibull (3P) $\alpha$ =3.1576 β=0.13327 γ=-0.09618         35       Burr       No fit (data min < 0)	32	Triangular	m=0.01092 a=-0.09082 b=0.18177
35         Burr         No fit (data min < 0)	33	Uniform	a=-0.0405 b=0.0888
36         Chi-Squared         No fit (data min < 0)	34	Weibull (3P)	α=3.1576 β=0.13327 γ=-0.09618
37 Chi-Squared (2P)       No fit         38 Dagum       No fit (data min < 0)	35	Burr	No fit (data min < 0)
38 Dagum No fit (data min < 0) 39 Erlang No fit (data min < 0) 40 Exponential No fit (data min < 0) 41 Fatigue Life No fit (data min < 0) 42 Frechet No fit (data min < 0) 43 Gamma No fit (data min < 0)	36	Chi-Squared	No fit (data min < 0)
39 Erlang No fit (data min < 0) 40 Exponential No fit (data min < 0) 41 Fatigue Life No fit (data min < 0) 42 Frechet No fit (data min < 0) 43 Gamma No fit (data min < 0)	37	Chi-Squared (2P)	No fit
40 Exponential No fit (data min < 0) 41 Fatigue Life No fit (data min < 0) 42 Frechet No fit (data min < 0) 43 Gamma No fit (data min < 0)	38	Dagum	No fit (data min < 0)
41 Fatigue Life No fit (data min < 0)  42 Frechet No fit (data min < 0)  43 Gamma No fit (data min < 0)	39	Erlang	No fit (data min < 0)
42 Frechet No fit (data min < 0) 43 Gamma No fit (data min < 0)	40	Exponential	No fit (data min < 0)
43 Gamma No fit (data min < 0)	41	Fatigue Life	No fit (data min < 0)
	42	Frechet	No fit (data min < 0)
44   Gen. Gamma   No fit (data min < 0)	43	Gamma	No fit (data min < 0)
	44	Gen. Gamma	No fit (data min < 0)

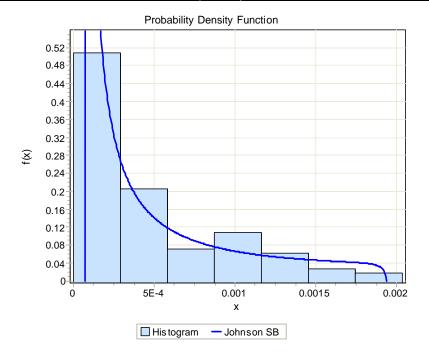
45	Inv. Gaussian	No fit (data min < 0)
46	Johnson SB	No fit
47	Levy	No fit (data min < 0)
48	Log-Gamma	No fit
49	Log-Logistic	No fit (data min < 0)
50	Log-Pearson 3	No fit
51	Lognormal	No fit (data min < 0)
52	Nakagami	No fit
53	Pareto	No fit
54	Pareto 2	No fit
55	Pearson 5	No fit (data min < 0)
56	Pearson 6	No fit (data min < 0)
57	Rayleigh	No fit (data min < 0)
58	Reciprocal	No fit
59	Rice	No fit
60	Student's t	No fit
61	Weibull	No fit (data min < 0)

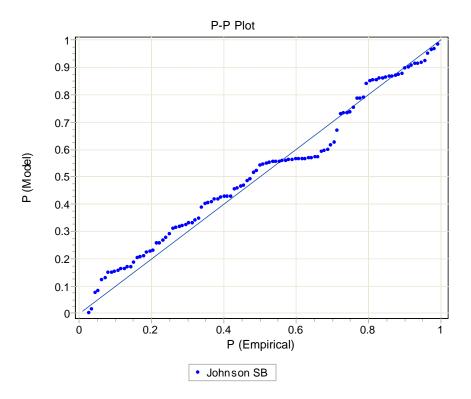
#	<u>Distribution</u>	Kolmog Smirr		Ander Darli		Chi-Squ	<u>iared</u>
		Statistic	Rank	Statistic	Rank	Statistic	Rank
1	Beta	0.18749	14	6.0022	16	65.507	24
2	Burr (4P)	0.14126	2	3.4075	4	21.088	3
3	Cauchy	0.12432	1	2.3221	1	11.158	1
4	Dagum (4P)	0.1495	3	3.2656	2	20.042	2
5	Erlang (3P)	0.20298	19	6.4782	18	52.982	15

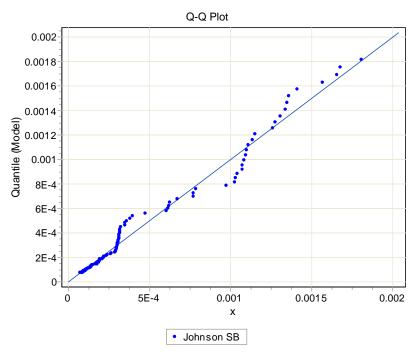
6	<u>Error</u>	0.18138	6	5.4814	8	38.582	7
7	Error Function	0.42494	32	25.435	30	72.846	26
8	Exponential (2P)	0.46029	33	28.647	32	307.42	30
9	Fatigue Life (3P)	0.18514	9	5.8198	13	51.877	14
10	Frechet (3P)	0.20903	22	10.414	24	N/A	
11	Gamma (3P)	0.18489	8	5.9751	15	60.296	21
12	Gen. Extreme Value	0.17344	5	7.572	21	25.232	6
13	Gen. Gamma (4P)	0.18584	13	5.8839	14	57.324	20
14	Gen. Pareto	0.21677	23	28.028	31	N/A	
15	Gumbel Max	0.20292	18	5.1465	6	21.102	4
16	Gumbel Min	0.27389	28	17.702	28	62.951	22
17	<u>Hypersecant</u>	0.19394	16	5.8064	12	40.035	9
18	Inv. Gaussian (3P)	0.20551	20	6.4974	19	52.994	16
19	Johnson SU	0.19323	15	4.4268	5	40.593	10
20	Kumaraswamy	0.22194	24	8.2487	23	41.802	12
21	<u>Laplace</u>	0.18138	7	5.4814	7	38.582	8
22	Levy (2P)	0.47234	34	38.512	33	629.37	31
23	Log-Logistic (3P)	0.15436	4	3.3249	3	23.127	5
24	<u>Logistic</u>	0.20004	17	6.2912	17	46.309	13
25	Lognormal (3P)	0.18547	11	5.7109	11	55.603	19
26	Normal	0.20727	21	7.3325	20	66.434	25
27	Pearson 5 (3P)	0.1856	12	5.5943	9	54.127	18
28	Pearson 6 (4P)	0.18536	10	5.6005	10	54.126	17
29	<u>Pert</u>	0.25427	27	11.761	25	108.52	27
30	Power Function	0.30973	30	21.197	29	157.53	29
31	Rayleigh (2P)	0.34907	31	14.254	27	63.314	23
32	<u>Triangular</u>	0.27598	29	12.874	26	137.08	28
33	<u>Uniform</u>	0.23444	26	39.802	34	N/A	
34	Weibull (3P)	0.22216	25	8.2008	22	41.8	11
35	Burr	No fit (data min < 0)					
36	Chi-Squared	No fit (data min < 0)					
37	Chi-Squared (2P)	No fit					
38	Dagum	No fit (d	ata min	(0)			
39	Erlang	No fit (d	ata min	(0)			

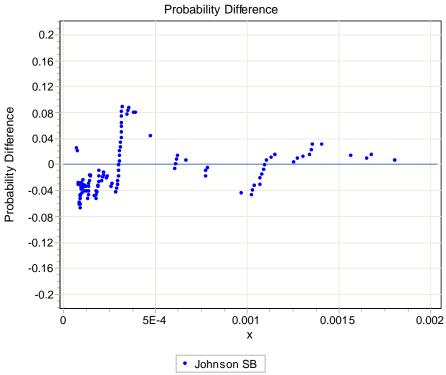
40	Exponential	No fit (data min < 0)
41	Fatigue Life	No fit (data min < 0)
42	Frechet	No fit (data min < 0)
43	Gamma	No fit (data min < 0)
44	Gen. Gamma	No fit (data min < 0)
45	Inv. Gaussian	No fit (data min < 0)
46	Johnson SB	No fit
47	Levy	No fit (data min < 0)
48	Log-Gamma	No fit
49	Log-Logistic	No fit (data min < 0)
50	Log-Pearson 3	No fit
51	Lognormal	No fit (data min < 0)
52	Nakagami	No fit
53	Pareto	No fit
54	Pareto 2	No fit
55	Pearson 5	No fit (data min < 0)
56	Pearson 6	No fit (data min < 0)
57	Rayleigh	No fit (data min < 0)
58	Reciprocal	No fit
59	Rice	No fit
60	Student's t	No fit
61	Weibull	No fit (data min < 0)

#### FS (ewma)









#	Distribution	Parameters	

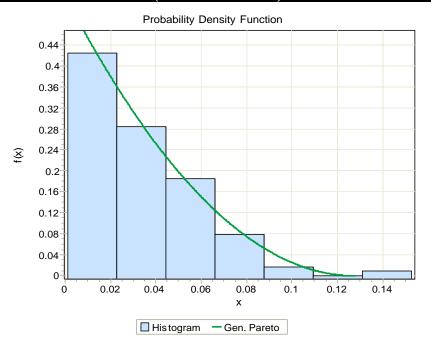
1	Beta	α <sub>1</sub> =0.51173 α <sub>2</sub> =1.7268 a=9.7960E-7 b=0.00245
2	Burr	k=0.63705 α=2.053 β=1.8827E-4
3	Burr (4P)	k=312.91 α=0.45005 β=231.87 γ=9.7960E-7
4	Cauchy	σ=1.3447Ε-4 μ=2.0096Ε-4
5	Dagum	k=1.3296 α=1.5584 β=2.1310E-4
6	Dagum (4P)	k=1.9299 α=1.4653 β=1.5212E-4 γ=-2.1798E-6
7	Error	k=1.4407 σ=4.7527E-4 μ=4.6658E-4
8	Error Function	h=1487.8
9	Exponential	λ=2143.3
10	Exponential (2P)	λ=2147.8 γ=9.7960Ε-7
11	Fatigue Life	α=1.7761 β=1.5799Ε-4
12	Fatigue Life (3P)	α=0.97108 β=3.3663Ε-4 γ=-2.5317Ε-5
13	Frechet	α=1.0244 β=1.6067Ε-4
14	Frechet (3P)	α=1.2931 β=1.8865Ε-4 γ=9.7960Ε-7
15	Gamma	α=0.96376 β=4.8412E-4
16	Gamma (3P)	α=1.1273 β=4.1359Ε-4 γ=3.4929Ε-7
17	Gen. Extreme Value	k=0.34148 σ=2.2326E-4 μ=2.2541E-4
18	Gen. Gamma	k=1.0399 α=1.0114 β=4.8412E-4
19	Gen. Gamma (4P)	k=0.31984 α=9.8994 β=2.5857E-7 γ=-2.0173E-6
20	Gen. Pareto	k=0.16139 σ=3.6852E-4 μ=2.7141E-5
21	Gumbel Max	σ=3.7057Ε-4 μ=2.5268Ε-4
22	Gumbel Min	σ=3.7057Ε-4 μ=6.8047Ε-4
23	Hypersecant	σ=4.7527Ε-4 μ=4.6658Ε-4
24	Inv. Gaussian	λ=4.4967Ε-4 μ=4.6658Ε-4
25	Inv. Gaussian (3P)	λ=3.5687Ε-4 μ=4.7353Ε-4 γ=8.8791Ε-7
26	Johnson SB	γ=1.0215 δ=0.44789 $ λ=0.00186 ξ=7.4402E-5$
27	Kumaraswamy	α <sub>1</sub> =0.95679 α <sub>2</sub> =11.015 a=9.7960E-7 b=0.00607
28	Laplace	λ=2975.6 μ=4.6658Ε-4
29	Levy	σ=0.8994

30	Levy (2P)	σ=0.89815 γ=8.7590Ε-7
31	Log-Logistic	α=1.4932 β=2.7793Ε-4
32	Log-Logistic (3P)	α=1.7291 β=2.8057Ε-4 γ=-3.5864Ε-6
33	Log-Pearson 3	α=4.7126 β=-0.501 γ=-5.8094
34	Logistic	σ=2.6203Ε-4 μ=4.6658Ε-4
35	Lognormal	σ=1.0827 μ=-8.1704
36	Lognormal (3P)	σ=0.94574 μ=-8.0871 γ=-1.2948Ε-5
37	Normal	σ=4.7527Ε-4 μ=4.6658Ε-4
38	Pareto	α=0.1765 β=9.7960E-7
39	Pareto 2	α=40.763 β=0.01856
40	Pearson 5	α=2.909 β=7.2573E-4
41	Pearson 5 (3P)	α=1.1411 β=2.3470Ε-4 γ=1.0074Ε-6
42	Pearson 6	$\alpha_1$ =2.1134 $\alpha_2$ =2.4345 $\beta$ =3.3774E-4
43	Pearson 6 (4P)	α <sub>1</sub> =1.6515 α <sub>2</sub> =3.5113
		β=7.1802E-4 γ=5.3632E-7
44	Pert	m=2.6047E-6 a=4.7183E-7 b=0.00299
45	Power Function	α=0.35563 a=9.7960E-7 b=0.00204
46	Rayleigh	σ=3.7227E-4
47	Rayleigh (2P)	σ=4.6563Ε-4 γ=8.6572Ε-7
48	Reciprocal	a=9.7430E-7 b=0.00206
49	Rice	v=2.3135E-5 σ=4.6990E-4
50	Triangular	m=9.7596E-7 a=7.2417E-7 b=0.0021
51	Uniform	a=-3.5661E-4 b=0.00129
52	Weibull	α=1.0446 β=4.7567Ε-4
53	Weibull (3P)	α=1.0236 β=4.7051Ε-4 γ=8.7593Ε-7
54	Chi-Squared	No fit
55	Chi-Squared (2P)	No fit
56	Erlang	No fit
57	Erlang (3P)	No fit
58	Johnson SU	No fit
59	Log-Gamma	No fit
60	Nakagami	No fit
61	Student's t	No fit

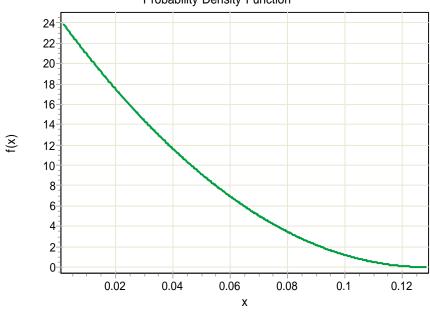
#	<u>Distribution</u>	Kolmogorov Smirnov		Anderson Darling		Chi-Squared	
		Statistic	Rank	Statistic	Rank	Statistic	Rank
1	<u>Beta</u>	0.21794	31	10.532	33	N/A	1
2	<u>Burr</u>	0.10003	3	1.7194	2	19.157	8
3	Burr (4P)	0.29899	39	17.711	42	N/A	1
4	Cauchy	0.24879	34	12.863	37	42.588	28
5	<u>Dagum</u>	0.10186	5	1.8715	3	12.868	4
6	Dagum (4P)	0.09866	2	1.6841	1	14.256	5
7	Error	0.30096	40	11.301	35	51.81	37
8	Error Function	0.55199	50	50.814	51	208.83	42
9	Exponential	0.16203	22	3.5128	14	23.682	14
10	Exponential (2P)	0.16236	23	3.6611	17	23.727	15
11	Fatigue Life	0.31858	42	14.442	40	44.209	30
12	Fatigue Life (3P)	0.14767	18	2.6167	9	21.777	11
13	Frechet	0.10534	6	3.8983	20	10.557	3
14	Frechet (3P)	0.10114	4	5.5637	25	N/A	1
15	<u>Gamma</u>	0.15693	21	3.4915	13	23.388	13
16	Gamma (3P)	0.17873	28	3.9001	21	46.159	32
17	Gen. Extreme Value	0.14772	19	3.5236	15	28.247	19
18	Gen. Gamma	0.18582	29	3.9404	22	49.559	33
19	Gen. Gamma (4P)	0.13179	15	2.6469	10	30.178	21
20	Gen. Pareto	0.13169	14	6.4713	29	N/A	Λ.
21	Gumbel Max	0.22422	32	6.8115	30	25.65	18
22	Gumbel Min	0.34476	44	22.244	44	49.787	34
23	Hypersecant	0.30717	41	11.849	36	50.16	35
24	Inv. Gaussian	0.14599	17	6.1307	28	32.261	22
25	Inv. Gaussian (3P)	0.11978	12	4.0024	23	17.647	7
26	Johnson SB	0.08876	1	8.8882	31	N/A	<b>\</b>
27	Kumaraswamy	0.16606	24	3.8541	19	24.093	16
28	Laplace	0.33535	43	14.063	39	56.886	38
29	Levy	1	53	N/A	<u> </u>	N/A	1

Log-Logistic (3P)	30	Levy (2P)	1	52	N/A		N/A	
10g-Pearson 3	31	Log-Logistic	0.11035	7	2.0848	5	8.8356	2
Logistic   0.2951   37   11.014   34   45.425   31   35   Lognormal   0.11294   9   2.0154   4   35.281   24   36   Lognormal (3P)   0.12443   13   2.3916   8   21.192   10   10   37   Normal   0.28008   36   10.329   32   43.384   29   38   Pareto   0.52366   49   37.143   46   7.6282   1   39   Pareto 2   0.15645   20   3.4205   12   21.102   9   40   Pearson 5   0.20412   30   17.285   41   37.45   25   41   Pearson 5 (3P)   0.11364   10   5.5781   26   N/A   42   Pearson 6   0.11386   11   2.1803   6   28.67   20   43   Pearson 6 (4P)   0.13407   16   2.6801   11   32.352   23   44   Pert   0.22777   33   5.6584   27   51.619   36   45   Power Function   0.29549   38   13.614   38   N/A   46   Rayleigh (2P)   0.44884   47   46.513   49   123.69   41   48   Reciprocal   0.55471   51   43.109   47   38.203   26   49   Rice   0.45147   48   46.817   50   N/A   48   Reciprocal   0.55471   51   43.109   47   38.203   26   49   Rice   0.45147   48   46.817   50   N/A   50   Triangular   0.3779   46   21.001   43   78.295   39   51   Uniform   0.25301   35   44.242   48   N/A   50   Triangular   0.3779   46   21.001   43   78.295   39   51   Uniform   0.25301   35   44.242   48   N/A   50   N/A   50   Triangular   0.3779   46   21.001   43   78.295   39   51   Uniform   0.25301   35   44.242   48   N/A   50   Triangular   0.3779   46   21.001   43   78.295   39   51   Uniform   0.25301   35   44.242   48   N/A   50   Triangular   0.3769   25   3.6375   16   22.821   12   54   Chi-Squared   No fit   55   Chi-Squared   No fit   57   Erlang   3P   No fit   57   Erlang   3P   No fit   59   Log-Gamma   No fit   59   Log-Gamma   No fit   59   Log-Gamma   No fit   59   Log-Gamma   No fit   50   No fit	32	Log-Logistic (3P)	0.11052	8	2.2075	7	16.795	6
10   10   10   10   10   10   10   10	33	Log-Pearson 3	0.17395	26	4.306	24	40.673	27
10   10   10   10   10   10   10   10	34	Logistic	0.2951	37	11.014	34	45.425	31
Normal   0.28008   36   10.329   32   43.384   29   29   38   Pareto   0.52366   49   37.143   46   7.6282   1   39   Pareto 2   0.15645   20   3.4205   12   21.102   9   40   Pearson 5   0.20412   30   17.285   41   37.45   25   41   Pearson 6   0.11364   10   5.5781   26   N/A   42   Pearson 6   0.11386   11   2.1803   6   28.67   20   43   Pearson 6   0.13407   16   2.6801   11   32.352   23   44   Pert   0.22777   33   5.6584   27   51.619   36   45   Power Function   0.29549   38   13.614   38   N/A   46   Rayleigh   0.34835   45   35.391   45   102.96   40   47   Rayleigh (2P)   0.44884   47   46.513   49   123.69   41   48   Reciprocal   0.55471   51   43.109   47   38.203   26   49   Rice   0.45147   48   46.817   50   N/A   50   Triangular   0.3779   46   21.001   43   78.295   39   51   Uniform   0.25301   35   44.242   48   N/A   52   Weibull   0.17465   27   3.7696   18   24.494   17   53   Weibull (3P)   0.16897   25   3.6375   16   22.821   12   54   Chi-Squared   No fit   55   Chi-Squared (2P)   No fit   57   Erlang (3P)   No fit   58   Johnson SU   No fit   59   Log-Gamma   No fit   59   Log-Gamma   No fit   59   Log-Gamma   No fit   50   No	35	Lognormal	0.11294	9	2.0154	4	35.281	24
38         Pareto         0.52366         49         37.143         46         7.6282         1           39         Pareto 2         0.15645         20         3.4205         12         21.102         9           40         Pearson 5         0.20412         30         17.285         41         37.45         25           41         Pearson 5 (3P)         0.11364         10         5.5781         26         N/A           42         Pearson 6         0.11386         11         2.1803         6         28.67         20           43         Pearson 6 (4P)         0.13407         16         2.6801         11         32.352         23           44         Pert         0.22777         33         5.6584         27         51.619         36           45         Power Function         0.29549         38         13.614         38         N/A           46         Rayleigh         0.34835         45         35.391         45         102.96         40           47         Rayleigh (2P)         0.44884         47         46.513         49         123.69         41           48         Reciprocal         0.55471         51	36	Lognormal (3P)	0.12443	13	2.3916	8	21.192	10
39 Pareto 2     0.15645     20     3.4205     12     21.102     9       40 Pearson 5     0.20412     30     17.285     41     37.45     25       41 Pearson 5 (3P)     0.11364     10     5.5781     26     N/A       42 Pearson 6     0.11386     11     2.1803     6     28.67     20       43 Pearson 6 (4P)     0.13407     16     2.6801     11     32.352     23       44 Pert     0.22777     33     5.6584     27     51.619     36       45 Power Function     0.29549     38     13.614     38     N/A       46 Rayleigh     0.34835     45     35.391     45     102.96     40       47 Rayleigh (2P)     0.44884     47     46.513     49     123.69     41       48 Reciprocal     0.55471     51     43.109     47     38.203     26       49 Rice     0.45147     48     46.817     50     N/A       50 Triangular     0.3779     46     21.001     43     78.295     39       51 Uniform     0.25301     35     44.242     48     N/A       52 Weibull     0.17465     27     3.7696     18     24.494     17       54 Ch	37	Normal	0.28008	36	10.329	32	43.384	29
40 Pearson 5	38	<u>Pareto</u>	0.52366	49	37.143	46	7.6282	1
41 Pearson 5 (3P)	39	Pareto 2	0.15645	20	3.4205	12	21.102	9
42         Pearson 6         0.11386         11         2.1803         6         28.67         20           43         Pearson 6 (4P)         0.13407         16         2.6801         11         32.352         23           44         Pert         0.22777         33         5.6584         27         51.619         36           45         Power Function         0.29549         38         13.614         38         N/A           46         Rayleigh         0.34835         45         35.391         45         102.96         40           47         Rayleigh (2P)         0.44884         47         46.513         49         123.69         41           48         Reciprocal         0.55471         51         43.109         47         38.203         26           49         Rice         0.45147         48         46.817         50         N/A           50         Triangular         0.3779         46         21.001         43         78.295         39           51         Uniform         0.25301         35         44.242         48         N/A           52         Weibull (3P)         0.16897         25         3.6375<	40	Pearson 5	0.20412	30	17.285	41	37.45	25
43         Pearson 6 (4P)         0.13407         16         2.6801         11         32.352         23           44         Pert         0.22777         33         5.6584         27         51.619         36           45         Power Function         0.29549         38         13.614         38         N/A           46         Rayleigh         0.34835         45         35.391         45         102.96         40           47         Rayleigh (2P)         0.44884         47         46.513         49         123.69         41           48         Reciprocal         0.55471         51         43.109         47         38.203         26           49         Rice         0.45147         48         46.817         50         N/A           50         Triangular         0.3779         46         21.001         43         78.295         39           51         Uniform         0.25301         35         44.242         48         N/A           52         Weibull (3P)         0.16897         25         3.6375         16         22.821         12           54         Chi-Squared         No fit           55 <td>41</td> <td>Pearson 5 (3P)</td> <td>0.11364</td> <td>10</td> <td>5.5781</td> <td>26</td> <td>N/A</td> <td></td>	41	Pearson 5 (3P)	0.11364	10	5.5781	26	N/A	
144   Pert	42	Pearson 6	0.11386	11	2.1803	6	28.67	20
1.5   Power Function   0.29549   38   13.614   38   N/A     46   Rayleigh   0.34835   45   35.391   45   102.96   40     47   Rayleigh (2P)   0.44884   47   46.513   49   123.69   41     48   Reciprocal   0.55471   51   43.109   47   38.203   26     49   Rice   0.45147   48   46.817   50   N/A     50   Triangular   0.3779   46   21.001   43   78.295   39     51   Uniform   0.25301   35   44.242   48   N/A     52   Weibull   0.17465   27   3.7696   18   24.494   17     53   Weibull (3P)   0.16897   25   3.6375   16   22.821   12     54   Chi-Squared   No fit     55   Chi-Squared (2P)   No fit     56   Erlang   No fit     57   Erlang (3P)   No fit     58   Johnson SU   No fit     59   Log-Gamma   No fit     60   Nakagami   No fit	43	Pearson 6 (4P)	0.13407	16	2.6801	11	32.352	23
46         Rayleigh         0.34835         45         35.391         45         102.96         40           47         Rayleigh (2P)         0.44884         47         46.513         49         123.69         41           48         Reciprocal         0.55471         51         43.109         47         38.203         26           49         Rice         0.45147         48         46.817         50         N/A           50         Triangular         0.3779         46         21.001         43         78.295         39           51         Uniform         0.25301         35         44.242         48         N/A           52         Weibull         0.17465         27         3.7696         18         24.494         17           53         Weibull (3P)         0.16897         25         3.6375         16         22.821         12           54         Chi-Squared         No fit           55         Chi-Squared (2P)         No fit           56         Erlang         No fit           57         Erlang (3P)         No fit           59         Log-Gamma         No fit           60         N	44	<u>Pert</u>	0.22777	33	5.6584	27	51.619	36
47 Rayleigh (2P)	45	Power Function	0.29549	38	13.614	38	N/A	
48         Reciprocal         0.55471         51         43.109         47         38.203         26           49         Rice         0.45147         48         46.817         50         N/A           50         Triangular         0.3779         46         21.001         43         78.295         39           51         Uniform         0.25301         35         44.242         48         N/A           52         Weibull         0.17465         27         3.7696         18         24.494         17           53         Weibull (3P)         0.16897         25         3.6375         16         22.821         12           54         Chi-Squared         No fit           55         Chi-Squared (2P)         No fit           56         Erlang         No fit           57         Erlang (3P)         No fit           58         Johnson SU         No fit           59         Log-Gamma         No fit           60         Nakagami         No fit	46	Rayleigh	0.34835	45	35.391	45	102.96	40
49 Rice         0.45147         48         46.817         50         N/A           50 Triangular         0.3779         46         21.001         43         78.295         39           51 Uniform         0.25301         35         44.242         48         N/A           52 Weibull         0.17465         27         3.7696         18         24.494         17           53 Weibull (3P)         0.16897         25         3.6375         16         22.821         12           54 Chi-Squared         No fit           55 Chi-Squared (2P)         No fit           57 Erlang (3P)         No fit           58 Johnson SU         No fit           59 Log-Gamma         No fit           60 Nakagami         No fit	47	Rayleigh (2P)	0.44884	47	46.513	49	123.69	41
50         Triangular         0.3779         46         21.001         43         78.295         39           51         Uniform         0.25301         35         44.242         48         N/A           52         Weibull         0.17465         27         3.7696         18         24.494         17           53         Weibull (3P)         0.16897         25         3.6375         16         22.821         12           54         Chi-Squared         No fit           55         Chi-Squared (2P)         No fit           56         Erlang         No fit           57         Erlang (3P)         No fit           58         Johnson SU         No fit           59         Log-Gamma         No fit           60         Nakagami         No fit	48	Reciprocal	0.55471	51	43.109	47	38.203	26
51         Uniform         0.25301         35         44.242         48         N/A           52         Weibull         0.17465         27         3.7696         18         24.494         17           53         Weibull (3P)         0.16897         25         3.6375         16         22.821         12           54         Chi-Squared         No fit           55         Chi-Squared (2P)         No fit           56         Erlang         No fit           57         Erlang (3P)         No fit           58         Johnson SU         No fit           59         Log-Gamma         No fit           60         Nakagami         No fit	49	Rice	0.45147	48	46.817	50	N/A	
52         Weibull         0.17465         27         3.7696         18         24.494         17           53         Weibull (3P)         0.16897         25         3.6375         16         22.821         12           54         Chi-Squared         No fit           55         Chi-Squared (2P)         No fit           56         Erlang         No fit           57         Erlang (3P)         No fit           58         Johnson SU         No fit           59         Log-Gamma         No fit           60         Nakagami         No fit	50	<u>Triangular</u>	0.3779	46	21.001	43	78.295	39
53         Weibull (3P)         0.16897         25         3.6375         16         22.821         12           54         Chi-Squared         No fit           55         Chi-Squared (2P)         No fit           56         Erlang         No fit           57         Erlang (3P)         No fit           58         Johnson SU         No fit           59         Log-Gamma         No fit           60         Nakagami         No fit	51	<u>Uniform</u>	0.25301	35	44.242	48	N/A	
54 Chi-Squared No fit 55 Chi-Squared (2P) No fit 56 Erlang No fit 57 Erlang (3P) No fit 58 Johnson SU No fit 59 Log-Gamma No fit 60 Nakagami No fit	52	<u>Weibull</u>	0.17465	27	3.7696	18	24.494	17
55 Chi-Squared (2P) No fit 56 Erlang No fit 57 Erlang (3P) No fit 58 Johnson SU No fit 59 Log-Gamma No fit 60 Nakagami No fit	53	Weibull (3P)	0.16897	25	3.6375	16	22.821	12
56 Erlang No fit 57 Erlang (3P) No fit 58 Johnson SU No fit 59 Log-Gamma No fit 60 Nakagami No fit	54	Chi-Squared	No fit					
57 Erlang (3P) No fit 58 Johnson SU No fit 59 Log-Gamma No fit 60 Nakagami No fit	55	Chi-Squared (2P)	No fit					
58 Johnson SU No fit 59 Log-Gamma No fit 60 Nakagami No fit	56	Erlang	No fit					
59 Log-Gamma No fit 60 Nakagami No fit	57	Erlang (3P)	No fit					
60 Nakagami No fit	58	Johnson SU	No fit					
	59	Log-Gamma	No fit					
61 Student's t No fit	60	Nakagami	No fit					
	61	Student's t	No fit					

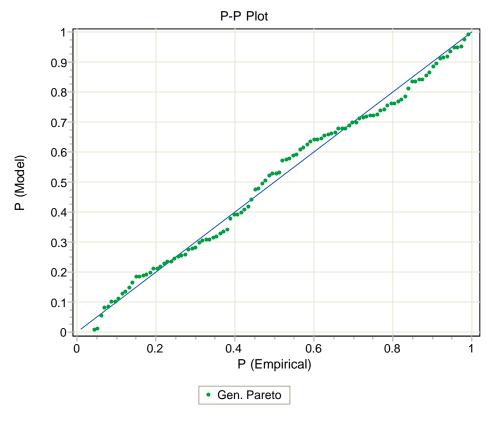
#### F (constant-variance)

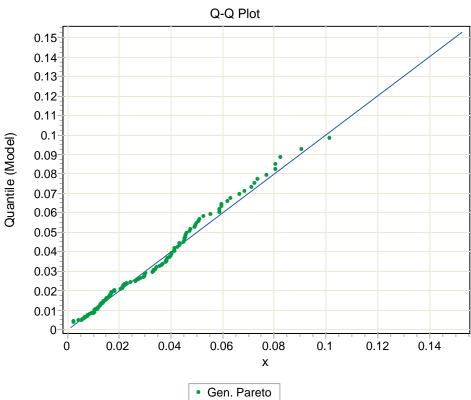


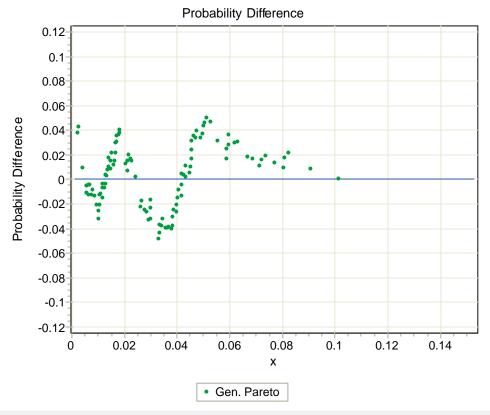
#### Probability Density Function



— Gen. Pareto (-0.333; 0.04195; 0.00214)







#	Distribution	Parameters
1	Beta	$\alpha_1$ =1.2106 $\alpha_2$ =7.7599 $\alpha$ =0.001 $\alpha$ =0.24192
2	Burr	k=553.03 α=1.3522 β=3.9076
3	Burr (4P)	k=332.89 α=1.2807 β=3.2866 γ=8.4239E-4
4	Cauchy	σ=0.01524 μ=0.02782
5	Dagum	k=0.22678 α=4.5948 β=0.05802
6	Dagum (4P)	k=0.2552 α=3.7293 β=0.05817 γ=0.00118
7	Erlang	m=1 β=0.01881
8	Erlang (3P)	m=2 β=0.02201 γ=4.3402E-4
9	Error	k=1.0 σ=0.02514 μ=0.03362
10	Error Function	h=28.123
11	Exponential	λ=29.747
12	Exponential (2P)	λ=30.829 γ=0.00118

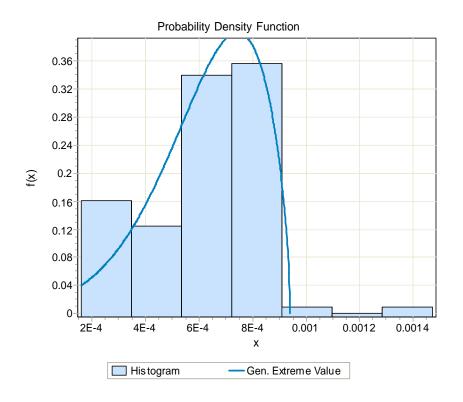
13	Fatigue Life	α=1.0971 β=0.02033
14	Fatigue Life (3P)	α=0.62281 β=0.03539 γ=-0.00867
15	Frechet	α=1.1452 β=0.01435
16	Frechet (3P)	α=6.9191 β=0.11779 γ=-0.0966
17	Gamma	α=1.7875 β=0.01881
18	Gamma (3P)	α=1.5075 β=0.02201 γ=4.3402Ε-4
19	Gen. Extreme Value	k=0.04644 σ=0.01861 μ=0.02198
20	Gen. Gamma	k=0.96605 α=1.7254 β=0.01881
21	Gen. Gamma (4P)	k=1.8645 α=0.49181 β=0.05928 γ=0.00118
22	Gen. Pareto	k=-0.333 σ=0.04195 μ=0.00214
23	Gumbel Max	σ=0.0196 μ=0.0223
24	Gumbel Min	σ=0.0196 μ=0.04493
25	Hypersecant	σ=0.02514 μ=0.03362
26	Inv. Gaussian	λ=0.06009 μ=0.03362
27	Inv. Gaussian (3P)	λ=0.12141 μ=0.04434 γ=-0.01072
28	Johnson SB	$\gamma$ =11.437 $\delta$ =2.3539 $\lambda$ =6.8057 $\xi$ =-0.0236
29	Kumaraswamy	α <sub>1</sub> =0.64337 α <sub>2</sub> =1.7946 a=0.00118 b=0.15577
30	Laplace	λ=56.245 μ=0.03362
31	Levy	σ=0.01312
32	Levy (2P)	σ=0.01508 γ=-5.8119Ε-4
33	Log-Logistic	α=1.7535 β=0.02343
34	Log-Logistic (3P)	α=2.5354 β=0.03289 γ=-0.0057
35	Log-Pearson 3	α=4.2058 β=-0.46559 γ=-1.7789
36	Logistic	σ=0.01386 μ=0.03362
37	Lognormal	σ=0.9506 μ=-3.737
38	Lognormal (3P)	σ=0.58964 μ=-3.316 γ=-0.00925
39	Normal	σ=0.02514 μ=0.03362
40	Pareto	α=0.33265 β=0.00118
41	Pareto 2	α=306.37 β=9.9552
42	Pearson 5	α=0.97045 β=0.01273

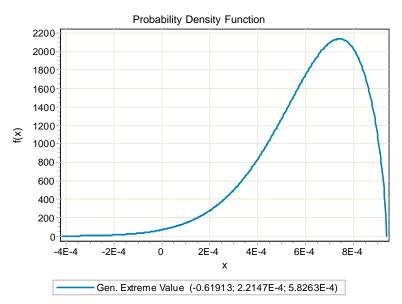
43	Pearson 5 (3P)	α=0.5639 β=0.00411 γ=0.00117
44	Pearson 6	$\alpha_1$ =1.5724 $\alpha_2$ =1.2516E+8 $\beta$ =2.6927E+6
45	Pearson 6 (4P)	$\alpha_1$ =1.5138 $\alpha_2$ =12274.0 $\beta$ =269.28 $\gamma$ =4.1091E-4
46	Pert	m=0.00363 a=0.00116 b=0.18836
47	Power Function	α=0.32693 a=0.00118 b=0.15267
48	Rayleigh	σ=0.02682
49	Rayleigh (2P)	σ=0.03413 γ=-0.00766
50	Reciprocal	a=0.00118 b=0.15267
51	Rice	v=4.3764E-5 σ=0.02964
52	Triangular	m=0.00118 a=0.001 b=0.15459
53	Uniform	a=-0.00993 b=0.07717
54	Weibull	α=1.2844 β=0.03628
55	Weibull (3P)	α=1.2759 β=0.03524 γ=8.4273Ε-4
56	Chi-Squared	No fit
57	Chi-Squared (2P)	No fit
58	Johnson SU	No fit
59	Log-Gamma	No fit
60	Nakagami	No fit
61	Student's t	No fit

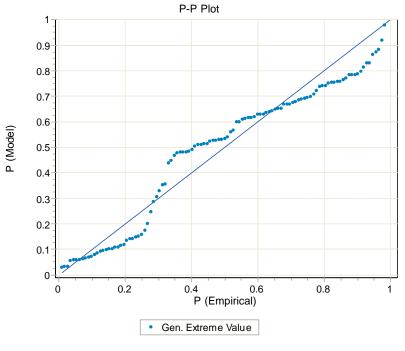
#	<u>Distribution</u>	Kolmogorov Smirnov		Anderson Darling		Chi-Squared	
		Statistic	Rank	Statistic	Rank	Statistic	Rank
1	<u>Beta</u>	0.0763	9	0.58732	7	6.8297	9
2	<u>Burr</u>	0.06819	5	0.45328	2	9.8018	23
3	Burr (4P)	0.0761	8	0.55278	5	6.6131	6
4	Cauchy	0.16534	40	3.5343	35	12.261	27
5	<u>Dagum</u>	0.078	12	0.43196	1	11.444	25
6	Dagum (4P)	0.06671	4	0.59383	8	6.3914	5
7	Erlang	0.31442	53	30.745	54	86.171	52
8	Erlang (3P)	0.18283	45	9.1671	44	26.339	40
9	<u>Error</u>	0.1676	42	3.3335	32	24.286	37

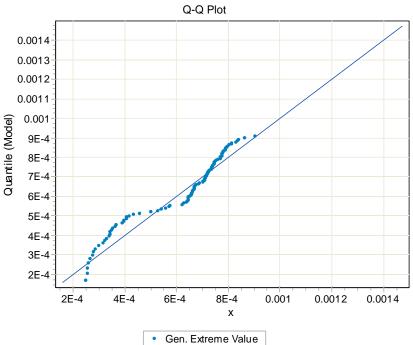
10	Error Function	0.52707	55	90.727	55	268.71	53
11	<u>Exponential</u>	0.12291	29	3.0382	31	17.412	30
12	Exponential (2P)	0.11261	27	2.4832	30	14.724	28
13	Fatigue Life	0.15959	38	4.7082	37	19.199	34
14	Fatigue Life (3P)	0.09129	24	0.76953	16	8.6626	19
15	Frechet	0.16757	41	6.3739	39	34.634	43
16	Frechet (3P)	0.08671	17	0.97296	22	11.638	26
17	<u>Gamma</u>	0.07777	11	0.77869	17	8.0272	16
18	Gamma (3P)	0.08715	19	0.67966	11	6.3444	3
19	Gen. Extreme Value	0.08251	15	0.78701	19	8.3892	18
20	Gen. Gamma	0.08778	20	0.68123	12	4.9701	1
21	Gen. Gamma (4P)	0.06546	2	0.71623	14	7.6428	12
22	Gen. Pareto	0.05755	1	15.664	50	N/A	
23	Gumbel Max	0.08462	16	0.78133	18	6.1398	2
24	Gumbel Min	0.15552	37	8.1203	42	19.632	35
25	Hypersecant	0.14521	36	2.4641	29	15.056	29
26	Inv. Gaussian	0.11176	26	7.5263	41	22.407	36
27	Inv. Gaussian (3P)	0.0902	23	0.79807	20	7.9695	15
28	Johnson SB	0.07976	13	0.71569	13	8.7292	20
29	Kumaraswamy	0.13051	33	4.4636	36	29.056	41
30	Laplace	0.1676	43	3.3335	33	24.286	38
31	<u>Levy</u>	0.28369	50	10.555	45	77.604	49
32	Levy (2P)	0.30362	51	11.68	46	72.743	48
33	Log-Logistic	0.13332	35	2.1734	28	17.928	32
34	Log-Logistic (3P)	0.0894	22	1.141	23	8.8529	21
35	Log-Pearson 3	0.06562	3	0.47467	3	7.7125	13
36	Logistic	0.12982	32	2.1452	27	8.2582	17
37	Lognormal	0.12167	28	2.0396	25	17.507	31
38	Lognormal (3P)	0.08936	21	0.85426	21	7.8656	14
39	Normal	0.10748	25	2.0942	26	7.2876	11
40	<u>Pareto</u>	0.37259	54	26.573	53	78.844	51
41	Pareto 2	0.13055	34	3.5012	34	18.382	33
42	Pearson 5	0.16049	39	6.6966	40	38.771	45
43	Pearson 5 (3P)	0.24426	47	15.124	48	77.773	50

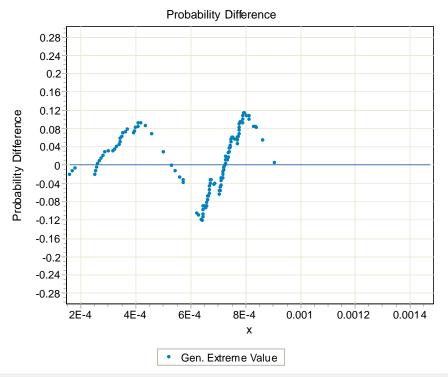
Pearson 6	0.08189	14	0.60218	9	6.7919	8	
Pearson 6 (4P)	0.08682	18	0.67452	10	6.3464	4	
<u>Pert</u>	0.06848	6	0.7312	15	8.8926	22	
Power Function	0.25785	48	12.66	47	72.483	47	
Rayleigh	0.17007	44	6.1987	38	24.538	39	
Rayleigh (2P)	0.12701	30	1.665	24	11.179	24	
Reciprocal	0.30462	52	15.683	51	71.64	46	
Rice	0.20456	46	8.3575	43	33.55	42	
Triangular	0.27628	49	15.541	49	34.875	44	
<u>Uniform</u>	0.12758 31 25.269 52 N/A						
Weibull	0.07519	7	0.50725	4	6.9089	10	
Weibull (3P)	0.07703	10	0.55543	6	6.6132	7	
Chi-Squared	No fit						
Chi-Squared (2P)	No fit						
Johnson SU	No fit						
Log-Gamma	No fit						
Nakagami	No fit						
Student's t	No fit						
	Pearson 6 (4P) Pert Power Function Rayleigh Rayleigh (2P) Reciprocal Rice Triangular Uniform Weibull Weibull (3P) Chi-Squared Chi-Squared (2P) Johnson SU Log-Gamma Nakagami	Pearson 6 (4P)         0.08682           Pert         0.06848           Power Function         0.25785           Rayleigh         0.17007           Rayleigh (2P)         0.12701           Reciprocal         0.30462           Rice         0.20456           Triangular         0.27628           Uniform         0.12758           Weibull         0.07519           Weibull (3P)         0.07703           Chi-Squared         No fit           Chi-Squared (2P)         No fit           Johnson SU         No fit           No fit         No fit           No fit         No fit	Pearson 6 (4P)         0.08682         18           Pert         0.06848         6           Power Function         0.25785         48           Rayleigh         0.17007         44           Rayleigh (2P)         0.12701         30           Reciprocal         0.30462         52           Rice         0.20456         46           Triangular         0.27628         49           Uniform         0.12758         31           Weibull         0.07519         7           Weibull (3P)         0.07703         10           Chi-Squared         No fit           Johnson SU         No fit           Log-Gamma         No fit           Nakagami         No fit	Pearson 6 (4P)         0.08682         18         0.67452           Pert         0.06848         6         0.7312           Power Function         0.25785         48         12.66           Rayleigh         0.17007         44         6.1987           Rayleigh (2P)         0.12701         30         1.665           Reciprocal         0.30462         52         15.683           Rice         0.20456         46         8.3575           Triangular         0.27628         49         15.541           Uniform         0.12758         31         25.269           Weibull         0.07519         7         0.50725           Weibull (3P)         0.07703         10         0.55543           Chi-Squared         No fit           Johnson SU         No fit           No fit         No fit           Nakagami         No fit	Pearson 6 (4P)         0.08682         18         0.67452         10           Pert         0.06848         6         0.7312         15           Power Function         0.25785         48         12.66         47           Rayleigh         0.17007         44         6.1987         38           Rayleigh (2P)         0.12701         30         1.665         24           Reciprocal         0.30462         52         15.683         51           Rice         0.20456         46         8.3575         43           Triangular         0.27628         49         15.541         49           Uniform         0.12758         31         25.269         52           Weibull         0.07519         7         0.50725         4           Weibull (3P)         0.07703         10         0.55543         6           Chi-Squared         No fit         No fit           Johnson SU         No fit           No fit         No fit	Pearson 6 (4P)         0.08682         18         0.67452         10         6.3464           Pert         0.06848         6         0.7312         15         8.8926           Power Function         0.25785         48         12.66         47         72.483           Rayleigh         0.17007         44         6.1987         38         24.538           Rayleigh (2P)         0.12701         30         1.665         24         11.179           Reciprocal         0.30462         52         15.683         51         71.64           Rice         0.20456         46         8.3575         43         33.55           Triangular         0.27628         49         15.541         49         34.875           Uniform         0.12758         31         25.269         52         N/A           Weibull         0.07519         7         0.50725         4         6.9089           Weibull (3P)         0.07703         10         0.55543         6         6.6132           Chi-Squared         No fit           Johnson SU         No fit           No fit           Nog-Gamma         No fit	











#	Distribution	Parameters
1	Beta	$\alpha_1$ =8200.6 $\alpha_2$ =17392.0 a=-0.02232 b=0.04927
2	Burr	k=10.093 α=3.5947 β=0.00129
3	Burr (4P)	k=90908.0 α=3421.1 β=1.4194 γ=-1.414
4	Cauchy	σ=8.9889Ε-5 μ=7.0090Ε-4
5	Dagum	k=0.15313 α=17.725 β=8.2039E-4
6	Dagum (4P)	k=0.15007 α=0.84066 β=0.86306 γ=1.5905E-4
7	Erlang	m=8 β=7.0979E-5
8	Erlang (3P)	m=156 β=1.6973E-5 γ=-0.00202
9	Error	k=1.2771 σ=2.0976E-4 μ=6.1990E-4
10	Error Function	h=3371.0
11	Exponential	λ=1613.2
12	Exponential (2P)	λ=2169.9 γ=1.5905E-4
13	Fatigue Life	α=0.4236 β=5.6841Ε-4
14	Fatigue Life (3P)	α=0.00586 β=0.03566 γ=-0.03504

15	Frechet	α=2.4119 β=4.5287Ε-4
16	Frechet (3P)	α=1.3999E+8 β=29368.0 γ=-29368.0
17	Gamma	α=8.7336 β=7.0979Ε-5
18	Gamma (3P)	α=133.1 β=1.8424Ε-5 γ=-0.00184
19	Gen. Extreme Value	k=-0.61913 σ=2.2147E-4 μ=5.8263E-4
20	Gen. Gamma	k=0.95352 α=7.8618 β=7.0979E-5
21	Gen. Gamma (4P)	k=2.1535 α=58.796 β=5.2299E-4 γ=-0.00284
22	Gen. Pareto	k=-1.8282 σ=0.00121 μ=1.9185E-4
23	Gumbel Max	σ=1.6355Ε-4 μ=5.2550Ε-4
24	Gumbel Min	σ=1.6355Ε-4 μ=7.1431Ε-4
25	Hypersecant	σ=2.0976Ε-4 μ=6.1990Ε-4
26	Inv. Gaussian	λ=0.00541 μ=6.1990Ε-4
27	Inv. Gaussian (3P)	λ=46.976 μ=0.01271 γ=-0.01209
28	Johnson SU	γ=0.09459 δ=2.047 λ=3.7883E-4 ξ=6.3963E-4
29	Kumaraswamy	α <sub>1</sub> =0.92175 α <sub>2</sub> =1.9949 a=1.5905E-4 b=0.00148
30	Laplace	λ=6742.0 μ=6.1990Ε-4
31	Levy	σ=1.4189
32	Levy (2P)	σ=2.0 γ=1.3519Ε-4
33	Log-Logistic	α=3.8175 β=5.7154Ε-4
34	Log-Logistic (3P)	α=2.2409E+8 β=24252.0 γ=-24252.0
35	Log-Pearson 3	α=2.9198 β=-0.24255 γ=-6.7505
36	Logistic	σ=1.1565Ε-4 μ=6.1990Ε-4
37	Lognormal	σ=0.4126 μ=-7.4587
38	Lognormal (3P)	σ=0.03438 μ=-5.0986 γ=-0.00549
39	Nakagami	m=2.5416 Ω=4.2788E-7
40	Normal	σ=2.0976Ε-4 μ=6.1990Ε-4
41	Pareto	α=0.77666 β=1.5905Ε-4
42	Pareto 2	α=2215.2 β=1.3729
43	Pearson 5	α=5.2079 β=0.00272
44	Pearson 5 (3P)	α=246.18 β=0.81794 γ=-0.00271

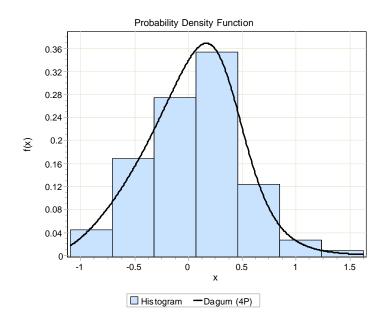
45	Pearson 6	$\alpha_1 = 7.3371$ $\alpha_2 = 90.266$ $\beta = 0.00753$
46	Pearson 6 (4P)	$\alpha_1$ =0.57414 $\alpha_2$ =1.2618 $\beta$ =9.0927E-4 $\gamma$ =1.5905E-4
47	Pert	m=5.1440E-4 a=1.3109E-4 b=0.00157
48	Power Function	α=0.64896 a=1.5905E-4 b=0.00147
49	Rayleigh	σ=4.9461Ε-4
50	Rayleigh (2P)	σ=3.6138Ε-4 γ=1.4513Ε-4
51	Reciprocal	a=1.5783E-4 b=0.00149
52	Rice	v=5.7984E-4 σ=2.2055E-4
53	Triangular	m=3.5102E-4 a=1.2735E-4 b=0.00149
54	Uniform	a=2.5658E-4 b=9.8322E-4
55	Weibull	α=2.8794 β=6.9457Ε-4
56	Weibull (3P)	α=3.3568 β=7.1100Ε-4 γ=-2.0141Ε-5
57	Chi-Squared	No fit
58	Chi-Squared (2P)	No fit
59	Johnson SB	No fit
60	Log-Gamma	No fit
61	Student's t	No fit

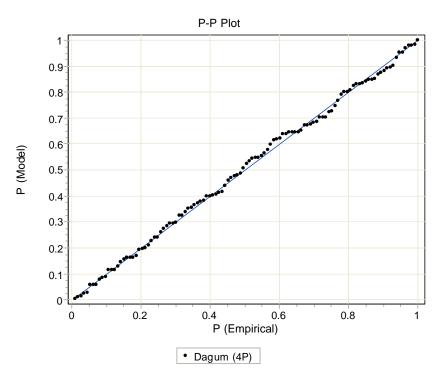
#	Distribution	Kolmogorov Smirnov		Anderson Darling		Chi-Squared	
		Statistic	Rank	Statistic	Rank	Statistic	Rank
1	Beta	0.19905	13	4.8684	5	35.833	10
2	Burr	0.20454	19	5.3003	15	57.237	31
3	Burr (4P)	0.27183	41	16.693	45	79.584	36

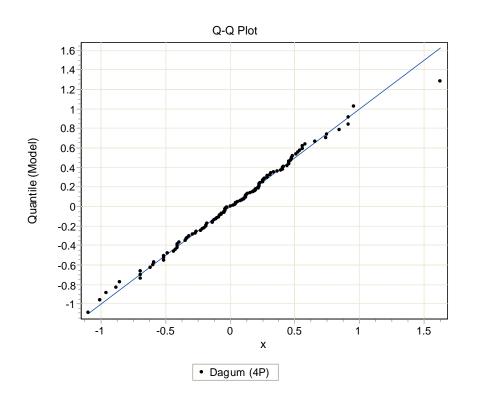
5         Dagum         0.16924         5         3.1675         1         31.924           6         Dagum (4P)         0.57503         53         45.359         53         N/A           7         Erlang         0.33652         52         13.598         41         76.533           8         Erlang (3P)         0.19676         9         5.1056         11         37.057           9         Error         0.2127         23         5.813         20         61.545           10         Error Function         0.85796         54         483.93         54         2176.7           11         Exponential         0.3118         49         23.725         48         198.06           12         Exponential (2P)         0.31219         50         16.339         44         32.121           13         Fatigue Life         0.2691         40         8.3569         33         41.237           14         Fatigue Life (3P)         0.20086         15         4.9095         6         35.837           15         Frechet         0.30702         46         11.721         40         29.676           16         Frechet (3P)         0.238	4
7         Erlang         0.33652         52         13.598         41         76.533           8         Erlang (3P)         0.19676         9         5.1056         11         37.057           9         Error         0.2127         23         5.813         20         61.545           10         Error Function         0.85796         54         483.93         54         2176.7           11         Exponential         0.3118         49         23.725         48         198.06           12         Exponential (2P)         0.31219         50         16.339         44         32.121           13         Fatigue Life         0.2691         40         8.3569         33         41.237           14         Fatigue Life (3P)         0.20086         15         4.9095         6         35.837           15         Frechet         0.30702         46         11.721         40         29.676           16         Frechet (3P)         0.23861         31         6.9845         24         52.58           17         Gamma         0.24138         32         7.7414         29         48.339	
8       Erlang (3P)       0.19676       9       5.1056       11       37.057         9       Error       0.2127       23       5.813       20       61.545         10       Error Function       0.85796       54       483.93       54       2176.7         11       Exponential       0.3118       49       23.725       48       198.06         12       Exponential (2P)       0.31219       50       16.339       44       32.121         13       Fatigue Life       0.2691       40       8.3569       33       41.237         14       Fatigue Life (3P)       0.20086       15       4.9095       6       35.837         15       Frechet       0.30702       46       11.721       40       29.676         16       Frechet (3P)       0.23861       31       6.9845       24       52.58         17       Gamma       0.24138       32       7.7414       29       48.339	
9         Error         0.2127         23         5.813         20         61.545           10         Error Function         0.85796         54         483.93         54         2176.7           11         Exponential         0.3118         49         23.725         48         198.06           12         Exponential (2P)         0.31219         50         16.339         44         32.121           13         Fatigue Life         0.2691         40         8.3569         33         41.237           14         Fatigue Life (3P)         0.20086         15         4.9095         6         35.837           15         Frechet         0.30702         46         11.721         40         29.676           16         Frechet (3P)         0.23861         31         6.9845         24         52.58           17         Gamma         0.24138         32         7.7414         29         48.339	35
10         Error Function         0.85796         54         483.93         54         2176.7           11         Exponential         0.3118         49         23.725         48         198.06           12         Exponential (2P)         0.31219         50         16.339         44         32.121           13         Fatigue Life         0.2691         40         8.3569         33         41.237           14         Fatigue Life (3P)         0.20086         15         4.9095         6         35.837           15         Frechet         0.30702         46         11.721         40         29.676           16         Frechet (3P)         0.23861         31         6.9845         24         52.58           17         Gamma         0.24138         32         7.7414         29         48.339	17
11       Exponential       0.3118       49       23.725       48       198.06         12       Exponential (2P)       0.31219       50       16.339       44       32.121         13       Fatigue Life       0.2691       40       8.3569       33       41.237         14       Fatigue Life (3P)       0.20086       15       4.9095       6       35.837         15       Frechet       0.30702       46       11.721       40       29.676         16       Frechet (3P)       0.23861       31       6.9845       24       52.58         17       Gamma       0.24138       32       7.7414       29       48.339	32
12     Exponential (2P)     0.31219     50     16.339     44     32.121       13     Fatigue Life     0.2691     40     8.3569     33     41.237       14     Fatigue Life (3P)     0.20086     15     4.9095     6     35.837       15     Frechet     0.30702     46     11.721     40     29.676       16     Frechet (3P)     0.23861     31     6.9845     24     52.58       17     Gamma     0.24138     32     7.7414     29     48.339	46
13     Fatigue Life     0.2691     40     8.3569     33     41.237       14     Fatigue Life (3P)     0.20086     15     4.9095     6     35.837       15     Frechet     0.30702     46     11.721     40     29.676       16     Frechet (3P)     0.23861     31     6.9845     24     52.58       17     Gamma     0.24138     32     7.7414     29     48.339	44
14     Fatigue Life (3P)     0.20086     15     4.9095     6     35.837       15     Frechet     0.30702     46     11.721     40     29.676       16     Frechet (3P)     0.23861     31     6.9845     24     52.58       17     Gamma     0.24138     32     7.7414     29     48.339	5
15     Frechet     0.30702     46     11.721     40     29.676       16     Frechet (3P)     0.23861     31     6.9845     24     52.58       17     Gamma     0.24138     32     7.7414     29     48.339	20
16     Frechet (3P)     0.23861     31     6.9845     24     52.58       17     Gamma     0.24138     32     7.7414     29     48.339	11
17 <u>Gamma</u> 0.24138 32 7.7414 29 48.339	3
	28
18 Gamma (3P) 0.21478 25 5.3868 16 33.7	24
	6
19         Gen. Extreme Value         0.13006         1         10.53         37         N/A	
20         Gen. Gamma         0.244444         33         7.0766         25         46.351	21
21 Gen. Gamma (4P) 0.2013 16 4.9626 8 35.889	12
22         Gen. Pareto         0.17173         6         19.084         47         N/A	
23 <u>Gumbel Max</u> 0.2679 39 11.168 39 91.674	40
24         Gumbel Min         0.13021         2         4.6079         2         36.615	15
25 <u>Hypersecant</u> 0.20909 21 5.7594 19 64.314	33
26         Inv. Gaussian         0.26207         36         10.7         38         88.429	39
27         Inv. Gaussian (3P)         0.19751         10         4.9193         7         36.81	16
28         Johnson SU         0.19899         12         5.0105         9         54.52	29
29         Kumaraswamy         0.29302         45         17.466         46         N/A	
30         Laplace         0.22698         28         7.2099         27         81.277	37
31 <u>Levy</u> 1 56 N/A N/A	
32 <u>Levy (2P)</u> 1 55 N/A N/A	
33         Log-Logistic         0.26507         37         8.0361         32         38.116	18
34         Log-Logistic (3P)         0.15874         4         5.1791         13         48.17	22
35 <u>Log-Pearson 3</u> 0.1854 7 8.5573 34 N/A	
36         Logistic         0.20385         18         5.2363         14         57.16	30
37         Lognormal         0.25884         35         7.9948         31         48.219	23

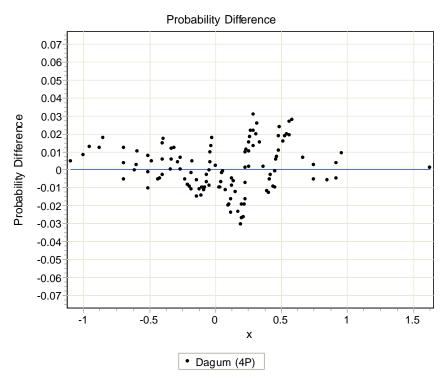
	Г				1			
38	Lognormal (3P)	0.20351	17	5.095	10	35.993	13	
39	<u>Nakagami</u>	0.21615	26	6.4027	23	65.45	34	
40	Normal	0.19766	11	4.8162	3	34.393	7	
41	<u>Pareto</u>	0.33172	51	27.447	52	172.67	43	
42	Pareto 2	0.31179	48	23.728	49	198.1	45	
43	Pearson 5	0.27715	42	9.2818	36	35.526	9	
44	Pearson 5 (3P)	0.21077	22	5.4697	17	35.466	8	
45	Pearson 6	0.24819	34	7.1504	26	50.184	26	
46	Pearson 6 (4P)	0.28289	43	26.167	51	N/A		
47	<u>Pert</u>	0.21393	24	6.354	22	52.511	27	
48	Power Function	0.31012	47	14.431	42	29.335	2	
49	Rayleigh	0.22622	27	8.6216	35	109.08	41	
50	Rayleigh (2P)	0.26712	38	7.6542	28	39.633	19	
51	Reciprocal	0.28916	44	16.19	43	153.38	42	
52	Rice	0.23299	29	5.7262	18	N/A		
53	<u>Triangular</u>	0.23547	30	7.9873	30	84.719	38	
54	<u>Uniform</u>	0.18649	8	24.61	50	N/A		
55	<u>Weibull</u>	0.2047	20	5.1601	12	48.743	25	
56	Weibull (3P)	0.20054	14	4.8231	4	36.189	14	
57	Chi-Squared	No fit						
58	Chi-Squared (2P)	No fit						
59	Johnson SB	No fit						
60	Log-Gamma	No fit						
61	Student's t	No fit						

#### Tr (constant-variance)









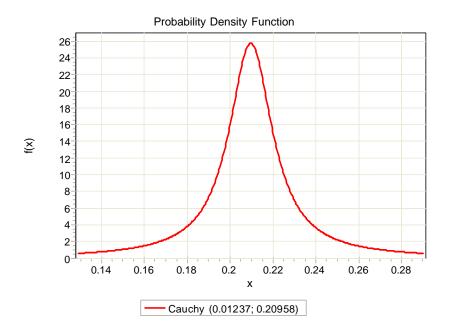
#	Distribution	Parameters	

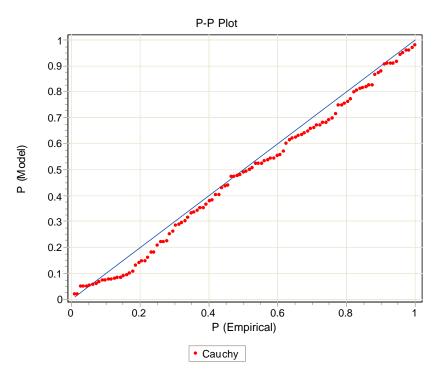
1	Beta	α <sub>1</sub> =1843.3 α <sub>2</sub> =1701.6 a=-28.062 b=25.997
2	Burr (4P)	k=2.4896 α=9.248 β=3.3524 γ=-2.9062
3	Cauchy	σ=0.26184 μ=0.09568
4	Chi-Squared (2P)	ν=1 γ=-1.0986
5	Dagum (4P)	k=0.26209 α=10.708 β=1.8023 γ=-1.3483
6	Erlang (3P)	m=120 β=0.04204 γ=-4.9798
7	Error	k=1.5209 σ=0.45555 μ=0.04825
8	Error Function	h=1.5522
9	Exponential (2P)	λ=0.87194 γ=-1.0986
10	Fatigue Life (3P)	α=0.01063 β=42.708 γ=-42.662
11	Frechet (3P)	α=1.2525E+8 β=5.7017E+7 γ=-5.7017E+7
12	Gamma (3P)	α=121.31 β=0.04177 γ=-5.0273
13	Gen. Extreme Value	k=-0.35391 σ=0.46406 μ=-0.09499
14	Gen. Gamma (4P)	k=1.4955 α=106.36 β=0.31036 γ=-6.9776
15	Gen. Pareto	k=-1.1595 σ=1.7337 μ=-0.75454
16	Gumbel Max	σ=0.35519 μ=-0.15677
17	Gumbel Min	σ=0.35519 μ=0.25327
18	Hypersecant	σ=0.45555 μ=0.04825
19	Inv. Gaussian (3P)	λ=1.0368E+5 μ=27.751 γ=-27.703
20	Johnson SU	$\gamma$ =-0.03089 $\delta$ =2.6482 $\lambda$ =1.1214 $\xi$ =0.03421
21	Kumaraswamy	α <sub>1</sub> =3.5224 α <sub>2</sub> =645.95 a=-1.4205 b=8.8011
22	Laplace	λ=3.1044 μ=0.04825
23	Levy (2P)	σ=0.85413 γ=-1.1591
24	Log-Logistic (3P)	α=1.1482E+8 β=2.9211E+7 γ=-2.9211E+7
25	Logistic	σ=0.25116 μ=0.04825
26	Lognormal (3P)	σ=0.03951 μ=2.4435 γ=-11.472
27	Normal	σ=0.45555 μ=0.04825
28	Pearson 5 (3P)	α=235.97 β=1660.6 γ=-7.0175
29	Pearson 6 (4P)	$\alpha_1 = 17613.0 \ \alpha_2 = 19223.0$

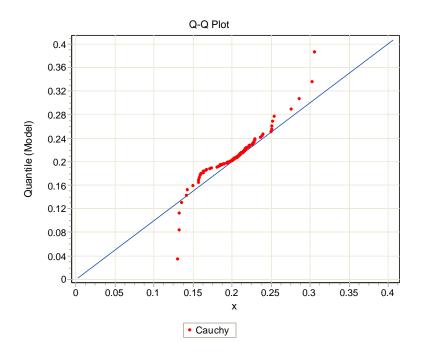
		β=47.495 γ=-43.471
30	Pert	m=-0.06435 a=-1.1701 b=1.7507
31	Power Function	α=0.80355 a=-1.0986 b=1.626
32	Rayleigh (2P)	σ=0.88706 γ=-1.1214
33	Triangular	m=-0.04082 a=-1.1755 b=1.6619
34	Uniform	a=-0.74078 b=0.83728
35	Weibull (3P)	α=3.5546 β=1.6362 γ=-1.4285
36	Burr	No fit (data min < 0)
37	Chi-Squared	No fit (data min < 0)
38	Dagum	No fit (data min < 0)
39	Erlang	No fit (data min < 0)
40	Exponential	No fit (data min < 0)
41	Fatigue Life	No fit (data min < 0)
42	Frechet	No fit (data min < 0)
43	Gamma	No fit (data min < 0)
44	Gen. Gamma	No fit (data min < 0)
45	Inv. Gaussian	No fit (data min < 0)
46	Johnson SB	No fit
47	Levy	No fit (data min < 0)
48	Log-Gamma	No fit
49	Log-Logistic	No fit (data min < 0)
50	Log-Pearson 3	No fit
51	Lognormal	No fit (data min < 0)
52	Nakagami	No fit
53	Pareto	No fit
54	Pareto 2	No fit
55	Pearson 5	No fit (data min < 0)
56	Pearson 6	No fit (data min < 0)
57	Rayleigh	No fit (data min < 0)
58	Reciprocal	No fit
59	Rice	No fit
60	Student's t	No fit
61	Weibull	No fit (data min < 0)

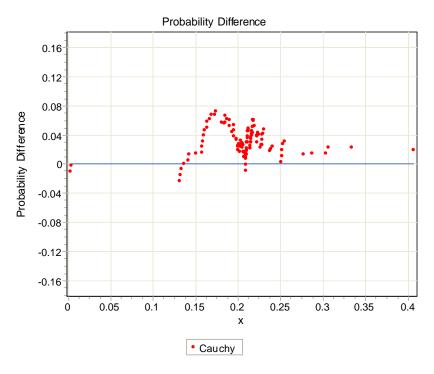
#	<u>Distribution</u>	Kolmogorov Smirnov		Anderson Darling		Chi-Squared	
		Statistic	Rank	Statistic	Rank	Statistic	Rank
1	<u>Beta</u>	0.04971	3	0.3848	7	3.5887	8
2	Burr (4P)	0.04868	2	0.24051	2	3.5194	6
3	Cauchy	0.08702	22	1.7372	22	5.9492	20
4	Chi-Squared (2P)	0.45938	34	37.396	35	165.42	30
5	Dagum (4P)	0.03932	1	0.14944	1	1.9003	1
6	Erlang (3P)	0.06216	15	0.54576	17	5.7981	19
7	<u>Error</u>	0.06508	18	0.36358	6	3.2274	5
8	Error Function	0.08935	23	1.1416	20	5.4636	17
9	Exponential (2P)	0.3051	33	18.843	31	127.38	29
10	Fatigue Life (3P)	0.05054	4	0.41272	10	3.2132	4
11	Frechet (3P)	0.10085	26	2.3771	23	20.195	26
12	Gamma (3P)	0.06594	19	0.64668	18	7.0077	21
13	Gen. Extreme Value	0.05178	7	4.2711	28	N/A	Λ
14	Gen. Gamma (4P)	0.05433	10	0.47531	12	3.8347	10
15	Gen. Pareto	0.09522	24	27.383	33	N/A	Λ.
16	Gumbel Max	0.11757	29	3.3262	26	14.977	25
17	Gumbel Min	0.06718	20	2.8961	25	2.2169	2
18	Hypersecant	0.07567	21	0.54092	16	5.7481	18
19	Inv. Gaussian (3P)	0.05248	8	0.41413	11	3.5847	7
20	Johnson SU	0.05734	14	0.33783	4	4.7691	14
21	Kumaraswamy	0.05563	13	0.50976	15	5.0899	15
22	<u>Laplace</u>	0.10396	27	1.1774	21	10.167	23
23	Levy (2P)	0.46605	35	28.03	34	256.78	31
24	Log-Logistic (3P)	0.05318	9	0.31995	3	3.9149	12
25	Logistic	0.06334	17	0.3609	5	3.9196	13
26	Lognormal (3P)	0.05449	11	0.48603	13	3.835	11
27	Normal	0.05083	6	0.39285	8	3.5918	9
28	Pearson 5 (3P)	0.06273	16	0.66467	19	8.1005	22
29	Pearson 6 (4P)	0.05067	5	0.4079	9	3.2127	3
30	<u>Pert</u>	0.11399	28	2.4923	24	14.756	24

31	Power Function	0.25167	32	16.578	30	N/A		
32	Rayleigh (2P)	0.157	30	4.7568	29	29.171	28	
33	<u>Triangular</u>	0.17157	31	3.9008	27	21.985	27	
34	<u>Uniform</u>	0.10032 25 20.647 32 N/A						
35	Weibull (3P)	0.05516	12	0.49596	14	5.3397	16	
36	Burr	No fit (d	ata min	< 0)				
37	Chi-Squared	No fit (d	ata min	< 0)				
38	Dagum	No fit (d	No fit (data min < 0)					
39	Erlang	No fit (d	No fit (data min < 0)					
40	Exponential	No fit (d	No fit (data min < 0)					
41	Fatigue Life	No fit (d	ata min	< 0)				
42	Frechet	No fit (data min < 0)						
43	Gamma	No fit (d	ata min	< 0)				
44	Gen. Gamma	No fit (d	ata min	< 0)				
45	Inv. Gaussian	No fit (d	ata min	< 0)				
46	Johnson SB	No fit						
47	Levy	No fit (d	ata min	< 0)				
48	Log-Gamma	No fit						
49	Log-Logistic	No fit (d	ata min	< 0)				
50	Log-Pearson 3	No fit						
51	Lognormal	No fit (d	ata min	< 0)				
52	Nakagami	No fit						
53	Pareto	No fit						
54	Pareto 2	No fit						
55	Pearson 5	No fit (d	ata min	< 0)				
56	Pearson 6	No fit (d	ata min	< 0)				
57	Rayleigh	No fit (d	ata min	< 0)				
58	Reciprocal	No fit						
59	Rice	No fit						
60	Student's t	No fit						
61	Weibull	No fit (d	ata min	< 0)				









#	Distribution	Parameters
1	Beta	α <sub>1</sub> =5220.3 α <sub>2</sub> =6999.0 a=-4.2663 b=6.1999
2	Burr	k=4.4432 α=5.2214 β=0.28335
3	Burr (4P)	k=5.329 α=6.7009 β=0.38348 γ=-0.08323
4	Cauchy	σ=0.01237 μ=0.20958
5	Dagum	k=0.28211 α=14.597 β=0.23546
6	Dagum (4P)	k=0.63769 α=90.297 β=1.5711 γ=-1.3534
7	Erlang	m=18 β=0.01084
8	Erlang (3P)	m=261 β=0.00307 γ=-0.59735
9	Error	k=1.0 σ=0.04715 μ=0.20508
10	Error Function	h=14.996
11	Exponential	λ=4.8761
12	Exponential (2P)	λ=4.9359 γ=0.00248
13	Fatigue Life	α=1.0053 β=0.12717
14	Fatigue Life (3P)	α=0.00438 β=10.685 γ=-10.48
15	Frechet	α=0.96569 β=0.10579
16	Frechet (3P)	α=2.9332E+8 β=1.9151E+7 γ=-1.9151E+7
17	Gamma	α=18.915 β=0.01084
18	Gamma (3P)	α=263.25 β=0.00304 γ=-0.59707
19	Gen. Extreme Value	k=-0.39742 σ=0.04114 μ=0.19343
20	Gen. Gamma	k=0.80679 α=10.594 β=0.01084
21	Gen. Gamma (4P)	k=1.3014 α=200.83 β=0.01497 γ=-0.67543
22	Gen. Pareto	k=-1.262 σ=0.1632 μ=0.13293
23	Gumbel Max	σ=0.03677 μ=0.18386
24	Gumbel Min	σ=0.03677 μ=0.2263
25	Hypersecant	σ=0.04715 μ=0.20508
26	Inv. Gaussian	λ=3.8791 μ=0.20508
27	Inv. Gaussian (3P)	λ=6.6077 μ=0.20971 γ=-9.5332E-4

28	Johnson SU	$\gamma$ =0.1435 $\delta$ =1.2827 $\lambda$ =0.04295 $\xi$ =0.21161
29	Kumaraswamy	$\alpha_1$ =5.4683 $\alpha_2$ =1935.5 a=-0.05854 b=1.0668
30	Laplace	λ=29.991 μ=0.20508
31	Levy	σ=0.09098
32	Levy (2P)	σ=0.07766 γ=0.00243
33	Log-Logistic	α=1.7008 β=0.18918
34	Log-Logistic (3P)	α=9.7528E+7 β=2.0501E+6 γ=-2.0501E+6
35	Log-Pearson 3	α=0.09769 β=-1.8931 γ=-1.4733
36	Logistic	σ=0.026 μ=0.20508
37	Lognormal	σ=0.58903 μ=-1.6582
38	Lognormal (3P)	σ=0.02231 μ=0.74278 γ=-1.8966
39	Nakagami	m=5.1655 Ω=0.04426
40	Normal	σ=0.04715 μ=0.20508
41	Pareto	α=0.23041 β=0.00248
42	Pareto 2	α=192.48 β=38.869
43	Pearson 5	α=32.113 β=6.4957
44	Pearson 5 (3P)	α=33.72 β=6.9822 γ=-0.00463
45	Pearson 6	$\alpha_1$ =8.6279 $\alpha_2$ =4.3806E+8 $\beta$ =1.0259E+7
46	Pearson 6 (4P)	$\alpha_1$ =488.94 $\alpha_2$ =10238.0 $\beta$ =21.873 $\gamma$ =-0.84085
47	Pert	m=0.20552 a=-0.01229 b=0.41464
48	Power Function	α=1.3094 a=7.0355E-4 b=0.40641
49	Rayleigh	σ=0.16363
50	Rayleigh (2P)	σ=0.14955 γ=-0.00113
51	Reciprocal	a=0.00248 b=0.40643
52	Rice	v=0.19943 σ=0.0474
53	Triangular	m=0.20958 a=-0.00538 b=0.41068
54	Uniform	a=0.12341 b=0.28676
55	Weibull	α=1.3462 β=0.28705
56	Weibull (3P)	α=5.5122 β=0.28228 γ=-0.05895
57	Chi-Squared	No fit
58	Chi-Squared (2P)	No fit

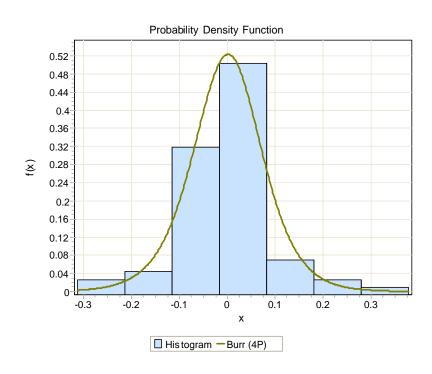
59	Johnson SB	No fit
60	Log-Gamma	No fit
61	Student's t	No fit

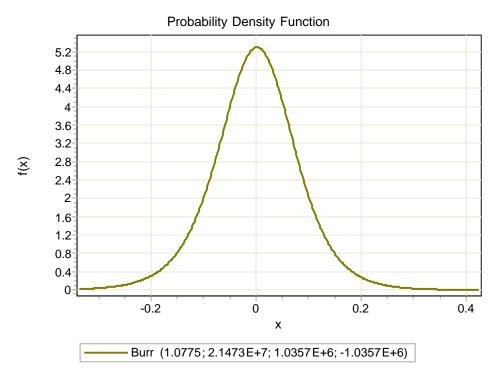
#	<u>Distribution</u>	Kolmogorov Smirnov		Anderson Darling		Chi-Squared	
		Statistic	Rank	Statistic	Rank	Statistic	Rank
1	<u>Beta</u>	0.17161	13	5.7497	10	57.252	15
2	Burr	0.1933	25	6.8766	19	58.393	19
3	Burr (4P)	0.19177	24	7.3467	20	69.671	30
4	Cauchy	0.07249	1	0.88493	1	5.9143	1
5	<u>Dagum</u>	0.19991	28	6.2733	14	46.385	10
6	Dagum (4P)	0.13008	6	2.651	2	29.383	5
7	Erlang	0.27726	39	12.358	31	89.522	31
8	Erlang (3P)	0.18297	20	6.5794	16	67.137	27
9	<u>Error</u>	0.12987	5	2.6642	4	25.411	2
10	Error Function	0.97936	56	1060.9	56	N/A	<b>\</b>
11	<u>Exponential</u>	0.45495	50	34.316	48	345.69	45
12	Exponential (2P)	0.45361	49	34.388	49	342.1	43
13	Fatigue Life	0.50323	53	41.607	52	414.87	47
14	Fatigue Life (3P)	0.17164	14	5.7458	9	57.252	16
15	<u>Frechet</u>	0.42511	48	33.426	47	502.6	48
16	Frechet (3P)	0.24655	34	12.722	32	100.49	32
17	<u>Gamma</u>	0.19147	22	7.9029	25	61.798	23
18	Gamma (3P)	0.19156	23	6.6896	18	67.511	28
19	Gen. Extreme Value	0.1588	11	19.527	37	N/A	1
20	Gen. Gamma	0.25341	36	14.675	34	113.63	34
21	Gen. Gamma (4P)	0.1718	15	6.0173	13	57.255	18
22	Gen. Pareto	0.20765	31	29.235	45	N/A	
23	Gumbel Max	0.22673	33	11.33	30	69.251	29
24	Gumbel Min	0.20663	29	8.6657	28	51.753	13
25	Hypersecant	0.14781	7	3.7832	7	37.623	9
26	Inv. Gaussian	0.18383	21	9.3868	29	60.565	21

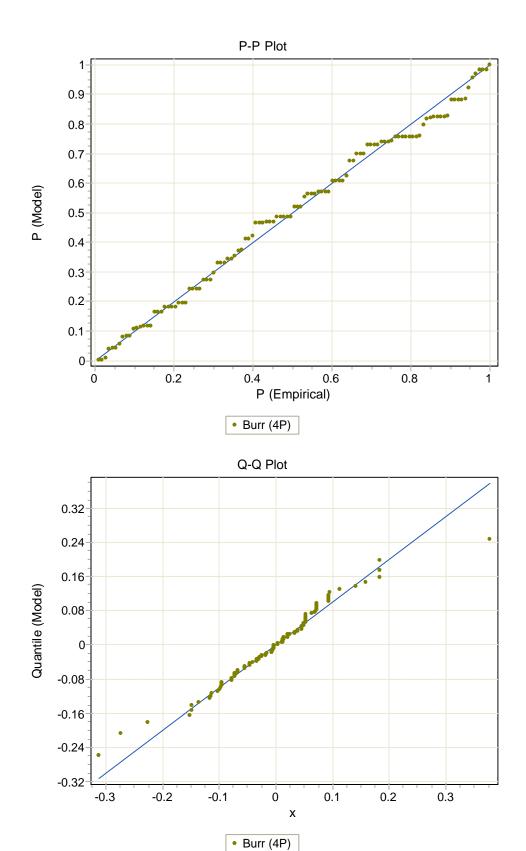
27	Inv. Gaussian (3P)	0.16888	12	8.1861	27	50.01	12
28	Johnson SU	0.12402	2	2.7713	5	29.332	4
29	Kumaraswamy	0.19649	27	7.9184	26	66.912	26
30	Laplace	0.12987	4	2.6642	3	25.411	3
31	Levy	0.39636	45	33.265	46	637.9	49
32	Levy (2P)	0.41874	47	37.772	51	675.42	50
33	Log-Logistic	0.34147	42	26.738	44	276.6	41
34	Log-Logistic (3P)	0.12733	3	2.8787	6	33.007	6
35	Log-Pearson 3	0.49902	52	103.75	54	N/A	
36	Logistic	0.15307	10	4.5054	8	48.848	11
37	Lognormal	0.29174	41	21.395	39	250.98	40
38	Lognormal (3P)	0.17436	18	5.7701	11	58.504	20
39	Nakagami	0.17286	16	6.6044	17	56.131	14
40	Normal	0.17352	17	5.8161	12	57.253	17
41	<u>Pareto</u>	0.58095	54	47.316	53	N/A	
42	Pareto 2	0.45974	51	34.85	50	347.98	46
43	Pearson 5	0.15219	9	7.8099	23	35.023	8
44	Pearson 5 (3P)	0.15099	8	7.7686	22	35.021	7
45	Pearson 6	0.25236	35	13.242	33	107.64	33
46	Pearson 6 (4P)	0.1808	19	6.283	15	66.007	24
47	<u>Pert</u>	0.26351	37	15.29	36	165.33	37
48	Power Function	0.4041	46	26.281	43	202.09	38
49	Rayleigh	0.28917	40	19.761	38	150.94	35
50	Rayleigh (2P)	0.34855	43	23.374	40	246.11	39
51	Reciprocal	0.75968	55	108.16	55	308.05	42
52	Rice	0.20752	30	7.3749	21	61.001	22
53	Triangular	0.26513	38	15.13	35	154.7	36
54	<u>Uniform</u>	0.22239	32	24.739	41	N/A	
55	Weibull	0.3728	44	26.055	42	343.49	44
56	Weibull (3P)	0.19559	26	7.8389	24	66.91	25
57	Chi-Squared	No fit					
58	Chi-Squared (2P)	No fit					
59	Johnson SB	No fit					
60	Log-Gamma	No fit					

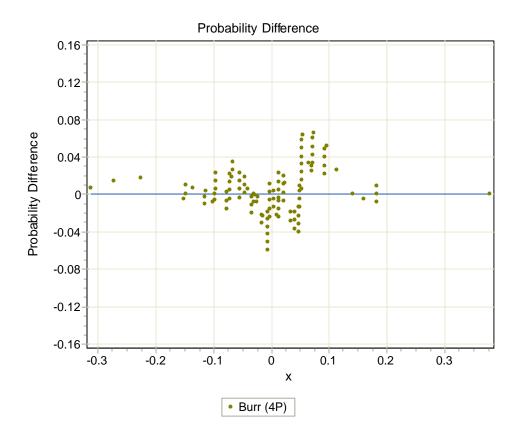
61 Student's t No fit

#### RoSD (constant-variance)









#	Distribution	Parameters
1	Beta	$\alpha_1$ =8673.4 $\alpha_2$ =16349.0 a=-10.414 b=19.625
2	Burr (4P)	k=1.0775 α=2.1473E+7 β=1.0357E+6 γ=-1.0357E+6
3	Cauchy	σ=0.0456 μ=0.00557
4	Dagum (4P)	k=108.9 α=15.533 β=1.198 γ=-1.6734
5	Erlang (3P)	m=213 β=0.00626 γ=-1.3365
6	Error	k=1.0 σ=0.09079 μ=-0.00181
7	Error Function	h=7.7884
8	Exponential (2P)	λ=3.2144 γ=-0.31291
9	Fatigue Life (3P)	α=0.00765 β=11.813 γ=-11.815
10	Frechet (3P)	α=2.0152 β=0.25672 γ=-0.3228
11	Gamma (3P)	α=193.54 β=0.00657 γ=-1.2744

Sear-Barton Face   E-0.53068 G=0.50707 pc-0.5208	12	Gen. Extreme Value	k=-0.35048 σ=0.08769 μ=-0.02906
13   Gen. Gamma (4P)   β=0.00556 γ=-0.66493     14   Gen. Pareto   k=-1.1516 σ=0.32605 μ=-0.15335     15   Gumbel Max   σ=0.07079 μ=-0.04267     16   Gumbel Min   σ=0.07079 μ=-0.00181     17   Hypersecant   σ=0.09079 μ=-0.00181     18   Inv. Gaussian (3P)   λ=84796.0 μ=8.8515 γ=8.8532     19   Johnson SU   γ=-0.0299 δ=1.5544   λ=0.1131 ξ=0.00449     20   Kumaraswamy   α <sub>1</sub> =4.2325 α <sub>2</sub> =1192.8   α=0.37509 b=1.7922     21   Laplace   λ=15.577 μ=-0.00181     22   Levy (2P)   σ=0.26144 γ=-0.33007     23   Log-Logistic (3P)   α=8.1885E+7 β=3.8564E+6 γ=3.8564E+6     24   Logistic   σ=0.05005 μ=-0.00181     25   Lognormal (3P)   σ=0.0284 μ=1.1629 γ=-3.2023     26   Normal   σ=0.09079 μ=-0.00181     27   Pearson 5 (3P)   α=312.44 β=501.76 γ=-1.6137     28   Pearson 6 (4P)   α <sub>1</sub> =30936.0 α <sub>2</sub> =22843.0 β=7.6575 γ=-10.373     29   Pert   m=-0.0167 a=-0.32604 b=0.39807     30   Power Function   α=0.91541 a=-0.31291 b=0.37826     31   Rayleigh (2P)   σ=0.23163 γ=-0.31667     32   Triangular   m=-0.00576 a=-0.32285 b=0.38592     33   Uniform   a=-0.15906 b=0.15544     34   Weibull (3P)   α=4.2515 β=0.40766 γ=-0.37582     35   Burr   No fit (data min < 0)     36   Chi-Squared   No fit (data min < 0)     37   Chi-Squared   No fit (data min < 0)     38   Dagum   No fit (data min < 0)     39   Erlang   No fit (data min < 0)     40   Exponential   No fit (data min < 0)	12	Gen. Extreme value	·
Solution	13	Gen. Gamma (4P)	
Gumbel Min   σ=0.07079   μ=0.03905	14	Gen. Pareto	k=-1.1516 σ=0.32605 μ=-0.15335
17 Hypersecant $\sigma$ =0.09079 $\mu$ =-0.00181 18 Inv. Gaussian (3P) $\lambda$ =84796.0 $\mu$ =8.8515 $\gamma$ =8.8532 19 Johnson SU $\gamma$ =-0.0299 $\delta$ =1.5544 $\lambda$ =0.1131 $\xi$ =-0.00449 20 Kumaraswamy $\alpha_1$ =4.2325 $\alpha_2$ =1192.8 $\alpha$ =-0.37509 b=1.7922 21 Laplace $\lambda$ =15.577 $\mu$ =-0.00181 22 Levy (2P) $\sigma$ =0.26144 $\gamma$ =-0.33007 23 Log-Logistic (3P) $\alpha$ =8.1885E+7 $\beta$ =3.8564E+6 $\gamma$ =-3.8564E+6 24 Logistic $\sigma$ =0.05005 $\mu$ =-0.00181 25 Lognormal (3P) $\sigma$ =0.0284 $\mu$ =1.1629 $\gamma$ =-3.2023 26 Normal $\sigma$ =0.09079 $\mu$ =-0.00181 27 Pearson 5 (3P) $\alpha$ =312.44 $\beta$ =501.76 $\gamma$ =-1.6137 28 Pearson 6 (4P) $\alpha$ =312.44 $\beta$ =501.76 $\gamma$ =-1.6137 29 Pert $\alpha$ =-0.0167 $\alpha$ =-0.32604 $\alpha$ =0.39807 30 Power Function $\alpha$ =0.91541 $\alpha$ =-0.31291 $\alpha$ =0.37826 31 Rayleigh (2P) $\alpha$ =0.23163 $\gamma$ =-0.31667 32 Triangular $\alpha$ =-0.0576 $\alpha$ =-0.32285 $\alpha$ =0.38592 33 Uniform $\alpha$ =-0.15906 $\alpha$ =0.15544 34 Weibull (3P) $\alpha$ =4.2515 $\alpha$ =0.40766 $\alpha$ =-0.37582 35 Burr No fit (data min < 0) 36 Chi-Squared (2P) No fit 38 Dagum No fit (data min < 0) 39 Erlang No fit (data min < 0)	15	Gumbel Max	σ=0.07079 μ=-0.04267
18   Inv. Gaussian (3P)   \$\lambda_{=84796.0}\$   \$\mu_{=8.8515}\$   \$\gamma_{=}\$   \$\sigma_{=}\$   \$\sigma_{=}	16	Gumbel Min	σ=0.07079 μ=0.03905
19 Johnson SU $\gamma$ =-0.0299 δ=1.5544 $\lambda$ =0.1131 $\xi$ =-0.00449 20 Kumaraswamy $\alpha_1$ =4.2325 $\alpha_2$ =1192.8 $\alpha$ =-0.37509 b=1.7922 21 Laplace $\lambda$ =15.577 $\mu$ =-0.00181 22 Levy (2P) $\sigma$ =0.26144 $\gamma$ =-0.33007 23 Log-Logistic (3P) $\alpha$ =8.1885E+7 $\beta$ =3.8564E+6 $\gamma$ =-3.8564E+6 (24 Logistic $\sigma$ =0.05005 $\mu$ =-0.00181 25 Lognormal (3P) $\sigma$ =0.0284 $\mu$ =1.1629 $\gamma$ =-3.2023 26 Normal $\sigma$ =0.09079 $\mu$ =-0.00181 27 Pearson 5 (3P) $\alpha$ =312.44 $\beta$ =501.76 $\gamma$ =-1.6137 28 Pearson 6 (4P) $\alpha$ 1=30936.0 $\alpha$ 2=22843.0 $\alpha$ 2=7.6575 $\alpha$ 2=10.373 29 Pert $\alpha$ 30 Power Function $\alpha$ 4=0.91541 $\alpha$ 4=-0.31291 $\alpha$ 5=0.37826 31 Rayleigh (2P) $\alpha$ 5=0.23163 $\alpha$ 5=0.31667 32 Triangular $\alpha$ 5=0.15906 $\alpha$ 5=0.35892 33 Uniform $\alpha$ 6=0.15906 $\alpha$ 6=0.15544 34 Weibull (3P) $\alpha$ 6=4.2515 $\alpha$ 6=0.40766 $\alpha$ 6=0.37582 35 Burr No fit (data min < 0) 36 Chi-Squared No fit (data min < 0) 37 Chi-Squared (2P) No fit (data min < 0) 39 Erlang No fit (data min < 0) 39 Erlang No fit (data min < 0)	17	Hypersecant	σ=0.09079 μ=-0.00181
30   30   30   3   3   3   3   3   3	18	Inv. Gaussian (3P)	λ=84796.0 μ=8.8515 γ=-8.8532
20       Rumaraswamy       a=-0.37509 b=1.7922         21       Laplace       λ=15.577 μ=-0.00181         22       Levy (2P)       σ=0.26144 γ=-0.33007         23       Log-Logistic (3P)       α=8.1885E+7 β=3.8564E+6 γ=-3.8564E+6         24       Logistic       σ=0.05005 μ=-0.00181         25       Lognormal (3P)       σ=0.0284 μ=1.1629 γ=-3.2023         26       Normal       σ=0.09079 μ=-0.00181         27       Pearson 5 (3P)       α=312.44 β=501.76 γ=-1.6137         28       Pearson 6 (4P)       α=39936.0 α2=22843.0 β=7.6575 γ=-10.373         29       Pert       m=-0.0167 a=-0.32604 b=0.39807         30       Power Function       α=0.91541 a=-0.31291 b=0.37826         31       Rayleigh (2P)       σ=0.23163 γ=-0.31667         32       Triangular       m=-0.00576 a=-0.32285 b=0.38592         33       Uniform       a=-0.15906 b=0.15544         34       Weibull (3P)       α=4.2515 β=0.40766 γ=-0.37582         35       Burr       No fit (data min < 0)	19	Johnson SU	,
22 Levy (2P) σ=0.26144 γ=-0.33007 23 Log-Logistic (3P) α=8.1885E+7 β=3.8564E+6 γ=-3.8564E+6 24 Logistic σ=0.05005 μ=-0.00181 25 Lognormal (3P) σ=0.0284 μ=1.1629 γ=-3.2023 26 Normal σ=0.09079 μ=-0.00181 27 Pearson 5 (3P) α=312.44 β=501.76 γ=-1.6137 28 Pearson 6 (4P) α <sub>1</sub> =30936.0 α <sub>2</sub> =22843.0 β=7.6575 γ=-10.373 29 Pert m=-0.0167 a=-0.32604 b=0.39807 30 Power Function α=0.91541 a=-0.31291 b=0.37826 31 Rayleigh (2P) σ=0.23163 γ=-0.31667 32 Triangular m=-0.00576 a=-0.32285 b=0.38592 33 Uniform a=-0.15906 b=0.15544 34 Weibull (3P) α=4.2515 β=0.40766 γ=-0.37582 35 Burr No fit (data min < 0) 36 Chi-Squared No fit (data min < 0) 37 Chi-Squared (2P) No fit 38 Dagum No fit (data min < 0) 40 Exponential No fit (data min < 0)	20	Kumaraswamy	
23 Log-Logistic (3P) $α=8.1885E+7$ $β=3.8564E+6$ $γ=-3.8564E+6$ 24 Logistic $σ=0.05005$ $µ=-0.00181$ 25 Lognormal (3P) $σ=0.0284$ $µ=1.1629$ $γ=-3.2023$ 26 Normal $σ=0.09079$ $µ=-0.00181$ 27 Pearson 5 (3P) $α=312.44$ $β=501.76$ $γ=-1.6137$ 28 Pearson 6 (4P) $β=7.6575$ $γ=-10.373$ 29 Pert $m=-0.0167$ $a=-0.32604$ $b=0.39807$ 30 Power Function $α=0.91541$ $a=-0.31291$ $b=0.37826$ 31 Rayleigh (2P) $σ=0.23163$ $γ=-0.31667$ 32 Triangular $m=-0.00576$ $a=-0.32285$ $b=0.38592$ 33 Uniform $a=-0.15906$ $b=0.15544$ 34 Weibull (3P) $α=4.2515$ $β=0.40766$ $γ=-0.37582$ 35 Burr No fit (data min < 0) 36 Chi-Squared No fit (data min < 0) 37 Chi-Squared (2P) No fit 38 Dagum No fit (data min < 0) 40 Exponential No fit (data min < 0)	21	Laplace	λ=15.577 μ=-0.00181
24 Logistic $\sigma$ =0.05005 μ=-0.00181  25 Lognormal (3P) $\sigma$ =0.0284 μ=1.1629 γ=-3.2023  26 Normal $\sigma$ =0.09079 μ=-0.00181  27 Pearson 5 (3P) $\sigma$ =312.44 $\sigma$ =501.76 γ=-1.6137  28 Pearson 6 (4P) $\sigma$ =312.44 $\sigma$ =501.76 γ=-1.6137  29 Pert $\sigma$ =0.0167 a=-0.32604 b=0.39807  30 Power Function $\sigma$ =0.91541 a=-0.31291 b=0.37826  31 Rayleigh (2P) $\sigma$ =0.23163 γ=-0.31667  32 Triangular $\sigma$ =0.00576 a=-0.32285 b=0.38592  33 Uniform $\sigma$ =0.15906 b=0.15544  34 Weibull (3P) $\sigma$ =4.2515 $\sigma$ =0.40766 γ=-0.37582  35 Burr No fit (data min < 0)  36 Chi-Squared No fit (data min < 0)  37 Chi-Squared (2P) No fit  38 Dagum No fit (data min < 0)  40 Exponential No fit (data min < 0)	22	Levy (2P)	σ=0.26144 γ=-0.33007
25 Lognormal (3P) $\sigma$ =0.0284 μ=1.1629 γ=-3.2023 26 Normal $\sigma$ =0.09079 μ=-0.00181 27 Pearson 5 (3P) $\alpha$ =312.44 $\beta$ =501.76 γ=-1.6137 28 Pearson 6 (4P) $\alpha$ =30936.0 $\alpha$ =222843.0 $\beta$ =7.6575 $\gamma$ =-10.373 29 Pert $\alpha$ =-0.0167 a=-0.32604 b=0.39807 30 Power Function $\alpha$ =0.91541 a=-0.31291 b=0.37826 31 Rayleigh (2P) $\alpha$ =0.23163 $\gamma$ =-0.31667 32 Triangular $\alpha$ =-0.00576 a=-0.32285 b=0.38592 33 Uniform $\alpha$ =-0.15906 b=0.15544 34 Weibull (3P) $\alpha$ =4.2515 $\beta$ =0.40766 $\gamma$ =-0.37582 35 Burr No fit (data min < 0) 36 Chi-Squared No fit (data min < 0) 37 Chi-Squared (2P) No fit 38 Dagum No fit (data min < 0) 40 Exponential No fit (data min < 0) No fit (data min < 0)	23	Log-Logistic (3P)	α=8.1885E+7 β=3.8564E+6 γ=-3.8564E+6
26       Normal       σ=0.09079 μ=-0.00181         27       Pearson 5 (3P)       α=312.44 β=501.76 γ=-1.6137         28       Pearson 6 (4P)       α₁=30936.0 α₂=22843.0 β=7.6575 γ=-10.373         29       Pert       m=-0.0167 a=-0.32604 b=0.39807         30       Power Function       α=0.91541 a=-0.31291 b=0.37826         31       Rayleigh (2P)       σ=0.23163 γ=-0.31667         32       Triangular       m=-0.00576 a=-0.32285 b=0.38592         33       Uniform       a=-0.15906 b=0.15544         34       Weibull (3P)       α=4.2515 β=0.40766 γ=-0.37582         35       Burr       No fit (data min < 0)	24	Logistic	σ=0.05005 μ=-0.00181
27 Pearson 5 (3P) α=312.44 β=501.76 γ=-1.6137  28 Pearson 6 (4P) α <sub>1</sub> =30936.0 α <sub>2</sub> =22843.0 β=7.6575 γ=-10.373  29 Pert m=-0.0167 a=-0.32604 b=0.39807  30 Power Function α=0.91541 a=-0.31291 b=0.37826  31 Rayleigh (2P) σ=0.23163 γ=-0.31667  32 Triangular m=-0.00576 a=-0.32285 b=0.38592  33 Uniform a=-0.15906 b=0.15544  34 Weibull (3P) α=4.2515 β=0.40766 γ=-0.37582  35 Burr No fit (data min < 0)  36 Chi-Squared No fit (data min < 0)  37 Chi-Squared (2P) No fit  38 Dagum No fit (data min < 0)  39 Erlang No fit (data min < 0)  40 Exponential No fit (data min < 0)	25	Lognormal (3P)	σ=0.0284 μ=1.1629 γ=-3.2023
28       Pearson 6 (4P) $\alpha_1$ =30936.0 $\alpha_2$ =22843.0 $\beta$ =7.6575 $\gamma$ =-10.373         29       Pert       m=-0.0167 a=-0.32604 b=0.39807         30       Power Function $\alpha$ =0.91541 a=-0.31291 b=0.37826         31       Rayleigh (2P) $\sigma$ =0.23163 $\gamma$ =-0.31667         32       Triangular       m=-0.00576 a=-0.32285 b=0.38592         33       Uniform       a=-0.15906 b=0.15544         34       Weibull (3P) $\alpha$ =4.2515 $\beta$ =0.40766 $\gamma$ =-0.37582         35       Burr       No fit (data min < 0)	26	Normal	σ=0.09079 μ=-0.00181
28       Featson 6 (4F) $\beta$ =7.6575 $\gamma$ =-10.373         29       Pert $m$ =-0.0167 $\alpha$ =-0.32604 $\beta$ =0.39807         30       Power Function $\alpha$ =0.91541 $\alpha$ =-0.31291 $\beta$ =0.37826         31       Rayleigh (2P) $\alpha$ =0.23163 $\gamma$ =-0.31667         32       Triangular $\alpha$ =-0.00576 $\alpha$ =-0.32285 $\alpha$ =0.38592         33       Uniform $\alpha$ =-0.15906 $\alpha$ =-0.3544         34       Weibull (3P) $\alpha$ =4.2515 $\alpha$ =0.40766 $\alpha$ =-0.37582         35       Burr       No fit (data min < 0)	27	Pearson 5 (3P)	α=312.44 β=501.76 γ=-1.6137
30       Power Function       α=0.91541 a=-0.31291 b=0.37826         31       Rayleigh (2P)       σ=0.23163 γ=-0.31667         32       Triangular       m=-0.00576 a=-0.32285 b=0.38592         33       Uniform       a=-0.15906 b=0.15544         34       Weibull (3P)       α=4.2515 β=0.40766 γ=-0.37582         35       Burr       No fit (data min < 0)	28	Pearson 6 (4P)	I I
31 Rayleigh (2P) σ=0.23163 γ=-0.31667  32 Triangular m=-0.00576 a=-0.32285 b=0.38592  33 Uniform a=-0.15906 b=0.15544  34 Weibull (3P) α=4.2515 β=0.40766 γ=-0.37582  35 Burr No fit (data min < 0)  36 Chi-Squared No fit (data min < 0)  37 Chi-Squared (2P) No fit  38 Dagum No fit (data min < 0)  39 Erlang No fit (data min < 0)  40 Exponential No fit (data min < 0)	29	Pert	m=-0.0167 a=-0.32604 b=0.39807
32 Triangular m=-0.00576 a=-0.32285 b=0.38592 33 Uniform a=-0.15906 b=0.15544 34 Weibull (3P) α=4.2515 β=0.40766 γ=-0.37582 35 Burr No fit (data min < 0) 36 Chi-Squared No fit (data min < 0) 37 Chi-Squared (2P) No fit 38 Dagum No fit (data min < 0) 39 Erlang No fit (data min < 0) 40 Exponential No fit (data min < 0)	30	Power Function	α=0.91541 a=-0.31291 b=0.37826
33 Uniform a=-0.15906 b=0.15544  34 Weibull (3P) α=4.2515 β=0.40766 γ=-0.37582  35 Burr No fit (data min < 0)  36 Chi-Squared No fit (data min < 0)  37 Chi-Squared (2P) No fit  38 Dagum No fit (data min < 0)  39 Erlang No fit (data min < 0)  40 Exponential No fit (data min < 0)	31	Rayleigh (2P)	σ=0.23163 γ=-0.31667
34 Weibull (3P) α=4.2515 β=0.40766 γ=-0.37582  35 Burr No fit (data min < 0)  36 Chi-Squared No fit (data min < 0)  37 Chi-Squared (2P) No fit  38 Dagum No fit (data min < 0)  39 Erlang No fit (data min < 0)  40 Exponential No fit (data min < 0)	32	Triangular	m=-0.00576 a=-0.32285 b=0.38592
35       Burr       No fit (data min < 0)	33	Uniform	a=-0.15906 b=0.15544
36       Chi-Squared       No fit (data min < 0)	34	Weibull (3P)	α=4.2515 β=0.40766 γ=-0.37582
37 Chi-Squared (2P) No fit  38 Dagum No fit (data min < 0)  39 Erlang No fit (data min < 0)  40 Exponential No fit (data min < 0)	35	Burr	No fit (data min < 0)
38 Dagum No fit (data min < 0) 39 Erlang No fit (data min < 0) 40 Exponential No fit (data min < 0)	36	Chi-Squared	No fit (data min < 0)
39 Erlang No fit (data min < 0) 40 Exponential No fit (data min < 0)	37	Chi-Squared (2P)	No fit
40 Exponential No fit (data min < 0)	38	Dagum	No fit (data min < 0)
	39	Erlang	No fit (data min < 0)
41   Fatigue Life   No fit (data min < 0)	40	Exponential	No fit (data min < 0)
	41	Fatigue Life	No fit (data min < 0)

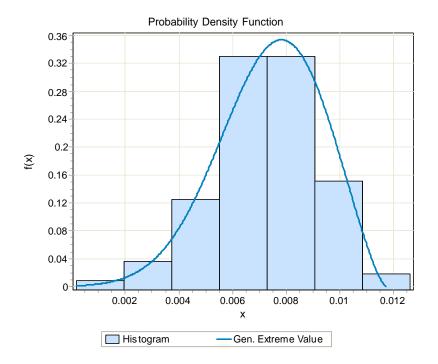
42       Frechet       No fit (data min < 0)         43       Gamma       No fit (data min < 0)         44       Gen. Gamma       No fit (data min < 0)         45       Inv. Gaussian       No fit (data min < 0)         46       Johnson SB       No fit         47       Levy       No fit (data min < 0)         48       Log-Gamma       No fit (data min < 0)         50       Log-Pearson 3       No fit         51       Lognormal       No fit (data min < 0)         52       Nakagami       No fit         53       Pareto       No fit
44       Gen. Gamma       No fit (data min < 0)
45 Inv. Gaussian No fit (data min < 0)  46 Johnson SB No fit  47 Levy No fit (data min < 0)  48 Log-Gamma No fit  49 Log-Logistic No fit (data min < 0)  50 Log-Pearson 3 No fit  51 Lognormal No fit (data min < 0)  52 Nakagami No fit  53 Pareto No fit
46       Johnson SB       No fit         47       Levy       No fit (data min < 0)
47         Levy         No fit (data min < 0)
48         Log-Gamma         No fit           49         Log-Logistic         No fit (data min < 0)
49 Log-Logistic No fit (data min < 0) 50 Log-Pearson 3 No fit 51 Lognormal No fit (data min < 0) 52 Nakagami No fit 53 Pareto No fit
50 Log-Pearson 3 No fit  51 Lognormal No fit (data min < 0)  52 Nakagami No fit  53 Pareto No fit
51 Lognormal No fit (data min < 0) 52 Nakagami No fit 53 Pareto No fit
52 Nakagami No fit 53 Pareto No fit
53 Pareto No fit
54 Pareto 2 No fit
55 Pearson 5 No fit (data min < 0)
S6   Pearson 6   No fit (data min < 0)
S7   Rayleigh   No fit (data min < 0)
58 Reciprocal No fit
59 Rice No fit
60 Student's t No fit
61 Weibull No fit (data min < 0)

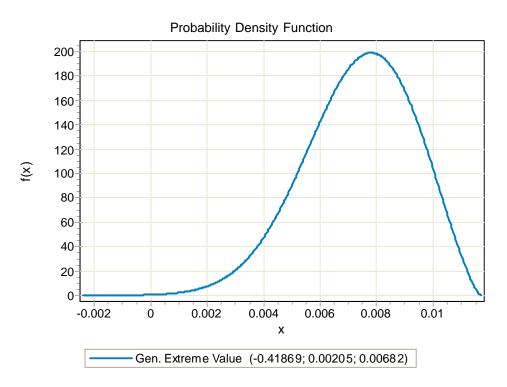
#	<u>Distribution</u>	Kolmogorov Smirnov		Anderson Darling		Chi-Squared	
		Statistic	Rank	Statistic	Rank	Statistic	Rank
1	Beta	0.09656	12	1.2453	8	12.833	14
2	Burr (4P)	0.06838	1	0.48203	1	10.495	5
3	Cauchy	0.08768	8	1.5509	18	18.75	22
4	Dagum (4P)	0.16381	27	5.9902	25	43.427	24
5	Erlang (3P)	0.09742	15	1.4583	16	11.342	10
6	<u>Error</u>	0.08365	6	0.6787	5	10.726	8
7	Error Function	0.10356	20	1.2723	13	10.724	7
8	Exponential (2P)	0.4071	33	27.887	33	184.68	28

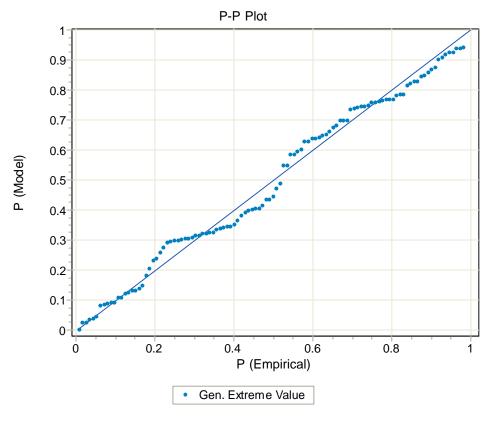
9	Fatigue Life (3P)	0.09618	10	1.251	9	12.831	13
10	Frechet (3P)	0.25023	30	16.324	29	N/A	
11	Gamma (3P)	0.09669	14	1.4169	15	13.448	16
12	Gen. Extreme Value	0.09553	9	5.0211	24	N/A	
13	Gen. Gamma (4P)	0.10185	19	1.7455	19	13.464	17
14	Gen. Pareto	0.14263	24	25.15	32	N/A	
15	Gumbel Max	0.15124	25	3.6519	22	15.964	20
16	Gumbel Min	0.11555	23	3.7271	23	9.0804	4
17	Hypersecant	0.0767	3	0.52921	4	6.2037	1
18	Inv. Gaussian (3P)	0.09823	17	1.2531	10	14.566	18
19	Johnson SU	0.07954	5	0.51449	3	6.7392	3
20	Kumaraswamy	0.11344	21	2.223	21	17.553	21
21	<u>Laplace</u>	0.08365	7	0.6787	6	10.726	9
22	Levy (2P)	0.51588	34	36.58	34	241.4	29
23	Log-Logistic (3P)	0.06955	2	0.50373	2	10.497	6
24	Logistic	0.07902	4	0.68051	7	6.2955	2
25	Lognormal (3P)	0.09641	11	1.3269	14	12.457	11
26	Normal	0.09785	16	1.2719	12	12.835	15
27	Pearson 5 (3P)	0.10061	18	1.51	17	14.813	19
28	Pearson 6 (4P)	0.09656	13	1.2614	11	12.831	12
29	<u>Pert</u>	0.20309	28	8.1823	26	54.852	26
30	Power Function	0.32053	32	23.251	31	N/A	
31	Rayleigh (2P)	0.27302	31	13.509	28	65.922	27
32	<u>Triangular</u>	0.24586	29	9.1402	27	54.265	25
33	<u>Uniform</u>	0.155	26	22.667	30	N/A	
34	Weibull (3P)	0.11484	22	2.2046	20	19.04	23
35	Burr	No fit (d	ata min	< 0)		1	
36	Chi-Squared	No fit (d	ata min	ı < 0)			
37	Chi-Squared (2P)	No fit					
38	Dagum	No fit (d	ata min	(0)			
39	Erlang	No fit (d	ata min	ı < 0)			
40	Exponential	No fit (d	ata min	ı < 0)			
41	Fatigue Life	No fit (d	ata min	ı < 0)			
42	Frechet	No fit (d	ata min	ı < 0)			

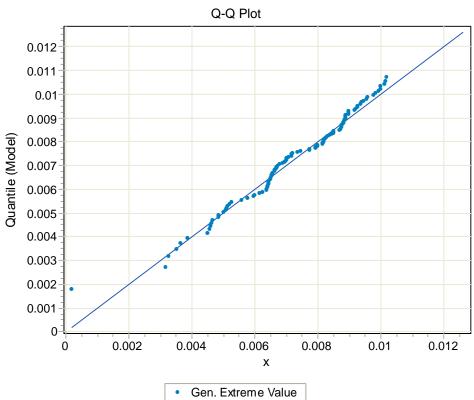
43	Gamma	No fit (data min < 0)
44	Gen. Gamma	No fit (data min < 0)
45	Inv. Gaussian	No fit (data min < 0)
46	Johnson SB	No fit
47	Levy	No fit (data min < 0)
48	Log-Gamma	No fit
49	Log-Logistic	No fit (data min < 0)
50	Log-Pearson 3	No fit
51	Lognormal	No fit (data min < 0)
52	Nakagami	No fit
53	Pareto	No fit
54	Pareto 2	No fit
55	Pearson 5	No fit (data min < 0)
56	Pearson 6	No fit (data min < 0)
57	Rayleigh	No fit (data min < 0)
58	Reciprocal	No fit
59	Rice	No fit
60	Student's t	No fit
61	Weibull	No fit (data min < 0)

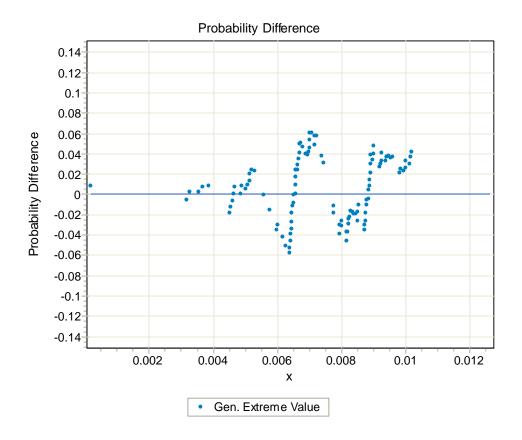
#### RoSD (ewma)











#	Distribution	Parameters
1	Beta	$\begin{array}{c} \alpha_1 = 884.8 \ \alpha_2 = 94.012 \\ a = -0.17982 \ b = 0.02725 \end{array}$
2	Burr	k=1.2263E+14 α=0.91967 β=1.6529E+13
3	Burr (4P)	k=192.54 α=5.9621 β=0.02676 γ=-0.00292
4	Cauchy	σ=0.00125 μ=0.00753
5	Dagum	k=0.187 α=16.441 β=0.00946
6	Dagum (4P)	k=0.21929 α=0.89273 β=0.95053 γ=1.7769E-4
7	Erlang	m=13 β=5.2716E-4
8	Erlang (3P)	m=149 β=1.6514E-4 γ=-0.01729
9	Error	k=1.3966 σ=0.00197 μ=0.00738
10	Error Function	h=358.61
11	Exponential	λ=135.59

12	Exponential (2P)	λ=138.94 γ=1.7769Ε-4
13	Fatigue Life	α=0.62518 β=0.0061
14	Fatigue Life (3P)	α=0.00586 β=0.33489 γ=-0.32752
15	Frechet	α=1.8385 β=0.0051
16	Frechet (3P)	α=4.2726 β=0.00775 γ=-0.00149
17	Gamma	α=13.99 β=5.2716Ε-4
18	Gamma (3P)	α=141.87 β=1.7099Ε-4 γ=-0.01686
19	Gen. Extreme Value	k=-0.41869 σ=0.00205 μ=0.00682
20	Gen. Gamma	k=0.90715 α=10.878 β=5.2716E-4
21	Gen. Gamma (4P)	k=8.4806 α=6.2373 β=0.03258 γ=-0.03271
22	Gen. Pareto	k=-1.3131 σ=0.00837 μ=0.00376
23	Gumbel Max	σ=0.00154 μ=0.00649
24	Gumbel Min	σ=0.00154 μ=0.00826
25	Hypersecant	σ=0.00197 μ=0.00738
26	Inv. Gaussian	λ=0.10318 μ=0.00738
27	Inv. Gaussian (3P)	λ=0.09155 μ=0.00733 γ=1.1415Ε-4
28	Johnson SU	$\gamma$ =0.77047 $\delta$ =2.5318 $\lambda$ =0.00439 $\xi$ =0.00884
29	Kumaraswamy	α <sub>1</sub> =5.9407 α <sub>2</sub> =369.66 a=-0.00296 b=0.02716
30	Laplace	λ=717.23 μ=0.00738
31	Levy	σ=0.00516
32	Levy (2P)	σ=0.0067 γ=1.7764Ε-4
33	Log-Logistic	α=3.0018 β=0.00692
34	Log-Logistic (3P)	α=1.4463E+5 β=159.56 γ=-159.55
35	Log-Pearson 3	α=0.14214 β=-1.1752 γ=-4.8013
36	Logistic	σ=0.00109 μ=0.00738
37	Lognormal	σ=0.44111 μ=-4.9684
38	Lognormal (3P)	σ=0.02921 μ=-2.689 γ=-0.06056
39	Nakagami	m=4.2445 Ω=5.8246E-5
40	Normal	σ=0.00197 μ=0.00738
41	Pareto	α=0.2727 β=1.7769Ε-4

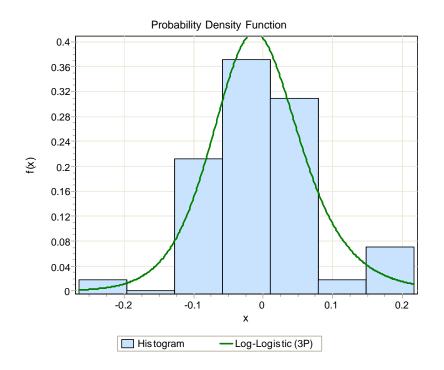
Pareto 2	α=522.05 β=3.8482
Pearson 5	α=1.8262 β=0.00943
Pearson 5 (3P)	α=12.492 β=0.08537 γ=6.8529Ε-5
Pearson 6	$\alpha_1$ =10.404 $\alpha_2$ =1.3260E+6 $\beta$ =930.85
Pearson 6 (4P)	$\alpha_1 = 193.1  \alpha_2 = 885.51$ $\beta = 0.12023  \gamma = -0.01888$
Pert	m=0.00782 a=-3.4638E-4 b=0.01295
Power Function	α=1.6779 a=8.3257E-6 b=0.01261
Rayleigh	σ=0.00588
Rayleigh (2P)	σ=0.00533 γ=9.8462Ε-5
Reciprocal	a=1.7754E-4 b=0.01263
Rice	v=0.00709 σ=0.002
Triangular	m=0.00843 a=-2.7003E-6 b=0.01288
Uniform	a=0.00396 b=0.01079
Weibull	α=2.2858 β=0.00884
Weibull (3P)	α=5.9576 β=0.01113 γ=-0.00297
Chi-Squared	No fit
Chi-Squared (2P)	No fit
Johnson SB	No fit
Log-Gamma	No fit
Student's t	No fit
	Pearson 5 Pearson 5 (3P) Pearson 6 Pearson 6 (4P) Pert Power Function Rayleigh Rayleigh (2P) Reciprocal Rice Triangular Uniform Weibull Weibull (3P) Chi-Squared Chi-Squared (2P) Johnson SB Log-Gamma

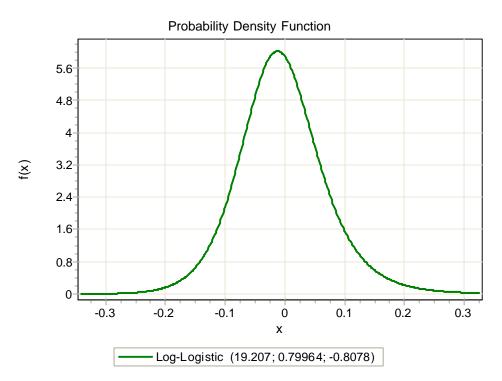
#	<b>Distribution</b>	Kolmogorov Smirnov		Anderson Darling		Chi-Squared	
		Statistic	Rank	Statistic	Rank	Statistic	Rank
1	<u>Beta</u>	0.07736	9	0.79488	7	22.857	20
2	Burr	0.39711	47	27.214	46	197.02	45
3	Burr (4P)	0.07183	5	0.75878	4	22.299	18
4	Cauchy	0.12245	24	2.6064	23	18.551	9
5	Dagum	0.07181	4	0.72136	1	22.812	19
6	Dagum (4P)	0.57409	54	43.779	52	15.399	6
7	Erlang	0.20889	37	7.9601	33	32.934	31

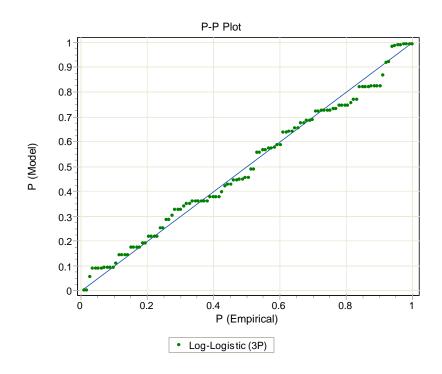
_							
8	Erlang (3P)	0.10309	17	1.3126	17	17.628	7
9	<u>Error</u>	0.10826	20	1.2106	14	23.436	22
10	Error Function	0.93772	56	741.78	56	N/A	
11	Exponential	0.40427	50	28.059	49	191.99	43
12	Exponential (2P)	0.39905	48	27.752	47	178.67	42
13	Fatigue Life	0.30398	46	16.979	40	106.79	39
14	Fatigue Life (3P)	0.08318	12	0.9292	9	25.013	26
15	Frechet	0.29102	45	20.172	42	104.87	38
16	Frechet (3P)	0.16611	35	8.5145	35	N/A	<b>.</b>
17	<u>Gamma</u>	0.10919	21	2.0157	21	11.193	2
18	Gamma (3P)	0.09021	13	1.1108	13	20.555	15
19	Gen. Extreme Value	0.06698	1	8.359	34	N/A	
20	Gen. Gamma	0.15557	34	3.2674	25	23.026	21
21	Gen. Gamma (4P)	0.06969	2	0.73495	2	23.821	24
22	Gen. Pareto	0.1068	19	20.198	43	N/A	
23	Gumbel Max	0.14158	30	4.1009	27	19.669	13
24	Gumbel Min	0.10553	18	1.2406	15	8.3296	1
25	Hypersecant	0.11406	23	1.3961	18	14.586	4
26	Inv. Gaussian	0.12345	25	4.6158	30	18.663	11
27	Inv. Gaussian (3P)	0.11084	22	4.2025	28	13.527	3
28	Johnson SU	0.07452	7	0.74373	3	15.281	5
29	Kumaraswamy	0.07152	3	0.77301	6	22.298	17
30	Laplace	0.14215	31	2.3874	22	26.538	27
31	Levy	0.50569	51	32.984	50	330.43	47
32	Levy (2P)	0.56901	53	42.946	51	360.95	48
33	Log-Logistic	0.22054	39	10.224	37	77.3	36
34	Log-Logistic (3P)	0.08267	11	0.93629	10	17.671	8
35	Log-Pearson 3	0.27096	41	153.57	55	N/A	
36	Logistic	0.09998	16	1.0515	12	23.807	23
37	Lognormal	0.19698	36	6.995	32	60.634	34
38	Lognormal (3P)	0.07696	8	0.95199	11	28.326	29
39	<u>Nakagami</u>	0.09808	14	1.4847	19	19.402	12
40	Normal	0.08176	10	0.91669	8	25.007	25
41	<u>Pareto</u>	0.53572	52	45.341	53	115.56	40

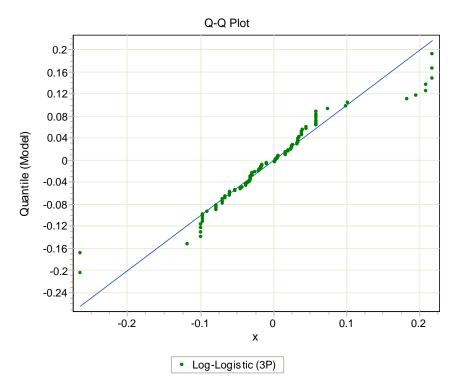
42	Pareto 2	0.40424	49	28.058	48	192.11	44		
43	Pearson 5	0.28909	44	21.049	44	126.77	41		
44	Pearson 5 (3P)	0.12567	26	4.837	31	26.685	28		
45	Pearson 6	0.14759	32	2.8163	24	18.558	10		
46	Pearson 6 (4P)	0.09957	15	1.2508	16	19.773	14		
47	<u>Pert</u>	0.12906	28	3.4613	26	33.142	32		
48	Power Function	0.28334	43	17.225	41	N/A			
49	Rayleigh	0.21949	38	11.002	38	81.895	37		
50	Rayleigh (2P)	0.27589	42	13.286	39	69.801	35		
51	Reciprocal	0.70527	55	104.49	54	265.2	46		
52	Rice	0.13165	29	1.8773	20	N/A			
53	<u>Triangular</u>	0.15002	33	4.3136	29	29.454	30		
54	<u>Uniform</u>	0.12864	27	24.141	45	N/A			
55	<u>Weibull</u>	0.23337	40	9.0887	36	49.919	33		
56	Weibull (3P)	0.07214	6	0.76643	5	22.297	16		
57	Chi-Squared	No fit							
58	Chi-Squared (2P)	No fit							
59	Johnson SB	No fit							
60	Log-Gamma	No fit	No fit						
61	Student's t	No fit							

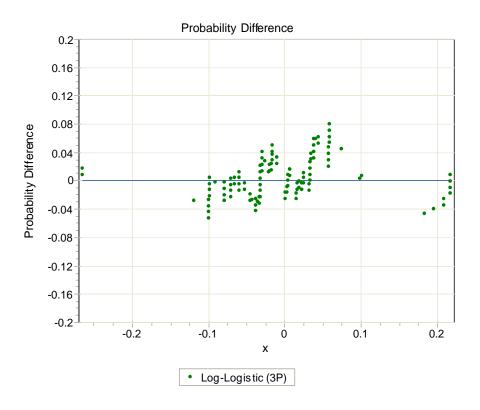
#### RoTD (constant-variance)











#	Distribution	Parameters
1	Beta	$\alpha_1$ =158.71 $\alpha_2$ =709.67 $\alpha_2$ =-1.1396 $\alpha_2$ =1.0874
2	Burr (4P)	k=0.55451 α=76381.0 β=2460.1 γ=-2460.1
3	Cauchy	σ=0.03979 μ=-0.01091
4	Dagum (4P)	k=109.12 α=21.394 β=1.4441 γ=-1.8496
5	Erlang (3P)	m=138 β=0.00697 γ=-0.96505
6	Error	k=1.0822 σ=0.08259 μ=-0.00233
7	Error Function	h=8.5612
8	Exponential (2P)	λ=3.8097 γ=-0.26482
9	Fatigue Life (3P)	α=0.05954 β=1.3718 γ=-1.3766
10	Frechet (3P)	α=7.1305 β=0.3817 γ=-0.41688
11	Gamma (3P)	α=135.93 β=0.00704 γ=-0.95942
12	Gen. Extreme Value	k=-0.11337 σ=0.06845 μ=-0.03488
13	Gen. Gamma (4P)	k=1.4231 α=50.909

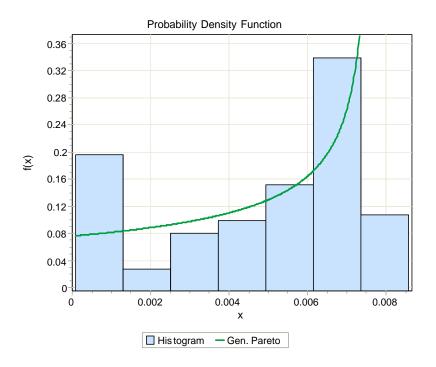
		β=0.05272 γ=-0.83481		
14	Con Poreto	<u> </u>		
	Gen. Pareto	k=-0.63933 σ=0.18677 μ=-0.11627		
15	Gumbel Max	σ=0.0644 μ=-0.03951		
16	Gumbel Min	σ=0.0644 μ=0.03484		
17	Hypersecant	σ=0.08259 μ=-0.00233		
18	Inv. Gaussian (3P)	λ=387.41 μ=1.3743 γ=-1.3767		
19	Johnson SU	$\gamma$ =-0.37943 $\delta$ =1.8005 $\lambda$ =0.12329 $\xi$ =-0.03287		
20	Kumaraswamy	$\alpha_1$ =3.8895 $\alpha_2$ =281.52 a=-0.31679 b=1.1587		
21	Laplace	λ=17.122 μ=-0.00233		
22	Levy (2P)	σ=0.21901 γ=-0.28397		
23	Log-Logistic (3P)	α=19.207 β=0.79964 γ=-0.8078		
24	Logistic	σ=0.04554 μ=-0.00233		
25	Lognormal (3P)	σ=0.06297 μ=0.25925 γ=-1.3009		
26	Normal	$\sigma$ =0.08259 $\mu$ =-0.00233		
27	Pearson 5 (3P)	α=290.46 β=402.49 γ=-1.3931		
28	Pearson 6 (4P)	$\alpha_1$ =1433.0 $\alpha_2$ =411.09 $\beta$ =0.41771 $\gamma$ =-1.4619		
29	Pert	m=0.00609 a=-0.29152 b=0.25906		
30	Power Function	α=1.481 a=-0.27197 b=0.21687		
31	Rayleigh (2P)	σ=0.19853 γ=-0.27078		
32	Triangular	m=-0.01568 a=-0.27593 b=0.24987		
33	Uniform	a=-0.14539 b=0.14073		
34	Weibull (3P)	α=3.9168 β=0.34722 γ=-0.31898		
35	Burr	No fit (data min < 0)		
36	Chi-Squared	No fit (data min < 0)		
37	Chi-Squared (2P)	No fit		
38	Dagum	No fit (data min < 0)		
39	Erlang	No fit (data min < 0)		
40	Exponential	No fit (data min < 0)		
41	Fatigue Life	No fit (data min < 0)		
42	Frechet	No fit (data min < 0)		
43	Gamma	No fit (data min < 0)		

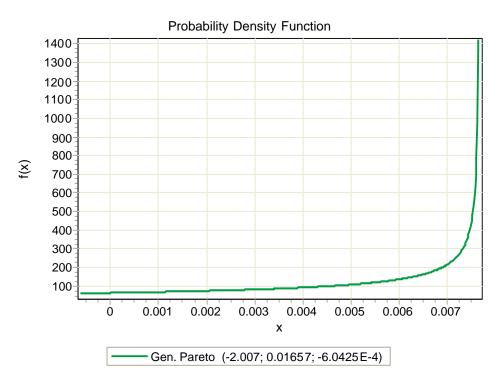
44	Gen. Gamma	No fit (data min < 0)
45	Inv. Gaussian	No fit (data min < 0)
46	Johnson SB	No fit
47	Levy	No fit (data min < 0)
48	Log-Gamma	No fit
49	Log-Logistic	No fit (data min < 0)
50	Log-Pearson 3	No fit
51	Lognormal	No fit (data min < 0)
52	Nakagami	No fit
53	Pareto	No fit
54	Pareto 2	No fit
55	Pearson 5	No fit (data min < 0)
56	Pearson 6	No fit (data min < 0)
57	Rayleigh	No fit (data min < 0)
58	Reciprocal	No fit
59	Rice	No fit
60	Student's t	No fit
61	Weibull	No fit (data min < 0)

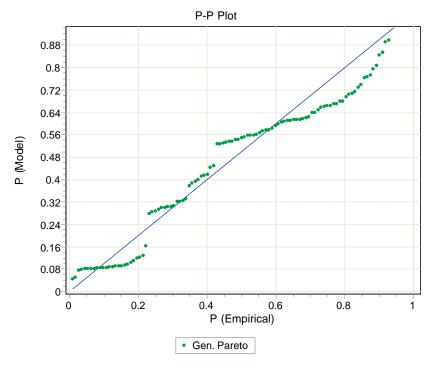
#	<u>Distribution</u>	Kolmogorov Smirnov		Anderson Darling		Chi-Squared	
		Statistic	Rank	Statistic	Rank	Statistic	Rank
1	Beta	0.13213	19	2.5691	19	12.261	12
2	Burr (4P)	0.08687	2	0.8519	1	7.508	1
3	Cauchy	0.10764	10	1.6026	7	13.11	19

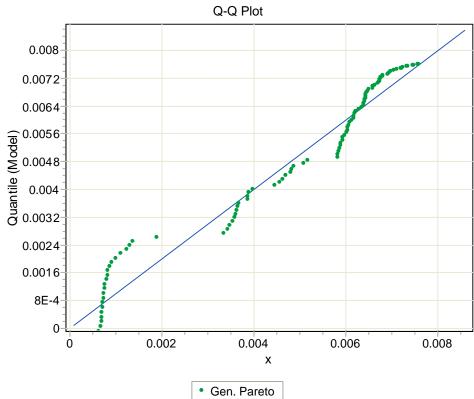
4	Dagum (4P)	0.1538	25	5.3943	24	31.348	25
5	Erlang (3P)	0.127	14	2.4334	15	11.319	6
6	<u>Error</u>	0.10013	6	1.3546	5	11.859	9
7	Error Function	0.14273	22	2.9938	21	24.924	24
8	Exponential (2P)	0.4405	33	27.869	31	208.91	30
9	Fatigue Life (3P)	0.12772	16	2.4179	14	12.328	14
10	Frechet (3P)	0.09181	3	9.0681	27	N/A	_
11	Gamma (3P)	0.12856	17	2.4534	16	12.329	15
12	Gen. Extreme Value	0.10644	8	1.7808	8	9.6686	5
13	Gen. Gamma (4P)	0.1289	18	2.4824	17	12.306	13
14	Gen. Pareto	0.14343	23	31.899	32	N/A	
15	Gumbel Max	0.10331	7	2.3454	11	8.236	3
16	Gumbel Min	0.15464	26	9.3161	28	14.704	21
17	Hypersecant	0.09724	4	1.4187	6	12.474	18
18	Inv. Gaussian (3P)	0.13685	21	2.5531	18	12.373	17
19	Johnson SU	0.09931	5	1.1621	3	8.2627	4
20	Kumaraswamy	0.15643	27	4.0803	23	18.067	23
21	Laplace	0.10662	9	1.3544	4	11.934	10
22	Levy (2P)	0.49157	34	36.264	34	408.45	31
23	Log-Logistic (3P)	0.08007	1	1.0252	2	7.9989	2
24	Logistic	0.1115	11	1.7935	9	13.954	20
25	Lognormal (3P)	0.12733	15	2.3956	13	12.341	16
26	Normal	0.13403	20	2.814	20	12.199	11
27	Pearson 5 (3P)	0.12525	12	2.3158	10	11.393	8
28	Pearson 6 (4P)	0.12697	13	2.374	12	11.356	7
29	<u>Pert</u>	0.20921	30	6.8344	26	38.994	27
30	Power Function	0.34311	32	17.211	30	103.16	29
31	Rayleigh (2P)	0.28381	31	12.596	29	51.723	28
32	<u>Triangular</u>	0.20059	29	6.7795	25	37.931	26
33	<u>Uniform</u>	0.19067	28	34.307	33	N/A	
34	Weibull (3P)	0.15311	24	4.0347	22	18.059	22
35	Burr	No fit (data min < 0)					
36	Chi-Squared	No fit (data min < 0)					
37	Chi-Squared (2P)	No fit					

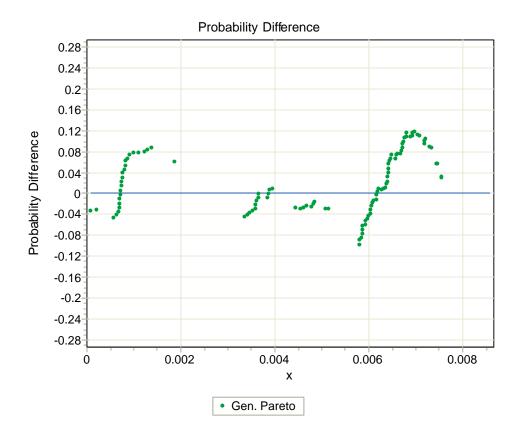
38	Dagum	No fit (data min < 0)
39	Erlang	No fit (data min < 0)
40	Exponential	No fit (data min < 0)
41	Fatigue Life	No fit (data min < 0)
42	Frechet	No fit (data min < 0)
43	Gamma	No fit (data min < 0)
44	Gen. Gamma	No fit (data min < 0)
45	Inv. Gaussian	No fit (data min < 0)
46	Johnson SB	No fit
47	Levy	No fit (data min < 0)
48	Log-Gamma	No fit
49	Log-Logistic	No fit (data min < 0)
50	Log-Pearson 3	No fit
51	Lognormal	No fit (data min < 0)
52	Nakagami	No fit
53	Pareto	No fit
54	Pareto 2	No fit
55	Pearson 5	No fit (data min < 0)
56	Pearson 6	No fit (data min < 0)
57	Rayleigh	No fit (data min < 0)
58	Reciprocal	No fit
59	Rice	No fit
60	Student's t	No fit
61	Weibull	No fit (data min < 0)











## **Fitting Results**

#	Distribution	Parameters
1	Beta	α <sub>1</sub> =1.0486 α <sub>2</sub> =0.79844 a=7.3922E-5 b=0.00858
2	Burr	k=4960.1 α=1.8505 β=0.53959
3	Burr (4P)	k=53218.0 α=361.12 β=0.65537 γ=-0.62983
4	Cauchy	σ=0.00112 μ=0.00619
5	Dagum	k=0.03917 α=32.198 β=0.00819
6	Dagum (4P)	k=0.05963 α=26.068 β=0.00856 γ=-5.5759E-4
7	Erlang	m=3 β=0.00127
8	Erlang (3P)	m=91 β=2.7112E-4 γ=-0.01966
9	Error	k=4.9113 σ=0.00249 μ=0.00491
10	Error Function	h=283.42
11	Exponential	λ=203.81

12	Exponential (2P)	λ=207.28 γ=8.2155Ε-5
13	Fatigue Life	α=1.1181 β=0.00286
14	Fatigue Life (3P)	α=0.00426 β=0.58252 γ=-0.57761
15	Frechet	α=0.9667 β=0.00206
16	Frechet (3P)	α=2.5814E+8 β=6.8903E+5 γ=-6.8903E+5
17	Gamma	α=3.8675 β=0.00127
18	Gamma (3P)	α=96.696 β=2.6434Ε-4 γ=-0.02062
19	Gen. Extreme Value	k=-0.68331 σ=0.00275 μ=0.00453
20	Gen. Gamma	k=0.81255 α=2.8757 β=0.00127
21	Gen. Gamma (4P)	k=25.538 α=0.04859 $β$ =0.00838 $γ$ =2.8015E-5
22	Gen. Pareto	k=-2.007 σ=0.01657 μ=-6.0425E-4
23	Gumbel Max	σ=0.00195 μ=0.00378
24	Gumbel Min	σ=0.00195 μ=0.00603
25	Hypersecant	σ=0.00249 μ=0.00491
26	Inv. Gaussian	λ=0.01898 μ=0.00491
27	Inv. Gaussian (3P)	λ=0.00352 μ=0.00492 γ=7.9696Ε-5
28	Johnson SB	γ=-0.52246 δ=0.42445 $ λ$ =0.00794 $ ξ$ =-3.5520E-4
29	Kumaraswamy	α <sub>1</sub> =1.1246 α <sub>2</sub> =0.99362 a=7.5381E-5 b=0.00858
30	Laplace	λ=566.83 μ=0.00491
31	Levy	σ=1.1883
32	Levy (2P)	σ=1.0705 γ=8.2128Ε-5
33	Log-Logistic	α=1.5657 β=0.00368
34	Log-Logistic (3P)	$\alpha$ =2.0444E+8 $\beta$ =2.6299E+5 $\gamma$ =-2.6299E+5
35	Log-Pearson 3	α=1.5717 β=-0.75068 γ=-4.4162
36	Logistic	σ=0.00138 μ=0.00491
37	Lognormal	σ=0.93689 μ=-5.5961
38	Lognormal (3P)	σ=0.03424 μ=-2.6047 γ=-0.06903
39	Nakagami	m=2.0936 Ω=3.0244E-5
40	Normal	σ=0.00249 μ=0.00491
41	Pareto	α=0.26241 β=8.2155Ε-5

42	Pareto 2	α=626.61 β=3.0724
43	Pearson 5	α=0.85379 β=0.00159
44	Pearson 5 (3P)	α=0.97437 β=0.00199 γ=7.9416Ε-5
45	Pearson 6	$\alpha_1$ =1.4724 $\alpha_2$ =1.0722E+8 $\beta$ =3.9397E+5
46	Pearson 6 (4P)	$\alpha_1$ =222.84 $\alpha_2$ =2757.2 $\beta$ =0.45639 $\gamma$ =-0.03203
47	Pert	m=0.00662 a=-0.00557 b=0.00864
48	Power Function	α=1.1585 a=5.2946E-5 b=0.00858
49	Rayleigh	σ=0.00391
50	Rayleigh (2P)	σ=0.00421 γ=-4.9885Ε-4
51	Reciprocal	a=8.2042E-5 b=0.0086
52	Rice	v=0.00397 σ=0.00269
53	Triangular	m=0.00693 a=-0.00127 b=0.00887
54	Uniform	a=5.8521E-4 b=0.00923
55	Weibull	α=1.2026 β=0.00588
56	Weibull (3P)	α=1.2908E+8 β=2.4024E+5 γ=-2.4024E+5
57	Chi-Squared	No fit
58	Chi-Squared (2P)	No fit
59	Johnson SU	No fit
60	Log-Gamma	No fit
61	Student's t	No fit

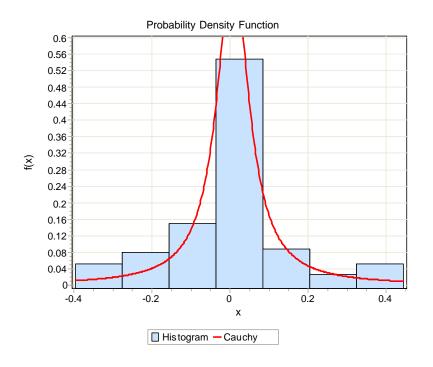
## **Goodness of Fit - Summary**

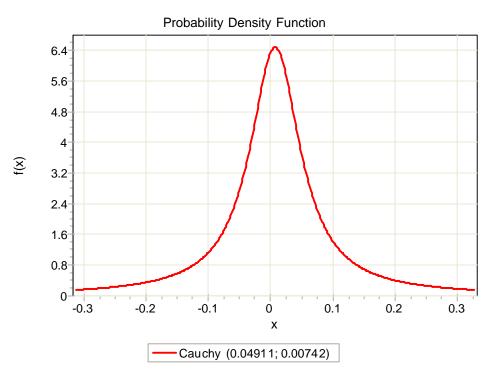
#	<u>Distribution</u>	Kolmog Smirn				Chi-Squared	
		Statistic	Rank	Statistic	Rank	Statistic	Rank
1	Beta	0.15854	5	7.9365	23	N/A	<b>L</b>
2	<u>Burr</u>	0.25817	35	9.8434	30	62.899	38

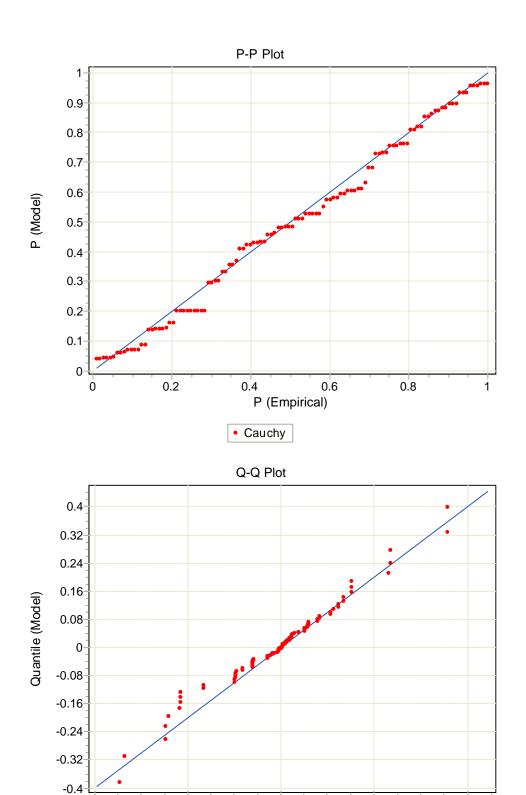
4       Cauchy       0.191       13       9.0982       28       30.014       8         5       Dagum       0.22918       27       5.4218       9       31.946       10         6       Dagum (4P)       0.2123       16       4.4379       6       27.702       6         7       Erlang       0.41564       52       31.747       50       151.35       45         8       Erlang (3P)       0.21407       17       6.0479       14       53.973       31         9       Error       0.19305       14       5.7472       10       44.788       20         10       Error Function       0.68723       54       221.82       54       678.65       47         11       Exponential       0.27441       39       12.833       37       39.7       15         12       Exponential       0.27441       39       12.833       37       39.7       17         13       Eatigue Life       0.33207       50       17.222       42       64.523       39         14       Eatigue Life       0.33201       47       15.062       39       56.103       34         15	3	Burr (4P)	0.15629	4	3.7646	5	7.3505	1
6       Dagum (4P)       0.2123       16       4.4379       6       27.702       6         7       Erlang       0.41564       52       31.747       50       151.35       45         8       Erlang (3P)       0.21407       17       6.0479       14       53.973       31         9       Error       0.19305       14       5.7472       10       44.788       20         10       Error Function       0.68723       54       221.82       54       678.65       47         11       Exponential       0.27441       39       12.833       37       39.7       15         12       Exponential (2P)       0.27537       40       12.737       36       41.79       17         13       Eatigue Life       0.33279       50       17.222       42       64.523       39         14       Eatigue Life (3P)       0.22276       25       5.8822       12       48.142       25         15       Frechet       0.31201       47       15.062       39       56.103       34         16       Frechet (3P)       0.22229       22       7.3706       21       75.806       42 <tr< td=""><td>4</td><td>Cauchy</td><td>0.191</td><td>13</td><td>9.0982</td><td>28</td><td>30.014</td><td>8</td></tr<>	4	Cauchy	0.191	13	9.0982	28	30.014	8
7       Erlang       0.41564       52       31.747       50       151.35       45         8       Erlang (3P)       0.21407       17       6.0479       14       53.973       31         9       Error       0.19305       14       5.7472       10       44.788       20         10       Error Function       0.68723       54       221.82       54       678.65       47         11       Exponential       0.27441       39       12.833       37       39.7       15         12       Exponential (2P)       0.27537       40       12.737       36       41.79       17         13       Fatigue Life       0.33279       50       17.222       42       64.523       39         14       Fatigue Life (3P)       0.22276       25       5.8822       12       48.142       25         15       Frechet       0.31201       47       15.062       39       56.103       34         16       Frechet (3P)       0.22229       22       7.3706       21       75.806       42         17       Gamma       0.27548       42       17.332       43       55.767       33	5	<u>Dagum</u>	0.22918	27	5.4218	9	31.946	10
8       Erlang (3P)       0.21407       17       6.0479       14       53.973       31         9       Error       0.19305       14       5.7472       10       44.788       20         10       Error Function       0.68723       54       221.82       54       678.65       47         11       Exponential       0.27441       39       12.833       37       39.7       15         12       Exponential (2P)       0.27537       40       12.737       36       41.79       17         13       Fatigue Life       0.33279       50       17.222       42       64.523       39         14       Fatigue Life (3P)       0.22276       25       5.8822       12       48.142       25         15       Frechet       0.31201       47       15.062       39       56.103       34         16       Frechet (3P)       0.22229       22       7.3706       21       75.806       42         17       Gamma       0.27548       42       17.332       43       55.767       33         18       Gamma (3P)       0.2227       24       6.1033       15       57.709       36 <t< td=""><td>6</td><td>Dagum (4P)</td><td>0.2123</td><td>16</td><td>4.4379</td><td>6</td><td>27.702</td><td>6</td></t<>	6	Dagum (4P)	0.2123	16	4.4379	6	27.702	6
9       Error       0.19305       14       5.7472       10       44.788       20         10       Error Function       0.68723       54       221.82       54       678.65       47         11       Exponential       0.27441       39       12.833       37       39.7       15         12       Exponential (2P)       0.27537       40       12.737       36       41.79       17         13       Fatigue Life       0.33279       50       17.222       42       64.523       39         14       Fatigue Life (3P)       0.22276       25       5.8822       12       48.142       25         15       Frechet       0.31201       47       15.062       39       56.103       34         16       Frechet (3P)       0.22229       22       7.3706       21       75.806       42         17       Gamma       0.27548       42       17.332       43       55.767       33         18       Gamma (3P)       0.2227       24       6.1033       15       57.709       36         19       Gen. Gamma       0.27538       41       11.346       32       43.99       18 <t< td=""><td>7</td><td>Erlang</td><td>0.41564</td><td>52</td><td>31.747</td><td>50</td><td>151.35</td><td>45</td></t<>	7	Erlang	0.41564	52	31.747	50	151.35	45
The temperature   The temper	8	Erlang (3P)	0.21407	17	6.0479	14	53.973	31
Exponential   0.27441   39   12.833   37   39.7   15	9	<u>Error</u>	0.19305	14	5.7472	10	44.788	20
Exponential (2P)	10	Error Function	0.68723	54	221.82	54	678.65	47
13       Fatigue Life       0.33279       50       17.222       42       64.523       39         14       Fatigue Life (3P)       0.22276       25       5.8822       12       48.142       25         15       Frechet       0.31201       47       15.062       39       56.103       34         16       Frechet (3P)       0.22229       22       7.3706       21       75.806       42         17       Gamma       0.27548       42       17.332       43       55.767       33         18       Gamma (3P)       0.2227       24       6.1033       15       57.709       36         19       Gen. Extreme Value       0.14601       3       6.2052       16       N/A         20       Gen. Gamma       0.27538       41       11.346       32       43.99       18         21       Gen. Gamma (4P)       0.22826       26       5.3848       8       34.042       11         22       Gen. Pareto       0.11753       1       31.48       48       N/A         23       Gumbel Max       0.28316       44       15.28       40       52.549       29         24       Gumbe	11	<u>Exponential</u>	0.27441	39	12.833	37	39.7	15
14       Fatigue Life (3P)       0.22276       25       5.8822       12       48.142       25         15       Frechet       0.31201       47       15.062       39       56.103       34         16       Frechet (3P)       0.22229       22       7.3706       21       75.806       42         17       Gamma       0.27548       42       17.332       43       55.767       33         18       Gamma (3P)       0.2227       24       6.1033       15       57.709       36         19       Gen. Extreme Value       0.14601       3       6.2052       16       N/A         20       Gen. Gamma       0.27538       41       11.346       32       43.99       18         21       Gen. Gamma (4P)       0.22826       26       5.3848       8       34.042       11         22       Gen. Pareto       0.11753       1       31.48       48       N/A         23       Gumbel Max       0.28316       44       15.28       40       52.549       29         24       Gumbel Min       0.17128       8       3.1057       2       10.828       3         25       Hyperseca	12	Exponential (2P)	0.27537	40	12.737	36	41.79	17
15   Frechet   0.31201   47   15.062   39   56.103   34     16   Frechet (3P)   0.22229   22   7.3706   21   75.806   42     17   Gamma   0.27548   42   17.332   43   55.767   33     18   Gamma (3P)   0.2227   24   6.1033   15   57.709   36     19   Gen. Extreme Value   0.14601   3   6.2052   16   N/A     20   Gen. Gamma   0.27538   41   11.346   32   43.99   18     21   Gen. Gamma (4P)   0.22826   26   5.3848   8   34.042   11     22   Gen. Pareto   0.11753   1   31.48   48   N/A     23   Gumbel Max   0.28316   44   15.28   40   52.549   29     24   Gumbel Min   0.17128   8   3.1057   2   10.828   3     25   Hypersecant   0.25256   33   7.4264   22   20.082   4     26   Inv. Gaussian   0.30123   45   41.771   52   49.983   26     27   Inv. Gaussian   0.31973   48   19.661   46   68.456   40     28   Johnson SB   0.13166   2   31.543   49   N/A     29   Kumaraswamy   0.22014   19   8.9308   27   N/A     30   Laplace   0.28093   43   9.1117   29   22.849   5     31   Levy   1   55   N/A   N/A     32   Levy (2P)   1   56   N/A   N/A     33   Log-Logistic   0.2515   32   11.525   33   40.926   16     34   Log-Logistic   (3P)   0.18768   11   6.5782   18   37.384   13     35   Log-Pearson 3   0.18963   12   6.7731   20   114.61   44	13	Fatigue Life	0.33279	50	17.222	42	64.523	39
16         Frechet (3P)         0.22229         22         7.3706         21         75.806         42           17         Gamma         0.27548         42         17.332         43         55.767         33           18         Gamma (3P)         0.2227         24         6.1033         15         57.709         36           19         Gen. Extreme Value         0.14601         3         6.2052         16         N/A           20         Gen. Gamma         0.27538         41         11.346         32         43.99         18           21         Gen. Gamma (4P)         0.22826         26         5.3848         8         34.042         11           22         Gen. Pareto         0.11753         1         31.48         48         N/A           23         Gumbel Max         0.28316         44         15.28         40         52.549         29           24         Gumbel Min         0.17128         8         3.1057         2         10.828         3           25         Hypersecant         0.25256         33         7.4264         22         20.082         4           26         Inv. Gaussian (3P)         0.31973	14	Fatigue Life (3P)	0.22276	25	5.8822	12	48.142	25
17   Gamma   0.27548   42   17.332   43   55.767   33   18   Gamma (3P)   0.2227   24   6.1033   15   57.709   36   19   Gen. Extreme Value   0.14601   3   6.2052   16   N/A   20   Gen. Gamma   0.27538   41   11.346   32   43.99   18   21   Gen. Gamma (4P)   0.22826   26   5.3848   8   34.042   11   22   Gen. Pareto   0.11753   1   31.48   48   N/A   23   Gumbel Max   0.28316   44   15.28   40   52.549   29   24   Gumbel Min   0.17128   8   3.1057   2   10.828   3   25   Hypersecant   0.25256   33   7.4264   22   20.082   4   26   Inv. Gaussian   0.30123   45   41.771   52   49.983   26   27   Inv. Gaussian   0.31973   48   19.661   46   68.456   40   28   Johnson SB   0.13166   2   31.543   49   N/A   29   Kumaraswamy   0.22014   19   8.9308   27   N/A   30   Laplace   0.28093   43   9.1117   29   22.849   5   31   Levy   1   55   N/A   N/A   32   Levy (2P)   1   56   N/A   N/A   33   Log-Logistic   0.2515   32   11.525   33   40.926   16   34   Log-Logistic   0.18768   11   6.5782   18   37.384   13   35   Log-Pearson 3   0.18963   12   6.7731   20   114.61   44   44   44   44   44   44   44	15	Frechet	0.31201	47	15.062	39	56.103	34
18         Gamma (3P)         0.2227         24         6.1033         15         57.709         36           19         Gen. Extreme Value         0.14601         3         6.2052         16         N/A           20         Gen. Gamma         0.27538         41         11.346         32         43.99         18           21         Gen. Gamma (4P)         0.22826         26         5.3848         8         34.042         11           22         Gen. Pareto         0.11753         1         31.48         48         N/A           23         Gumbel Max         0.28316         44         15.28         40         52.549         29           24         Gumbel Min         0.17128         8         3.1057         2         10.828         3           25         Hypersecant         0.25256         33         7.4264         22         20.082         4           26         Inv. Gaussian (3P)         0.31973         48         19.661         46         68.456         40           28         Johnson SB         0.13166         2         31.543         49         N/A           29         Kumaraswamy         0.22014         19	16	Frechet (3P)	0.22229	22	7.3706	21	75.806	42
19         Gen. Extreme Value         0.14601         3         6.2052         16         N/A           20         Gen. Gamma         0.27538         41         11.346         32         43.99         18           21         Gen. Gamma (4P)         0.22826         26         5.3848         8         34.042         11           22         Gen. Pareto         0.11753         1         31.48         48         N/A           23         Gumbel Max         0.28316         44         15.28         40         52.549         29           24         Gumbel Min         0.17128         8         3.1057         2         10.828         3           25         Hypersecant         0.25256         33         7.4264         22         20.082         4           26         Inv. Gaussian         0.30123         45         41.771         52         49.983         26           27         Inv. Gaussian (3P)         0.31973         48         19.661         46         68.456         40           28         Johnson SB         0.13166         2         31.543         49         N/A           29         Kumaraswamy         0.22014         19<	17	<u>Gamma</u>	0.27548	42	17.332	43	55.767	33
20         Gen. Gamma         0.27538         41         11.346         32         43.99         18           21         Gen. Gamma (4P)         0.22826         26         5.3848         8         34.042         11           22         Gen. Pareto         0.11753         1         31.48         48         N/A           23         Gumbel Max         0.28316         44         15.28         40         52.549         29           24         Gumbel Min         0.17128         8         3.1057         2         10.828         3           25         Hypersecant         0.25256         33         7.4264         22         20.082         4           26         Inv. Gaussian         0.30123         45         41.771         52         49.983         26           27         Inv. Gaussian (3P)         0.31973         48         19.661         46         68.456         40           28         Johnson SB         0.13166         2         31.543         49         N/A           29         Kumaraswamy         0.22014         19         8.9308         27         N/A           30         Laplace         0.28093         43	18	Gamma (3P)	0.2227	24	6.1033	15	57.709	36
21         Gen. Gamma (4P)         0.22826         26         5.3848         8         34.042         11           22         Gen. Pareto         0.11753         1         31.48         48         N/A           23         Gumbel Max         0.28316         44         15.28         40         52.549         29           24         Gumbel Min         0.17128         8         3.1057         2         10.828         3           25         Hypersecant         0.25256         33         7.4264         22         20.082         4           26         Inv. Gaussian         0.30123         45         41.771         52         49.983         26           27         Inv. Gaussian (3P)         0.31973         48         19.661         46         68.456         40           28         Johnson SB         0.13166         2         31.543         49         N/A           29         Kumaraswamy         0.22014         19         8.9308         27         N/A           30         Laplace         0.28093         43         9.1117         29         22.849         5           31         Levy (2P)         1         56	19	Gen. Extreme Value	0.14601	3	6.2052	16	N/A	
22         Gen. Pareto         0.11753         1         31.48         48         N/A           23         Gumbel Max         0.28316         44         15.28         40         52.549         29           24         Gumbel Min         0.17128         8         3.1057         2         10.828         3           25         Hypersecant         0.25256         33         7.4264         22         20.082         4           26         Inv. Gaussian         0.30123         45         41.771         52         49.983         26           27         Inv. Gaussian (3P)         0.31973         48         19.661         46         68.456         40           28         Johnson SB         0.13166         2         31.543         49         N/A           29         Kumaraswamy         0.22014         19         8.9308         27         N/A           30         Laplace         0.28093         43         9.1117         29         22.849         5           31         Levy         1         55         N/A         N/A           32         Levy (2P)         1         56         N/A         N/A           3	20	Gen. Gamma	0.27538	41	11.346	32	43.99	18
23         Gumbel Max         0.28316         44         15.28         40         52.549         29           24         Gumbel Min         0.17128         8         3.1057         2         10.828         3           25         Hypersecant         0.25256         33         7.4264         22         20.082         4           26         Inv. Gaussian         0.30123         45         41.771         52         49.983         26           27         Inv. Gaussian (3P)         0.31973         48         19.661         46         68.456         40           28         Johnson SB         0.13166         2         31.543         49         N/A           29         Kumaraswamy         0.22014         19         8.9308         27         N/A           30         Laplace         0.28093         43         9.1117         29         22.849         5           31         Levy         1         55         N/A         N/A           32         Levy (2P)         1         56         N/A         N/A           33         Log-Logistic         0.2515         32         11.525         33         40.926         16 <td>21</td> <td>Gen. Gamma (4P)</td> <td>0.22826</td> <td>26</td> <td>5.3848</td> <td>8</td> <td>34.042</td> <td>11</td>	21	Gen. Gamma (4P)	0.22826	26	5.3848	8	34.042	11
24         Gumbel Min         0.17128         8         3.1057         2         10.828         3           25         Hypersecant         0.25256         33         7.4264         22         20.082         4           26         Inv. Gaussian         0.30123         45         41.771         52         49.983         26           27         Inv. Gaussian (3P)         0.31973         48         19.661         46         68.456         40           28         Johnson SB         0.13166         2         31.543         49         N/A           29         Kumaraswamy         0.22014         19         8.9308         27         N/A           30         Laplace         0.28093         43         9.1117         29         22.849         5           31         Levy         1         55         N/A         N/A           32         Levy (2P)         1         56         N/A         N/A           33         Log-Logistic         0.2515         32         11.525         33         40.926         16           34         Log-Logistic (3P)         0.18768         11         6.5782         18         37.384         13	22	Gen. Pareto	0.11753	1	31.48	48	N/A	
25         Hypersecant         0.25256         33         7.4264         22         20.082         4           26         Inv. Gaussian         0.30123         45         41.771         52         49.983         26           27         Inv. Gaussian (3P)         0.31973         48         19.661         46         68.456         40           28         Johnson SB         0.13166         2         31.543         49         N/A           29         Kumaraswamy         0.22014         19         8.9308         27         N/A           30         Laplace         0.28093         43         9.1117         29         22.849         5           31         Levy         1         55         N/A         N/A           32         Levy (2P)         1         56         N/A         N/A           33         Log-Logistic         0.2515         32         11.525         33         40.926         16           34         Log-Logistic (3P)         0.18768         11         6.5782         18         37.384         13           35         Log-Pearson 3         0.18963         12         6.7731         20         114.61         4	23	Gumbel Max	0.28316	44	15.28	40	52.549	29
26       Inv. Gaussian       0.30123       45       41.771       52       49.983       26         27       Inv. Gaussian (3P)       0.31973       48       19.661       46       68.456       40         28       Johnson SB       0.13166       2       31.543       49       N/A         29       Kumaraswamy       0.22014       19       8.9308       27       N/A         30       Laplace       0.28093       43       9.1117       29       22.849       5         31       Levy       1       55       N/A       N/A         32       Levy (2P)       1       56       N/A       N/A         33       Log-Logistic       0.2515       32       11.525       33       40.926       16         34       Log-Logistic (3P)       0.18768       11       6.5782       18       37.384       13         35       Log-Pearson 3       0.18963       12       6.7731       20       114.61       44	24	Gumbel Min	0.17128	8	3.1057	2	10.828	3
27         Inv. Gaussian (3P)         0.31973         48         19.661         46         68.456         40           28         Johnson SB         0.13166         2         31.543         49         N/A           29         Kumaraswamy         0.22014         19         8.9308         27         N/A           30         Laplace         0.28093         43         9.1117         29         22.849         5           31         Levy         1         55         N/A         N/A           32         Levy (2P)         1         56         N/A         N/A           33         Log-Logistic         0.2515         32         11.525         33         40.926         16           34         Log-Logistic (3P)         0.18768         11         6.5782         18         37.384         13           35         Log-Pearson 3         0.18963         12         6.7731         20         114.61         44	25	Hypersecant	0.25256	33	7.4264	22	20.082	4
28         Johnson SB         0.13166         2         31.543         49         N/A           29         Kumaraswamy         0.22014         19         8.9308         27         N/A           30         Laplace         0.28093         43         9.1117         29         22.849         5           31         Levy         1         55         N/A         N/A           32         Levy (2P)         1         56         N/A         N/A           33         Log-Logistic         0.2515         32         11.525         33         40.926         16           34         Log-Logistic (3P)         0.18768         11         6.5782         18         37.384         13           35         Log-Pearson 3         0.18963         12         6.7731         20         114.61         44	26	Inv. Gaussian	0.30123	45	41.771	52	49.983	26
29       Kumaraswamy       0.22014       19       8.9308       27       N/A         30       Laplace       0.28093       43       9.1117       29       22.849       5         31       Levy       1       55       N/A       N/A         32       Levy (2P)       1       56       N/A       N/A         33       Log-Logistic       0.2515       32       11.525       33       40.926       16         34       Log-Logistic (3P)       0.18768       11       6.5782       18       37.384       13         35       Log-Pearson 3       0.18963       12       6.7731       20       114.61       44	27	Inv. Gaussian (3P)	0.31973	48	19.661	46	68.456	40
30       Laplace       0.28093       43       9.1117       29       22.849       5         31       Levy       1       55       N/A       N/A         32       Levy (2P)       1       56       N/A       N/A         33       Log-Logistic       0.2515       32       11.525       33       40.926       16         34       Log-Logistic (3P)       0.18768       11       6.5782       18       37.384       13         35       Log-Pearson 3       0.18963       12       6.7731       20       114.61       44	28	Johnson SB	0.13166	2	31.543	49	N/A	
31       Levy       1       55       N/A       N/A         32       Levy (2P)       1       56       N/A       N/A         33       Log-Logistic       0.2515       32       11.525       33       40.926       16         34       Log-Logistic (3P)       0.18768       11       6.5782       18       37.384       13         35       Log-Pearson 3       0.18963       12       6.7731       20       114.61       44	29	Kumaraswamy	0.22014	19	8.9308	27	N/A	
32     Levy (2P)     1     56     N/A     N/A       33     Log-Logistic     0.2515     32     11.525     33     40.926     16       34     Log-Logistic (3P)     0.18768     11     6.5782     18     37.384     13       35     Log-Pearson 3     0.18963     12     6.7731     20     114.61     44	30	Laplace	0.28093	43	9.1117	29	22.849	5
33     Log-Logistic     0.2515     32     11.525     33     40.926     16       34     Log-Logistic (3P)     0.18768     11     6.5782     18     37.384     13       35     Log-Pearson 3     0.18963     12     6.7731     20     114.61     44	31	Levy	1	55	N/A	L	N/A	
34 <u>Log-Logistic (3P)</u> 0.18768 11 6.5782 18 37.384 13 35 <u>Log-Pearson 3</u> 0.18963 12 6.7731 20 114.61 44	32	Levy (2P)	1	56	N/A	L	N/A	
35 <u>Log-Pearson 3</u> 0.18963 12 6.7731 20 114.61 44	33	Log-Logistic	0.2515	32	11.525	33	40.926	16
	34	Log-Logistic (3P)	0.18768	11	6.5782	18	37.384	13
36         Logistic         0.23908         30         6.6252         19         51.114         28	35	Log-Pearson 3	0.18963	12	6.7731	20	114.61	44
	36	Logistic	0.23908	30	6.6252	19	51.114	28

37	Lognormal	0.26412	37	12.214	35	48.125	24
38	Lognormal (3P)	0.2213	20	5.9592	13	52.75	30
39	<u>Nakagami</u>	0.23316	29	25.062	47	45.186	21
40	Normal	0.22189	21	5.7836	11	48.112	23
41	<u>Pareto</u>	0.39891	51	34.055	51	44.701	19
42	Pareto 2	0.27432	38	12.836	38	39.669	14
43	Pearson 5	0.32313	49	15.384	41	56.631	35
44	Pearson 5 (3P)	0.30801	46	17.503	44	54.963	32
45	Pearson 6	0.22242	23	8.924	26	51.105	27
46	Pearson 6 (4P)	0.2318	28	6.2255	17	61.375	37
47	Pert	0.16041	6	2.7697	1	30.941	9
48	Power Function	0.21489	18	4.9039	7	35.858	12
49	Rayleigh	0.24804	31	10.145	31	81.926	43
50	Rayleigh (2P)	0.25515	34	8.2683	24	71.765	41
51	Reciprocal	0.57426	53	86.197	53	245.26	46
52	Rice	0.26243	36	11.735	34	N/A	L
53	<u>Triangular</u>	0.1838	9	3.6487	4	28.693	7
54	<u>Uniform</u>	0.18502	10	18.956	45	N/A	
55	<u>Weibull</u>	0.20754	15	8.7518	25	46.512	22
56	Weibull (3P)	0.16578	7	3.3142	3	8.8311	2
57	Chi-Squared	No fit					
58	Chi-Squared (2P)	No fit					
59	Johnson SU	No fit					
60	Log-Gamma	No fit					
61	Student's t	No fit					

## BOPO (constant-variance)







• Cauchy

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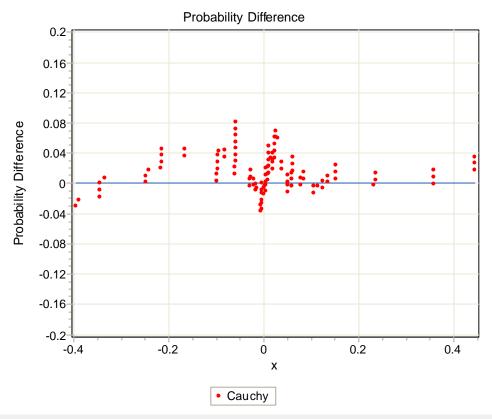
х

0.2

0.4

-0.2

-0.4



# **Fitting Results**

#	Distribution	Parameters
1	Beta	$\alpha_1$ =3339.0 $\alpha_2$ =3922.2 $\alpha$ =-12.142 $\alpha$ =14.265
2	Burr (4P)	k=0.88488 α=1.1072E+7 β=8.2439E+5 γ=-8.2439E+5
3	Cauchy	σ=0.04911 μ=0.00742
4	Dagum (4P)	k=0.58051 α=17.22 β=1.114 γ=-1.062
5	Erlang (3P)	m=135 β=0.01345 γ=-1.8133
6	Error	k=1.175 σ=0.15515 μ=0.00134
7	Error Function	h=4.5574
8	Exponential (2P)	λ=2.5184 γ=-0.39573
9	Fatigue Life (3P)	α=0.02694 β=5.7278 γ=-5.7285
10	Frechet (3P)	α=9.5419E+7 β=1.5464E+7 γ=-1.5464E+7
11	Gamma (3P)	α=169.76 β=0.01195 γ=-2.0278
12	Gen. Extreme Value	k=-0.29512 σ=0.14245 μ=-0.04778

13	Gen. Gamma (4P)	k=2.1648 α=38.136
13	Octi. Gaiiiiia (41 )	$\beta$ =0.38549 $\gamma$ =-2.0644
14	Gen. Pareto	k=-1.0254 σ=0.49146 μ=-0.24131
15	Gumbel Max	σ=0.12097 μ=-0.06848
16	Gumbel Min	σ=0.12097 μ=0.07117
17	Hypersecant	σ=0.15515 μ=0.00134
18	Inv. Gaussian (3P)	λ=7894.5 μ=5.73 γ=-5.7287
19	Johnson SU	γ=-0.15217 δ=1.8769 $ λ=0.25006 ξ=-0.02205$
20	Kumaraswamy	α <sub>1</sub> =3.4809 α <sub>2</sub> =292.2 a=-0.50674 b=2.3649
21	Laplace	λ=9.1149 μ=0.00134
22	Levy (2P)	σ=0.29014 γ=-0.42099
23	Log-Logistic (3P)	α=211.2 β=16.413 γ=-16.411
24	Logistic	σ=0.08554 μ=0.00134
25	Lognormal (3P)	σ=0.04213 μ=1.2975 γ=-3.6621
26	Normal	σ=0.15515 μ=0.00134
27	Pearson 5 (3P)	α=281.59 β=732.27 γ=-2.6086
28	Pearson 6 (4P)	$\alpha_1$ =8169.7 $\alpha_2$ =2106.2 $\beta$ =1.6271 $\gamma$ =-6.313
29	Pert	m=-0.01666 a=-0.44254 b=0.52539
30	Power Function	α=0.89635 a=-0.39573 b=0.44435
31	Rayleigh (2P)	σ=0.3089 γ=-0.40729
32	Triangular	m=0.00498 a=-0.42767 b=0.47671
33	Uniform	a=-0.26739 b=0.27008
34	Weibull (3P)	α=3.5182 β=0.56766 γ=-0.51223
35	Burr	No fit (data min < 0)
36	Chi-Squared	No fit (data min < 0)
37	Chi-Squared (2P)	No fit
38	Dagum	No fit (data min < 0)
39	Erlang	No fit (data min < 0)
40	Exponential	No fit (data min < 0)
41	Fatigue Life	No fit (data min < 0)
42	Frechet	No fit (data min < 0)

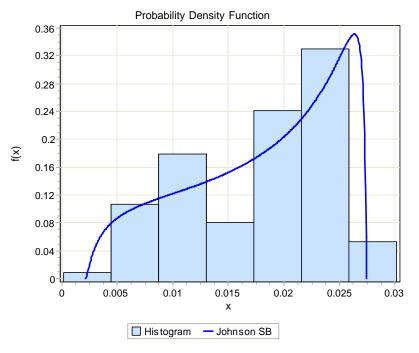
43	Gamma	No fit (data min < 0)
44	Gen. Gamma	No fit (data min < 0)
45	Inv. Gaussian	No fit (data min < 0)
46	Johnson SB	No fit
47	Levy	No fit (data min < 0)
48	Log-Gamma	No fit
49	Log-Logistic	No fit (data min < 0)
50	Log-Pearson 3	No fit
51	Lognormal	No fit (data min < 0)
52	Nakagami	No fit
53	Pareto	No fit
54	Pareto 2	No fit
55	Pearson 5	No fit (data min < 0)
56	Pearson 6	No fit (data min < 0)
57	Rayleigh	No fit (data min < 0)
58	Reciprocal	No fit
59	Rice	No fit
60	Student's t	No fit
61	Weibull	No fit (data min < 0)

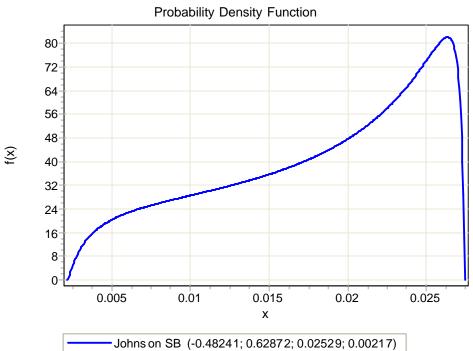
## Goodness of Fit – Summary

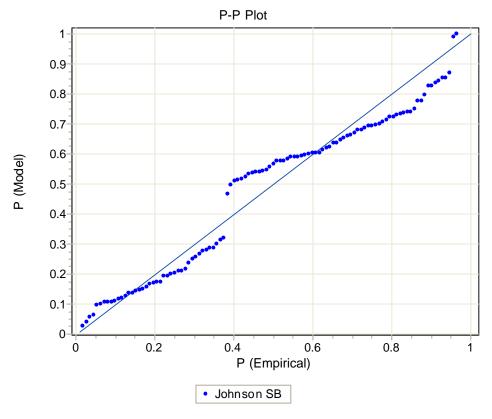
#	Distribution		Kolmogorov Smirnov		Anderson Darling		Chi-Squared	
		Statistic	Rank	Statistic	Rank	Statistic	Rank	
1	Beta	0.14769	15	3.9939	10	54.089	16	
2	Burr (4P)	0.11821	4	2.5995	7	34.649	9	
3	Cauchy	0.08121	1	0.60814	1	1.6155	1	
4	Dagum (4P)	0.12457	7	2.4661	5	29.389	6	
5	Erlang (3P)	0.1503	19	4.1189	19	66.054	24	
6	<u>Error</u>	0.10889	3	1.974	3	24.564	4	
7	Error Function	0.14537	11	4.0181	15	50.997	14	
8	Exponential (2P)	0.39379	33	22.023	31	152.65	29	
9	Fatigue Life (3P)	0.14559	13	4.0065	11	57.965	19	

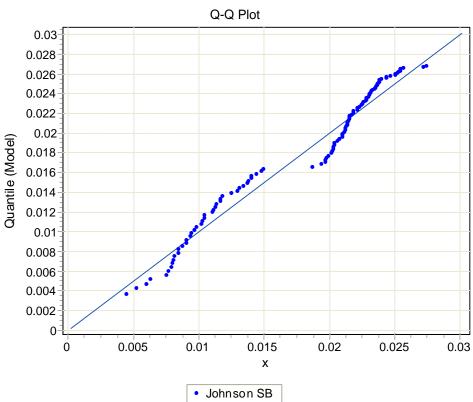
Camma (3P)	10	Frechet (3P)	0.19821	28	6.629	25	42.289	11
12   Gen. Extreme Value   0.14192   10   15.717   30   N/A     13   Gen. Gamma (4P)   0.14557   12   4.0274   16   50.981   13     14   Gen. Pareto   0.17653   24   38.893   34   N/A     15   Gumbel Max   0.20185   29   7.1795   26   33.289   7     16   Gumbel Min   0.19753   27   7.3181   27   21.112   3     17   Hypersecant   0.11949   6   2.3667   4   35.274   10     18   Inv. Gaussian (3P)   0.14859   18   4.0088   13   54.103   17     19   Johnson SU   0.13486   9   2.8903   8   29.188   5     20   Kumaraswamy   0.15827   23   4.623   21   68.227   26     21   Laplace   0.1   2   1.4173   2   18.503   2     22   Levy (2P)   0.44344   34   29.545   33   375.84   31     23   Log-Logistic (3P)   0.11835   5   2.5894   6   34.608   8     24   Logistic   0.13082   8   2.9662   9   48.601   12     25   Lognormal (3P)   0.14805   16   4.0283   17   65.963   23     26   Normal   0.14858   17   4.0171   14   54.115   18     27   Pearson 5 (3P)   0.1557   21   4.1694   20   51.751   15     28   Pearson 6 (4P)   0.14593   14   4.0073   12   57.968   20     29   Pert   0.18319   25   6.521   24   69.421   27     30   Power Function   0.26089   31   14.128   29   156.25   30     31   Rayleigh (2P)   0.26447   32   9.3346   28   91.14   28     32   Triangular   0.20302   30   6.3598   23   58.132   21     33   Uniform   0.18688   26   29.375   32   N/A     34   Weibull (3P)   0.15793   22   4.6262   22   67.769   25     35   Burr   No fit (data min < 0)	11		0.15281	20	4.1055	18	59.53	22
14   Gen. Pareto   0.17653   24   38.893   34   N/A     15   Gumbel Max   0.20185   29   7.1795   26   33.289   7     16   Gumbel Min   0.19753   27   7.3181   27   21.112   3     17   Hypersecant   0.11949   6   2.3667   4   35.274   10     18   Inv. Gaussian (3P)   0.14859   18   4.0088   13   54.103   17     19   Johnson SU   0.13486   9   2.8903   8   29.188   5     20   Kumaraswamy   0.15827   23   4.623   21   68.227   26     21   Laplace   0.1   2   1.4173   2   18.503   2     22   Levy (2P)   0.44344   34   29.545   33   375.84   31     23   Log-Logistic (3P)   0.11835   5   2.5894   6   34.608   8     24   Logistic   0.13082   8   2.9662   9   48.601   12     25   Lognormal (3P)   0.14805   16   4.0283   17   65.963   23     26   Normal   0.14858   17   4.0171   14   54.115   18     27   Pearson 5 (3P)   0.1557   21   4.1694   20   51.751   15     28   Pearson 6 (4P)   0.14593   14   4.0073   12   57.968   20     29   Pert   0.18319   25   6.521   24   69.421   27     30   Power Function   0.26089   31   14.128   29   156.25   30     31   Rayleigh (2P)   0.26447   32   9.3346   28   91.14   28     32   Triangular   0.20302   30   6.3598   23   58.132   21     33   Uniform   0.18688   26   29.375   32   N/A     34   Weibull (3P)   0.15793   22   4.6262   22   67.769   25     35   Burr   No fit (data min < 0)   0.15793   22   4.6262   22   67.769   25     36   Chi-Squared   No fit (data min < 0)   0.15793   20   4.6262   22   67.769   25     37   Chi-Squared   No fit (data min < 0)   0.15793   27   4.6262   27   67.769   25     38   Dagum   No fit (data min < 0)   0.15793   27   4.6262   28   67.769   25     39   Erlang   No fit (data min < 0)   0.15793   27   4.6262   28   67.769   25     39   Erlang   No fit (data min < 0)   0.15793   27   4.6262   28   67.769   25     30   Erlang   No fit (data min < 0)   0.15793   27   4.6262   28   67.769   27     30   Erlang   No fit (data min < 0)   0.15793   27   4.6262   28   67.769   27     39   Erlang   No fit (data min < 0)   0.15793   0.15793   0.15793   0	12	Gen. Extreme Value	0.14192	10	15.717	30	N/A	
15   Gumbel Max   0.20185   29   7.1795   26   33.289   7     16   Gumbel Min   0.19753   27   7.3181   27   21.112   3     17   Hypersecant   0.11949   6   2.3667   4   35.274   10     18   Inv. Gaussian (3P)   0.14859   18   4.0088   13   54.103   17     19   Johnson SU   0.13486   9   2.8903   8   29.188   5     20   Kumaraswamy   0.15827   23   4.623   21   68.227   26     21   Laplace   0.1   2   1.4173   2   18.503   2     22   Levy (2P)   0.44344   34   29.545   33   375.84   31     23   Log-Logistic (3P)   0.11835   5   2.5894   6   34.608   8     24   Logistic   0.13082   8   2.9662   9   48.601   12     25   Lognormal (3P)   0.14805   16   4.0283   17   65.963   23     26   Normal   0.14858   17   4.0171   14   54.115   18     27   Pearson 5 (3P)   0.1557   21   4.1694   20   51.751   15     28   Pearson 6 (4P)   0.14593   14   4.0073   12   57.968   20     29   Pert   0.18319   25   6.521   24   69.421   27     30   Power Function   0.26089   31   14.128   29   156.25   30     31   Rayleigh (2P)   0.26447   32   9.3346   28   91.14   28     32   Triangular   0.20302   30   6.3598   23   58.132   21     33   Uniform   0.18688   26   29.375   32   N/A    34   Weibull (3P)   0.15793   22   4.6262   22   67.769   25     35   Burr   No fit (data min < 0)     37   Chi-Squared   No fit (data min < 0)     40   Exponential   No fit (data min < 0)     41   Fatigue Life   No fit (data min < 0)	13	Gen. Gamma (4P)	0.14557	12	4.0274	16	50.981	13
16   Gumbel Min   0.19753   27   7.3181   27   21.112   3     17   Hypersecant   0.11949   6   2.3667   4   35.274   10     18   Inv. Gaussian (3P)   0.14859   18   4.0088   13   54.103   17     19   Johnson SU   0.13486   9   2.8903   8   29.188   5     20   Kumaraswamy   0.15827   23   4.623   21   68.227   26     21   Laplace   0.1   2   1.4173   2   18.503   2     22   Levy (2P)   0.44344   34   29.545   33   375.84   31     23   Log-Logistic (3P)   0.11835   5   2.5894   6   34.608   8     24   Logistic   0.13082   8   2.9662   9   48.601   12     25   Lognormal (3P)   0.14805   16   4.0283   17   65.963   23     26   Normal   0.14858   17   4.0171   14   54.115   18     27   Pearson 5 (3P)   0.1557   21   4.1694   20   51.751   15     28   Pearson 6 (4P)   0.14593   14   4.0073   12   57.968   20     29   Pert   0.18319   25   6.521   24   69.421   27     30   Power Function   0.26089   31   14.128   29   156.25   30     31   Rayleigh (2P)   0.26447   32   9.3346   28   91.14   28     32   Triangular   0.20302   30   6.3598   23   58.132   21     33   Uniform   0.18688   26   29.375   32   N/A     34   Weibull (3P)   0.15793   22   4.6262   22   67.769   25     35   Burr   No fit (data min < 0)     36   Chi-Squared   No fit (data min < 0)     37   Chi-Squared   No fit (data min < 0)     40   Exponential   No fit (data min < 0)     41   Fatigue Life   No fit (data min < 0)	14	Gen. Pareto	0.17653	24	38.893	34	N/A	
Hypersecant   0.11949   6   2.3667   4   35.274   10	15	Gumbel Max	0.20185	29	7.1795	26	33.289	7
18	16	Gumbel Min	0.19753	27	7.3181	27	21.112	3
19   Johnson SU	17	Hypersecant	0.11949	6	2.3667	4	35.274	10
Numaraswamy   0.15827   23	18	Inv. Gaussian (3P)	0.14859	18	4.0088	13	54.103	17
Laplace	19	Johnson SU	0.13486	9	2.8903	8	29.188	5
22   Levy (2P)	20	Kumaraswamy	0.15827	23	4.623	21	68.227	26
23   Log-Logistic (3P)   0.11835   5   2.5894   6   34.608   8     24   Logistic   0.13082   8   2.9662   9   48.601   12     25   Lognormal (3P)   0.14805   16   4.0283   17   65.963   23     26   Normal   0.14858   17   4.0171   14   54.115   18     27   Pearson 5 (3P)   0.1557   21   4.1694   20   51.751   15     28   Pearson 6 (4P)   0.14593   14   4.0073   12   57.968   20     29   Pert   0.18319   25   6.521   24   69.421   27     30   Power Function   0.26089   31   14.128   29   156.25   30     31   Rayleigh (2P)   0.26447   32   9.3346   28   91.14   28     32   Triangular   0.20302   30   6.3598   23   58.132   21     33   Uniform   0.18688   26   29.375   32   N/A     34   Weibull (3P)   0.15793   22   4.6262   22   67.769   25     35   Burr   No fit (data min < 0)     36   Chi-Squared   No fit (data min < 0)     37   Chi-Squared (2P)   No fit     38   Dagum   No fit (data min < 0)     40   Exponential   No fit (data min < 0)     41   Fatigue Life   No fit (data min < 0)     42   Frechet   No fit (data min < 0)	21	<u>Laplace</u>	0.1	2	1.4173	2	18.503	2
24       Logistic       0.13082       8       2.9662       9       48.601       12         25       Lognormal (3P)       0.14805       16       4.0283       17       65.963       23         26       Normal       0.14858       17       4.0171       14       54.115       18         27       Pearson 5 (3P)       0.1557       21       4.1694       20       51.751       15         28       Pearson 6 (4P)       0.14593       14       4.0073       12       57.968       20         29       Pert       0.18319       25       6.521       24       69.421       27         30       Power Function       0.26089       31       14.128       29       156.25       30         31       Rayleigh (2P)       0.26447       32       9.3346       28       91.14       28         32       Triangular       0.20302       30       6.3598       23       58.132       21         33       Uniform       0.18688       26       29.375       32       N/A         34       Weibull (3P)       0.15793       22       4.6262       22       67.769       25         35	22	Levy (2P)	0.44344	34	29.545	33	375.84	31
25         Lognormal (3P)         0.14805         16         4.0283         17         65.963         23           26         Normal         0.14858         17         4.0171         14         54.115         18           27         Pearson 5 (3P)         0.1557         21         4.1694         20         51.751         15           28         Pearson 6 (4P)         0.14593         14         4.0073         12         57.968         20           29         Pert         0.18319         25         6.521         24         69.421         27           30         Power Function         0.26089         31         14.128         29         156.25         30           31         Rayleigh (2P)         0.26447         32         9.3346         28         91.14         28           32         Triangular         0.20302         30         6.3598         23         58.132         21           33         Uniform         0.18688         26         29.375         32         N/A           34         Weibull (3P)         0.15793         22         4.6262         22         67.769         25           35         Burr <td< td=""><td>23</td><td>Log-Logistic (3P)</td><td>0.11835</td><td>5</td><td>2.5894</td><td>6</td><td>34.608</td><td>8</td></td<>	23	Log-Logistic (3P)	0.11835	5	2.5894	6	34.608	8
26       Normal       0.14858       17       4.0171       14       54.115       18         27       Pearson 5 (3P)       0.1557       21       4.1694       20       51.751       15         28       Pearson 6 (4P)       0.14593       14       4.0073       12       57.968       20         29       Pert       0.18319       25       6.521       24       69.421       27         30       Power Function       0.26089       31       14.128       29       156.25       30         31       Rayleigh (2P)       0.26447       32       9.3346       28       91.14       28         32       Triangular       0.20302       30       6.3598       23       58.132       21         33       Uniform       0.18688       26       29.375       32       N/A         34       Weibull (3P)       0.15793       22       4.6262       22       67.769       25         35       Burr       No fit (data min < 0)	24	Logistic	0.13082	8	2.9662	9	48.601	12
27       Pearson 5 (3P)       0.1557       21       4.1694       20       51.751       15         28       Pearson 6 (4P)       0.14593       14       4.0073       12       57.968       20         29       Pert       0.18319       25       6.521       24       69.421       27         30       Power Function       0.26089       31       14.128       29       156.25       30         31       Rayleigh (2P)       0.26447       32       9.3346       28       91.14       28         32       Triangular       0.20302       30       6.3598       23       58.132       21         33       Uniform       0.18688       26       29.375       32       N/A         34       Weibull (3P)       0.15793       22       4.6262       22       67.769       25         35       Burr       No fit (data min < 0)	25	Lognormal (3P)	0.14805	16	4.0283	17	65.963	23
28         Pearson 6 (4P)         0.14593         14         4.0073         12         57.968         20           29         Pert         0.18319         25         6.521         24         69.421         27           30         Power Function         0.26089         31         14.128         29         156.25         30           31         Rayleigh (2P)         0.26447         32         9.3346         28         91.14         28           32         Triangular         0.20302         30         6.3598         23         58.132         21           33         Uniform         0.18688         26         29.375         32         N/A           34         Weibull (3P)         0.15793         22         4.6262         22         67.769         25           35         Burr         No fit (data min < 0)	26	Normal	0.14858	17	4.0171	14	54.115	18
29       Pert       0.18319       25       6.521       24       69.421       27         30       Power Function       0.26089       31       14.128       29       156.25       30         31       Rayleigh (2P)       0.26447       32       9.3346       28       91.14       28         32       Triangular       0.20302       30       6.3598       23       58.132       21         33       Uniform       0.18688       26       29.375       32       N/A         34       Weibull (3P)       0.15793       22       4.6262       22       67.769       25         35       Burr       No fit (data min < 0)	27	Pearson 5 (3P)	0.1557	21	4.1694	20	51.751	15
30   Power Function   0.26089   31   14.128   29   156.25   30     31   Rayleigh (2P)   0.26447   32   9.3346   28   91.14   28     32   Triangular   0.20302   30   6.3598   23   58.132   21     33   Uniform   0.18688   26   29.375   32   N/A     34   Weibull (3P)   0.15793   22   4.6262   22   67.769   25     35   Burr   No fit (data min < 0)     36   Chi-Squared   No fit (data min < 0)     37   Chi-Squared (2P)   No fit     38   Dagum   No fit (data min < 0)     39   Erlang   No fit (data min < 0)     40   Exponential   No fit (data min < 0)     41   Fatigue Life   No fit (data min < 0)     42   Frechet   No fit (data min < 0)	28	Pearson 6 (4P)	0.14593	14	4.0073	12	57.968	20
31         Rayleigh (2P)         0.26447         32         9.3346         28         91.14         28           32         Triangular         0.20302         30         6.3598         23         58.132         21           33         Uniform         0.18688         26         29.375         32         N/A           34         Weibull (3P)         0.15793         22         4.6262         22         67.769         25           35         Burr         No fit (data min < 0)	29	<u>Pert</u>	0.18319	25	6.521	24	69.421	27
32       Triangular       0.20302       30       6.3598       23       58.132       21         33       Uniform       0.18688       26       29.375       32       N/A         34       Weibull (3P)       0.15793       22       4.6262       22       67.769       25         35       Burr       No fit (data min < 0)	30	Power Function	0.26089	31	14.128	29	156.25	30
33 Uniform 0.18688 26 29.375 32 N/A  34 Weibull (3P) 0.15793 22 4.6262 22 67.769 25  35 Burr No fit (data min < 0)  36 Chi-Squared No fit (data min < 0)  37 Chi-Squared (2P) No fit  38 Dagum No fit (data min < 0)  39 Erlang No fit (data min < 0)  40 Exponential No fit (data min < 0)  41 Fatigue Life No fit (data min < 0)  42 Frechet No fit (data min < 0)	31	Rayleigh (2P)	0.26447	32	9.3346	28	91.14	28
34 Weibull (3P) 0.15793 22 4.6262 22 67.769 25  35 Burr No fit (data min < 0)  36 Chi-Squared No fit (data min < 0)  37 Chi-Squared (2P) No fit  38 Dagum No fit (data min < 0)  39 Erlang No fit (data min < 0)  40 Exponential No fit (data min < 0)  41 Fatigue Life No fit (data min < 0)  42 Frechet No fit (data min < 0)	32	Triangular	0.20302	30	6.3598	23	58.132	21
35       Burr       No fit (data min < 0)	33	<u>Uniform</u>	0.18688	26	29.375	32	N/A	
36       Chi-Squared       No fit (data min < 0)	34	Weibull (3P)	0.15793	22	4.6262	22	67.769	25
37       Chi-Squared (2P)       No fit         38       Dagum       No fit (data min < 0)	35	Burr	No fit (d	ata min	< 0)			
38 Dagum No fit (data min < 0) 39 Erlang No fit (data min < 0) 40 Exponential No fit (data min < 0) 41 Fatigue Life No fit (data min < 0) 42 Frechet No fit (data min < 0)	36	Chi-Squared	No fit (d	ata min	< 0)			
39 Erlang No fit (data min < 0) 40 Exponential No fit (data min < 0) 41 Fatigue Life No fit (data min < 0) 42 Frechet No fit (data min < 0)	37	Chi-Squared (2P)	No fit					
40 Exponential No fit (data min < 0) 41 Fatigue Life No fit (data min < 0) 42 Frechet No fit (data min < 0)	38	Dagum	No fit (d	ata min	< 0)			
41 Fatigue Life No fit (data min < 0)  42 Frechet No fit (data min < 0)	39	Erlang	No fit (data min < 0)					
42 Frechet No fit (data min < 0)	40	Exponential	No fit (data min < 0)					
	41	Fatigue Life	No fit (data min < 0)					
43 Gamma No fit (data min < 0)	42	Frechet	No fit (d	ata min	< 0)			
	43	Gamma	No fit (d	ata min	< 0)			

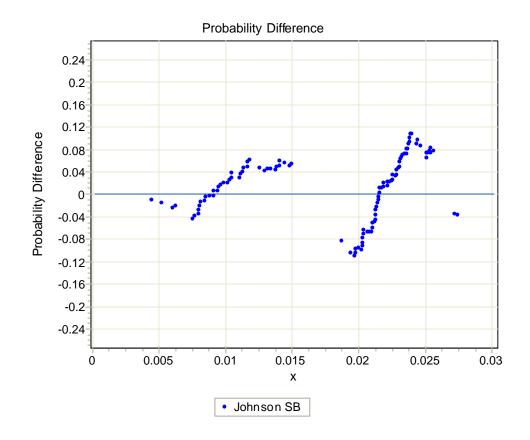
44	Gen. Gamma	No fit (data min < 0)
45	Inv. Gaussian	No fit (data min < 0)
46	Johnson SB	No fit
47	Levy	No fit (data min < 0)
48	Log-Gamma	No fit
49	Log-Logistic	No fit (data min < 0)
50	Log-Pearson 3	No fit
51	Lognormal	No fit (data min < 0)
52	Nakagami	No fit
53	Pareto	No fit
54	Pareto 2	No fit
55	Pearson 5	No fit (data min < 0)
56	Pearson 6	No fit (data min < 0)
57	Rayleigh	No fit (data min < 0)
58	Reciprocal	No fit
59	Rice	No fit
60	Student's t	No fit
61	Weibull	No fit (data min < 0)











# **Fitting Results**

#	Distribution	Parameters
1	Beta	α <sub>1</sub> =3.3625 α <sub>2</sub> =1.9468 a=-0.00363 b=0.03057
2	Burr	k=932.16 α=2.9743 β=0.20077
3	Burr (4P)	k=12613.0 α=75.374 β=0.46345 γ=-0.38768
4	Cauchy	σ=0.00329 μ=0.02143
5	Dagum	k=0.09443 α=21.409 β=0.02625
6	Dagum (4P)	k=0.1058 α=22.367 β=0.02777 γ=-0.00176
7	Erlang	m=7 β=0.00252
8	Erlang (3P)	m=136 β=5.8335E-4 γ=-0.06119
9	Error	k=4.5778 σ=0.00675 μ=0.01809
10	Error Function	h=104.79
11	Exponential	λ=55.292

12	Exponential (2P)	λ=55.858 γ=1.8337Ε-4
13	Fatigue Life	α=0.93906 β=0.01199
14	Fatigue Life (3P)	$\alpha$ =0.00633 $\beta$ =1.0656 $\gamma$ =-1.0475
15	Frechet	α=1.4041 β=0.01075
16	Frechet (3P)	α=2.1428 β=0.01378 γ=-4.6107Ε-4
17	Gamma	α=7.183 β=0.00252
18	Gamma (3P)	α=114.45 β=6.4116Ε-4 γ=-0.05544
19	Gen. Extreme Value	k=-0.58217 σ=0.00741 μ=0.01671
20	Gen. Gamma	k=0.89519 α=5.7623 β=0.00252
21	Gen. Gamma (4P)	k=12.356 α=0.18402 β=0.02862 γ=-0.00144
22	Gen. Pareto	k=-1.7285 σ=0.03835 μ=0.00403
23	Gumbel Max	σ=0.00526 μ=0.01505
24	Gumbel Min	σ=0.00526 μ=0.02112
25	Hypersecant	σ=0.00675 μ=0.01809
26	Inv. Gaussian	λ=0.12991 μ=0.01809
27	Inv. Gaussian (3P)	λ=0.08329 μ=0.01812 γ=1.2496Ε-4
28	Johnson SB	$\gamma$ =-0.48241 $\delta$ =0.62872 $\lambda$ =0.02529 $\xi$ =0.00217
29	Kumaraswamy	$\alpha_1$ =2.8732 $\alpha_2$ =2.091 a=-0.00295 b=0.03066
30	Laplace	λ=209.57 μ=0.01809
31	Levy	σ=0.26398
32	Levy (2P)	σ=0.01467 γ=1.8319Ε-4
33	Log-Logistic	α=2.2905 β=0.01603
34	Log-Logistic (3P)	α=2.0656E+8 β=8.3107E+5 γ=-8.3107E+5
35	Log-Pearson 3	α=0.27376 β=-1.1714 γ=-3.807
36	Logistic	σ=0.00372 μ=0.01809
37	Lognormal	σ=0.61014 μ=-4.1277
38	Lognormal (3P)	σ=0.03405 μ=-1.6314 γ=-0.17765
39	Nakagami	m=2.7319 Ω=3.7223E-4
40	Normal	σ=0.00675 μ=0.01809
41	Pareto	α=0.2234 β=1.8337Ε-4

42       Pareto 2 $\alpha$ =370.29 β=6.6918         43       Pearson 5 $\alpha$ =8.8328 β=0.14035         44       Pearson 5 (3P) $\alpha$ =4.6881 β=0.07005 γ=8.9465E-5         45       Pearson 6 $\alpha_1$ =5.7971 $\alpha_2$ =1302.2 β=4.0675         46       Pearson 6 (4P) $\alpha_1$ =171.94 $\alpha_2$ =4675.7 β=2.4396 γ=-0.07191         47       Pert $m$ =0.02078 a=-0.00549 b=0.03068         48       Power Function $\alpha$ =1.4645 a=1.3285E-4 b=0.03013         49       Rayleigh $\sigma$ =0.01443         50       Rayleigh (2P) $\sigma$ =0.01371 γ=-1.0193E-4         51       Reciprocal       a=1.8330E-4 b=0.03015         52       Rice $v$ =0.01642 $\sigma$ =0.00715         53       Triangular $m$ =0.02322 a=-6.4911E-4 b=0.03074         54       Uniform       a=0.0064 b=0.02977         55       Weibull $\alpha$ =1.7547 $\beta$ =0.02207			
44 Pearson 5 (3P) $\alpha$ =4.6881 β=0.07005 γ=8.9465E-5 45 Pearson 6 $\alpha_1$ =5.7971 $\alpha_2$ =1302.2 β=4.0675 46 Pearson 6 (4P) $\alpha_1$ =171.94 $\alpha_2$ =4675.7 β=2.4396 γ=-0.07191 47 Pert $\alpha_2$ =0.02078 a=-0.00549 b=0.03068 48 Power Function $\alpha$ =1.4645 a=1.3285E-4 b=0.03013 49 Rayleigh $\alpha$ =0.01443 50 Rayleigh (2P) $\alpha$ =0.01371 γ=-1.0193E-4 51 Reciprocal $\alpha$ =1.8330E-4 b=0.03015 52 Rice $\alpha$ =0.01642 $\alpha$ =0.00715 53 Triangular $\alpha$ =0.02322 a=-6.4911E-4 b=0.03074 54 Uniform $\alpha$ =0.0064 b=0.02977 55 Weibull $\alpha$ =1.7547 β=0.02207	42	Pareto 2	α=370.29 β=6.6918
45 Pearson 6 $\alpha_1$ =5.7971 $\alpha_2$ =1302.2 $\beta$ =4.0675 46 Pearson 6 (4P) $\alpha_1$ =171.94 $\alpha_2$ =4675.7 $\alpha_2$ =2.4396 $\alpha_3$ =-0.07191 47 Pert $\alpha_3$ =0.02078 $\alpha_3$ =-0.00549 $\alpha_3$ =0.03068 48 Power Function $\alpha_3$ =1.4645 $\alpha_3$ =1.3285E-4 $\alpha_3$ =0.03013 49 Rayleigh $\alpha_3$ =0.01443 50 Rayleigh (2P) $\alpha_3$ =0.01371 $\alpha_3$ =1.0193E-4 51 Reciprocal $\alpha_3$ =1.8330E-4 $\alpha_3$ =0.03015 52 Rice $\alpha_3$ =0.01642 $\alpha_3$ =0.00715 53 Triangular $\alpha_3$ =0.02322 $\alpha_3$ =-6.4911E-4 $\alpha_3$ =0.03074 54 Uniform $\alpha_3$ =0.0064 $\alpha_3$ =0.02207	43	Pearson 5	α=8.8328 β=0.14035
46 Pearson 6 (4P) $\alpha_1=171.94$ $\alpha_2=4675.7$ $\beta=2.4396$ $\gamma=-0.07191$ 47 Pert $m=0.02078$ $a=-0.00549$ $b=0.03068$ 48 Power Function $\alpha=1.4645$ $a=1.3285E-4$ $b=0.03013$ 49 Rayleigh $\sigma=0.01443$ 50 Rayleigh (2P) $\sigma=0.01371$ $\gamma=-1.0193E-4$ 51 Reciprocal $a=1.8330E-4$ $b=0.03015$ 52 Rice $v=0.01642$ $\sigma=0.00715$ 53 Triangular $m=0.02322$ $a=-6.4911E-4$ $b=0.03074$ 54 Uniform $a=0.0064$ $b=0.02977$ 55 Weibull $\alpha=1.7547$ $\beta=0.02207$	44	Pearson 5 (3P)	α=4.6881 β=0.07005 γ=8.9465Ε-5
46       Pearson 6 (4P) $\beta$ =2.4396 $\gamma$ =-0.07191         47       Pert       m=0.02078 a=-0.00549 b=0.03068         48       Power Function $\alpha$ =1.4645 a=1.3285E-4 b=0.03013         49       Rayleigh $\sigma$ =0.01443         50       Rayleigh (2P) $\sigma$ =0.01371 $\gamma$ =-1.0193E-4         51       Reciprocal       a=1.8330E-4 b=0.03015         52       Rice       v=0.01642 $\sigma$ =0.00715         53       Triangular       m=0.02322 a=-6.4911E-4 b=0.03074         54       Uniform       a=0.0064 b=0.02977         55       Weibull $\alpha$ =1.7547 $\beta$ =0.02207	45	Pearson 6	$\alpha_1 = 5.7971 \ \alpha_2 = 1302.2 \ \beta = 4.0675$
48 Power Function $\alpha$ =1.4645 a=1.3285E-4 b=0.03013 49 Rayleigh $\sigma$ =0.01443 50 Rayleigh (2P) $\sigma$ =0.01371 γ=-1.0193E-4 51 Reciprocal a=1.8330E-4 b=0.03015 52 Rice $\nu$ =0.01642 $\sigma$ =0.00715 53 Triangular m=0.02322 a=-6.4911E-4 b=0.03074 54 Uniform a=0.0064 b=0.02977 55 Weibull $\alpha$ =1.7547 $\beta$ =0.02207	46	Pearson 6 (4P)	
49 Rayleigh $\sigma$ =0.01443 50 Rayleigh (2P) $\sigma$ =0.01371 $\gamma$ =-1.0193E-4 51 Reciprocal $\sigma$ =1.8330E-4 $\sigma$ =0.03015 52 Rice $\sigma$ =0.01642 $\sigma$ =0.00715 53 Triangular $\sigma$ =0.02322 $\sigma$ =-6.4911E-4 $\sigma$ =0.03074 54 Uniform $\sigma$ =0.0064 $\sigma$ =0.02977 55 Weibull $\sigma$ =1.7547 $\sigma$ =0.02207	47	Pert	m=0.02078 a=-0.00549 b=0.03068
50 Rayleigh (2P) σ=0.01371 γ=-1.0193E-4 51 Reciprocal a=1.8330E-4 b=0.03015 52 Rice v=0.01642 σ=0.00715 53 Triangular m=0.02322 a=-6.4911E-4 b=0.03074 54 Uniform a=0.0064 b=0.02977 55 Weibull α=1.7547 β=0.02207	48	Power Function	α=1.4645 a=1.3285E-4 b=0.03013
51 Reciprocal a=1.8330E-4 b=0.03015  52 Rice v=0.01642 σ=0.00715  53 Triangular m=0.02322 a=-6.4911E-4 b=0.03074  54 Uniform a=0.0064 b=0.02977  55 Weibull α=1.7547 β=0.02207	49	Rayleigh	σ=0.01443
52 Rice v=0.01642 σ=0.00715 53 Triangular m=0.02322 a=-6.4911E-4 b=0.03074 54 Uniform a=0.0064 b=0.02977 55 Weibull α=1.7547 β=0.02207	50	Rayleigh (2P)	σ=0.01371 γ=-1.0193Ε-4
53 Triangular m=0.02322 a=-6.4911E-4 b=0.03074 54 Uniform a=0.0064 b=0.02977 55 Weibull α=1.7547 β=0.02207	51	Reciprocal	a=1.8330E-4 b=0.03015
54 Uniform a=0.0064 b=0.02977 55 Weibull α=1.7547 β=0.02207	52	Rice	v=0.01642 σ=0.00715
55 Weibull α=1.7547 β=0.02207	53	Triangular	m=0.02322 a=-6.4911E-4 b=0.03074
S   THE TY   0102207	54	Uniform	a=0.0064 b=0.02977
56 W. 1. 11 (2D)	55	Weibull	α=1.7547 β=0.02207
$\alpha = 103.78 \ \beta = 0.56216 \ \gamma = -0.54094$	56	Weibull (3P)	α=103.78 β=0.56216 γ=-0.54094
57 Chi-Squared No fit	57	Chi-Squared	No fit
58 Chi-Squared (2P) No fit	58	Chi-Squared (2P)	No fit
59 Johnson SU No fit	59	Johnson SU	No fit
60 Log-Gamma No fit	60	Log-Gamma	No fit
61 Student's t No fit	61	Student's t	No fit

## **Goodness of Fit - Summary**

#	<b>Distribution</b>	Kolmogorov Smirnov		Anderson Darling		<u>Chi-Squared</u>	
		Statistic	Rank	Statistic	Rank	Statistic	Rank
1	Beta	0.16413	14	2.7095	6	41.535	22
2	Burr	0.21319	26	4.7808	17	44.765	25
3	Burr (4P)	0.13714	4	2.8886	9	32.511	13
4	Cauchy	0.22486	32	9.0356	35	44.435	24
5	Dagum	0.16568	15	2.469	4	31.662	11
6	Dagum (4P)	0.14878	8	2.3066	3	32.126	12
7	Erlang	0.27081	46	8.1542	32	67.927	36

8         Erlang (3P)         0.20677         22         4.7161         15         46.051         26           9         Error         0.18129         18         3.2051         11         27.33         6           10         Error Function         0.82436         55         390.68         55         998.4         47           11         Exponential         0.29734         48         19.933         46         50.246         29           12         Exponential (2P)         0.29341         47         19.875         45         43.864         23           13         Fatigue Life         0.31319         51         20.767         48         39.405         21           14         Fatigue Life (3P)         0.1994         19         4.4293         14         34.985         17           15         Frechet         0.26215         42         15.618         42         20.134         5           16         Frechet (3P)         0.24887         38         11.301         40         N/A           17         Gamma         0.24523         36         7.3573         30         56.272         32           18         Gamma (3P)			I		I	I	I	
10   Error Function   0.82436   55   390.68   55   998.4   47     11   Exponential   0.29734   48   19.933   46   50.246   29     12   Exponential (2P)   0.29341   47   19.875   45   43.864   23     13   Eatigue Life   0.31319   51   20.767   48   39.405   21     14   Eatigue Life (3P)   0.1994   19   4.4293   14   34.985   17     15   Frechet   0.26215   42   15.618   42   20.134   5     16   Frechet (3P)   0.24887   38   11.301   40   N/A     17   Gamma   0.2453   36   7.3573   30   56.272   32     18   Gamma (3P)   0.21808   27   4.872   19   48.007   27     19   Gen. Extreme Value   0.13804   5   6.499   25   N/A     20   Gen. Gamma   0.24625   37   5.8561   23   77.617   40     21   Gen. Gamma   0.24625   37   5.8561   23   77.617   40     21   Gen. Gamma   0.24625   37   5.8561   23   77.617   40     22   Gen. Pareto   0.12184   2   24.414   50   N/A     23   Gumbel Max   0.26769   45   9.9831   39   76.883   38     24   Gumbel Min   0.13956   6   3.0589   10   33.012   14     25   Hypersecant   0.22265   30   6.9336   26   56.35   33     26   Inv. Gaussian   0.26692   44   12.248   41   76.912   39     27   Inv. Gaussian   0.25942   41   8.9101   34   5.2834   1     28   Johnson SB   0.11811   1   17.683   44   N/A     29   Kumaraswamy   0.16194   12   2.6835   5   38.027   18     30   Laplace   0.25025   39   9.2465   37   61.573   35     31   Levy   0.99692   56   818.52   56   1.0378E+5   48     32   Log-Logistic   0.22322   31   8.7214   33   12.124   2     34   Log-Logistic   0.22322   31   8.7214   33   12.124   2     35   Log-Pearson 3   0.15929   11   117.32   53   N/A     36   Logistic   0.21267   24   5.7498   21   53.512   31     37   Lognormal   0.23532   33   7.2251   27   18.33   4     38   Lognormal   0.21301   25   9.0789   36   61.51   34     40   Normal   0.20053   20   4.4087   12   34.864   16	8	Erlang (3P)	0.20677	22	4.7161	15	46.051	26
Exponential   0.29734   48   19.933   46   50.246   29     Exponential (2P)   0.29341   47   19.875   45   43.864   23     Fatigue Life   0.31319   51   20.767   48   39.405   21     Fatigue Life   0.26215   42   15.618   42   20.134   5     Frechet   0.26215   42   15.618   42   20.134   5     Frechet   0.24887   38   11.301   40   N/A     Gamma   0.2453   36   7.3573   30   56.272   32     Ramma (3P)   0.21808   27   4.872   19   48.007   27     Gen. Extreme Value   0.13804   5   6.499   25   N/A     Gen. Gamma   0.24625   37   5.8561   23   77.617   40     Gen. Gamma   0.24625   37   5.8561   23   77.617   40     Gen. Gamma   0.24625   37   5.8561   23   77.617   40     Gen. Gamma   0.14854   7   2.2569   2   33.174   15     Gen. Pareto   0.12184   2   24.414   50   N/A     Gumbel Max   0.26769   45   9.9831   39   76.883   38     Gumbel Min   0.13956   6   3.0589   10   33.012   14     Gen. Gaussian   0.26692   44   12.248   41   76.912   39     Inv. Gaussian   0.25942   41   8.9101   34   5.2834   1     Inv. Gaussian (3P)   0.25942   41   8.9101   34   5.2834   1     Subnson SB   0.11811   1   17.683   44   N/A     Kumaraswamy   0.16194   12   2.6835   5   38.027   18     30   Laplace   0.25025   39   9.2465   37   61.573   35     31   Levy   0.99692   56   818.52   56   1.0378E+5   48     32   Levy (2P)   0.51594   52   33.448   51   104.42   45     33   Log-Logistic   0.22322   31   8.7214   33   12.124   2     34   Log-Logistic   0.21267   24   5.7498   21   53.512   31     35   Log-Pearson 3   0.15929   11   117.32   53   N/A     36   Lognormal   0.23532   33   7.2251   27   18.33   4     40   Normal   0.20653   20   4.4087   12   34.864   16	9	Error	0.18129	18	3.2051	11	27.33	6
Exponential (2P)	10	Error Function	0.82436	55	390.68	55	998.4	47
13       Eatigue Life       0.31319       51       20.767       48       39.405       21         14       Fatigue Life (3P)       0.1994       19       4.4293       14       34.985       17         15       Frechet       0.26215       42       15.618       42       20.134       5         16       Frechet (3P)       0.24887       38       11.301       40       N/A         17       Gamma       0.2453       36       7.3573       30       56.272       32         18       Gamma (3P)       0.21808       27       4.872       19       48.007       27         19       Gen. Extreme Value       0.13804       5       6.499       25       N/A         20       Gen. Gamma       0.24625       37       5.8561       23       77.617       40         21       Gen. Gamma (4P)       0.14854       7       2.2569       2       33.174       15         22       Gen. Pareto       0.12184       2       24.414       50       N/A         23       Gumbel Max       0.26769       45       9.9831       39       76.883       38         24       Gumbel Min       0	11	Exponential	0.29734	48	19.933	46	50.246	29
14   Eatigue Life (3P)   0.1994   19   4.4293   14   34.985   17     15   Frechet   0.26215   42   15.618   42   20.134   5     16   Frechet (3P)   0.24887   38   11.301   40   N/A     17   Gamma   0.2453   36   7.3573   30   56.272   32     18   Gamma (3P)   0.21808   27   4.872   19   48.007   27     19   Gen. Extreme Value   0.13804   5   6.499   25   N/A     20   Gen. Gamma   0.24625   37   5.8561   23   77.617   40     21   Gen. Gamma (4P)   0.14854   7   2.2569   2   33.174   15     22   Gen. Pareto   0.12184   2   24.414   50   N/A     23   Gumbel Max   0.26769   45   9.9831   39   76.883   38     24   Gumbel Min   0.13956   6   3.0589   10   33.012   14     25   Hypersecant   0.22265   30   6.9336   26   56.35   33     26   Inv. Gaussian   0.26692   44   12.248   41   76.912   39     27   Inv. Gaussian   0.25942   41   8.9101   34   5.2834   1     28   Johnson SB   0.11811   1   17.683   44   N/A     29   Kumaraswamy   0.16194   12   2.6835   5   38.027   18     30   Laplace   0.25025   39   9.2465   37   61.573   35     31   Levy   0.99692   56   818.52   56   1.0378E+5   48     32   Levy (2P)   0.51594   52   33.448   51   104.42   45     33   Log-Logistic   0.22322   31   8.7214   33   12.124   2     34   Log-Logistic   0.22322   31   8.7214   33   12.124   2     35   Log-Pearson 3   0.15929   11   117.32   53   N/A     36   Logistic   0.21367   24   5.7498   21   53.512   31     37   Lognormal   0.23532   33   7.2251   27   18.33   4     38   Lognormal   0.23532   33   7.2251   27   18.33   4     39   Nakagami   0.21301   25   9.0789   36   61.51   34     40   Normal   0.20053   20   4.4087   12   34.864   16	12	Exponential (2P)	0.29341	47	19.875	45	43.864	23
15   Frechet	13	Fatigue Life	0.31319	51	20.767	48	39.405	21
16         Frechet (3P)         0.24887         38         11.301         40         N/A           17         Gamma         0.2453         36         7.3573         30         56.272         32           18         Gamma (3P)         0.21808         27         4.872         19         48.007         27           19         Gen. Extreme Value         0.13804         5         6.499         25         N/A           20         Gen. Gamma         0.24625         37         5.8561         23         77.617         40           21         Gen. Gamma (4P)         0.14854         7         2.2569         2         33.174         15           22         Gen. Pareto         0.12184         2         24.414         50         N/A           23         Gumbel Max         0.26769         45         9.9831         39         76.883         38           24         Gumbel Min         0.13956         6         3.0589         10         33.012         14           25         Hypersecant         0.22265         30         6.9336         26         56.35         33           26         Inv. Gaussian         0.26692         44	14	Fatigue Life (3P)	0.1994	19	4.4293	14	34.985	17
17   Gamma   0.2453   36   7.3573   30   56.272   32   18   Gamma (3P)   0.21808   27   4.872   19   48.007   27   19   Gen. Extreme Value   0.13804   5   6.499   25   N/A   20   Gen. Gamma   0.24625   37   5.8561   23   77.617   40   21   Gen. Gamma (4P)   0.14854   7   2.2569   2   33.174   15   22   Gen. Pareto   0.12184   2   24.414   50   N/A   23   Gumbel Max   0.26769   45   9.9831   39   76.883   38   24   Gumbel Min   0.13956   6   3.0589   10   33.012   14   14   15   14   15   14   15   15	15	Frechet	0.26215	42	15.618	42	20.134	5
18 Gamma (3P)         0.21808         27         4.872         19         48.007         27           19 Gen. Extreme Value         0.13804         5         6.499         25         N/A           20 Gen. Gamma         0.24625         37         5.8561         23         77.617         40           21 Gen. Gamma (4P)         0.14854         7         2.2569         2         33.174         15           22 Gen. Pareto         0.12184         2         24.414         50         N/A           23 Gumbel Max         0.26769         45         9.9831         39         76.883         38           24 Gumbel Min         0.13956         6         3.0589         10         33.012         14           25 Hypersecant         0.22265         30         6.9336         26         56.35         33           26 Inv. Gaussian         0.26692         44         12.248         41         76.912         39           27 Inv. Gaussian (3P)         0.25942         41         8.9101         34         5.2834         1           28 Johnson SB         0.11811         1         17.683         44         N/A           29 Kumaraswamy         0.16194         12 <td>16</td> <td>Frechet (3P)</td> <td>0.24887</td> <td>38</td> <td>11.301</td> <td>40</td> <td>N/A</td> <td></td>	16	Frechet (3P)	0.24887	38	11.301	40	N/A	
19 Gen. Extreme Value   0.13804   5   6.499   25   N/A   20 Gen. Gamma   0.24625   37   5.8561   23   77.617   40   21 Gen. Gamma (4P)   0.14854   7   2.2569   2   33.174   15   22 Gen. Pareto   0.12184   2   24.414   50   N/A   23 Gumbel Max   0.26769   45   9.9831   39   76.883   38   24 Gumbel Min   0.13956   6   3.0589   10   33.012   14   25 Hypersecant   0.22265   30   6.9336   26   56.35   33   26 Inv. Gaussian   0.26692   44   12.248   41   76.912   39   27 Inv. Gaussian (3P)   0.25942   41   8.9101   34   5.2834   1   28 Johnson SB   0.11811   1   17.683   44   N/A   29 Kumaraswamy   0.16194   12   2.6835   5   38.027   18   30 Laplace   0.25025   39   9.2465   37   61.573   35   31 Levy   0.99692   56   818.52   56   1.0378E+5   48   32 Levy (2P)   0.51594   52   33.448   51   104.42   45   33 Log-Logistic   0.22322   31   8.7214   33   12.124   2   34 Log-Logistic   0.2322   31   8.7214   33   12.124   2   34 Log-Logistic   0.21267   24   5.7498   21   53.512   31   35 Log-Pearson 3   0.15929   11   117.32   53   N/A   36 Logistic   0.21267   24   5.7498   21   53.512   31   37 Lognormal   0.23532   33   7.2251   27   18.33   4   38 Lognormal (3P)   0.20565   21   4.7703   16   39.403   20   39 Nakagami   0.21301   25   9.0789   36   61.51   34   40 Normal   0.20053   20   4.4087   12   34.864   16	17	<u>Gamma</u>	0.2453	36	7.3573	30	56.272	32
20         Gen. Gamma         0.24625         37         5.8561         23         77.617         40           21         Gen. Gamma (4P)         0.14854         7         2.2569         2         33.174         15           22         Gen. Pareto         0.12184         2         24.414         50         N/A           23         Gumbel Max         0.26769         45         9.9831         39         76.883         38           24         Gumbel Min         0.13956         6         3.0589         10         33.012         14           25         Hypersecant         0.22265         30         6.9336         26         56.35         33           26         Inv. Gaussian         0.26692         44         12.248         41         76.912         39           27         Inv. Gaussian (3P)         0.25942         41         8.9101         34         5.2834         1           28         Johnson SB         0.11811         1         17.683         44         N/A           29         Kumaraswamy         0.16194         12         2.6835         5         38.027         18           30         Laplace         0.25025	18	Gamma (3P)	0.21808	27	4.872	19	48.007	27
21         Gen. Gamma (4P)         0.14854         7         2.2569         2         33.174         15           22         Gen. Pareto         0.12184         2         24.414         50         N/A           23         Gumbel Max         0.26769         45         9.9831         39         76.883         38           24         Gumbel Min         0.13956         6         3.0589         10         33.012         14           25         Hypersecant         0.22265         30         6.9336         26         56.35         33           26         Inv. Gaussian         0.26692         44         12.248         41         76.912         39           27         Inv. Gaussian (3P)         0.25942         41         8.9101         34         5.2834         1           28         Johnson SB         0.11811         1         17.683         44         N/A           29         Kumaraswamy         0.16194         12         2.6835         5         38.027         18           30         Laplace         0.25025         39         9.2465         37         61.573         35           31         Levy         0.99692	19	Gen. Extreme Value	0.13804	5	6.499	25	N/A	
22         Gen. Pareto         0.12184         2         24.414         50         N/A           23         Gumbel Max         0.26769         45         9.9831         39         76.883         38           24         Gumbel Min         0.13956         6         3.0589         10         33.012         14           25         Hypersecant         0.22265         30         6.9336         26         56.35         33           26         Inv. Gaussian         0.26692         44         12.248         41         76.912         39           27         Inv. Gaussian (3P)         0.25942         41         8.9101         34         5.2834         1           28         Johnson SB         0.11811         1         17.683         44         N/A           29         Kumaraswamy         0.16194         12         2.6835         5         38.027         18           30         Laplace         0.25025         39         9.2465         37         61.573         35           31         Levy         0.99692         56         818.52         56         1.0378E+5         48           32         Levy (2P)         0.51594	20	Gen. Gamma	0.24625	37	5.8561	23	77.617	40
23         Gumbel Max         0.26769         45         9.9831         39         76.883         38           24         Gumbel Min         0.13956         6         3.0589         10         33.012         14           25         Hypersecant         0.22265         30         6.9336         26         56.35         33           26         Inv. Gaussian         0.26692         44         12.248         41         76.912         39           27         Inv. Gaussian (3P)         0.25942         41         8.9101         34         5.2834         1           28         Johnson SB         0.11811         1         17.683         44         N/A           29         Kumaraswamy         0.16194         12         2.6835         5         38.027         18           30         Laplace         0.25025         39         9.2465         37         61.573         35           31         Levy         0.99692         56         818.52         56         1.0378E+5         48           32         Levy (2P)         0.51594         52         33.448         51         104.42         45           33         Log-Logistic (3P)	21	Gen. Gamma (4P)	0.14854	7	2.2569	2	33.174	15
24         Gumbel Min         0.13956         6         3.0589         10         33.012         14           25         Hypersecant         0.22265         30         6.9336         26         56.35         33           26         Inv. Gaussian         0.26692         44         12.248         41         76.912         39           27         Inv. Gaussian (3P)         0.25942         41         8.9101         34         5.2834         1           28         Johnson SB         0.11811         1         17.683         44         N/A           29         Kumaraswamy         0.16194         12         2.6835         5         38.027         18           30         Laplace         0.25025         39         9.2465         37         61.573         35           31         Levy         0.99692         56         818.52         56         1.0378E+5         48           32         Levy (2P)         0.51594         52         33.448         51         104.42         45           33         Log-Logistic         0.22322         31         8.7214         33         12.124         2           34         Log-Pearson 3	22	Gen. Pareto	0.12184	2	24.414	50	N/A	
25         Hypersecant         0.22265         30         6.9336         26         56.35         33           26         Inv. Gaussian         0.26692         44         12.248         41         76.912         39           27         Inv. Gaussian (3P)         0.25942         41         8.9101         34         5.2834         1           28         Johnson SB         0.11811         1         17.683         44         N/A           29         Kumaraswamy         0.16194         12         2.6835         5         38.027         18           30         Laplace         0.25025         39         9.2465         37         61.573         35           31         Levy         0.99692         56         818.52         56         1.0378E+5         48           32         Levy (2P)         0.51594         52         33.448         51         104.42         45           33         Log-Logistic         0.22322         31         8.7214         33         12.124         2           34         Log-Logistic (3P)         0.17022         16         4.4245         13         30.573         10           35         Log-Pearson 3 <td>23</td> <td>Gumbel Max</td> <td>0.26769</td> <td>45</td> <td>9.9831</td> <td>39</td> <td>76.883</td> <td>38</td>	23	Gumbel Max	0.26769	45	9.9831	39	76.883	38
26         Inv. Gaussian         0.26692         44         12.248         41         76.912         39           27         Inv. Gaussian (3P)         0.25942         41         8.9101         34         5.2834         1           28         Johnson SB         0.11811         1         17.683         44         N/A           29         Kumaraswamy         0.16194         12         2.6835         5         38.027         18           30         Laplace         0.25025         39         9.2465         37         61.573         35           31         Levy         0.99692         56         818.52         56         1.0378E+5         48           32         Levy (2P)         0.51594         52         33.448         51         104.42         45           33         Log-Logistic         0.22322         31         8.7214         33         12.124         2           34         Log-Logistic (3P)         0.17022         16         4.4245         13         30.573         10           35         Log-Pearson 3         0.15929         11         117.32         53         N/A           36         Logistic         0.21267 <td>24</td> <td>Gumbel Min</td> <td>0.13956</td> <td>6</td> <td>3.0589</td> <td>10</td> <td>33.012</td> <td>14</td>	24	Gumbel Min	0.13956	6	3.0589	10	33.012	14
27         Inv. Gaussian (3P)         0.25942         41         8.9101         34         5.2834         1           28         Johnson SB         0.11811         1         17.683         44         N/A           29         Kumaraswamy         0.16194         12         2.6835         5         38.027         18           30         Laplace         0.25025         39         9.2465         37         61.573         35           31         Levy         0.99692         56         818.52         56         1.0378E+5         48           32         Levy (2P)         0.51594         52         33.448         51         104.42         45           33         Log-Logistic         0.22322         31         8.7214         33         12.124         2           34         Log-Logistic (3P)         0.17022         16         4.4245         13         30.573         10           35         Log-Pearson 3         0.15929         11         117.32         53         N/A           36         Logistic         0.21267         24         5.7498         21         53.512         31           37         Lognormal         0.23532	25	Hypersecant	0.22265	30	6.9336	26	56.35	33
28         Johnson SB         0.11811         1         17.683         44         N/A           29         Kumaraswamy         0.16194         12         2.6835         5         38.027         18           30         Laplace         0.25025         39         9.2465         37         61.573         35           31         Levy         0.99692         56         818.52         56         1.0378E+5         48           32         Levy (2P)         0.51594         52         33.448         51         104.42         45           33         Log-Logistic         0.22322         31         8.7214         33         12.124         2           34         Log-Logistic (3P)         0.17022         16         4.4245         13         30.573         10           35         Log-Pearson 3         0.15929         11         117.32         53         N/A           36         Logistic         0.21267         24         5.7498         21         53.512         31           37         Lognormal         0.23532         33         7.2251         27         18.33         4           38         Lognormal         0.20565 <td< td=""><td>26</td><td>Inv. Gaussian</td><td>0.26692</td><td>44</td><td>12.248</td><td>41</td><td>76.912</td><td>39</td></td<>	26	Inv. Gaussian	0.26692	44	12.248	41	76.912	39
29         Kumaraswamy         0.16194         12         2.6835         5         38.027         18           30         Laplace         0.25025         39         9.2465         37         61.573         35           31         Levy         0.99692         56         818.52         56         1.0378E+5         48           32         Levy (2P)         0.51594         52         33.448         51         104.42         45           33         Log-Logistic         0.22322         31         8.7214         33         12.124         2           34         Log-Logistic (3P)         0.17022         16         4.4245         13         30.573         10           35         Log-Pearson 3         0.15929         11         117.32         53         N/A           36         Logistic         0.21267         24         5.7498         21         53.512         31           37         Lognormal         0.23532         33         7.2251         27         18.33         4           38         Lognormal (3P)         0.20565         21         4.7703         16         39.403         20           39         Nakagami         <	27	Inv. Gaussian (3P)	0.25942	41	8.9101	34	5.2834	1
30       Laplace       0.25025       39       9.2465       37       61.573       35         31       Levy       0.99692       56       818.52       56       1.0378E+5       48         32       Levy (2P)       0.51594       52       33.448       51       104.42       45         33       Log-Logistic       0.22322       31       8.7214       33       12.124       2         34       Log-Logistic (3P)       0.17022       16       4.4245       13       30.573       10         35       Log-Pearson 3       0.15929       11       117.32       53       N/A         36       Logistic       0.21267       24       5.7498       21       53.512       31         37       Lognormal       0.23532       33       7.2251       27       18.33       4         38       Lognormal (3P)       0.20565       21       4.7703       16       39.403       20         39       Nakagami       0.21301       25       9.0789       36       61.51       34         40       Normal       0.20053       20       4.4087       12       34.864       16	28	Johnson SB	0.11811	1	17.683	44	N/A	
31     Levy     0.99692     56     818.52     56     1.0378E+5     48       32     Levy (2P)     0.51594     52     33.448     51     104.42     45       33     Log-Logistic     0.22322     31     8.7214     33     12.124     2       34     Log-Logistic (3P)     0.17022     16     4.4245     13     30.573     10       35     Log-Pearson 3     0.15929     11     117.32     53     N/A       36     Logistic     0.21267     24     5.7498     21     53.512     31       37     Lognormal     0.23532     33     7.2251     27     18.33     4       38     Lognormal (3P)     0.20565     21     4.7703     16     39.403     20       39     Nakagami     0.21301     25     9.0789     36     61.51     34       40     Normal     0.20053     20     4.4087     12     34.864     16	29	Kumaraswamy	0.16194	12	2.6835	5	38.027	18
32         Levy (2P)         0.51594         52         33.448         51         104.42         45           33         Log-Logistic         0.22322         31         8.7214         33         12.124         2           34         Log-Logistic (3P)         0.17022         16         4.4245         13         30.573         10           35         Log-Pearson 3         0.15929         11         117.32         53         N/A           36         Logistic         0.21267         24         5.7498         21         53.512         31           37         Lognormal         0.23532         33         7.2251         27         18.33         4           38         Lognormal (3P)         0.20565         21         4.7703         16         39.403         20           39         Nakagami         0.21301         25         9.0789         36         61.51         34           40         Normal         0.20053         20         4.4087         12         34.864         16	30	<u>Laplace</u>	0.25025	39	9.2465	37	61.573	35
33 Log-Logistic 0.22322 31 8.7214 33 12.124 2 34 Log-Logistic (3P) 0.17022 16 4.4245 13 30.573 10 35 Log-Pearson 3 0.15929 11 117.32 53 N/A 36 Logistic 0.21267 24 5.7498 21 53.512 31 37 Lognormal 0.23532 33 7.2251 27 18.33 4 38 Lognormal (3P) 0.20565 21 4.7703 16 39.403 20 39 Nakagami 0.21301 25 9.0789 36 61.51 34 40 Normal 0.20053 20 4.4087 12 34.864 16	31	Levy	0.99692	56	818.52	56	1.0378E+5	48
34         Log-Logistic (3P)         0.17022         16         4.4245         13         30.573         10           35         Log-Pearson 3         0.15929         11         117.32         53         N/A           36         Logistic         0.21267         24         5.7498         21         53.512         31           37         Lognormal         0.23532         33         7.2251         27         18.33         4           38         Lognormal (3P)         0.20565         21         4.7703         16         39.403         20           39         Nakagami         0.21301         25         9.0789         36         61.51         34           40         Normal         0.20053         20         4.4087         12         34.864         16	32	Levy (2P)	0.51594	52	33.448	51	104.42	45
35         Log-Pearson 3         0.15929         11         117.32         53         N/A           36         Logistic         0.21267         24         5.7498         21         53.512         31           37         Lognormal         0.23532         33         7.2251         27         18.33         4           38         Lognormal (3P)         0.20565         21         4.7703         16         39.403         20           39         Nakagami         0.21301         25         9.0789         36         61.51         34           40         Normal         0.20053         20         4.4087         12         34.864         16	33	Log-Logistic	0.22322	31	8.7214	33	12.124	2
36         Logistic         0.21267         24         5.7498         21         53.512         31           37         Lognormal         0.23532         33         7.2251         27         18.33         4           38         Lognormal (3P)         0.20565         21         4.7703         16         39.403         20           39         Nakagami         0.21301         25         9.0789         36         61.51         34           40         Normal         0.20053         20         4.4087         12         34.864         16	34	Log-Logistic (3P)	0.17022	16	4.4245	13	30.573	10
37         Lognormal         0.23532         33         7.2251         27         18.33         4           38         Lognormal (3P)         0.20565         21         4.7703         16         39.403         20           39         Nakagami         0.21301         25         9.0789         36         61.51         34           40         Normal         0.20053         20         4.4087         12         34.864         16	35	Log-Pearson 3	0.15929	11	117.32	53	N/A	
38     Lognormal (3P)     0.20565     21     4.7703     16     39.403     20       39     Nakagami     0.21301     25     9.0789     36     61.51     34       40     Normal     0.20053     20     4.4087     12     34.864     16	36	Logistic	0.21267	24	5.7498	21	53.512	31
39 <u>Nakagami</u> 0.21301 25 9.0789 36 61.51 34 40 <u>Normal</u> 0.20053 20 4.4087 12 34.864 16	37	Lognormal	0.23532	33	7.2251	27	18.33	4
40 <u>Normal</u> 0.20053 20 4.4087 12 34.864 16	38	Lognormal (3P)	0.20565	21	4.7703	16	39.403	20
	39	Nakagami	0.21301	25	9.0789	36	61.51	34
41 <u>Pareto</u> 0.51979 53 43.361 52 89.221 43	40	Normal	0.20053	20	4.4087	12	34.864	16
	41	<u>Pareto</u>	0.51979	53	43.361	52	89.221	43

01 28				
82 8				
91 3				
97 37				
73 30				
36 19				
N/A				
64 41				
98 42				
69 46				
N/A				
35 7				
N/A				
97 44				
25 9				
No fit				

### APPENDIX 6.1 Unit Root Tests

#### LROS (LEVEL)

ADF Test Statistic	4.218319	1% Critical Value*	-3.4911
		5% Critical Value	-2.8879
		10% Critical Value	-2.5807

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(LROS)
Method: Least Squares
Date: 08/22/11 Time: 15:56
Sample(adjusted): 2001:04 2010:04

Included observations: 109 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LROS(-1)	0.334757	0.079358	4.218319	0.0001
D(LROS(-1))	0.649611	0.174966	3.712785	0.0003
D(LROS(-2))	-0.721331	0.183773	-3.925129	0.0002
C	1.643692	0.411725	3.992211	0.0001
R-squared	0.343550	Mean dependent var		-0.098775
Adjusted R-squared	0.324794	S.D. dependent var		0.760272
S.E. of regression	0.624722	Akaike info criterion		1.932988
Sum squared resid	40.97918	Schwarz criterion		2.031753
Log likelihood	-101.3478	F-statistic		18.31707
Durbin-Watson stat	1.731350_	Prob(F-stati	stic)	0.000000

PP Test Statistic	5.526277	1% Critical Value*	-3.4900
		5% Critical Value	-2.8874
		10% Critical Value	-2.5804

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel: (Newey-West suggests: 4)
4
Residual variance with no correction 0.467389
Residual variance with correction 0.411888

Phillips-Perron Test Equation Dependent Variable: D(LROS) Method: Least Squares Date: 08/22/11 Time: 15:57 Sample(adjusted): 2001:02 2010:04

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LROS(-1)	0.343259	0.072847	4.712047	0.0000
C	1.684706	0.383711	4.390562	0.0000
R-squared 0.169229 Mean dependent var		ident var	-0.096834	

Adjusted R-squared	0.161607	S.D. dependent var	0.753466
S.E. of regression	0.689902	Akaike info criterion	2.113320
Sum squared resid	51.88018	Schwarz criterion	2.162140
Log likelihood	-115.2892	F-statistic	22.20339
Durbin-Watson stat	1.055792_	Prob(F-statistic)	_0.000007

#### LROS (1st DIFFERENCE)

PP Test Statistic	-1.529401	1% Critical Value*	-3.4906
		5% Critical Value	-2.8877
		10% Critical Value	-2.5805

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel:	( Newey-West suggests: 4 )
Residual variance with no correction	0.460129
Residual variance with correction	0.496359

Phillips-Perron Test Equation Dependent Variable: D(LROS,2)

Method: Least Squares
Date: 08/23/11 Time: 14:45
Sample(adjusted): 2001:03 2010:04

Included observations: 110 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LROS(-1))	-0.146032	0.170024	-0.858892	0.3923
C	-0.067662	0.065547	-1.032267	0.3043
R-squared Adjusted R-squared S.E. of regression Sum squared resid Log likelihood Durbin-Watson stat	0.006784 -0.002412 0.684580 50.61420 -113.3896 1.206966	Mean depen S.D. depend Akaike info d Schwarz crit F-statistic Prob(F-statis	lent var criterion erion	-0.062508 0.683756 2.097992 2.147092 0.737695 0.392303
ADF Test Statistic	-2.435343	1% Critical 5% Critical	Value	-4.0452 -3.4515 -3.1509

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(LROS,2)

Method: Least Squares Date: 08/23/11 Time: 14:47 Sample(adjusted): 2001:05 2010:04

Included observations: 108 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LROS(-1))	-0.681214	0.279720	-2.435343	0.0166
D(LROS(-1),2)	0.554442	0.228284	2.428735	0.0169
D(LROS(-2),2)	0.084581	0.191434	0.441827	0.6595

327

C	0.191697	0.136931	1.399950	0.1645
_@TREND(2001:01)	-0.004733	0.002138	-2.213935	0.0290
R-squared Adjusted R-squared S.E. of regression Sum squared resid Log likelihood Durbin-Watson stat	0.102354 0.067494 0.666368 45.73679 -106.8470 1.446046	Mean depen S.D. depend Akaike info d Schwarz crit F-statistic Prob(F-statis	dent var criterion cerion	-0.063667 0.690063 2.071242 2.195414 2.936145 0.024114

#### LRoSD (LEVEL)

ADF Test Statistic	DF Test Statistic -3.244152 1% Critic	1% Critical Value*	-4.0429
		5% Critical Value	-3.4504
		10% Critical Value	-3.1503

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(LROSD)

Method: Least Squares 

Sample(adjusted): 2001:02 2010:04 Included observations: 111 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LROSD(-1)	-0.547314	0.168708	-3.244152	0.0016
C ` ´	-2.472530	0.784508	-3.151696	0.0021
@TREND(2001:01)	-0.004699	0.001433	-3.278540	0.0014
R-squared	0.107248	Mean dependent var		-0.034469
Adjusted R-squared	0.090716	S.D. dependent var		0.389923
S.E. of regression	0.371817	Akaike info criterion		0.885825
Sum squared resid	14.93076	Schwarz criterion		0.959055
Log likelihood	-46.16327	F-statistic		6.487137
Durbin-Watson stat	_ 1.329005_	Prob(F-stati	stic)	0.002185

## 1<sup>ST</sup> DIFFERENCE

ADF Test Statistic	-5.879333	1% Critical Value*	-4.0452
		5% Critical Value	-3.4515
		10% Critical Value	-3.1509

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation Dependent Variable: D(LROSD,2)

Method: Least Squares Date: 08/23/11 Time: 14:50

Sample(adjusted): 2001:05 2010:04

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LROSD(-1))	-2.506282	0.426287	-5.879333	0.0000
D(LROSD(-1),2)	0.470641	0.363985	1.293022	0.1989
D(LROSD(-2),2)	0.320288	0.248258	1.290138	0.1999
С	0.063817	0.072547	0.879662	0.3811
@TREND(2001:01)	-0.001804	0.001112	-1.621821	0.1079
R-squared	0.465331	Mean deper	ndent var	-0.033955
Adjusted R-squared	0.444567	S.D. depend	dent var	0.481574
S.E. of regression	0.358904	Akaike info	criterion	0.833668
Sum squared resid	13.26766	Schwarz crit	terion	0.957841
Log likelihood	-40.01806	F-statistic		22.41061
Durbin-Watson stat	1.419057	Prob(F-stati	stic)	0.000000

#### **LEVEL**

PP Test Statistic	-3.705670	1% Critical Value*	-4.0429
		5% Critical Value	-3.4504
		10% Critical Value	-3.1503

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel:	( Newey-West suggests: 4 )
4	
Residual variance with no correction	0.134511
Residual variance with correction	0.142888

Phillips-Perron Test Equation Dependent Variable: D(LROSD)

Method: Least Squares
Date: 08/23/11 Time: 14:51
Sample(adjusted): 2001:02 2010:04

Included observations: 111 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LROSD(-1)	-0.547314	0.168708	-3.244152	0.0016
C	-2.472530	0.784508	-3.151696	0.0021
@TREND(2001:01)	-0.004699	0.001433	-3.278540	0.0014
R-squared	0.107248	Mean dependent var		-0.034469
Adjusted R-squared	0.090716	S.D. dependent var		0.389923
S.E. of regression	0.371817	Akaike info	criterion	0.885825
Sum squared resid	14.93076	Schwarz crit	terion	0.959055
Log likelihood	-46.16327	F-statistic		6.487137
Durbin-Watson stat	_ 1.329005_	Prob(F-stati	stic)	0.002185

## 1<sup>ST</sup> DIFFERENCE

PP Test Statistic	-7.319482	1% Critical Value*	-4.0437
		5% Critical Value	-3.4508
		10% Critical Value	-3.1505

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel: (Newey-West suggests: 4)
4
Residual variance with no correction 0.123009
Residual variance with correction 0.092723

Phillips-Perron Test Equation Dependent Variable: D(LROSD,2)

Method: Least Squares
Date: 08/23/11 Time: 14:51
Sample(adjusted): 2001:03 2010:04

Included observations: 110 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LROSD(-1))	-2.045722	0.220102	-9.294432	0.0000
` C ` '	0.056316	0.069219	0.813587	0.4177
@TREND(2001:01)	-0.001628	0.001068	-1.524265	0.1304
R-squared	0.455006	Mean deper	ndent var	-0.034701
Adjusted R-squared	0.444819	S.D. dependent var		0.477260
S.E. of regression	0.355609	Akaike info criterion		0.796922
Sum squared resid	13.53095	Schwarz criterion		0.870571
Log likelihood	-40.83069	F-statistic		44.66614
Durbin-Watson stat	_ 1.411154_	Prob(F-stati	stic)	0.000000

#### LROSD LEVEL

ADF Test Statistic	-3.244152	1% Critical Value*	-4.0429
		5% Critical Value	-3.4504
		10% Critical Value	-3.1503

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(LROSD)

Method: Least Squares Date: 08/23/11 Time: 14:52 Sample(adjusted): 2001:02 2010:04

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LROSD(-1)	-0.547314	0.168708	-3.244152	0.0016
C	-2.472530	0.784508	-3.151696	0.0021
@TREND(2001:01)	-0.004699	0.001433	-3.278540	0.0014
R-squared	0.107248	Mean deper	ndent var	-0.034469
Adjusted R-squared	0.090716	S.D. dependent var		0.389923
S.E. of regression	0.371817	Akaike info criterion		0.885825
Sum squared resid	14.93076	Schwarz criterion		0.959055
Log likelihood	-46.16327	F-statistic		6.487137
Durbin-Watson stat	1.329005_	Prob(F-stati	stic)	0.002185

## 1<sup>ST</sup> DIFFERENCE

<b>ADF Test Statistic</b>	-5.879333	1% Critical Value*	-4.0452
		5% Critical Value	-3.4515
		10% Critical Value	-3.1509

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation Dependent Variable: D(LROSD,2)

Method: Least Squares Date: 08/23/11 Time: 14:53 Sample(adjusted): 2001:05 2010:04

Included observations: 108 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LROSD(-1))	-2.506282	0.426287	-5.879333	0.0000
D(LROSD(-1),2)	0.470641	0.363985	1.293022	0.1989
D(LROSD(-2),2)	0.320288	0.248258	1.290138	0.1999
C	0.063817	0.072547	0.879662	0.3811
@TREND(2001:01)	-0.001804	0.001112	-1.621821	0.1079
R-squared	0.465331	Mean deper		-0.033955
Adjusted R-squared	0.444567	S.D. depend		0.481574
S.E. of regression	0.358904	Akaike info		0.833668
Sum squared resid	13.26766	Schwarz crit	terion	0.957841
Log likelihood	-40.01806	F-statistic		22.41061
Durbin-Watson stat	1.419057	Prob(F-stati	stic)	0.000000

**LEVEL** 

PP Test Statistic	-3.705670	1% Critical Value*	-4.0429
		5% Critical Value	-3.4504
		10% Critical Value	-3.1503

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel: (Newey-West suggests: 4)
4
Residual variance with no correction 0.134511
Residual variance with correction 0.142888

Phillips-Perron Test Equation Dependent Variable: D(LROSD)

Method: Least Squares
Date: 08/23/11 Time: 14:53
Sample(adjusted): 2001:02 2010:04

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LROSD(-1)	-0.547314	0.168708	-3.244152	0.0016
С	-2.472530	0.784508	-3.151696	0.0021
@TREND(2001:01)	-0.004699	0.001433	-3.278540	0.0014
R-squared	0.107248	Mean deper	ndent var	-0.034469

Adjusted R-squared	0.090716	S.D. dependent var	0.389923
S.E. of regression	0.371817	Akaike info criterion	0.885825
Sum squared resid	14.93076	Schwarz criterion	0.959055
Log likelihood	-46.16327	F-statistic	6.487137
Durbin-Watson stat	1.329005_	Prob(F-statistic)	_0.002185
	<u> </u>		

#### 1<sup>ST</sup> DIFFERENCE

PP Test Statistic	-7.319482	1% Critical Value*	-4.0437
		5% Critical Value	-3.4508
		10% Critical Value	-3.1505

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel: ( Newey-West suggests: 4 )

4

Residual variance with no correction 0.123009
Residual variance with correction 0.092723

Phillips-Perron Test Equation Dependent Variable: D(LROSD,2)

Method: Least Squares Date: 08/23/11 Time: 14:55 Sample(adjusted): 2001:03 2010:04

Included observations: 110 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LROSD(-1))	-2.045722	0.220102	-9.294432	0.0000
С	0.056316	0.069219	0.813587	0.4177
@TREND(2001:01)	-0.001628	0.001068	-1.524265	0.1304
R-squared	0.455006	Mean deper	ndent var	-0.034701
Adjusted R-squared	0.444819	S.D. dependent var		0.477260
S.E. of regression	0.355609	Akaike info	criterion	0.796922
Sum squared resid	13.53095	Schwarz crit	terion	0.870571
Log likelihood	-40.83069	F-statistic		44.66614
Durbin-Watson stat	_ 1.411154_	Prob(F-stati	stic)	0.000000

#### LROTD LEVEL

ADF Test Statistic	2.124770	1% Critical Value*	-2.5846
		5% Critical Value	-1.9430
		10% Critical Value	-1.6173

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(LROTD)

Method: Least Squares Date: 08/23/11 Time: 14:56 Sample(adjusted): 2001:04 2010:04

Variable	Coefficient	Std. Error	t-Statistic	Prob.
variable	_ 000	O.u	· olaliolio_	

LROTD(-1)	0.008981	0.004227	2.124770	0.0359
D(LROTD(-1))	0.039604	0.103331	0.383270	0.7023
D(LROTD(-2))	-0.363618	0.149525	-2.431818	0.0167
R-squared Adjusted R-squared S.E. of regression Sum squared resid Log likelihood	0.068364 0.050786 0.241841 6.199617 1.579593	Mean deper S.D. depend Akaike info Schwarz cri Durbin-Wats	ndent var dent var criterion terion	-0.040521 0.248226 0.026063 0.100136 1.909651

PP Test Statistic	1.982646	1% Critical Value*	-2.5843
		5% Critical Value	-1.9429
		10% Critical Value	-1.6172

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel:	( Newey-West suggests: 4 )
Residual variance with no correction	0.059513
Residual variance with correction	0.057792

Phillips-Perron Test Equation Dependent Variable: D(LROTD)

Method: Least Squares
Date: 08/23/11 Time: 14:57
Sample(adjusted): 2001:02 2010:04

Included observations: 111 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LROTD(-1)	0.008046	0.004132	1.947153	0.0541
R-squared Adjusted R-squared S.E. of regression Sum squared resid Log likelihood	0.007988 0.007988 0.245060 6.605969 0.905792	Mean depen S.D. depend Akaike info o Schwarz crit Durbin-Wats	lent var criterion erion	-0.039649 0.246044 0.034339 0.058749 1.716886
1 <sup>ST</sup> DIFFERENCE ADF Test Statistic	-4.735118	1% Critical 5% Critical 10% Critical	Value	-4.0452 -3.4515 -3.1509

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation Dependent Variable: D(LROTD,2)

Method: Least Squares Date: 08/23/11 Time: 14:57

Sample(adjusted): 2001:05 2010:04

Included observations: 108 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LROTD(-1))	-1.324076	0.279629	-4.735118	0.0000
D(LROTD(-1),2)	0.342118	0.227694	1.502533	0.1360
D(LROTD(-2),2)	-0.041153	0.153612	-0.267903	0.7893
С	0.046814	0.048654	0.962169	0.3382
@TREND(2001:01)	-0.001635	0.000763	-2.141887	0.0346
R-squared	0.472040	Mean deper	ndent var	-0.010117
Adjusted R-squared	0.451537	S.D. depend	lent var	0.323803
S.E. of regression	0.239803	Akaike info	criterion	0.027195
Sum squared resid	5.923080	Schwarz crit	erion	0.151368
Log likelihood	3.531468	F-statistic		23.02264
Durbin-Watson stat	1.935164	Prob(F-statis	stic)	0.000000
PP Test Statistic	-8.954848	1% Critical	Value*	-4.0437
		5% Critical	Value	-3.4508
		10% Critical	Value	-3.1505

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel: ( Newey-West suggests: 4 )
4

Residual variance with no correction0.057626Residual variance with correction0.048675

Phillips-Perron Test Equation Dependent Variable: D(LROTD,2)

Method: Least Squares Date: 08/23/11 Time: 14:58 Sample(adjusted): 2001:03 2010:04

Included observations: 110 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LROTD(-1))	-0.924108	0.101696	-9.086918	0.0000
C	0.047878	0.047505	1.007858	0.3158
@TREND(2001:01)	-0.001514	0.000741	-2.043484	0.0435
R-squared	0.436535	Mean deper	ndent var	-0.008369
Adjusted R-squared	0.426002	S.D. dependent var		0.321261
S.E. of regression	0.243396	Akaike info	criterion	0.038641
Sum squared resid	6.338859	Schwarz criterion		0.112290
Log likelihood	0.874745	F-statistic		41.44814
Durbin-Watson stat	_ 1.907368_	Prob(F-statis	stic)	0.000000

#### LBOPO (LEVEL)

ADF Test Statistic	1.444668	1% Critical Value*	-2.5846
		5% Critical Value	-1.9430
		10% Critical Value	-1.6173

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(LBOPO)

Method: Least Squares
Date: 08/23/11 Time: 15:00
Sample(adjusted): 2001:04 2010:04

Included observations: 109 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.	
LBOPO(-1)	0.013568	0.009392	1.444668	0.1515	
D(LBOPO(-1))	-0.749813	0.217911	-3.440910	0.0008	
D(LBOPO(-2))	-0.054852	0.222986	-0.245989	0.8062	
R-squared	0.106162	Mean deper	ident var	-0.044643	
Adjusted R-squared	0.089297	S.D. depend	lent var	0.419667	
S.E. of regression	0.400491	Akaike info	criterion	1.034887	
Sum squared resid	17.00167	Schwarz crit	1.108960		
Log likelihood	-53.40132	Durbin-Wats	on stat	1.251267	
PP Test Statistic	-1.668364	1% Critical	Value*	-4.0429	
		5% Critical	Value	-3.4504	
		10% Critical	Value	-3.1503	
*MacKinnon critical values for rejection of hypothesis of a unit root.					

Lag truncation for Bartlett kernel:	( Newey-West suggests: 4 )
Residual variance with no correction	0.163058
Residual variance with correction	0.163058

Phillips-Perron Test Equation Dependent Variable: D(LBOPO)

Method: Least Squares
Date: 08/23/11 Time: 15:00
Sample(adjusted): 2001:02 2010:04

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LBOPO(-1)	-0.238937	0.143216	-1.668364	0.0981
C ` ´	-0.763516	0.504815	-1.512469	0.1333
@TREND(2001:01)	-0.004589	0.001966	-2.333943	0.0214
R-squared	0.048644	Mean deper	ndent var	-0.043862
Adjusted R-squared	0.031026	S.D. dependent var		0.415877
S.E. of regression	0.409374	Akaike info criterion		1.078281
Sum squared resid	18.09943	Schwarz crit	terion	1.151512
Log likelihood	-56.84462	F-statistic		2.761058
Durbin-Watson stat	1.404670	Prob(F-stati	stic)	0.067692

#### 1<sup>ST</sup> DIFFERENCE

ADF Test Statistic	-5.270874	1% Critical Value*	-4.0452
		5% Critical Value	-3.4515
		10% Critical Value	-3.1509

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(LBOPO,2)

Method: Least Squares Date: 08/23/11 Time: 15:01 Sample(adjusted): 2001:05 2010:04

Included observations: 108 after adjusting endpoints

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Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LBOPO(-1))	-2.532419	0.480455	-5.270874	0.0000
D(LBOPO(-1),2)	0.701529	0.367431	1.909283	0.0590
D(LBOPO(-2),2)	0.472230	0.221585	2.131140	0.0355
C	0.070171	0.079147	0.886592	0.3774
@TREND(2001:01)	-0.002290	0.001215	-1.884610	0.0623
R-squared	0.437263	Mean dependent var		-0.038148
Adjusted R-squared	0.415409	S.D. dependent var		0.512624
S.E. of regression	0.391944	Akaike info criterion		1.009797
Sum squared resid	15.82290	Schwarz criterion		1.133970
Log likelihood	-49.52903	F-statistic		20.00851
Durbin-Watson stat	1.370912	Prob(F-stati	stic)	0.000000
		-		
PP Test Statistic	-6.746960	1% Critical	Value*	-4.0437
		5% Critical	Value	-3.4508
		10% Critical	Value	-3.1505

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel: (Newey-West suggests: 4)
4
Residual variance with no correction 0.150522
Residual variance with correction 0.115003

Phillips-Perron Test Equation Dependent Variable: D(LBOPO,2)

Method: Least Squares Date: 08/23/11 Time: 15:01 Sample(adjusted): 2001:03 2010:04

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LBOPO(-1))	-1.732860	0.203739	-8.505309	0.0000
C	0.069948	0.076556	0.913681	0.3629
@TREND(2001:01)	-0.002129	0.001181	-1.802375	0.0743
R-squared	0.412140	Mean dependent var		-0.035686
Adjusted R-squared	0.401151	S.D. dependent var		0.508330
S.E. of regression	0.393372	Akaike info criterion		0.998774
Sum squared resid	16.55738	Schwarz criterion		1.072424

Log likelihood	-51.93259	F-statistic	37.50799
Durbin-Watson stat	1.291229_	Prob(F-statistic)	_0.000000
LTR (LEVEL) ADF Test Statistic	-3.681239	1% Critical Value* 5% Critical Value 10% Critical Value	-4.0444 -3.4512 -3.1507

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(LTR)
Method: Least Squares
Date: 08/23/11 Time: 15:01
Sample(adjusted): 2001:04 2010:04

Included observations: 109 after adjusting endpoints

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Variable	Coefficient	Std. Error	t-Statistic	Prob.	
LTR(-1)	-0.857329	0.232891	-3.681239	0.0004	
D(LTR(-1))	0.850821	0.228706	3.720153	0.0003	
D(LTR(-2))	1.945342	0.358940	5.419684	0.0000	
C	-1.226472	0.365227	-3.358110	0.0011	
@TREND(2001:01)	-0.003133	0.001216	-2.575928	0.0114	
R-squared	0.262786	Mean dependent var		-0.040869	
Adjusted R-squared	0.234432	S.D. dependent var		0.443909	
S.E. of regression	0.388406	Akaike info criterion		0.991256	
Sum squared resid	15.68937	Schwarz criterion		1.114712	
Log likelihood	-49.02344	F-statistic		9.267910	
Durbin-Watson stat	_ 2.094475_	Prob(F-statistic)		_0.000002	
	-				
PP Test Statistic	-1.576034	1% Critical	Value*	-4.0429	
		-o/ O ::: 1		0.4504	

		10% Critical Value	-3.1503
		100/ Critical Value	2.4502
		5% Critical Value	-3.4504
PP Test Statistic	-1.576034	1% Critical value"	-4.0429

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel:	( Newey-West suggests: 4 )
4	
Residual variance with no correction	0.185806
Residual variance with correction	0.211440

Phillips-Perron Test Equation Dependent Variable: D(LTR) Method: Least Squares Date: 08/23/11 Time: 15:02 Sample(adjusted): 2001:02 2010:04

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LTR(-1)	-0.092605	0.100038	-0.925696	0.3567
C` ´	-0.058508	0.169422	-0.345340	0.7305
@TREND(2001:01)	0.002341_	0.001320	-1.773298 <sub>_</sub>	0.0790

0.439948
1.208881
1.282112
1.745039
_0.179526
-4.0437
-3.4508
-3.1505

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(LTR,2) Method: Least Squares Date: 08/23/11 Time: 15:03 Sample(adjusted): 2001:03 2010:04

Included observations: 110 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LTR(-1))	-1.019613	0.096733	-10.54052	0.0000
C	0.082013	0.086071	0.952857	0.3428
@TREND(2001:01)	-0.002174	0.001337	-1.625979	0.1069
R-squared	0.509427	Mean deper	ndent var	-0.003221
Adjusted R-squared	0.500257	S.D. depend		0.623316
S.E. of regression	0.440637	Akaike info criterion		1.225705
Sum squared resid	20.77527	Schwarz criterion		1.299355
Log likelihood	-64.41379	F-statistic		55.55611
Durbin-Watson stat	1.992778	Prob(F-statistic)		0.000000
PP Test Statistic	-10.54578	1% Critical	Value*	-4.0437
		5% Critical	Value	-3.4508
		10% Critical	Value	-3.1505

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel:	( Newey-West suggests: 4 )
Residual variance with no correction	0.188866
Residual variance with correction	0.203071

Phillips-Perron Test Equation Dependent Variable: D(LTR,2) Method: Least Squares Date: 08/23/11 Time: 15:04 Sample(adjusted): 2001:03 2010:04

Included observations: 110 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LTR(-1))	-1.019613	0.096733	-10.54052	0.0000
C	0.082013	0.086071	0.952857	0.3428

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@TREND(2001:01)	-0.002174	0.001337	-1.625979	0.1069
R-squared	0.509427	Mean dependent var		-0.003221
Adjusted R-squared	0.500257	S.D. dependent var		0.623316
S.E. of regression	0.440637	Akaike info criterion		1.225705
Sum squared resid	20.77527	Schwarz criterion		1.299355
Log likelihood	-64.41379	F-statistic		55.55611
Durbin-Watson stat	1.992778_	Prob(F-statistic)		0.000000

#### LROF LEVEL

ADF Test Statistic	0.576038	1% Critical Value*	-4.0444
		5% Critical Value	-3.4512
		10% Critical Value	-3.1507

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(LROF)

Method: Least Squares
Date: 08/23/11 Time: 15:08
Sample(adjusted): 2001:04 2010:04

Included observations: 109 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LROF(-1)	0.070013	0.121543	0.576038	0.5658
D(LROF(-1))	-0.151571	0.243741	-0.621853	0.5354
D(LROF(-2))	-0.384040	0.239231	-1.605309	0.1115
С	0.539803	0.770511	0.700579	0.4851
@TREND(2001:01)	-0.002003	0.002009	-0.997005	0.3211
R-squared	0.055810	Mean dependent var		-0.055113
Adjusted R-squared	0.019495	S.D. dependent var		0.472887
S.E. of regression	0.468255	Akaike info criterion		1.365178
Sum squared resid	22.80332	Schwarz criterion		1.488635
Log likelihood	-69.40223	F-statistic		1.536819
<b>Durbin-Watson stat</b>	_ 1.293580_	Prob(F-stati	stic)	0.196968
	-			
PP Test Statistic	1.141308	1% Critical	Value*	-4.0429
		5% Critical	Value	-3.4504
		10% Critical	Value	-3.1503

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel:	( Newey-West suggests: 4 )
4	
Residual variance with no correction	0.210915
Residual variance with correction	0.181039

Phillips-Perron Test Equation
Dependent Variable: D(LROF)
Method: Least Squares
Date: 08/23/11 Time: 15:08

Sample(adjusted): 2001:02 2010:04

Included observations: 111 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LROF(-1)	0.020361	0.113153	0.179943	0.8575
С	0.216008	0.718920	0.300462	0.7644
@TREND(2001:01)	-0.002302	0.001914	-1.202381	0.2318
R-squared	0.030744	Mean dependent var		-0.054636
Adjusted R-squared	0.012795	S.D. dependent var		0.468597
S.E. of regression	0.465590	Akaike info criterion		1.335631
Sum squared resid	23.41156	Schwarz criterion		1.408861
Log likelihood	-71.12751	F-statistic		1.712837
<b>Durbin-Watson stat</b>	_ 1.233587_	Prob(F-stati	stic)	_0.185216

## LROF (1<sup>ST</sup> DIFFERENCE)

PP Test Statistic	-3.201995	1% Critical Value*	-4.0437
		5% Critical Value	-3.4508
		10% Critical Value	-3.1505

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel:	( Newey-West suggests: 4 )
Residual variance with no correction	0.212725
Residual variance with correction	0.185343

Phillips-Perron Test Equation Dependent Variable: D(LROF,2)

Method: Least Squares Date: 08/23/11 Time: 15:09 Sample(adjusted): 2001:03 2010:04

Included observations: 110 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LROF(-1))	-1.051915	0.229211	-4.589292	0.0000
C	0.091224	0.091025	1.002182	0.3185
@TREND(2001:01)	-0.002603	0.001407	-1.850320	0.0670
R-squared	0.180169	Mean deper	ndent var	-0.040940
Adjusted R-squared	0.164845	S.D. depend	dent var	0.511718
S.E. of regression	0.467642	Akaike info criterion		1.344669
Sum squared resid	23.39978	Schwarz criterion		1.418318
Log likelihood	-70.95678	F-statistic		11.75734
Durbin-Watson stat	_ 1.211468_	Prob(F-statis	stic)	0.000024
	-			
ADF Test Statistic	-4.384068	1% Critical	Value*	-4.0452
		5% Critical	Value	-3.4515
		10% Critical	Value	-3.1509

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(LROF,2)

Method: Least Squares

Date: 08/23/11 Time: 15:09 Sample(adjusted): 2001:05 2010:04

Included observations: 108 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LROF(-1))	-2.007134	0.457825	-4.384068	0.0000
D(LROF(-1),2)	0.858584	0.357414	2.402214	0.0181
D(LROF(-2),2)	0.431728	0.233202	1.851304	0.0670
С	0.108766	0.093676	1.161081	0.2483
@TREND(2001:01)	-0.003107	0.001443	-2.153688	0.0336
R-squared	0.224385	Mean deper	ndent var	-0.041707
Adjusted R-squared	0.194264	S.D. dependent var		0.516388
S.E. of regression	0.463524	Akaike info criterion		1.345273
Sum squared resid	22.13001	Schwarz criterion		1.469446
Log likelihood	-67.64474	F-statistic		7.449476
Durbin-Watson stat	1.240663	Prob(F-stati	stic)	0.000026
	-			
LF LEVEL		404 0 141 1		
ADF Test Statistic	-3.656628		Value*	-4.0429
		5% Critical		-3.4504
		10% Critical	Value	-3.1503

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(LF)
Method: Least Squares
Date: 08/23/11 Time: 15:11
Sample(adjusted): 2001:02 2010:04

Included observations: 111 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LF(-1)	-0.238877	0.065327	-3.656628	0.0004
Ċ ´	-1.659645	0.460597	-3.603246	0.0005
@TREND(2001:01)	-0.002342	0.000813	-2.878893	0.0048
R-squared	0.114991	Mean deper	ndent var	-0.011602
Adjusted R-squared	0.098602	S.D. depend	dent var	0.228909
S.E. of regression	0.217331	Akaike info	criterion	-0.188139
Sum squared resid	5.101119	Schwarz crit	erion	-0.114909
Log likelihood	13.44172	F-statistic		7.016323
Durbin-Watson stat	2.377237	Prob(F-stati	stic)	_0.001365
	F			<del></del>

PP Test Statistic	-3.464070	1% Critical Value*	-4.0429
		5% Critical Value	-3.4504
		10% Critical Value	-3.1503

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel: ( Newey-West suggests: 4 )
4
Residual variance with no correction 0.045956
Residual variance with correction 0.041027

Phillips-Perron Test Equation
Dependent Variable: D(LF)
Method: Least Squares
Date: 08/23/11 Time: 15:11
Sample(adjusted): 2001:02 2010:04
Included observations: 111 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LF(-1)	-0.238877	0.065327	-3.656628	0.0004
C	-1.659645	0.460597	-3.603246	0.0005
@TREND(2001:01)	-0.002342	0.000813	-2.878893	0.0048
R-squared	0.114991	Mean deper	ndent var	-0.011602
Adjusted R-squared	0.098602	S.D. depend	dent var	0.228909
S.E. of regression	0.217331	Akaike info	criterion	-0.188139
Sum squared resid	5.101119	Schwarz crit	terion	-0.114909
Log likelihood	13.44172	F-statistic		7.016323

1 <sup>ST</sup> DIFFERENCE ADF Test Statistic	-5.614584	1% Critical Value* 5% Critical Value	-4.0452 -3.4515
		10% Critical Value	-3.1509

Prob(F-statistic)

0.001365

2.377237

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(LF,2) Method: Least Squares Date: 08/23/11 Time: 15:11 Sample(adjusted): 2001:05 2010:04

**Durbin-Watson stat** 

Included observations: 108 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LF(-1))	-1.439365	0.256362	-5.614584	0.0000
D(LF(-1),2)	0.060224	0.217837	0.276465	0.7827
D(LF(-2),2)	-0.010153	0.145319	-0.069866	0.9444
С	0.027174	0.044580	0.609546	0.5435
@TREND(2001:01)	-0.000789	0.000686	-1.148714	0.2533
R-squared	0.678044	Mean dependent var		-2.22E-05
Adjusted R-squared	0.665541	S.D. dependent var		0.380551
S.E. of regression	0.220082	Akaike info criterion		-0.144445
Sum squared resid	4.988904	Schwarz crit	terion	-0.020273
Log likelihood	12.80005	F-statistic		54.22985
Durbin-Watson stat	1.996406	Prob(F-statistic)		0.000000
	- <del></del> -			=
PP Test Statistic	-15.42924	1% Critical	Value*	-4.0437
		5% Critical	Value	-3.4508
		10% Critical	Value	-3.1505

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel: ( Newey-West suggests: 4 )

4

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Phillips-Perron Test Equation Dependent Variable: D(LF,2) Method: Least Squares Date: 08/23/11 Time: 15:12 Sample(adjusted): 2001:03 2010:04

Included observations: 110 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LF(-1))	-1.353298	0.090526	-14.94925	0.0000
C	0.025319	0.042185	0.600192	0.5496
@TREND(2001:01)	-0.000732	0.000652	-1.122777	0.2640
R-squared	0.676233	Mean dependent var		0.000479
Adjusted R-squared	0.670181	S.D. dependent var		0.377088
S.E. of regression	0.216561	Akaike info criterion		-0.194994
Sum squared resid	5.018162	Schwarz criterion		-0.121345
Log likelihood	13.72467	F-statistic		111.7422
Durbin-Watson stat	_ 2.048510_	Prob(F-stati	stic)	0.000000

#### **FS LEVEL**

ADF Test Statistic	-3.598643	1% Critical Value*	-4.0444
		5% Critical Value	-3.4512
		10% Critical Value	-3.1507

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(LFS) Method: Least Squares Date: 09/20/11 Time: 15:28 Sample(adjusted): 2001:04 2010:04

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LFS(-1)	-0.557082	0.154803	-3.598643	0.0005
D(LFS(-1))	-0.255930	0.272143	-0.940426	0.3492
D(LFS(-2))	0.191524	0.255516	0.749556	0.4552
C	-3.642371	1.018142	-3.577467	0.0005
@TREND(2001:01)	-0.016912	0.004610	-3.668720	0.0004
R-squared	0.188253	Mean dependent var		-0.066256
Adjusted R-squared	0.157032	S.D. dependent var		0.582368
S.E. of regression	0.534691	Akaike info criterion		1.630531
Sum squared resid	29.73304	Schwarz criterion		1.753987
Log likelihood	-83.86394	F-statistic		6.029676
Durbin-Watson stat	_ 1.299925_	Prob(F-stati	stic)	0.000210

PP Test Statistic	-4.641050	1% Critical Value*	-4.0429
		5% Critical Value	-3.4504
		10% Critical Value	-3.1503

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel: ( Newey-West suggests: 4 )
4

Residual variance with no correction 0.274454
Residual variance with correction 0.281461

Phillips-Perron Test Equation Dependent Variable: D(LFS) Method: Least Squares Date: 09/20/11 Time: 15:29 Sample(adjusted): 2001:02 2010:04

Included observations: 111 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LFS(-1)	-0.600402	0.133008	-4.514027	0.0000
C	-3.921527	0.884807	-4.432072	0.0000
@TREND(2001:01)	-0.018194	0.003918	-4.643106	0.0000
R-squared	0.169122	Mean dependent var		-0.065527
Adjusted R-squared	0.153735	S.D. dependent var		0.577340
S.E. of regression	0.531110	Akaike info criterion		1.598961
Sum squared resid	30.46444	Schwarz criterion		1.672191
Log likelihood	-85.74233	F-statistic		10.99149
Durbin-Watson stat	_ 1.345537_	Prob(F-stati	stic)	0.000045

## 1<sup>ST</sup> DIFFERENCE

ADF Test Statistic	-4.079865	1% Critical Value*	-4.0468
		5% Critical Value	-3.4523
		10% Critical Value	-3.1514

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(LFS,2) Method: Least Squares Date: 09/20/11 Time: 15:29 Sample(adjusted): 2001:07 2010:04

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LFS(-1))	-3.471260	0.850827	-4.079865	0.0001
D(LFS(-1),2)	1.670672	0.748213	2.232883	0.0278
D(LFS(-2),2)	1.309487	0.617434	2.120853	0.0364
D(LFS(-3),2)	0.717153	0.454285	1.578641	0.1176
D(LFS(-4),2)	0.214117	0.266858	0.802362	0.4243
С	-0.093398	0.127731	-0.731207	0.4664

@TREND(2001:01)	-0.000391	0.001872	-0.209052	0.8348
R-squared	0.359122	Mean deper	ndent var	-0.055339
Adjusted R-squared	0.320281	S.D. depend	dent var	0.685620
S.E. of regression	0.565260	Akaike info	criterion	1.760693
Sum squared resid	31.63234	Schwarz crit	terion	1.936581
Log likelihood	-86.31675	F-statistic		9.245918
Durbin-Watson stat	_ 1.236015_	Prob(F-statistic)		0.000000
PP Test Statistic	-5.664024	1% Critical 5% Critical 10% Critical	Value	-4.0437 -3.4508 -3.1505

<sup>\*</sup>MacKinnon critical values for rejection of hypothesis of a unit root.

Lag truncation for Bartlett kernel:	( Newey-West suggests: 4 )
4	
Residual variance with no correction	0.304577
Residual variance with correction	0.257679

Phillips-Perron Test Equation Dependent Variable: D(LFS,2) Method: Least Squares
Date: 09/20/11 Time: 15:29
Sample(adjusted): 2001:03 2010:04
Included observations: 110 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(LFS(-1))	-1.693156	0.235832	-7.179509	0.0000
` C` ''	0.005063	0.109698	0.046151	0.9633
@TREND(2001:01)	-0.001463	0.001689	-0.866018	0.3884
R-squared	0.335833	Mean dependent var		-0.051911
Adjusted R-squared	0.323419	S.D. dependent var		0.680289
S.E. of regression	0.559568	Akaike info criterion		1.703592
Sum squared resid	33.50349	Schwarz criterion		1.777241
Log likelihood	-90.69754	F-statistic		27.05208
Durbin-Watson stat	1.223548_	Prob(F-stati	stic)	0.000000

# **APPENDIX 6.2 Regression Results**

Dependent Variable: LROS Method: Least Squares Date: 08/23/11 Time: 15:26 Sample(adjusted): 2001:02 2010:04

Included observations: 111 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
С	1.281917	1.196592	1.071307	0.2865
LFS	-0.784835	0.100632	-7.799032	0.0000
LROSD	2.129038	0.311270	6.839854	0.0000
LROTD	0.219086	0.102065	2.146535	0.0341
D(LBOPO)	0.798280	0.271323	2.942177	0.0040
LTR	0.692927	0.152517	4.543268	0.0000
R-squared	0.739522	Mean depende	ent var	-5.286916
Adjusted R-squared	0.727118	S.D. dependent var		1.393865
S.E. of regression	0.728129	Akaike info criterion		2.255860
Sum squared resid	55.66798	Schwarz criterion		2.402321
Log likelihood	-119.2002	F-statistic		59.62093
Durbin-Watson stat	0.424671	Prob(F-statisti	ic)	0.000000

Dependent Variable: D(LROF) Method: Least Squares Date: 08/23/11 Time: 15:37 Sample(adjusted): 2001:02 2010:04

Variable	Coefficient	Std. Error	t-Statistic	Prob.
С	0.565114	0.447220	1.263615	0.2091
LF	0.033094	0.063860	0.518219	0.6054
D(LBOPO)	0.754363	0.068886	10.95094	0.0000
LTR	0.204839	0.053004	3.864593	0.0002
R-squared	0.730310	Mean dependent var		-0.054636
Adjusted R-squared	0.722749	S.D. dependent var		0.468597
S.E. of regression	0.246738	Akaike info criterion		0.074393
Sum squared resid	6.514135	Schwarz criterion		0.172034
Log likelihood	-0.128830	F-statistic		96.58394
Durbin-Watson stat	2.228165	Prob(F-statistic)		0.000000

Dependent Variable: LROS Method: Least Squares Date: 08/23/11 Time: 15:33 Sample(adjusted): 2001:03 2010:04

Included observations: 110 after adjusting endpoints

Convergence achieved after 15 iterations

White Heteroskedasticity-Consistent Standard Errors & Covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
С	4.001626	1.650942	2.423844	0.0171
LFS	0.134162	0.287630	0.466440	0.6419
LROSD	1.321568	0.430591	3.069194	0.0027
LROTD	0.219408	0.176210	1.245152	0.2159
D(LBOPO)	0.178289	0.209748	0.850019	0.3973
LTR	0.426427	0.136518	3.123592	0.0023
AR(1)	0.970464	0.036382	26.67402	0.0000
R-squared	0.952311	Mean dependent var		-5.283856
Adjusted R-squared	0.949533	S.D. dependent var		1.399869
S.E. of regression	0.314480	Akaike info criterion		0.585726
Sum squared resid	10.18643	Schwarz criterion		0.757575
Log likelihood	-25.21495	F-statistic		342.8025
Durbin-Watson stat	1.758621	Prob(F-statistic)		0.000000
Inverted AR Roots	97			

Dependent Variable: LROS Method: Least Squares Date: 08/23/11 Time: 15:33 Sample(adjusted): 2001:03 2010:04

Included observations: 110 after adjusting endpoints

Convergence achieved after 15 iterations

Newey-West HAC Standard Errors & Covariance (lag truncation=4)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
С	4.001626	1.377139	2.905754	0.0045
LFS	0.134162	0.298345	0.449687	0.6539
LROSD	1.321568	0.550240	2.401801	0.0181
LROTD	0.219408	0.190241	1.153319	0.2514
D(LBOPO)	0.178289	0.122574	1.454544	0.1488
LTR	0.426427	0.145265	2.935500	0.0041
AR(1)	0.970464	0.031431	30.87580	0.0000
R-squared	0.952311	Mean dependent var		-5.283856
Adjusted R-squared	0.949533	S.D. dependent var		1.399869
S.E. of regression	0.314480	Akaike info criterion		0.585726
Sum squared resid	10.18643	Schwarz criterion		0.757575
Log likelihood	-25.21495	F-statistic		342.8025
Durbin-Watson stat	1.758621	Prob(F-statistic)		0.000000
Inverted AR Roots	97			

Dependent Variable: D(LROF) Method: Least Squares Date: 08/23/11 Time: 15:42 Sample(adjusted): 2001:02 2010:04

# Included observations: 111 after adjusting endpoints White Heteroskedasticity-Consistent Standard Errors & Covariance

Variable	Coefficient	Std. Error	t-Statistic	Prob.
С	0.565114	0.620988	0.910023	0.3649
LF	0.033094	0.092251	0.358734	0.7205
D(LBOPO)	0.754363	0.139723	5.398978	0.0000
LTR	0.204839	0.094719	2.162606	0.0328
R-squared	0.730310	Mean dependent var		-0.054636
Adjusted R-squared	0.722749	S.D. dependent var		0.468597
S.E. of regression	0.246738	Akaike info criterion		0.074393
Sum squared resid	6.514135	Schwarz criterion		0.172034
Log likelihood	-0.128830	F-statistic		96.58394
Durbin-Watson stat	2.228165	Prob(F-statisti	c)	0.000000

Dependent Variable: D(LROF)

Method: Least Squares Date: 08/23/11 Time: 15:43 Sample(adjusted): 2001:02 2010:04

Newey-West HAC Standard Errors & Covariance (lag truncation=4)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
С	0.565114	0.651459	0.867459	0.3876
LF	0.033094	0.092371	0.358270	0.7208
D(LBOPO)	0.754363	0.168890	4.466587	0.0000
LTR	0.204839	0.059909	3.419183	0.0009
R-squared	0.730310	Mean dependent var		-0.054636
Adjusted R-squared	0.722749	S.D. dependent var		0.468597
S.E. of regression	0.246738	Akaike info criterion		0.074393
Sum squared resid	6.514135	Schwarz criterion		0.172034
Log likelihood	-0.128830	F-statistic		96.58394
Durbin-Watson stat	2.228165	Prob(F-statistic)		0.000000