Barriers and the Transition to Modern Growth[*]

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Abstract

This paper argues that barriers affect both the beginning date and the subsequent pace of modern growth, and taking into account this fact enriches our knowledge of cross-country income differences. The model matches the observed inverted U-shape of cross-country income differences, which implies that a substantial fraction of current income differences is transitional. Hence, the model requires smaller barriers to account for current income differences relative to models that focus only on steady states. Empirically, I find that differences in the beginning dates of modern growth explain large differences in incomes.

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1 Introduction

Models of international income differences focus on the steady state effect of barriers to capital accumulation and technology adoption[1] ignoring an important long run development fact: countries that have experienced modern growth (a sustained increase in per capita output) also experienced a long period of extensive growth (growth in aggregate terms but stagnation in per capita terms) before it. Moreover, countries entered modern growth at different points in time. A parallel literature studies development paths but with no reference to international income differences.[2] In this paper, I bring elements from both literatures and study the international income differences implied by differences in development paths. I argue that some countries are poor because of bad institutions or policies that act as barriers to technology adoption and capital accumulation. My key contribution to this literature is to study the implications of an overlooked consequence of these barriers: the delay in the process of transition from extensive growth to modern growth. I introduce barriers into the Hansen and Prescott (1999) model and show that barriers in this model lower the level of income along the balanced growth path and, more importantly, delay the economy's turning point from extensive to modern growth. Because of this second effect, cross-country income differences exhibit an inverted U-shape pattern over time. A key implication is that a substantial fraction of existing income differences is transitional, and so smaller barriers are required to account for the observed large cross-country income differences relative to models that focus only on steady states. This transitional effect increases significantly when I include the fact that today's low-income countries have higher population growth rates during their early development stage than did today's high income countries.

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[1] This literature generally focuses on policies that distort capital accumulation (e.g., Mankiw et al., (1992), Chari et al., (1997), Parente et al., (2000)), technology adoption (e.g., Parente and Prescott (1994)), and level of total factor productivity (e.g., Hall and Jones (1999), Prescott (1998) ,Parente and Prescott (1999), and Baier et al., (2003)). McGrattan and Schmit (1998) provided a survey of papers on cross country income differences.

[2] An exception is the work of Lucas (2002) which uses the model by Tamura (1996) to study the evolution of the relative income distribution by assigning turning points exogenously, and finds that income inequality exhibits an inverted U-shape. In my model the turning point is endogenously determined. Models on transition from stagnation to modern growth includes Becker et al., (1990), Goodfriend and McDermott (1995), Galor and Weil (1998), Jones (1999) and Hansen and Prescott (1999). These models differ in several aspects regarding the driving forces of the transition to modern growth and whether such transition is inevitable or not.

To illustrate the strength of this model in explaining cross-country income differences, I examine the countries in Maddison's (2001) dataset. I run two tests. First, I derive the relative size of barriers that are required to explain the observed differences in turning points between any two groups of countries. I then compare the income differences predicted by the model with the data, and find that the model accounts for about 80 percent of the income differences for most of the country groups. Second, I choose Japan to study the case of a reduction in barriers and Argentina to study the case of an increase in barriers due to documented institutional changes. The data that I used to change the barriers are historical data on the relative prices of investment goods. The predicted long run development experience for each economy matches closely the data.

The remainder of the paper is organized as follows. Section 2 documents the long run development facts as motivation and section 3 presents the model. I discuss the role of barriers in section 4 and show the potential of the model to account for international income differences in Section 5. The empirical studies are in Section 6, and a conclusion follows in section 7.

2 Motivation

Maddison's (2001) dataset covers 29 Western European countries, 4 Western Offshoots, 7 Eastern European countries, 15 Successor States of the Former USSR, 44 Latin American countries, 57 African countries, 41 East Asian countries, and 15 West Asian countries. I divide these countries into seven groups where Group 1 includes Western Offshoots and 12 Western European countries, Group 2 includes other 17 Western European countries, Group 3 includes Eastern European countries and the Former USSR, Group 4 includes Latin American countries, Group 5 is Japan only, Group 6 is Africa and finally Group 7 includes all Asian countries except Japan. Appendix 4 provides the details.

Figure (1) shows that per capita income for all seven groups remained stagnant for a long period before starting to grow at different points in time. This stagnation is not because there was no growth in total output but because the increase in population offset the increase in output. A "Malthusian" model therefore matches well world experiences prior to the 19th century. But subsequently countries started to leave this type of stagnation and enter modern growth, I refer to the time of entry into modern growth as the turning point. World income differences were small prior to the 19th century but because of differences in turning points, they started to diverge during the 19th century, a feature emphasized by Pritchett (1997). Figure (2) plots this for individual countries, which also shows large differences in the turning points. As a result of the different turning points, income differences between two countries exhibit an inverted U-shape pattern over time, a feature of the data emphasized by Lucas (2002).

The data suggest that the timing of modern growth is crucial for understanding the observed income differences. To proceed, I use a version of Hansen and Prescott (1999) model to determine the timing of modern growth. The Hansen-Prescott model has the advantage that it determines the turning point endogenously and behaves asymptotically like the one-sector Solow model. There are two reasons, however, why the Hansen-Prescott model cannot be directly used as a model for understanding international income differences. The turning point in their model depends on initial land per worker and initial technologies. But these factors also determine income prior to the turning point. Therefore, it cannot simultaneously account for both the large differences in turning points and the small differences in pre-1800 income levels. Moreover, the model predicts that an economy with a lower level of income in the pre-modern growth stage reaches its turning point sooner, which is not consistent with the data. I argue that different institutions for investment incentives can reconcile these facts. This is because capital has a small role to play prior to modern growth, therefore, differences in investment incentives also have a small role in determining the income differences along the Malthusian path. But they can delay the adoption of the capital-intensive Solow technology and so explain large post-modern growth income differences as a result of the differences in the turning points.

3 The Model

I focus on barriers to capital accumulation as an explanation why countries are poor and, in the context of this paper, why modern growth begins later in some countries.[3] Barriers can take the form of taxes on investment goods, corruption or other institutional factors

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[3] I show later barriers to technology adoption can be introduced in the model in a similar way.

that increase the relative price of investment goods, which in turn discourages capital accumulation. In this paper, I follow Parente and Prescott and model barriers by assuming that they reduce the efficiency of transforming forgone consumption goods into usable capital goods.

Technology Output in this economy can be produced using either the Malthus or the Solow technologies. Both technologies are subject to exogenous technological change and both have constant returns to scale. The two production functions are as follows:

$$Y_{mt} = A_m \gamma_m^t K_{mt}^{\phi} N_{mt}^{\mu} L_{mt}^{1-\mu-\phi}; \qquad Y_{st} = A_s \gamma_s^t K_{st}^{\theta} N_{st}^{1-\theta} \tag{1}$$

where K_{it}, N_{it} and L_{it} denote capital, labor and land used in technology i at time t, $\phi \in (0,1)$ is the capital share, $\mu \in (0,1)$ is the labor share and $1-\mu-\phi \in (0,1)$ is the land share in the Malthus technology, $\theta \in (0,1)$ is the capital share for the Solow technology, $\gamma_m > 1$ and $\gamma_s > 1$ are the growth rates while A_m and A_s are the initial level of total factor productivity (TFP). Land does not enter the Solow technology. Capital is assumed to depreciate completely each period.[4] Land is a fixed factor. Output of the two sectors are identical and can be used for consumption or investment. Feasibility requires

$$C_t + X_{mt} + X_{st} = Y_{mt} + Y_{st} \tag{2}$$

where C_t is aggregate consumption, while X_{mt} and X_{st} are aggregate investments.

Firms in each sector are assumed to behave competitively and rent all factors of production from households. A representative firm in sector i takes the wage rate and rental rates for capital and land as given, and chooses labor, capital and land input to maximize profits.

$$\underset{N_{it},K_{it},L_{it}}{Max} \; Y_{it} - w_t N_{it} - r_{Kit} K_{it} - r_{Lt} L_{it} \qquad s.t.(1) \qquad i = m, s \tag{3}$$

Household Sector The model has two-period overlapping generations. Let N_t be the number of young agents and c_{1t} be the consumption level for young agents in period t. Population dynamics are given by $N_{t+1} = g(c_{1t})N_t$, where $g(.)$ is an exogenous function that will be specified later. In period 0, there are N_{-1} old agents, each is endowed with

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[4] In the quantitative work a period is interpreted to be 35 years, so this assumption is empirically reasonable.

$\frac{K_0}{N_{-1}}$ units of capital and $\frac{L}{N_{-1}}$ units of land. Young agents are born with one unit of labor time, which they supply inelastically. They make a consumption-saving decision on how much land and capital to purchase. They become old in the second period where their sources of income are from renting land and capital to firms and from the sale of land to the next generation. The barriers are modelled as policy parameters that discourage young agents from investing. More specifically, for every unit of consumption good a young agent gives up, he gets $\frac{1}{\pi_m}$ units of Malthus capital and $\frac{1}{\pi_s}$ unit of Solow capital.[5] In equilibrium, π_m and π_s are the relative prices of Malthus and Solow capital goods to consumption goods. In my international income comparison that follows, π_m and π_s are allowed to vary across countries. For each generation t, young agents choose consumption (c_{1t}, c_{2t+1}) and investment $(x_{mt}, x_{st}, l_{t+1})$ to maximize lifetime utility,

$$U(c_{1t}, c_{2t}) = u(c_{1t}) + \beta u(c_{2t+1}) \tag{4}$$

subject to the budget constraints

$$\begin{aligned} c_{1t} &= w_t - (x_{mt} + x_{st} + q_t l_{t+1}) & (5) \\ c_{2t+1} &= r_{kmt+1}\frac{x_{mt}}{\pi_m} + r_{kst+1}\frac{x_{st}}{\pi_s} + (q_{t+1} + r_{Lt+1})l_{t+1} & (6) \end{aligned}$$

where β is the discount factor and q_t is the price of land in period t.

Equilibrium For given sizes of barriers, the competitive equilibrium and the dynamics are similar to those of Hansen and Prescott. Readers are referred to Appendix 1 for precise definition and proofs. I look for an equilibrium where the dynamics of the model are characterized by three development stages. Stage one is the pre-modern growth stage where the Solow technology is not used and the economy is on a Malthus balanced growth path (MBGP).[6] The exogenous population growth function is chosen such that all the improvement in Malthus TFP is absorbed by population growth. Hence, there is no growth in per capita terms. Stage two is the transition stage where the level of TFP in the Solow technology is sufficiently high relative to the barriers. It becomes profitable to use the Solow

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[5] I allow for different barriers in the two sectors to capture the possiblility that policies are biased.

[6] Because land is always supplied inelastically, in equilibrium it is always profitable to operate the Malthus technology. Too see this, suppose r_{Lt}, r_{kmt}, r_{Kst} and w_t are equilibrium prices such that the Malthus technology is not operated. Then since land can only be used in the Malthus technology, there is an excess supply of land, which implies that these prices cannot be an equilibrium.

technology and the economy is in transition to modern growth. In stage 3, only the Solow technology is used and the economy converges to a Solow balanced growth path (SBGP) asymptotically. The dynamics of the model capture the experience of a rich country that starts off with stagnant output per worker, then modern growth begins with an increase in labor being allocated to the industrial sector, and finally, the economy converges to a balanced growth path where output per worker is growing at a constant rate.

4 Barriers to Development

This section highlights the role of barriers in the three development stages. Along the MBGP, the barriers to Malthus capital reduce the capital-output ratio by a factor π_m. Let v_{m1} be the capital-output ratio for an economy with $\pi_m = 1$, the output per worker along the MBGP is

$$\hat{y}_m = \left[A_m \gamma_m^t \left(\frac{v_{m1}}{\pi_m} \right)^{\phi} \left(\frac{L}{N_t} \right)^{1-\mu-\phi} \right]^{1/(1-\phi)} \tag{7}$$

which is constant given the assumption that population is growing at rate $\gamma_m^{1-\mu-\phi}$. The barriers to Malthus capital reduce this constant level by a factor of $\pi_m^{\phi/(1-\phi)}$.

When on the MBGP, firms can determine when it is profitable to start using the Solow technology. This requires profit to be positive when the wage and rental rate of capital are at their MBGP levels. The condition is on the level of TFP in the Solow technology:

$$A_{st} \geq (\frac{\pi_s}{\pi_m} \frac{\hat{r}_m}{\theta})^{\theta} (\frac{\hat{w}_m}{1-\theta})^{1-\theta} \tag{8}$$

where $\hat{w}_m$ and $\hat{r}_m$ are the constant wage and rental rate of capital along the MBGP. Let the turning point t^* be the period that the Solow technology is first used. the condition implies

$$A_s \gamma_s^{t^*} \geq B \pi_s^{\theta} \pi_m^{-\phi(1-\theta)/(1-\phi)} \left(\frac{L}{N_0} \right)^{(1-\mu-\phi)(1-\theta)/(1-\phi)} > A_s \gamma_s^{t^*-1} \tag{9}$$

where B is a function of technologies and preference parameters. The existence of the turning point is independent of the relative sizes of the growth rates for the two technologies. Since the threshold is a constant, the Solow technology will be used at some point as long as it is growing. Therefore, the model predicts that modern growth is inevitable in all countries. If countries have access to different types of technologies, of course their turning points are different. But even if they have access to the same technologies, their turning points can still be different, depending on their level of barriers and land per worker.

The two barriers have opposite effects on the turning point. The barriers to Solow capital delay the turning point but the barriers to Malthus capital speed it up. Intuitively, if policies favor the Malthus sector relative to the Solow sector, the economy stays on the MBGP longer. When policies are neutral, referred to as the case of symmetric barriers to capital accumulation, $\pi_m = \pi_s = \pi$. The effect of symmetric barriers on the turning point is $\pi^{(\theta-\phi)/(1-\phi)}$, which delays the turning point if and only if the Solow technology is more capital intensive than the Malthus technology, which I henceforth assume. A higher level of land per worker delays the turning point because it implies higher wages which makes it more expensive to start using the Solow technology. So the model has the prediction that a country with a better endowment of natural resources enjoys a higher living standard in the pre-modern growth stage but it also stays longer in that stage.

When both technologies are used, the allocation of inputs must equalize marginal products across sectors. Let n^*_{mt} be the equilibrium fraction of labor in the Malthus sector. It solves

$$f\left(n_{mt}\right) = \frac{\mu}{1-\theta}\pi_s^{\theta}\pi_m^{-\phi}\psi^{\phi}\left(1-\left(1-\psi\right)n_{mt}\right)^{\theta-\phi} - \frac{A_{st}}{A_{mt}}I_t^{\theta-\phi}N_t^{1-\theta-\mu}n_{mt}^{1-\phi-\mu} = 0 \tag{10}$$

where $\psi = \frac{(1-\theta)\phi}{\theta\mu}$ and I_t is total value of investment by the young population at time $t-1$. Assume the Solow technology is growing faster than the Malthus technology, the fraction of labor in the Malthus sector is decreasing and converges to zero. The two barriers have opposite effects on this process of structural transformation. The barriers to Solow capital slow down this process while the barriers to Malthus capital speed up this process. The effect of the symmetric barriers is captured by $\pi^{\theta-\phi}$, which slow down the process.

Asymptotically, the economy behaves like a one-sector Solow growth economy. Assume the population growth rate converges to a constant rate, the economy converges to the SBGP. The barriers to Solow capital reduce the capital-output ratio by a factor π_s. Let v_{s1} be the capital-output ratio for an economy with $\pi_s = 1$. The output per worker along the SBGPof an economy with barriers π_s is

$$\hat{y}_{st} = \left(A_s\gamma_s^t\left(\frac{v_{s1}}{\pi_s}\right)^{\theta}\right)^{1/(1-\theta)} \tag{11}$$

which is lower by a factor of $\pi_s^{\theta/(1-\theta)}$.

5 International Income Differences

Can the model account for the large observed international income differences? To answer this question, I consider two economies that are identical except for the level of their barriers: economy 1 has $\pi_{m1} = \pi_{s1} = 1$, and economy 2 has $\pi_{m2} = \pi_m$ and $\pi_{s2} = \pi_s$.

5.1 Analytical Results

In what follows I refer to the ratio of output per worker in economy 1 to output per worker in economy 2 as their income ratio. Equations (7) and (11) imply that the income ratio is $\pi_m^{\phi/(1-\phi)}$ along the MBGP, and $\pi_s^{\theta/(1-\theta)}$ along the SBGP. For the case of symmetric barriers ($\pi_m = \pi_s = \pi$), the model predicts higher income ratio along the SBGP than the MBGP. In other words, even if barriers remain unchanged, the model predicts an increase in the income ratio because both economies experience a structural transformation with more capital allocated to the more capital-intensive sector.

That the two-sector barrier model generates the same long-run income ratio as the standard one-sector barrier model (as in Parente and Prescott (1994)), but crucially for my purpose, it implies a different turning point for each economy. Equation (9) implies a relationship between the two turning points,[7]

$$t_2^* - t_1^* = \frac{\ln\left(\pi_s^{\theta}\pi_m^{-\phi(1-\theta)/(1-\phi)}\right)}{\ln \gamma_s}. \tag{12}$$

If the two economies have the same barriers to Malthus capital, then modern growth is delayed by $\frac{\theta \ln \pi_s}{\ln \gamma_s}$ periods in economy 2. The income ratio first increases from one, when both economies use only the Malthus technology, then converges to $\pi_s^{\theta/(1-\theta)}$. On the other hand, if their barriers to Solow capital are the same, modern growth is delayed by $\left[\frac{\phi(1-\theta)}{(1-\phi)}\right]\frac{\ln \pi_m}{\ln \gamma_s}$ periods in economy 1. The income ratio first decreases from $\pi_m^{\phi/(1-\phi)}$, then converges to one. For the case of symmetric barriers, the relationship between their turning points is

$$t_2^* - t_1^* = \left(\frac{\theta - \phi}{1-\phi}\right)\frac{\ln \pi}{\ln \gamma_s} \tag{13}$$

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[7] To be more precise, the difference in their turning point should be the minimum integer that is greater than $\left[\ln\left(\pi_s^{\theta}\pi_m^{-\phi(1-\theta)/(1-\phi)}\right)\right]/\ln \gamma_s$.

The turning point for economy 2 occurs $\frac{\theta-\phi}{1-\phi}\frac{\ln\pi}{\ln\gamma_s}$ periods later. The income ratio first increases from $\pi^{\phi/(1-\phi)}$, then converges to $\pi^{\theta/(1-\theta)}$.[8]

5.2 Quantitative Results

The benchmark economy with barriers equal to one is identical to that of Hansen and Prescott. I therefore follow their calibration strategies. Appendix 2 provides a brief review of the procedure. With the same calibrated parameters, I then compute the equilibrium path of a distorted economy with barriers bigger than one. For simplification, I assume barriers to Malthus capital equal to one. This does not change the quantitative results regarding income differences along the transition. The reason is that given that the capital share in the Malthus technology is calibrated to 0.1, the barriers to Malthus capital have very small effects on both the level of income and the turning point.

In order to set a value for the barriers to capital accumulation in the Solow technology I use Jones's (1994) estimate of the maximum relative machinery price in the Summer-Hetson data set to that of the US for the period 1960-85, which is equal to 4. It turns out, however, that for the main focus of this paper, which is the contribution of the turning point to the differences in income, the precise value chosen for the barrier is not important. Other authors, in particular Chari, Kehoe and McGrattan (1997) and Restuccia and Urrutia (2001) use the relative price of investment to consumption goods as a measure for barriers. Restuccia and Urrutia construct a panel of the relative prices for the period 1960-85 using the Summer-Hetson data set. They find that the differences in relative prices across countries are large. The ratio between the average of the top and bottom five percent of the distribution of relative prices is 11.3 in 1960 and 6.5 in 1985.[9] I report the results for values of barriers larger than 4 and show that my results are not substantially altered.

Figures (3) - (5) summarize the quantitative results for the case of barriers equals 4. Figures (3) shows that while the benchmark economy starts to allocate labor to the Solow

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[8] The capital shares have interesting roles in this model. Increasing the capital share of the Malthus (Solow) technology increases the income ratio along the MBGP (SBGP) and delays the turning point in economy 1 (2).

[9] One concern with the investment to consumption measure is that it may overstate the size of barriers if consumption goods are cheaper in the poor countries because of non-tradable consumption goods that are produced by labor-intensive technologies. This issue has been addressed by both sets of authors and they find that this bias is small.

sector in period 1, the Solow technology is still inactive in the distorted economy until period 3.[10] The Solow technology is actually profitable in the distorted economy between periods 2 and 3 since the right hand side of equation (12) is equal to 1.3 periods. This explains why the distorted economy allocates 70 percent (compare to 10 percent for the benchmark economy) of its labor to the Solow sector during the first period that the Solow technology is used. The inverted U-shape income ratio in the data is replicated in Figure (4). The model predicts the income ratio increases from 1 to a maximum of 3.2 before declining to 2.5. Thus, a bigger income ratio is obtained (a 26 percent difference) relative to the balanced growth path level. Figure (5) shows that the growth rate of output per worker is not monotonic as in the one-sector Solow growth model. It first increases and then decreases to its balanced growth path rate. The increasing growth rate is a feature of the data emphasized by Romer (1986).[11] It is interesting to note that this model can produce such an outcome with two constant return to scale technologies.

As the barriers delay the turning point for the distorted economy, the growth rate for the benchmark economy is higher than that of the distorted economy before it starts to decrease. Thus, their income ratio increases during this period. After this point, the model predicts faster growth in the distorted economy so that the income ratio decreases. The income ratio converges to a constant when both economies converge to their SBGPs. The inverted U-shape income ratio predicted by the model provides an answer to the question raised by Restuccia and Urrutia (2001) namely, ‘why the income differences remain constant in light of the decline in the level of barriers?’ This is indeed a puzzle if one focuses on the one-sector barrier model, which predicts income differences should fall when the barriers fall. However, these two empirical observations can coexist in the two-sector barriers model. This is because, as I will show in section 6.2, a decline in the level of barriers may decrease only the slope of the increasing income ratio but leaving its level constant.

In this model, cross-country income differences are generated by differences in balanced

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[10] The fraction of capital in the Malthus sector is proportional to the fraction of labor in the Malthus sector.

[11] Romer (1986) tests the trend of the growth rate using raw data from Maddison (1979) for countries with data no later than 1870. These countries include: United Kingdom, France, Denmark, United States, Germany, Sweden, Italy, Australia, Norway, Japan and Canada. He rejects the null hypothesis that there is a nonpositive trend in the growth rate for 8 out of the 11 countries at the 10 percent level.

growth path levels and differences in turning points. Income differences along the balanced growth path are smaller than along the transition from Malthus to Solow. Table (1) shows that as barriers increase, the percentage difference between the income ratio along the SBGP and the maximum income ratio increases. This is partly due to the longer delay of modern growth. For example, when the barriers are increased from 8 to 16, the delay in modern growth increases from 2 to 3 periods, and the percentage difference in the income ratio rises from 33 percent to 40 percent. To address the factor 30 income differences in the data, Table (2) reports the corresponding combination of capital shares and barriers that can generate a maximum income ratio of this magnitude.[12] It shows that the required size of barriers needed for a factor 30 income difference is much lower than in models that focus on the balanced growth path. For example, for θ equal to 0.4, the required level of barriers is reduced by 40 percent. The reduction holds true for other levels of θ as well. It is interesting to note that a factor 30 income difference is associated with a three- or four-period delay in the model. In other words, given that rich countries entered modern growth in 1820, the model predicts that a country that entered modern growth in 1960 would be 30 times poorer by today's standards.

I have been focusing on the barriers to capital accumulation to show that the timing of modern growth is important for understanding the large international income differences. Alternatively, some have argued that some countries are poor because there are barriers that deter technology adoption, which in turn lowers the level of TFP. For example, Parente and Prescott (1999) have studied the role of unions as barriers to adopting better technology. The simplest way to incorporate this barrier to technology adoption into this model is to assume the TFP for the Solow technology is A_{st}/π_A. The interpretation is that the best Solow technology is not being adopted or the barriers reduce the efficiency of using the Solow technology. At a general level, these two types of models are isomorphic in that one can choose the size of barriers such that they imply the same output per worker ratio along the balanced growth path for the two models. In particular, set $\pi_A = \pi_s^{\theta}$,where π_A and π_s are the barriers to technology adoption and capital accumulation. Then, the delay in

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[12] To be consistent with the calibration procedure, γ_s and β have to be adjusted when θ is increased. Therefore, increasing θ need not necessarily increase the delay in modern growth as noted earlier in section 5.1.

turning points implied by these two models is the same and same quantitative results apply. The key differences lie in the representation of these two barriers in the empirical studies.

5.3 Population Profile

The previous quantitative exercise assumed population profiles are the same for both economies. My focus was to study the effect of barriers holding other factors constant. The analysis in Appendix 3 shows that my main result is sensitive to changes in the population profile. In particular, when the maximum population growth rate is increased by 1 percent for both economies, the maximum income ratio increases from 3.2 to 3.5, a near 10 percent increase. In view of this, it is of interest to see what the data imply for the population profiles for a broader set of countries. Figure (6) uses the data from Table A4.1 and plots the population profile for the seven groups of countries. The x-axis is the GDP per capita in a given year relative to year 1700, which represents the stage of development of each group. The data suggest that whereas the shapes of the population profile are similar across countries, the peaks are very different. More precisely, late developers have higher peaks than early developers. While the population profiles of these countries do not affect their turning points, they may affect the path of relative income.

In this paper, I focus on the role of barriers taking the profile of the population growth rate as given without decomposing it into fertility and mortality. The interaction between mortality and fertility have been widely studied. Recent work has emphasized the role of mortality on the return to human capital and/or the role of mortality on the altuistic parent's precautionary demand for children (e.g., Ehrich and Lui (1991), Jones (1999), and Tamura (2002b)). They argue that the falling mortality rate is the key driving force for the falling fertility rate. This literature provides an explanation to why population growth is increasing during the early development stage, and falling in the late development stage.[13] The question then is why the population growth rate for the late developers reaches a higher peak then for the early developers.[14] Coale (1979) has documented for the case of Europe,

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[13] In a paper with human capital accumulation, Tamura (2002b) argues that the reason that the declining population growth (and industrialization) happend sooner in the early developers is due to that fact that the TFP of industry relative to agriculture is much higher for the early developers.

[14] Doepke (1999) endogenizes the fertility dynamics for the Hansen-Prescott (1999) model. However, by assuming countries have the same population growth rate at their common turning point, the differences in

and Dyson and Murphy (1985) have documented for the case of other countries, that fertility rates were also increasing during this period.[15] On the other hand, Livi-Bacci (1997) shows that mortality rates at the early development stage for the late developers are more or less the same as European mortality rates. However, the fertility rates in developing countries are considerably larger than those experienced in European countries, which suggests that the difference in the peaks of population growth rate is due mainly to differential fertility rates. Cultural, religious and policy differences that affect the fertility decision may all be important for understanding Figure (6). While understanding what accounts for these differences is of interest in its own right, I will simply take these differences as exogenous and examine their consequences for development.

I now allow the peak population growth rate to be one percent higher in the distorted economy. In other words, all the parameters are the same as before except m (the parameter corresponding to the peak population growth rate) is equal to 2.8 for the distorted economy. As shown in Figure (7), the income ratio increases by more than 20 percent from period 6 to 9, and the maximum income ratio increases from 3.2 to 4, which is a 25 percent increase. Thus, it confirms the intuition that differences in the population profiles between the early and the later developers are important in accounting for their income differences.

6 Empirical Studies

In this section, I use the size of barriers implied by the difference in turning points to compare the predicted income ratio with the income ratio in the data. To highlights the role of barriers, I assume countries have the same preference and access to the same technology throughout. Equation (9) then tells the relationship between the turning point, barriers and the initial level of land per worker. With information on the difference in turning points, I still have three unknowns, the ratio of the barriers to Malthus capital, the ratio of the

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the peaks of the population growth rates cannot be addressed.

[15] This increase in the total fertility rate can be decomposed into changes in marriage behavior and changes in marital fertility. Wrigley and Schofield (1981) provide evidence that in England, the marriage rate increased and age of first marriage decreased during the initial stage of industrialization. Evidence from the demography literature (see Dyson and Murphy (1985)), suggests that marital fertility was increasing during the early development stage and that this increase was mainly due to changes in postpartum sexual abstinence and duration of breast-feeding.

barriers to Solow capital, and the ratio of the initial level of land per worker. One way to get a solution is to make two assumptions: (1) countries have the same initial level of land per worker (l_0), and (2) the barriers are symmetric, or the barriers for Malthus capital (π_m) are equal to one. The size of barriers can then be derived from equation (12) . An alternative way is to observe that the effects of l_0 and π_m can be summarized by the income ratio along the MBGP. This is because the turning point is derived by comparing the revenue of using the Solow technology (the Solow TFP level) with its cost (the cost of capital and labor along the MBGP), which is precisely what equation (8) says. I can rewrite this equation using the equilibrium conditions for the prices as,

$$A_s \gamma_s^{t^*} \geq \pi_s^{\theta} \hat{y}_m^{1-\theta} D > A_s \gamma_s^{t^*-1} \tag{14}$$

where $D = \left(\frac{\phi}{v_{m1}\theta}\right)^{\theta} \left(\frac{\mu}{1-\theta}\right)^{1-\theta}$. Thus l_0 and π_m are irrelevant for the difference in turning points once the constant income ratio is known. The barriers to Solow capital can then be derived from equation (14) using the observed difference in turning points and the income ratio along the MBGP.

However, although I do not need separate information on π_m and l_0 to derive the barriers to Solow capital, I need to know π_m and l_0 separately to calculate the subsequent income path. In order to do this, I set π_m equal to one because the capital share in the Malthus technology is sufficiently small to make π_m virtually irrelevant. l_0 can then be derived from equation (7) . I now proceed to conduct the empirical exercises using the second way, i.e. to derive the barriers to Solow capital and the initial land per worker using information on the difference in turning points and the ratio of pre-modern growth income.[16]

6.1 The Seven Groups

I consider the seven groups in Figure 1 and calculate their average annual growth rates using Table A4.1. The growth rate for Group 1 was below 0.2% for 1500-1600, 1600-1700, and 1700-1800. It increased to 1% during 1820-1870. Following Maddison, I use 1820 as

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[16] In Ngai (2000), I have used the first method to study the income differences between UK, Japan and Africa (using data from Lucas (1998)). For Africa and the UK, I found that the π implied by the difference in turning points can account for about 70 percent of their current income differences. For Japan and the UK, I considered the institution reforms in Japan and showed that the model can account for both the Japanese miracle and the slowdown.

the turning point for Group 1. Since one period in the model is 35 years, the turning points for all other groups will be assigned to year 1855, 1890,..etc. Groups 2-5 all experienced a growth rate of 1% during 1870-1913. But during 1820-1870, Groups 2 and 3 both have growth rates around 0.5%, while the growth rates of Groups 4 and 5 are below 0.2%. Thus, I assign 1855 as the turning point for both Groups 2 and 3, and 1890 for both Groups 4 and 5. The turning point for Group 6 is 1925 since it reached a 1% growth rate during 1913-1950. Finally, the turning point for Group 7 is 1960 given it reached a 1% growth rate during 1950-1973. The average income ratio between Group 1 and another group during 1500-1700 is used to match the income ratio along the MBGP of the model. Denote $d_i = t_i^* - t_1^*$ the difference in the turning points of Group i and Group 1, π_i the level of the barriers in Group i relative to that of Group 1, and z_i the average relative income of Group 1 to that of Group i during 1500-1700 which can be calculated from Table A4.1 as

z_2	z_3	z_4	z_5	z_6	z_7
1.2	1.7	2.0	1.7	2.3	1.6

Given z_i and d_i, the initial land per worker for Group i is equal to $z_i^{(1-\phi)/(1-\mu-\phi)}$, and the range of π_i implied by equation (14) is

$$z_i \left(\gamma_s^{1/(1-\theta)}\right)^{d_i} \geq \pi_i^{\theta/(1-\theta)} > z_i \left(\gamma_s^{1/(1-\theta)}\right)^{d_i - 1}. \tag{15}$$

where $\pi_i^{\theta/(1-\theta)}$ and $\gamma_s^{1/(1-\theta)}$ are the predicted income ratio and growth rate along the SBGP.

The benchmark economy is now calibrated to Group 1, thus the peak of the population is adjusted to 1.3% as implied by Figure (6). This implies that population in 1995 is about five times that in 1820 which matches that of Group 1. The other calibrated parameters are the same as before except that $\theta = 0.5$.[17] This value of θ is larger than before but in accordance with many authors, e.g. Parente and Prescott, who have argued that capital shares should be higher than the canonical values because of unmeasured investment. The parameters for any Group i are identical to that of Group 1 except for the barriers and the initial land per worker. I calculate the barriers as the minimum value implied by equation

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[17] Thus, γ_s and β are adjusted accordingly to match the growth rate and interest rate. The choice of θ will mainly affect the level of π but not the main result on income ratio. It is because the difference in turning points is already given by the data. Moeover γ_s is adjusted to match the growth rate and so the income ratio along the SBGP $\left(\pi^{\theta/(1-\theta)}\right)$ is always within the range given by equation (15) regardless of the level of θ.

(15). The barriers derived are

π_2	π_3	π_4	π_5	π_6	π_7
1.5	2.0	4.0	3.5	9.5	13.5

I assume these barriers remain the same throughout the sample years. The results are summarized in Table 3 which report the income ratio implied by the model as a percentage of that in the data for the specified periods between 1820 to 1995. The periods are chosen because data between years 1820-1950 are available only for years 1820, 1870, 1913 and 1950 for all seven groups. I use linear interpolation between the periods in the model to compare the model with the data. For Group 2 (other Western European countries), the income ratio predicted by the model matches the data very well for both the beginning and end of the sample years, accounting for 80 percent of the income ratio. It falls short of the data for the period 1913-1960 which covers the World War II. Given the implied relative barrier is 1.5 for Group 2, the model predicts Group 2 is converging to Group 1 and reaches an income ratio of 1.5 along the SBGP. In the data, the ratio of GDP per capita for Group 1 to that of Group 2 fluctuates between 1.55 to 1.67 for the period 1990-2001. The income ratio predicted by the model for Group 4 (Latin America) is about 20 percent more than in the data except for the period 1913-1960, during which the model's prediction is 30-40 percent more than in the data. It could be due to the fact that the turning point for Group 4 is somewhere between the year 1855 to 1890, thus the barrier derived is on the high side.

For other groups, the model performs well for the period 1820-1960. But for the period 1960-1995, the predicted incomes for Groups 5 and 7 are too low relative to that of Group 1, and the predicted incomes for Groups 3 and 6 are too high relative to that of Group 1. The reason that the predicted income is too high for Group 3 (Eastern Europe and Former USSR) for the period 1960-1995 is that the GDP per capita for Group 3 actually fell in 1998 to half its level of 1990 for political reasons which are not in my model. The reason that the predicted incomes are too low for Groups 5 and 7 is that I have assumed the barriers to remain the same throughout the sample years. But these two groups contain most of the countries (such as Japan and South Korea) that have experienced "growth miracles" during this period. These miracle experiences are often associated with institutional reforms that may have lowered the size of barriers.

So far I have assumed all groups are identical except for the level of initial income and barriers. But as shown in Fig(6), they have different population growth rates during their early development stage. In fact, for Group 6 (Africa), the peak population growth rate is 3 percent during the period 1950-1995. Figure (8) reports the results of allowing a higher peak population growth rate for Africa.[18] It is interesting to note that, in contrast to the balanced growth path approach, the model predicts that the income ratio between Africa and Group 1 will continue to worsen even if relative barriers are unchanged. A large fraction of this increase is due to the high population growth rate.[19] With this adjustment, the percentage of income ratio (Group1/Africa) predicted by the model increases from 71 percent to 84 percent for the period 1960-1995.

To sum up, the barriers that match the observed differences in turning points and the pre-modern growth income ratio achieve the following: (1) it predicts 'convergence' among the Western countries; (2) it accounts for a significant portion of the income ratio between the current poor and current rich, especially when the different population profiles are taking into account.

6.2 Institutional Change

I now address the issue of institution changes that may have changed the size of barriers and focus on individual countries. The benchmark economy is now interpreted as the UK (a member of Group 1), which has similar population profile as that of the whole of Group 1. I make comparison with Japan and Argentina, two countries that experienced well-documented institutional changes.

The growth rate of the UK was below 0.3% before the period 1820-1870, thus 1820 is also used as the turning point for the UK.[20] I need a measure for the barriers along the long run development path. The Penn World Table covers the price of capital starting from

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[18] I chose to study Africa as one unit since Maddison (1995, 2001) contains very few data for African countries prior to 1950, for four sample countries in year 1913, and two out of the four sample countries in year 1900. Moreover, they all have very similar population profiles.

[19] Note that the implied ratio of output per worker is higher than that of output per capita during the period 1960-2135. In particular, for the benchmark case, the maximum ratio for output per worker is 12 and for output per capita is 11. For the case with adjusted population profile, the maximum ratio for output per worker is 18 and for output per capita is 14.

[20] There are some disagreement on the turning point of the UK, but since I am using the Maddison data, I follow his choice of 1820.

year 1950. Collins and Williamson (1999) construct a panel database for 1870-1950 for eleven OECD countries. Apart from Japan, the other countries are all from Group 1 who have very similar long run development experiences.[21] I focus on comparing the experience of Japan with that of the UK (one of the 10 countries). Modern growth began in Japan around the end of the 19th century. However, Japan's GDP per capita exceeded that of the UK in 1990. This rapid rate of catch up is due to the exceptionally high growth rate during the postwar period. Its average growth rate was 8.1% during the period 1950-73, compared to 2.4% for the UK. Growth slowed down after 1973 when its growth rate dropped to 2.3% for (for 1973-2000).

To see the model's predictions on the development experience of Japan relative to that of the UK, I need a measure of relative barriers in Japan since the beginning of the modern growth era. Based on the difference in their turning points and the ratio of their pre-modern growth income, equation (15) can be used to derived the relative barriers in Japan when both economies are in the pre-modern growth regime. However, this level of barriers did not remain constant over time. The historical record suggests two episodes that significantly lowered barriers in Japan. They are the Meiji Restoration in 1868 which ended Shogunate Japan, and the postwar economic and institutional reforms. According to Yamamura (1977), the new Meiji government adopted policies to encourage the absorption and dissemination of western technologies and skills, and help the growth of private industries. Following these policy changes, the fraction of workers employed in industry increased significantly in 1907. Postwar Japan also underwent many major reforms such as introducing numerous tax-exemptions or tax-reliefs for investment; industry-financing programs; allowing the purchase of new foreign patents; dissolving the *zaibatsu* system and the deconcentration of many *zaibatsu* subsidiaries[22]; and trade liberalization (see Tsuru (1961) and Rotwein (1964)). According to Ohkawa and Rosovsky (1963), these reforms led to a steep rise in the rate of private investment and a rapid shift of resources from the agricultural to the nonagricultural sector.

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[21] The other ten countries are Australia, Canada, Denmark, Finland, Germany, Italy, Norway, Sweden, Great Britain and the US.

[22] The "*zaibatsu*" refers to a relatively small number of family-dominated company systems holding assets through large segments of the Japanese economy. These groups had become a major force in Japanese economic and political life before World War II.

These reductions in barriers are consistent with the data reported in Collins and Williamson. Based on their Tables (1a) and (1b), Figure (9a) plots the relative price of capital goods and equipment in Japan where the relative price in 1900 is normalized to 100. Figure (9b) plots the ratio of the relative prices in Japan to that of the UK using their Tables (2a) and (2b). These two figures show that there are significant reductions in the relative prices in Japan and their ratio relative to that of the UK. This evidence is consistent with the view that barriers in Japan were reduced after the Meiji Restoration. For the postwar period, Jones (1994) shows that the relative price of equipment in Japan relative to the UK is equal to 0.7 (in 1980).

In view of these facts, I carry out the following exercise to account for the experience of Japan relative to that of the UK. As Japan experienced a two-period delay compared to the UK and its income for the period 1500-1700 is 54 percent of that in the UK, equation (15) implies the range of the relative size of barriers in Japan to be between 3.8 and 7.7 in the pre-modern growth regime. Together with the evidence from Figures (9a) and (9b), the relative size of barriers in Japan is then set to 4 initially. To capture the impact of the Meiji restoration, the barriers are reduced by half in 1890, which matches the data in Figures (9a) and (9b). Finally, the postwar reforms are captured by reducing the barriers to 0.7 in 1960 based on the evidence in Jones (1994).

Table 4 compares the predictions of the model with the data for the specified periods between 1820 to 2000. The model tracks the trend of the income ratio between the UK and Japan very closely. It predicts both the divergence between Japan and UK prior to 1890 and the later convergence but the catch up of the Japanese economy happens at a slower rate than in the data. There are two points to note. First, the income ratio for the period 1850-1925 is fairly stable though the barriers are reduced by half in 1890. This is because the model predicts an inverted U-shape for the time path of the income ratio for a given level of barriers. Therefore, if the level of barriers is reduced before the maximum income ratio is reached, it will only cause the income ratio to increase at a smaller rate but not necessarily reduce it. This is an interesting property of the model and is consistent with the finding of Restuccia and Urrutia (2001) that the range of the relative price of investment is decreasing for the period 1960-85 while the magnitude of the income ratio is not. Second, the model implies both the Japanese miracle and the slowdown for the period 1960-2000 with one single change in the level of barriers in 1960.[23] Within a version of the neoclassical growth model, Parente and Prescott (1994) interpret the miracle in Japan as a reduction in its barriers to less than that of the US, while the subsequent slowdown is associated with an increase in its relative size. They argue that Japan is converging to three different balanced growth paths, corresponding to the period before the miracle, during the miracle, and the slowdown after the miracle, and they assume the existence of three different levels of barriers each corresponding to a steady state. I find, however, that the slowdown of the Japanese economy after the miracle years can be obtained without increasing the relative barriers, as part of the normal process of transition. The difference in our results highlight the difference in my approach and the standard balanced growth approach in accounting for international income differences. If we focus on balanced growth paths, differences of this magnitudes can only be explained by exogenous shocks that change the balanced growth equilibrium.

In contrast to the case of Japan, modern growth began in Argentina around the middle of 19th century. The GDP per capita of the UK relative to that of the Argentina declined from 2.1 in 1870 to 1.4 during 1900-1929, but started to rise since then. Díaz-Alejandro (1970), and more recently Talyor (1994), dated this as the end of the Belle Époque. They argue this is due to the dramatic rise in the price of capital goods in the post-1935 era as a result of the interventionist political regime. These interventions include rationing, controls and other distortions on machinery and equipment. Based on Table 3 in Taylor (1994), Figure (10) plots the relative price of investment and two of its major components (machinery and Durable Producers' Equipment) in Argentina during the period 1935-1960. It suggests that the average relative price of investment (equipment and machinery) for the period 1939-60 is about 67 (92 and 65) percent higher than in the period 1935-38. Using the data from Collins and Williamson (1999), the average relative price of capital (equipment) for the period 1939-50 is about 20 (15) percent higher than in the period 1935-39. Therefore, the initial relative level of barriers in Argentina is set to be 1.5 (to generate a one-period delay) and increased in 1925 by 40 percent to 2.1.[24] Table 5 compares the prediction of the

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[23] The removal of barriers can only partly replicate the postwar miracle of Japan as the destruction of the capital during the war is also an important factor.

[24] Díaz-Alejandro (1970) reports that the relative price of new machinery and equipment was between 2.5 and 3.3 times higher in Buenos Aires than in two major US cities for 1962. On the other hand, the relative

model with the data in Maddison (1995 and 2000) for the specified period between 1870 and 1995. The model closely tracks the trend of the income ratio, i.e. it predicts convergence prior to 1925 then divergence and slowdown in the Argentine economy for the period 1925-1960 due to the increased barriers in 1925. The shortcoming is that the predicted growth rate for the period 1960-1995 is too high compared to the data.

To conclude, the institutional changes can explain why Japan reaches a higher income level than Argentina even though modern growth began later in Japan.

7 Conclusion

Recent studies have emphasized differences in the barriers to capital accumulation and technology adoption as determinants of cross-country income differences, but they have generally focused on steady states. In this paper I focus on the role of the barriers in determining the beginning date and pace of modern economic growth. A fundamental property of the model is that cross-country income differences exhibit an inverted U-shape pattern over time, an important feature of long run economic data. A key implication of my model is that a substantial fraction of existing income differences are transitional. The transitional effect increases significantly when I include the fact that today's low-income countries had higher population growth rates during the early development stage than did the currently rich countries. I find interesting results in my empirical tests. I divide all countries in Maddison's dataset into seven groups and I find that the barriers that account for their differences in turning points also account for a significant portion of their income differences. The case of Japan and Argentina relative to the UK, which I used to study the effect of institutional change, illustrates how the model can explain both the growth miracle and subsequent slowdown along the same development path.

The model abstracts from the fact that home production (the non-market sector) plays an important role in the early development stage of economy. Parente, Rogerson and Wright (2000) extend the standard barrier model to include home production. They find that the measured income disparity along the balanced growth path increases significantly if market

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price of machinery is 1.7 times higher in Argentina than in the UK in 1980 (Jones (1994)). Restuccia and Urrutia (2001) shows that the relative price of investment is 1.5 times higher in Argentina than in the UK for the period 1960-85.

and home produced goods are close substitutes and the capital share of the home production technology is small. Incorporating home production in this model is expected to work in a similar way as in their model.

Another interesting extension not pursued here is to allow for mortality risk and human capital accumulation, as in Tamura (2002b). One well-known development fact is the positive correlation between average years of schooling and life expectancy across time and countries. In the context of this paper, the exogenous barriers delay modern growth and in turn delay the improvement in mortality, thus providing an endogenous barrier to human and physical capital accumulation which can explain even more income differences.

Appendix 1. Competitive Equilibrium This appendix derives the competitive equilibrium which satisfies the three development stages under assumptions A1-A6 specified below.

Given π_m, π_s, N_0, K_0 and L, a competitive equilibrium consists of prices $\{q_t, w_t, r_{Kmt}, r_{Kst}, r_{Lt}\}$; firm allocations $\{K_{mt,}K_{st}, N_{mt}, N_{st}, L_{mt}, Y_{mt}, Y_{st}\}$; and household allocations $\{c_{1t}, c_{2t+1}, x_{mt}, x_{st}, l_{t+1}\}$, such that (1) given prices, household and firm allocations maximize utility and profit; (2) all markets clear

$$Y_{mt}+Y_{st} = N_t c_{1t}+N_{t-1}c_{2t}+N_t x_t; \quad N_{mt}+N_{st} = N_t; \quad K_{mt}+K_{st} = K_t; \quad L_{mt} = L = N_{t-1}l_t$$

and (3) the laws of motion hold

$$K_{mt+1} = N_t\frac{x_{mt}}{\pi_m}; \quad K_{st+1} = N_t\frac{x_{st}}{\pi_s}; \quad N_{t+1} = g(c_{1t})N_t.$$

The model can be solved for constant intertemporal elasticity of substitution utility, but I assume

$$A1 : u(c) = \ln c$$

In equilibrium, $c_{1t} = \frac{w_t}{1+\beta}$, and $R_t = \frac{q_t+r_{Lt}}{q_{t-1}}\ if\ l_t > 0$; $R_t = \frac{r_{kst}}{\pi_s}\ if\ x_{st} > 0$; and $R_t = \frac{r_{kmt}}{\pi_m}\ if\ x_{mt} > 0$.

Malthus Balanced Growth Path (MBGP) Function $g\left(.\right)$ is chosen so that output per worker $(\hat{y}_m)$ and capital per worker $(\hat{k}_m)$ are constant, where $\hat{y}_m = A_m\gamma_m^t\hat{k}_m^\phi\left(\frac{L}{N_t}\right)^{1-\mu-\phi}$ is constant if assume

$$A2 : g(\hat{c}_{1m}) = \gamma_m^{1/(1-\phi-\mu)} \text{ and } g(c_1) > g(\hat{c}_{1m}) \quad \forall c_1 \in [c_{1m}, c_{1m}+\epsilon]\ where\ \epsilon > 0,$$

then $\frac{\hat{k}_m}{\hat{y}_m} = \frac{v_{m1}}{\pi_m}$ where $v_{m1} = \frac{1+\beta-\mu-\sqrt{(1+\beta-\mu)^2-4\mu\phi\beta(1+\beta)}}{2(1+\beta)\gamma_m^{1/(1-\mu-\phi)}}$, is the ratio for an economy with $\pi_m = 1$, and

$$\hat{y}_m = \left[A_m\gamma_m^t\left(v_{m1}/\pi_m\right)^\phi\left(L/N_t\right)^{1-\mu-\phi}\right]^{1/(1-\phi)}$$

The price and rental rate of land grow at $\gamma_m^{1/(1-\phi-\mu)}$. The wage rate and rental rate of capital are constant.

Transition A firm can write down his profit function if it starts using the Solow technology,

$$\Psi(r_{kmt}, w_t) = \max_{K_{st},N_{st}}\left(A_s\gamma_s^t K_{st}^\theta N_{st}^{1-\theta} - r_{kst}K_{st} - w_t N_{st}\right)$$

The optimal decision of the firm implies $\frac{K_{st}}{N_{st}} = \frac{\theta w_t}{(1-\theta)r_{kst}}$, so profit function becomes:

$$\Psi(r_{kmt}, w_t) = \max_{N_{st}}\left[A_s\gamma_s^t(\frac{\theta w_t}{(1-\theta)r_{kst}})^\theta - \frac{w_t}{1-\theta}\right]N_{st}$$

For household to invest in both capitals, $\frac{r_{kst}}{\pi_s} = \frac{r_{kmt}}{\pi_m}$,

$$\Psi(r_{kmt}, w_t) = \max_{N_{st}}\left[A_{st}\left(\frac{\pi_m}{\pi_s}\frac{\theta w_t}{(1-\theta)r_{kmt}}\right)^\theta - \frac{w_t}{1-\theta}\right]N_{st}$$

When on the MBGP, the firm will use the Solow technology if $\Psi(\hat{r}_m, \hat{w}_m) \geq 0$,

$$A_{st} \geq \left(\frac{\pi_s}{\pi_m}\frac{\hat{r}_m}{\theta}\right)^{\theta}\left(\frac{\hat{w}_m}{1-\theta}\right)^{1-\theta}$$

Given $K_0 = \left[N_0^{\mu} L^{1-\phi-\mu}\left(\frac{v_{m1}}{\pi_m}\right)\right]^{1/(1-\phi)}$, both $\hat{r}_m$ and $\hat{w}_m$ are functions of N_0, the turning point (t^*),

$$A_s\gamma_s^{t^*} \geq B\pi_s^{\theta}\pi_m^{-\phi(1-\theta)/(1-\phi)}\left(L/N_0\right)^{(1-\mu-\phi)(1-\theta)/(1-\phi)} > A_s\gamma_s^{t^*-1}$$

where $B = \left(\frac{\phi}{\theta}\right)^{\theta}\left(\frac{\mu}{1-\theta}\right)^{1-\theta}\left[v_{m1}^{(\phi-\theta)}A_m^{(1-\theta)}\right]^{1/(1-\phi)}$.

Given q_{t-1}, N_t, L, and $I_t \equiv N_{t-1}(w_{t-1} - c_{1t-1}) - q_{t-1}L$, profit maximization implies

$$\frac{\theta Y_{st}}{\pi_s K_{st}} = \frac{\phi Y_{mt}}{\pi_m K_{mt}}; \quad w_t = (1-\theta)\frac{Y_{st}}{N_{st}} = \mu\frac{Y_{mt}}{N_{mt}}; \quad r_{Lt} = (1-\phi-\mu)\frac{Y_{mt}}{L}$$

which imply $k_{mt} = \frac{\pi_s}{\pi_m}\psi k_{st}$, where $k_{mt} = \frac{K_{mt}}{N_{mt}}, k_{st} = \frac{K_{st}}{N_{st}}$, and $\psi = \frac{(1-\theta)\phi}{\theta\mu} < 1$ if assume

$$A3 : \theta > \phi$$

Market clearing implies $\pi_m k_{mt} = \frac{\psi I_t/N_t}{1-(1-\psi)m_t}$, where $n_{mt} = \frac{N_{mt}}{N_t}$. Labor indifference implies

$$k_{mt}^{\theta-\phi} = \frac{\mu}{1-\theta}\frac{A_{mt}}{A_{st}}\left(\psi\frac{\pi_s}{\pi_m}\right)^{\theta}\left(\frac{L}{N_{mt}}\right)^{1-\phi-\mu}$$

Thus the equilibrium n_{mt}^* solves $f\left(n_{mt}^*\right) = 0$ where

$$f\left(n_{mt}\right) = \frac{\mu}{1-\theta}\pi_s^{\theta}\pi_m^{-\phi}\psi^{\phi}\left(1-(1-\psi)\,n_{mt}\right)^{\theta-\phi} - \frac{A_{st}}{A_{mt}}I_t^{\theta-\phi}N_t^{1-\theta-\mu}n_{mt}^{1-\phi-\mu}$$

and $1-\mu-\phi > 0$ and $t \geq t^*$ implies $f\,' < 0, f\left(0\right) > 0$ and $f\left(1\right) < 0$, thus there exists an unique $n_{mt}^* \in [0,1)$. Moreover, n_{mt}^* converges to zero if $\frac{A_{st}}{A_{mt}}I_t^{\theta-\phi}N_t^{1-\mu-\theta}$ is increasing in t which is true if assume

$$\begin{array}{lll} A4 & : & \gamma_s \geq \gamma_m \\ A5 & : & \exists \bar{t}, n, \quad s.t. \quad g\left(c_{1t}\right) \leq n \quad \forall t > \bar{t} \qquad\qquad if\ 1-\theta < \mu \\ & : & g\left(c_{1t}\right) \geq 1 \qquad\qquad\qquad\qquad\qquad\quad if\ 1-\theta \geq \mu \end{array}$$

Solow Balanced Growth Path (SBGP) As $n_{mt} \to 0$, both $r_{Lt} \to 0$ and $q_t \to 0$. Assume,

$$A6 : \lim_{c_1 \to \infty} g\left(c_1\right) = g,$$

the economy converges to a SBGP where output per worker $\left(y_{st}\right)$ is growing at a constant rate. The capital-output ratio equal v_{s1}/π_s, where $v_{s1} = \frac{\beta(1-\theta)}{(1+\beta)g\gamma_s^{1/(1-\theta)}}$ is the ratio for an economy with $\pi_s = 1$, and

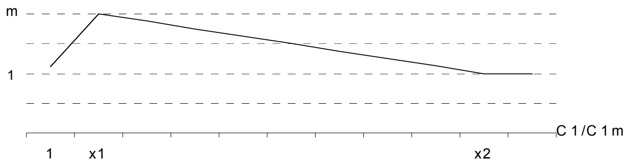
$$y_{st} = \left(A_s\gamma_s^t\left(v_{s1}/\pi_s\right)^{\theta}\right)^{1/(1-\theta)}$$

The wage and consumption also grow at $\gamma_s^{1/(1-\theta)}$.

Appendix 2. Calibration This appendix give a brief review of the calibration in Hansen and Prescott.

The economy with barriers equal to one is calibrated to match the development experience of England before 1800 and the postwar development experience of the industrialized countries. A period in this economy is 35 years in real time. Agents will therefore live for 70 years working for the first 35 years of their life-span. The initial conditions, A_m, A_s, L and N_0 are set to be one arbitrarily. Given N_0, K_0 is chosen such that the economy is initially on the MBGP. The capital share of the Solow technology is chosen to match factor share in postwar US. The capital share of the Malthus technology is set to 0.1. Labor shares are assume to be the same for both technologies. The population growth rate for the pre-1800 period in the UK is used to calibrate γ_m, and the relationship between the population growth rate and the GDP per capita for the industrial economies is used to calibrate the function $g(.)$. A general pattern in the long run population data can be summarized by

Figure A3.1. Population Growth Function



It says that population growth rate first increases until the living standard is x_1 times its Malthusian level and the decreases to a constant level when the living standard is x_2 times its Malthusian level. The $g(.)$ is then calibrated to this shape with $x_1 = 2$, $x_2 = 18$ and $m = 2$ where $m = 2$ corresponds to a 2% average annual population growth rate. Finally, γ_s and β are chosen so that the growth rate is around 2% for postwar periods, and interest rate is around 2% in Malthus era and 4-4.5% for the postwar periods. To summarize, the parameter values are

θ	μ	ϕ	γ_m	γ_s	β	x_1	x_2	m
0.4	0.6	0.1	1.03	1.52	1	2	18	2

Given L, N_0 and K_0, q_t is solved using the shooting algorithm described in Hansen and Prescott.

Appendix 3. Sensitivity Analysis I examine the robustness of the shape of Figure (6) with respect to changes in parameters of the model. These parameters are initial population, quality of land, initial TFP levels for the Malthus and Solow technologies, input shares for Malthus technology, population growth rate along the Malthus balanced growth path, and the population growth function $g(c_1)$. Figure A1 shows that doubling initial population, quality of land and $\frac{A_m}{A_s}$ all have insignificant effects on the shape of the income ratio curve. Given the Malthus sector almost disappear three periods after modern growth begins, both capital and land shares of the Malthus technology have an insignificant effect on the income ratio curve. Doubling the population growth rate along the Malthus balanced growth path from 0.3 percent to 0.6 percent will increase γ_m from 1.03 to 1.07.

This again is insignificant in determining the income ratio curve since consumption is doubled two periods after modern growth begins, and after this point γ_m does not enter into $g(c_1)$. I check the robustness of shape of income difference by varying x_1, x_2 and m. Figure A2 shows that both x1 and x2 have an insignificant effect on the maximum income ratio but m has a significant effect. By increasing the maximum annual population growth rate from 2% to 3% ($m = 2$ to $m = 2.81$), the maximum income ratio is increased from 3.2 to 3.5 (a nearly 10 percent increase).

Figure A3.1. Initial Conditions

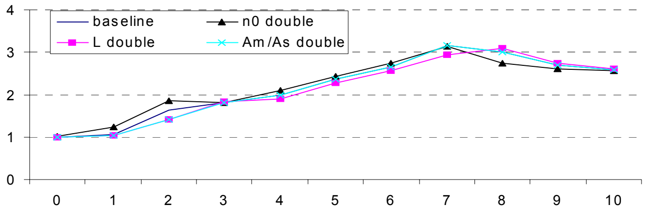
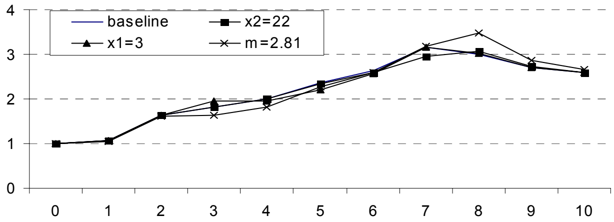


Figure A3.2. Population Profile



Appendix 4. Data Appendix Data are from Maddison (2001) which includes population, GDP and GDP per capita for 124 individual countries, as well as regional, subregional and the world total. The seven groups are different from the seven regions of Maddison, the definition of each group are as follow:

Group 1 includes 4 Western Offshoots and 12 Western European countries. The Western Offshoots are Australia, New Zealand, Canada, and United States. The Western European Countries-Austria, Belgium Denmark, Finland, France, Germany, Italy, Netherlands, Norway, Sweden, Switzerland and the United Kingdom.

Group 2 includes 17 other western European countries. They are Ireland, Greece, Portugal, Spain and other 13 small western European countries.

Group 3 includes 7 Eastern European countries and 15 Successor States of the Former USSR. The Eastern European countries are Albania, Bulgaria, Czechoslovakia (a) Czech Republic and Slovakia from 1990), Hungary, Poland, Romania and Former Yugoslavia. The Successor States of the Former USSR are Armenia, Azerbaijan, Belarus, Estonia, Georgia, Kazakhstan, Kyrgyzstan, Latvia, Lithuania, Moldova, Russian Federation, Tajikistan, Turkmenistan, Ukraine, and Uzbekistan.

Group 4 includes 44 Latin American countries. They are Argentina, Brazil, Chile, Colombia, Mexico, Peru, Uruguay, Venezuela, Bolivia, Costa Rica, Cuba, Dominican Republic, Ecuador, El Salvador, Guatemala, Haiti,Honduras, Jamaica, Nicaragua, Panama, Paraguay, Puerto Rico, Trinidad & Tobago, and other 21 small Caribbean countries.

Group 5 includes Japan only.

Group 6 includes 57 African countries. They are Algeria, Angola, Benin, Botswana, Burkina Faso, Burundi, Cameroon, Cape Verde, Central African Republic, Chad, Comoros, Congo, Cote d'Ivoire, Djibouti, Egypt, Eritrea &Ethiopia, Gabon, Gambia, Ghana, Guinea, Guinea-Bissau, Kenya, Lesotho, Liberia, Madagascar, Malawi, Mali, Mauritania, Mauritius, Morocco, Mozambique, Namibia, Niger, Nigeria, Reunion, Rwanda, Senegal, Seychelles, Sierra Leone, Somalia, South Africa, Sudan, Swaziland, Tanzania, Togo, Tunisia, Uganda, Zaire, Zambia, Zimbabwe, and other 6 countries.

Group 7 includes 40 East Asian countries and 15 West Asian countries. The East Asian countries are China, India, Indonesia, Philippines, South Korea, Thailand, Taiwan, Bangladesh, Burma, Hong Kong, Malaysia, Nepal, Pakistan, Singapore, Sri Lanka, Afghanistan, Cambodia, Laos, Mongolia, North Korea, Vietnam, and 19 small countries. The West Asian countries are Bahrain, Iran, Iraq, Israel, Jordan, Kuwait, Lebanon, Oman, Qatar, Saudi Arabia, Syria, Turkey, UAE, West Bank and Gaza, and Yemen.

Table A4.1. GDP per Capita and Population for Seven Groups

	1500	1700	1820	1870	1913	1950	1960	1970	1980	1990	1998
GDP per capita (1990 International Dollars)											
Group 1	775	1041	1269	2168	4203	6753	9075	12593	15934	19554	22518
Group 2	657	850	994	1253	1986	2374	3413	6824	9164	11725	13980
Group 3	483	592	667	917	1501	2601	3663	5183	6236	6446	3893
Group 4	416	437	665	698	1511	2554	3167	4016	5413	5055	5795
Group 5	500	520	669	737	1387	1926	3988	9715	13429	18778	20541
Group 6	400	400	418	444	585	876	1046	1332	1496	1396	1384
Group 7	572	571	575	543	640	713	1032	1536	2036	2781	3565
Population (millions)											
Group 1	51	71	125	208	339	433	485	537	575	611	645
Group 2	9	13	18	25	33	48	52	56	62	64	66
Group 3	30	45	91	141	236	267	313	351	382	411	412
Group 4	18	12	21	40	81	166	218	286	362	443	508
Group 5	15	27	31	34	52	84	94	104	117	124	126
Group 6	46	61	74	90	125	227	283	361	473	627	767
Group 7	268	375	679	731	926	1382	1687	2093	2580	3103	3516

Table A4.2 GDP per Capita for Individual Countries (1990 International Dollars)

	1600	1700	1820	1870	1913	1950	1973	1998
United Kingdom	974	1250	1707	3191	4921	6907	12022	18714
Argentina*	430	505	623	1311	3797	4987	7973	9219
Japan	520	570	669	737	1387	1926	11439	20413
China	600	600	600	530	552	439	839	3117
India	550	550	533	533	673	619	853	1746

*The GDP per capita for Argentina is the same as Other Latin American for 1600-1820 (Maddison Table B-21)

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Table 1: Income Ratio ($\theta = 4$)

Barriers	Delay	BGP Level	Maximum Level	Percent Increased
4	2	2.5	3.2	28
8	2	4	5.3	33
16	3	6.3	8.8	40
32	4	10	14.1	41
64	4	16	23	44

Table 2: Combinations of θ and Barriers for Factor 30 Income Ratio

θ	Delay	Barriers (BGP)	Barriers (Transition)	Percent Reduced
0.4	4	164	96	41
0.45	4	64	40	38
0.5	4	30	18	40
0.55	3	16	10	38
0.6	3	10	6.5	35

Table 3: Percentage of Income Ratio Predicted by the Model

	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7
1820-1870	90	88	115	104	102	98
1870-1913	77	78	123	99	103	107
1913-1950	66	78	140	119	100	102
1950-1960	55	78	130	113	89	94
1960-1995	83	70	115	272	71	126

Table 4: Prediction for Japan

	Growth Rate of GDP per Capita (Japan)		Ratio of GDP per Capita (UK/Japan)	
	Data	**Model**	**Data**	**Model**
1820-1850	0.2	0.1	2.9	2.9
1850-1890	0.9	1.4	4.2	3.4
1890-1925	1.8	2.6	3.5	2.9
1925-1960	2.2	2.1	2.9	2.3
1960-1995	4.7	3.6	1.2	1.7
Miracle and Slowdown				
1960-1973	8.4	5.1	1.5	1.8
1973-2000	2.3	3.1	0.9	1.3

Table 5. Prediction for Argentina

	Growth Rate of GDP per Capita (Argentina)		Ratio of GDP per Capita (UK/Argentina)	
	Data	**Model**	**Data**	**Model**
1870			2.4	2.1
1870-1900	2.5	1.8	1.8	1.7
1900-1925	1.4	1.6	1.4	1.6
1925-1960	1.0	1.4	1.5	1.7
1960-1995	1.1	1.8	1.8	1.9

Figure 1: GDP per Capita for 7 Groups
Log (1990 International Dollars)

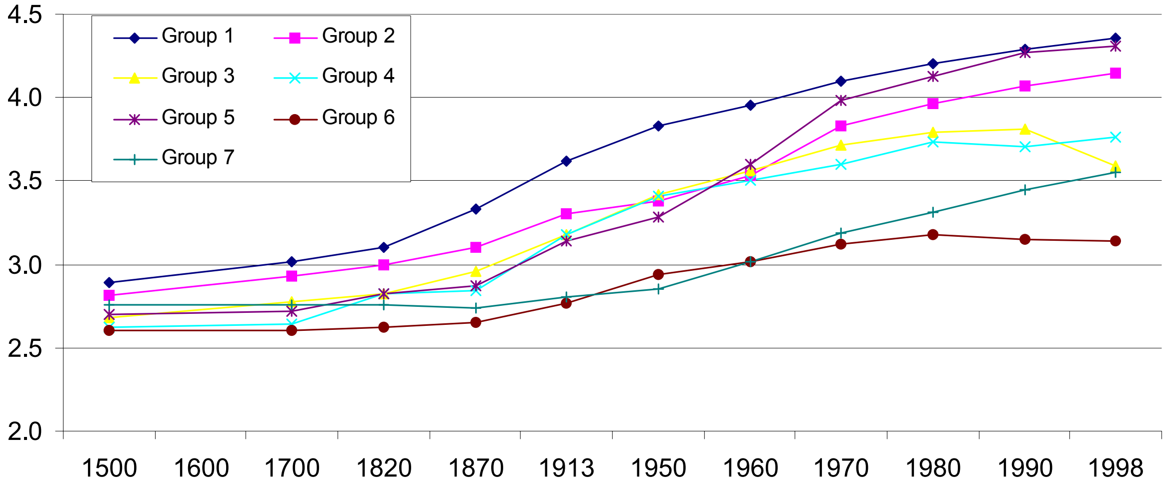

Figure 2: GDP per Capita for Individual Countries
Log (1990 International Dollars)

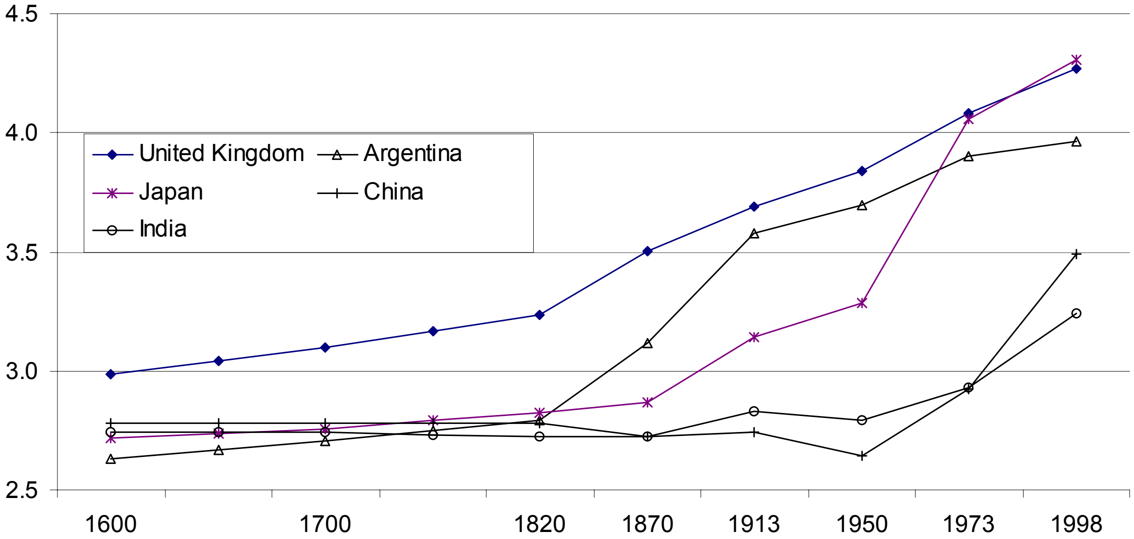

Figure 3: Fraction of Labor in the Malthus Sector

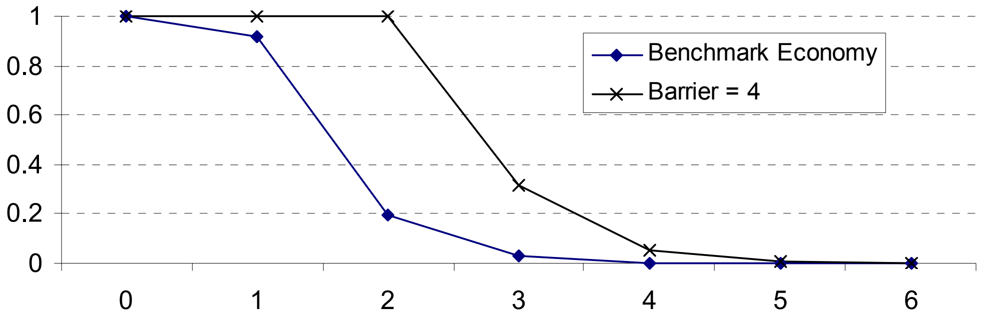

Figure 4: Ratio of Output per Worker

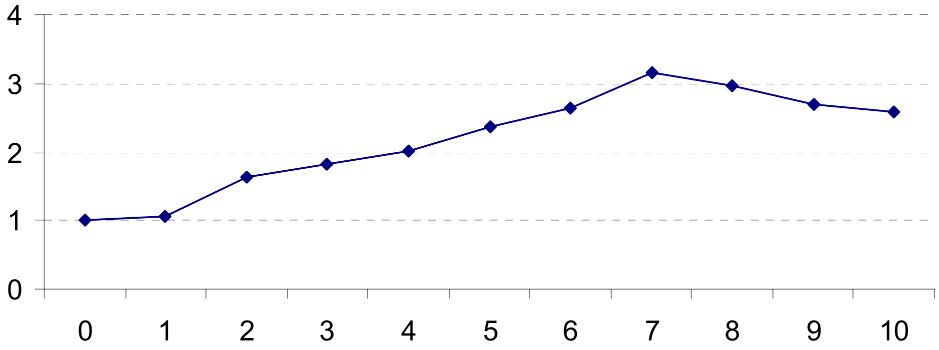

Figure 5: Growth rate of Output per Worker

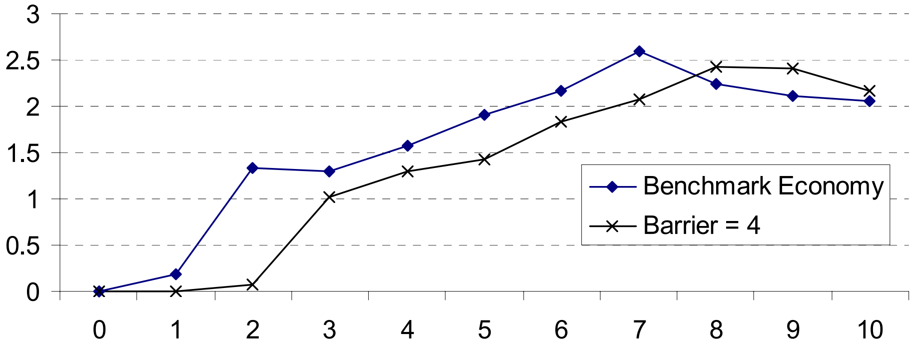

Figure 6: Population profile

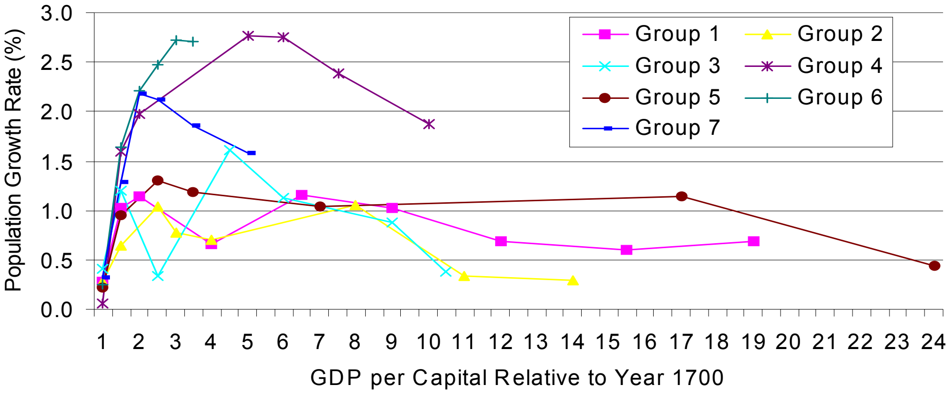

Figure 7: Ratio of Output per Worker (Different Population Profiles)

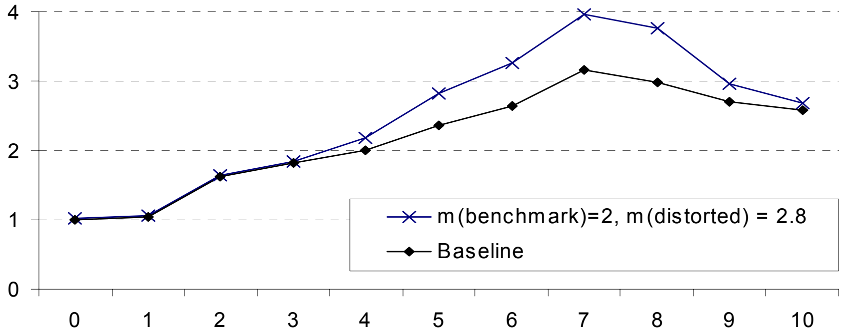

Figure 8: Predicted Ratio of GDP per capita (Group1/Africa)

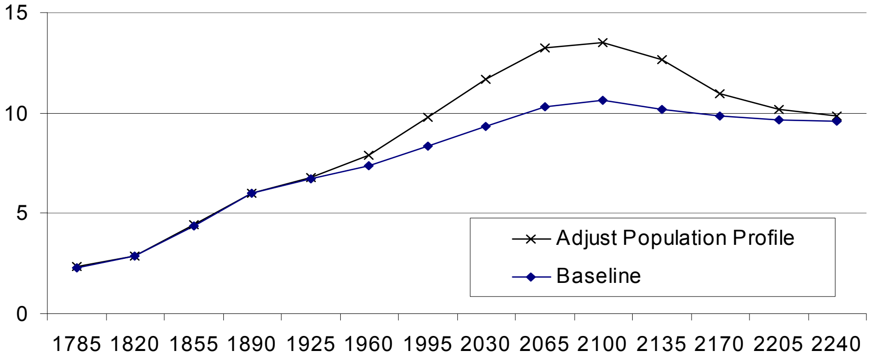

Figure 9: Historical Relative Prices in Japan

Figure 9a. Relative Prices in Japan (1900 =100)

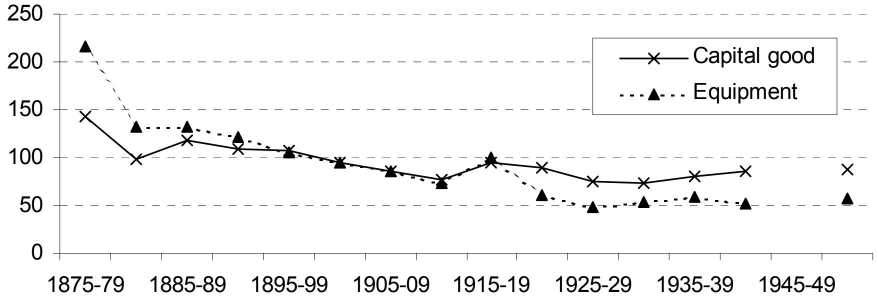

Figure 9b. Ratio of Relative Prices (Japan/UK)

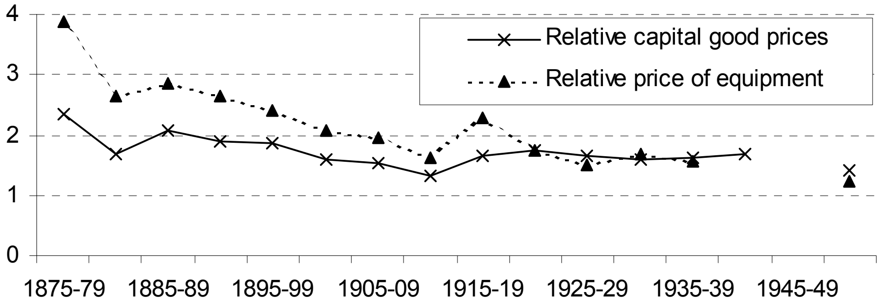

Figure 10: Relative Prices in Argentina (1935-38 = 100)