# Essays on Exchange Rates and Optimal Monetary Policy for Open Economies

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*To the memory of my father*

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# Declaration

I declare the following:

The material contained in this thesis is my own work.

The thesis has not been submitted for a degree at any other university.

**Abstract**

The thesis consists of three chapters of self-contained empirical and theoretical studies.

In Chapter 1, I examine whether the Balassa-Samuelson effect is indeed the reason behind the behaviour of the currencies of transition economies. So far, in the literature, transition Economies appear to be subject to the Balassa-Samuelson effect. This implies that their currencies experience a prolonged appreciation in real terms as their convergence goes on. However, in the current literature, the effects of the capital account have not been analyzed extensively. In this paper I show that the capital account, rather than productivity, is a key determinant of the appreciation of the currencies of transition economies. I find that a long-run relationship exists between the real exchange rate, productivity, the real interest rate differential and the capital account. Moreover, those variables are found to cointegrate in a nonlinear fashion according to a smooth transition autoregressive model. This implies that a multivariate smooth transition error correction model is the appropriate model to describe their short-run and long-run dynamics.

In Chapter 2, I examine the importance of a real exchange rate target in the monetary policy of a central bank. I address that question both empirically and theoretically. Using monthly data I estimate of a structural VAR model for the Eurozone providing evidence in favour of real exchange rate targeting. I examine this case theoretically using a two-country DSGE model; I find that when the home central bank includes a real exchange rate target in its interest rate rule, it achieves lower welfare losses compared to the Taylor rule. Contrary to similar papers, I compute the optimized coefficients in the interest rate rules considered. I show that the benefits from real exchange rate targeting at home rise as persistence in inflation and output increases. In the robustness analysis I show that a rise in the fraction of backward looking consumers affects negatively the performance of the real exchange rate targeting rule and positively that of the Taylor rule. Asymmetries in the degree of rule-of-thumb behavior in consumption have important effects, as regards the performance of a real exchange rate targeting rule. The performance of both rules is not sensitive to variations in the degree of backward looking price setting behavior .

In Chapter 3, I show, using both empirical and theoretical analysis, that changes in monetary policy in one country can have important effects on other economies. My new empirical evidence shows that changes in the monetary policy behaviour of the Fed since the start of the Euro, well captured by a Markov-switching Taylor rule, have had significant effects on the behaviour of inflation and output in the Eurozone even though ECB's monetary policy is found to be fairly stable. Using a two-country DSGE model, I examine this case theoretically; monetary policy in one of the countries (labelled foreign) switches regimes according to a Markov-switching process and this has non-negligible effects in the other (home) country. Switching by the foreign central bank renders commitment to a time invariant interest rate rule suboptimal for the home central bank. This is because home agents expectations change as foreign monetary policy

changes which affects the dynamics of home inflation and output. Optimal policy in the home country instead reacts to the regime of the foreign monetary policy and so implies a time-varying reaction of the home Central Bank. Following this time-varying optimal policy at home eliminates the effects in the home country of foreign regime shifts, and also reduces dramatically the effects in the foreign country. Therefore, changes in foreign monetary regimes should not be neglected in considering monetary policy at home.

## INTRODUCTION

In this thesis I focus on exchange rate modelling and the design of optimal monetary policy for open economies. In the first chapter I examine the behaviour of the real exchange rate in transition economies. In the second chapter I examine the importance of the real exchange rate in the conduct of monetary policy. In the third chapter I examine the international effects of regime switches in monetary policy.

In particular, in Chapter 1 I examine whether the Balassa-Samuelson effect is the main reason for the appreciation of the currencies in transition economies, or not. The Balassa-Samuelson effect, originally introduced by Balassa (1964) and Samuelson (1964) states that countries that experience high levels of productivity will have their currencies appreciated and their price levels higher. A vast empirical literature focusing either on transition or on industrialized economies supports this perspective. However, there is still a number of researchers that strongly reject it.

The industrial development of the countries under consideration is of great importance once the Balassa-Samuelson effect is tested. Countries that are close trade partners are unlikely to validate this effect. In particular Lothian and Taylor (2008) testing for productivity effects on the real exchange rate between the United States, the United Kingdom and France found that for the Sterling-dollar exchange rate there is evidence in favour of the Balassa-Samuelson effect. However, the sterling-franc real exchange is not subject to this effect as the United Kingdom and France appear to be close trade partners. Nevertheless, testing the validity of these effects in industrialized economies, one should be careful as any potential effect may either die out very quickly due to technological diffusion, or it may be the case that productivity explains a part but not the entire variation of the real exchange rate. Therefore, it becomes clear why in industrial countries only a small part of the total variation in the real exchange rates is explained by productivity fluctuations.

In this paper I test whether a long-run relationship exists among the real exchange rate, productivity, the real interest rate differential and the capital account. I find that productivity does not cause appreciation of the currency in all cases. However, the capital account causes an appreciation of the currency in almost all cases. More importantly, I show that whether the capital account will cause an appreciation or not depends on the composition of capital inflows. In particular, I find that the capital account causes an appreciation of the currency as long as foreign direct investment exceeds portfolio investment. When portfolio investment is the major element of the capital account, then the latter no longer causes an appreciation of the currency. Therefore, this paper suggests that the Balassa-Samuelson effect may not be the appropriate explanation of the behaviour of the currencies in transition economies. Rather, it is the kind of investment that a transition economy receives the driving force behind the appreciation of its currency.

I show that the real exchange, productivity, the real interest rate differential and the capital account share a common trend. Notably, I find that they are cointegrated in a nonlinear fashion. In particular, I show that the residuals from their long-run relationship are subject to nonlinearities. This implies that standard linear error correction models are no longer valid. Therefore, I make use of a nonlinear error correction model. The form of this model is determined by the type of nonlinearity that is found for the cointegrating residuals.

One of the weaknesses of many studies so far is the failure to capture the effect of the fundamentals, apart from productivity, on the the real exchange rate. Clarida and Taylor (2001) and Taylor and Sarno (2001) used decomposition techniques in order to deal with this problem. Instead, the approach used here tries to tackle this problem by introducing one of the fundamentals, in the long-run equation for the real exchange rate. For this reason I allow for the capital account to be one of the determinants of the real exchange rate. This approach is close to the fundamental equilibrium exchange rate approach (FEER), which was applied to transition economies by Amadkov et al. (2002) and Coudert and Couharde (2002). This approach focuses only on the current account effects. In particular, the divergence between the underlying current account and the medium-run current account determines whether the exchange rate has been appreciated or depreciated. However, a weakness of the FEER approach is that productivity fluctuations are not taken into account. Therefore, I move further the analysis by incorporating both effects (i.e. productivity and the capital account) in the determination of the real exchange rate.

The effect the capital account may have on the real exchange rate depends on many factors. If a country's capital inflows translate into higher domestic consumption then the country faces higher current account deficits, and its currency is likely to appreciate. In this case there will be a capital account surplus, further increasing the current account deficit. For transition economies, these capital inflows can enhance productivity, thereby strengthening further their currencies through the Balassa-Samuelson channel. However, any negative effects the inflows may have cannot be neglected. These can be inflationary pressures, higher current account deficits, rapid monetary expansion (Calvo et al., 1996, Agenor, 2004, Calvo, 2005).

The way the capital account is likely to affect the real exchange rate cannot be known in advance. By the balance of payments identity, the capital account is implicitly a proxy for the current account. The composition of the capital flows determines the way the capital account affects the real exchange rate. Long term capital flows are the foreign direct investment which fosters growth, as it is generally taken to be determined by long-term profitability considerations and often leads to the transfer of state-of-the-art technology (Agenor, 2004). As a result, this kind of flows are less subject to market sentiment. On the contrary, capital flows focusing on portfolio investment and short term bank lending (financial products) are a short-run investment and are subject to any kind of market volatility. Therefore, if the latter sort of investment is the key determinant of capital flows to a transition economy, then it is likely that productivity is less strengthened in this country compared to others. In this case, I expect the capital account to have an overall positive effect on the real exchange rate (i.e. depreciation), and as a result productivity will matter less in its fluctuations.

In Chapter 2, I examine, both empirically and theoretically, the ability of real exchange rate targeting in achieving lower inflation and output gap fluctuations. Using monthly data I estimate of a structural VAR model with short-run restrictions for the Eurozone. I find that when the ECB reacts contemporaneously to the real exchange rate it controls both inflation and the output gap better. I examine this case theoretically using a two-country DSGE model. I find that when the home central bank includes a real exchange rate target in its interest rate rule, it achieves a better control of inflation and output gap. I compare the performance of a rule with a real exchange rate target to that of the Taylor rule using a welfare criterion. The latter is derived from a second order approximation to the agents utility function as in Rotemberg and Woodford (1998). Contrary to similar papers, I compute the optimized coefficients in the interest rate rules considered. Real exchange rate targeting yields lower welfare losses. Notably, the benefits from real exchange rate targeting at home rise as persistence in inflation and output increases. In the robustness analysis I show that a rise in the fraction of backward looking consumers affects negatively the performance of the real exchange rate targeting rule and positively that of the Taylor rule. The performance of both rules, though, is not sensitive to variations in the degree of backward looking price setting behavior .

In an empirical exercise, Clarida, Gali and Gertler (1998), estimated simple interest rate rules for the G3 and E3 economies. Their estimates showed that the standard Taylor rule appears to be a good approximation to the policy rate of the central banks of the countries considered. In particular, the coefficients on the inflation and the output gap targets seemed to move close to Taylor's (1993) suggestion. Interest rate smoothing is proved to be statistically significant. Extending their analysis through adding more targets, they found that their coefficients were either very small, or statistically insignificant[1].

In a theoretical small open economy model, Svensson (2000) argues that additional targets could be included in an interest rate rule of an open economy. Those could include variables like the exchange rate (either real, or nominal), or even foreign variables. On the other hand, McCallum and Nelson (1999) argue against exchange rate targeting. Their conclusion is that the central bank should not react to exchange rate movements since the it reacts to it indirectly through its inflation target. In their model, however, McCallum and Nelson assume perfect exchange rate pass through. I assume that the pass through is imperfect. In our model, firms set one price for the Home country and one for the Foreign country for the good they produce. Weerapana (2000), on the other hand, argues in favor of an exchange rate target. Simulating a two country sticky price model, he finds that an exchange rate target yields lower welfare losses. However, Weerapana does not specify whether the central bank achieves a better control of inflation.

Exchange rate targeting is perfectly aligned with the effort of a central bank to keep the exchange rate within certain bands. In this case Benigno and Benigno (2001) show that a nominal exchange rate target is welfare improving. Their focus, though, is on different exchange rate regimes and how their choice affects welfare. My focus is different. I try to explore whether adding a real exchange rate target in a simple interest rate rule, allows the Central Bank to achieve a better control of inflation and, at the same time, lower welfare losses. Achieving lower welfare losses does not necessarily imply lower CPI or PPI inflation variation. The reason is that my two country

[1]The additional targets considered were the real exchange rate, lagged inflation rate, money supply and the federal funds rate.

model is very rich in its dynamics. As is shown, welfare is affected not only by the variances of PPI inflation, the output gap and the real exchange, but also by their covariances. Additionally, since I introduce endogenous output and inflation persistence, I show that welfare is affected by the lags of output and inflation.

Benigno and Benigno (2008) show that in exchange rate regimes where the Central Bank reacts either to the change in the nominal exchange rate or its level, it achieves a better control of the terms of trade, in terms of volatility. However, they focus only on shocks to the natural terms of trade without proceeding to an impulse response analysis in the face of monetary policy, real exchange rate and productivity shocks. Restricting the analysis in only one kind of shock may be misleading, since the choice of an exchange rate regime may not be sustainable in the face of alternative policy or real shocks. Additionally, they do not conduct a welfare analysis. Benigno and Benigno (2001) , however, proceed to the evaluation of the alternative interest rate rules based on a welfare criterion. Their welfare analysis shows that interest rate rules with a nominal exchange rate target perform worse than a standard Taylor rule. However, they do not perform a robustness exercise, in order to test whether the rule is robustly optimal. Their model does not include endogenous inflation and output persistence and asymmetry, between the two countries, is considered only in the coefficients of the interest rate rules. I consider alternative kinds of asymmetries, as far as the structural parameters are concerned. The reason, as I show, is that asymmetries have important implications for monetary policy.

Leitemo and Soderstrom (2001) find that the inclusion of a real exchange rate target into the Taylor rule gives only slight improvements in terms of volatility of the important variables in the economy. They consider a small open economy model with model uncertainty. Altering parameters in the model, and more importantly, increasing the degree of asymmetries has non-negligible effects in terms of welfare losses. Leitemo and Soderstrom, like Weerapana (2000) do not derive the welfare loss from the utility function of households, but they, rather, impose an ad hoc version of it. Moreover, in this paper the coefficients in the interest rate rule of the home country are not imposed exogenously, but rather they are the ones that minimize the welfare loss function. Under this approach I show that the differences in welfare losses can be very large.

In Chapter 3, I build on the same model as in Chapter 2 and I show, both empirically and theoretically, that changes in monetary policy in one country have important effects on other economies. In the empirical analysis, I find that the monetary policy of the US has changed since the start of the Euro. This change affected the dynamics of inflation and output in the Eurozone significantly. However, the monetary policy of the ECB is found to be fairly stable. In the theoretical analysis, I show that changes in the monetary policy of one country (labelled foreign) have non-negligible effects on the dynamics of the key macroeconomic variables in the other (home) country. This result is further enhanced as long as the home country does not take into account changes in foreign monetary policy. However, both economies benefit when the home central bank reacts optimally to foreign monetary policy regime shifts.

A popular way of modelling regime changes in monetary policy is by assuming that the interest rate rule coefficients change according to a Markov switching process. Using this approach Davig and Leeper (2007), Liu et al. (2008, 2009), Farmer et al. (2011) and Bekaert et al. (2011) construct closed economy DSGE models in order to analyze the effects of regime shifts in monetary policy on inflation and output.[2] These papers conclude that the expectation of a future regime shift in monetary policy has significant effects on inflation and output today. Those effects can be either stabilizing or destabilizing depending on what is the expected future policy.

The existing literature on Markov-switching DSGE models, though, is restricted to a closed economy framework. As a result, so far, the cross country effects of regime shifts in monetary policy have not been analyzed. Therefore, it is important that we have an open economy framework, so that to analyze the effects in one country of a change in monetary policy of another country.

The first contribution of the paper is to provide empirical evidence regarding the international effects of changes in monetary policy. I estimate a SVAR model for the US and the Eurozone using real time monthly data spanning from 1999 through 2010. The empirical model includes seven variables, namely inflation, output gap and the nominal interest rate for both the Eurozone and the US, as well as the real exchange rate. I perform parameter stability tests using the Andrews sup-Wald test, as in Boivin and Giannoni (2002) and the Andrews-Ploberger test.[3] Both tests find that there have been statistically significant changes in the coefficients in the US interest

[2]In all of these papers the theoretical analysis is motivated by the empirical estimates about the way monetary policy was conducted in the US from 1970 until recently.

[3]I use the Andrews-Ploberger test because of its virtue of identifying the break date.

rate equation. This implies that there has been a change in the systematic behaviour of the Fed. However, coefficients in the Eurozone interest rate equation are stable throughout the sample. The Andrews-Ploberger test identifies the break date in June 2004. Therefore, I split the sample into two sub-samples, namely before and after that date. The impulse response analysis shows that the responses of inflation and output gap in the Eurozone are completely different in the two samples.

But what drives the changes in the impulse responses of inflation and output in the Eurozone? In order to answer that question, I perform a countrefactual analysis in the VAR model. I find that the main reason for the change in the impulse responses of those variables was the change in the US monetary policy. I examine also whether changes in the conditions in the Euro area can account for that. I find that their contribution at causing changes in the impulse responses is tiny.

Given the weakness of the SVAR model in uncovering a Taylor rule, a last step in the empirical analysis is to explore whether there have been indeed changes in Fed's contemporaneous reaction to inflation and output gap fluctuations. For this reason I estimate a Taylor rule for the US whose coefficients change over time according to a Markov-switching process. The estimated rule findings validate that the monetary policy of the Fed has changed since the start of the Euro and are in line with the stability tests from the SVAR model. The rule changes state only once. Notably, the regime change date is very close to the break date identified by the Andrews-Ploberger test in the US interest rate equation. Keeping those findings in mind, I proceed to the construction of a two-country DSGE model.

The theoretical model is similar to that of Benigno and Benigno (2001) and Benigno (2004). I extend their approach by allowing the coefficients in the foreign interest rate rule only to change according to a Markov-switching process. The home country instead adopts a time-invariant Taylor rule with some interest rate smoothing. I show that even though the home monetary policy is constantly (and with a constant coefficient) hawkish[4], home inflation exhibits changes in its volatility over time. Specifically, if there is a positive probability that foreign monetary policy will be dovish[5] in the future, then not only foreign inflation will be more volatile, but also home inflation. This is because both home and foreign agents incorporate this probability in their future

[4]Throughout the paper hawkish refers to the case where the coefficient on inflation in the interest rate rule is greater than one. In the literature, this implies that the central bank cares a lot about inflation stabilization.

[5]Throughout the paper dovish refers to the case where the coefficient on inflation in the interest rate rule is less than one. In the literature, this implies that the central bank is more tolerant of inflation fluctuations.

inflation expectations.[6] The increase in the volatility of home inflation in this case comes from the home agents expectation of an increasing volatility in the real exchange rate and relative prices. Therefore, commitment to a regime independent interest rate rule proves not to be enough to stabilize the home economy.

Hence, as a next step, I examine the optimal policy of the home country. I solve the optimal policy problem of the home central bank conditional on foreign monetary policy switching regimes over time. I extend Soderlind's (1998) algorithm for solving optimal policy problems in linear rational expectations models to a Markov-switching framework. I show that a time invariant interest rate rule is suboptimal for the home country. The home central bank must be always hawkish. How much hawkish the home central bank should be, depends on the regime which the foreign monetary policy lies in. More specifically, I find that as the probability that the foreign central bank becomes dovish rises, the home central bank should increase the coefficient on inflation further. The opposite holds as the probability that the foreign central bank becomes hawkish increases. The intuition behind this result is that when home agents expect that foreign monetary policy will become dovish, they anticipate an increase in the volatility of home inflation. Hence, the home central bank must react in such a way so that to offset this effect on home agents expectations. And this, as I show, is achieved by increasing the coefficient on home inflation in the home interest rate rule. Additionally, the coefficient on output gap must increase as well, as the foreign monetary policy becomes dovish. This means that when the foreign country changes its policy, then the home must adjust (change) its policy appropriately. Regime switching monetary policy proves to be Pareto superior for the home country. More importantly, I show that when the home central bank reacts optimally to changes in foreign monetary policy, the effects of changes in the latter are eliminated in the home country, and reduced dramatically in the foreign.

[6]Throughout the paper I assume that the probability of a regime switch is the same for both home and foreign agents.

# Chapter 1: Asymmetries, productivity and capital account effects in the determination of the Real Exchange rate: The case of Transition Economies

Konstantinos Mavromatis

**Abstract**

Transition Economies appear to be subject to the Balassa-Samuelson effect in many studies. This implies that their currencies experience a prolonged appreciation in real terms as their convergence goes on. However, in the current literature, the effects of the capital account have not been analyzed extensively. In this paper I show that the capital account, rather than productivity, is a key determinant of the appreciation of the currencies of transition economies. I find that a long-run relationship exists between the real exchange rate, productivity, the real interest rate differential and the capital account. Moreover, those variables are found to cointegrate in a nonlinear fashion according to a smooth transition autoregressive model. This implies that a multivariate smooth transition error correction model is the appropriate model to describe their short-run and long-run dynamics.

Keywords: Balassa-Samuelson effect, capital account, nonlinear error correction

JEL Classification: E52, F42.

# 1 Introduction

The Balassa-Samuelson effect, originally introduced by Balassa (1964) and Samuelson (1964) states that countries that experience high levels of productivity will have their currencies appreciated and their price levels higher. A vast empirical literature focusing either on transition or on industrialized economies supports this perspective. However, there is still a number of researchers that strongly reject it.

The industrial development of the countries under consideration is of great importance once the Balassa-Samuelson effect is tested. Countries that are close trade partners are unlikely to validate this effect. In particular Lothian and Taylor (2008) testing for productivity effects on the real exchange rate between the United States, the United Kingdom and France found that for the Sterling-dollar exchange rate there is evidence in favour of the Balassa-Samuelson effect. However, the sterling-franc real exchange is not subject to this effect as the United Kingdom and France appear to be close trade partners. Nevertheless, testing the validity of these effects in industrialized economies, one should be careful as any potential effect may either die out very quickly due to technological diffusion, or it may be the case that productivity explains a part but not the entire variation of the real exchange rate. Therefore, it becomes clear why in industrial countries only a small part of the total variation in the real exchange rates is explained by productivity fluctuations.

The analysis of the productivity effects in transition economies has attracted much research as these economies were on their path towards their accession to the EU until recently. Taylor and Sarno (2001) analyzing a sample of nine Transition Economies, found that productivity explains a very significant part of the fluctuations in the real exchange rate. Additionally, Halpern and Wyplosz (1997) using the dollar wage and panel data techniques found significant evidence in favour of the Balassa-Samuelson effect.

In this paper I test whether a long-run relationship exists among the real exchange rate, productivity, the real interest rate differential and the capital account. I find that productivity does not cause appreciation of the currency in all cases. However, the capital account causes an appreciation of the currency in almost all cases. More importantly, I show that whether the capital account will cause an appreciation or not depends on the composition of capital inflows. In particular, I find that the capital account causes an appreciation of the currency as long as foreign direct investment exceeds portfolio investment. When portfolio investment is the major element of the capital account, then the latter no longer causes an appreciation of the currency. Therefore, this paper suggests that the Balassa-Samuelson effect may not be the appropriate explanation of the behaviour of the currencies in transition economies. Rather, it is the kind of investment that a transition economy receives the driving force behind the appreciation of its currency.

I show that the real exchange, produc5ivity, the real interest rate differential and the capital account share a common trend. Notably, I find that they are cointegrated in a nonlinear fashion. In particular, I show that the residuals from their long-run relationship are subject to nonlinearities. This implies that standard linear error correction models are no longer valid. Therefore, I make use of a nonlinear error correction model. The form of this model is determined by the type of nonlinearity that is found for the cointegrating residuals.

The econometric analysis of the exchange rates in transition economies can be hard due to statistical problems. A first problem is that the time period available is very short, as the procedure of transition for these countries started in the early to mid '90s, and it would be inappropriate to include observations when these economies were still planned economies. A second problem relates to the fact that since these economies were in transition, this implies that their economic variables approached a long term equilibrium schedule instead of fluctuating around it (Fernandez, Osbat and Schnatz, 2007). Apart from these problems one should take into account what is the right model to describe the behaviour of the real exchange rate. The PPP-Puzzle was the starting point towards rejecting typical models that failed to capture the way the real exchange rate adjusts. In particular, the PPP-Puzzle put the question about how can one reconcile the enormous short-term volatility of real exchange rates with the extremely slow rate at which shocks appear to die out (Rogoff, 1996). Until then, the existing models based on real shocks could not account for the short-term exchange rate volatility. After this problem was officially stated, a vast literature came out as far as the econometric modeling of the real exchange rate is concerned. A number of authors (Lothian and Taylor, 1997, Taylor and Sarno, 1998, Michael, Nobay and Peel, 1997, Bec, Salem and MacDonald, 2006, Obstfeld and Taylor, 1997, Sercu, Uppal and Van Hulle, 1995, Lothian and Taylor, 1996) suggested that the most appropriate way to deal with these problems was to model the real exchange rate in a nonlinear fashion. The focus turned on the effect transactions costs have on the adjustment of the exchange rate. Reestimating the half-lives and the response functions, they showed that there was increasing evidence of the nonlinear behaviour of the real exchange rate, as the speed of adjustment turned out to be higher, the larger the shock.

One of the weaknesses of many studies so far is the failure to capture the effect of the fundamentals, apart from productivity, on the the real exchange rate. Clarida and Taylor (2001) and Taylor and Sarno (2001) used decomposition techniques in order to deal with this problem. Instead, the approach used here tries to tackle this problem by introducing one of the fundamentals, in the long-run equation for the real exchange rate. For this reason I allow for the capital account to be one of the determinants of the real exchange rate. This approach is close to the fundamental equilibrium exchange rate approach (FEER), which was applied to transition economies by Amadkov et al. (2002) and Coudert and Couharde (2002). This approach focuses only on the current account effects. In particular, the divergence between the underlying current account and the medium-run current account determines whether the exchange rate has been appreciated or depreciated. However, a weakness of the FEER approach is that productivity fluctuations are not taken into account. Therefore, I move further the analysis by incorporating both effects (i.e. productivity and the capital account) in the determination of the real exchange rate.

The effect the capital account may have on the real exchange rate depends on many factors. If a country's capital inflows translate into higher domestic consumption then the country faces higher current account deficits, and its currency is likely to appreciate. In this case there will be a capital account surplus, further increasing the current account deficit. For transition economies, these capital inflows can enhance productivity, thereby strengthening further their currencies through the Balassa-Samuelson channel. However, any negative effects the inflows may have cannot be neglected. These can be inflationary pressures, higher current account deficits, rapid monetary expansion (Calvo et al., 1996, Agenor, 2004, Calvo, 2005).

The way the capital account is likely to affect the real exchange rate cannot be known in advance. By the balance of payments identity, the capital account is implicitly a proxy for the current account. The composition of the capital flows determines the way the capital account affects the real exchange rate. Long term capital flows are the foreign direct investment which fosters growth, as it is generally taken to be determined by long-term profitability considerations and often leads to the transfer of state-of-the-art technology (Agenor, 2004). As a result, this kind of flows are less subject to market sentiment. On the contrary, capital flows focusing on portfolio investment and short term bank lending (financial products) are a short-run investment and are subject to any kind of market volatility. Therefore, if the latter sort of investment is the key determinant of capital flows to a transition economy, then it is likely that productivity is less strengthened in this country compared to others. In this case, I expect the capital account to have an overall positive effect on the real exchange rate (i.e. depreciation), and as a result productivity will matter less in its fluctuations.

The paper is organized as follows. In section 2 I describe the environment in transition economies and construct an index that shows the composition of the capital account of each country. This index allows me to explore the key determinant of the capital account of each country. In section 3, I present the dataset. In section 4, I present the econometric strategy. In section 5, I show the empirical results. Section 6 concludes.

## 2 The environment in Transition Economies

Transition economies are characterized by high levels of productivity and, hence, their currencies are subject to the Balassa-Samuelson effect. In the literature, researchers assume that there are two sectors, that of tradables and that of the nontradables. The increase in productivity takes place in the tradables sector. As tradables are considered only goods that are exported. A rise in productivity causes a rise in wages in the tradables sector. As a result, as wages remain unchanged in the nontradables, there will be a rise in the price of nontradables. Assuming PPP holds in the tradables sector, this will cause a rise in the price level, and consequently an appreciation of the domestic currency. Taking the CPI as the price level to test for the Balassa-Samuelson effect is not always the appropriate measure for the actual effects of productivity. The reason is that the CPI for some economies may include services (which are nontradables) to a high proportion, biasing thus the conclusions. It may be the case that even highly industrialized economies have a CPI where services weigh highly. Therefore, other measures for the price levels may be used. I tackle this problem by using the producer price index, excluding construction.

Before the start of the procedure of their convergence, the currencies of these economies experience depreciation, which sometimes may be very abrupt. After their depreciation, the real exchange rate falls (i.e. appreciation). This is because of the fact that increased labour mobility is observed towards the more productive sectors, as inefficient production units shut down. Moreover, this appreciation of the real exchange rate is further supported by the inflows of direct investment. As productivity levels tend to converge to those of the industrialized countries, the rates of appreciation of the currencies seem to slow down. An example is the Slovenian Tolar. It experienced a very strong depreciation until 1994. After that period the currency was appreciating until 1998. From 1998 on the currency appears to be more stabilized. Apparently, this is because the country joined the common currency. As a result, the overshooting behaviour of the real exchange rate seems to well describe the behaviour of the currencies of economies in transition. The real exchange rate is modeled as follows:

$$q = e + p * - p \tag{1}$$

where $q$ denotes the real exchange rate, e the nominal exchange rate, defined as the domestic price of the foreign currency, $p*$ the foreign (German) price level and $p$ the domestic price level. Since, the purpose of this paper is to analyze the channels through which the real exchange rate is determined by productivity, the real interest rate differential and the capital account, I proceed further to the main economic relationship to be analyzed. Its form is given by the following equation:

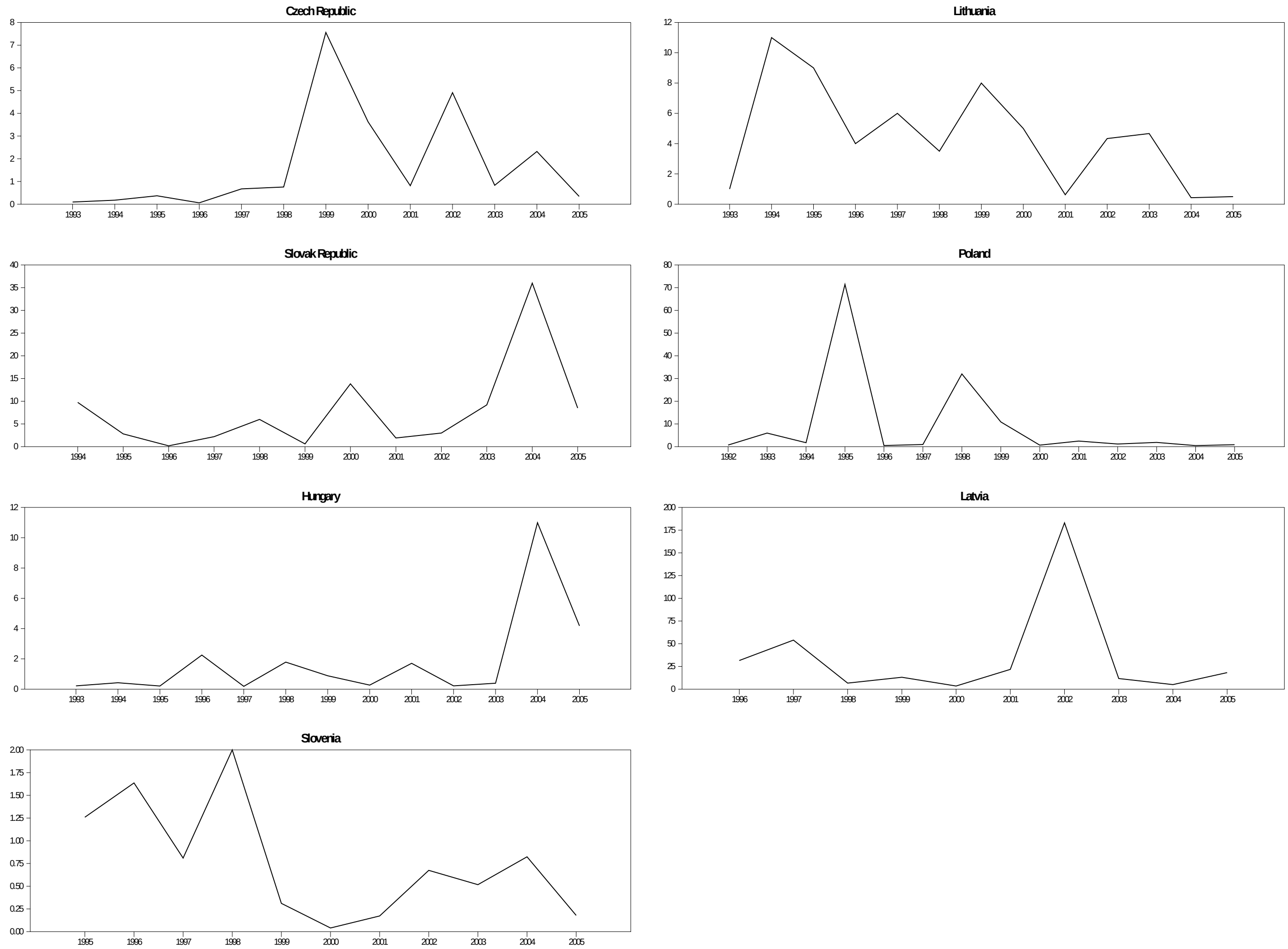
$$q_t = a_1 y_t + a_2 (r - r*)_t + a_3 CA_t \tag{2}$$

where $y_t$ denotes productivity, $(r - r*)_t$ denotes the real interest rate differential. r is the real interest rate and r* the foreign (German) interest rate and $CA_t$ denotes the capital account. One could expect the coefficient on productivity to be negative for the reasons explained above. However, this is not always the case, as I am showing later. The sign of the coefficient on the capital account, however, depends on the kind of investment each economy receives, as it will be shown below. Consequently, what matters is the composition of the flows of capital into the transition economies. In order to be able to determine the composition of the capital inflows, I create an index given by the ratio of foreign direct investment to portfolio investment in the reporting economy. Foreign direct investment fosters productivity, while portfolio investment does not. The index is the following:

$$g = \frac{FDI}{PI} \tag{3}$$

where $FDI$ denotes the foreign direct investment (inflows) and $PI$ the net portfolio investment (inflows). Transition economies have large current account deficits (capital account surpluses). As I show later countries with a high $g - ratio$ have their capital account surpluses with an overall (i.e. $a_3CA_t$) negative effect on the real exchange rate (i.e. appreciation), whereas countries with a low $g - ratio$ have their capital account surpluses with an overall positive effect on their real exchange rate (i.e. depreciation). The reason for this is that a rise in portfolio investment causes a rise in their capital account surplus, leaving productivity unaffected. As portfolio investment rises, the current account deficit is further increased (by the balance of payments identity). As a result, given the fact that for countries with a low $g - ratio$ a positive coefficient on the capital account is found, the overall effect (i.e. $a_3CA_t$) of the capital account on the exchange rate will be positive (i.e. depreciation). However, as foreign direct investment enhances productivity, it alleviates the negative effects caused by the rise in the capital account surplus. Since for the countries with a high $g - ratio$ a negative coefficient on the capital account is found, the overall effect (i.e. $a_3CA_t$) of the capital account on the real exchange rate will be negative (i.e. appreciation). The $g - ratio$ of each country is shown in figure 1 below. For all the countries the ratio is well above one in almost all periods of the sample. The only exception is Slovenia where the ratio is marginally above one initially, but then it fluctuates constantly below one. Finally, the $g - ratio$ for the Czech Republic is below one until 1998, but rises abruptly and fluctuates constantly above one from 1998 through 2005. Therefore, from the graphs it seems that all the countries but Slovenia had foreign direct investment being the major component of their capital accounts. Not surprisingly, as I show later, only for Slovenia the coefficient on the capital account in the long-run relationship is found to be positive.

**Figure 1:** $g - ratios$




# 3 The Data

Monthly data were gathered from the IMF, international financial statistics, the OECD, Main Economic Indicators, Eurostat. The end of period nominal exchange rate of the currency of each country against the German Mark is used. The 10 year government bond rate is used as the nominal interest rate, while producer price indexes are used as a proxy for the price levels. Industrial production, excluding construction, over the number of people employed in industry is used as a proxy for productivity. The countries for which data are collected are the Czech Republic, Lithuania, Slovak Republic, Hungary, Poland and Latvia. The sample spans form M10 1993 to

M12 2005 for the Czech Republic, M12 1993 to M12 2005 for Lithuania, M02 1994 to M12 2005 for Slovak Republic, M10 1992 to M12 2005 for Slovenia, M10 1993 to M12 2005 for Hungary, M03 1992 to M12 2005 for Poland and M02 1996 to M12 2005 for Latvia.

# 4 Econometric Modeling Strategy

## 4.1 Cointegration and the modeling of nonlinearities.

The first step of my empirical strategy is to test whether equation (2) constitutes a long-run relationship among the four variables considered. For this reason I apply the Johansen test for cointegration. The latter will show whether the variables have an error correction representation.

If cointegration is found, the next step is to test for nonlinearities in the cointegrating residuals. If this is case, then the cointegrating residuals will adjust in a nonlinear fashion following deviations from the long-run relationship (2). More importantly, the larger the deviations from equation (2), the faster the cointegrating residuals will adjust towards the long-run equilibrium.[1]

The procedure for the detection of nonlinearities in the cointegrating residuals $u_t$ is the one suggested by Terasvirta (1994). The null hypothesis of linearity is tested against the alternative of a stationary LSTAR or ESTAR model. Adjustment in the LSTAR model is asymmetric, whereas it is symmetric in the ESTAR. The two models receive the following form:

$$u_t = c_0 + c_1 u_{t-1} + (c_0^{'} + c_1^{'} u_{t-1})(1 - exp(-k_E(u_{t-d} - c_E)^2)) + e_{1t} \tag{4}$$

$$u_t = c_0 + c_1 u_{t-1} + (c_0^{'} + c_1^{'} u_{t-1})(1 + exp(-k_L(u_{t-d} - c_L)))^{-1} + e_{2t} \tag{5}$$

where $(1 + exp(-k_L(u_{t-d})))^{-1}$ and $(1 - exp(-k_E(u_{t-d})^2))$ is the transition function in the LSTAR and the ESTAR model respectively. It captures the nonlinear behaviour of the cointegrating residuals and describes the nonlinear adjustment. The economic interpretation, however, of the transition function is very important and different depending on whether it is an exponential or a logistic one. The exponential transition function describes symmetric behaviour either around

[1] Taylor and Sarno (2001) and Lothian and Taylor (2008) testing for productivity effects on the real exchange rate in transition economies showed that the adjustment is nonlinear and symmetric.

a long-run equilibrium or outside the thresholds. When it receives a zero value, then it depicts a middle regime, where the series lies within the thresholds and is well described by a nonstationary autoregressive model. When the threshold variable is different than the threshold, then the transition function takes values different from zero, and the model receives a nonlinear form.

The logistic transition function describes an asymmetric behavior. The transition function, in this case, takes values close to zero when the transition variable is close enough to the threshold . However, when the transition function receives values close to one when the transition variable lies far away from the threshold. In this paper the threshold variable is the cointegrating residuals.

The transition parameters $k_E$ and $k_L$, respectively, describe the speed at which each of the cointegrating residuals switch between the two regimes.

A reparameterization of models (4) and (5) yields the following

$$\Delta u_t = w_0 + w_1 u_{t-1} + (w_0^{'} + w_1^{'} u_{t-1})(1 - exp(-k_E(u_{t-d} - c_E)^2)) + e_{2t} \tag{6}$$

$$\Delta u_t = w_0 + w_1 u_{t-1} + (w_0^{'} + w_1^{'} u_{t-1})(1 + exp(-k_L(u_{t-d} - c_L)))^{-1} + e_{2t} \tag{7}$$

The above reparameterization is used when testing the power of the unit root tests when the true data generating process is the ESTAR or the LSTAR model.

### 4.2 Testing for nonlinearities

Testing for linearity against the alternative of either an ESTAR or LSTAR stationary model could be a test of $k_E = 0$ or $k_L = 0$. However, as noted by Davies (1987), Hansen(1996), Hansen(1997), Luukonen,Saikkonen and Terasvirta(1988) and Terasvirta(1994), such a test has very low power. The reason is that the problem of nonidentifiability arises. In particular, such a test would imply that the parameters $b_0^{'}$ , $b_1^{'}$ and $\sum_{j=1}^{q-1} b_j^{'}$ could not be identified under the null. Similarly, a joint test of statistical significance of the parameters mentioned above would mean that the transition parameter $k_L$ would not be identified either. Researchers, however, have tackled this problem by suggesting Taylor approximations of the transition function.[2] The third order Taylor approximation of the transition function of the LSTAR model ($F = (1 + exp(-k_L(u_{t-d} - c_L)))^{-1}$) around zero is given as follows:

$$T_3 = g_1(z) + g_3(z) \tag{8}$$

where $z = -k_L(u_{t-d} - c_L)$, $g_1(z) = \frac{\vartheta F}{\vartheta z_{|z=0}}$ and $g_3(z) = (1/6)\frac{\vartheta^3 F}{\vartheta z^3_{|z=0}}$ , as in Terasvirta (1994). I then substitute equation (8) into the LSTAR model (7), and end up to the following model:

$$u_t = b_0 + b_1 u_{t-1} + (b_0^{'} + b_1^{'} u_{t-1})T_3 + e_{2t} \tag{9}$$

In order to derive an LM type test for linearity, as in Terasvirta (1994), we conclude to the following auxiliary regression:

$$\hat{e}_{2t} = \beta_1 z_{1t}^{'} + \beta_2 u_{t-1} u_{t-d} + \beta_3 u_{t-1} u_{t-d}^2 + \beta_4 u_{t-1} u_{t-d}^3 \tag{10}$$

where $\beta_1$ is a $(p+1) \times 1$ vector of coefficients, $z_{1t} = (1, u_t, \ldots, u_{t-p})$ .

The test for linearity against nonlinearity consists of a test described below:

$$H_L : \beta_2 = \beta_3 = \beta_4 = 0$$

Apart from its LM version, the above test may performed as an F test. After rejecting the null hypothesis at the above test, one can move further towards testing whether an ESTAR or an LSTAR is the appropriate model. To carry out such a procedure, one must go through a sequence of tests described below:

[2]A third order Taylor expansion seems to be the best approximation since the transition function of the LSTAR model has a single inflection point and the third order Taylor expansion has a single inflection point itself (Escribano and Jorda, 2001). On the other hand the transition function of an ESTAR model has two inflection points. As a result, a higher order Taylor approximation is needed to best fit the transition function. As I am showing later, the LSTAR model was found to be the correct one for all countries but the Czech Republic. Hence, I present here the Taylor approximation of the logistic transition function only.

$$H_{01} : \beta_4 = 0$$

$$H_{02} : \beta_3 = 0 \mid \beta_4 = 0$$

$$H_{03} : \beta_2 = 0 \mid \beta_4 = \beta_3 = 0$$

The above sequence of tests can be performed as an LM test. However, the$LM$ test can be also performed using its $F$ version. The true significance level of the test may then be reasonably close to its nominal value, whereas the power may often be higher than that of the asymptotic $\chi^2$ test (Terasvirta, 1994 and Harvey, 1990). The delay parameter $d$ for each country was derived according to the procedure suggested by Terasvirta (1994). That is, the linearity test was carried out for 16 values of the delay parameter ($d = 1, ..., 16$). The value of $d$, which is finally chosen, is that corresponding to the linearity test with the lowest $p - value$. I then proceeded by performing the sequence of tests described above ($H_{01}, H_{02}, H_{03}$) for the parameters of equation (9). According to the procedure suggested by Terasvirta, the ESTAR model is the appropriate one, if and only if the $p - value$ from the $H_{02}$ test is the lowest among the three tests.

### 4.3 Multivariate Smooth Transition Modeling

Given a nonlinear behaviour in the cointegrating errors, a VECM is no longer valid to model the joint dynamics of the variables. For this reason a nonlinear version of the VECM is a better approximation. In this paper I will make use of a multivariate smooth transition error correction model.[3] For simplicity I will assume that the variables under consideration share a common nonlinearity. The latter is determined by the appropriate model (e.g. ESTAR or LSTAR) to be specified for the cointegrating residuals, which will be the transition variable.

[3]A significant empirical work has been done the last ten years macroeconometrics using multivariate smooth transition models. Camacho (2004), Van Dijk (1999) and Anderson and Vahid (1998) used MSTR models to model business cycles, arbitrage activity and stock price dynamics. They also performed Monte Carlo experiments for the size and the power of the linearity tests in a multivariate setting. Kavkler et al. (2007) using smooth transition vector error-correction models for the real exchange rate tried to test PPP for the Slovenian Tolar and the Czech Kruna against the Franch franc, the German Mark and the Italian Lira.

#### 4.3.1 Smooth Transition Equilibrium Correction Models

The equilibrium correction model receives the following form:

$$\Delta x_t = B_0 + B_i(L)\Delta x_{t-p} + \pi_{1,1}u_{t-1} + (\Gamma_0 + \Gamma_i(L)\Delta x_{t-p} + \pi_{2,1}u_{t-1})G(u_{t-d}; k, c) + \omega_{it} \tag{11}$$

where $B_i(L) = B_{i,1} + \ldots + B_{i,p}L^{p-1}$ and $\Gamma_i(L) = \Gamma_{i,1} + \ldots + \Gamma_{i,p}L^{p-1} \quad i = 1, 2, 3, 4$ is a $(4 \times (p-1))$ matrix, $\omega_{it}$ is a $(4 \times 1)$ vector of equation specific errors and $\Delta x = (\Delta f_t, \Delta y_t, \Delta(r - r*)_t, \Delta CA_t)$. $G(u_{t-d}; k, c)$ is the transition function. Its form, exponential or logistic, is determined by the type of nonlinearity that will be found for the cointegrating residuals.[4]

Van Dijk (1999) specifies a more parsimonious nonlinear error correction model, where the differenced lags of the variables do not enter the model as regressors. However, in our specification we allow for those lags into the regression, since they may capture the short run dynamics of the the dependent variable, especially during very small deviations from the long run equilibrium.

#### 4.3.2 System and equation specific tests

I perform system and equation specific linearity tests. This allows me to explore whether the nonlinear error correction model is indeed a correct specification. Moreover, linearity tests will show whether each variable in the model reacts nonlinearly to deviations from the long-run relationship (2). In this case the speed of adjustment is higher, the larger the deviations from the long-run equilibrium. The auxiliary model receives the following form:

$$\Delta x_t = A_{0,0} + A_0(L)\Delta x_{t-p} + A_1(L)\Delta x_{t-p}u_{t-d} + A_2(L)\Delta x_{t-p}u_{t-d}^2 + A_3(L)\Delta x_{t-p}u_{t-d}^3 + \eta_t \tag{12}$$

where $A_i(L) = A_{i,1} + \ldots + A_{i,p}L^{p-1}, \quad i = 0, 1, 2, 3,$ is a $(4 \times (p-1))$ matrix and $\eta_t$ is a combined error consisting of the errors from the initial smooth transition error correction model and the

[4] The number of lags in this model is determined according to AIC and BIC information criteria. Note, however, that it is important to keep the model as parsimonious as possible, given the difficulty of achieving convergence.

errors from the Taylor expansion. The normality assumption is needed if the specification tests are derived as Lagrange Multiplier tests. The test for linearity has a null where the coefficients of the nonlinear elements are equal to zero, that is $H_0: \; A_i = 0, \quad i = 1, 2, 3$. The LM statistic will have an asymptotic $\chi^2$ distribution with $12p$ degrees of freedom.[5] I denote the system linearity test as $LM_0'$. However, as Kavkler et. al (2007) observes, the system linearity test rejects the null of linearity when in at least one auxiliary regression linearity is rejected. Therefore, the system linearity test may be misleading. They suggest that it is better if one looks at the individual regression LM test to draw secure results. As a result, it may be the case that the system linearity test rejects the null more frequently than the nominal significance level. This means that a greater power of the system linearity test may be due to the fact that its size exceeds the nominal one, and, thus, the power is distorted.

The residuals of the multivariate smooth transition model are also be subjected to some diagnostic tests, so that to be able to evaluate the estimated model. Eitrheim and Terasvirta (1996) provided tests for no serial correlation[6], parameter constancy and no remaining nonlinearity for univariate models. However, these tests can be easily generalized into a multivariate setting as in Anderson and Vahid (1998). Tsay (1996) derived a test for neglected nonlinearities in univariate nonlinear time series models, which also can be easily generalized into a multivariate framework.

## 5 Empirical Results

As a first step, stationarity tests[7] were performed for each of the four variables, namely the real exchange rate $q_t$ the real interest rate differential $(r_t - r_t^*)$, the proxy for productivity $y_t$ and the capital account $CA_t$. The results suggest that all four variables are integrated of the same order and, hence, I can proceed to test for the existence of a common trend among them. The

[5] Van Dijk (1999) considering the general case with $k$ variables notes that the $LM$ statistic will have an asymptotic $\chi^2$ distribution with $3pk^2$ degrees of freedom.

[6] In particular, an LM test was perfrormed in an autoregressive series of residuals given by:

$$\eta_t = J(L)\eta_t + \xi_t, \quad \xi_t \sim N[0, \Omega] \tag{13}$$

where $\eta_t = (\eta_{dft}, \eta_{dyt}, \eta_{d(i-i*)t}, \eta_{dCAt})$, $J(L) = (J_1 L + \cdots + J_p L)$ is a $(4 \times 4)$ matrix polynomial in the lag operator $L$ and $\xi_t$ is serially uncorrelated. The test for serial correlation in the residuals is a test of $H_0: \; J_1 = \cdots = J_p = 0$, and can be performed as an LM test.

[7] The results for the unit root tests are presented in appendix B.

cointegration results are illustrated at table 1.

Table 1: Cointegration Results

Panel A: Cointegration Test Statistics

| | D-F test | P-P test | Johansen:λmax(λtrace) (r= 0 vs r=1) | Johansen:λmax (r=1 vs r=2) | KPSS |
|---|---|---|---|---|---|
| Czech Republic | -1.89476 | -2.19917 | 41.9160* (53.33) | 13.9422 | 0.193 |
| Lithuania | -1.28379 | -4.02824* | 33.418** (86.22) | 30.3204* | 0.659 |
| Slovak Republic | -2.80348 | -6.11011* | 69.254* (114.02) | 17.7785 | 0.138 |
| Slovenia | -0.47987 | -0.52102 | 53.9523* (80.05) | 11.4872 | 0.508 |
| Hungary | -2.21785 | -2.57816 | 79.897* (110.18) | 4.8731 | 0.279 |
| Poland | -2.51272 | -5.19596* | 53.8157* (102.3) | 9.6525 | 0.258 |
| Latvia | -2.69280*** | -3.39467** | 29.41*** (56.02) | 15.3050 | 0.079 |

Panel B: Estimated cointegrating vectors

| | $y_t$ | $(r - r*)_t$ | $CA_t$ |
|---|---|---|---|
| Czech Republic | -1.963 | -0.137 | -0.079 |
| Lithuania | 0.651 | 0.407 | -0.016 |
| Slovak Republic | 1.561 | 0.169 | -1.190 |
| Slovenia | 0.123 | - | 0.017 |
| Hungary | 1.637 | 0.119 | -0.890 |
| Poland | -0.516 | -0.169 | -0.066 |
| Latvia | -0.726 | 0.237 | -0.329 |

Notes:Panel A:***Significant at 10%. **Significant at 5%. *Significant at 1%. For the Johansen test with an intercept in the ECM, the critical values for the $\lambda_{max}$ test statistics, for $H_0: \ r = 0 \, vs \, r = 1$, are 25.52 for the 1%, 20.97 for the 5% and 18.6 for the 10% significance level respectively. For the $\lambda_{trace}$ the 1% critical value is 35.65, the 5% critical value is 29.68 and the 10% critical value is 26.79 respectively (Johansen and Juselius, 1990). For the KPSS test the 10 %, 5% and 1% critical values are 0.347, 0.463 and 0.739 respectively. Panel B:Maximum Likelihood estimates. The cointegrating vectors are derived from the from the Johansen procedudure and normalized in order to receive the parameter values corresponding to equation (2).

The coefficient on the proxy for productivity $y_t$ is found to be negative only for the Czech Republic, Poland and Latvia. Assuming that this is a good approximation to productivity, this implies that currencies of those three countries are subject to the Balassa-Samuelson effect. On the other hand the coefficient on productivity is positive for Lithuania, the Slovak Republic, Slovenia and Hungary. Therefore, the Balassa-Samuleson effect does not seem to be present in these countries.

As far as the effect of the capital account is concerned, its coefficient is negative for all countries, but Slovenia. This is the country with the lowest $g - ratio$. The reason why the $g - ratio$ is low for Slovenia may be the fact that its capital market was getting more integrated than that of the other countries over that period. For the Czech Republic, Poland and Latvia a negative coefficient on the capital account, accompanied with a capital account surplus generates a negative total effect (i.e. $a_3CA_t$) on the real exchange rate (i.e. appreciation). Therefore, as real exchange rate appreciation is the equilibrium behaviour of the exchange rate of these economies, it follows that the currencies of the economies where the coefficient on the capital account is negative, will have two sources of appreciation (i.e. productivity and the current account deficit or the capital account surplus). On the other hand, since the coefficient on the capital account is positive for Slovenia, this implies that the total effect of the capital account surplus (i.e. $a_3CA_t$) will be positive (i.e. depreciation).

The real interest rate differential has a positive effect for all countries considered, but the Czech Republic, Poland and Latvia. As Slovenia was about to join the Eurozone in December 2006, its capital market was getting more integrated. The ratio of the domestic to the German real interest rate for Slovenia was almost unity during the two year period before the country's accession to the Eurozone. In the period before 2004, the Slovenian real interest rate ratio was greater than one, and hence, the capital account surplus was rising faster.

As a next step I proceed to test for potential STAR nonlinearity in the cointegrating residuals, following the procedure described in section 4.2. The results of the tests and the delay parameters for each country are presented at table 2 below. The LSTAR model was found to be the appropriate one for all the countries but the Czech Republic, where an ESTAR model was found as the appropriate. Therefore, for those countries where the LSTAR was found to be the correct way to model nonlinearity, an asymmetric adjustment towards the long-run relationship is implied. Following Terasvirta Rule, the null of linearity was rejected in all cases. An ESTAR or an LSTAR model for the cointegrating residuals for each country was then estimated. The results are presented at table 3.

In the estimated STAR models the standard errors and the t-ratios should be treated with caution, especially when one looks at the transition parameters $k_E$ and $k_L$. If the significance of the transition parameter is tested, new standard errors must be computed. Under the null the autoregressive parameter in the linear part of the STAR model (i.e. $b_1^{'}$) is not identified, and the series follows a random walk (or a near-random walk), since it lies within the middle regime. Therefore, new standard errors were computed for the transition parameter through Monte-Carlo simulation. 10000 samples of $T$ observations[8] were generated, assuming that the true data generating process is the ESTAR or the LSTAR model, depending on which of the two was found to be the appropriate from the sequence of tests.

The number of lags in the STAR model could be based upon the standard AIC or BIC criteria. However, since nonlinearities exist, it is better to look at the partial autocorrelation functions. In particular, Granger and Terasvirta (1993) and Terasvirta (1994) suggest the PACF as a way of choosing the order of autocorrelation, $p$, than an information criterion. A general to specific procedure was followed, in addition to PACF. I keep on excluding lags until the lag where the Ljung-Box Q-test could no longer reject the null of no autocorrelation in the residuals.[9] In almost none of the cases could I find autocorrelation beyond the first lag.[10]

[8] $T$ is the number of observations in each sample.
[9] The PACF of the residuals in the LSTAR model for the cointegrating residuals are shown at Figure 4 in the appendix.
[10] However, one should question the power of the Ljung-Box test in the case of nonlinearities.

Table 2: Linearity LM tests

| | $H_L$(d) | $H_{01}$ | $H_{02}$ | $H_{03}$ |
|---|---|---|---|---|
| Czech Republic | 0.0246 ($\hat{d} = 12$) | 0.3730 | 0.0045 | 0.2781 |
| Lithuania | 0.017($\hat{d} = 16$) | 0.6928 | 0.1429 | 0.0048 |
| Slovak Republic | 0.0000 ($\hat{d} = 1$) | 0.0980 | 0.0122 | 0.0000 |
| Slovenia | 0.0000 ($\hat{d} = 3$) | 0.0000 | 0.2195 | 0.0351 |
| Hungary | 0.0413 ($\hat{d} = 9$) | 0.0199 | 0.2391 | 0.2365 |
| Poland | 0.0003 ($\hat{d} = 1$) | 0.8947 | 0.7105 | 0.0000 |
| Latvia | 0.0804 ($\hat{d} = 12$) | 0.0295 | 0.2071 | 0.5421 |

Notes: P-values reported. Numbers in parentheses refer to the delay parameter.

The power of the Dickey fuller test is computed. I perform a number of Monte-Carlo experiments on an artificial data generating process identical to both an autoregressive and a STAR model, with independent and identically distributed Gaussian innovations. The artificial series were initialized at zero. 10000 samples of 100 + T observations were generated where the first 100 observations were discarded, leaving 10000 samples of T observations.[11] The Augmented Dickey-Fuller statistic was calculated in each of the 10000 samples and rejection frequencies over the 10000 samples were computed taking the five percent significant values calculated by MacKinnon (1991).[12] At table 4 I show the power of the ADF for each country.

Finally, in order to test for global stationarity in the estimated STAR models for the cointegrating residuals, I performed the Kapetanios et al. (2003) test (from now on KSS test) for global stationarity. The test results show that the nonlinear models for the cointegrated residuals are all globally stationary. The null hypothesis is rejected in all cases against the alternative of global stationarity. The results from the KSS are shown in Appendix B.

[11] T is the number of observations that applies to each country

[12] In order for the results to be comparable across countries in all simulations the autoregressive parameter was set to be arbitrarily equal to 0.80.

Table 3: Estimated STAR models for the cointegrating residuals

| | $c_0$ | $c_1$ | $c_0^{'}$ | $c_1^{'}$ | $c_E, c_L$ | $k_E, k_L$ | $R^2$ | $LM$ | $TS$ |
|---|---|---|---|---|---|---|---|---|---|
| Czech | - | 0.722 | - | -0.049 | 0.007 | 169.33 | 0.842 | {0.887} | {0.738} |
| | | (0.000) | | (0.099) | (0.000) | [0.037] | | | |
| Lithuania | - | 0.145 | 0.034 | 0.854 | -0.137 | 6.587 | 0.466 | {0.559} | {0.104} |
| | | (0.018) | (0.015) | (0.003) | (0.000) | [0.000] | | | |
| Slovak | 0.011 | 0.917 | - | -0.800 | - | 1.970 | 0.244 | {0.282} | {0.930} |
| | (0.007) | (0.000) | | (0.000) | | [0.000] | | | |
| Slovenia | - | 0.834 | -0.079 | 2.171 | - | 2.211 | 0.688 | {0.744} | {0.271} |
| | | (0.000) | (0.000) | (0.006) | (0.005) | [0.034] | | | |
| Hungary | - | 1.079 | - | -0.484 | - | 2.426 | 0.3608 | {0.821} | {0.326} |
| | | (0.000) | | (0.001) | | [0.000] | | | |
| Poland | - | 0.812 | - | - | - | 0.615 | 0.3740 | {0.951} | {0.422} |
| | | (0.000) | | | | [0.000] | | | |
| Latvia | - | 0.810 | - | 0.017 | -0.041 | 33.391 | 0.6023 | {0.136} | {0.496} |
| | | (0.000) | | (0.005) | (0.000) | [0.000] | | | |

Notes: Standard errors in parentheses. Numbers in square brackets below transition parameters estimates show the marginal significance levels. LM is the test for autoregressive conditional heteroskedasticity in the residuals. TS is the test for remaining nonlinearities. For the LM and the TS tests only $p - values$ are reported.

Table 4: Power of the Dickey-Fuller test

| | (AR true process) | (STAR true process) |
|---|---|---|
| Czech | 99.48 | 49.71 |
| Lithuania | 89.56 | 50.33 |
| Slovak | 88.96 | 61.52 |
| Slovenia | 89.65 | 49.43 |
| Hungary | 99.47 | 52.11 |
| Poland | 89.35 | 14.62 |
| Latvia | 96.78 | 13.88 |

*Estimation of MSTEqC models*

Specification in the multivariate smooth transition equilibrium correction models is conducted in the same manner as in section 4.3. However, what makes the analysis easy is that I do not need to do any testing procedure in order to find out which is the appropriate transition variable, since the specification I use makes such a sequence of tests unnecessary[13]. The estimation of the nonlinear error correction models shows that the four variables considered here adjust to any deviations from the long-run equilibrium in a nonlinear fashion.

The specification tests for the multivariate smooth transition equilibrium correction model were implemented as described in section 4.3.1. Additionally, a neglected nonlinearities test, as suggested by Tsay (1996), was performed in each equation of the system individually. According to the test there is not remaining nonlinearity in the system. The $LM$ test for autoregressive conditional heteroskedasticity can be performed in the same way as in the linear case. Individual $LM$ tests were performed in the residuals from each equation. The number of lags included in each test was determined according to the partial autocorrelations in the equation specific residuals. According to the test, in all cases conditional heteroskedasticity is rejected in the residuals.[14] The equation specific residuals do not exhibit significant autocorrelation except from those for the regression with the differenced productivity as the dependent variable, which appear to be highly autocorrelated for almost all the countries. However, I did not include more lags in the system to account for that in order to keep the model parsimonious. The estimation results as well as the results from the $LM$ test for autoregressive conditional heteroskedasticity, residual autocorrelation and neglected nonlinearities are shown in appendix C and D.

## 6 Conclusion

In this paper I show that the Balassa-Samuelson effect cannot describe the behaviour of the real exchange rate for all economies in transition. Using an appropriate proxy for productivity I show that its effect on the real exchange rate is negative (i.e. causes appreciation) for only three countries, namely, the Czech Republic, Poland and Latvia. However, the effect of the capital account is negative (i.e. causes appreciation) for all countries but Slovenia. I show that this is

[13]However, one could do that sequence of tests in the MSTEqM as a way of testing whether this model is the correct one to model the variables. On the other hand, one could go short by just performing a common nonlinearities test in model (11) as described by Anderson and Vahid (1998).

[14]Note that the test for linearity can be robust to both autoregressive conditional heteroskedasticity and unspecified heteroskedasticity, by adjusting the $LM$ statistic so that to account for that.

mainly caused by the composition of the capital account. When foreign direct investment exceeds portfolio investment, for a long period of time, the effect of the capital account is negative. The opposite holds when portfolio investment exceeds foreign direct investment, as was the case for Slovenia. Therefore, long-run investment seems to have caused the appreciation of those currencies, rather than the Balassa-Samuelson effect itself.

I test for cointegration among the real exchange rate, the real interest rate differential, the proxy for productivity and the capital account. I show that they cointegrate in a nonlinear fashion. This implies that a linear vector error correction model is no longer valid, as it implies a linear adjustment for each variable following deviation from the long-run equilibrium. For this reason I use a nonlinear multivariate error correction model, whose nonlinearity is determined by that found for the cointegrated residuals. For simplicity, I assume that the four variables under consideration share a common nonlinearity. The threshold variable is the cointegrated residuals. Adjustment towards the long-run equilibrium happens in both a linear and a nonlinear fashion.

Finally, specification tests in the multivariate error correction model show that this is a correct specification to capture the dynamics of the variables considered.

# Chapter 2: Rule-of-thumb behavior and Real Exchange Rate targeting

Konstantinos Mavromatis

**Abstract**

How important is for the central bank to have a real exchange rate target? In this paper I address that question both empirically and theoretically. Using monthly data I estimate of a structural VAR model for the Eurozone providing evidence in favour of real exchange rate targeting. I examine this case theoretically using a two-country DSGE model; I find that when the home central bank includes a real exchange rate target in its interest rate rule, it achieves lower welfare losses compared to the Taylor rule. Contrary to similar papers, I compute the optimized coefficients in the interest rate rules considered. I show that the benefits from real exchange rate targeting at home rise as persistence in inflation and output increases. In the robustness analysis I show that a rise in the fraction of backward looking consumers affects negatively the performance of the real exchange rate targeting rule and positively that of the Taylor rule. Asymmetries in the degree of rule-of-thumb behavior in consumption have important effects, as regards the performance of a real exchange rate targeting rule. The performance of both rules is not sensitive to variations in the degree of backward looking price setting behavior .

Keywords: Taylor rule, real exchange rate targeting, asymmetries, output and inflation endogenous persistence, optimal monetary policy

JEL Classification: E52, F41, F42.

# 1 Introduction

Research on monetary policy analysis over the last years has focused on whether simple monetary policy rules keep their properties once put into an international framework, or not. Svensson (2000), Weerapana (2000), Benigno (2004) and Benigno and Benigno (2001) have analyzed extensively the properties of simple interest rate rules in both small and large open economy models. The general conclusion so far is that interest rate rules that are optimal for closed economy are not necessarily optimal for open economy models at the same time. The reason is that an open economy model is richer in its dynamics. This implies that the central bank needs to adjust its instruments in such a way so that to minimize as much as possible the welfare losses, in the face of various exogenous shocks.

In this paper I examine, both empirically and theoretically, the ability of real exchange rate targeting in achieving lower inflation and output gap fluctuations. Using monthly data I estimate of a structural VAR model with short-run restrictions for the Eurozone. I find that when the ECB reacts contemporaneously to the real exchange rate it controls both inflation and the output gap better. I examine this case theoretically using a two-country DSGE model. I find that when the home central bank includes a real exchange rate target in its interest rate rule, it achieves a better control of inflation and output gap. I compare the performance of a rule with a real exchange rate target to that of the Taylor rule using a welfare criterion. The latter is derived from a second order approximation to the agents utility function as in Rotemberg and Woodford (1998). Contrary to similar papers, I compute the optimized coefficients in the interest rate rules considered. Real exchange rate targeting yields lower welfare losses. Notably, the benefits from real exchange rate targeting at home rise as persistence in inflation and output increases. In the robustness analysis I show that a rise in the fraction of backward looking consumers affects negatively the performance of the real exchange rate targeting rule and positively that of the Taylor rule. The performance of both rules, though, is not sensitive to variations in the degree of backward looking price setting behavior .

In an empirical exercise, Clarida, Gali and Gertler (1998), estimated simple interest rate rules for the G3 and E3 economies. Their estimates showed that the standard Taylor rule appears to be a good approximation to the policy rate of the central banks of the countries considered. In particular, the coefficients on the inflation and the output gap targets seemed to move close to Taylor's (1993) suggestion. Interest rate smoothing is proved to be statistically significant. Extending their analysis through adding more targets, they found that their coefficients were either very small, or statistically insignificant[1].

In a theoretical small open economy model, Svensson (2000) argues that additional targets could be included in an interest rate rule of an open economy. Those could include variables like the exchange rate (either real, or nominal), or even foreign variables. On the other hand, McCallum and Nelson (1999) argue against exchange rate targeting. Their conclusion is that the central bank should not react to exchange rate movements since the it reacts to it indirectly through its inflation target. In their model, however, McCallum and Nelson assume perfect exchange rate pass through. I assume that the pass through is imperfect. In our model, firms set one price for the Home country and one for the Foreign country for the good they produce. Weerapana (2000), on the other hand, argues in favor of an exchange rate target. Simulating a two country sticky price model, he finds that an exchange rate target yields lower welfare losses. However, Weerapana does not specify whether the central bank achieves a better control of inflation.

Exchange rate targeting is perfectly aligned with the effort of a central bank to keep the exchange rate within certain bands. In this case Benigno and Benigno (2001) show that a nominal exchange rate target is welfare improving. Their focus, though, is on different exchange rate regimes and how their choice affects welfare. My focus is different. I try to explore whether adding a real exchange rate target in a simple interest rate rule, allows the Central Bank to achieve a better control of inflation and, at the same time, lower welfare losses. Achieving lower welfare losses does not necessarily imply lower CPI or PPI inflation variation. The reason is that my two country model is very rich in its dynamics. As is shown, welfare is affected not only by the variances of PPI inflation, the output gap and the real exchange, but also by their covariances. Additionally, since I introduce endogenous output and inflation persistence, I show that welfare is affected by the lags of output and inflation.

Benigno and Benigno (2008) show that in exchange rate regimes where the Central Bank reacts

[1] The additional targets considered were the real exchange rate, lagged inflation rate, money supply and the federal funds rate.

either to the change in the nominal exchange rate or its level, it achieves a better control of the terms of trade, in terms of volatility. However, they focus only on shocks to the natural terms of trade without proceeding to an impulse response analysis in the face of monetary policy, real exchange rate and productivity shocks. Restricting the analysis in only one kind of shock may be misleading, since the choice of an exchange rate regime may not be sustainable in the face of alternative policy or real shocks. Additionally, they do not conduct a welfare analysis. Benigno and Benigno (2001) , however, proceed to the evaluation of the alternative interest rate rules based on a welfare criterion. Their welfare analysis shows that interest rate rules with a nominal exchange rate target perform worse than a standard Taylor rule. However, they do not perform a robustness exercise, in order to test whether the rule is robustly optimal. Their model does not include endogenous inflation and output persistence and asymmetry, between the two countries, is considered only in the coefficients of the interest rate rules. I consider alternative kinds of asymmetries, as far as the structural parameters are concerned. The reason, as I show, is that asymmetries have important implications for monetary policy.

Leitemo and Soderstrom (2001) find that the inclusion of a real exchange rate target into the Taylor rule gives only slight improvements in terms of volatility of the important variables in the economy. They consider a small open economy model with model uncertainty. Altering parameters in the model, and more importantly, increasing the degree of asymmetries has non-negligible effects in terms of welfare losses. Leitemo and Soderstrom, like Weerapana (2000) do not derive the welfare loss from the utility function of households, but they, rather, impose an ad hoc version of it. Moreover, in our paper the coefficients in the interest rate rule of the home country are not imposed exogenously, but rather they are the ones that minimize the welfare loss function. Under this approach we show that the differences in welfare losses can be very large.

The paper is organized as follows. In section 2 I estimate a structural VAR model using data for the Eurozone and the US. I compute impulse responses under different restrictions in the companion matrix. In section 3 I develop a two-country DSGE model. In section 4 I present the log linearized version of the model. In section 5 I introduce monetary policy by presenting alternative interest rate rules the central bank may follow. In section 6 I present the calibration and simulation results. In section 7 I perform a robustness analysis. Section 8 concludes.

## 2 Empirical evidence and motivation

In this section I present a structural VAR model for the Eurozone. The SVAR model consists of four variables, namely inflation, output gap, interest rate and real exchange rate. The output gap[2] is proxied by the hp filter. The SVAR representation is specified as

$$B_0 X_t = \sum_{i=1}^{k} B_i X_{t-i} + U_t$$

where $B_i$, $i = 1, 2, 3, 4$ are $4 \times 4$ matrices and $X_t = (y_t, \pi_t, q_t, i_t)'$, where $i_t$ is the nominal interest rate, $q_t$ the real exchange rate, $y_t$ the output gap, $\pi_t$ the inflation rate and $U_t = [u_{y,t}\ u_{\pi,t}\ u_{q,t}\ u_{i,t}]'$ is the matrix of fundamental errors with a variance-covariance matrix$\Sigma_u = E(u_t, u_t^{'})$ assumed to be diagonal.

In order to consider real exchange rate targeting, along with inflation and output gap targeting I imposed the restriction that real exchange rate shock enters contemporaneously into the interest rate equation. Matrix $B_{0t}$ receives the following form

$$B_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ a_y^{\pi} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ a_y^i & a_{\pi}^i & a_q^i & 1 \end{bmatrix}$$

When the Central Bank targets the real exchange rate, real exchange rate shocks affect the nominal interest rate contemporaneously. In this case $a_q^i \neq 0$.

[2]The output gap was proxied using the hp filter. The latter's accuracy in capturing the actual output gap has been criticized. One reason is that the natural rate of output is proxied by a deterministic trend. However, the former may be a function of technology, monetary and demand shocks, and thus, more volatile. For a more detailed survey on the criticism on the output gap measures see Gali (2002), Gali and Gertler (1999), Sbordone (1999), Gertler, Gali and Lopez-Salido (2000) and the references therein.

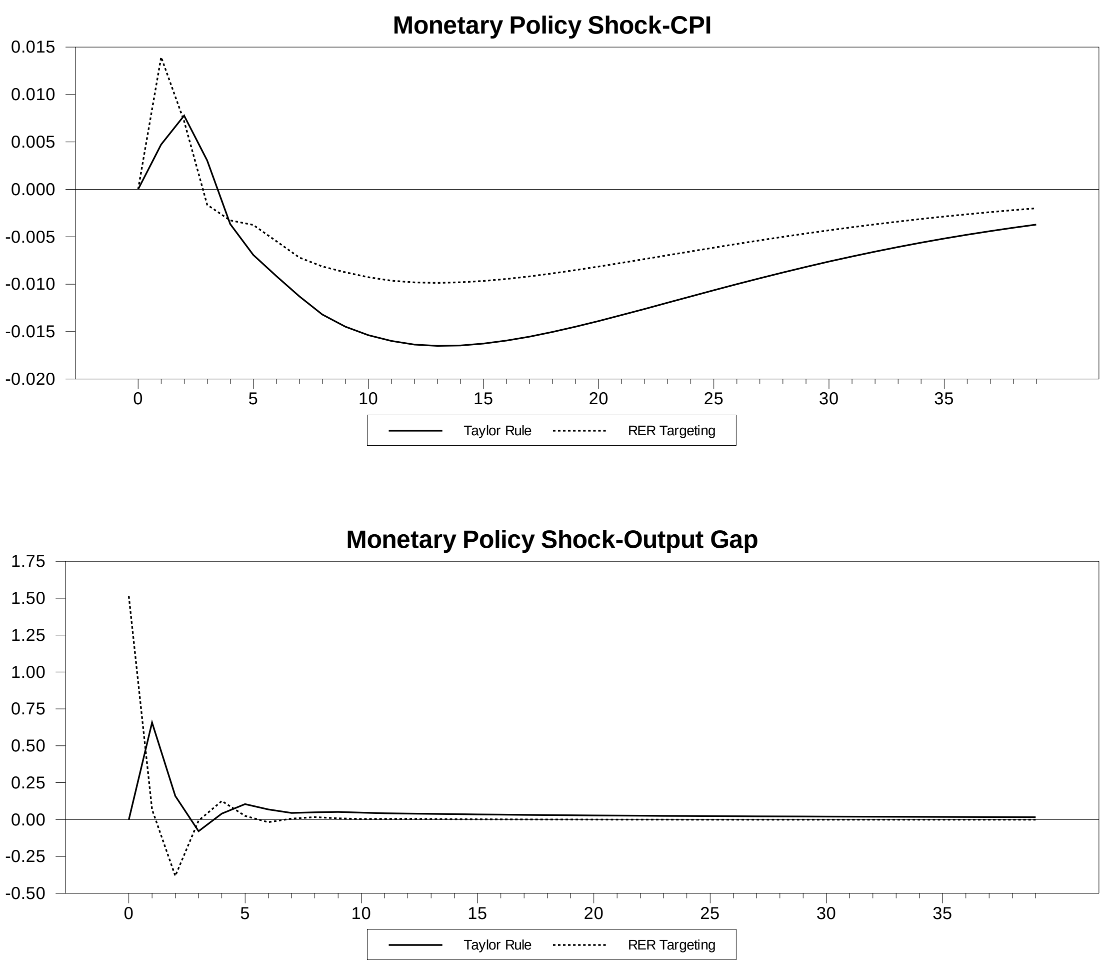
## 2.1 Data

Monthly data are gathered from the IMF International Financial Statistics for the Eurozone and US CPI , the end of period spot exchange rate of the Euro against the US dollar respectively. The interbank overnight rate is used as proxies for the nominal interest rate. The dataset spans from 1999:1 to 2009:3.

## 2.2 Impulse response analysis

The goal in this paper is to show the importance of the real exchange in the interest rate rule in terms of inflation and output gap variation. Specifically, I want to show that the central bank is able to achieve better control of inflation and, if possible, the output gap.

**Figure 1: Impulse Responses**



The impulse response functions for the inflation rate and the output gap in Eurozone are computed under a one standard deviation policy shock. The results are illustrated in figure 1.

The impulse responses show that the ECB achieves a better control of the inflation rate whenever the real exchange rate is introduced into the interest rate rule. Following a monetary policy shock, the CPI inflation rate initially jumps higher than under the Taylor rule, but reverts faster back to its initial level. The output gap has similar dynamics. It jumps higher under real exchange rate targeting, but is less persistent than under the Taylor rule .

# 3 The model

## 3.1 Households

In this section, I specify the structure of the baseline, two country stochastic general equilibrium model. Each country is populated by a continuum of infinitely lived and identical households in the interval $[0, 1]$. Foreign variables are denoted with an asterisk.

Persistence has been found to be an important feature of output in Eurozone and the US.[3] For this reason I introduce endogenous persistence in consumption by assuming that there are two kinds of households as in Amato and Laubach (2003). Let $\psi$ denote the probability that the household is able to choose its consumption optimally, and which is independent of the household's history. Therefore, by the law of large numbers, in each period a fraction $\psi$ of households will reoptimise, whereas the remaining fraction $1 - \psi$ will not. The latter will choose its consumption in period $t$ according to the following rule of thumb

$$C_t^R = C_{t-1} \tag{1}$$

where $C_t$ denotes aggregate per capita consumption in period $t$. The remaining $1-\psi$ of households choose $C_t^O$ so as to maximize their utility. Thus, per capita consumption in period $t$ is given by

$$C_t = \psi C_t^O + (1 - \psi) C_t^R \tag{2}$$

[3]Smets and Wouters (2005), Sahuc and Smets (2008) and Adjemian et al. (2008) using Bayesian techniques to estimate DSGE models for the Eurozone and the US find that output persistence in both regions is high.

As in Laubach and Amato, this modification to the consumer's problem is based on the assumption that it is costly to reoptimise every period[4]. The households who choose consumption optimally choose $C_t^O$ to maximize their utility function. They derive utility from consumption and disutility from labor supply. The utility function, thus, is specified as

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ \frac{(C_s)^{1-\sigma}}{1-\sigma} - \frac{(L_s)^{1+\gamma}}{1+\gamma} \right] \tag{3}$$

where $\sigma$ is the degree of relative risk aversion.

Home agents consume home and foreign goods. Therefore, per capita consumption $C_t$ is a composite consumption index described as

$$\begin{aligned} C_t &= \left[ \delta^{\frac{1}{\rho}} C_{H,t}^{\frac{\rho-1}{\rho}} + (1-\delta)^{\frac{1}{\rho}} C_{F,t}^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \\ & \qquad\qquad\qquad\qquad\qquad\qquad \rho > 1 \\ C_t^* &= \left[ (\delta^*)^{\frac{1}{\rho}} (C_{F,t}^*)^{\frac{\rho-1}{\rho}} + (1-\delta^*)^{\frac{1}{\rho}} (C_{H,t}^*)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \end{aligned} \tag{4}$$

where $\rho$ captures the intratemporal elasticity of substitution between home and foreign goods. $\delta > \frac{1}{2}$ is a parameter of home bias in preferences.$C_H$ and $C_F$ is the home and foreign goods consumption index respectively, in the home country. In the foreign country $C_H^*$ and $C_F^*$ is the home and foreign goods consumption index respectively. Consumption indices in the two countries are defined as

$$\begin{aligned} C_{H,t} &= \left[ \int_0^1 c_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \; C_{F,t} = \left[ \int_0^1 c_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} \\ C_{H,t}^* &= \left[ \int_0^1 c_t^*(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \; C_{F,t}^* = \left[ \int_0^1 c_t^*(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} \end{aligned} \tag{5}$$

The aggregate consumption price index for the home and foreign country is specified as

$$\begin{aligned} P_t &= \left[ \delta (P_{H,t})^{1-\rho} + (1-\delta) P_{F,t}^{1-\rho} \right]^{\frac{1}{1-\rho}} \\ P_t^* &= \left[ \delta^* (P_{F,t}^*)^{1-\rho} + (1-\delta^*) P_{H,t}^{*\,1-\rho} \right]^{\frac{1}{1-\rho}} \end{aligned} \tag{6}$$

[4] Amato and Laubach note that Rule (4) has the important feature that rule-of-thumb consumers learn from optimizing households with one period delay. Hence, although Rule (4) is not optimal, it has three important properties. First agents are not required to compute anything. Second, rule-of-thumb households learn from optimizing ones, because last period's decisions by the latter are part of $C_{t-1}$. Third, the differences between $C_t^R$ and $C_t^O$ are bounded, and will be zero in the steady state.

where $P_H$ and $P_F$ are price indices for home and foreign goods, expressed in the domestic currency. The price indices for the home and foreign country are defined as

$$P_{H,t} = \left[\int_0^1 p_t(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}} \quad , \quad P_{F,t} = \left[\int_0^1 p_t(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}} \tag{7}$$

$$P^*_{H,t} = \left[\int_0^1 p^*_t(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}} \quad , \quad P^*_{F,t} = \left[\int_0^1 p^*_t(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}}$$

Capital markets are complete. The consumers of both countries purchase state uncontingent bonds denominated in the domestic currency, $B_t$ for domestic agents and $B_t^*$ for foreign agents at price $Q_t$. That is $B_t$ denotes the home agent's holdings of a one period nominal bond paying one unit of the home currency.

The home agent maximizes her utility subject to the period budget constraint

$$P_t C_t + Q_{t,t+1} B_{t+1} = B_t + W_t L_t + \Pi_t \tag{8}$$

where $W_t$ is the nominal wage and $\Pi_t$ are nominal profits the individual receives.

### 3.2 First order conditions

Maximizing the utility function (6) subject to the budget constraint (11) yields the following first order conditions

$$Q_{t,t+1} = \frac{\beta P_t}{P_{t+1}} \left(\frac{C_t^O}{C_{t+1}^O}\right)^{\sigma} \tag{9}$$

$$L_t = (C_t^O)^{-\frac{\sigma}{\gamma}} w_t^{\frac{1}{\gamma}} \tag{10}$$

where the first equation is the usual Euler equation while the second determines the labor supply schedule.

Individual demands for each good $i = h,\ f$ produced in the home and in the foreign country respectively are expressed as

$$c_{h,t}(h) = \left(\frac{p_t^h(h)}{P_{H,t}}\right)^{-\theta} \left(\frac{P_{H,t}}{P_t}\right)^{-\rho} \delta C_t \tag{11}$$

$$c_{f,t}(h) = \left(\frac{p_t^*(h)}{P_{F,t}}\right)^{-\theta} \left(\frac{P_{F,t}}{P_t}\right)^{-\rho} (1-\delta) C_t \tag{12}$$

### 3.3 Risk sharing

The fraction of foreign households who choose their consumption optimally $(\psi^*)$ , maximize their utility subject to their budget constraint specified as

$$P_t^* C_t^* + \frac{Q_{t,t+1} B_{t+1}^*}{z_t} = \frac{B_t^*}{z_t} + W_t^* L_t^* + \Pi_t^* \tag{13}$$

where $z_t$ is the nominal exchange rate defined as the domestic currency price of the foreign currency. Therefore, the Euler equation from the foreign agent's maximization problem is

$$Q_{t,t+1} = \frac{\beta P_t^* z_t}{P_{t+1}^* z_{t+1}} \left(\frac{C_t^{O*}}{C_{t+1}^{O*}}\right)^{\sigma} \tag{14}$$

International financial markets are complete. Domestic and foreign households trade in the state contingent one period nominal bonds denominated in the domestic currency. Therefore, combining (12) and (17) , I receive the following optimal risk sharing condition

$$\left(\frac{C_t^{O*}}{C_t^{O}}\right)^{-\sigma} = \varpi q_t \tag{15}$$

where $\varpi \equiv \left(\frac{C_0^f + x}{C_0^h + x}\right)^{-\sigma} \frac{P_0}{z_0 P_0^*}$ depends on initial conditions and $q_t = \frac{z_t P_t^*}{P_t}$ is the real exchange rate.

### 3.4 Price setting

There is local currency pricing in both countries. That is, each firm sets one price for its goods consumed domestically and another for the same good consumed abroad. Prices are sticky with a price setting behavior *àℓa* Calvo (1983). At each date, each firm changes its price with a probability $1-\omega$ , regardless of the time since it last adjusted its price. The probability of not changing the price, thus, is $\omega$. The probability of not changing the price in the subsequent $s$ periods is $\omega^s$. Consequently, the price decision at time $t$ determines profits for the next $s$ periods. The price level for home goods at date $t$ will be defined as

$$P_{H,t} = \left[\omega P_{H,t-1}^{1-\theta} + (1-\omega)\widetilde{p}_t(h)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{16}$$

In the literature on inflation dynamics in the Eurozone and the US its has been found that persistence is one of the key features. Therefore, I introduce endogenous inflation persistence by assuming that firms that are given the opportunity to adjust their prices will either follow a rule of thumb (backward looking firms) or will chose the price that maximizes their expected discounted profits (forward looking firms), as in Gali et al. (2001). The price $\tilde{p}_t(h)$ that will be set at date $t$ is specified as

$$\tilde{p}_t(h) = \zeta p_t^B(h) + (1-\zeta)\, p_t^{For}(h) \tag{17}$$

where $\zeta \in (0,1)$ is the fraction of backward looking firms, $p_t^B(h)$ and $p_t^{For}(h)$ is the price set by the backward and the forward looking firms, respectively. A continuum of firms is assumed for the home economy indexed by $h \in [0,1]$. Each firm produces a differentiated good, with a technology

$$Y_t(h) = A_t L_t(h) \tag{18}$$

where $A_t$ is a country specific productivity shock at date $t$ which is assumed to follow a log stationary process

The structure of productivity shocks across the two countries receives the following form

$$\begin{bmatrix} \alpha_t \\ \alpha_t^* \end{bmatrix} = \begin{bmatrix} \rho_{\alpha_t} & \rho_{\alpha_t \alpha_t^*} \\ \rho_{\alpha_t^* \alpha_t} & \rho_{\alpha_t^*} \end{bmatrix} \begin{bmatrix} \alpha_{t-1} \\ \alpha_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_{\alpha,t} \\ \varepsilon_{\alpha^*,t}^* \end{bmatrix}$$

where $\begin{bmatrix} \varepsilon_{\alpha,t} \\ \varepsilon^*_{\alpha^*,t} \end{bmatrix} \sim N(0, \Sigma^2)$, with $\Sigma^2 = \begin{bmatrix} \sigma^2_{\varepsilon_a} & 0 \\ 0 & \sigma^2_{\varepsilon^*_{\alpha^*}} \end{bmatrix}$.

*Backward looking firms.*

Backward looking firms set their prices according to the following rule

$$p_t^B(h) = P_{H,t-1} + \pi_{H,t-1} \quad and \quad p_t^{B*}(h) = P^*_{H,t-1} + \pi^*_{H,t-1} \tag{19}$$

*Forward looking firms.*

Forward looking firms set their prices by maximizing their expected discounted profits. Their maximization problem comprises of two decisions. The one concerns the price for the domestic market and the other the price charged in the foreign market, when it exports. Hence their maximization problem is described as

$$maxE_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} \left\{ \widetilde{p}_t(h) y^h_{t+s}(h) + \varepsilon_t \widetilde{p}_t^{\,*}(h) y^f_{t+s}(h) - W^h_{t+s} L^h_{t+s} \right\} \tag{20}$$

where $y_t^i(h),\ i = h, f$ is the demand for the home good for home and foreign agents specified as

$$y_t^h(p_t(h)) = \left( \frac{\tilde{p}_t(h)}{P_{H,t}} \right)^{-\theta} \left( \frac{P_{H,t}}{P_t} \right)^{-\rho} \delta^* C_t, \tag{21}$$

$$y_t^f(p_t^*(h)) = \left( \frac{\tilde{p}_t^*(h)}{P^*_{H,t}} \right)^{-\theta} \left( \frac{P^*_{H,t}}{P^*_t} \right)^{-\rho} (1 - \delta^*) C^*_t \tag{22}$$

The firm maximizes its objective function (20) subject to (21) in order to find the optimal price for the home good in the home economy. It maximizes subject to (22), in order to find the optimal price for the home good in the foreign economy. The firm chooses a price for the home good in the home economy that satisfies the first order condition

$$E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} y_{t+s}(p_t(h)) \left\{ p_t(h) - \frac{\theta}{\theta - 1} MC_{t+s} \right\} = 0$$

where $MC_{t+s} = \frac{W_{t+s}}{A_{t+s}}$ denotes the nominal marginal cost and $\frac{\theta}{\theta-1}$ captures the optimal markup.

The optimal price for the home good in the home country is specified as

$$p_t(h) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} MC_{t+s} y_{t+s}^h (p_t(h))}{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} y_{t+s}^h (p_t(h))} \tag{23}$$

Respectively, the optimal price for the home good in the foreign country is specified as

$$p_t^*(h) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} MC_{t+s} y_{t+s}^f (p_t^*(h))}{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} y_{t+s}^f (p_t^*(h)) z_{t+s}} \tag{24}$$

*Aggregate price level*

Dividing (19) by $P_{H,t-1}$:

$$\Pi_{H,t}^{1-\theta} = \omega + (1 - \omega) \left( \frac{\widetilde{p}_t(h)}{P_{H,t-1}} \right)^{1-\theta} \tag{25}$$

where $\Pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$.

Similarly, for the foreign goods consumed in the home economy:

$$\Pi_{F,t}^{1-\theta} = \omega + (1 - \omega) \left( \frac{\widetilde{p}_t(f)}{P_{F,t-1}} \right)^{1-\theta} \tag{26}$$

The aggregate price level dynamics are specified, thus, as

$$\Pi_t^{1-\rho} = \delta \left[ \left( \frac{P_{H,t-1}}{P_{t-1}} \right) \Pi_{H,t} \right]^{1-\rho} + (1 - \delta) \left[ \left( \frac{P_{F,t-1}}{P_{t-1}} \right) \Pi_{F,t} \right]^{1-\rho} \tag{27}$$

## 4 Log linearized model

A log linearized version of the relationships found in the previous section serves in providing us with a way to deal with the problem of no closed form solution. Additionally, this is a way to end up in a state space form which can be estimated using real time series data.

### 4.1 Supply side

I use a first order Taylor approximation around the steady state of zero inflation rate. Log linearized variables are denoted with a hat.

After loglinearizing the first order condition (10), the production function (18) the demand schedules faced by each firm (21) and (22) and optimal price setting rules (23) and (24), I receive the two relations describing the domestically consumed home goods inflation rate and the respective of the home goods consumed in the foreign country

$$\pi_{H,t} = b_{\pi_{H,-1}}\pi_{H,t-1} + b_{\pi^*_{H,-1}}\pi^*_{H,t-1} + \beta E_t \pi_{H,t+1} + b_{\pi^*_H}\pi^*_{H,t} + b_C \hat{C}_t + \dots$$
$$\dots + b_T \hat{T}_t + b_{T^*}\hat{T}^*_t + b_q \hat{q}_t + b_a a_t \tag{28}$$

$$\pi^*_{H,t} = b_{\pi_{H,-1}}\pi_{H,t-1} + b_{\pi^*_{H,-1}}\pi^*_{H,t-1} + \beta E_t \pi^*_{H,t+1} + b^*_{\pi_H}\pi_{H,t} + b^*_C \hat{C}_t + \dots$$
$$\dots + b^*_T \hat{T}_t + b^*_{T^*}\hat{T}^*_t + b^*_q \hat{q}_t + b^*_a a_t \tag{29}$$

where $T_t = \frac{P_{F,t}}{P_{H,t}}$ and $T^*_t = \frac{P^*_{H,t}}{P^*_{F,t}}$ denote relative prices in the home and foreign country respectively. The log linearized aggregate price level relation (27) is specified as

$$\pi_t = \pi_{H,t} + (1-\delta)(\pi_{F,t} - \pi_{H,t}) \tag{30}$$

which can be further simplified as[5]

$$\pi_t = \pi_{H,t} + (1-\delta)\Delta\hat{T}_t$$

### 4.2 Demand side

In this section I proceed to the loglinearization of the Euler equation

[5]To end up to that expression, I used equation $\hat{T}_t = \hat{T}_{t-1} + \pi_{F,t} - \pi_{H,t}$ for the relative price which is reported later in the text.

$$\hat{C}_t^O = \kappa(i_t - E_t\pi_{t+1}) + E_t\hat{C}_{t+1}^O \tag{31}$$

where $\kappa = -\frac{1}{\sigma}$, and using (2) the Euler equation receives the forward form, which includes both backward and forward looking elements

$$\hat{C}_t = \frac{\kappa\psi}{2-\psi}(i_t - E_t\pi_{t+1}) + \frac{1}{2-\psi}E_t\hat{C}_{t+1} + \frac{1-\psi}{2-\psi}\hat{C}_{t-1} \tag{32}$$

Goods market clearing assumes the following two conditions

$$Y = C_H + C_H^* + G_t \quad and \quad Y^* = C_F + C_F^* + G_t^*$$

where $G_t$ and $G_t^*$ capture government expenditures for home and foreign country respectively, assumed to follow an exogenous stationary $AR(1)$ process $g_t = \rho_g g_{t-1} + \varepsilon_{g,t}$ and $g_t^* = \rho_{g^*} g_{t-1}^* + \varepsilon_{g,t}^*$, $\varepsilon_{g,t} \sim N(0, \sigma_{\varepsilon_g}^2)$ and $\varepsilon_{g,t}^* \sim N(0, \sigma_{\varepsilon_g}^{*2})$.

Combining equation (35) and the market clearing conditions, I derive the aggregate demand equation:

$$\begin{aligned}\hat{Y}_t = \eta_1\hat{Y}_{t-1} + \eta_2 E_t\hat{Y}_{t+1} + \eta_3(i_t - E_t\pi_{t+1}) + \eta_4\hat{q}_t + \eta_5\hat{q}_{t+1} + \eta_6\hat{q}_{t-1} + \dots \\ \dots + \eta_7\Delta\hat{T}_t + \eta_8 E_t\Delta\hat{T}_{t+1} + \eta_9\Delta\hat{T}_t^* + \eta_{10}E_t\Delta\hat{T}_{t+1}^*\end{aligned} \tag{33}$$

where $\eta_i$ , $i = 1, .., 9$ are defined in detail in appendix B.

### 4.3 Real exchange rate and relative prices

The real exchange rate dynamics are specified by the following relationship

$$\Delta\hat{q}_t = \Delta z_t + \pi_t^* - \pi_t \tag{34}$$

In the home country the price of imported goods relative to that of home goods is specified as $T_t = \frac{P_{F,t}}{P_{H,t}}$, whereas in the foreign country the relative price of home exported goods to foreign goods is specified as $T_t^* = \frac{P_{H,t}^*}{P_{F,t}^*}$. Loglinearizing those two expressions we receive the following

$$\hat{T}_t = \hat{T}_{t-1} + \pi_{F,t} - \pi_{H,t} \quad \hat{T}_t^* = \hat{T}_{t-1}^* + \pi_{H,t}^* - \pi_{F,t}^*$$

### 4.4 Flexible price equilibrium

At the flexible price equilibrium firms adjust their prices in each period. Each firm will set its marginal cost equal to the optimal marginal cost (i.e. $-log\left(\frac{\theta}{\theta-1}\right)$) which is constant over time and equal across firms. Since firms adjust their prices every period, monetary policy will not have any real effects into the economy. The real marginal cost is specified by the following equations

$$mc_t = -log\left(\frac{\theta}{\theta-1}\right) = -\mu$$

$$mc_t = w_t - \alpha_t - \nu$$

where $w_t$ is the real wage, $\alpha_t$ (log) productivity and $\nu$ a subsidy to labor.[6] Solving for the case with flexible prices, I receive the following set of equations describing the equilibrium processes for output, consumption, labor, real interest rate[7], given by:

$$y_t^n = \psi_c \bar{c}_{t-1} + \psi_\zeta \zeta + \psi_a \alpha_t + \psi_{a^*} \alpha_t^* + \psi_g g_t + \psi_{g^*} g_t^* \tag{35}$$

$$c_t^n = \tilde{\psi}_c \bar{c}_{t-1} + \psi_\zeta \zeta + \left(\frac{\gamma\delta^* + \sigma}{\delta(\gamma+\sigma) - \gamma(1-\delta^*)}\right) \psi_\alpha \alpha_t - \left(\frac{\gamma}{\sigma}\psi_{\alpha^*}\right)\alpha_t^* - \left(\frac{\gamma}{\sigma}\psi_g\right) g_t - \left(\frac{\gamma}{\sigma}\psi_{g^*}\right) g_t^* \tag{36}$$

$$l_t^n = \tilde{\psi}_c \bar{c}_{t-1} + \psi_\zeta \zeta + \left(\frac{\gamma(\delta^*(1-\sigma) - (1-\delta)) - \sigma(1-\delta)\psi_\alpha}{\delta(\gamma+\sigma) - \gamma(1-\delta^*)}\right) \alpha_t - \psi_{a^*}\alpha_t^* + \psi_g g_t + \psi_{g^*} g_t^* \tag{37}$$

$$r_t^n = \tilde{\tilde{\psi}}_c \bar{c}_{t-1} + \left(\frac{(\gamma\delta^* + \sigma)(1-\rho_a)\psi_a}{\kappa\delta(\gamma+\sigma) - \gamma(1-\delta^*)}\right) \alpha_t - \left(\frac{\gamma(1-\rho_{a^*})\psi_{a^*}}{\kappa\sigma}\right) \alpha_t^* - \left(\frac{\gamma(1-\rho_g)\psi_g}{\kappa\sigma}\right) g_t - \left(\frac{\gamma(1-\rho_{g^*})\psi_{g^*}}{\kappa\sigma}\right) g_t^* \tag{38}$$

## 5 Monetary Policy

Monetary policy is conducted through nominal interest rate rules by the central bank.

[6] This subsidy serves in rendering the flexible price equilibrium efficient. This is achieved by setting the subsidy equal to the mark-up (i.e. $\nu = \mu$), in order to remove the distortion associated with monopolistic competition.

[7] The flexible price expression for the real exchange rate can be easily derived using the risk sharing condition.

Open economy monetary policy literature has often rejected the importance of the exchange rate in the interest rate feedback rules, either because it is argued that its effect is already there, indirectly through its pass through on prices and then in inflation (Ball, 1999; Taylor, 1999), or because data do not support its significance (Clarida, Gali and Gertler, 1998). However, a weakness of many empirical studies is that they do not estimate a structural model, but, rather, they estimate an interest rate rule. This strategy is able, of course, to provide some information about the range of values of the coefficients, but its weakness rests on the fact that it does not take into account the interactions among the fundamental variables in the economy.

## 5.1 Policy rules

In this section I focus on two different policy rules. Each rule leads to a different system of equations and, thus, different conditions that are necessary for determinacy.

### 5.1.1 Taylor rule and real exchange rate targeting

The standard Taylor rule is known to perform quite well in a wide range of models. However, one weakness is that, the Taylor rule, in its standard form does not introduce history dependence. The latter is crucial in forward looking models. An interest rate rule with history dependence allows the central bank to control private sectors expectations better and, hence, to achieve lower welfare losses. Therefore, I consider the Taylor rule with some interest rate inertia.

As already mentioned, my main goal is to show that the real exchange rate has important information for the conduct of monetary policy in open economies. Therefore, I consider the Taylor rule expanded by a target for the real exchange rate as well. The two rules receive the following form

$$i_t = \phi_x x_t + \phi_\pi \pi_t + \phi_i i_{t-1} \tag{39}$$

$$i_t = \phi_x x_t + \phi_\pi \pi_t + \phi_q q_t + \phi_i i_{t-1} \tag{40}$$

where $x_t = y_t - y_t^n$ denotes the output gap.

### 5.2 Welfare

The central bank sets the interest rate in such a way to minimize a measure of social loss derived by a second order Taylor expansion of the consumer's utility function as in Rotemberg and Woodford (1998), Amato and Laubach (2003) and Pappa (2004). It is summarized as[8]

$$W_t = -\frac{1}{2}u_c C\Xi\{\lambda_1(\hat{Y}_t - y_t^n)^2 + \lambda_2(\hat{Y}_t^* - y_t^{*n})^2 + \lambda_3(\hat{q}_t - q_t^n)^2 + \lambda_4\Delta\hat{q}_t^2 + \lambda_5\Delta\hat{Y}_t^{*2} + \lambda_6\Delta\hat{Y}_t^2 + \dots$$

$$+\pi_{H,t}^2 + \lambda_7(\pi_{H,t} - \pi_{H,t-1})^2 + \lambda_8(\pi_{H,t}^*)^2 + \lambda_9(\pi_{H,t}^* - \pi_{H,t-1}^*)^2 + \lambda_{10}(\hat{q}_t + \hat{Y}_t)^2 + \lambda_{11}(\hat{q}_t + \hat{Y}_t^*)^2 + \dots$$

$$\lambda_{12}(\hat{q}_{t-1} + \hat{Y}_t)^2 + \lambda_{13}(\hat{q}_{t-1} + \hat{Y}_t^*)^2 + \lambda_{13}(\hat{q}_{t-1} + \hat{Y}_t^*)^2 + \dots$$
$$\lambda_{14}(\hat{Y}_{t-1}^* - y_{t-1}^{*n})(\hat{q}_{t-1} - q_{t-1}^n) + \lambda_{15}(y_{t-1} - y_{t-1}^n)(y_{t-1}^* - y_{t-1}^{*n}) + \lambda_{16}(\hat{C}_t - c_t^n)(\hat{q}_t - q_t^n) + \dots$$

$$\lambda_{17}(\hat{Y}_t + \hat{Y}_{t-1}^*)^2 + \lambda_{18}(\hat{Y}_{t-1} + \hat{Y}_t^*)^2 + \lambda_{19}(\hat{Y}_{t-1} - y_{t-1}^n)(q_{t-1} - q_{t-1}^n) + \dots$$

$$+\lambda_{20}(\hat{Y}_t^* - \hat{Y}_t^{*n})(\hat{Y}_{t-1}^* - \hat{Y}_{t-1}^{*n}) + \lambda_{21}(\hat{Y}_{t-1}^* + \hat{q}_t)^2 + \lambda_{22}(\hat{Y}_{t-1} + \hat{q}_t)^2 + \lambda_{23}(\hat{Y}_{t-1} - y_{t-1}^n)(\hat{q}_{t-1} - q_{t-1}^n) + \dots$$

$$\lambda_{24}(\hat{C}_{t-1}^* - c_{t-1}^{*n})(\hat{q}_{t-1} - q_{t-1}^n) + \lambda_{25}(\hat{q}_t - q_t^n)(\hat{q}_{t-1} - q_{t-1}^n) + \lambda_{26}(\hat{Y}_{t-1} - y_{t-1}^n)(\hat{Y}_t - y_t^n) + t.i.p. + O(||\xi||^3) \quad (41)$$

where the coefficients $\lambda_i$, $i = 1, ..., 21$ are functions of the structural parameters.

## 6 Calibration

In this section I proceed to the calibration and simulation of the model in order to evaluate alternative monetary policy rules. Alternative monetary policy rules will be compared according to the value of the welfare loss they generate.[9] The values of most the parameters are taken from similar studies. However, since the model is very rich, in terms of parameterization, I have made my own choice of some other parameters. This is the reason why I proceed at section 7 in a

[8]The derivation of the loss function is given in detail in the Appendix.

[9]As usual, optimal monetary policy is defined as one that minimizes the welfare loss as measured by (41).

robustness check, in order to figure out whether the results from the baseline calibration are highly sensitive to alternative values of the structural parameters.

## 6.1 Calibration results

In this section I calibrate the model to investigate how the variables of the model respond to shocks. In table 1 below I provide the values of the calibrated parameters.

Table 1: Parameter Values

| | | |
|---|---|---|
| *Structural parameters* | | |
| $\sigma$ | 3 | ( Amato & Laubach, 2003) |
| $\theta$ | 10 | (Benigno & Benigno, 2006) |
| $\rho$ | 4.5 | (Benigno & Benigno, 2006) |
| $\gamma$ | 3 | (Pappa, 2004) |
| $\omega = \omega^*$ | 0.75 | |
| $\delta = \delta^*$ | 0.8 | (Pappa, 2004) |
| $\zeta = \zeta^*$ | 0.4 | (Amato & Laubach, 2003) |
| $\psi = \psi^*$ | 0.5 | |
| *Interest rate weight* | | |
| $\lambda_r$ | 0.236 | (Amato & Laubach, 2003) |

The coefficients imposed in the foreign country policy rule are those estimated by Clarida et al. (1998) for the US in the post Volcker period, specified as $\phi^*_\pi = 2.15$, $\phi^*_x = 0.93$ and $\phi^*_i = 0.85$. In order to derive secure inference about the policy implications of the different interest rate rules, I computed the optimized coefficients for each rule. That is, the coefficients that minimize the welfare loss function subject to the equations describing the behaviour of the private sector.[10]

[10]For a detailed description of the optimal problem see Benigno and Benigno (2006) and Giannoni (2010).

Table 2: Optimized coefficients (all shocks)

| | $\phi_\pi$ | $\phi_x$ | $\phi_q$ | $\phi_i$ | *Loss* |
|---|---|---|---|---|---|
| | $\psi = \psi^* = 0.5, \quad \zeta = \zeta^* = 0.2$ | | | | |
| Taylor Rule | 1.40352 | 0.997814 | - | 0.969422 | 3.1582 |
| RER targeting | 1.52539 | 0.648157 | -0.0158189 | 0.878512 | 0.6833 |
| | $\psi = \psi^* = 1.0, \quad \zeta = \zeta^* = 0.0$ | | | | |
| Taylor Rule | 1.38497 | 1.63529 | - | -1.30846 | 0.0210 |
| RER targeting | 1.45215 | 0.776324 | -0.122784 | -0.63034 | 0.0185 |

Table 3: Standard deviations

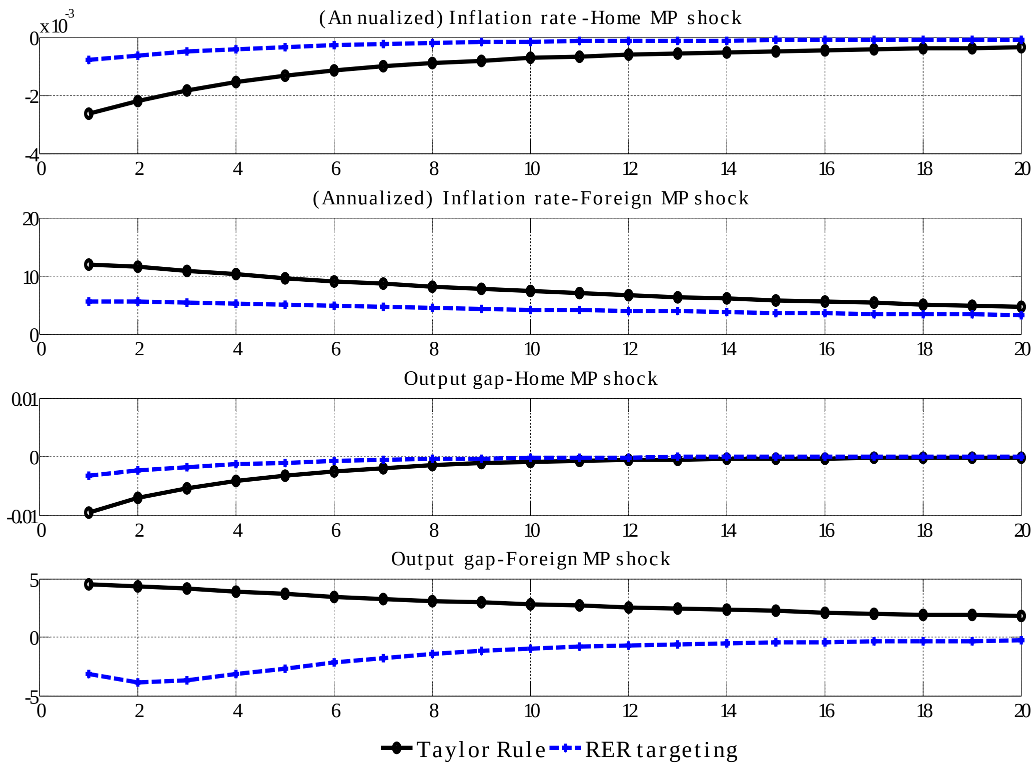
| | $\sigma_\pi$ | $\sigma_{\pi_H}$ | $\sigma_x$ | $\sigma_q$ |
|---|---|---|---|---|
| $\psi = \psi^* = 0.5, \quad \zeta = \zeta^* = 0.4$ (Persistence in inflation and output) | | | | |
| RER targeting | 0.0770 | 0.0784 | 0.4773 | 2.0274 |
| Taylor Rule | 0.2015 | 0.2126 | 1.7403 | 6.8933 |
| $\psi = \psi^* = 1.0, \quad \zeta = \zeta^* = 0.0$ (No persistence) | | | | |
| RER targeting | 0.0290 | 0.0275 | 0.0222 | 1.0543 |
| Taylor Rule | 0.0299 | 0.0289 | 0.0392 | 1.6144 |

The results in table 2 show that a real exchange rate target yields lower welfare losses. When persistence in inflation and output is set to zero (i.e. $\psi = \psi^* = 1.0, \quad \zeta = \zeta^* = 0.0$) the differences in welfare are very small. However, when I allow for persistence the differences in welfare increase abruptly.

All the key macroeconomic variables are more volatile under the Taylor rule as shown at table 3. When the home central bank follows a simple Taylor rule then real exchange rate volatility is higher compared to the case where a real exchange rate target is adopted. Given local currency pricing, a highly volatile real exchange rate has a direct impact on both home CPI and PPI inflation rates. Therefore, since a real exchange rate target decreases its volatility, the home central bank is able to control inflation fluctuations better. Moreover the differences in inflation volatility increase as the degree of persistence on inflation and output goes up. In particular, home CPI inflation is 2.6 times more volatile under the Taylor under persistence, while it is only 1.03 times higher without persistence. The same conclusion holds for the output gap.

The impulse responses in figure 2 from the baseline calibration, presented at table 1, give a picture of the main argument. That is, a real exchange target into an interest rate rule allows the policy maker to have a better control of CPI inflation. CPI inflation falls less when the central bank reacts to the real exchange rate, following a one standard deviation home monetary shock, and jumps less, following a foreign monetary policy shock. The output gap is also better stabilized under real exchange rate targeting. Both variables revert back to the steady state much faster under real exchange rate targeting.

**Figure 2: Impulse responses-Monetary policy shock**



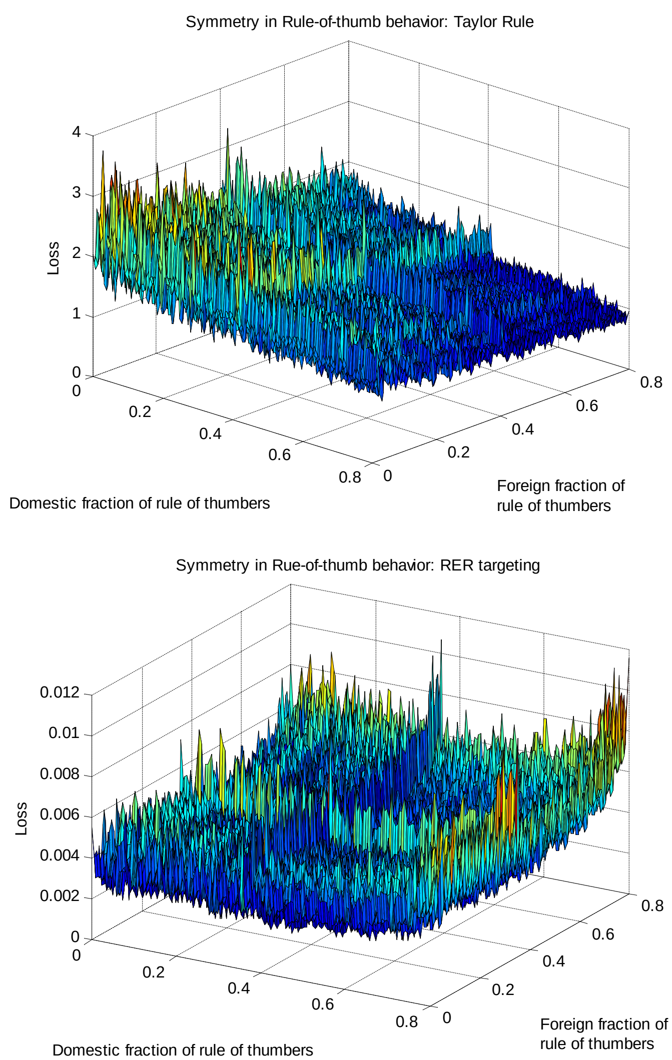

## 7 Robustness analysis

In this section I proceed to a sensitivity analysis of our results. In particular, I look at the behaviour of the welfare loss as structural parameters change over time.

### 7.1 Rule of thumb consumers

As a first exercise I focus on the importance of rule of thumb behavior in consumption. My approach is twofold. First, I keep symmetry between the two countries and see how loss varies as the fraction of rule of thumb consumers increases jointly in both countries. In every step of the simulations, optimized coefficients are computed and stored so as to compute the corresponding value for the welfare loss. As already mentioned, I do that, because the welfare loss is derived from a second order approximation of the utility function, being, thus, highly sensitive to small changes in the structural parameters. Second, I relax the symmetry assumption and allow for asymmetries in the degrees of rule of thumb behavior.

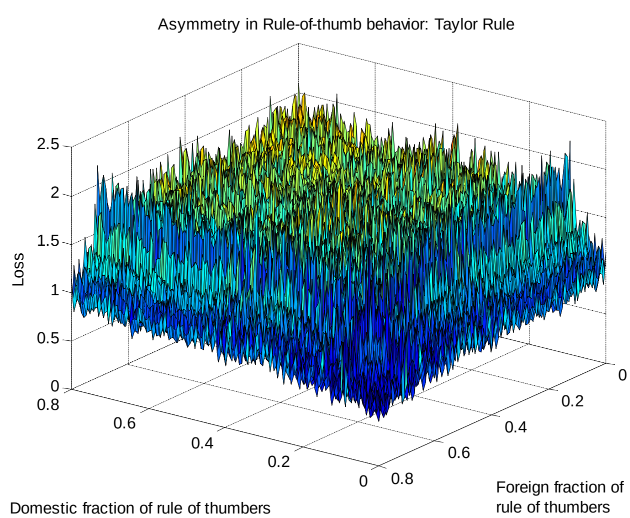
**Figure 3: Variations in Rule-of-thumb Consumers - Symmetric Case**



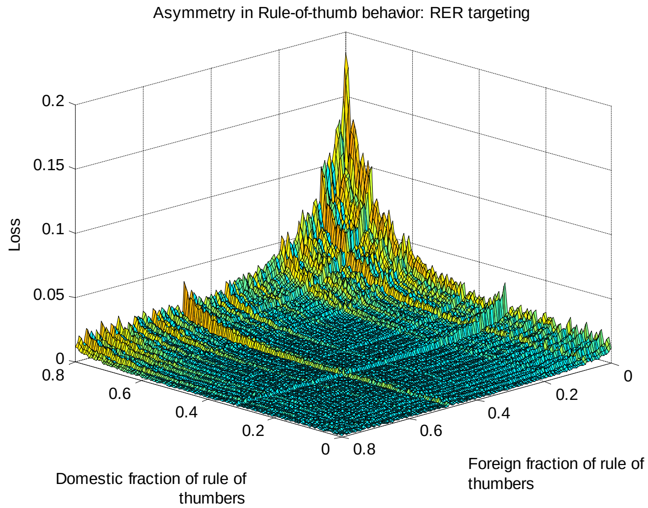
As the fraction of rule of thumb consumers rises symmetrically in both countries both interest rate rule lead to higher welfare losses. The two interest rate rules exhibit asymmetric behavior with respect to variations in rule-of-thumb behavior. The Taylor rule leads to higher welfare losses as the degree of rule-of-thumb behavior falls, whereas the real exchange rate targeting rule leads to to higher welfare losses as this degree goes up. Moreover, real exchange rate targeting interest rate rule seems to be preferred over the Taylor rule at all levels of output persistence. Under real exchange rate targeting, welfare losses start to increase abruptly for values of $1 - \psi > 0.6$.

As a next step I do a similar exercise, but now allowing for an asymmetric variation in the degrees of rule of thumb behavior between the two countries. In particular, as the domestic fraction of rule of thumbers increases the foreign falls[11].

**Figure 4: Variations in Rule-of-thumb Consumers - Asymmetric Case**




[11]The graphs should be interpreted with this ordering. Otherwise, they may lead to the wrong conclusions.



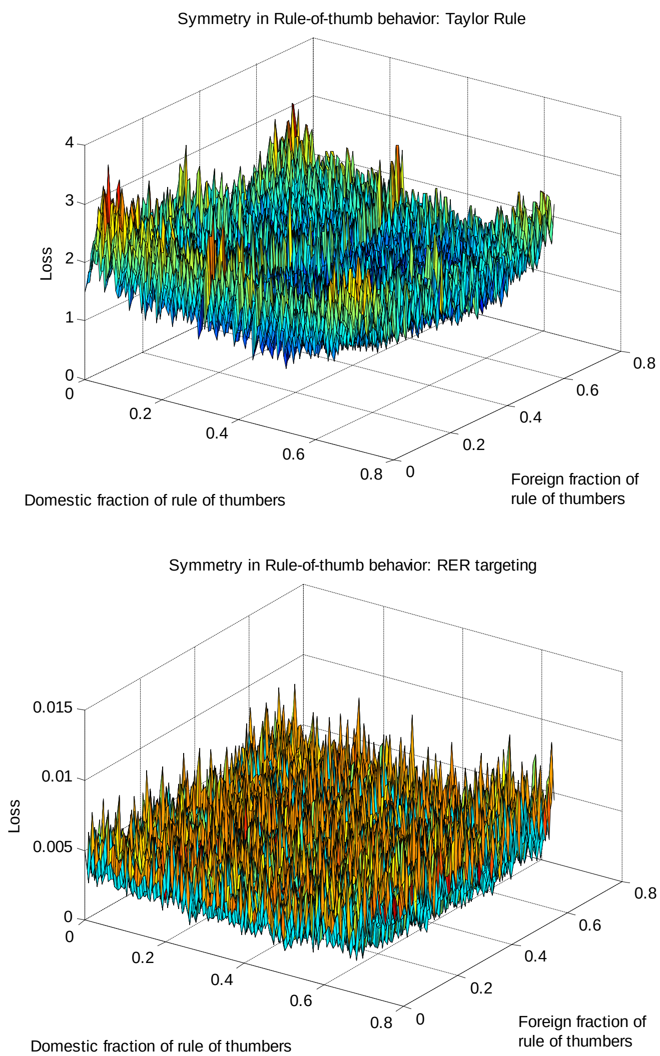

In the asymmetric case, under real exchange rate targeting welfare losses are always lower than in the Taylor rule. The difference, however, with the symmetric case is that the gap between the losses from the two interest rate rule is now much wider than in the symmetric case. The Taylor rule may yield even twenty times high welfare losses when there is asymmetry in the fraction of backward looking consumers in the two countries. On the other hand, losses from the real exchange rate targeting rule seem to increase abruptly for high levels of domestic backward looking behavior associated with low levels in the foreign country.

Looking at the four plots I conclude that adding a real exchange rate target in the interest rate rule leads to lower welfare losses for a wide range of backward looking behavior in consumption. However, the performance of the real exchange rate targeting rule worsens abruptly when, at least, the domestic fraction of backward looking consumers is high.

### 7.2 Rule of thumb price setters

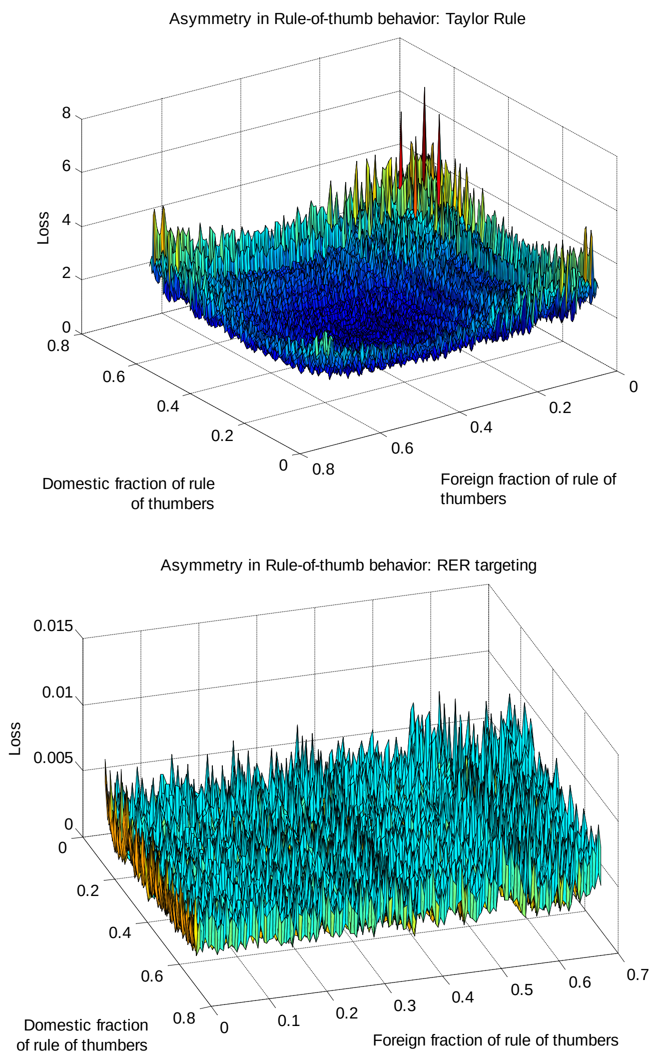
In this part I turn our focus on the effects of rule of thumb behavior in price setting. The strategy I follow is exactly the same as in the previous section. I first look at the symmetric case and then at the asymmetric.

**Figure 5: Variations in Rule-of-thumb price setters - Symmetric Case**




Symmetry in the fraction of backward looking firms leads to the same conclusions as in the case of rule-of-thumb consumption. The Taylor rule yields higher welfare losses. Real exchange rate targeting is able to keep losses fluctuating within a certain band, that includes losses that are lower than those in the Taylor rule.

**Figure 6: Variations in Rule-of-thumb Consumers - Asymmetric Case**

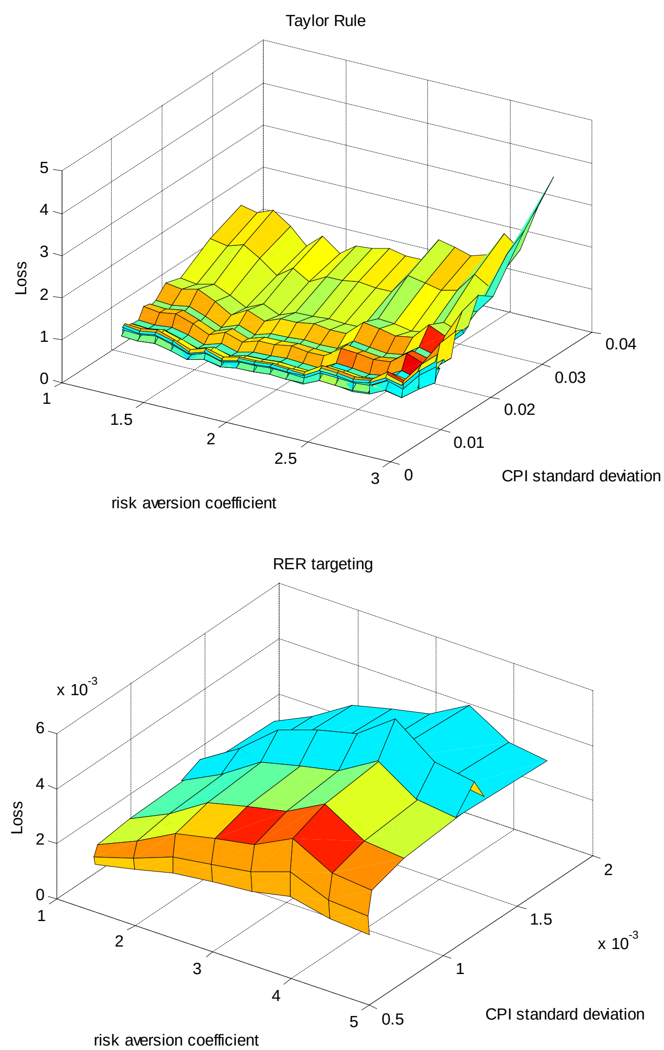



The results from the asymmetric case are similar to the symmetric one, as far as rule-of-thumb price setting behavior is concerned. The Taylor rule performs always worse. When the home central bank follows the optimized Taylor rule, welfare loss rises as the fraction of foreign backward looking firms goes down, for a given level of the domestic ratio. This is not the case under real exchange rate targeting. As the fraction of foreign backward looking firms goes down, welfare loss falls, for a given level of the domestic fraction The main result, though, is that losses are considerably lower when an exchange rate target is adopted.

### 7.3 Risk aversion coefficient

As a final robustness check, I look at the degree of relative risk aversion. Given its importance in the model, I expect that changes in this parameter will have remarkable impact in the performance of each rule. I allow this parameter to vary between 1.1 and 4. The results for each rule are summarized in the two figures below.

**Figure 7: Variations in Risk Aversion**




The two figures above show that for degrees of risk aversion that are in the band of actual data estimates (i.e. between 1 and 5), the Taylor rule performs worse.Moreover, welfare losses increase abruptly as risk aversion coefficient rises above 2.5. On the other hand welfare loss from real exchange rate targeting seems to increase smoothly as risk aversion rises. CPI volatility is considerably lower compared to its volatility under the Taylor rule.

# 8 Conclusion

In this paper I estimate a structural VAR for the nominal interest rate, CPI inflation, the output gap and the real exchange rate. From the impulse response analysis and I find that the ECB achieves a better control of CPI inflation when it allows its policy rate to react contemporaneously to exchange rate movements.

Relying on the above finding I constructed a two country DSGE model for the Eurozone and the US. I modelled the foreign monetary policy using the estimates of Clarida, Gali and Gertler (1998) for the coefficients in the output gap and CPI inflation in the interest rate rule for the US. Taking this policy as given and contrary to past papers, I compute the optimized coefficients in the interest rate rule of the home central bank. Adding the real exchange rate into the interest rate rule leads to robustly lower welfare losses. The gap in losses between the Taylor rule and the real exchange rate targeting rule is wider, the higher the degree of persistence on inflation and output.

Therefore both empirical and theoretical evidence in this paper suggest that an interest rate rule with a real exchange rate target is Pareto superior to the Taylor rule.

# Chapter 3: Markov Switching Monetary Policy in a two-country DSGE Model

Konstantinos Mavromatis

**JOB MARKET PAPER**

**Abstract**

In this paper I show, using both empirical and theoretical analysis, that changes in monetary policy in one country can have important effects on other economies. My new empirical evidence shows that changes in the monetary policy behaviour of the Fed since the start of the Euro, well captured by a Markov-switching Taylor rule, have had significant effects on the behaviour of inflation and output in the Eurozone even though ECB's monetary policy is found to be fairly stable. Using a two-country DSGE model, I examine this case theoretically; monetary policy in one of the countries (labelled foreign) switches regimes according to a Markov-switching process and this has non-negligible effects in the other (home) country. Switching by the foreign central bank renders commitment to a time invariant interest rate rule suboptimal for the home central bank. This is because home agents expectations change as foreign monetary policy changes which affects the dynamics of home inflation and output. Optimal policy in the home country instead reacts to the regime of the foreign monetary policy and so implies a time-varying reaction of the home Central Bank. Following this time-varying optimal policy at home eliminates the effects in the home country of foreign regime shifts, and also reduces dramatically the effects in the foreign country. Therefore, changes in foreign monetary regimes should not be neglected in considering monetary policy at home.

Keywords: Markov-switching DSGE, Optimal monetary policy, Dynamic programming, SVAR, real-time data.
JEL Classification: E52, F41, F42.

# 1 Introduction

Regime changes in the conduct of monetary policy have been documented largely over the last ten years. They refer to changes in the way a central bank reacts to the key macroeconomic variables, i.e. inflation and output. An example of this kind of change in monetary policy is that of the US. In particular, Clarida et al. (2001), Lubik and Schorfheide (2004) and Boivin and Giannoni (2006) show that the reaction of the Fed towards inflation fluctuations until the late '70s was less aggressive compared to that from the early '80s onwards. As a result many authors attribute high inflation volatility in the US during the '70s to the way the Fed was reacting over that period to inflation fluctuations.[1] Moreover, according to these authors, changes in monetary policy are the main reason for the changes in the impulse responses of inflation and output. Even though there is ample empirical and theoretical evidence regarding the effects of changes in monetary policy in a closed economy setup, there is very little evidence about the international effects.

In this paper I show, both empirically and theoretically, that changes in monetary policy in one country have important effects on other economies. In the empirical analysis, I find that the monetary policy of the US has changed since the start of the Euro. This change affected the dynamics of inflation and output in the Eurozone significantly. However, the monetary policy of the ECB is found to be fairly stable. In the theoretical analysis, I show that changes in the monetary policy of one country (labelled foreign) have non-neglible effects on the dynamics of the key macroeconomic variables in the other (home) country. This result is further enhanced as long as the home country does not take into account changes in foreign monetary policy. However, both economies benefit when the home central bank reacts optimally to foreign monetary policy regime shifts.

A popular way of modelling regime changes in monetary policy is by assuming that the interest rate rule coefficients change according to a Markov switching process. Using this approach Davig and Leeper (2007), Liu et al. (2008, 2009), Farmer et al. (2011) and Bekaert et al. (2011) construct closed economy DSGE models in order to analyze the effects of regime shifts in monetary policy on inflation and output.[2] These papers conclude that the expectation of a future regime shift in

[1] There is a huge literature over the causes of a change in inflation volatility in the US. Some authors, such as Stock and Watson (2003), attribute that change to different shock sizes, rather than to changes in the way monetary policy was conducted .

[2] In all of these papers the theoretical analysis is motivated by the empirical estimates about the way monetary policy was conducted

monetary policy has significant effects on inflation and output today. Those effects can be either stabilizing or destabilizing depending on what is the expected future policy.

The existing literature on Markov-switching DSGE models, though, is restricted to a closed economy framework. As a result, so far, the cross country effects of regime shifts in monetary policy have not been analyzed. Therefore, it is important that we have an open economy framework, so that to analyze the effects in one country of a change in monetary policy of another country.

The first contribution of the paper is to provide empirical evidence regarding the international effects of changes in monetary policy. I estimate a SVAR model for the US and the Eurozone using real time monthly data spanning from 1999 through 2010. The empirical model includes seven variables, namely inflation, output gap and the nominal interest rate for both the Eurozone and the US, as well as the real exchange rate. I perform parameter stability tests using the Andrews sup-Wald test, as in Boivin and Giannoni (2002) and the Andrews-Ploberger test.[3] Both tests find that there have been statistically significant changes in the coefficients in the US interest rate equation. This implies that there has been a change in the systematic behaviour of the Fed. However, coefficients in the Eurozone interest rate equation are stable throughout the sample. The Andrews-Ploberger test identifies the break date in June 2004. Therefore, I split the sample into two sub-samples, namely before and after that date. The impulse response analysis shows that the responses of inflation and output gap in the Eurozone are completely different in the two samples.

But what drives the changes in the impulse responses of inflation and output in the Eurozone? In order to answer that question, I perform a countrefactual analysis in the VAR model. I find that the main reason for the change in the impulse responses of those variables was the change in the US monetary policy. I examine also whether changes in the conditions in the Euro area can account for that. I find that their contribution at causing changes in the impulse responses is tiny.

Given the weakness of the SVAR model in uncovering a Taylor rule, a last step in the empirical analysis is to explore whether there have been indeed changes in Fed's contemporaneous reaction to inflation and output gap fluctuations. For this reason I estimate a Taylor rule for the US whose coefficients change over time according to a Markov-switching process. The estimated rule findings validate that the monetary policy of the Fed has changed since the start of the Euro and are in

in the US from 1970 until recently.

[3] I use the Andrews-Ploberger test because of its virtue of identifying the break date.

line with the stability tests from the SVAR model. The rule changes state only once. Notably, the regime change date is very close to the break date identified by the Andrews-Ploberger test in the US interest rate equation. Keeping those findings in mind, I proceed to the construction of a two-country DSGE model.

The theoretical model is similar to that of Benigno and Benigno (2001) and Benigno (2004). I extend their approach by allowing the coefficients in the foreign interest rate rule only to change according to a Markov-switching process. The home country instead adopts a time-invariant Taylor rule with some interest rate smoothing. I show that even though the home monetary policy is constantly (and with a constant coefficient) hawkish[4], home inflation exhibits changes in its volatility over time. Specifically, if there is a positive probability that foreign monetary policy will be dovish[5] in the future, then not only foreign inflation will be more volatile, but also home inflation. This is because both home and foreign agents incorporate this probability in their future inflation expectations.[6] The increase in the volatility of home inflation in this case comes from the home agents expectation of an increasing volatility in the real exchange rate and relative prices. Therefore, commitment to a regime independent interest rate rule proves not to be enough to stabilize the home economy.

Hence, as a next step, I examine the optimal policy of the home country. I solve the optimal policy problem of the home central bank conditional on foreign monetary policy switching regimes over time. I extend Soderlind's (1998) algorithm for solving optimal policy problems in linear rational expectations models to a Markov-switching framework. I show that a time invariant interest rate rule is suboptimal for the home country. The home central bank must be always hawkish. How much hawkish the home central bank should be, depends on the regime which the foreign monetary policy lies in. More specifically, I find that as the probability that the foreign central bank becomes dovish rises, the home central bank should increase the coefficient on inflation further. The opposite holds as the probability that the foreign central bank becomes hawkish increases. The intuition behind this result is that when home agents expect that foreign monetary policy will become dovish, they anticipate an increase in the volatility of home inflation.

[4]Throughout the paper hawkish refers to the case where the coefficient on inflation in the interest rate rule is greater than one. In the literature, this implies that the central bank cares a lot about inflation stabilization.

[5]Throughout the paper dovish refers to the case where the coefficient on inflation in the interest rate rule is less than one. In the literature, this implies that the central bank is more tolerant of inflation fluctuations.

[6]Throughout the paper I assume that the probability of a regime switch is the same for both home and foreign agents.

Hence, the home central bank must react in such a way so that to offset this effect on home agents expectations. And this, as I show, is achieved by increasing the coefficient on home inflation in the home interest rate rule. Additionally, the coefficient on output gap must increase as well, as the foreign monetary policy becomes dovish. This means that when the foreign country changes its policy, then the home must adjust (change) its policy appropriately. Regime switching monetary policy proves to be Pareto superior for the home country. More importantly, I show that when the home central bank reacts optimally to changes in foreign monetary policy, the effects of changes in the latter are eliminated in the home country, and reduced dramatically in the foreign.

The paper is organized as follows. In section 2 a SVAR model is estimated using real time data for the Eurozone and the US, in order to motivate the theoretical model. In section 3 a two country DSGE model is constructed, allowing for regime switching in monetary policy of the foreign country. In section 4, I describe how Markov switching monetary policy is introduced into the model. In section 5, the model is presented in its loglinear form. In section 6 the solution technique of the Markov-Switching DSGE (MSDSGE) is described. In section 7 the model is calibrated and simulated. In section 8 the optimal policy problem of the home central bank is solved, in order to find what the optimal reaction of the latter should be, conditional on foreign monetary policy switching regimes. Section 9 concludes.

# 2 Stylized facts

## 2.1 A SVAR model for the Eurozone and the US

In this section I present a structural VAR model for the Eurozone and the US.

The SVAR model consists of seven variables, namely output gap, inflation rate and nominal interest rates in the Eurozone and the US, and the real exchange rate. Such a model may lead to better policy implications because the regions under consideration are close trade partners and, hence, it is likely that changes or shocks in the monetary policy of one region have important effect on the other. The SVAR model has the following form.

$$A_0 X_t = \Gamma_0 + \Sigma_{i=1}^{p} \Gamma_i X_{t-i} + u_t \tag{1}$$

where $A_0$ is nonsingular, while the variance-covariance matrix of the fundamental disturbances $\Sigma_u = E(u_t, u_t')$ is assumed to be diagonal. The short-run restrictions imposed allow for contemporaneous effects of the CPI rate and the output gap on the policy rate in each region. Therefore, the complete representation of the SVAR model is summarized as follows.

$$\begin{pmatrix} 1 & a_{12} & 0 & a_{14} & 0 & a_{16} & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ a_{31} & a_{32} & 1 & 0 & 0 & 0 & 0 \\ a_{41} & 0 & 0 & 1 & a_{45} & 0 & 0 \\ 0 & a_{52} & 0 & a_{54} & 1 & a_{56} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & a_{75} & a_{76} & 1 \end{pmatrix} \begin{pmatrix} CPI_{Euro} \\ Gap_{Euro} \\ i_{Euro} \\ RER \\ CPI_{US} \\ Gap_{US} \\ i_{US} \end{pmatrix}_t =$$

$$= \begin{pmatrix} \gamma_{10} \\ \gamma_{20} \\ \gamma_{30} \\ \gamma_{40} \\ \gamma_{50} \\ \gamma_{60} \\ \gamma_{70} \end{pmatrix} + \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} & \gamma_{14} & \gamma_{15} & \gamma_{16} & \gamma_{17} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} & \gamma_{24} & \gamma_{25} & \gamma_{26} & \gamma_{27} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} & \gamma_{34} & \gamma_{35} & \gamma_{36} & \gamma_{37} \\ \gamma_{41} & \gamma_{42} & \gamma_{43} & \gamma_{44} & \gamma_{45} & \gamma_{46} & \gamma_{47} \\ \gamma_{51} & \gamma_{52} & \gamma_{53} & \gamma_{54} & \gamma_{55} & \gamma_{56} & \gamma_{57} \\ \gamma_{61} & \gamma_{62} & \gamma_{63} & \gamma_{64} & \gamma_{65} & \gamma_{66} & \gamma_{67} \\ \gamma_{71} & \gamma_{72} & \gamma_{73} & \gamma_{74} & \gamma_{75} & \gamma_{76} & \gamma_{77} \end{pmatrix} \begin{pmatrix} CPI_{Euro} \\ Gap_{Euro} \\ i_{Euro} \\ RER \\ CPI_{US} \\ Gap_{US} \\ i_{US} \end{pmatrix}_{t-1} + \begin{pmatrix} u_{1,t} \\ u_{2,t} \\ u_{3,t} \\ u_{4,t} \\ u_{5,t} \\ u_{6,t} \\ u_{7,t} \end{pmatrix}$$

The reduced form of the VAR model is specified as

$$X_t = A_0^{-1}\Gamma_0 + A_0^{-1}\Sigma_{i=1}^{p}\Gamma_i X_{t-i} + \varepsilon_t$$

where $\varepsilon_t = A_0^{-1}u_t$ are the reduced form errors with a variance-covariance matrix $\Sigma_\varepsilon = E(\varepsilon_t, \varepsilon_t') = A_0^{-1}E(u_t, u_t')A_0^{-1} = A_0^{-1}\Sigma_u A_0^{-1\prime}$.

The target in this section is to ascertain whether there have been changes in the way monetary policy was conducted until today by both the ECB and the Fed. Therefore, for each equation of the SVAR model, the stability of its the coefficients is tested.[7] The first test the Andrews sup-Wald test. The second is the Andrews-Ploberger test.[8] The former has the virtue that it has power against various alternatives, as far as the process of the structural parameters is concerned. The

[7]Evidence of parameter instability in monetary VAR models is mixed. Boivin and Giannoni (2002), Bernanke, Gertler and Watson (1997) and Boivin (2005) find evidence of parameter instability, while Christiano, Eichenbaum and Evans (1999) find the opposite.

[8]Note that the heteroskedasticity robust version of both tests was used.

latter is able to identify the timing of the break, if there is one. If there is evidence of parameter instability, then the impulse responses computed using the model estimated for the whole sample are no longer valid. Therefore, if this is the case, I will split the sample in smaller sub-samples, depending on the timing of the break, estimated by the Andrews-Ploeberger test.

Given that some authors have argued in favour of changes in the size of shocks hitting the economy, rather than changes in the structural parameters, being the reason for changes in the transmission of monetary policy, heteroskedasticity tests in the estimated residuals are also performed. For each equation specific estimated residual the $LM$ test for $ARCH$ effects is used.

## 2.2 Data

Real-time monthly data[9] were gathered from the ECB statistical warehouse and the Federal Reserve Bank of Philadelphia. The dataset spans from 1999:1 though 2010:6. GDP is proxied by total industrial production. CPI for each region is used as the inflation rate. As far as the policy rates are concerned, the Federal Funds rate for the US and the interbank overnight rate for the Eurozone are used. Finally, the nominal exchange rate is measured by the end of period euro-dollar rate.

## 2.3 Empirical results

### 2.3.1 Stability and heteroskedasticity tests

Prior to the estimation of the SVAR model[10], I perform stability tests in each equation's coefficients in the reduced form VAR model. At table 1 below the $p-values$ from both tests are reported[11]. Stability tests show that at 1% significance level, the systematic behaviour of the Fed has changed over the sample considered. Four out of seven coefficients in the equation for the Fed Funds rate have changed over time. On the other hand, monetary policy in the Eurozone has not changed at 1% significance level. At 5% significance level, though, the coefficients on lagged foreign inflation and the real exchange rate appear to have changed. As for the output gap in the Eurozone, it

[9] For the importance of using real-time data for monetary policy prescriptions see Orphanides (2003) and the references therein.

[10] The lag length of the VAR model was chosen based on the $AIC$ and the $BIC$ criterion. Both criteria showed that 2 lags is optimal.

[11] I report $p-values$ obtained only from the Andrews-Ploberger test in order to save space. The results from the Andrews-Quandt test lead to the same conclusions.

is stable. I derive the same result for CPI in the US. On the other hand the coefficients in the Eurozone CPI and the US output gap equations are subject to breaks at 5 %significance levels. Although, it is easy to interpret breaks in the coefficients in the interest rate equations as changes in the way monetary policy is conducted, breaks in the CPI and the output gap equations are less easy to interpret.

Table 1: Stability Tests on Reduced-form VAR coefficients

| | Regressors | | | | | | |
|---|---|---|---|---|---|---|---|
| Dep. vrb | $CPI_{Euro}$ | $Gap_{Euro}$ | $i_{Euro}$ | $RER$ | $CPI_{US}$ | $Gap_{US}$ | $i_{US}$ |
| $CPI_{Euro}$ | 0.0181* | 0.9491 | 0.0189* | 0.0415* | 0.0174* | 0.4007 | 0.0353 |
| $Gap_{Euro}$ | 0.7225 | 0.2944 | 0.7338 | 0.7030 | 0.7407 | 0.3018 | 0.6947 |
| $i_{Euro}$ | 0.0508 | 0.6871 | 0.1231 | 0.0432* | 0.0497* | 0.5500 | 0.0825 |
| $RER$ | 0.0008** | 0.5122 | 0.0002** | 0.0015** | 0.0007** | 0.7031 | 0.0047* |
| $CPI_{US}$ | 0.5558 | 0.4223 | 0.2338 | 0.6056 | 0.5608 | 0.4859 | 0.1903 |
| $Gap_{US}$ | 0.0112* | 0.0561 | 0.0132* | 0.0429* | 0.0112* | 0.1491 | 0.0388* |
| $i_{US}$ | 0.0025** | 0.6122 | 0.0000** | 0.0030** | 0.0026** | 0.2339 | 0.1093 |

Notes: $p - values$ reported. ** Significant at 1% s.l., * Significant at 5% s.l.

As regards Eurozone CPI, it is found that the coefficients on the lagged Eurozone and US CPI rates are subject to breaks. This could be attributed to changes in the degree of openness in the Eurozone, or home bias. Taking into account the structure of a hybrid New-Keynesian Phillips curve, the break in the coefficient on lagged interest rate in the Eurozone CPI equation , could be due to either a change in the frequency of price adjustments, or a change in the degree of backward lookingness in price setting behaviour, or a change in the degree of risk aversion, or change in the degree of habits in consumption, or a combination of all the above. Finally, the changes in the coefficients on lagged Eurozone CPI rate, on lagged Eurozone interest rate, on lagged real exchange rate, on lagged US CPI rate and on lagged US interest rate in the US output gap equation could be attributed to changes in the degree of openness of the US economy, the degree of risk aversion, the degree of endogenous persistence in output, or to a combination of those three factors. I keep, however, the fact that US monetary policy is found to have changed which is the main motivation of this paper.

Finally, the Andrews-Ploberger test showed that the break in the US interest rate equation coefficients took place in June 2004.[12] I use this estimate to split the initial sample into two sub-samples when I will be doing the impulse response analysis in the next section.

The last test performed was on the variance of the estimated equation specific residuals. As already mentioned, I test for this using the $LM$ test for $ARCH$ effects. The results are shown at table 2. Results at table 2 show that at 5% significance level only the variance of the residuals from the Eurozone interest rate equation has changed over time.

Table 2: Heteroskedacticity tests

| | $p-values$ |
|---|---|
| $CPI_{Euro}$ | 0.6088 |
| $Gap_{Euro}$ | 0.1550 |
| $i_{Euro}$ | 0.0105 |
| $RER$ | 0.5734 |
| $CPI_{US}$ | 0.2365 |
| $Gap_{US}$ | 0.4856 |
| $i_{US}$ | 0.4261 |

### 2.3.2 Impulse responses

In this section the impulse responses are computed. I split the initial sample into two sub-samples, according the results from the Andrews-Ploberger test. Namely, until and after June 2004.[13] The impulse responses of the variables are computed for each sub-sample. At figure 1 below I present the responses of CPI in the Eurozone following a contractionary monetary policy shock, a positive cost-push shock, a positive demand shock and a positive RER shock in both the Eurozone and the US.

The impulse responses are different in the two samples. In particular, CPI inflation is more volatile and persistent in the second sample for all kinds of shocks considered[14]. Moreover, the sign

[12]Ben Bernanke in his speech at the annual meeting of the American economic association in 2010 mentions that the FOMC increased its target for the federal funds rate in June 2004.

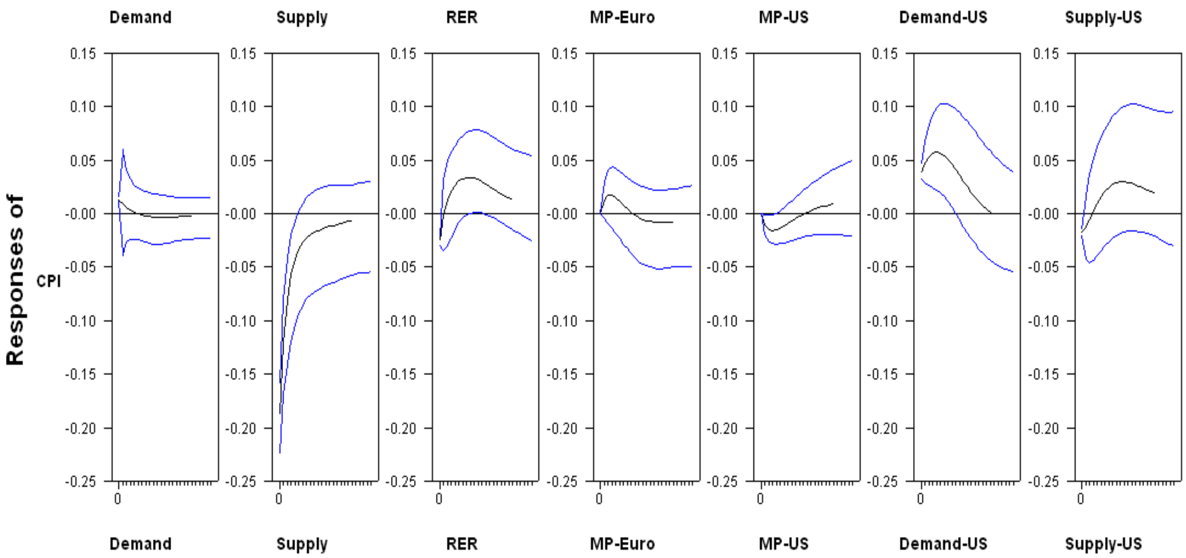
[13]From now on I will refer to the sample spanning from 1999:1 to 2004:6 as Sample 1. Sample 2 will represent the sample spanning from 2004:7 to 2010:6.

[14]Impulse responses of the output gap lead to the same conclusion. The latter is less volatile and persistent after all kinds of shocks, in the first sample.

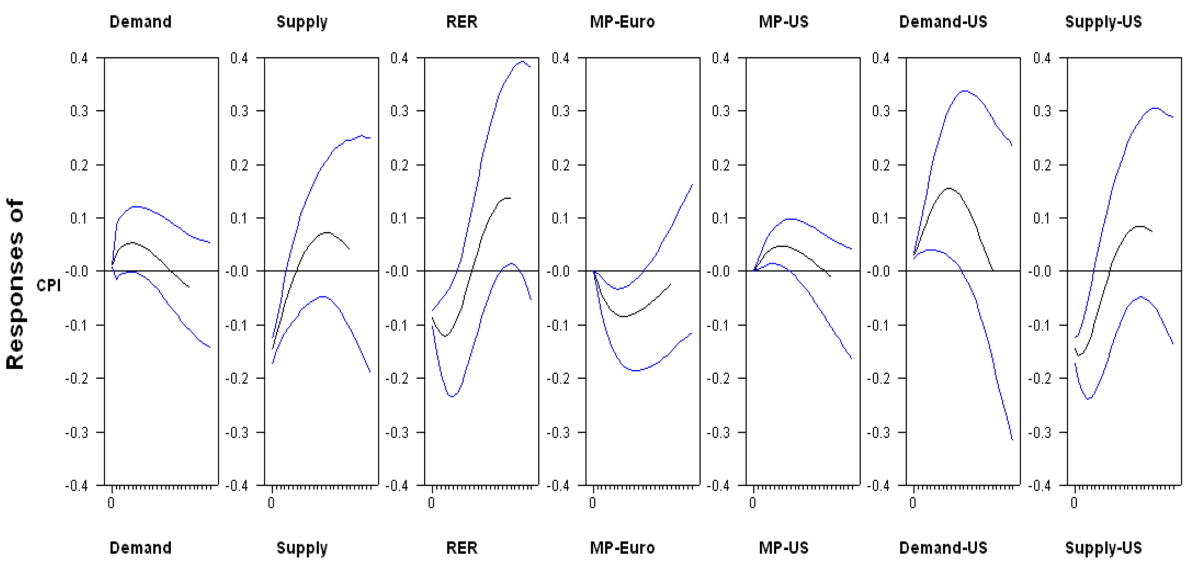
of the initial impact seems to change as well, following a monetary policy shock in the Eurozone and the US. For example, CPI initially jumps in sample 1, after a monetary policy shock in the Eurozone. On the contrary, it falls in sample 2.

**Figure 1: Impulse Responses of Eurozone CPI to alternative shocks**

*Sample 1: 1999:1 - 2004:6*




*Sample 2: 2004:7 - 2010:6*




Notes: Blue lines: 95% posterior confidence interval. Demand: demand shock in the Eurozone. Supply: supply shock in the Eurozone. RER: real exchange rate shock. MP-Euro: monetary policy shock in the Eurozone. MP-US: monetary policy shock in the US. Demand-US: demand shock in the US. Supply-US: supply shock in the US.

*Counterfactual Analysis with the SVAR*

In the previous section, I showed that the responses of Eurozone CPI to monetary policy shocks has changed over time. Given that stability tests suggest that coefficients in equations other than that of the US interest rate have changed as well, it may be that the changes in the impulse responses are due to changes in the coefficients in the nonpolicy part of the VAR rather than the policy one.

For this reason, I now investigate the source of the change in the impulse responses of inflation and output in both countries. I perform a counterfactual exercise on the structural VAR model. I implement two experiments. At the first, I am trying to figure out whether the observed changes in the impulse responses are explained by the change in the US monetary policy, keeping all other coefficients constant. At the second, I allow only for the coefficients in the US output gap and the Eurozone CPI equation to change. This allows me to explore the extent to which the differences in the impulse responses can be attributed to changes in the coefficients in the nonpolicy block of the SVAR model, rather than the policy one.

To address the above two questions, let $T$ characterize US monetary policy, $K$ characterize Eurozone CPI and US GDP and $N$ characterize the remaining part of the economy. In particular, $T_S$ is the set of the estimated parameters of the US interest rate equation, $K_S$ is the set of the estimated parameters in the Eurozone CPI and US GDP equation and $N_S$ is the set of the estimated parameters of the remaining part of the VAR. Subscript $S$ refers to the period within which those parameters have been estimated. For instance a combination $(T_{pre-2004:6},\ K_{pre-2004:6},\ N_{pre-2004:6})$ denotes the set of all the estimated parameters in the Sample 1. This set of parameters characterizes completely the impulse response functions computed for that sample. On the other hand a combination $(T_{post-2004:6},\ K_{post-2004:6},\ N_{post-2004:6})$ denotes the set of all the estimated parameters in Sample 2.

In order to answer the first question (i.e. whether the change in the impulse responses is due to a change in the US monetary policy) I will use $(T_{post-2004:6},\ K_{pre-2004:6},\ N_{pre-2004:6})$. That is, keeping all other coefficients fixed and allowing only the coefficients in the US interest rate equation to change, I will compute the new impulse response functions. The same strategy will be followed in order to answer the second question. Since, now, the focus is on the effect of changes in the parameters in the Eurozone CPI and the US GDP equations, I will keep all other coefficients fixed. In particular, the new impulse response functions are obtained using the combination $(T_{pre-2004:6},\ K_{post-2004:6},\ N_{pre-2004:6})$. Table 3 gives a picture of the two experiments. In the left column, I indicate the impulse response functions that will be used in each experiment. In the right column I refer to the coefficients used for the computation of each impulse response function.

Table 3: Counterfactual Analysis

| *Experiment 1: Changes only in US interest rate equation coefficients* | |
|---|---|
| Impulse Response | Set of coefficients used |
| Sample 1 | $(T_{pre-2004:6},\ K_{pre-2004:6},\ N_{pre-2004:6})$ |
| Counterfactual | $(T_{post-2004:6},\ K_{pre-2004:6},\ N_{pre-2004:6})$ |
| Sample 2 | $(T_{post-2004:6},\ K_{post-2004:6},\ N_{post-2004:6})$ |

| *Experiment 2: Changes only in US GDP and Euro CPI equation coefficients* | |
|---|---|
| Impulse Response | Set of coefficients used |
| Sample 1 | $(T_{pre-2004:6},\ K_{pre-2004:6},\ N_{pre-2004:6})$ |
| Counterfactual | $(T_{pre-2004:6},\ K_{post-2004:6},\ N_{pre-2004:6})$ |
| Sample 2 | $(T_{post-2004:6},\ K_{post-2004:6},\ N_{post-2004:6})$ |

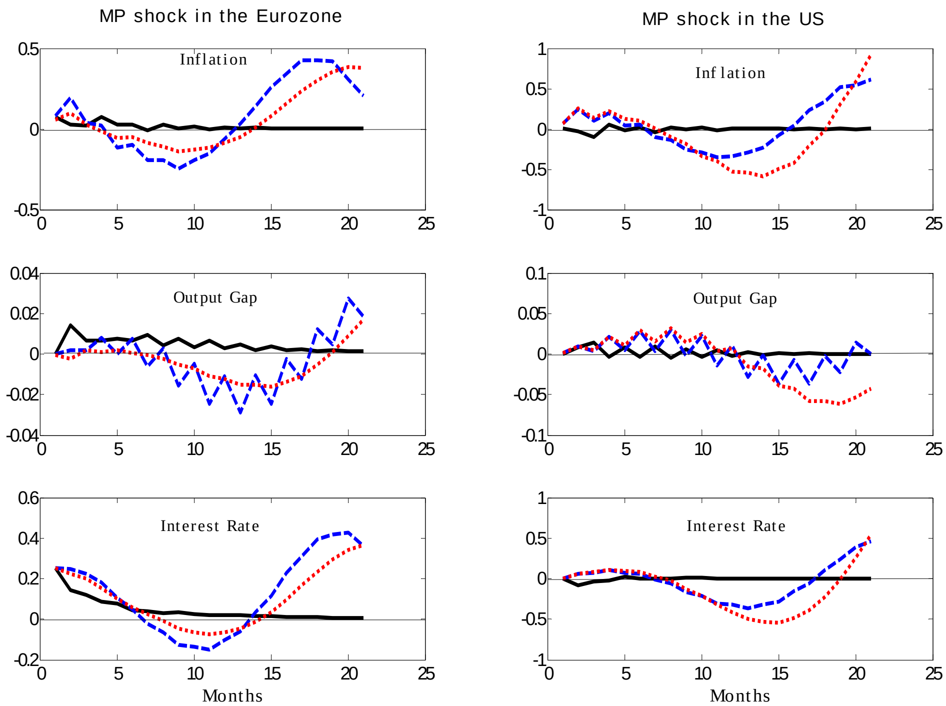
The impulse responses from experiments 1 and 2 are illustrated in panel (a) and (b) in figure 2. The impulse response functions in panel (a) in figure 2 show that changes in the US interest rate coefficients account more for the change in the impulse responses in the Sample 1. In fact, the blue dashed line (counterfactual impulse response) moves close to the red dotted line, which is the impulse response function in Sample 2.

On the other hand, as shown in panel (b), when only the coefficients in the US output gap and the Eurozone CPI equations change, the impulse response functions in Sample 1 do not seem to be affected significantly. The blue dashed line, now, moves very close to the black solid line in all cases. Therefore, the two experiments show that it is indeed the change in the US systematic reaction that caused the change in the impulse response functions of inflation and output gap in the Eurozone.[15]

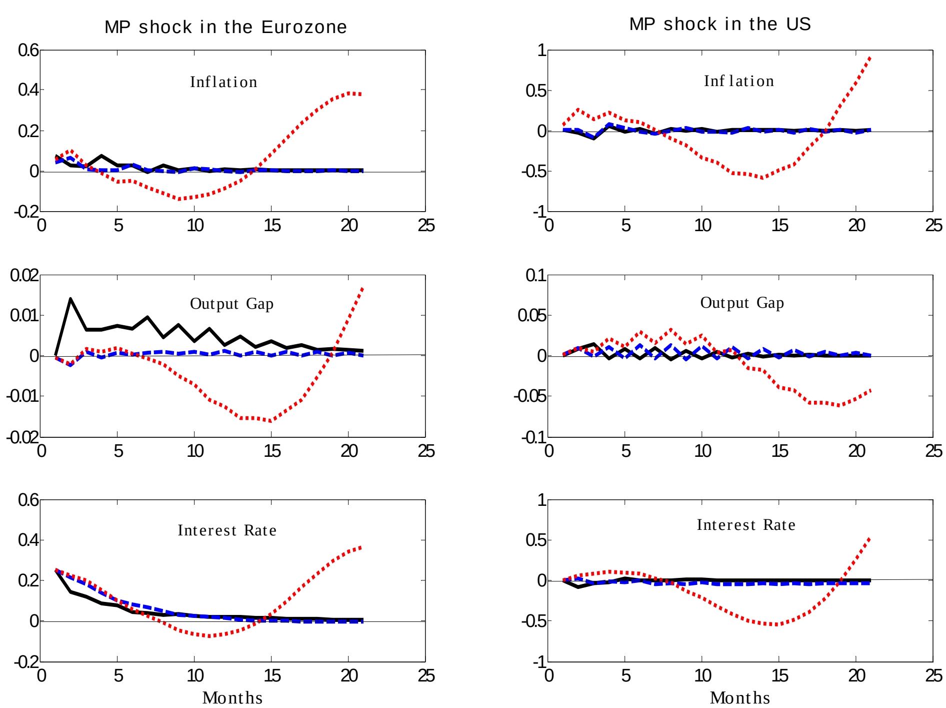
[15] Note that the results are the same for US CPI inflation and the output gaps of both countries. I do not present them here, in order to save space.

**Figure 2: VAR Counterfactual Exercise**

*Panel (a): Experiment 1 - Changes only in US interest rate equation coefficients*




*Panel (b): Experiment 2 - Changes only in US GDP and Euro CPI equation coefficients*




Notes: Red dotted line: Impulse responses from Sample 2. Blue dashed line: Counterfactual impulse responses. Black solid line: impulse responses from Sample 1.

### 2.3.3 Robustness checks

In order to check the sensitivity of the results found so far, various robustness exercises are implemented. The first one considers alternative measures for the output gap. The procedure followed is similar to that in CGG (2000). In particular, instead of using the hp-filter, the output gap was measured as the deviation of *log* industrial output from a fitted quadratic function of time. The results do not differ significantly.[16] Both the $AIC$ and the $BIC$ information criteria show that two is the optimal choice of lags in the VAR model. The parameter stability tests do not differ significantly from those reported at table 1 above. The Andrews-Ploberger test locates a break in the parameters in the Federal Funds rate equation in June 2004, as was the case when the hp-filter was used. However, what seems to change now is the coefficients only on the lags of the Euro-rate at 1% significance level. The coefficients on the rest the parameters remain unchanged.[17] The $LM$ test for $ARCH$ effects provides the same results as before. That is, only the the variance of the errors in the Euro-rate equation changes at 1% significance level. Finally, the impulse responses lead to the same conclusion as above. Both the CPI and the output gap in the Eurozone responses are different in the two sub-samples.

As a second exercise, a more parsimonious SVAR model was constructed. Given that the dataset is small, it is likely that the impulse responses may not be accurate, the higher the number of the free parameters to be estimated in matrix $A$ in (1). Therefore, a new SVAR model was estimated allowing for $a_{31}, a_{32}, a_{75}, a_{76}$ to be the only free parameters to be estimated. The key results, found so far, do not change. The impulse responses of the CPI and the the output gap in the Eurozone show that both are more volatile and persistent in sample 2.[18]

Moreover, the importance of additional targets in the interest rate rule of both central banks was tested. That is, it was assumed that the each of rest the variables in the system has a contemporaneous effect on the interest rate of each region. At first, the strategy followed was to

[16] I do not show the results of the robustness exercise here, in order to save space.

[17] Remember that when the $hp-filter$ was used, the Andrews-Ploberger test found that the coefficients on the US and the Euro CPI, the Eurozone output gap and the real exchange rate change, as well, apart from those on the lags of the Euro-rate.

[18] Setting $a_{12} = a_{16} = a_{52} = a_{56} = a_{75} = a_{76} = 0$ has negligible effects on the impulse responses. Setting, though, $a_{14} = a_{54} = 0$ has non-neglible effects on the impulse responses. That is, allowing for a contemporaneous effect of real exchange rate shocks on the CPI in either country changes the behavior of both the output gap and inflation. In the first sub-sample, the Eurozone output gap is less volatile after a shock to the RER than when $a_{14}, a_{54} \neq 0$. The same holds for the Eurozone CPI. In the second subsample, the Eurozone CPI is much less volatile after a shock to the RER. Following a demand shock, though, the latter is more volatile. The output gap in the Eurozone is more volatile after a RER shock whenver $a_{14} = a_{54} = 0$. However, as regards the rest of the shocks, the effects of not allowing for contermporaneous effects of RER shocks to the CPI are negligible. Finally, note that still the main conclusion does not change. All variables are more volatile in the secong sub-sample.

test the importance of each of the parameters in matrix $A$ individually, so that to avoid the cost of loosing degrees of freedom. Then, the case where both banks reacting to foreign variables or the RER, jointly, was considered. In this case, both central banks achieve a better control of inflation but only in sample 1. It is enough that only one of the two banks adopts a target for the real exchange rate. However, the opposite holds in sample 2, where RER targeting does worse than the initial specification in matrix $A_0$. Reacting to foreign inflation yields non-neglible gains[19] to both regions. But this holds only for sample 1. Moreover, the sign of the initial responses of some variables, after some shocks, seems to be reversed. When both banks react to the foreign interest rate, there are significant gains regarding inflation fluctuations, in sample 1, especially after a monetary policy shock in the Eurozone. On the contrary, this no longer holds in sample 2 where reacting to the foreign rate seems not preferable. Finally, foreign output gap targeting allows for lower inflation and output fluctuations in both regions, regardless of the sample.

The possibility, though, of both central banks targeting at the same time foreign variables and/or the real exchange rate was also considered. The differences with the initial results are negligible.

#### 2.3.4 A Markov switching interest rate rule for the US

Taking into account the stability test results of section 2.4.1 and given the weakness of the SVAR models in uncovering a Taylor rule, I now estimate a Markov-switching interest rate rule for the US. This allows me to explore whether there were indeed changes in the reaction of the Fed against inflation and output gap fluctuations. The rule is specified as

$$i_t = \alpha_0(s_t) + \alpha_\pi(s_t)\pi_t + \alpha_x(s_t)x_t + \varepsilon_t \tag{2}$$

where $\pi_t$ is inflation and $x_t$ is the output gap. $s_t$ indicates the monetary policy regime and follows a two-state Markov chain. The sample I use is the same as that used for the estimation of the structural VAR model above. Table 4 reports the parameter estimates.

[19] By gains, I mean lower inflation and output gap fluctuations.

Table 4: Monetary policy rule estimates

| *States* | *Hawkish* | *Dovish* |
|---|---|---|
| | $s_t = 1$ | $s_t = 2$ |
| $\alpha_\pi$ | 1.1621 (0.00) | 0.3298 (0.05) |
| $\alpha_x$ | 1.5640 (0.01) | 0.9499 (0.02) |
| $\sigma_\varepsilon$ | 0.555436 | 0.735924 |

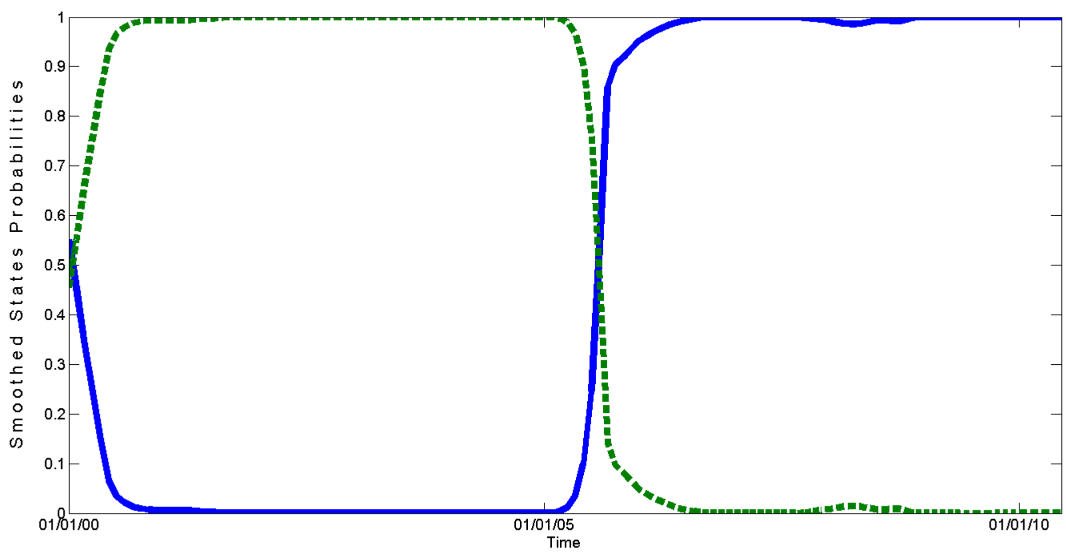
Log likelihood value = -188.5974. P-values in parentheses

the estimated transition matrix is as follows:

$$P = \begin{bmatrix} 0.99 & 0.01 \\ 0.01 & 0.99 \end{bmatrix} \tag{3}$$

Figure 3 below plots the estimated transition probabilities for each regime.

**Figure 3: Smoothed States Probabilities**




Notes: Blue solid line: Dovish (State 2). Green dashed line: Hawkish (State 1).

The estimated Markov-switching Taylor rule shows that the Fed started being hawkish since the start of the Euro and then switched to be more reluctant to inflation fluctuations from 2005 onwards. The regime change date is very close to what stability tests in section 2.3.1 suggest about the coefficients in the US interest rate equation. Note that the SVAR model specified cannot uncover a Taylor rule. However, the Markov-switching specification in this section does. Moreover, it ensures that there was indeed a change in the coefficients in the interest rate rule of the Fed throughout the sample considered.

#### 2.3.5 Key Results

From the empirical analysis above, I keep the following key messages. The first is that there were changes in US monetary policy since the adoption of the common currency in Europe which have affected the bahaviour of key macroeconomic variables not only in the US, but also in the Eurozone. Moreover, this change in US monetary policy has affected the way macroeconomic aggregates react to various kinds of domestic and foreign shocks. Therefore, changes in the way monetary policy is conducted in the foreign country (US) have important implications on the behaviour of the home country (Eurozone) macroeconomic variables, even though domestic monetary policy does not change. The degree of openness and, hence, terms of trade effects are likely to be one of the main driving forces for this result. The second is that, there were changes in the behavior of the private sector, as well. The counterfactual analysis, though, shows that their effect is small at changing the behavior of inflation and output in either region. Finally, a markov-switching interest rate rule for the US is in line with the stability tests in the SVAR model and provides evidence in favour of changes in the coefficients on inflation and output gap. Keeping those facts I proceed to the construction of a two country DSGE model, in order to explore theoretically what are the international effects of regime changes in foreign monetary policy. I then solve for the optimal policy problem of the home Central Bank, conditional on foreign monetary policy switching regimes over time.

# 3 The model

## 3.1 Households

In this section, I specify the structure of the baseline, two country stochastic general equilibrium model. Each country is populated by a continuum of infinitely lived and identical households in the interval $[0, 1]$. Foreign variables are denoted with an asterisk.

Persistence has been found to be an important feature of output in Eurozone and the US.[20] For this reason I introduce endogenous persistence in consumption by assuming that there are two kinds of households as in Amato and Laubach (2003). Let $\psi$ denote the probability that the household is able to choose its consumption optimally, and which is independent of the household's history. Therefore, by the law of large numbers, in each period a fraction $\psi$ of households will reoptimise, whereas the remaining fraction $1-\psi$ will not. The latter will choose its consumption in period $t$ according to the following rule of thumb

$$C_t^R = C_{t-1} \tag{4}$$

where $C_t$ denotes aggregate per capita consumption in period $t$. The remaining $1-\psi$ of households choose $C_t^O$ so as to maximize their utility. Thus, per capita cunsumption in period $t$ is given by

$$C_t = \psi C_t^O + (1-\psi) C_t^R \tag{5}$$

As in Laubach and Amato, this modification to the consumer's problem is based on the assumption that it is costly to reoptimise every period[21]. The households who choose consumption optimally choose $C_t^O$ to maximize their utility function. They derive utility from consumption and disutility from labor supply. The utility function, thus, is specified as

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ \frac{(C_s)^{1-\sigma}}{1-\sigma} - \frac{(L_s)^{1+\gamma}}{1+\gamma} \right] \tag{6}$$

where $\sigma$ is the degree of relative risk aversion.

Home agents consume home and foreign goods. Therefore, per capita consumption $C_t$ is a composite consumption index described as

[20] Smets and Wouters (2005), Sahuc and Smets (2008) and Adjemian et al. (2008) using Bayesian techniques to estimate DSGE models for the Eurozone and the US find that output persistence in both regions is high.

[21] Amato and Laubach note that Rule (4) has the important feature that rule-of-thumb consumers learn from optimizing households with one period delay. Hence, although Rule (4) is not optimal, it has three important properties. First agents are not required to compute anything. Second, rule-of-thumb households learn from optimizing ones, because last period's decisions by the latter are part of $C_{t-1}$. Third, the differences between $C_t^R$ and $C_t^O$ are bounded, and will be zero in the steady state.

$$C_t = \left[\delta^{\frac{1}{\rho}} C_{H,t}^{\frac{\rho-1}{\rho}} + (1-\delta)^{\frac{1}{\rho}} C_{F,t}^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$$

$$\rho > 1 \tag{7}$$

$$C_t^* = \left[(\delta^*)^{\frac{1}{\rho}} (C_{F,t}^*)^{\frac{\rho-1}{\rho}} + (1-\delta^*)^{\frac{1}{\rho}} (C_{H,t}^*)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$$

where $\rho$ captures the intratemporal elasticity of substitution between home and foreign goods. $\delta > \frac{1}{2}$ is a parameter of home bias in preferences.$C_H$ and $C_F$ is the home and foreign goods consumption index respectively, in the home country. In the foreign country $C_H^*$ and $C_F^*$ is the home and foreign goods consumption index respectively. Consumption indices in the two countries are defined as

$$C_{H,t} = \left[\int_0^1 c_t(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}, \; C_{F,t} = \left[\int_0^1 c_t(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}$$

$$\tag{8}$$

$$C_{H,t}^* = \left[\int_0^1 c_t^*(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}, \; C_{F,t}^* = \left[\int_0^1 c_t^*(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}$$

The aggregate consumption price index for the home and foreign country is specified as

$$P_t = \left[\delta (P_{H,t})^{1-\rho} + (1-\delta) P_{F,t}^{1-\rho}\right]^{\frac{1}{1-\rho}}$$

$$\tag{9}$$

$$P_t^* = \left[\delta^* (P_{F,t}^*)^{1-\rho} + (1-\delta^*) {P_{H,t}^*}^{1-\rho}\right]^{\frac{1}{1-\rho}}$$

where $P_H$ and $P_F$ are price indices for home and foreign goods, expressed in the domestic currency. The price indices for the home and foreign country are defined as

$$P_{H,t} = \left[\int_0^1 p_t(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}} \quad , \quad P_{F,t} = \left[\int_0^1 p_t(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}} \tag{10}$$

$$P_{H,t}^* = \left[\int_0^1 p_t^*(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}} \quad , \quad P_{F,t}^* = \left[\int_0^1 p_t^*(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}}$$

Capital markets are complete. The consumers of both countries purchase state uncontingent bonds denominated in the domestic currency, $B_t$ for domestic agents and $B_t^*$ for foreign agents at price $Q_t$. That is $B_t$ denotes the home agent's holdings of a one period nominal bond paying one unit of the home currency.

The home agent maximizes her utility subject to the period budget constraint

$$P_tC_t + Q_{t,t+1}B_{t+1} = B_t + W_tL_t + \Pi_t \tag{11}$$

where $W_t$ is the nominal wage and $\Pi_t$ are nominal profits the individual receives.

### 3.2 First order conditions

Maximizing the utility function (6) subject to the budget constraint (11) yields the following first order conditions

$$Q_{t,t+1} = \frac{\beta P_t}{P_{t+1}} \left( \frac{C_t^O}{C_{t+1}^O} \right)^{\sigma} \tag{12}$$

$$L_t = (C_t^O)^{-\frac{\sigma}{\gamma}} w_t^{\frac{1}{\gamma}} \tag{13}$$

where the first equation is the usual Euler equation while the second determines the labor supply schedule.

Individual demands for each good $i = h,\ f$ produced in the home and in the foreign country respectively are expressed as

$$c_{h,t}(h) = \left( \frac{p_t^h(h)}{P_{H,t}} \right)^{-\theta} \left( \frac{P_{H,t}}{P_t} \right)^{-\rho} \delta C_t \tag{14}$$

$$c_{f,t}(h) = \left( \frac{p_t^*(h)}{P_{F,t}} \right)^{-\theta} \left( \frac{P_{F,t}}{P_t} \right)^{-\rho} (1-\delta) C_t \tag{15}$$

### 3.3 Risk sharing

The fraction of foreign households who choose their consumption optimally $(\psi^*)$ , maximize their utility subject to their budget constraint specified as

$$P_t^* C_t^* + \frac{Q_{t,t+1} B_{t+1}^*}{z_t} = \frac{B_t^*}{z_t} + W_t^* L_t^* + \Pi_t^* \tag{16}$$

where $z_t$ is the nominal exchange rate defined as the domestic currency price of the foreign currency. Therefore, the Euler equation from the foreign agent's maximization problem is

$$Q_{t,t+1} = \frac{\beta P_t^* z_t}{P_{t+1}^* z_{t+1}} \left( \frac{C_t^{O*}}{C_{t+1}^{O*}} \right)^{\sigma} \tag{17}$$

International financial markets are complete. Domestic and foreign households trade in the state contingent one period nominal bonds denominated in the domestic currency. Therefore, combining (12) and (17) , I receive the following optimal risk sharing condition

$$\left( \frac{C_t^{O*}}{C_t^{O}} \right)^{-\sigma} = \varpi q_t \tag{18}$$

where $\varpi \equiv \left( \frac{C_0^f + x}{C_0^h + x} \right)^{-\sigma} \frac{P_0}{z_0 P_0^*}$ depends on initial conditions and $q_t = \frac{z_t P_t^*}{P_t}$ is the real exchange rate.

### 3.4 Price setting

There is local currency pricing in both countries. That is, each firm sets one price for its goods consumed domestically and another for the same good consumed abroad. Prices are sticky with a price setting behavior *àℓa* Calvo (1983). At each date, each firm changes its price with a probability $1-\omega$ , regardless of the time since it last adjusted its price. The probability of not changing the price, thus, is $\omega$. The probability of not changing the price in the subsequent $s$ periods is $\omega^s$. Consequently, the price decision at time $t$ determines profits for the next $s$ periods. The price level for home goods at date $t$ will be defined as

$$P_{H,t} = \left[ \omega P_{H,t-1}^{1-\theta} + (1-\omega) \widetilde{p}_t(h)^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{19}$$

In the literature on inflation dynamics in the Eurozone and the US its has been found that persistence is one of the key features. Therefore, I introduce endogenous inflation persistence by assuming that firms that are given the opportunity to adjust their prices will either follow a rule of thumb (backward looking firms) or will chose the price that maximizes their expected discounted profits (forward looking firms), as in Gali et al. (2001). The price $\tilde{p}_t(h)$ that will be set at date $t$ is specified as

$$\tilde{p}_t(h) = \zeta p_t^B(h) + (1-\zeta)\, p_t^{For}(h) \tag{20}$$

where $\zeta \in (0,1)$ is the fraction of backward looking firms, $p_t^B(h)$ and $p_t^{For}(h)$ is the price set by the backward and the forward looking firms, respectively. A continuum of firms is assumed for the home economy indexed by $h \in [0,1]$. Each firm produces a differentiated good, with a technology

$$Y_t(h) = A_t L_t(h) \tag{21}$$

where $A_t$ is a country specific productivity shock at date $t$ which is assumed to follow a log stationary process

The structure of productivity shocks across the two countries receives the following form

$$\begin{bmatrix} \alpha_t \\ \alpha_t^* \end{bmatrix} = \begin{bmatrix} \rho_{\alpha_t} & \rho_{\alpha_t \alpha_t^*} \\ \rho_{\alpha_t^* \alpha_t} & \rho_{\alpha_t^*} \end{bmatrix} \begin{bmatrix} \alpha_{t-1} \\ \alpha_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_{\alpha,t} \\ \varepsilon_{\alpha^*,t}^* \end{bmatrix}$$

where $\begin{bmatrix} \varepsilon_{\alpha,t} \\ \varepsilon_{\alpha^*,t}^* \end{bmatrix} \sim N(0, \Sigma^2)$, with $\Sigma^2 = \begin{bmatrix} \sigma_{\varepsilon_a}^2 & 0 \\ 0 & \sigma_{\varepsilon_{\alpha^*}^*}^2 \end{bmatrix}$.

*Backward looking firms.*

Backward looking firms set their prices according to the following rule

$$p_t^B(h) = P_{H,t-1} + \pi_{H,t-1} \quad \textit{and} \quad p_t^{B*}(h) = P_{H,t-1}^* + \pi_{H,t-1}^* \tag{22}$$

*Forward looking firms.*

Forward looking firms set their prices by maximizing their expected discounted profits. Their maximization problem comprises of two decisions. The one concerns the price for the domestic market and the other the price charged in the foreign market, when it exports. Hence their maximization problem is described as

$$maxE_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} \left\{ \widetilde{p_t}(h) y^h_{t+s}(h) + \varepsilon_t \widetilde{p_t}^*(h) y^f_{t+s}(h) - W^h_{t+s} L^h_{t+s} \right\} \tag{23}$$

where $y^i_t(h),\ i = h, f$ is the demand for the home good for home and foreign agents specified as

$$y^h_t(p_t(h)) = \left( \frac{\tilde{p}_t(h)}{P_{H,t}} \right)^{-\theta} \left( \frac{P_{H,t}}{P_t} \right)^{-\rho} \delta^* C_t, \tag{24}$$

$$y^f_t(p^*_t(h)) = \left( \frac{\tilde{p}^*_t(h)}{P^*_{H,t}} \right)^{-\theta} \left( \frac{P^*_{H,t}}{P^*_t} \right)^{-\rho} (1 - \delta^*) C^*_t \tag{25}$$

The firm maximizes its objective function (23) subject to (24) in order to find the optimal price for the home good in the home economy. It maximizes subject to (25), in order to find the optimal price for the home good in the foreign economy. The firm chooses a price for the home good in the home economy that satisfies the first order condition

$$E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} y_{t+s}(p_t(h)) \left\{ p_t(h) - \frac{\theta}{\theta - 1} MC_{t+s} \right\} = 0$$

where $MC_{t+s} = \frac{W_{t+s}}{A_{t+s}}$ denotes the nominal marginal cost and $\frac{\theta}{\theta-1}$ captures the optimal markup. The optimal price for the home good in the home country is specified as

$$p_t(h) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} MC_{t+s} y^h_{t+s}(p_t(h))}{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} y^h_{t+s}(p_t(h))} \tag{26}$$

Respectively, the optimal price for the home good in the foreign country is specified as

$$p^*_t(h) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} MC_{t+s} y^f_{t+s}(p^*_t(h))}{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} y^f_{t+s}(p^*_t(h)) z_{t+s}} \tag{27}$$

*Aggregate price level*

Dividing (19) by $P_{H,t-1}$:

$$\Pi^{1-\theta}_{H,t} = \omega + (1 - \omega) \left( \frac{\widetilde{p_t}(h)}{P_{H,t-1}} \right)^{1-\theta} \tag{28}$$

where $\Pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$.

Similarly, for the foreign goods consumed in the home economy:

$$\Pi_{F,t}^{1-\theta} = \omega + (1-\omega)\left(\frac{\widetilde{p}_t(f)}{P_{F,t-1}}\right)^{1-\theta} \tag{29}$$

The aggregate price level dynamics are specified, thus, as

$$\Pi_t^{1-\rho} = \delta\left[\left(\frac{P_{H,t-1}}{P_{t-1}}\right)\Pi_{H,t}\right]^{1-\rho} + (1-\delta)\left[\left(\frac{P_{F,t-1}}{P_{t-1}}\right)\Pi_{F,t}\right]^{1-\rho} \tag{30}$$

# 4 Markov Switching Monetary Policy

Monetary policy in each country is conducted through nominal interest rate rules by each central bank. Only foreign monetary policy is assumed to switch regimes over time. I first show that even though domestic monetary policy does not change its policy, a switch in the foreign monetary policy has important effects on home domestic output and inflation. In section 8, it is shown that optimal monetary policy for the home country suggests it changes the coefficients in its interest rate rule, depending on which regime foreign monetary policy lies in and, of course, on the probabilities of a switch.

## 4.1 Policy rules

In this subsection I describe how Markov switching is introduced into the model. A markov swtiching interest rate rule for the foreign country is specified as

$$i_t^* = i_{t-1}^{*\rho_{s_t}^*}\left(\xi_{s_t}^*\left(\frac{\pi_t^*}{\tilde{\pi}^*}\right)^{\phi_{\pi^*,s_t}^*}\tilde{y}_t^{*\phi_{y^*,s_t}^*}\right)^{1-\rho_{s_t}^*} e^{\varepsilon_t^*} \tag{31}$$

where $s_t$ captures the realized policy regime taking values 1 or 2. Regime follows a Markov process with transition probabilities $p_{ji} = P\left[s_t = i | s_{t-1} = j\right]$, where $i, j = 1, 2$. $\xi_t$ is a scale parameter, $\tilde{\pi}^*$ is the inflation target and $\tilde{y}_t^*$ is the output gap. This specification implies that the policy maker and the private sector does not observe the current regime. Therefore, private sector expectations about future inflation, for example, are specified as $E\left[\pi_{t+1}|\Omega_t^{-s}\right]$, where $\Omega_t^{-s} =$

$\left\{s_{t-1},\ldots,\varepsilon_t,\varepsilon_{t-1},\ldots,\varepsilon_t^*,\varepsilon_{t-1}^*,\ldots\right\}$ captures its information set. Having assumed a two regime markov process for monetary policy, the transition probability matrix $P$ receives the form

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

where $p_{11}$ measures the probability of staying at date $t$ in regime 1 and $p_{12}$ the probability of moving to regime 2 at date $t$ while being in regime 1 at date $t-1$. $p_{22}$ measures the probability of staying in regime 2 at date $t$ and $p_{21}$ the probability of moving to regime 1 at date $t$ while being in regime 2 at date $t-1$.

Monetary policy may switch because of various reasons. One of them could be the switch of the interests of the central banker. There may be periods, for example, that he is more interested in output gap fluctuations rather than inflation. As a result, the weight on inflation in the interest rate rule could be lower. A monetary policy switch may also be justified by the change of the central banker. As already mentioned, there is a number of papers arguing that the US monetary policy has been more tolerant as regards inflation fluctuations in the pre-Volcker period.

The empirical findings in section 2 showed that there was a change in impulse response functions and the volatility of inflation in the Eurozone, even though the monetary policy of the latter remained unchanged. I keep this finding, at first, and assume that the interest rate of home central bank has time invariant coefficients. A standard Taylor rule with interest rate smoothing is adopted which can be summarized as

$$i_t = i_{t-1}^{\rho}\left(\left(\frac{\pi_t}{\tilde{\pi}}\right)^{\phi_\pi}\tilde{y}_t^{\phi_y}\right)^{1-\rho}e^{\varepsilon_t} \tag{32}$$

## 5 Log linearized model

A log linearized version of the relationships found in the previous section serves in providing a way to deal with the problem of no closed form solution. The model is loglinearized around a specific steady state. Given the markov-switching nature of the model, it is necessary to provide the necessary and sufficient conditions which guarantee that the steady state of the model is unique, and, thus, independent of regime changes. This can be summarized in the following proposition, which is a simple extension to that in Liu, Waggoner and Zha (2008) for the closed economy case

**Proposition:** *The steady state equilibrium values of aggregate output, consumption and the real wage in both countries are independent of monetary policy and are thus invariant to monetary policy regime shifts. Moreover, as long as domestic monetary policy does not change regimes, it is enough that*

$$\xi^*_{s_t} = \frac{1}{\beta}\tilde{\pi}^* \bar{y}^{*-\phi^*_{y^*,s_t}},$$

*where $\bar{y^*}$ is the steady state foreign output gap, so that the steady state nominal variables are given by $\pi = \tilde{\pi}, \quad \pi^* = \tilde{\pi}^*$, $R = \frac{\lambda}{\beta}\tilde{\pi}$ and $R^* = \frac{\lambda^*}{\beta}\tilde{\pi}^*$, and which are independent of regime changes as well.*

**Proof.** See appendix E. □

### 5.1 Supply side

I use a first order Taylor approximation around the steady state of zero inflation rate. Log linearized variables are denoted with a hat.

After loglinearizing the first order condition (12), the production function (21) the demand schedules faced by each firm (24) and (25) and optimal price setting rules (26) and (27), I receive the two relations describing the domestically consumed home goods inflation rate and the respective of the home goods consumed in the foreign country

$$\pi_{H,t} = b_{\pi_{H,-1}}\pi_{H,t-1} + b_{\pi^*_{H,-1}}\pi^*_{H,t-1} + \beta E_t \pi_{H,t+1} + b_{\pi^*_H}\pi^*_{H,t} + b_C \hat{C}_t + \dots$$

$$\dots + b_T \hat{T}_t + b_{T^*}\hat{T}^*_t + b_q \hat{q}_t + b_a a_t \tag{33}$$

$$\pi^*_{H,t} = b_{\pi_{H,-1}}\pi_{H,t-1} + b_{\pi^*_{H,-1}}\pi^*_{H,t-1} + \beta E_t\pi^*_{H,t+1} + b^*_{\pi_H}\pi_{H,t} + b^*_C\hat{C}_t + \dots$$

$$\dots + b^*_T\hat{T}_t + b^*_{T^*}\hat{T}^*_t + b^*_q\hat{q}_t + b^*_a a_t \tag{34}$$

where $T_t = \frac{P_{F,t}}{P_{H,t}}$ and $T^*_t = \frac{P^*_{H,t}}{P^*_{F,t}}$ denote relative prices in the home and foreign country respectively. The log linearized aggregate price level relation (30) is specified as

$$\pi_t = \pi_{H,t} + (1-\delta)(\pi_{F,t} - \pi_{H,t}) \tag{35}$$

which can be further simplified as[22]

$$\pi_t = \pi_{H,t} + (1-\delta)\Delta\hat{T}_t$$

### 5.2 Demand side

In this section I proceed to the loglinearization of the Euler equation

$$\hat{C}^O_t = \kappa(i_t - E_t\pi_{t+1}) + E_t\hat{C}^O_{t+1} \tag{36}$$

where $\kappa = -\frac{1}{\sigma}$, and using (5) the Euler equation receives the forward form, which includes both backward and forward looking elements

$$\hat{C}_t = \frac{\kappa\psi}{2-\psi}(i_t - E_t\pi_{t+1}) + \frac{1}{2-\psi}E_t\hat{C}_{t+1} + \frac{1-\psi}{2-\psi}\hat{C}_{t-1} \tag{37}$$

Goods market clearing assumes the following two conditions

$$Y = C_H + C^*_H + G_t \quad and \quad Y^* = C_F + C^*_F + G^*_t$$

where $G_t$ and $G^*_t$ capture government expenditures for home and foreign country respectively, assumed to follow an exogenous stationary $AR(1)$ process $g_t = \rho_g g_{t-1} + \varepsilon_{g,t}$ and $g^*_t = \rho_{g^*} g^*_{t-1} + \varepsilon^*_{g,t}$, $\varepsilon_{g,t} \sim N(0, \sigma^2_{\varepsilon_g})$ and $\varepsilon^*_{g,t} \sim N(0, \sigma^{*2}_{\varepsilon_g})$.

[22]To end up to that expression, I used equation $\hat{T}_t = \hat{T}_{t-1} + \pi_{F,t} - \pi_{H,t}$ for the relative price which is reported later in the text.

Combining equation (35) and the market clearing conditions, I derive the aggregate demand equation:

$$\hat{Y}_t = \eta_1 \hat{Y}_{t-1} + \eta_2 E_t \hat{Y}_{t+1} + \eta_3 (i_t - E_t \pi_{t+1}) + \eta_4 \hat{q}_t + \eta_5 \hat{q}_{t+1} + \eta_6 \hat{q}_{t-1} + \dots$$
$$\dots + \eta_7 \Delta \hat{T}_t + \eta_8 E_t \Delta \hat{T}_{t+1} + \eta_9 \Delta \hat{T}^*_t + \eta_{10} E_t \Delta \hat{T}^*_{t+1} \tag{38}$$

where $\eta_i$ , $i = 1, .., 9$ are defined in detail in appendix F.

### 5.3 Real exchange rate and relative prices

The real exchange rate dynamics are specified by the following relationship

$$\Delta \hat{q}_t = \Delta z_t + \pi^*_t - \pi_t \tag{39}$$

In the home country the price of imported goods relative to that of home goods is specified as $T_t = \frac{P_{F,t}}{P_{H,t}}$, whereas in the foreign country the relative price of home exported goods to foreign goods is specified as $T^*_t = \frac{P^*_{H,t}}{P^*_{F,t}}$. Loglinearizing those two expressions we receive the following

$$\hat{T}_t = \hat{T}_{t-1} + \pi_{F,t} - \pi_{H,t} \quad \hat{T}^*_t = \hat{T}^*_{t-1} + \pi^*_{H,t} - \pi^*_{F,t}$$

### 5.4 Flexible price equilibrium

At the flexible price equilibrium firms adjust their prices in each period. Each firm will set its marginal cost equal to the optimal marginal cost (i.e. $-log\left(\frac{\theta}{\theta-1}\right)$) which is constant over time and equal across firms. Since firms adjust their prices every period, monetary policy will not have any real effects into the economy. The real marginal cost is specified by the following equations

$$mc_t = -log\left(\frac{\theta}{\theta - 1}\right) = -\mu$$

$$mc_t = w_t - \alpha_t - \nu$$

where $w_t$ is the real wage, $\alpha_t$ (log) productivity and $\nu$ a subsidy to labor.[23] Solving for the case with flexible prices, I receive the following set of equations describing the equilibrium processes for

[23]This subsidy serves in rendering the flexible price equilibrium efficient. This is achieved by setting the subsidy equal to the mark-up (i.e. $\nu = \mu$), in order to remove the distortion associated with monopolistic competition.

output, consumption, labor, real interest rate[24], given by:

$$y_t^n = \psi_c \bar{c}_{t-1} + \psi_\zeta \zeta + \psi_a \alpha_t + \psi_{a^*} \alpha_t^* + \psi_g g_t + \psi_{g^*} g_t^* \tag{40}$$

$$c_t^n = \tilde{\psi}_c \bar{c}_{t-1} + \psi_\zeta \zeta + \left( \frac{\gamma \delta^* + \sigma}{\delta(\gamma + \sigma) - \gamma(1 - \delta^*)} \right) \psi_\alpha \alpha_t - \left( \frac{\gamma}{\sigma} \psi_{\alpha^*} \right) \alpha_t^* - \left( \frac{\gamma}{\sigma} \psi_g \right) g_t - \left( \frac{\gamma}{\sigma} \psi_{g^*} \right) g_t^* \tag{41}$$

$$l_t^n = \tilde{\psi}_c \bar{c}_{t-1} + \psi_\zeta \zeta + \left( \frac{\gamma(\delta^*(1-\sigma) - (1-\delta)) - \sigma(1-\delta)\psi_\alpha}{\delta(\gamma + \sigma) - \gamma(1 - \delta^*)} \right) \alpha_t - \psi_{a^*} \alpha_t^* + \psi_g g_t + \psi_{g^*} g_t^* \tag{42}$$

$$r_t^n = \tilde{\tilde{\psi}}_c \bar{c}_{t-1} + \left( \frac{(\gamma \delta^* + \sigma)(1 - \rho_a)\psi_a}{\kappa \delta(\gamma + \sigma) - \gamma(1 - \delta^*)} \right) \alpha_t - \left( \frac{\gamma(1 - \rho_{a^*})\psi_{a^*}}{\kappa \sigma} \right) \alpha_t^* - \left( \frac{\gamma(1 - \rho_g)\psi_g}{\kappa \sigma} \right) g_t - \left( \frac{\gamma(1 - \rho_{g^*})\psi_{g^*}}{\kappa \sigma} \right) g_t^* \tag{43}$$

## 5.5 Welfare

The Central Bank sets the interest rate in such a way to minimize a measure of social loss derived by a second order Taylor expansion to the consumer's utility function as in Rotemberg and Woodford (1998), Amato and Laubach (2003), Pappa (2004) and Benigno and Benigno (2006). It is summarized as[25]

$$W_t = -\tfrac{1}{2} u_c C \Xi \{ \lambda_1 (\hat{Y}_t - y_t^n)^2 + \lambda_2 (\hat{Y}_t^* - y_t^{*n})^2 + \lambda_3 (\hat{q}_t - q_t^n)^2 + \lambda_4 \Delta \hat{q}_t^2 + \lambda_5 \Delta \hat{Y}_t^{*2} + \lambda_6 \Delta \hat{Y}_t^2 + \dots$$

$$+ \pi_{H,t}^2 + \lambda_7 (\pi_{H,t} - \pi_{H,t-1})^2 + \lambda_8 (\pi_{H,t}^*)^2 + \lambda_9 (\pi_{H,t}^* - \pi_{H,t-1}^*)^2 + \lambda_{10} (\hat{q}_t + \hat{Y}_t)^2 + \lambda_{11} (\hat{q}_t + \hat{Y}_t^*)^2 + \dots$$

$$\lambda_{12} (\hat{q}_{t-1} + \hat{Y}_t)^2 + \lambda_{13} (\hat{q}_{t-1} + \hat{Y}_t^*)^2 + \lambda_{13} (\hat{q}_{t-1} + \hat{Y}_t^*)^2 + \dots$$

$$\lambda_{14} (\hat{Y}_{t-1}^* - y_{t-1}^{*n})(\hat{q}_{t-1} - q_{t-1}^n) + \lambda_{15} (y_{t-1} - y_{t-1}^n)(y_{t-1}^* - y_{t-1}^{*n}) + \lambda_{16} (\hat{C}_t - c_t^n)(\hat{q}_t - q_t^n) + \dots$$

$$\lambda_{17} (\hat{Y}_t + \hat{Y}_{t-1}^*)^2 + \lambda_{18} (\hat{Y}_{t-1} + \hat{Y}_t^*)^2 + \lambda_{19} (\hat{Y}_{t-1} - y_{t-1}^n)(q_{t-1} - q_{t-1}^n) + \dots$$

$$+ \lambda_{20} (\hat{Y}_t^* - \hat{Y}_t^{*n})(\hat{Y}_{t-1}^* - \hat{Y}_{t-1}^{*n}) + \lambda_{21} (\hat{Y}_{t-1}^* + \hat{q}_t)^2 + \lambda_{22} (\hat{Y}_{t-1} + \hat{q}_t)^2 + \lambda_{23} (\hat{Y}_{t-1} - y_{t-1}^n)(\hat{q}_{t-1} - q_{t-1}^n) + \dots$$

$$\lambda_{24} (\hat{C}_{t-1}^* - c_{t-1}^{*n})(\hat{q}_{t-1} - q_{t-1}^n) + \lambda_{25} (\hat{q}_t - q_t^n)(\hat{q}_{t-1} - q_{t-1}^n) + \lambda_{26} (\hat{Y}_{t-1} - y_{t-1}^n)(\hat{Y}_t - y_t^n) + t.i.p. + O(||\xi||^3) \tag{44}$$

where the coefficients $\lambda_i$, $i = 1, ..., 21$ are functions of the structural parameters.

[24] The flexible price expression for the real exchange rate can be easily derived using the risk sharing condition.
[25] The derivation of the loss function is given in detail in the Appendix G.

# 6 Model Solution

Given the Markov-Switching structure of the model, standard solution techniques cannot be applied in order to find a solution. In the recent literature on markov-swithing DSGE models, various alternative techniques for solving such models have been suggested (Farmer, Waggoner and Zha, 2011; Farmer, Waggoner and Zha, 2008; Davig and Leeper, 2007; Svensson and Williams, 2005). The technique I use is that of Farmer, Waggoner and Zha (2011). The virtue of that technique is that it is able to find all possible minimal state variable (MSV) solutions. Moreover, the algorithm is able to find whether the MSV solution is stationary (mean square stable) in the sense of Costa, Fragoso and Marques (2004).[26] The model can be written in the following state space form

$$A(s_t)X_t = B(s_t)X_{t-1} + \Psi(s_t)\varepsilon_t + \Pi(s_t)\eta_t \tag{45}$$

where $X_t = [y_{t+1}, y^*_{t+1}, \pi_{H,t+1}, \pi^*_{H,t+1}, \pi_{F,t+1}, \pi^*_{F,t+1}, q_t, z_{t+1}, T_{t+1}, T_t, y_t, y^*_t, \pi_{H,t}, \ldots$
$\ldots, \pi^*_{H,t}, \pi_{F,t}, \pi^*_{F,t}, q_{t-1}, z_t, T^*_{t+1}, T^*_t, i_t, i^*_t, a_t, a^*_t]$, $\varepsilon_t$ is a $6 \times 1$ vector of i.i.d. stationary exogenous shocks and $\eta_t$ is an $8 \times 1$ vector of endogenous random variables.

According to that technique the MSV equilibrium of the model takes the form

$$X_t = g_{1,s_t}X_{t-1} + g_{2,s_t}\varepsilon_t \tag{46}$$

In order for the above minimal state variable solution to be stationary it must be that the the eigenvalues of

$$(P \otimes I_{24^2})diag\left[\Gamma_1 \otimes \Gamma_1, \Gamma_2 \otimes \Gamma_2\right] \tag{47}$$

where $\Gamma_j = A(j)V_j$ for $j = 1, 2$. And where $V_j$ is a $24 \times 10$ matrix resulting from the Schur decomposition of $A(j)^{-1}B(j)$. In the present model the largest eigenvalue was found to be equal to 0.9174, implying, thus, that the MSV solution is stationary. The impulse responses and the moments of the variables of interest are then derived from that stationary solution.

[26] For an extensive argument regarding the merits of the solution technique used in this paper over the alternative ones see Farmer et al. (2011) and the references therein.

# 7 Parameterization

In this section, the model is simulated so that to explore what regime switching implies about the dynamic behavior of the key macroeconomic variables. In order to make my argument clearer the impulse responses of inflation and output are compared to those when there is no regime switching, as in Liu et al. (2009). Throughout this section I assume that it is only the foreign central bank switching regimes. The home central bank is assumed to commit to the Taylor rule, independently of what the foreign central bank does. Therefore, whenever I refer to the hawkish regime, I mean an inflation coefficient in the interest rate rule of the foreign central bank that is greater than one. Whenever I refer to the dovish regime, I mean an inflation coefficient in the interest rate rule of the foreign country that is less than one.

Since it is only the foreign central bank that switches regimes in its monetary policy I have to choose four different parameters for its interest rate rule, depending on the regime. The values assigned are those from the Markov-switching interest rate rule for the the US estimated in section 2. That is, $\phi^*_{\pi,1} = 1.1621$ , $\phi^*_{\pi,2} = 0.3298$, $\phi^*_{x,1} = 1.5640$ , $\phi^*_{x,2} = 0.9499$. I also assume some interest rate smoothing with $\rho^*_1 = \rho^*_2 = 0.6$[27].

As far as the rest of the parameters in the model are concerned, they are regime invariant. Those parameters are the subjective discount factor $\beta$, the degree of relative risk aversion $\sigma$, the elasticity of substitution between goods produced domestically $\theta$, the elasticity of substitution between home and foreign goods $\rho$, the Frisch elasticity of labor supply $1/\gamma$, the degree of price stickiness for the home and the foreign country respectively $\omega$ and $\omega^*$, the fractions of rule of thumb firms for each country $\zeta$ and $\zeta^*$, the fractions of rule of thumb consumers $1-\psi$ and $1-\psi^*$, the home bias parameters $\delta$ and $\delta^*$ and the coefficients on the home country interest rate rule $\phi_\pi$, $\phi_x$ and $\rho_i$. The values of the parameters are chosen according to the existing empirical and theoretical literature in models similar to mine. They are summarized at table 5.

[27]Note that the results presented in this section hold also for $\rho_1 = \rho_2 = 0$

Table 5: Parameter Values

| | | |
|---|---|---|
| Structural Parameters | | |
| $\beta$ | 0.99 | |
| $\sigma$ | 1.5 | |
| $\theta$ | 10 | (Obstfeld & Rogoff, 2000) |
| $\rho$ | 3 | (Obstfeld & Rogoff, 2000) |
| $\gamma$ | 3 | (Pappa, 2004) |
| $\omega = \omega^*$ | 0.75 | (Adjemian, Paries & Smets, 2008) |
| $\delta = \delta^*$ | 0.67 | |
| $\zeta = \zeta^*$ | 0.5 | (Adjemian, Paries & Smets, 2008) |
| $\psi = \psi^*$ | 0.4 | (Adjemian, Paries & Smets, 2008) |
| Policy Rule Coefficients | | |
| Home | | |
| $\phi_\pi = 1.5$ | $\phi_y = 0.5$ | $\rho = 0.75$ |
| Foreign | | |
| Regime 1: $\phi^*_{\pi^*,1} = 1.1621$ | $\phi^*_{y^*,1} = 1.5640$ | $\rho^*_1 = 0.6$ |
| Regime 2: $\phi^*_{\pi^*,2} = 0.3298$ | $\phi^*_{y^*,2} = 0.9499$ | $\rho^*_2 = 0.6$ |
| Probabilities | | |
| $p11 = 0.99$ | $p22 = 0.99$ | |

### 7.1 Impulse responses

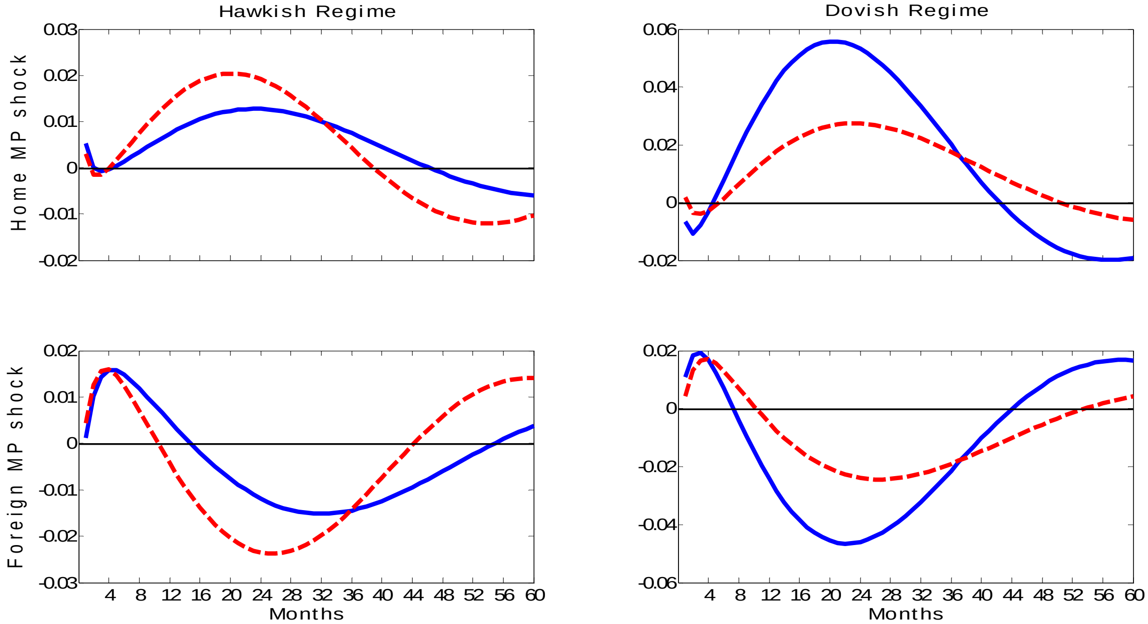
To gauge how the possibility of a future switch in foreign monetary affects the dynamics of the macroeconomic variables in the home country, I compute the impulse responses in the Markov-switching model following a one standard deviation monetary policy shock in both countries.[28] In order to emphasize the importance of expectation effects, the impulse responses from the regime switching model (red dashed line) are compared to those from the constant parameter model (blue solid line).[29]

[28]The results reported in this section hold for demand and productivity shocks in either country as well. I do not report them in order to save space.

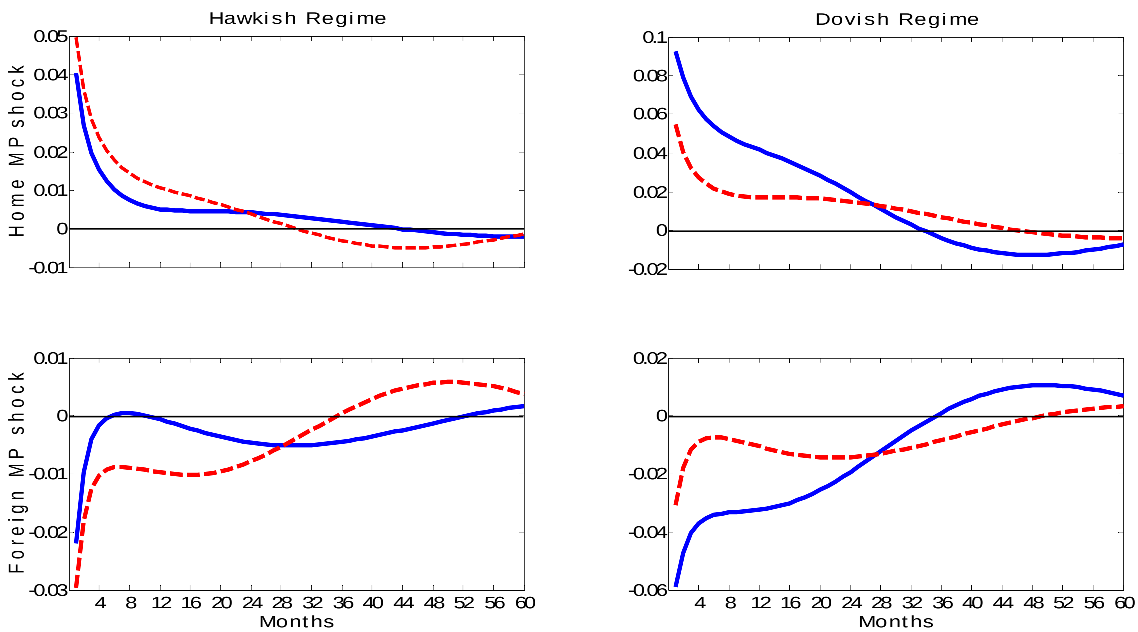
[29]As already mentioned, by constant parameter, I mean the absorbing state, i.e. when there is a zero probability of switching to another regime.

## Figure 4: Home and Foreign inflation responses to a MP shock

*(a) Home CPI*




*(b) Foreign CPI*




Notes: The red dashed line impulse responses are from the Markov switching model. The blue solid line responses are from the constant parameter model. Impulse responses in the hawkish regime are illustrated on the left panel in each graph. Impulse responses in the dovish regime are illustrated on the right panel in each graph.

In figure 4 the impulse responses of the CPI rate are plotted for each of the two regimes. As it is evident, inflation responses, in both countries are dampened[30] in the dovish regime when the probability of a switch to the hawkish regime becomes non zero (red dashed line) after both a home and a foreign monetary policy shock. Inflation fluctuates at considerably lower levels than in the absorbing state (blue line). This change in the behavior of inflation is due to the expectations formation effect. Agents in both countries assign a positive probability on the foreign monetary policy becoming hawkish, affecting, the behavior of inflation in the home (and the foreign) country. Home and foreign inflation are better controlled. As far as home inflation is concerned, this result is brought about solely, by home agents expectations, without any change in the policy of the home central bank. This is one of the key results in this paper.

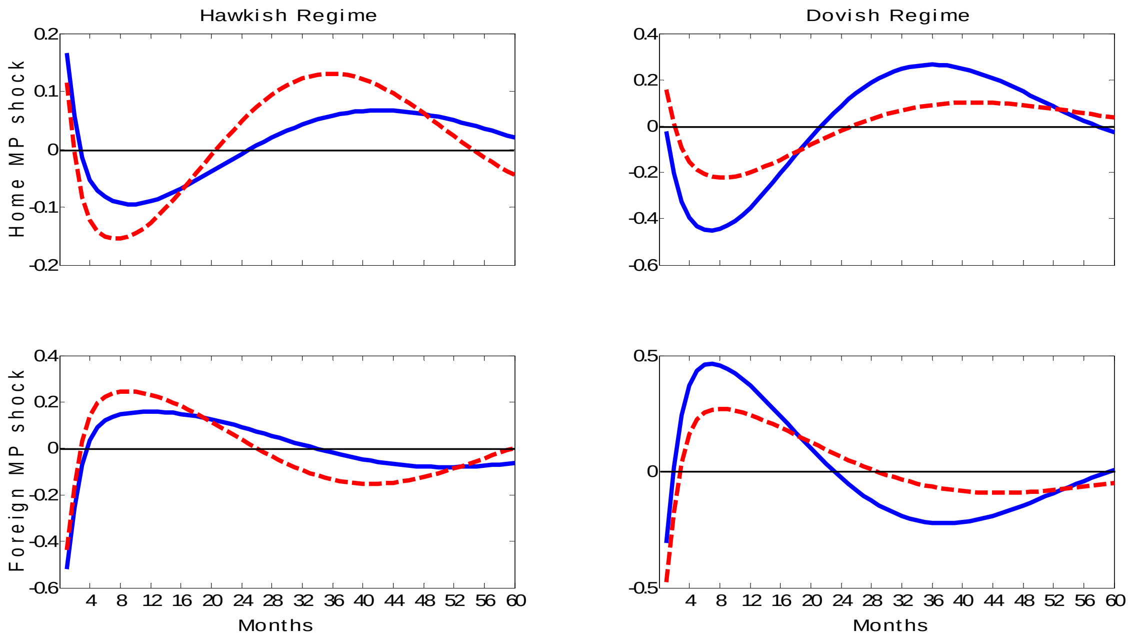
> **Result 1:** *In the dovish regime, the response of home inflation to monetary policy shocks is dampened. This result is purely expectations driven and independent of monetary policy in the home country. It is enough, that agents in the home country assign a positive probability on the foreign monetary policy becoming hawkish in the future, while it being currently dovish.*

On the other hand, there is an amplifying effect on inflation in the hawkish regime. Inflation responses in both countries seem to be slightly amplified. It is evident that the stabilizing effect, generated in the dovish regime, is stronger than the amplifying effect. This can be observed by looking at the distance between the red dashed and the blue solid impulse responses in the hawkish and the dovish regime, respectively. However, as I am showing later, this does not imply that the overall stabilizing effect on either home or foreign inflation is stronger than the amplifying effect. Note also, the asymmetry in the responses of inflation in each regime, for both countries. This is because of the asymmetry in expectation effects which arises because of the existence of the hawkish regime. The latter is strong enough, so that to make the stabilizing effect stronger than the amplifying. Additionally, the possibility of a future switch to hawkish regime helps anchor agent's expectations (Liu et al., 2009).

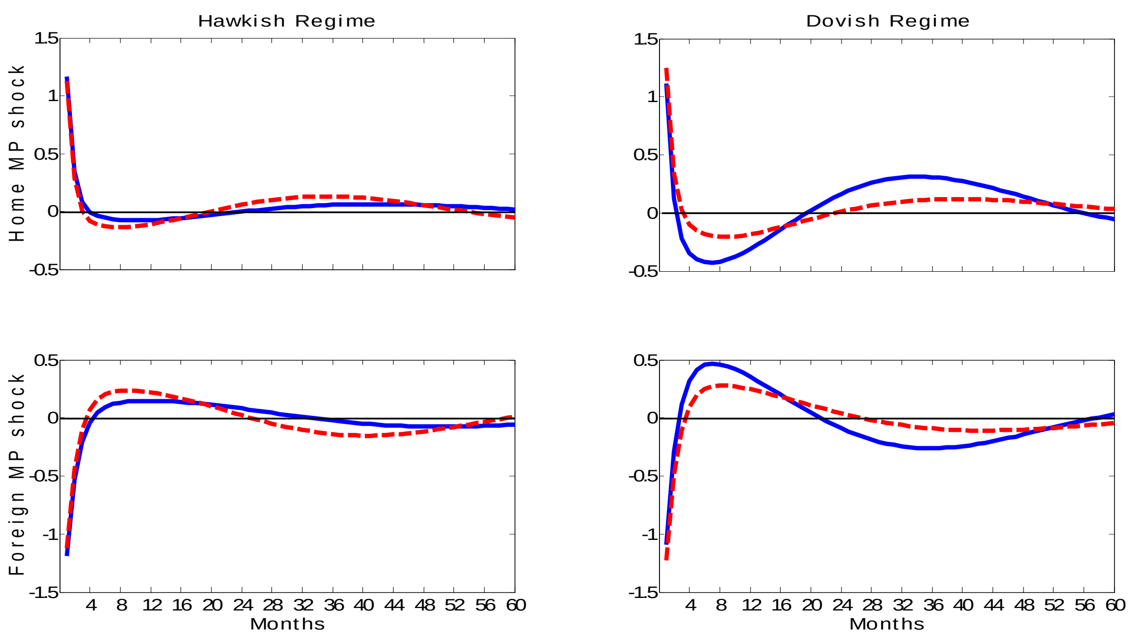
[30]From now on, I will use the term "stabilizing effect" for the case where the effects of a shock, as measured by the impulse responses, are dampened, and the term "amplifying effect" when the effects of a shock are amplified.

**Figure 5: Home and Foreign output responses to a MP shock**

*(a) Home output*




*(b) Foreign output*




Notes: The red dashed line impulse responses are from the Markov switching model. The blue solid line responses are from the constant parameter model. Impulse responses in the hawkish regime are illustrated on the left panel in each graph. Impulse responses in the dovish regime are illustrated on the right panel in each graph.

The same reasoning applies to output responses, illustrated in figure 5. Output impulse responses in both countries exhibit a pattern similar to those of inflation. Following a home or foreign monetary policy shock Output in either country is clearly less volatile in the dovish regime for a positive probability of moving to the hawkish regime (red dashed line). Home and foreign output responses, in the dovish regime, are dampened, while they are amplified in the hawkish regime compared to the constant parameter case (blue solid lines). The stabilizing effect is clearly stronger. Home output fluctuations are controlled better when home agents attach a positive probability to the foreign monetary policy becoming hawkish in the future, while being currently dovish.

The conclusion drawn until here concerns the two monetary policy shocks only. The dynamics of the model are rich enough and one cannot derive any inference by focusing only on one shock. In order to make this point clearer, I compute the changes in volatilities on inflation and output relative to the absorbing state, at table 6 below.

Table 6: Inflation and Output relative volatilities

| | *Inflation* | | *Output* | | *Losses* | |
|---|---|---|---|---|---|---|
| | *Home* | *Foreign* | *Home* | *Foreign* | *Home* | *Foreign* |
| *Hawkish* | 1.1714 | 1.7205 | 1.2709 | 1.3255 | 1.6289 | 1.4633 |
| *Dovish* | 0.7078 | 0.4495 | 0.7456 | 0.7455 | 0.5610 | 0.4942 |

Table 6 shows that there are significant decreases in inflation and output volatility, relative to the absorbing state (i.e. no regime switching case), when foreign monetary policy is dovish. In particular, home country's inflation is 0.7078 times or approximately 30% lower than in the case where the probability of staying in the dovish regime is one. This fall is larger for the foreign country, 0.45 times or 55% lower. On the other hand, a positive probability of a switch to the dovish regime increases home inflation relative to the absorbing state by 17%, while foreign inflation is increased by 72%. The stabilizing effect, thus, on home inflation is much stronger than the amplifying effect. The opposite hods for foreign inflation, where the amplifying effect is much stronger than the stabilizing.

The overall amplifying effect seems to dominate in output fluctuations, as well. In particular, home output is 27% more volatile in the hawkish regime relative to the absorbing state, while it is 25% less volatile in the dovish regime. Foreign output is 33% more volatile in the hawkish regime and 25% less volatile in the dovish regime.

Markov-switching closed economy models examine the effectiveness of regime switching monetary policy by looking at the change in volatilities of inflation and output only. Given the structure of those models, judging such a policy relying on changes in volatility, or on changes in a welfare measure leads to the same conclusions. In an open economy model, as the one in this paper, judging Markov-switching monetary policy by simply looking at the changes in volatilies of inflation and output could lead to the wrong conclusions. As the welfare measure (42) shows the dynamics in the model are far more rich than those in a closed economy model. Therefore, alternative policies would be better compared based on an appropriate welfare measure, rather than by observing changes in volatilities of some variables. I use the relative changes in the welfare measure (42) as a guide, in order to figure out whether Markov-switching monetary policy generates strong enough stabilizing effects[31] for both economies. As is clear in table 6, the relative fall in home welfare loss in the dovish regime is smaller, in absolute terms, than its relative increase in the hawkish regime. In particular, in the dovish regime, a non-zero probability of a switch to the hawkish regime causes home welfare loss to be 0.5610 times or approximately 44% lower relative to the absorbing state. On the other hand, it is 1.6289 times or 63% higher relative to the absorbing state, in the hawkish regime. Foreign welfare loss rises by 46% in the hawkish regime, and falls by approximately 50% in the dovish regime, relative to the absorbing state. The above results can be summarized as follows.

**Result 2:** *Markov switching monetary policy in the foreign country generates a stabilizing (dovish regime) and an amplifying (hawkish regime) effect on output and inflation. The stabilizing effect is stronger than the amplifying effect for home inflation. As regards home output and foreign inflation and output, the amplifying effect is stronger*

[31] By strong enough stabilizing effects, I mean that the latter is much stronger than the amplifying effects, that is effects caused by the increase in volatility relative to the absorbing state in the hawkish regime.

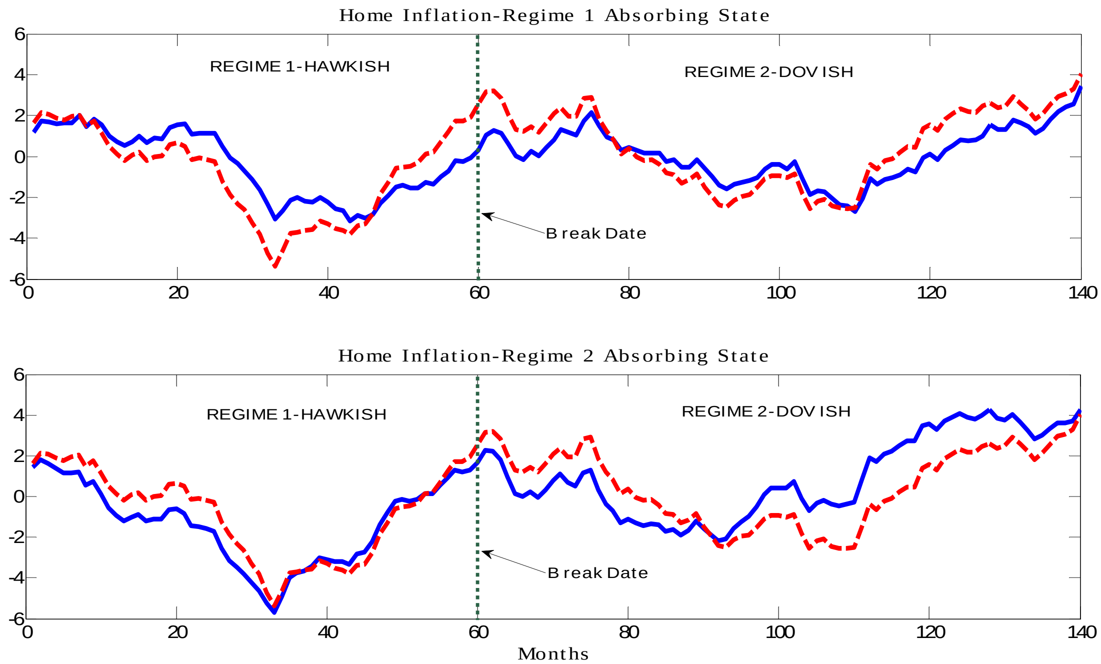
**Result 3:** *The overall stabilizing effects are stronger in the foreign country and weaker in the home, in terms of the welfare measure (44).*

So far I have shown that changes in the volatilities and the impulse responses of key macroeconomic variables of the home country may be caused by changes in the way monetary policy is conducted in the foreign country only. In figures 6 and 7 below I show the simulated paths of inflation in each country. The model was simulated for 140 periods allowing for a random date of regime switching in foreign monetary policy. I assume that the initial regime is the hawkish. The regime changing date is 60 (switch to the dovish regime). For convenience a green dotted vertical line is drawn on the regime changing date. In the upper panel in both figures, along with inflation in the MSDSGE model (red line) I plot home (foreign) inflation, had foreign monetary policy stayed in the hawkish regime forever (blue solid line). In the bottom panel inflation in the MSDGE model (red dashed line) is compared to inflation, had foreign monetary policy been always dovish (blue solid line).

As the upper panel in figure 5 illustrates, inflation in the home country appears to be fluctuating within a wider band while still being in regime 1. On the regime change date (period 60) home inflation jumps well above the blue solid line. It keeps fluctuating at higher levels compared to its behaviour in the constant parameter case, the only exception being from period 80 until 110 where its behaviour resembles that in the no regime switching case. The higher volatility of home inflation is due the expectations formation effect. As the probability of a switch in foreign monetary policy rises, inflation in the hawkish regime starts to fluctuate more. This implies that the home Central Bank should change its policy as well, in order to eliminate as much as possible the additional volatility on domestic inflation.

At the lower panel in figure 6, inflation in the MSDGE model (red dashed line) is illustrated along with inflation when the dovish regime is the absorbing state (blue solid line). Home inflation in the regime switching case resembles that in the constant parameter. From the regime change date, its behaviour changes. It fluctuates at slightly higher levels than the absorbing state until period 90, but from that period onwards it fluctuates at consistently lower levels. This is because home agents incorporate in their expectations the probability of a switch to the hawkish regime in foreign monetary policy.

**Figure 6: Home inflation**

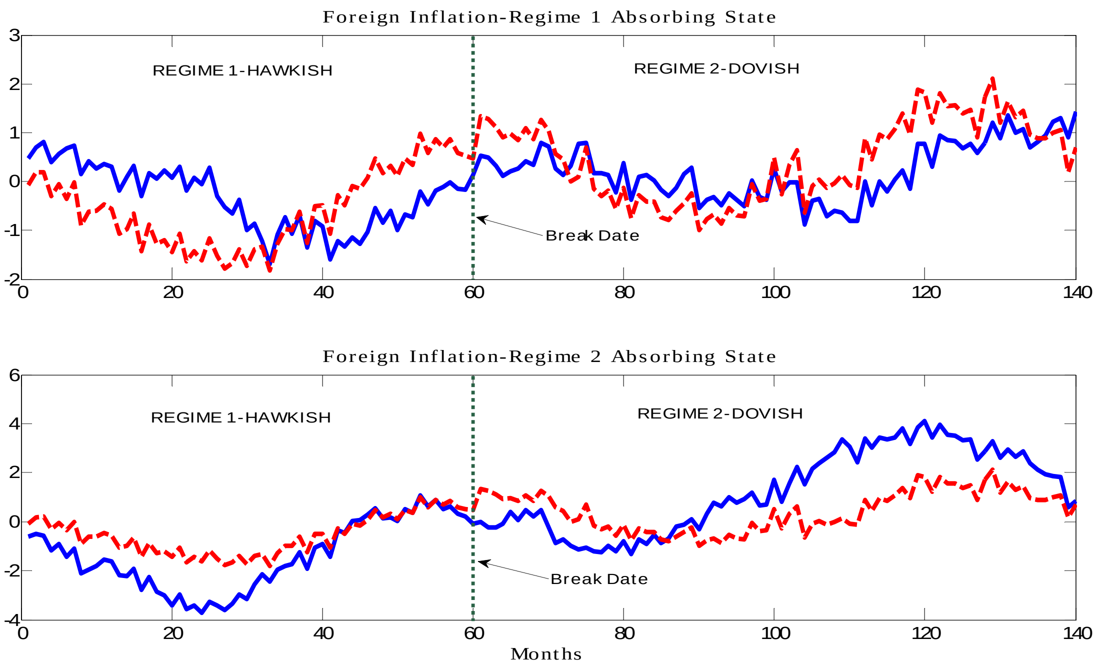



Notes: Top panel: Blue solid line: home inflation when the foreign central bank is hawkish forever. Red dashed line: home inflation in the Markov switching model. Bottom panel: Blue solid line: home inflation when the foreign central bank is dovish forever. Red dashed line: home inflation in the Markov switching model.

The path of foreign inflation is shown in figure 7. At the top panel, foreign inflation fluctuates within a slightly wider region for most of the period in regime 1 (i.e. until date 60). As already mentioned, the reason for this effect is the expectation formation effect becoming stronger as the probability of a regime switch increases and as the regime change date approaches. From date 60 onwards (Regime 2), foreign inflation keeps fluctuating at a constatly wider region than otherwise. Again the blue solid line shows how inflation fluctuates when the foreign central bank stays in the hawkish regime forever. The red dashed line shows how inflation behaves when the foreign central bank switches from being hawkish to dovish. Notice in regime 1 (hawkish) the effect on foreign inflation dynamics of the positive probability of a switch to the dovish regime. Inflation falls until period 30. But after that period it is constantly higher than in the constant parameter case. When foreign monetary policy switches to the dovish regime, foreign inflation is more volatile than in the absorbing state.

On the other hand, foreign inflation is considerably stabilized relative to the case where the foreign Central Bank is always dovish, as is shown in the bottom panel of figure 6. The red dashed line fluctuates at a narrower band than the blue line.

**Figure 7: Foreign inflation**




Notes: Top panel: Blue solid line: foreign inflation when the foreign central bank is hawkish forever. Red dashed line: foreign inflation in the Markov switching model. Bottom panel: Blue solid line: foreign inflation when the foreign central bank is dovish forever. Red dashed line: foreign inflation in the Markov switching model.

## 7.2 Alternative interest rate rules.

Having analyzed the effects of foreign policy regime switching under standard Taylor rules, I turn now the focus to alternative rules. I allow for different or additional targets in the home country's interest rate rule. In particular, I first look at what PPI instead of CPI inflation targeting implies for the home country. Second, I examine the importance of having a real exchange rate target in the home interest rate rule. Third, I introduce foreign variables in the rule. Throughout this section I assume that the interest rate rule of the foreign country is exactly the same as it was in the previous section. That is, the foreign Central Bank keeps targeting foreign CPI and output gap.

*Targeting PPI inflation.*

When a CPI target is replaced by a target for PPI the interest rate rule of the home central bank is specified as

$$i_t = \rho i_{t-1} + (1-\rho)\left(\phi_{\pi_H}\pi_{H,t} + \phi_y \tilde{y}_t\right) \tag{48}$$

As a first exercise, I compare the performance of rule (48) to the benchmark rule in which the home central bank targets CPI inflation and the output gap.

Table 7: Inflation and Output relative volatilities (Rule (48) vs Benchmark)

| | *Inflation (CPI)* | | *Output* | | *Losses* | |
|---|---|---|---|---|---|---|
| | *Home* | *Foreign* | *Home* | *Foreign* | *Home* | *Foreign* |
| *Hawkish* | 0.9532 | 0.7746 | 0.9610 | 0.9282 | 0.9212 | 0.9588 |
| *Dovish* | 0.9887 | 0.9101 | 0.9431 | 0.9225 | 0.8830 | 1.0067 |

The result from table 7 show that it is better for the home country to target PPI rather than CPI inflation.[32] Home loss is lower by 8% in the hawkish regime and 12% in the dovish. Foreign loss in the hawkish regime is lower compared to that under the benchmark rule where CPI inflation is targeted by the home Central Bank. On the other hand foreign loss is almost unchanged in the dovish regime. Home output and CPI inflation are marginally less volatile in both regimes. The foreign country has considerable benefits regarding CPI inflation volatility in the hawkish regime. Foreign inflation volatility is 0.7746 times lower in the hawkish and 0.9101 times lower in the dovish regime.

The intuition behind the results above is that, by targeting home PPI inflation, the home central bank isolates the latter from the effects of additional volatility in CPI inflation resulting from higher volatility in imported goods inflation ($\pi_{F,t}$). Imported goods inflation is more volatile in both regimes, by 1.0378 in the hawkish and by 1.0192 in the dovish. Which effect will dominate depends also on the degree of openness of the home country. Not surprisingly, with a degree of home bias in consumption equal to 0.67, the stabilizing effect on home PPI in both regime dominates, leading to lower volatility in CPI inflation.

Lower home output volatility is justified by the lower volatility in the home real interest rate in both regimes. In particular, it is 0.9203 times less volatile in the hawkish regime and 0.9165 less volatile in the dovish regime.

[32]The coefficients in rule (46) are exactly the same as in the baseline calibration, that is $\phi_{\pi_H} = 1.5$, $\phi_y = 0.5$ and $\rho = 0.6$.

*Targeting the Real Exchange Rate.*

I now extend the benchmark interest rate rule of the home Central Bank by adding a real exchange rate target. The rule has the following form

$$i_t = \rho i_{t-1} + (1-\rho)\left(\phi_\pi \pi_{H,t} + \phi_y \tilde{y}_t + \phi_q q_t\right) \tag{49}$$

As above, I compare the performance of rule (49) to that used in the baseline calibration.[33] Note, though, the substantial differences between rule (49) and the Taylor rule in the baseline calibration. In the former, the home Central Bank targets the home PPI inflation and the real exchange rate.[34] The only common feature is the output gap target.

Table 8: Inflation and Output relative volatilities (Rule (49) vs Benchmark)

| | *Inflation (CPI)* | | *Output* | | *Losses* | |
|---|---|---|---|---|---|---|
| | *Home* | *Foreign* | *Home* | *Foreign* | *Home* | *Foreign* |
| *Hawkish* | 0.9097 | 0.6770 | 0.9244 | 0.8915 | 0.8518 | 0.8904 |
| *Dovish* | 0.9978 | 0.8586 | 0.9421 | 0.9060 | 0.8743 | 1.1126 |

When the home central bank targets the home PPI inflation along with a target for the real exchange rate the benefits in terms of welfare losses, compared to the benchmark case, are significant. Home loss is almost 15% lower in the hawkish regime and approximately 13% lower in the dovish regime relative to the Taylor rule. The main driving force for the lower volatility in both regimes seems to be the real exchange rate. The latter is almost 7% less volatile in the hawkish regime, and 43% less volatile in the dovish. The most crucial conclusion from rule (49) is that the amplifying effects of a possibility of a switch to the dovish regime in the future are considerably decreased.

*Targeting foreign variables.*

One of the important questions in open economy monetary economics has been that of whether central banks should target foreign variables or not. Empirically, it seems that such targets can provide the central banks some information in order to control better the overall volatility in the domestic economy (Clarida, Gali and Gertler, 1998). One may question the implementability of such rules. Targeting foreign variables implies that the home Central Bank has sufficient in-

[33]The coefficient on the real exchange rate is $\phi_q = 0.1$.

[34]The performance of rule (47) with a CPI inflation target, instead, was also checked. The accrued benefits, however, were negligible.

formation about those, so that to be sure about which direction should it move its instrument. Additionally, in practice, it is not even certain the size and the sign of the effect such variables have on domestic economy. I, however, abstract from this criticism by sticking to the initial assumptions of the model. The class of such rules considered receive the following form[35]

$$i_t = \rho i_{t-1} + (1-\rho)\left(\phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_{y^*} \tilde{y}_t^*\right) \tag{50}$$

$$i_t = \rho i_{t-1} + (1-\rho)\left(\phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_{\pi^*} \pi_t^*\right) \tag{51}$$

$$i_t = \rho i_{t-1} + (1-\rho)\left(\phi_\pi \pi_t + \phi_y \tilde{y}_t + \sum_{s=0}^{p} \phi_{i^*,p} i_{t-s}^*\right) \tag{52}$$

The results for the performance of each of the above interest rate rules above are summarized at table 9 below

Table 9: Inflation and Output relative volatilities (vs Benchmark)

| | *Inflation (CPI)* | | *Output* | | *Losses* | |
|---|---|---|---|---|---|---|
| | *Home* | *Foreign* | *Home* | *Foreign* | *Home* | *Foreign* |
| *Rule* 50 $\phi_{y^*} = -0.1$ | | | | | | |
| *Hawkish* | 0.8508 | 0.3928 | 0.7775 | 0.6716 | 0.6033 | 0.6407 |
| *Dovish* | 0.9879 | 0.7395 | 0.8269 | 0.7264 | 0.6578 | 1.1251 |
| *Rule* 51 $\phi_{\pi^*} = 0.5$ | | | | | | |
| *Hawkish* | 0.9371 | 0.9726 | 0.9471 | 0.8988 | 0.8922 | 0.9549 |
| *Dovish* | 0.9537 | 0.8427 | 0.9737 | 0.9496 | 0.9393 | 1.0918 |
| *Rule* 52 $\phi_{i^*,p} = -0.1$ | | | | | | |
| *Hawkish* | 0.7699 | 0.3683 | 0.6045 | 0.4366 | 0.3735 | 0.3037 |
| *Dovish* | 0.9335 | 0.6651 | 0.7083 | 0.5441 | 0.4846 | 0.8195 |

The results at table 9 suggest that rule (52) performs much better than any other alternative rule considered in this section. Home country's welfare loss is considerably lower compared to that in the baseline calibration, in both regimes. Welfare loss of the foreign country is dramatically lower than under the benchmark interest rate rule in both regimes.

As for output relative volatilities, they are much lower compared to the benchmark case for both

[35]The coefficients on inflation, the output gap and smoothing are $\phi_\pi = 1.5$, $\phi_y = 0.5$ and $\rho = 0.75$.

countries in both regimes. As regards home inflation it is 7% less volatile in the dovish regime and 23% less volatile in the hawkish. The effects on foreign inflation are more pronounced. The latter is approximately 63% less volatile in the hawkish regime and 34% less volatile in the dovish.

But the main criterion to judge the overall effects in each country is welfare loss. Since the latter is considerably lower for both countries, it follows that both benefit when the home Central Bank adopts rule (52) instead of the standard Taylor rule.

A direct reaction of the home Central Bank to foreign interest rate fluctuations implies higher weights on both home inflation and output. In fact, by using the UIP condition in rule (52) where the home Central Bank reacts only contemporaneously to the foreign interest rate, I receive the following

$$i_t = (\frac{\rho}{1+\phi_{i^*,0}})i_{t-1} + (1-\rho)\left[\frac{\phi_\pi}{1+\phi_{i^*,0}}\pi_t + \frac{\phi_y}{1+\phi_{i^*,0}}\tilde{y}_t + \frac{\phi_{i^*,0}}{1+\phi_{i^*,0}}\Delta\hat{z}_{t+1}\right]$$

A negative $\phi_{i^*,0}$ implies higher weights on output and inflation, hence a more aggressive reaction against their fluctuations. As I am showing in the next section, it is optimal for the home central bank to raise the coefficients on inflation and output as the probability of shifting to the dovish regime in the future increases.

## 8 Optimal policy with regime switches

So far in the analysis, the parameters in the interest rate rule of the home country have been assumed to be constant over time, independently of what the foreign monetary policy is and have been set arbitrarily, corresponding to the standard Taylor rule suggested by Taylor (1993). In this section I am looking for the optimal policy conditional on the coefficients in the interest rate rule of the foreign country. I am not interested in the cooperative allocation.[36] In this paper I focus on the optimal discretionary policy for the home central bank conditional on regime switches in foreign monetary policy. For this reason, I will make use of dynamic programming techniques. The algorithm I use is that of Soderlind (1998), but extended to a Markov-switching framework.

[36]For an example about the cooperative solution in a two-country model see Benigno and Benigno (2006).

### 8.1 Formulation

The procedure followed in this section is similar to that in Zampolli (2006). The policy maker chooses the control $i_t$ (i.e. the interest rate rule) which minimizes the expected value of the intertemporal loss function, stated in the previous section and summarized as

$$\sum_{t=0}^{\infty} \beta^t W(h_t, i_t) \tag{53}$$

subject to $h_0$, $s_0$ given, and the model describing the economy

$$h_{t+1} = A(s_{t+1})h_t + B(s_{t+1})i_t + C\varepsilon_{t+1} \qquad t \geq 0 \tag{54}$$

where $L(h_t, i_t)$ is the period loss function, $\beta$ is the discount factor, $h_t$ is a $24 \times 1$ vector of state variables, $i_t$ is the control variable (i.e. the interest rate) and $\varepsilon_t$ is a $6 \times 1$ vector of white noise shocks with variance covariance matrix $\Sigma_\varepsilon$ and $C$ is a $24 \times 6$

The loss function (42) epxanded by a weight on interest rate stabilization can be conveniently expressed as follows

$$W(h_t, i_t) = h_t^{'} R h_t + i_t Q i_t \tag{55}$$

where $R$ is a $24 \times 24$ positive definite matrix and $Q$ is a scalar. The matrices $A$ and $B$, as already mentioned, are stochastic and take on different values depending on the regime $s_t$, $t = 1, 2$.

### 8.2 The Bellman equation

The policy maker in a markov-switching environment needs to find the interest rate rule that is state-contigent. This rule describes the way that the control variable, the interest rate, should be set as a function of both the state variables and the regime occurring at date $t$. Therefore, as in Zampolli (2006) a Bellman equation is associated with each regime. In other words, the policy maker solves her minimization problem conditional on the regime. The regime $j$ dependent Bellman equation is specified, thus, as follows

$$V(h_t, j) = max_{i_t} \left\{ W(h_t, i_t) + \beta \Sigma_{i=1}^{2} p_{ji} E_t \left[ V(h_{t+1}, i) \right] \right\} \tag{56}$$

where $V(h_t, j)$ is a function of the state variables $h_t$, the regime prevailing at date $t$ and represents the continuation value of the optimal dynamic programming problem at $t$.

The value function for this problem is

$$V(h_t, j) = h_t' P_j h_t + d_j, \quad j = 1, 2 \tag{57}$$

where $P_j$ is a $24 \times 24$ symmetric positive semi-definite matrix, while $d_i$ is a scalar. The optimal policy is given by

$$i(h_t, j) = -F_j h_t, \quad j = 1, 2 \tag{58}$$

where $F_j$ is a $24 \times 1$ matrix, depending on $P_j$. That is, matrix $F_j$ specifies the coefficients in the policy rule of the central bank. Those coefficients are regime specific. Maximizing, thus, the Bellman subject to the constraints, the matrix $F_j$ is specified as

$$F_j = \left(Q + \beta p_{j1} B_1' P_i B_1 + \beta p_{j2} B_2' P_i B_2\right)^{-1} \beta \left(p_{j1} A_1' P_i B_1 + p_{j2} A_2' P_i B_2\right) \tag{59}$$

where matrix $P_i$ has been already determined by a set of interrelated Riccati equations, which specify a system with the following form

$$\begin{aligned} P_j = {} & R + \beta p_{j1} A_1' P_i A_1 + \beta p_{j2} A_2' P_i A_2 - \ldots \\ & -\beta^2 \left(p_{j1} A_1' P_i B_1 + p_{j2} A_2' P_i B_2\right) \left(Q + \beta p_{j1} B_1' P_i B_1 + \beta p_{j2} B_2' P_i B_2\right)^{-1} \left(p_{j1} B_1' P_i A_1 + p_{j2} B_2' P_i A_2\right) \end{aligned} \tag{60}$$

### 8.3 How should home central bank react?

Having specified the formulation of the policy problem of the home central bank, in this section, I find the optimal rule conditional on regime shifts in foreign monetary policy. Figures 8 and 9 summarize the key results.

The first result from the two figures above is that the home central bank must change the coefficients in its interest rate rule as foreign monetary policy changes over time. Therefore, it is not optimal fro the home country to adopt a regime invariant interest rate rule. The second is that, the weight on PPI inflation must increase as the probability of foreign monetary policy switching to the dovish regime increases.[37] The opposite holds as the probability of foreign monetary policy switching to the hawkish regime increases. In this case the weight on PPI inflation falls. The weight on the output gap changes similarly. That is, it rises as the probability of switching to the dovish regime increases, and falls as the probability of moving to the hawkish regime increases.

**Figure 8: Coefficients when the foreign central bank is hawkish**

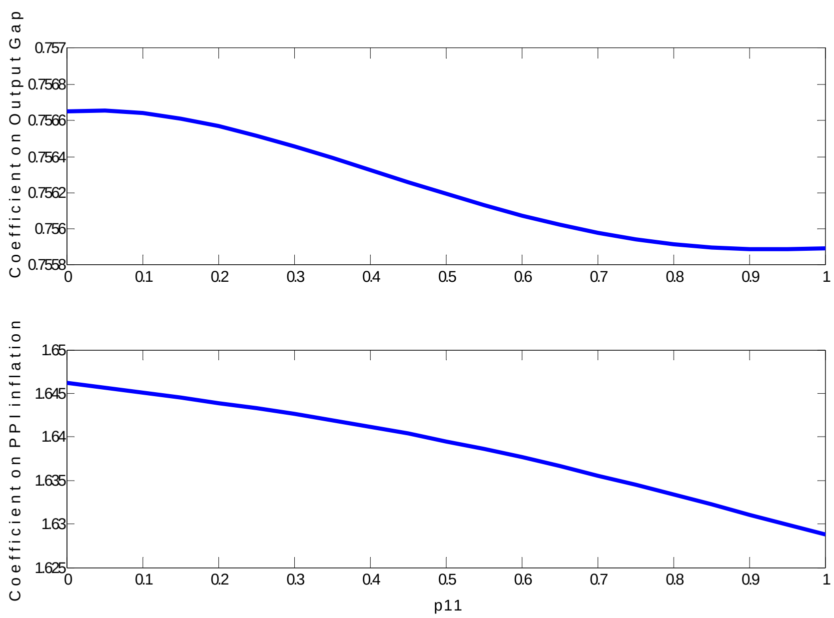
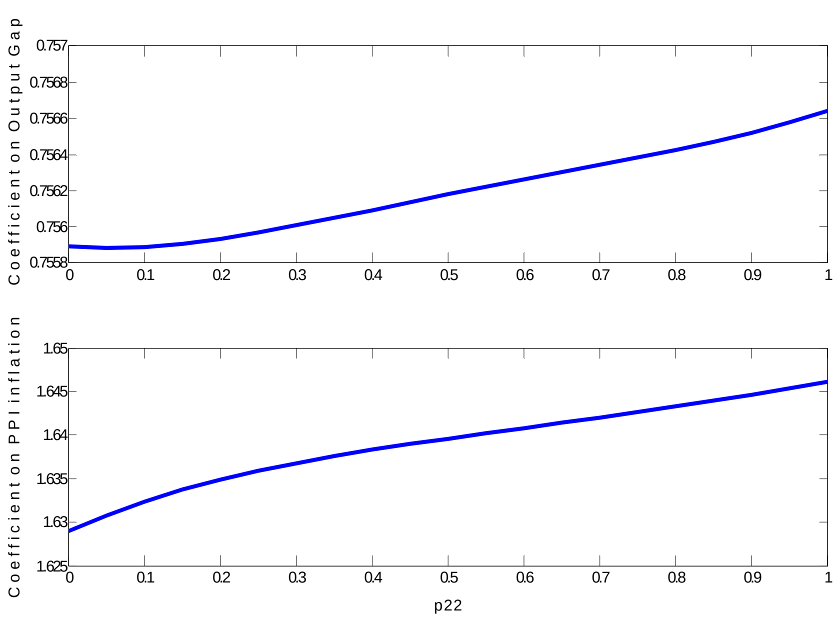



**Figure 9: Coefficients when the foreign central bank is dovish**




[37] As in Svensson (1998), CPI inflation $\pi_t$ is not included in the optimal reaction function of the home Central Bank. This is due to the fact that it is not an independent state variable, but, rather, a linear combination of other state variables, i.e. $\pi_{H,t}$ and $\pi_{F,t}$.

From the computation of optimal policy of the home central bank I end up to the following two results:

**Result 4:** *As the probability of the foreign monetary policy switching to the dovish regime increases, the home central bank should become more aggressive to home PPI inflation fluctuations. As the probability of the foreign monetary policy switching to the hawkish regime increases, the home central bank should become less aggressive to home PPI inflation fluctuations.*

**Result 5:** *The home central bank must attach a weight on home PPI inflation that is always greater than one. That is, it must be always hawkish. Moreover, it must be even more aggressive to PPI inflation fluctuations, as the foreign central bank becomes dovish.*

### 8.4 The importance of always reacting optimally.

In this section I focus on the importance, in terms of welfare, of an optimal reaction of the home central bank to changes in foreign monetary policy. I assume that the home Central Bank always reacts optimally conditional on foreign monetary policy. Again, I compute the relative welfare losses. That is, the losses in each regime are expressed relative to those when each corresponding regime is an absorbing state.

Table 10: Relative Losses

| | *Losses* | |
|---|---|---|
| | *Home* | *Foreign* |
| *Hawkish* | 1.0003 | 1.0023 |
| *Dovish* | 1.0000 | 0.9990 |

The results at table 10 show that when the home central bank reacts always optimally to foreign monetary policy, the home country is entirely unaffected by regime shifts in foreign monetary policy. Home welfare loss remains unchanged in the dovish regime relative to the constant parameter case. In the hawkish regime the increase in home loss is tiny. More importantly, the foreign country benefits when the home central bank reacts optimally to changes in its policy. Foreign welfare loss is only 0.2% higher in the hawkish regime and 0.1% lower in the dovish regime, compared to the absorbing state. Therefore, optimal reaction in the home country is enough to eliminate the large fluctuations in overall volatility in both countries.

Finally, as a last exercise, I compare rule (52) with the case where the home central bank reacts optimally. Given that this rule yields the lowest home welfare losses (relative to the Taylor rule considered in the baseline calibration) than any other of the alternative rules considered in this paper, the comparison of its performance relative to the optimal reaction of the home central bank is enough to show how much simple rules are away from the optimal case.

Table 11: *Rule* (52) *vs* *Optimal*

| | *Losses* | |
|---|---|---|
| | *Home* | *Foreign* |
| *Hawkish* | 3.4663 | 2.0603 |
| *Dovish* | 3.6832 | 5.2785 |

As table 11 shows rule (52) yields losses that are 3.5 times higher in the home country and 2 times higher in the foreign, in the hawkish regime. As regards losses in the dovish regime, they are 3.7 and 5 times higher in the home and the foreign country respectively, relative to the losses accruing under the optimal reaction function.

## 9 Conclusion

In this paper, I show that regime shifts in the monetary policy of one country have important effects on other economies. My new empirical evidence shows that the monetary policy of the Fed has changed since the start of the Euro and is found to be the main reason for the changes in the dynamics of inflation and output gap in the Eurozone. Furthermore, changes in the monetary policy of the Fed are well captured by a Taylor rule whose coefficients change according a two-state Markov-switching process. The monetary policy of the ECB, though, is found to be fairly stable.

Taking into account the empirical findings, I examine the international effects of changes in monetary policy theoretically. I construct a two country DSGE model in which foreign monetary policy switches regimes over time. I give further insight regarding the effects of regime switching in monetary policy both domestically and abroad. Home monetary policy was initially assumed to be time invariant and follow the Taylor rule with some interest rate smoothing. Home inflation is found to be affected both in terms of volatility and in terms of its response to alternative shocks, by regime shifts in foreign monetary policy (and, consequently, by the change in inflation expectations). Foreign monetary policy regime shifts generate a stabilization and an amplifying effect on output and inflation, both in the foreign and the home country. Which effect arises depends on which regime the foreign monetary policy lies in. When the latter is dovish there is a stabilization effect. That is, impulse responses of inflation and output are dampened, given a positive probability of the foreign monetary policy becoming hawkish. On the contrary, when foreign monetary policy is hawkish there is an amplifying effect in both countries, given a positive probability of the foreign monetary policy becoming dovish. That is, the impulse responses are more volatile. Moreover, there is an asymmetry on the size of each effect. In particular, I show that the stabilization effect is stronger in the foreign, but weaker in the home country, based on a welfare measure, derived by a second order approximation of the agents utility function.

Finally, through the solution of the optimal policy problem of the home central banker, conditional on foreign monetary policy switching regimes over time, I show that it is optimal to follow a time varying interest rate rule. When the home central bank reacts optimally, the effects of regime switches in foreign monetary policy on the home country are completely eliminated. Moreover, the foreign country seems to benefit a lot, in terms of its welfare measure, when the home country reacts optimally to changes in its policy.

## CONCLUSION

The thesis consists of three self-contained empirical and theoretical studies. These are (i) Asymmetries, productivity and capital account effects in the determination of the Real Exchange rate: The case of Transition Economies, (ii) Rule-of-thumb behavior and Real Exchange Rate targeting and (iii) Markov Switching Monetary Policy in a two-country DSGE Model.

In Chapter 1, I show that the Balassa-Samuelson effect cannot describe the behaviour of the real exchange rate for all economies in transition. Using an appropriate proxy for productivity I show that its effect on the real exchange rate is negative (i.e. causes appreciation) for only three countries, namely, the Czech Republic, Poland and Latvia. However, the effect of the capital account is negative (i.e. causes appreciation) for all countries but Slovenia. I show that this is mainly caused by the composition of the capital account. When foreign direct investment exceeds portfolio investment, for a long period of time, the effect of the capital account is negative. The opposite holds when portfolio investment exceeds foreign direct investment, as was the case for Slovenia. Therefore, long-run investment seems to have caused the appreciation of those currencies, rather than the Balassa-Samuelson effect itself.

I test for cointegration among the real exchange rate, the real interest rate differential, the proxy for productivity and the capital account. I show that they cointegrate in a nonlinear fashion. This implies that a linear vector error correction model is no longer valid, as it implies a linear adjustment for each variable following deviation from the long-run equilibrium. For this reason I use a nonlinear multivariate error correction model, whose nonlinearity is determined by that found for the cointegrated residuals. For simplicity, I assume that the four variables under consideration share a common nonlinearity. The threshold variable is the cointegrated residuals. Adjustment towards the long-run equilibrium happens in both a linear and a nonlinear fashion.

Finally, specification tests in the multivariate error correction model show that this is a correct specification to capture the dynamics of the variables considered.

In chapter 2, I examine the importance of real exchange rate targeting in monetary policy. I estimate a structural VAR for the nominal interest rate, CPI inflation, the output gap and the real exchange rate. From the impulse response analysis and I find that the ECB achieves a better control of CPI inflation when it allows its policy rate to react contemporaneously to exchange rate movements.

Relying on the above finding I constructed a two country DSGE model for the Eurozone and the US. I modelled the foreign monetary policy using the estimates of Clarida, Gali and Gertler (1998) for the coefficients in the output gap and CPI inflation in the interest rate rule for the US. Taking this policy as given and contrary to past papers, I compute the optimized coefficients in the interest rate rule of the home central bank. Adding the real exchange rate into the interest rate rule leads to robustly lower welfare losses. The gap in losses between the Taylor rule and the real exchange rate targeting rule is wider, the higher the degree of persistence on inflation and output.

Therefore both empirical and theoretical evidence in this paper suggest that an interest rate rule with a real exchange rate target is Pareto superior to the Taylor rule in a model.

In chapter 3, I show that regime shifts in the monetary policy of one country have important effects on other economies. My new empirical evidence shows that the monetary policy of the Fed has changed since the start of the Euro and is found to be the main reason for the changes in the dynamics of inflation and output gap in the Eurozone. Furthermore, changes in the monetary policy of the Fed are well captured by a Taylor rule whose coefficients change according a two-state Markov-switching process. The monetary policy of the ECB, though, is found to be fairly stable.

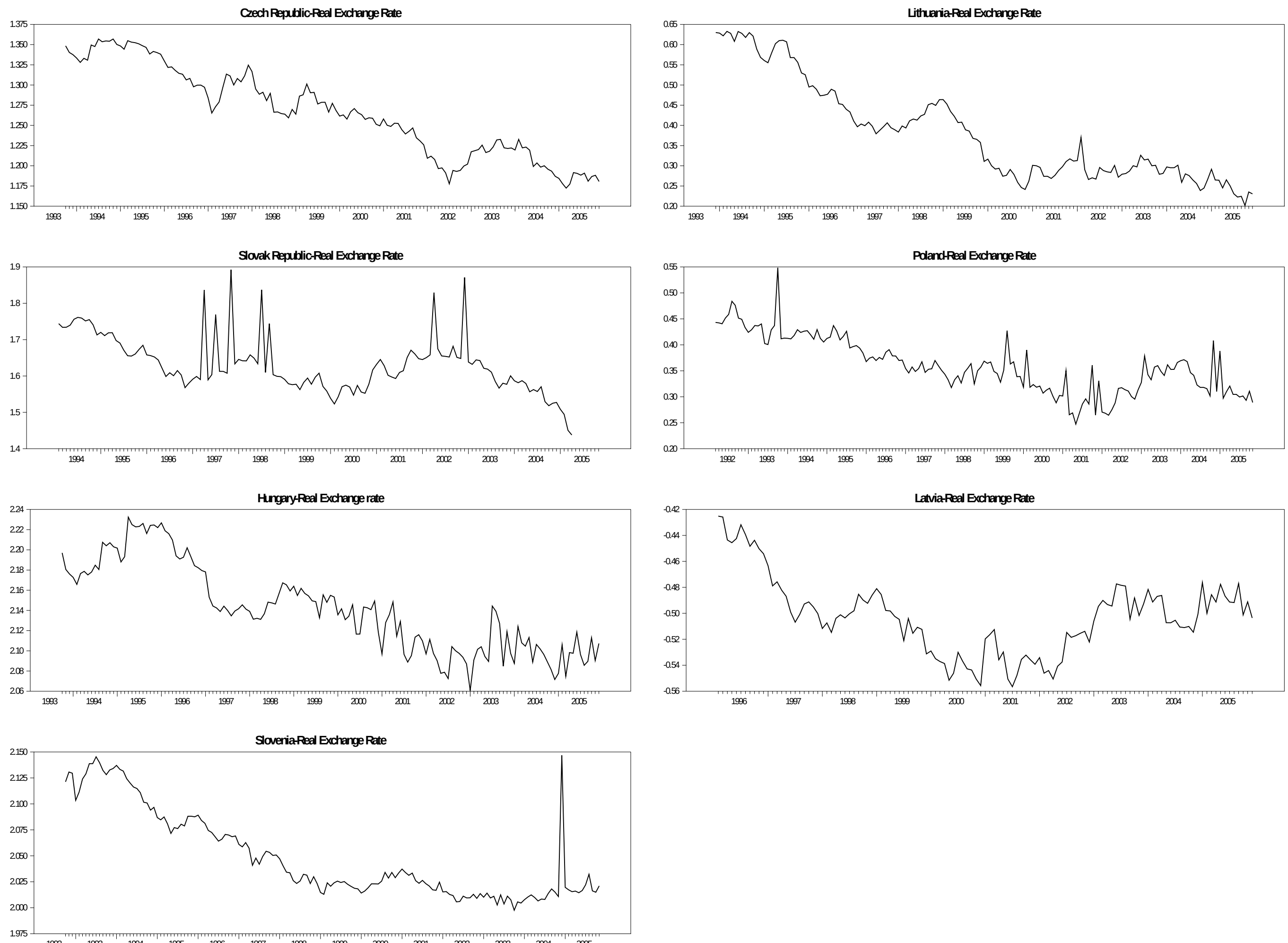
Taking into account the empirical findings, I examine the international effects of changes in monetary policy theoretically. I construct a two country DSGE model in which foreign monetary policy switches regimes over time. I give further insight regarding the effects of regime switching in monetary policy both domestically and abroad. Home monetary policy was initially assumed to be time invariant and follow the Taylor rule with some interest rate smoothing. Home inflation is found to be affected both in terms of volatility and in terms of its response to alternative shocks, by regime shifts in foreign monetary policy (and, consequently, by the change in inflation expectations). Foreign monetary policy regime shifts generate a stabilization and an amplifying effect on output and inflation, both in the foreign and the home country. Which effect arises depends on which regime the foreign monetary policy lies in. When the latter is dovish there is a stabilization effect. That is, impulse responses of inflation and output are dampened, given a positive probability of the foreign monetary policy becoming hawkish. On the contrary, when foreign monetary policy is hawkish there is an amplifying effect in both countries, given a positive probability of the foreign monetary policy becoming dovish. That is, the impulse responses are more volatile. Moreover, there is an asymmetry on the size of each effect. In particular, I show that the stabilization effect is stronger in the foreign, but weaker in the home country, based on a welfare measure, derived by a second order approximation of the agents utility function.

Finally, through the solution of the optimal policy problem of the home central banker, conditional on foreign monetary policy switching regimes over time, I show that it is optimal to follow a time varying interest rate rule. When the home central bank reacts optimally, the effects of regime switches in foreign monetary policy on the home country are completely eliminated. Moreover, the foreign country seems to benefit a lot, in terms of its welfare measure, when the home country reacts optimally to changes in its policy.

# Appendix for Chapter 1

# Appendix A : Real Exchange Rates

**Figure 2: Real Exchange Rates relative to the German Mark**



## Appendix B

Dickey-Fuller Tests (Levels)

| | Czech | Lithuania | Slovak Re. | Slovenia | Poland | Hungary | Latvia |
|---|---|---|---|---|---|---|---|
| $q_t$ | -0.4620 | -1.8075 | -1.4961 | -2.6760 | -1.7270 | -0.9625 | -2.4863 |
| $y_t$ | -0.4552 | -1.9775 | -0.70707 | -2.5186 | -0.9661 | -0.9826 | 0.0671 |
| $r_t - r_t^*$ | -1.4860 | -3.44464* | -0.8555 | -1.7353 | -1.0174 | -1.3586 | -3.27506* |
| $CA_t$ | -1.7495 | -1.1004 | -1.4880 | -2.7234 | -2.0284 | -0.0961 | -2.3836 |

Notes: ** Significant at 1% significance level. * Significant at 5% significance level.

Dickey-Fuller Tests (First Differences)

| | Czech | Lithuania | Slovak Re. | Slovenia | Poland | Hungary | Latvia |
|---|---|---|---|---|---|---|---|
| $q_t$ | -5.61150** | -5.51958** | -6.61717** | -6.15045** | -9.59054** | -7.39139** | -5.39095** |
| $y_t$ | -11.4443** | -6.14024** | -9.43722** | -7.63840** | -11.8253** | -11.0622** | -6.72186** |
| $r_t - r_t^*$ | -4.19793** | -4.31926** | -5.72474** | -6.73867** | -4.66228** | -6.05173** | -5.35520** |
| $CA_t$ | -4.75613** | -4.60758** | -6.14546** | -5.35863** | -5.70488** | -8.24773** | -3.92114** |

Notes: ** Significant at 1% significance level. * Significant at 5% significance level.

### The KSS Test for Global Stationarity.

The LSTAR model given in (5) describes a series that is globally stationary under the assumption that $c_1 + c_1^{'} < 1$[38]. Although $c_1$ can receive values greater than one, $c_1^{'}$ must be less than zero. Consequently, at the reparameterized model (6) we must have $b_0 + b_1^{'} < 0$. This means that we may allow the autoregressive paramter in the linear part of the model to be equal to one, representing, thus, the random walk behaviour of the series in the middle regime (locally nonstationary), but it must be that whenver the series is outside the thresholds (or for large deviations), it exhibits a strong mean reverting behaviour.

Kapetanios, Shin and Snell (2003) suggested a t-test for global stationarity. In particular, they suggested a test where at the ESTAR model (4), the null of $k_E = 0$ (given that $c_1 = 1$) was tested against the alternative of $k_E \neq 0$ (given that $c_1 = 1$). However, as already mentioned, a problem in the ESTAR or the LSTAR model is that one cannot perform a test where the null as that just

[38] In fact, this is a necessary assumption for geometric ergodicity. In particular, in our case the Markov chain $u_t$ is geometrically ergodic if there are constants $\varphi < 1$, $\mu$, $\zeta < \infty$ and a set $K$ such that:

$$E[\|u_t\| \mid u_{t-1} = u] < \varphi\|u_t\| + \zeta \qquad \forall u \notin K,$$

$$E[\|u_t\| \mid u_{t-1} = u] \leq \mu \qquad \forall u \in K.$$

The set $K$ denotes the case where the Markov Chain lies within the thresholds, or the case where it is either on or fluctauates very closely to the long run equilibrium level. If $u \notin K$ , this corresponds to the case where the Chain is outside the bands (or thresholds), or the case where large shocks have occured and the series is described by mean reverting behaviour (since $\varphi < 1$).

described can be tested, since under the null parameter $c_1^{'}$ is not identified. Therefore, Kapetanios et al. (2003) used a first order Taylor approximation of the transition function, having imposed the resriction that $c_1 = 1$. This yielded the following auxilliary regression[39]:

$$\Delta u_t = \delta u_{t-1}^3 + error$$

where the null of no stationarity ($\delta = 0$) is tested against the alternative of global stationarity ($\delta < 0$). The test is conducted as a t-test[40]. We performed the same procedure, in order to test for global stationarity in the LSTAR model. Instead of taking the first order approximation, we took the thrid order, for the same reasons explained at section 4. The power of the test was then computed. As far as the properties of the statistic are concerned, we tend to believe that they should be exactly the same, as a Taylor expansion is used leading to a functional form similar to that used in Kapetanios et al. (2003). The only variation made, compared to the original test, was that in the auxilliary regression where the test was applied, only the statistically significant terms were used, based on the Terasvirta procedure applied at section 4, for the choice of the appropriate model. Kapetanios et al. derived critical values for their statistic using Monte-Carlo simualtion. However, these critical values do not apply in our case as we used a higher order Taylor expansion. Therefore, for each country new critical values were computed using Monte-Carlo techniques. In all countries the null of no stationarity was rejected, suggesting global stationarity (cointegration). Consequently, the variables in equation (2) are linearly cointegrated, but the adjustment, when deviations arise, is nonlinear. The estimated auxilliary regressions are shown at table 16 in the appendix. As expected the coefficients were negative[41].

[39] Having reparameterized the ESTAR model as we did for the LSTAR, given at (6)

[40] Kapetanios et al. (2003) derived the properties of the statistic.

[41] A variant of the test could be used here, instead of using the statistically significant terms from the auxilliary regression, based on the Terasvirta Rule. Hence, one could derive an F-test. The properties of the statistic could be derived in the same fashion as for the t-ratio. However, what is more important, one should be cuatious as far as what the alternative hypothesis is. In Kapetanios et al. (2003) the alternative is that $\delta < 0$. One suggestion, if one carries out the joint F-test version of the KSS test, is to construct hypotheses conditional upon the coefficients to be negative.

| Estimated Auxilliary Regressions for the KSS test. |
|---|
| Czech Republic |
| $\Delta u_t = -49.5512 u_{t-1} u^3_{t-2}$ |
| (0.0243) |
| Lithuania |
| $\Delta u_t = -2.315 u_{t-1} u_{t-10}$ |
| (0.0006) |
| Slovak Republic |
| $\Delta u_t = -3.358 u^2_{t-1}$ |
| (0.0005) |
| Slovenia |
| $\Delta u_t = -2.315 u^2_{t-1}$ |
| (0.0003) |
| Hungary |
| $\Delta u_t = -5185.15 u^2_{t-1}$ |
| (0.03173) |
| Poland |
| $\Delta u_t = -5.4734 u^2_{t-1}$ |
| (0.00002) |
| Latvia |
| $\Delta u_t = -233.43 u^2_{t-1}$ |
| (0.0096) |

Notes: Standard errors in parentheses, estimated through Monte-Carlo simulation, where 10000 samples of $100+T$ observations were generated. The first 100 observations were discarded.

The power of the test was then computed. As in the previous cases, 10000 samples of $100+T$ observations were generated, where the first 100 observations were dropped. The test appears to have strong power towards rejecting the false null of $\delta = 0$ for all countries, but Lithuania and Slovenia. Apart from the fact that the test indicates global stationarity, it also provides evidence in favour of nonlinear mean reversion[42]. The results are shown at the table below.

Power of the KSS test

| | KSS |
|---|---|
| Czech | 87.75 |
| Lithuania | 77.08 |
| Slovak | 100.00 |
| Slovenia | 75.83 |
| Hungary | 66.21 |
| Poland | 94.20 |
| Latvia | 100.00 |

**Appendix C**

Estimation Results for the Multivariate Smooth Transition Error Correction Model

[42]In the Monte-Carlo experiments the values of the coefficients were set to be equal to those found by the Gauss-Newton estimation.

Czech Republic: MSTeqC model estimation results

| dep variable | Coefficients | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $\Gamma_1$ | $\Gamma_2$ | $\Gamma_3$ | $\Gamma_4$ | $\pi_{1,1}$ | $\pi_{2,1}$ | $k_E, k_L$ | $c_E, c_L$ |
| $\Delta q_t$ | -0.431 | - | - | 0.164 | 0.456 | | | | -0.076 | 0.072 | 6.761 | 0.023 |
| | (0.031) | | | (0.045) | (0.052) | | | | (0.029) | (0.052) | (0.082) | (0.111) |
| $\Delta y_t$ | 1.870 | -0.247 | - | - | -2.910 | - | - | - | - | 0.341 | 6.761 | 0.023 |
| | (0.031) | (0.006) | | | (0.005) | | | | | (0.054) | (0.082) | (0.111) |
| $\Delta(r_t - r_t^*)$ | 1.379 | -0.183 | 0.330 | - | - | - | - | - | 0.235 | - | 6.761 | 0.023 |
| | (0.044) | (0.009) | (0.000) | | | | | | (0.062) | | (0.082) | (0.111) |
| $\Delta CA_t$ | - | - | - | | | | -0.576 | | 0.271 | 0.2672 | 6.761 | 0.023 |
| | | | | | | | (0.018) | | (0.021) | (0.035) | (0.082) | (0.111) |

Lithuania: MSTeqC model estimation results

| dep variable | Coefficients | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $\Gamma_1$ | $\Gamma_2$ | $\Gamma_3$ | $\Gamma_4$ | $\pi_{1,1}$ | $\pi_{2,1}$ | $k_E, k_L$ | $c_E, c_L$ |
| $\Delta q_t$ | -1.774 | - | 1.267 | - | 1.664 | - | -1.254 | - | 0.960 | -1.000 | 1.000 | - |
| | (0.000) | | (0.000) | | (0.000) | | (0.000) | | (0.000) | (0.000) | (0.000) | |
| $\Delta y_t$ | - | - | - | - | - | - | - | - | 1.252 | -1.000 | 1.000 | - |
| | | | | | | | | | (0.000) | (0.000) | (0.000) | |
| $\Delta(r_t - r_t^*)$ | 1.828 | -0.183 | 0.330 | - | 2.179 | - | -1.062 | - | 1.029 | -1.000 | 1.000 | - |
| | (0.008) | (0.009) | (0.000) | | (0.002) | | (0.030) | | (0.000) | (0.000) | (0.000) | |
| $\Delta CA_t$ | - | - | - | 0.271 | - | - | - | 0.763 | 0.271 | -1.000 | 1.000 | - |
| | | | | (0.012) | | | | (0.000) | (0.021) | (0.000) | (0.000) | |

Slovak Republic: MSTeqC model estimation results

| dep variable | Coefficients | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $\Gamma_1$ | $\Gamma_2$ | $\Gamma_3$ | $\Gamma_4$ | $\pi_{1,1}$ | $\pi_{2,1}$ | $k_E, k_L$ | $c_E, c_L$ |
| $\Delta q_t$ | - | - | - | - | - | - | - | - | - | -1.000 | 1.359 | -0.039 |
| | | | | | | | | | | (0.000) | (0.000) | (0.000) |
| $\Delta y_t$ | - | - | - | - | 0.4012 | - | - | 0.804 | 0.518 | -1.000 | 1.359 | -0.039 |
| | | | | | (0.007) | | | (0.065) | (0.000) | (0.000) | (0.000) | (0.000) |
| $\Delta(r_t - r_t^*)$ | - | - | -0.506 | - | 0.582 | - | -1.062 | - | 0.532 | -1.000 | 1.359 | -0.039 |
| | | | (0.001) | | (0.005) | | (0.030) | | (0.000) | (0.000) | (0.000) | (0.000) |
| $\Delta CA_t$ | -0.270 | - | - | -0.517 | 0.623 | - | - | 1.011 | 0.400 | -1.000 | 1.359 | -0.039 |
| | (0.000) | | | (0.003) | (0.000) | | | (0.001) | (0.000) | (0.000) | (0.000) | (0.000) |

Poland: MSTeqC model estimation results

| dep variable | Coefficients | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $\Gamma_0$ | $\Gamma_1$ | $\Gamma_2$ | $\Gamma_3$ | $\Gamma_4$ | $\pi_{1,1}$ | $\pi_{2,1}$ | $k_E, k_L$ | $c_E, c_L$ |
| $\Delta q_t$ | -0.367 | - | - | - | - | - | - | - | - | 0.637 | -1.000 | 0.106 | -16.169 |
| | (0.000) | - | | - | | | | | | (0.000) | (0.000) | (0.000) | (0.000) |
| $\Delta y_t$ | - | -0.253 | - | - | - | - | - | - | - | 0.987 | -1.000 | 0.106 | -16.169 |
| | | (0.001) | | | | | | | | (0.000) | (0.000) | (0.000) | (0.000) |
| $\Delta(r_t - r_t^*)$ | - | - | -0.215 | - | - | - | - | - | - | 1.023 | -1.000 | 0.106 | -16.169 |
| | - | | (0.005) | | | | | | | (0.000) | (0.000) | (0.000) | (0.000) |
| $\Delta CA_t$ | - | - | - | - | - | - | - | - | - | 0.987 | -1.000 | 0.106 | -16.169 |
| | | | | | | | | | | (0.000) | (0.000) | (0.000) | (0.000) |

Hangury: MSTeqC model estimation results

| dep variable | Coefficients | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $\Gamma_0$ | $\Gamma_1$ | $\Gamma_2$ | $\Gamma_3$ | $\Gamma_4$ | $\pi_{1,1}$ | $\pi_{2,1}$ | $k_E, k_L$ | $c_E, c_L$ |
| $\Delta q_t$ | -0.234 | - | - | -0.127 | - | - | - | - | - | - | -0.290 | 9.471 | 0.001 |
| | (0.005) | - | | (0.049) | | | | | | | (0.002) | (0.000) | (0.000) |
| $\Delta y_t$ | - | -0.258 | - | - | - | - | - | - | - | - | - | 9.471 | 0.001 |
| | | (0.001) | | | | | | | | | (0.407) | (0.000) | (0.000) |
| $\Delta(r_t - r_t^*)$ | - | 0.124 | 0.166 | - | -0.009 | - | - | - | - | - | -0.010 | 9.471 | 0.001 |
| | - | (0.043) | (0.047) | | (0.085) | | | | | | (0.036) | (0.000) | (0.000) |
| $\Delta CA_t$ | - | - | - | - | - | - | - | - | - | - | -0.320 | 9.471 | 0.001 |
| | | | | | | | | | | | (0.021) | (0.000) | (0.000) |

Latvia: MSTeqC model estimation results

| dep variable | Coefficients | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $\Gamma_0$ | $\Gamma_1$ | $\Gamma_2$ | $\Gamma_3$ | $\Gamma_4$ | $\pi_{1,1}$ | $\pi_{2,1}$ | $k_E, k_L$ | $c_E, c_L$ |
| $\Delta q_t$ | -0.325 | - | 0.036 | - | - | - | - | - | - | 0.981 | -1.000 | 1.439 | -0.156 |
| | (0.005) | - | (0.034) | | | | | | | (0.000) | (0.000) | (0.000) | (0.000) |
| $\Delta y_t$ | - | -0.199 | - | - | - | - | - | - | - | 0.550 | -1.000 | 1.439 | -0.156 |
| | | (0.047) | | | | | | | | (0.000) | (0.000) | (0.000) | (0.000) |
| $\Delta(r_t - r_t^*)$ | - | - | 0.202 | - | - | - | - | - | - | 0.946 | -1.000 | 1.439 | -0.156 |
| | | | (0.026) | | | | | | | (0.000) | (0.000) | (0.000) | (0.000) |
| $\Delta CA_t$ | - | - | - | - | - | - | - | - | - | 0.822 | -1.000 | 1.439 | -0.156 |
| | | | | | | | | | | (0.000) | (0.000) | (0.000) | (0.000) |

Slovenia: MSTeqC model estimation results

| dep variable | Coefficients | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $\Gamma_0$ | $\Gamma_1$ | $\Gamma_2$ | $\Gamma_3$ | $\Gamma_4$ | $\pi_{1,1}$ | $\pi_{2,1}$ | $k_E, k_L$ | $c_E, c_L$ |
| $\Delta q_t$ | - | - | - | -0.185 | - | - | - | - | - | - | -1.000 | 1.017 | -0.529 |
| | | | | (0.091) | | | | | | | (0.000) | (0.003) | (0.000) |
| $\Delta y_t$ | - | -0.269 | - | - | - | - | - | - | - | - | -1.000 | 1.017 | -0.529 |
| | | (0.000) | | | | | | | | | (0.000) | (0.003) | (0.000) |
| $\Delta(r_t - r_t^*)$ | - | - | -0.190 | - | - | - | - | - | -1.006 | - | -1.000 | 1.017 | -0.529 |
| | | | (0.022) | | | | | | (0.003) | | (0.000) | (0.003) | (0.000) |
| $\Delta CA_t$ | - | - | - | - | 0.005 | - | - | - | - | - | -1.000 | 1.017 | -0.529 |
| | | | | | (0.000) | | | | | | (0.000) | (0.003) | (0.000) |

## Appendix D

Specification tests in the Multivariate Smooth Transition Error Correction Model

Table5 : Heteroskedasticity tests in the MSTeqC model

| | $F_1^{ARCH}$ | $F_2^{ARCH}$ | $F_3^{ARCH}$ | $F_4^{ARCH}$ |
|---|---|---|---|---|
| Czech Republic | 0.653 | 0.915 | 0.021 | 0.854 |
| Lithuania | 0.031 | 0.713 | 0.579 | 0.107 |
| Slovak Republic | 0.105 | 0.925 | 0.088 | 0.904 |
| Slovenia | 0.657 | 0.732 | 0.035 | 0.936 |
| Hungary | 0.890 | 0.478 | 0.025 | 0.000 |
| Poland | 0.576 | 0.034 | 0.799 | 0.999 |
| Latvia | 0.555 | 0.021 | 0.791 | 0.079 |

Notes:Numbers reported are $p - values$. $F_1^{ARCH}$ is the test for the regression for $\Delta f_t$, $F_2^{ARCH}$ is the test for the regression for $\Delta y_t$, $F_3^{ARCH}$ is the test for the regression for $\Delta(i - i*)_t$, $F_3^{ARCH}$is the test for the regression for $\Delta CA_t$

Table 6 : Residual autocorrelation tests in the MSTeqC model

| | $F_1^{COR}$ | $F_2^{COR}$ | $F_3^{COR}$ | $F_4^{COR}$ |
|---|---|---|---|---|
| Czech Republic | 0.101 | 0.000 | 0.222 | 0.774 |
| Lithuania | 0.594 | 0.003 | 0.917 | 0.174 |
| Slovak Republic | 0.034 | 0.000 | 0.347 | 0.704 |
| Slovenia | 0.777 | 0.260 | 0.011 | 0.139 |
| Hungary | 0.476 | 0.000 | 0.686 | 0.007 |
| Poland | 0.313 | 0.000 | 0.727 | 0.999 |
| Latvia | 0.938 | 0.000 | 0.945 | 0.082 |

Notes:Numbers reported are $p - values$. $F_1^{COR}$ is the test for the regression for $\Delta f_t$, $F_2^{COR}$ is the test for the regression for $\Delta y_t$, $F_3^{COR}$ is the test for the regression for $\Delta(i - i*)_t$, $F_3^{COR}$is the test for the regression for $\Delta CA_t$

Table 7 : Neglected Nonlinearities tests in the MSTeqC model

| | $F_1^{NNL}$ | $F_2^{NNL}$ | $F_3^{NNL}$ | $F_4^{NNL}$ |
|---|---|---|---|---|
| Czech Republic | 0.281 | 0.029 | 0.908 | 0.999 |
| Lithuania | 0.617 | 0.079 | 0.962 | 0.090 |
| Slovak Republic | 0.042 | 0.265 | 0.189 | 0.397 |
| Slovenia | 0.126 | 0.733 | 0.020 | 0.092 |
| Hungary | 0.256 | 0.175 | 0.010 | 0.152 |
| Poland | 0.027 | 0.571 | 0.429 | 0.508 |
| Latvia | 0.424 | 0.432 | 0.066 | 0.571 |

Notes:Numbers reported are $p - values$. $F_1^{NNL}$ is the test for the regression for $\Delta f_t$,$F_2^{NNL}$ is the test for the regression for $\Delta y_t$,$F_3^{NNL}$ is the test for the regression for $\Delta(i - i*)_t$, $F_3^{NNL}$ is the test for the regression for $\Delta CA_t$

**Appendix for Chapters 2 and 3 (Joint)**

**Appendix E: The steady State**

In this section I compute the steady state of the the real variables, first and then through the proof of proposition 1, the steady state of the nominal variables.

Given that in the steady state each firm will change the same price in both countries, the law of one price holds and, hence, PPP holds as well. Therefore the real exchange rate is pegged to one.

$$Q = 1$$

Given an international risk sharing condition, PPP implies that at the steady state consumption levels will be equalized across the two countries. Hence

$$C = C^*$$

From the representative household's labor supply decision, I have for each country that

$$L^{\gamma} = C^{-\sigma}\frac{W}{P}$$

$$L^{*\gamma} = C^{*-\sigma}\frac{W^*}{P^*}$$

while from the firms production function in each country, I have that

$$Y = L \quad and \quad Y^* = L^*$$

As already mentioned, firms will set the same price in each country. From their maximization problem it follows that prices at the steady state will be specified as follows

$$p_H = Sp_H^* = P_H = \frac{\theta}{\theta - 1}\frac{W}{A}$$

$$\frac{p_F^*}{S} = p_F = P_F^* = \frac{\theta}{\theta - 1}\frac{W^*}{A^*}$$

and since the law of one price holds, the demand for the home and foreign produced good respectively will be specified as

$$Y_H = \left(\frac{P_H}{P}\right)^{-\rho} C$$

$$Y_F = \left(\frac{P_F^*}{P^*}\right)^{-\rho} C$$

Combining, thus, the above equations, along with the household's optimal labor decision I end up to the following expressions for the consumption levels in the steady state

$$C = \left[\frac{\theta - 1}{\theta}\left(\frac{P_H}{P}\right)^{1+\rho\gamma} A\right]^{\frac{1}{\gamma+\sigma}}$$

$$C^* = \left[\frac{\theta - 1}{\theta}\left(\frac{P_F^*}{P^*}\right)^{1+\rho\gamma} A^*\right]^{\frac{1}{\gamma+\sigma}}$$

As in Benigno (2004), note that both $\frac{P_H}{P}$ and $\frac{P_F}{P}$ are both functions of $T \equiv \frac{P_F}{P_H}$ , so that the two equations above uniquely determine $C$ and $T$. Having specified the steady state values of consumption output and relative prices, I can proceed to the proof of proposition in section 5.

**Proof of Proposition in section 5**

The foreign households intertemporal decision (14) implies that in the steady state the following will be true for the nominal interest rate

$$i^* = \frac{\pi^*}{\beta}$$

Additionally, the assumed interest rate rule of the foreign country (31) receives the following form in the steady state

$$i = \xi_s \left(\frac{\pi^*}{\tilde{\pi}^*}\right)^{\phi_{\pi^*}^*} y^{*\phi_{y^*,s}^*}$$

Combining the above two equations for the foreign interest rate, solving for $\xi_s$ and recalling that the interest rate in the steady state is such that foreign inflation $\pi^*$ hits its target $\tilde{\pi}^*$, I receive the following

$$\xi_s = \frac{1}{\beta}\pi^* y^{*\phi^*_{y^*,s}}$$

Therefore the steady state interest rate is

$$i^* = \frac{\tilde{\pi}^*}{\beta}$$

and, as already mentioned, inflation at the steady state is $\pi^* = \tilde{\pi}^*$. Nominal variables, thus, are independent of policy regime in the steady state. Moreover, as already shown above, the real variables (i.e. consumption, output, labor) are independent of policy regime, as well, in the steady state.

**Appendix F: Aggregate Supply and Aggregate Demand**

In this section I derive the PPI inflation rates (33) and (34) and the aggregate demand equation (38) reported in the text.

**Aggregate Supply**

Forward looking producers in the home country maximize their profits in the home market by choosing the optimal price specified as

$$p_t^{For}(h) = \frac{\theta}{\theta - 1}\frac{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} MC_{t+s} y^h_{t+s}(p_t(h))}{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} y^h_{t+s}(p_t(h))}$$

where $y^h_{t+s}(p_t(h))$ is specified in (24) in the text. The optimal price above rearranged can be written in the following form

$$E_t \sum_{s=0}^{\infty} (\omega\beta)^s \frac{C_{t+s}^{-\sigma} P_{H,t+s}}{P_{t+s}} \left[ \left\{ \frac{p_t^{For}(h)}{P_{H,t+s}} - \left(\frac{\theta}{\theta-1}\right) \frac{W_{t+s}}{A_{t+s} P_{H,t+s}} \right\} y^h_{t+s}(p_t(h)) \right] = 0$$

and its loglinear approximation is summarized as follows

$$E_t \sum_{s=0}^{\infty} (\omega\beta)^s \left[ \hat{\hat{p}}^{For}_{t,t+s}(h) - \left( \frac{\widehat{W_{t+s}}}{A_{t+s} P_{H,t+s}} \right) \right] = 0 \tag{61}$$

where $\hat{\hat{p}}_t^{For}(h) = ln\left(\frac{p_t^{For}(h)}{P_{H,t+s}}\right)$. Using the household's optimality condition (13) I can expand the marginal cost term in the above relationship as follows

$$\frac{\widehat{W_{t+s}}}{A_{t+s}P_{H,t+s}} = \gamma\left(\hat{y}_{t+s}(h) - a_t\right) + \frac{\sigma}{\psi}\hat{C}_{t+s} + \frac{(1-\psi)\sigma}{\psi}\hat{C}_{t+s-1} + a_{t+s} + (1-\delta)\hat{T}_{t+s}$$

where I have used the fact that $\hat{C_t^O} = \frac{1}{\psi}\hat{C}_t - \frac{1-\psi}{\psi}\hat{C}_{t-1}$ . Furthermore, by suing the demand for the home good $\hat{y}_{t+s}(h)$ can be expanded as follows

$$\hat{y}_{t+s}(h) = -\rho\delta\hat{\hat{p}}_{t,t+s}(h) + \rho\delta\left(1-\delta\right)\hat{T}_{t+s} + \hat{C}_{t+s} - \rho\left(1-\delta^*\right)\hat{\hat{p}}^*_{t,t+s}(h)\ldots$$

$$-\rho\delta^*\left(1-\delta^*\right)\hat{T}^*_{t+s} - \frac{(1-\delta^*)}{\sigma}\hat{q}_{t+s}$$

But $\hat{\hat{p}}_{t+s}(h)$ and $\hat{\hat{p}}^*_{t,t+s}(h)$ are specified as

$$\hat{\hat{p}}_{t+s}(h) = \zeta\hat{p}^{For}_{t+s}(h) + (1-\zeta)\,\hat{p}^{B}_{t+s}(h)$$

$$\hat{\hat{p}}^*_{t,t+s}(h) = \zeta\hat{p}^{*For}_{t+s}(h) + (1-\zeta)\,\hat{p}^{*B}_{t+s}(h)$$

for the home good in the home and the foreign market respectively. From (19) $\hat{\hat{p}}_t(h)$ and $\hat{\hat{p}}^*_t(h)$ can be expressed as follows

$$\hat{\hat{p}}_{t,t+s}(h) = \frac{\omega}{1-\omega}\pi_{H,t} - \sum_{i=1}^{s}\pi_{H,t+i}$$

$$\hat{\hat{p}}^*_{t,t+s}(h) = \frac{\omega^*}{1-\omega^*}\pi^*_{H,t} - \sum_{i=1}^{s}\pi^*_{H,t+i}$$

Combining the above relationships for the prices set at date $t$ , I can express the price set by the forward looking firms as follows

$$\hat{p}^{For}_t(h) - P_{H,t-1} = \frac{1}{(1-\omega)(1-\zeta)}\pi_{H,t} - \frac{\zeta}{(1-\omega)(1-\zeta)}\pi_{H,t-1}$$

Solving for $\hat{p}^{For}_{t,t+s}(h)$ in (61) and combining all the above relationships I end to the following relationship for PPI inflation

$$\pi_{H,t} = \frac{\zeta}{\left(\zeta + \omega\left(1-\zeta\right) + \theta\gamma\delta\omega\left(1-\zeta\right)\right)}\pi_{H,t-1} + \frac{\left(\omega-\omega^*\right)\left(\gamma\theta\left(1-\delta^*\right)\left(1-\zeta\right)\left(1-\omega\right)\right)}{\left(1-\omega^*\right)\left(\zeta + \omega\left(1-\zeta\right) + \theta\gamma\delta\omega\left(1-\zeta\right)\right)}\pi^*_{H,t} + \ldots$$

$$\frac{\left(1-\omega\beta\right)\left(1-\zeta\right)\left(1-\omega\right)}{\left(\zeta + \omega\left(1-\zeta\right) + \theta\gamma\delta\omega\left(1-\zeta\right)\right)}\hat{R}_t + \frac{\omega\gamma\theta\left(1-\delta^*\right)\left(1-\zeta\right)\left(1-\omega\right)}{\left(1-\omega^*\right)\left(\zeta + \omega\left(1-\zeta\right) + \theta\gamma\delta\omega\left(1-\zeta\right)\right)}\left(\beta E_t\pi^*_{H,t+1} - \pi^*_{H,t}\right)$$

where $\hat{R}_t$ is specified as

$$\hat{R}_t = (1+\gamma\rho\delta)(1-\delta)\hat{T}_t + \left(\gamma + \frac{\sigma}{\psi}\right)\hat{C}_t - \gamma\rho\delta^*(1-\delta^*)\hat{T}_t^* - \frac{\gamma(1-\delta^*)}{\sigma}\hat{q}_t - \frac{(1-\psi)\sigma}{\psi}\hat{C}_{t-1} - (\gamma+1)a_t$$

and from the resource constraint

$$\hat{C}_t = \hat{Y}_t - \rho\delta(1-\delta)\hat{T}_t + \rho(1-\delta^*)\delta^*\hat{T}_t^* + \left(\frac{1-\delta^*}{\sigma}\right)\hat{q}_t$$

The supply of home produced goods in the foreign country is derived by following similar steps. Home producers set their price in foreign country according to the following maximization rule

$$E_t\sum_{s=0}^{\infty}(\omega^*\beta)^s\frac{C_{t+s}^{-\sigma}P_{H,t+s}}{P_{t+s}}\left[\left\{\frac{p_t^{*For}(h)}{P_{H,t+s}^*}\frac{Z_{t+s}P_{H,t+s}^*}{P_{H,t+s}} - \left(\frac{\theta}{\theta-1}\right)\frac{W_{t+s}}{A_{t+s}P_{H,t+s}}\right\}y_{t+s}^f(p_t(h))\right] = 0$$

and its loglinear approximation is summarized as follows

$$E_t\sum_{s=0}^{\infty}(\omega\beta)^s\left[\hat{p}_{t,t+s}^{*For}(h) + \widehat{zh_t} - \left(\widehat{\frac{W_{t+s}}{A_{t+s}P_{H,t+s}}}\right)\right] = 0 \tag{62}$$

where $zh_t = \frac{Z_tP_{H,t}^*}{P_{H,t}}$. And after following similar steps as in the derivation of the supply in the home country I conclude to the following for the supply of home goods in the foreign country

$$\pi_{H,t}^* = \frac{\zeta}{(\zeta+\omega^*(1-\zeta)+\theta\gamma\delta\omega^*(1-\zeta))}\pi_{H,t-1}^* + \frac{(\omega^*-\omega)(\gamma\theta\delta(1-\zeta)(1-\omega^*))}{(1-\omega)(\zeta+\omega^*(1-\zeta)+\theta\gamma\delta\omega^*(1-\zeta))}\pi_{H,t} + \dots$$

$$\frac{(1-\omega^*\beta)(1-\zeta)(1-\omega^*)}{(\zeta+\omega^*(1-\zeta)+\theta\gamma\delta\omega^*(1-\zeta))}\hat{R}_t^* + \frac{\omega^*\gamma\theta\delta(1-\zeta)(1-\omega^*)}{(1-\omega)(\zeta+\omega^*(1-\zeta)+\theta\gamma\delta\omega^*(1-\zeta))}(\beta E_t\pi_{H,t+1} - \pi_{H,t})$$

Having used $\widehat{zh_t} = \hat{q}_t - \delta^*\hat{T}_t^* + (1-\delta)\hat{T}_t$, $\hat{R}_t^*$ is specified as

$$\hat{R}_t^* = (\gamma\rho\delta-1)(1-\delta)\hat{T}_t + \left(\gamma+\frac{\sigma}{\psi}\right)\hat{C}_t - \delta^*(\gamma\rho(1-\delta^*)-1)\hat{T}_t^* - \left(\frac{\gamma(1-\delta^*)}{\sigma}+1\right)\hat{q}_t - \frac{(1-\psi)\sigma}{\psi}\hat{C}_{t-1} - (\gamma+1)a_{t+s}$$

**Aggregate Demand**

The market clearing condition for home goods market satisfies the following

$$Y_t = C_{H,t} + C^*_{H,t}$$

or

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\rho} \delta C_{H,t} + \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\rho} (1-\delta^*) C^*_{H,t}$$

and after loglinearizing and solving for $\hat{C}_t$ , I receive the following

$$\hat{C}_t = \hat{Y}_t - \rho\delta(1-\delta)\hat{T}_t + \rho(1-\delta^*)\delta^*\hat{T}^*_t + \left(\frac{1-\delta^*}{\sigma}\right)\hat{q}_t$$

Using the Euler equation accruing from the optimizing households loglinearized first order condition (12) and the fact that $\hat{C^O_t} = \frac{1}{\psi}\hat{C}_t - \frac{1-\psi}{\psi}\hat{C}_{t-1}$, I end up to the aggregate demand equation for the home country

$$\hat{Y}_t = -\frac{\psi}{(2-\psi)\sigma}(i_t - E_t\pi_{t+1}) + \frac{1}{2-\psi}E_t\hat{Y}_{t+1} + \frac{1-\psi}{2-\psi}\hat{Y}_{t-1} - \frac{\rho\delta(1-\delta)}{2-\psi}E_t\hat{T}_{t+1} + \frac{\rho\delta^*(1-\delta^*)}{2-\psi}E_t\hat{T}^*_{t+1} + \dots$$

$$\frac{(1-\delta^*)}{(2-\psi)\sigma}E_t\hat{q}_{t+1} + \rho\delta(1-\delta)\hat{T}_t - \rho\delta^*(1-\delta^*)\hat{T}^*_t - \frac{1-\delta^*}{\sigma}\hat{q}_t - \frac{\rho\delta(1-\psi)(1-\delta)}{2-\psi}\hat{T}_{t-1} + \dots$$

$$\frac{\rho\delta^*(1-\psi)(1-\delta^*)}{2-\psi}\hat{T}^*_{t-1} + \frac{(1-\psi)(1-\delta^*)}{(2-\psi)\sigma}\hat{q}_{t-1}$$

and similarly for the foreign country

$$\hat{Y}^*_t = -\frac{\psi^*}{(2-\psi^*)\sigma}\left(i^*_t - E_t\pi^*_{t+1}\right) + \frac{1}{2-\psi^*}E_t\hat{Y}^*_{t+1} + \frac{1-\psi^*}{2-\psi^*}\hat{Y}^*_{t-1} - \frac{\rho\delta^*(1-\delta^*)}{2-\psi^*}E_t\hat{T}^*_{t+1} + \frac{\rho\delta(1-\delta)}{2-\psi^*}E_t\hat{T}_{t+1} - \dots$$

$$-\frac{(1-\delta)}{(2-\psi^*)\sigma}E_t\hat{q}_{t+1} + \rho\delta^*(1-\delta^*)\hat{T}^*_t - \rho\delta(1-\delta)\hat{T}_t + \frac{1-\delta}{\sigma}\hat{q}_t - \frac{\rho\delta^*(1-\psi^*)(1-\delta^*)}{2-\psi^*}\hat{T}^*_{t-1} + \dots$$

$$\frac{\rho\delta(1-\psi^*)(1-\delta)}{2-\psi^*}\hat{T}_{t-1} - \frac{(1-\psi^*)(1-\delta^*)}{(2-\psi^*)\sigma}\hat{q}_{t-1}$$

### Appendix G: The welfare criterion

In this section I derive the second order approximation (44) to the representative household's utility function (6) in the home country. The steps for the derivation of the welfare measure for the foreign country are exactly the same. I assume that there is a subsidy to labor. This implies that the steady state is efficient, given that the distortions form monopolistic competition are exhausted. Therefore, I derive the welfare criterion for each country using a second-order Taylor series expansion of (6) around the efficient steady state. Moreover, the welfare measure is expressed as deviations from the flexible price equilibrium, which is efficient as well, given the labor subsidy.

The second order approximation of the welfare of the representative optimizing household receives the following form

$$W_t = U + U_C(\hat{C_t^O} + \frac{1}{2}(1 + \frac{U_{CC}C}{U_C})\hat{C_t^O}^2) - U_L(\hat{L}_t + \frac{1}{2}(1 + \frac{U_{LL}L}{U_L})\hat{L}_t^2 \tag{63}$$

where $U_C = C^{-\sigma}$, $U_{CC} = C^{-\sigma-1}$, $U_L = L^{\gamma}$ and $U_{LL} = L^{\gamma-1}$. Using the fact that $\hat{y}_t(h) = a_t + \hat{L}_t$ and approximating it up to a second order I receive the following expression for labor

$$\hat{L}_t = 1 + \frac{y(h)}{L}E_t(\hat{y}_t(h)) + a_t + \frac{y(h)}{2L}var(\hat{y}_t(h)) + a_t^2 - \frac{1}{2}\hat{L}_t^2 \tag{64}$$

Moreover by Woodford (Ch. 6) I have that

$$var(\hat{y}(i)) = \delta\theta^2 var(\tilde{p}_t(h)) + (1-\delta)\theta^2 var(\tilde{p}_t^*(h)) \tag{65}$$

But $\tilde{p}_t(h)$ and $\tilde{p}_t(h)$ are determined according to (18) in the main text. Let $\bar{P}_{H,t} \equiv E_t\left[log(\tilde{p}_t(h))\right]$ and $\Delta_t \equiv var(log(\tilde{p}_t(h))$. Then,

$$\begin{aligned}\Delta_t &\equiv var(log(\tilde{p}_t(h) - P_{H,t-1}) \\ &= E_t\left[(log(\tilde{p}_t(h) - P_{H,t-1})^2 - (E_t\left[log(\tilde{p}_t(h) - P_{H,t-1}\right])^2\right] \\ &= \omega\Delta_{t-1} + (1-\omega)\zeta(log(p_t^B(h) - \bar{P}_{H,t-1})^2 + (1-\omega)(1-\zeta)(log(p_t^{For}(h) - \bar{P}_{H,t-1})^2 \\ &\quad -(\bar{P}_{H,t} - \bar{P}_{H,t-1})\end{aligned} \tag{66}$$

where $p_t^B(h)$ and $p_t^F(h)$ are the prices set by the backward and forward looking firms respectively. The same expression holds for $\tilde{p}_t^*(h)$. Before substituting the above expression in (62) and then in (61), note that $\bar{P}_{H,t} = log(\bar{P}_{H,t}) + O(||\xi||^2)$, so that $\bar{P}_{H,t} - \bar{P}_{H,t-1} = \pi_{H,t} + O(||\xi||^2)$. Additionally, the following relationships hold

$$\tilde{p}_t(h) = \zeta p_t^B(h) + (1-\zeta)\, p_t^{For}(h)$$

$$\tilde{p}_t(h) = \frac{\omega}{1-\omega}\pi_{H,t} + P_{H,t}$$

Using the above expressions for $\tilde{p}_t(h)$ I end up to the following expression for the price that is set by the forward looking firms

$$\hat{p}_t^{For}(h) - P_{H,t-1} = \frac{1}{(1-\omega)(1-\zeta)}\pi_{H,t} - \frac{\zeta}{(1-\omega)(1-\zeta)}\pi_{H,t-1}$$

Substituting the above expression into (64), I receive the following for $\Delta_t$

$$\sum_{t=0}^{\infty}\beta^t\Delta_t = \frac{1}{(1-\omega\beta)}\sum_{t=0}^{\infty}\beta^t\left[\frac{\omega}{1-\omega}\pi_{H,t}^2 + \frac{1-\zeta}{\zeta(1-\omega)}\left(\pi_{H,t}-\pi_{H,t-1}\right)^2\right] + t.i.p. + O(||\xi||^3) \qquad (67)$$

Similarly for the price set in the foreign country for the home good I receive the following

$$\sum_{t=0}^{\infty}\beta^t\Delta_t^* = \frac{1}{(1-\omega^*\beta)}\sum_{t=0}^{\infty}\beta^t\left[\frac{\omega^*}{1-\omega^*}\pi_{H,t}^{*2} + \frac{1-\zeta}{\zeta(1-\omega^*)}\left(\pi_{H,t}^*-\pi_{H,t-1}^*\right)^2\right] + t.i.p. + O(||\xi||^3) \qquad (68)$$

where $t.i.p.$ represents terms independent of policy and $O(||\xi||^3)$ stands for terms of order higher than two.

Additionally, note that for the home output the following relationship holds (and similarly for foreign output)

$$\hat{Y}_t = E_t(\hat{y}_t(h)) + \frac{1}{2}\left(\frac{\theta-1}{\theta}\right) var(\hat{y}_t(h)) + O(||\xi||^3)$$

Using the above expression to substitute for $E_t(\hat{y}_t(i))$ in equation (2), I receive the following expression for $\hat{L}_t$

$$\hat{L}_t \approx 1 + \frac{Y}{L}\hat{Y}_t - \frac{1}{2\theta}\frac{Y}{L}var(\hat{y}_t(h)) - \frac{1}{2}\hat{L}_t^2 + t.i.p. \qquad (69)$$

Finally, a second order approximation of the resource constraint of the model yields the following

$$\hat{C}_t \approx \frac{1}{2}\hat{Y}_t + \frac{1}{4}\hat{Y}_t^2 + \frac{1}{2}\hat{Y}_t^* + \frac{1}{4}\hat{Y}_t^{*2} + \frac{1}{2\sigma}\hat{q}_t + \frac{1}{4\sigma^2}\hat{q}_t^2 - \frac{1}{2\sigma}\hat{q}_t\hat{C}_t \tag{70}$$

Recalling that

$$C_t = \psi C_t^O + (1-\psi)C_t^R$$

and

$$C_t^R = C_{t-1}$$

so that

$$\hat{C_t^O} = \frac{1}{\psi}\hat{C}_t - \frac{1-\psi}{\psi}\hat{C}_{t-1} \tag{71}$$

Substituting, (69) into (61), I receive the following form for welfare

$$W_t = U + U_C(\frac{1}{\psi}\hat{C}_t - \frac{1-\psi}{\psi}\hat{C}_{t-1} + \frac{1}{2}(1+\frac{U_{CC}C}{U_C})(\frac{1}{\psi}\hat{C}_t^2 + \frac{1-\psi}{\psi}\hat{C}_{t-1}^2 + \frac{1-\psi}{\psi^2}\hat{C}_t\hat{C}_{t-1}))$$

$$-U_L(\hat{L}_t + \frac{1}{2}(1+\frac{U_{LL}L}{U_L})\hat{L}_t^2 + t.i.p. + O(||\xi||^3) \tag{72}$$

Substituting (67), (68), (69) and (70) into (72), I receive the following form for the welfare measure

$W_t = -\frac{1}{2}u_c C\Xi\{\lambda_1(\hat{Y}_t - y_t^n)^2 + \lambda_2(\hat{Y}_t^* - y_t^{*n})^2 + \lambda_3(\hat{q}_t - q_t^n)^2 + \lambda_4\Delta\hat{q}_t^2 + \lambda_5\Delta\hat{Y}_t^{*2} + \lambda_6\Delta\hat{Y}_t^2 + \ldots$
$+\pi_{H,t}^2 + \lambda_7(\pi_{H,t} - \pi_{H,t-1})^2 + \lambda_8(\pi_{H,t}^*)^2 + \lambda_9(\pi_{H,t}^* - \pi_{H,t-1}^*)^2 + \lambda_{10}(\hat{q}_t + \hat{Y}_t)^2 + \lambda_{11}(\hat{q}_t + \hat{Y}_t^*)^2 + \lambda_{12}(\hat{q}_{t-1} + \hat{Y}_t)^2 + \lambda_{13}(\hat{q}_{t-1} + \hat{Y}_t^*)^2 \ldots \lambda_{12}(\hat{q}_{t-1} + \hat{Y}_t)^2 + \lambda_{13}(\hat{q}_{t-1} + \hat{Y}_t^*)^2 + \lambda_{14}(\hat{Y}_{t-1}^* - y_{t-1}^{*n})(\hat{q}_{t-1} - q_{t-1}^n) + \lambda_{15}(y_{t-1} - y_{t-1}^n)(y_{t-1}^* - y_{t-1}^{*n}) + \lambda_{16}(\hat{C}_t - c_t^n)(\hat{q}_t - q_t^n) +$
$+\lambda_{17}(\hat{Y}_t + \hat{Y}_{t-1}^*)^2 + \lambda_{18}(\hat{Y}_{t-1} + \hat{Y}_t^*)^2 + \lambda_{19}(\hat{Y}_{t-1} - y_{t-1}^n)(q_{t-1} - q_{t-1}^n) + \lambda_{20}(\hat{Y}_t^* - \hat{Y}_t^{*n})(\hat{Y}_{t-1}^* - \hat{Y}_{t-1}^{*n}) + \lambda_{21}(\hat{Y}_{t-1}^* + \hat{q}_t)^2 + \lambda_{22}(\hat{Y}_{t-1} + \hat{q}_t)^2 + \lambda_{23}(\hat{Y}_{t-1} - y_{t-1}^n)(\hat{q}_{t-1} - q_{t-1}^n) + \lambda_{24}(\hat{C}_{t-1}^* - c_{t-1}^{*n})(\hat{q}_{t-1} - q_{t-1}^n) + \lambda_{25}(\hat{q}_t - q_t^n)(\hat{q}_{t-1} - q_{t-1}^n) + \lambda_{26}(\hat{Y}_{t-1} - y_{t-1}^n)(\hat{Y}_t - y_t^n) + t.i.p. + O(||\xi||^3)$

where

$\Xi = (\theta\omega)(\sigma/(-1+\sigma))^{-\rho}C^{-\sigma(1-\rho)}L^{\gamma(1+\rho)})/(1-\omega)(1-\omega\beta)$

$\lambda_1 = \Xi(((3(-1+\sigma-2\psi)+16(C-1))(L^\gamma+1))\gamma(\psi^2))/(16(\psi))) + ((3+3\sigma(-1+\psi)-\psi)(-1+\psi)/(16(\psi^2)))-$
$-(\sigma-1)(1-\psi)/(2\sigma(\psi^2)) - (1-\psi)(-1+\sigma)/(4\psi^2) - -(1-\psi)(-1+\sigma)/(4\psi^2))$

$\lambda_2 = -\Xi((3(1-\sigma+2\psi)/(16(\psi^2))) - ((3+3\sigma(-1+\psi)-\psi)(1-\psi)/(16\psi^2)) - (1-\psi)*(-2+2\sigma+\psi)/(8\sigma(\psi^2))-$
$-(1-\psi)(-1+\sigma)/(4\psi^2) - (1-\psi)(-1+\sigma)/(4\sigma\psi^2) - -(1-\psi)(-1+\sigma)/(4\psi^2))$

$\lambda_3 = \Xi(((5-5\sigma+2\psi)/(16((\sigma\psi)^2)))+(-1+\sigma)(1-\psi)/(2\sigma\psi^2)+(1-\psi)(-2+2\sigma+\psi)/(8\sigma\psi^2)+(1-\psi)*(-1+\sigma)/(4\sigma\psi^2))$

$\lambda_4 = -\Xi(-((\sigma-1)(1-\psi)/(2\sigma\psi^2)) + ((-1+\psi)(5+15\psi+\sigma(-5+13\psi))/(16(\sigma\psi)^2)) - (1-\psi)(-1+\sigma)/(4\sigma\psi^2))$

$\lambda_5 = -\Xi((3+3\sigma(-1+\psi)-\psi)*(1-\psi)/(16\psi^2) - (1-\psi)(-2+2\sigma+\psi)/(8\sigma\psi^2) - (1-\psi)(-1+\sigma)/(4\psi^2))$

$\lambda_6 = -\Xi(((3+3\sigma(-1+\psi)-\psi)(-1+\psi)/(16(\psi^2))) - ((-1+\sigma)(1-\psi)/(4\sigma(\psi^2))) - (1-\psi)(-1+\sigma)/(4\psi^2))$

$\lambda_7 = \upsilon\zeta/(\omega(1-\zeta))$,$\lambda_8 = \upsilon\omega^*(1-\omega)(1-\omega\beta)/(\omega(1-\omega^*)(1-\omega^*\beta))$, $\lambda_9 = \upsilon\zeta(1-\omega)(1-\omega\beta)/(\omega(1-\omega^*)(1-$
$\zeta)(1-\omega^*\beta))$ $\lambda_{10} = -\Xi(-1+\sigma+2\psi)/(8\sigma\psi^2)$, $\lambda_{11} = -\Xi(-1+\sigma+2\psi)/(8\sigma\psi^2)$, $\lambda_{12} = -\Xi((\sigma-1)(1-\psi)/(2\sigma\psi^2))$

$\lambda_{13} = -\Xi(1-\psi)(-1+\sigma)/(4\sigma\psi^2)$ $\lambda_{14} = -\Xi(-1+\psi)(1-\psi+\sigma(-1+5\psi))/(8\sigma\psi^2)$, $\lambda_{15} = \Xi(1+\sigma(-1+\psi)-$

$3\psi)(-1+\psi)/(8(\psi^2))$ $\lambda_{16} = -\Xi(-1+\sigma)/(2\sigma\psi^2)$ $\lambda_{17} = -\Xi(-1+\sigma)(1-\psi)/(4\psi^2)$, $\lambda_{18} = -\Xi(-1+\sigma)/(4\psi^2)$,

$\lambda_{19} = -\Xi(-1+\psi)(1-3\psi+\sigma(-1+5\psi))/(8\sigma\psi^2)$
$\lambda_{20} = -\Xi(((-1+\sigma)(1-\psi)/(4\psi^2)) + ((3+3\sigma(-1+\psi)-\psi)(1-\psi)/(8\psi^2)) - (1-\psi)(-2+2\sigma+\psi)/(8\sigma\psi^2) + (1-$

$\psi)(-1+\sigma)/(2\psi^2))$ $\lambda_{21} = -\Xi(1-\psi)(-2+2\sigma+\psi)/(8\sigma\psi^2)$, $\lambda_{22} = \Xi(1-\psi)(-1+\sigma)/(4\psi^2)$

$\lambda_{23} = -\Xi(-1+\psi)(1-3\psi+\sigma(-1+5\psi))/(8\sigma\psi^2)$, $\lambda_{24} = \Xi(-1+\sigma)((-1+\psi)^2)/(2\sigma\psi^2)$ $\lambda_{25} = -\Xi(((-1+\sigma)(1-$

$\psi)/(4(\sigma\psi)^2))-((\sigma-1)(1-\psi)/(\sigma\psi^2))+((-1+\psi)(5+15\psi+\sigma(-5+13\psi))/(8(\sigma\psi)^2))-(1-\psi)(-1+\sigma)/(4\sigma\psi^2))$ $\lambda_{26} =$

$-\Xi(((-1+\sigma)(1-\psi)/(2\psi^2))+((3+3\sigma(-1+\psi)-\psi)(-1+\psi)/(8\psi^2))-((-1+\sigma)(1-\psi)/(4\sigma\psi^2))+(1-\psi)(-1+\sigma)/(2\psi^2))$

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