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Whole class interaction in the

mathematics classroom:

A conversation analytic approach

by

Jennifer Jayne Ingram

A thesis submitted in partial fulfilment of the requirements for the

degree of

Doctor of Philosophy in Mathematics Education

University of Warwick, Institute of Education

February 2012

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Acknowledgements

A great number of people have enabled and supported me in the work reported in this thesis. My thanks first and foremost goes to my supervisors, Mary, Keith and Peter who guided, encouraged and supported me.

I would also like to thank the teachers and pupils who allowed me into their classrooms and who participated in this study and remain anonymous. The huge variety in the experiences they offered me gave me lots to think about and this research would not have been possible without them.

I would also like to thank my colleagues both at the University of Warwick and the members of BSRLM for their support and encouragement, and in many cases their insightful and thought provoking comments and questions which often led to significant moments in the development of this work.

Finally, I would like to express my deepest gratitude and appreciation to my family. My husband Jon and our wonderful children, Elisabeth, James and Daniel. I could not have done this without your love, support and patience.

Declaration

I hereby declare that this thesis is my own work and has not been submitted for a degree at another university.

A few publications include data collected as part of the study along with some of the ideas generated in the early stages of the analysis. These include:

Ingram, J., Briggs, M., Richards, K. and Johnston-Wilder, P. (2011) The Discursive Construction of Learning Mathematics. *In: Smith, C. ed. Proceedings of the British Society for Research into Learning Mathematics 31(2),* Leeds, June.

Ingram, J., Briggs, M. and Johnston-Wilder, P. (2011). Turn-taking in the mathematics classroom. *In: Pytlak, M., Swoboda, E. And Rowland, T. eds. Proceedings of the Seventh Congress of the European Society for Research in Mathematics Education, Rzeszow, Poland, 9th-13th February.*

Ingram, J. (2010). The affordances and constraints of turn-taking in the secondary mathematics classroom. *In: Joubert, M. ed. Proceedings of the British Society for Research into Learning Mathematics 30(3), Newcastle,* June.

Ingram, J., Briggs, M. And Johnston-Wilder, P. (2009). Simon Says: Direction in Dialogue. *In: Joubert, M. ed. Proceedings of the British Society for Research into Learning Mathematics 29(3),* Loughborough, November.

Summary

This thesis analyses whole-class interactions in the mathematics lessons of four mathematics teachers and their pupils. A conversation analytic approach was taken in analysing the transcripts of whole-class interactions, focusing on those interactions that were about mathematics. The sequential organisation of talk, in particular turn-taking and preference organisation, is examined for similarities and differences across the four classrooms and the implications these may have for the teaching and learning of mathematics are explored.

This research also examines the discursive construction of the mathematical tasks and activities in each of the classrooms. The analysis reveals that the teachers and pupils orient to the institutional setting in which the interaction occurs. The structure of interactions in formal classrooms offers opportunities that can support particular features of learning mathematics, such as using mathematical terminology, building in opportunities for pupils to think about the mathematics, explain their reasoning, and ask mathematically related questions. However, these structures also constrain the interactions and so features of learning mathematics only feature in interactions that deviate from the usual patterns of interaction in formal classrooms, such as argumentation and justification. Finally, this research offers evidence that the way mathematical tasks and activities are talked into being affects the nature of the mathematics that the pupils experience.

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Part 1: Background to the study

Chapter 1: Introduction

The main aim of this study is to develop a description of the interactional organisation of the secondary mathematics classroom. The description and analysis of the discourse and communication are then related to the learning of mathematics. In other words, this thesis begins to answer the question of what types and forms of interaction promote and support the learning of mathematics.

Classrooms are complex and dynamic environments and a conversation analytic approach has been adopted to develop a detailed and in-depth description of the structures of whole-class interaction. This approach relies on the detailed examination of brief extracts from the transcripts of the whole class discussions that occurred in each of the four teachers' lessons. The analysis of whole-class interaction has in the past been difficult to research. There are many speakers and the interaction is further complicated through multi-modal forms of communication. Video cameras, and in particular observation classrooms, are making this data easier to capture, yet the process remains complex and time consuming.

The importance of research into communication in the mathematics classroom is clear through the impact much of the existing research has had; however, the frameworks for describing, analysing or evaluating interaction in mathematics classrooms are still being developed. These frameworks may then help us to answer questions such as what kinds of interaction support or encourage the learning of mathematics.

The aim of this study is to create a detailed description of whole class interactions in secondary mathematics classrooms. By examining interaction in this way, we can develop more of an understanding of the mathematics that pupils experience in lessons, and the relationship between interactions and the teaching and learning of mathematics. It can also help to develop ways of interacting with pupils that support different mathematical activities. Initiatives aiming to change interaction in classrooms implicitly and explicitly evaluate patterns and features of interaction. Yet without a firm basis for making these evaluations they remain heavily influenced by the subjectivity of the observer. By focusing on the interaction itself, rather than the observer's opinion, we can begin to develop a basis for making evaluations about the nature of classroom interaction.

Data used in the analysis come from 17 hours of transcribed mathematics classroom interactions in four teachers' classrooms. These transcriptions were then analysed along two of the central themes of conversation analysis; turn-taking and preference organisation. These analyses were then combined with ideas drawn from discursive psychology to examine the discursive construction of mathematics. Seedhouse (1996) argues that in order to reveal the nature of activity in classrooms, a thorough analysis of classroom interaction is needed. This thesis examines the structures and features of the interactions as a means for negotiating not only mathematical meaning, but also the doing of mathematics itself.

A conversation analytic approach was taken in the analysis of the data collected in this study. This approach draws upon ethnomethodological

ways of viewing and treating data and the complex relationship between language and meaning. Barwell (2002) talks about the link between experience and meaning and the consequences this had on the discursive psychology methodology he works within his own research. Similar issues, particularly those relating to the interpretation of data, arose in my own experiences as I reviewed the literature on classroom interaction.

There has been a great deal of research into interaction and communication in classrooms, drawing upon a range of methodological approaches (for example Forman, *et al.*, 1998; Mendez, *et al.*, 2007; Morgan, *et al.*, 2002; Mueller, *et al.*, 2011). Many of these have focused on younger children than in this study (Mercer, 2000; Myhill, 2006), or interactions within small groups or dyads (Kumpulainen and Mutanen, 1999) rather than full classes, but their findings have influenced the way mathematics is taught and learnt for many teachers and have also influenced the development of ideas in this thesis. Few studies have used a conversation analytic approach (see Forrester and Pike, 1998 for an example of one such study), and many of those that have drawn from this approach do not carry all the methodological assumptions and principles of 'pure' conversation analysis, nor can this study.

There are also studies with similar theoretical assumptions or guiding principles to those used in conversation analytic research. Pragmatic research focus on language in use and what participants are doing in their turns at talk. Guides to pragmatics often include sections on conversation analysis (e.g. Levinson, 1983; Mey, 1993) but these analysis often draw upon the motives or intentions of the speakers or the context in which the

interactions occur in the analysis. Similarly, Barwell's (2002) use of discursive psychology is developed out of the conversation analytic tradition but includes contextual details in the analysis. Mehan's (1979) research on classroom interaction lays the foundation for a lot of the analysis in this thesis. The approach Mehan takes bears a lot of similarity to conversation analytic studies, and occurred at the time that the sociological work that conversation analysis developed out of was being done.

One of the key differences between the methodological approaches of the literature reviewed in chapter 2 and this study is the role of context in the analysis. This study examines not only how the interactions are sequentially organised, but also the reflexive relationship between the interactions and the institutional setting. That is, the opportunities and constraints that the organisation of talk offers in terms of teaching and learning mathematics, but also how the focus of the interaction as it relates to the teaching and learning of mathematics, affords and constraints the organisation of talk. Many studies into the relationship between talk and the teaching and learning of mathematics consider the influence of context, but one key difference in this study is the nature of this context and this is outlined in chapter 5 and explored in detail in chapter 10.

One of the largest issues faced in the writing of this thesis has been the bringing together of two disciplines, mathematics education and linguistics. This issue affects both the literature review and the analysis and interpretations of the transcripts of the lessons recorded. Whilst there are very few conversation analytic studies of mathematics classrooms, there

have been many analysing language classrooms. The Discourses (Gee, 1999) of the two disciplines have required careful navigating.

The different disciplines have different ways of talking about features of interaction, and emphasise different aspects of any data collected. This has been a particular challenge in chapters 2 and 9. In chapter 2, the literature on classroom interaction is reviewed but many authors describe similar features using different terminology or alternatively use words that take on different meanings depending on the methodology adopted. In chapter 9, the challenge was more that words like 'preference' and 'avoid' take on different meanings from those in ordinary conversation and these terms themselves are used with varying meanings throughout the literature. Similarly the word 'rule' that is used to describe turn-taking structures in the conversation analysis literature does not share the conscious acts of obeying that are implied through the everyday use of the word.

In the next chapter, the literature on language and communication in the mathematics classroom is reviewed. The literature presents a complex, varied and diverse range of features of discourse and the potential influences these have on the learning and teaching of mathematics. Communication of some form is always taking place in any classroom as even silence and inactivity are interpreted as having meaning and can influence participants in the classroom. The literature reviewed serves two purposes. Firstly, they provide a comparison of various approaches to classroom interaction as much of the research adopts discourse analysis methodologies, which contrast with the conversation analytic approach adopted in this thesis.

Secondly, it sets the context within which this study occurs as it builds on or exemplifies and explains many of the findings presented in the literature.

Chapter 3 presents the theoretical background to the study. This chapter outlines many of the key assumptions and guiding principles of the methodological approach adopted in this thesis. The methodological approach that is outlined in chapter 5, contrasts to many of the approaches taken in the existing literature on classroom discourse. Many of the differences arise from the different theoretical perspectives that the methodologies develop out of; in particular the way that data are treated and subsequently analysed in the research.

Chapters 4 and 6 focus on the research questions that underpin this study and the methods used to collect, represent, and analyse the data. Ethical considerations are also discussed in chapter 6 and are closely related to the choices of data representation.

Chapter 7 is somewhat unusual in that it presents three transcripts from the data collected. These transcripts are from three different teachers and offer contrasting interactions as is detailed in later chapters. Whilst it is usual to include the full data set in a piece of work using a conversation analytic approach, limitations on space in the thesis do not permit this. Consequently, the analysis focuses on these three extracts, drawing from the wider data to illustrate structures and patterns that are not evident in the presented transcripts.

Chapters 8, 9 and 10 present the analysis of the data collected in this study. The first two of these focus on key themes within the conversation analytic

approach, those of turn-taking and preference organisation. The beginning of each of these chapters includes a brief introduction to the literature on these themes as it relates to classroom interaction before examining the data collected in this study. Chapter 10 deviates a little from the previous two chapters, focusing on a more discursive psychology application of conversation analytic analysis and exploring the discursive construction of identities and mathematics in each of the extracts.

The analysis of the data is presented alongside the transcripts of the interactions so that the reader can evaluate for themselves the validity of the analysis. This is a key feature of conversation analytic research. The analysis is still an interpretive process but instead of making claims about what participants are thinking or intending, it draws on what participants do or say. Any analysis or description of interaction involves elements of interpretation by the researcher and the reader. "First, no description, however detailed or extensive, can exhaust the state of affairs it describes. Second, the description will be found to reference those aspects in a particular way" (Heritage, 1984) . Any claims made in this study are evidenced by extracts from the transcripts, though I am clearly influenced by my own views and experiences.

Throughout the thesis I have treated all teachers as male and all pupils as female, though this is not in fact the case in either the literature reviewed or in this study. This is both for methodological reasons as outlined in chapter 6 but also for ease of reference.

The original interest in this study arose in my own teaching in a secondary comprehensive school. There were government initiatives encouraging all teachers to increase the wait time between questions and answers and change the way questions are asked in lessons, and I was also influenced by my experiences working with the Association of Teachers of Mathematics and Mathematical Association, particularly on the role that questioning could have on the learning of mathematics. Many of the school's strategies for implementing changes in questioning and discussion practices of teachers seemed to me to over-simplify the issues and lead to a tick-box approach to teaching. Despite the good intentions of many of the teachers I worked with and my own, we found that changing our practice was not always simple and did not always have the effect we were expecting. Similar experiences are described in much of the literature (e.g. Mendez, *et al.*, 2007).

After the first few months of this study, I made the move into teacher education and have been privileged to see a large number and wide variety of mathematics lessons as a result. This enabled me to see that the ways in which teachers and pupils interacted in lessons varied enormously in many ways, but were also similar in others. These differences and similarities intrigued me and led to the final focus of this study on whole-class interaction. This variety is one feature of classroom interaction that makes research into classroom communication both interesting and essential.

Chapter 2: Literature Review

This chapter reviews the literature that sets the context for and has informed this study. It begins with a brief exploration of the relationship between mathematics and learning mathematics, focusing particularly on theories of learning that emphasise the role of social interaction and communication. The nature of mathematics and the how we learn mathematics carry assumptions that influence the collection and analysis of data and the impact of these is reflected in the literature discussed in the remainder of this chapter.

The key area of research relevant to this study is the relationship between language and learning and the relationship between language and the learning and teaching of mathematics in particular. The main part of this chapter explores what we know already about the nature of language and interactions but also reveals gaps where the relationships are not so clear. The chapter ends with a discussion of the role of language in the developing identities of pupils in relation to learning mathematics in light of more recent research exploring the relationship between identities and learning.

Scherer and Steinbring (2006) talk of a paradigmatic shift in the focus and approaches of mathematics education research, where research has moved from a focus on teachers or learners to the reciprocal relationship between teaching and learning. Central to much of this research is mathematical interaction and communication. This change in research focus follows developments in theories of learning, in particular the current prominence of constructivist views of learning. Learners actively construct their knowledge

and understanding through interaction with the environment, which includes peers and teachers. This in turn leads to a need for learners to communicate their ideas, interpretations and understanding, and to explain and justify these (Scherer and Steinbring, 2006) to both support the learning process and help the teacher to monitor and support it.

Research building on the work of Vygotsky (1978; 1986) emphasises the importance of social interaction with more knowledgeable others, drawing upon the notions of the genetic law of cultural development and the Zone of Proximal Development (Vygotsky, 1978). The role of culture and society in the learning process has been explored further leading to social-cultural and situated views of learning as enculturation into a community of practice (Cobb, 1994; Lave and Wenger, 1991). Learning mathematics is conceived as learning to talk and behave like a mathematician, including defining, justifying, conjecturing, and making connections both within and outside of mathematics (Barwell, 2005). Social-cultural theorists go as far as to claim that we cannot separate mental activity from the social, cultural, and historical context and therefore research needs to focus on the mediated interactions between an individual and the environment.

Yet the constraints within schools may mean that the types of mathematical thinking and behaviour as well as the mathematical knowledge that pupils are acquiring differ significantly from those of a mathematician. School mathematics is time restricted in that a bell goes at the end of the lesson and that is often the end of the task. It is rare that pupils spend a long period of time just exploring one task, often because of the expected classroom

management issues that accompany this as well as a perceived urgency to complete a syllabus (Myhill and Warren, 2005). There is a common belief, particularly with lower attaining pupils, that lessons need a variety of tasks because pupils cannot concentrate for long periods of time (DeGeest, *et al.*, 2003). It is rarer still that pupils are left with tasks unresolved yet each of these are common place in graduate level mathematics. Watson takes a broader and more complex view of school mathematics (2008) which closely resembles a broader view of mathematics but she also argues that this type of mathematics is what it is *possible* for school mathematics to be, and does not necessarily represent what most pupils are *offered* in schools, in particular low attaining pupils.

A great deal of research into the mathematics that school pupils experience has developed from Ernest's (1991) distinctions between different views of mathematics or issues surrounding social justice. Yes Ernest's classification over simplifies the complexity of how teachers and pupils view mathematics and categorising people as having a particular view of mathematics implies that views are relatively stable, not dynamically changing in response to particular contexts. This study takes a discursive approach to analysing the experiences of mathematics pupils have, particularly in chapter 10, building upon a wider range of literature that has examined the role of language in teaching and learning, and teaching and learning mathematics in particular. The approach adopted demonstrates the fluidity within which the nature of mathematics changes and develops through interactions with the local context.

The Role of Language

The term discourse is used in a number of different senses by different authors. It can refer to all interaction and communication between people, including written, spoken, visual, and gestural; where more than one mode of communication is used the term multi-modal is often used adjectively. It can also refer to simply spoken interaction; in some cases it is used to describe the whole act of communicating and considers context and meaning, and the term text is used to refer to the actual language used; it is also used to identify types of communication in particular contexts such as legal discourse or scientific discourse. Few authors define the way that they are using the term discourse (see Evans, *et al.*, 2006 for an exception). Much of Sfard's work (for example 2002; 1998) focuses on transcriptions of spoken interactions between a teacher and a pupil implying a narrow use of the term discourse, yet her theory of commognition describes a broader meaning encompassing written and gestural communication.

Gee (1999) distinguishes between discourse and Discourse where discourse refers to language-in-use and Discourse refers to social practices, routines, and activities of a particular group, and it is this group to which he ascribes the name Discourse community. Both Gee and Sfard emphasise the situatedness of language and meaning and argue that learning is socially constructed within a discourse community. This study takes this notion further through a narrower view of context focusing on the relationship of individual turns at talk and the surrounding talk, rather than the broader physical, social or cultural context in which the talk occurs.

Sfard takes the role of language and discourse in the learning of mathematics further and argues that "learning mathematics is tantamount to modifying and extending one's discourse" (2007, p.565). The idea of community of practice is consequently developed into discourse communities where people are brought together by discourse, where members communicate through the shared discourse and membership is achieved by participating in discursive activities. So learning mathematics is about participating in mathematical discourse, including the use of particular terminology, mediators, endorsed narratives and discursive routines shared by mathematicians (Sfard, 2008). Sfard names this theory commognition to emphasise the link between communication and cognition.

Drawing on these perspectives, the study of classroom interaction has become essential for gaining a better understanding of how and what pupils learn in mathematics classrooms as well as how they use the resources provided by different Discourses to construct and negotiate meaning, as well as construct 'identities' (Evans, *et al.*, 2006). For example, within the classroom, explanations, questions, and discussions are all impossible without some form of language, yet language, along with diagrams or graphs, is frequently interpreted by pupils in ways that differ from those intended by the teacher. Different individuals focus on different words or phrases or parts of a diagram or graph. What they see as important and what they disregard as superfluous will differ from individual to individual. Consequently, the meanings that pupils associate with particular words and images will differ. It is not surprising that communication often breaks down, it is perhaps more challenging to explain why communication in mathematics

lessons is so often successful (Cobb, 1988). This study seeks to offer some explanations as to how the structure and organisation of whole class interactions enable successful communication between teachers and their pupils, but also how communication and interaction enable different meanings to be constructed. Particularly through an analysis of how teachers shift the focus of attention through the construction of their turns at talk.

The Language of Mathematics

Mathematics as a discipline has many challenges that pupils and teachers need to overcome. Mathematics is often rigorous and essentially abstract. It has an extensive vocabulary that combines familiar words with either their every-day meanings or significantly different meanings, and new terminology with a mixture of historical roots. Written mathematics includes many symbols with their own rules of grammar (Morgan, 1998). Additionally, mathematicians often manipulate these symbols as if they were the mathematical object themselves (Pimm, 1987, p.19). Each of these is an aspect of the mathematical register which "consists of the use of symbols, specialist vocabulary, precision in expression, grammatical structures, formality and impersonality and a high level of lexical density and conciseness" (Lee, 2003, p.13). It also reflects modes of argument, styles of reasoning and to some extent ways of behaving. Part of learning mathematics is learning how to use the mathematical register and as with all languages (or registers), someone with experience of mathematical language will "know it when he or she sees it" (Morgan, 1998, p.11).

Although most mathematics teachers are proficient users of the mathematics register, lessons are usually conducted in a mixture of this and ordinary English, requiring pupils to switch between the two. Kumpulainen and Wray (2002, p.101) argue that we should be less dismissive of the role of everyday language in the development of mathematical understanding. In fact, the vast majority of utterances in transcripts of pupils discussing mathematics show that the language used is predominantly informal (Kumpulainen and Wray, 2002; Sfard, 2007). Pupils are quite capable of making insightful mathematical points and developing and understanding of mathematical ideas without using the technical vocabulary. Pupils can also develop language themselves which they use successfully, partially because of the personal relevance of the language.

Additionally, pupils need to learn to use many different registers throughout school, including the language of school (Pirie, 1998, p.22). Some of these different registers can cause difficulties where terms again have different meanings within the different register; the use of the term proof in mathematics and in science have very different meanings and forms the focus of the transcript from Richard's lesson in chapter 7. However, it is not only the register that changes from classroom to classroom, but also the norms of interaction and behaviour. Learning is not just about using particular registers, it is also about ways of acting and behaving.

The role of pronouns in classroom discourse has also featured in a wide variety of studies of classroom interaction. The pronoun 'we' is frequently

used by mathematics teachers but can refer to many different groups: the wider community of mathematicians; the wider community as a whole; the class as a whole; the teacher and an individual pupil; or the class excluding the teacher. This ambiguity is not restricted to the mathematics classroom and therefore is not new to pupils. It is however another mechanism that the teacher can use to control (or request) desirable behaviour in his pupils (Rowland, 1999).

There are many such subtle linguistic tools available to the teacher and they are often used unconsciously (such as using 'thank you' instead of 'please' to infer the expectation of a pupil doing something). The use of the phrase 'don't we' at the end of a statement such as 'we multiply out the brackets first, don't we' reinforce the particular method being taught but also has the effect of dissuading pupils disagreeing or asking questions. The 'don't we' implies that the pupil already knows this, as does everyone else in the room. If they admit they do not know it, then this will be a source of disapproval or embarrassment. This also indicates that no explanation needs to be given by the teacher. The use of we also stresses the generality of what is being said, it is not specific to that individual teacher or class.

Morgan (2006) draws upon the use of pronouns by two pupils in their written mathematics to examine how the pupils position themselves in relation to the reader of the text but also how the pupils position themselves in relation to the mathematics. Rowland (1999) examines the indexicality of pronouns in mathematics, focusing on how individual pupils use them to indicate both vagueness and generality. This examination focuses on the use of 'it' but

Rowland also points out that whilst 'it' is often used by both teachers and pupils to express generality it is also used when they are referring to something they cannot or are not confident in naming. Hence, the learning of a particular concept is linked to the ability to name and use the name in interactions. Bills (1999) also explores how pupils use pronouns in interview situations to indicate their confidence with the mathematics and as a way of adopting the style of language used in the classroom. These studies of pupils' uses of pronouns are built on in this study when examining how the teachers *and* pupils jointly construct mathematics in different ways, partly through their use of pronouns when describing tasks and the mathematics.

The role of interaction and language and the forms of interaction that are most beneficial for learning is debated widely in the literature (Atwood, *et al.*, 2010). Many authors claim that classroom discourse serves a multitude of purposes including shaping identities, and communicating beliefs about the nature of mathematics and beliefs about teaching and learning (e.g. Sfard, 2007; Sherin, 2002). Some authors argue that patterns of discourse reflect and serve to reinforce teachers' and pupils' beliefs about the nature of teaching and learning in school (e.g. Cobb and Hodge, 2011).

The literature concerning patterns of interaction, including the well know IRF sequence, focusing and funnelling, revoicing, and cued elicitation amongst others, are discussed and then developed within the wider context of classroom norms. Many of these patterns of interaction are evident in the transcripts presented in chapter 7 and have informed the analysis of these transcripts in the remaining chapters of this thesis.

Mathematical Discourse in the classroom

There have been many initiatives, both in the United States and the United Kingdom, to encourage more pupil talk in lessons. The role of the teachers has been described as that of facilitator, by listening carefully to his pupils, carefully asking questions and posing problems and careful management of whole class discussion, the pupils will develop mathematical skills and understanding (Stigler and Hiebert, 1999). Barnes argued that pupils need to play a highly active role in classroom interactions if they are to have genuine ownership of meaning (1992). It is important to consider whether quantitatively more pupil talk is a good thing in its own right (e.g. Mendez, et al., 2007). Obviously, the quality of talk is important, but is it possible for greater pupil talk to have a positive impact on learning irrespective of the quality of talk? Additionally, it is important to identify the different types of talk and what the techniques and strategies that teachers can employ to encourage these different types, which are the focus of much research into curriculum and pedagogic design. For example, DeGeest et al. found that teachers thought it was important for pupils to discuss mathematics with their peers, in pairs, groups, or whole class discussions (DeGeest, et al., 2003) and explored how the teachers supported pupils in doing this.

Whole-class discussion provides models for pupils showing how to discuss mathematics. Pupils rarely encounter mathematical discourse outside of their mathematics lessons. Their teacher is often their main model of mathematical language in use and mathematical behaviour, though some of their peers may offer examples of these too. In their mathematics lessons, pupils are not only learning facts, relationships and theorems but also

acceptable ways of communicating mathematics and what it means to behave mathematically and be a mathematician.

Discussion in classrooms can make thinking public and help pupils to clarify their own thinking. This opens it up to questioning, clarification, justification and extension and enables the collective negotiation of meaning (Bauersfeld and Cobb, 1995). It can also support pupils in making connections between their everyday experiences expressed in everyday language with mathematics and the language of mathematics. Whole-class discussions also support teachers in assessing their pupils' understanding; their mathematical knowledge, misconceptions and any gaps (Resnick, 1988).

In many classrooms there is little opportunity for pupils to talk aloud to themselves, particularly during whole-class discussions, as it is often seen as being disruptive or as a distraction to the other pupils (Pimm, 1987). It is also possible that many pupils do not wish to have this type of talk overheard by the teacher or some of their peers because of the fear of being judged on what they say in this exploratory stage (Mercer, 2000).

Pimm (1987) identifies two main reasons for pupils talking in mathematics lessons: talking to communicate with others (teachers or peers), and talking for themselves, though the latter predominately occurs during group work rather than when the class is working as a whole. Both of these offer an opportunity to make informal assessments of the pupils (Watson, 1998). Kumpulanien and Wray (2002) give two examples of this in practice in a study of pupils collaborating in a geometrical problem solving situation. In the first example, the pupils are clearly communicating with each other, and

working jointly to solve the problem. In the second example, one pupil quickly dominates the dialogue and the transcript indicates that he is 'thinking aloud', ignoring contributions and questions from the other pupil.

Mercer (1995) proposes three "ways of talking and thinking": disputational talk, cumulative talk and exploratory talk. The first of these 'is characterised by disagreement and individualised decision making'; the second is talk 'in which speakers build positively but uncritically on what the other has said'. However, in exploratory talk people engage in constructive criticism of each other's ideas and, knowledge is made more publicly accountable and reasoning is more visible in the talk. Progress then emerges from the eventual joint agreement reached.

Many discursive accounts of mathematics classrooms have sought to compare and contrast the discourse of particular classrooms with other discourses. For example, Elbers (2003) describes a collaboration between Streefland and Gertsen in developing a 'different' way of communication and teaching in mathematics and makes claims about the impact this has on mathematics done during lessons. Many of these accounts are discussed below. The conversation analysis approach used in this study focuses instead on how the discourse in a mathematics classroom accomplishes the 'doing' of school mathematics and also how the discourse constitutes school mathematics for both the teacher and the pupils (Barwell, 2003).

Classroom discussion needs careful planning and management by teachers if pupils are to develop their mathematical learning and understanding. Yet many strategies employed with this intention may not be successful. Pupils

may not share the teacher's understanding of the purpose of the discussion and often may not see what mathematical problems and issues the task set raises, and these are often a consequence of the norms of interaction within the class. Teachers also need to make decisions about whether to allow a classroom discussion to continue and develop when it does not fit with the acceptable mathematical view, leaving the pupils to reach their own conclusions or to interrupt and offer 'the correct answer'. The latter option reinforces the role of the teacher as 'expert' or authority figure, and can lead to poor recall or routine memorising as the pupils do not have a personal understanding (Mason, 2000).

Ritualised discourse (Williams and Baxter, 1996), for example, occurs when the pupils focus on the teacher's desire for participation, rather than on understanding the concept under discussion. Williams and Baxter found that for some pupils discourse became an end unto itself, and for other pupils it became just another extraneous requirement. The discussion-oriented environment the teacher sought to create became "part of the meaningless ritual of classroom life, rather than a tool for learning" (p.36). In this classroom, pupils seemed to lack motivation for actively listening, making sense of, and building off each other's ideas. If discussion lacks real purpose in the minds of pupils, then perhaps it is not surprising that talk becomes part of "doing school" rather than part of learning mathematics.

Interactional Patterns

Several empirical studies have identified the interactional regularities of classroom discourse (Bauersfeld, 1988; Nathan and Kim, 2009; Sinclair and Coulthard, 1975; Steinbring, 1989) and socio-linguistic research has

focussed on these exchange structures within classrooms. The majority of transcripts of mathematics lessons at all levels show little pupil-pupil interaction. Classroom discourse is generally dominated by the teacher and the teacher controls the speaking rights of the pupils (typically through the IRF sequence discussed below).

One particularly prevalent structure is commonly referred to as IRF or IRE, Initiation-Response-Feedback (Evaluation). The teacher initiates with a question, this is followed by a pupil response and then feedback from the teacher. The teacher is very much in control of the dialogue and the traditional IRF sequence prevails (Sinclair and Coulthard, 1975). Kyriacou and Issitt's (2008) systematic review found that the IRF pattern continues to dominate mathematics classroom discourse.

The IRF sequence itself is neither a good nor a bad thing. As with all pedagogic strategies, its advantages and disadvantages will depend on how it is used and the purposes for which it is used for in the particular situation (Wells, 1993, p.3). This complexity within the IRF structure has led to considerable research exploring particular variations (e.g. Chin, 2006). The data in this study also reveal much of this complexity, and the analysis examines the reflexive relationship between each of the aspects of the IRF. However, the majority of research into the IRF structure, particularly within mathematics education, has focused on variations of just one of these parts in isolation of the other two.

The IRF framework is so dominant, it can be difficult to use different styles, requiring a significant investment of time and energy on the part of the

teacher to retrain his pupils, though this can be explained through the norms of interactions within many classrooms. Sherin (2002) describes a teacher who 'goes beyond IRF' to insert questions encouraging pupils to explain their answers and to offer their own views on previous answers. However, in the included transcript the teacher includes these questions as part of a sequence of consecutive IRF exchanges. Conversation analysis offers descriptions and explanations for this dominance and the difficulty in deviating from this and this is discussed in more detail in chapters 8 and 9, but more significantly, CA enables a detailed description of interactional practices that offer opportunities or constrain different forms of participation and hence different opportunities for learning (Waring, 2009).

There has been a great deal of focus on the feedback aspect of this triad. One aspect discussed by Mehan (1979) is the methods employed by teachers when the response does not match the teacher's expectations. Repeating or rephrasing the question is one such method, and this is interpreted by pupils as indicating that the previous response was incorrect or inadequate. Likewise when a teacher moves on to a new question this can be taken to imply that the response was correct and appropriate. These interactional strategies and their interpretations are part of what conversation analysts describe as preference organisation and this is expanded further in chapter 9. Prompting incorrect or incomplete responses further is another method or simplifying the question until the response required is forthcoming. This last method is similar to 'cued elicitation' where the teacher asks a question whilst simultaneously providing heavy clues to the answer via bodily gestures and demonstrations (Edwards, *et al.*, 1987, p.110). Smith and

Higgins (2006) argue that it is the quality and nature of the feedback that supports and encourages a more interactive learning environment rather than a deviation from the IRF pattern, and the analysis in chapters 9 and 10 support this.

Edwards and Westgate (1987) discuss the pupils' focus on the feedback aspect of this triad as a way of judging what the teacher was really asking in the initiation part, hence the pupils' attention is on what comes next, not on the exchange itself (p.97). A conversation analytic approach would also argue that subsequent turns also reveal the purpose of the initiation, but does not only restrict this to the feedback aspect. Each part of the triad is reflexively related to the others, and therefore analyses of each aspect in isolation does not necessarily reveal the meaning of the initiation, response or feedback.

Chin (2006, p.1336) observed that teachers often restated pupils' responses which Brophy and Good argue "wastes time, lessens the value of pupil responses, and fails to hold students accountable for attending to what their classmates say" (1986, p.353). However these restatements or rephrasings can have a positive role (O'Connor and Michaels, 1993). They can be used to introduce or reinforce the use of technical vocabulary; they can be used to focus pupils' attention; they can be used to model responses, such as using complete sentences. Additionally, rephrasing pupil statements can result in a focus on the structure (or form) of what was being said as opposed to the meaning (Cazden, 2001). In some cases, echoing and revoicing can also be used to deflect the evaluative responsibility from the teacher to the class.

O'Connor and Michaels (1993) use the term 'revoicing' to describe the way in which a teacher can use a pupil's response, by paraphrasing, clarifying, or commenting on its relevance or importance. Using a response in this way, they argue, leaves the pupil at the centre of the discussion whilst identifying the response as appropriate and enabling other pupils to contribute to the discussion. It can 'animate' the pupil's response giving it status as something worth exploring or discussing (Goffman, 1981).

Revoicing refers to the ways in which teachers repeat and possibly reformulate their pupils' utterances. Many authors argue that revoicing is a discursive practice that alters the positioning of pupils which leads to a deeper conceptual understanding of mathematics (Enyedy, *et al.*, 2008; Planas and Morera, 2011). Revoicing includes echoing, rephrasing, and explication. Echoing is a simple repetition of the pupils' own words but often involves a change in emphasis or tone. Rephrasing involves keeping the intended meaning of the pupil's utterance, but changing the wording or structure, such as using mathematical terms to replace everyday ones. Explication involves expanding upon the pupil's response or clarifying it. This notion of revoicing has now been extended to include pupils revoicing their peers, particularly in small group work (Forman and Ansell, 2001; 2002; Planas and Morera, 2011).

O'Conner and Michaels (1993) argue that revoicing may be useful for (a) positioning pupils and their propositions within a participant framework, (b) reformulating pupils' ideas in more official language while still crediting them verbally, and (c) strengthening a weak voice that might otherwise be

overlooked, which includes pupils whose first language is not the language in which they are being taught (Planas and Morera, 2011). Forman found that where teachers used revoicing, pupils were more likely to "initiate explanations, provide answers or claims backed by explanatory grounds, warrants and backings, and to evaluate their own and each other's arguments (Forman, *et al.*, 1998, p.546). Revoicing not only affects pupils' positioning in relation to the other participants, it also affects their positioning in relation to the mathematics.

Planas and Morera (2011) describes two types of 'positive' revoicing. The first of these involves the explication or expansion of another pupil's partially constructed argumentation, which they argue reinforces mutual mathematical understanding amongst the group. The second use is as a mechanism through which pupils can ask for further explanations of a previous turn. However, Planas and Morera argue that whilst revoicing may have these positive effects on the interaction, they may also be interpreted as indicating error or disapproval which they attribute to who the speaker is (c.f. chapter 9).

Whilst most of the literature has focused on the benefits of revoicing there are potential dangers. Herbel-Eisenmann et al (2009) identify some of these. First, pupils may learn that they need not listen to each other, since the teacher will likely restate any important ideas or suggestions. Second, in recasting pupils' comments, the teacher may help to create an illusion of understanding; the teacher may recast ideas so as to align them with predetermined lesson goals, thereby masking pupils' true understandings.

Revoicing can force a pupil to agree or disagree with the teacher's rephrasing of a response, which can result in the alignment of pupils to the teachers' position. Whilst Forman puts this forward as a benefit in the joint negotiation of mathematical meanings, there are situations where this may not in fact be a benefit and this is discussed further in chapter 9.

Van Zee and Minstrell discuss a particular type of teacher response which they call a "reflective toss" where the teacher 'catches' the meaning of the pupil's original response and 'throw' the responsibility for thinking back to the pupils (1997, p.241). For example, "what do you mean by …", "could you say a little more about …". These techniques have been encouraged as methods for the teacher to extend the talk in the classroom and to attempt to clarify the pupils' understanding.

However, each of the analyses above of the third turn has taken a functional approach or categorisation approach (Nassaji and Wells, 2000). This third turn is affected by a variety of local influencing factors. Teachers are not only evaluating or giving feedback on the 'correctness' of the pupil's turn, but are also responding to how this second turn is produced (Lee, 2007). Attempts to categorise the third turn into echoes, evaluations, revoicings etc. are unable to capture the complexity of this move and the relationship it may have with learning.

An example of how different interpretations of turns can effect learning interactions is funnelling (Wood, *et al.*, 1976; Wood, 1998). The teacher leads the pupil(s) through a series of low-order explicit questions, each designed to 'funnel' pupils towards the desired answer. This breaks down

the task into manageable pieces and offers pupils a method that could be used in similar situations at the same time. The pupils' contribution to this process usually consists of recalling known facts or performing simple calculations, while it is the teacher who does the necessary reasoning to move from one step to the next. This can lead to a pupil understanding no more than before the exchange, but this is not necessarily the case (Anghileri, 2006). Funnelling is a common alternative to immediately correcting a mistake, and can then function as a face saving move. Funnelling also exemplifies the power relationship between a teacher and his pupils. The teacher is very much in control of the discussion, its structure and content whilst the pupil assumes a dependent role, filling in the gaps as required. These strategies and features of teacher-pupil interaction are extended in chapter 9, in particular the influence they may have on the role of mistakes in the teaching and learning of mathematics.

Wood (1998) describes a contrasting interactional pattern of focusing, where the teacher and the pupils share control of the discussion. Pupils are asked to explain their methods and solutions and their peers are encouraged to ask questions, query steps and ask for clarification from the pupil explaining. Sherin (2002) contrasts focusing questions with filtering questions, where "any new content raised by the teacher is based on a narrowing of ideas raised already by the students" (p.220).

Wood (1999) identified another pattern of interaction in some classroom discussions that involved argument. A pupil offers an explanation or a solution to a problem, this is then challenged by another pupil, the first pupil

then responds with a justification or further explanation, which is either accepted or rejected by the challenging pupil. If it is rejected a cycle of challenges and justifications involving many pupils begins until the teacher is satisfied that the issue has been settled (p. 179). This type of interaction only occurred in specific classrooms, where norms of listening and participating in classroom discussions had been established and examples of this pattern of interaction are explored in chapter 8.

Wait Time

Another key idea in the research relating to patterns of interaction is that of wait time and wait time has a direct impact on both the response given by pupils to teachers' initiation and also the nature and content of the feedback move. Rowe's (1974) initial research identified two types of wait time, the first occurring between the teacher finishing speaking and the pupil starting to speak, and the second occurring between the pupil(s) finishing speaking and the teacher starting to speak, which she labels 'wait time I' and 'wait time Il' respectively. Her analysis of more than 300 audio recordings of questioning revealed that the mean wait time of type one was around one second, at which point the teacher repeated, rephrased, or moved on. The mean wait time of type two was around 0.9 seconds. Heinze and Erhard (2006) found an average wait time I of 2.5 seconds, with no differences between the activities being undertaken in the interaction in their study of geometry lessons with high achieving students. Jones (1980) examined the time taken by pupils to answer different questions in an individual interview situation and found that the average time for convergent questions was 2.8 seconds and 6.9 seconds for divergent questions.

Many studies, including Rowe's included exploring the effects of extending wait time. After training teachers to leave a wait time of 3-5 seconds, Rowe found that, amongst other things, the length of the pupils' responses increased, the number of spontaneous responses increased, and the occurrence of "I don't know" or equivalent responses decreased. Possibly more interesting was the observed changes in teacher behaviour. She noted that both the number and type of questions changed. Teachers asked significantly fewer questions and there was a significantly larger proportion of higher-cognitive level questions. Additionally, teachers were able to respond more flexibly allowing a smoother discourse.

Rowe's research also revealed that pupil talk often came in bursts separated by pauses of around 3 seconds, possibly because what a pupil is required to do to answer some questions can be complex. For example they need to work out the explicit meaning(s) of the question, and then the implicit meaning(s), they need to work out what their response is going to be and then finally they need to translate this response into appropriate language. This raises another issue with the earlier classification of pupil responses, the suggestion is that a longer wait time, of both types, could have changed the cognitive level of the pupils' responses, with the shorter wait times preventing pupils from responding in full. As Tobin (1987) concluded "Wait time probably affects higher cognitive level achievement directly by providing additional time for pupil cognitive processing" (p.89).

Tincani and Crozier (2008) examined the relationship between wait time and responses from two pupils with challenging behaviour. In their study, the

wait times were specifically designed to be one second or four seconds in length, though in fact the lengths varied from 0.6 to 1.6 seconds and 3.4 to 4.6 seconds. So whilst they distinguished between brief and extended wait times, their categories overlap those categories described by Rowe. Their results indicate that the number of correct responses actually decreased when the wait time was increased, though at the beginning of the experiment extended wait time resulted in a higher number of correct responses. They also reported higher non-response rates and a higher range of disruptive responses with extended wait time. The authors ignored what they describe as "error correction sequences". Rowe (1986) and Black et al. (2004) have both found that making changes to the length of wait time in teacher's practice is difficult to do. The role of wait time in classroom interaction and the difficulties in changing the times between turns are explored further in chapter 8, where the analysis offers explanations for many of these research findings as consequences of the structure of classroom interaction.

Classroom Norms

The patterns of interaction and communication that regularly feature in classroom contribute to the construction of classroom norms, and the relationship between these is the focus of this study. Every classroom has its own rules and norms for communication. It is well documented that teachers use various linguistic devices in ways that differ from everyday context. For example, asking questions they already know the answer to, or indirect comments to request desired behaviour. Edwards and Mercer (1987) identify three main ground-rules, often implicit, for classroom discourse: the teacher asks the questions, the teacher knows the answers

and repeated questions imply wrong answers. Underlying the communication in a classroom are the rules and expectations of both the teacher and his pupils. These are often implicit and unconscious and taken-for-granted, but are the basis for the interactional patterns.

Many studies have focused on how these norms are developed or established in mathematics classrooms (Green and Weade, 1985; Yackel and Cobb, 1996). Yet what these authors mean by norms is not always clearly defined. Most authors describe norms as evolving in line with symbolic interactions. Green and Weade define norms as "when a particular set of verbal and nonverbal behaviours recur over time" become routinized (1985, p.15), emphasising the construction of norms through activity, which, similarly to Cobb and Yackel's work, draws upon a more ethnomethodological approach to norms (see chapter 3). In contrast, other authors (Patrick, *et al.*, 2001) have argued that classroom norms are 'comparably stable' and are initiated and established in a relatively short period of time. However, teaching and learning can still be quite fluid within the norms and expectations of any classroom (Atwood, *et al.*, 2010).

The rules and expectations are built over the academic year between the pupils and their teacher. They can include simple 'rules' such as putting a hand up to answer a question, but they can be specific to certain classrooms and include how to agree or disagree with a statement, offering an explanation or justification with an answer. Green and Weade (1985) distinguish between three types of classroom norms of interaction; those relating to the academic context, those relating to social participation, and

those relating to the nature of the activity. They go on to offer an example how the establishment by a teacher of a particular norm is used by the pupils to guide subsequent interactions. Pupils' views and expectations of what should happen in a mathematics classroom are often quite rigid ('we haven't done any maths', writing, right and wrong, finishing quickly) and are reflexively related to the classroom norms (Cobb, *et al.*, 2001).

The norms and expectations that a teacher encourages will have an enormous effect on the success of both whole-class and small group discussions. Ideas, information and solutions are often accepted without debate from a teacher because of their role-given authority (Mueller, *et al.*, 2011). The same is often true of statements given by pupils in group work, they are more easily accepted if they are given by a socially dominant member of the class (Abele, 1998). A teacher will need to establish norms that challenge these institutional assumptions if discussion is going to lead to pupils developing their mathematical understanding, yet there are issues with developing these norms.

Asking pupils to discuss things in pairs or small groups requires the teacher to relinquish some control and informal assessment opportunities. It is not possible for the teacher to overhear or participate in all these discussions. Those discussions that the teacher may overhear are influenced by the teacher's presence. Comments and questions often become directed towards the teacher, not other pupils because of the assumed authority of the teacher. Many pupils become more passive, and a few more active in the presence of a teacher altering the social dynamics of a group (Ford,

1999). These activities do offer pupils the chance to try out ideas, refine or dismiss them before sharing them with both the teacher and the class as a whole. Yet when discussing ideas as a whole, there are often not sufficient opportunities for each pupil to contribute and key ideas can be missed.

The notion of norms is closely related to the idea of participant frameworks (Goodwin, 1990) which include how participants are aligned with each other, as well as how they are positioned relative to the ideas under discussion. These frameworks are co-constructed by teachers and pupils as they animate and position themselves and each other. Teachers exert influence over the structure of participant frameworks both by revoicing pupils' utterances and by posing questions such as "did anyone do it differently?"

Establishing participant frameworks can be a powerful tool for engaging pupils in the examination of each other's ways of thinking. Pupils need encouragement, however, to explain their ideas and listen to and respond to each other. In addition to establishing appropriate norms and expectations related to classroom discourse, teachers can engage with pupils during discussions in ways that encourage these behaviours (O'Connor, *et al.*, 1996).

The theory of politeness (Bills, 2000; Brown and Levinson, 1987) offers another explanation of the ways in which both teachers and pupils communicate within established norms. It is argued that speakers avoid threats to the 'face' of those they are speaking to, through indirect comments and vagueness, where meanings are implicit rather than asserted directly. Orders, request, criticism and disagreement are all considered to be face

threatening acts, and the speaker generally attempts to avoid each of these or find ways of mitigating their effect. Issues of face are often most evident in the way that teachers and pupils handle errors and mistakes but are also relevant to how teachers make orders and requests.

Sociomathematical Norms

Yackel and Cobb (1996) develop their notion of sociomathematical norms out of their work on classroom social norms. These include understandings of what is mathematically different, mathematically sophisticated, mathematically efficient and mathematically elegant in a mathematics classroom. They also include an understanding of what it means for a mathematical explanation or justification to be acceptable (p. 461). In essence, pupils accept or reject explanations and justifications for mathematical reasons, not because of the authority or status of the person offering the explanation or justification.

Yackel and Cobb argue that sociomathematical norms go some way to explaining how pupils develop mathematical beliefs and values. The studies of Yackel and Cobb have focused on pupils between 6 and 8 years old, yet there is some evidence of some of the same sociomathematical norms in older age groups, in particular mathematical efficiency. Edwards' (2007) research on friendship in peer-group interactions focuses on pupils between 11 and 15 years old. Drawing on Selman's model of the stages of development in 'role-taking', she argues that this age group is becoming increasingly aware of other people's perspectives and pupils consider rules and norms before acting. Some norms and sociomathematical norms are established by this point, but because of the changing nature of

mathematics, in terms of the content and the skills needed by pupils, other sociomathematical norms need to be renegotiated or newly generated. Edwards identifies a new sociomathematical norm in this age group which she calls mathematical evidence and concerns the impact of written mathematics.

To think mathematically, pupils must learn how to justify their results, to explain why they think they are correct, and to convince their teacher and fellow pupils. This is the beginning of mastering 'mathematical proof', but also is an aspect of what Yackel and Cobb describe as being intellectually autonomous in mathematics (1996). A proof is a conclusive argument that a proposed result follows from an accepted theory. 'Follows' means the argument convinces qualified, sceptical mathematicians. Yackel and Cobb's analysis focuses on the role the teacher plays in establishing these sociomathematical norms but also offers evidence that pupils are capable from a young age of making judgements of what counts as mathematical or not.

The way in which mathematics is communicated, the interactional patterns used and the norms of interaction can reveal teachers' and pupils' different views and beliefs about mathematics and what it means to learn mathematics. Does the teacher focus on the product, the answer, or the process, the methods and strategies that a pupil uses. Is the teacher using language to support the transfer of mathematical knowledge to her pupils or is she using it to enable the pupils to generate new meanings for themselves (Wood, 1998, p.168). What pupils see as mathematics can depend upon

their interpretation of the mathematics offered by their teachers; what is emphasised, valued and assessed; combined with the beliefs about mathematics that those they interact with have, such as their parents.

Recent theories concerning learning (Lave and Wenger, 1991) argue that it is the practices of learning mathematics which define the mathematical knowledge that is learnt. Research shows that in mathematics lessons that emphasise individual work, pupils perceive maths to be a rule-bound subject (Boaler, *et al.*, 2000b) and similar in nature to the absolutist view of mathematics. These pupils also have difficulty using their mathematics in new and varied contexts. Those pupils from classes encouraging discussion, saw mathematics as inquiry based and strove for understanding. These pupils were more able to use their mathematics in different situations. In essence, pupils have qualitatively different types of knowledge and beliefs about mathematics and learning mathematics through their practices in the different learning environments.

Identity

These differences in pupil's experiences of mathematics have led to more recent work focusing on the identities pupils develop in relation to mathematics.

"Because learning transforms who we are and what we can do, it is an experience of identity (Wenger, 1998).

There are two contrasting views in the literature about the nature of identity. Firstly the view that identity is fixed and permanent and is based on

biological characteristics. There is an absolute self. The second sees identity as dynamic, constantly forming and developing in response to social and cultural contexts. It is this second viewpoint, social constructionist approaches to identity, including the discursive psychology approach that is discussed in this section and drawn upon in the analysis in chapter 10.

Lave and Wenger argue that learning is a social activity through which our identities change and develop. Our identities are dependent upon the social situation, they are multiple and dynamic (Lave and Wenger, 1991; Wenger, 1998). Identities can influence and are influenced by social contexts and groups. It is through pupils learning mathematics that they develop their identity in the community of mathematicians, through adopting behaviours that are consistent with the context of their mathematics classroom. The classroom norms help pupils see themselves in relation to their mathematics classroom and develop a Discourse identity (Gee, 2000). Pupils will adopt many identities throughout their schooling, varying from subject to subject and different from their peers. Some pupils will develop a feeling of belonging in the mathematics classroom, for others it will be a feeling of rejection (Wenger, 1998).

Mathematics is often viewed as the gatekeeper to further study or employment. As such, pupils often want to be successful at mathematics but do not wish to become mathematicians (Boaler, *et al.*, 2000b). This is particularly evident in the work by Sfard and Prusak (2005). They distinguish between two types of identity: *actual* describing the current identity and *designated* which describe the identities that for a variety of reasons are

expected to be the case in the future. So an actual identity would be described in terms of I am, whereas a designated identity would be described in terms of I want to be. They compare the identities of 'NewComers' and 'OldTimers' and found that 'NewComers' designated identities focused on professions whilst 'OldTimers' focused on 'being happy'. Consequently, 'OldTimers' did mathematics because of its gate keeping role whilst 'NewComers' saw mathematics as a tool for closing the gap between their actual and designated identities.

The traditional mathematics classroom emphasising facts and procedures encourages an environment where pupils do not need to behave as thinking agents, runs counter to many secondary school pupils developing identities as mathematicians. Boaler and Greeno (2000) found that pupils who learnt mathematics in a traditional manner discussed doing mathematics in a passive way, and their descriptions were at odds with their development of their identities outside of mathematics. On the other hand, pupils who learnt mathematics in classrooms where discussion was valued, spoke of doing mathematics actively and described it as creative subject. The former pupils expressed a conflict between what they wanted to become and what they thought it meant to become a mathematician, whereas in the latter case the two were seen as being compatible. Each of these studies have drawn upon pupils' own descriptions of either themselves or mathematics.

To become a mathematician you need to be able to communicate with other mathematicians. This involves making and understanding mathematical discourse, using mathematical language and notation and your

understanding of mathematical concepts to match approximately those of other mathematicians so they can be discussed meaningfully (Sfard, 2008). Black argues that the different ways in which pupils participate in whole-class discussions may lead to the "construction of different types of pupil identities" (2004, p.34) and many researchers argue that features of classroom interaction contribute to the development of pupils' identities in relation to mathematics (Enyedy, *et al.*, 2008; Forman, *et al.*, 1998). This study builds on this existing research by examining how these identities are discursively constructed through whole class interactions.

Summary

This chapter has reviewed the literature, largely within mathematics education, that explores the relationship between language and learning. This research has identified a variety of structures and classifications of interaction and these have been built upon in subsequent research to explore the impact they might have on the teaching and learning of mathematics. Key structures from this literature, such as the IRF sequence and wait time, are examined and deconstructed in this study in chapters 8 and 9, but many of the ideas outlined have also influenced the analysis of the data collected in this study. In particular, chapter 10 builds on the changing views on identity, adopting the notion of identity as something you do and consequently combines this with ideas about what it means to do mathematics.

The literature also establishes the importance of further research into the relationship between interaction and learning. The vast majority of the

research in this chapter has adopted some form of discourse analysis methodology. The underlying assumptions of these studies contrast in many ways to those underlying the present study which uses a conversation analytic approach and in the next chapter these assumptions are examined in more detail before the methodological approach adopted in this study is discussed.

Chapter 3: Theoretical Background

This section on the theoretical background is intended to explore the theories that have influenced the methodology, methods and analysis adopted in this thesis. The conversation analytic approach draws heavily on ethnomethodology but is also developed, partly as a reaction to and partly inspired by, other theoretical ideas such as symbolic interactionism, hermeneutics and the work of Goffman. Each of these is discussed in turn, focussing on the aspects of the theories that illustrate the assumptions made and the theoretical approach to the collection and subsequent analysis of data. A key underlying assumption in this research is that all data involves interpretation. We do not have direct access to the external world (assuming it exists) or to social facts. All individuals, including researchers and the participants in this study, view the world through lenses, or perspectives.

This section begins with a brief discussion of the ideas and assumptions attributed to the symbolic interactionists before focusing on two key concepts drawn from hermeneutics, those of indexicality and reflexivity. Finally, the section ends with a brief discussion of Garfinkel's ethnomethodology and the development by Garfinkel of many of the key ideas from symbolic interactionism and hermeneutics. The ethnomethodological understanding of indexicality and reflexivity pervade the entirety of this thesis. The work of Goffman is integrated into these discussions where his work complements or contrasts with the ideas, assumptions and beliefs that are currently being discussed. Whilst links to the conversation analytic approach and the works of Harvey Sacks are made in this section, the main discussion of these appears in the methodology section in chapter 5.

Symbolic Interactionism

Symbolic interactionism originated in the works of George Herbert Mead, but was named and developed by his student Herbert Blumer. It is a perspective within social psychology that was developed as a reaction to the behaviourist theory of stimulus and response. Blumer summarises the perspective with three premises:

- we act towards things based on the meaning those things have for us;
- that this meaning is socially constructed through interaction;
- and this meaning is continuously being negotiated and changed through interpretation

(Blumer, 1969, p. 6).

In other words, we do not merely 'respond' to others, we interpret their actions and intentions and respond accordingly. The negotiation and changing of meaning implies that there is no such thing as an immutable objective meaning (von Glasersfeld, 1989) but also that humans have an active part to play.

Symbolic interactionists focused their studies on unobservable phenomena, such as attitudes, using a wide variety of methods including in-depth interviews and surveys, using participants' responses to these to make claims about attitudes or intentions behind behaviour. Many of these analyses were quantitative in nature, drawing upon statistical analysis of large of samples of participants (Hutchby and Wooffitt, 1998, p.25). The three principles of symbolic interactionism underlie a great deal of sociological, educational and linguistic research, including this study, but it is

the focus of the study and the methods used where the key differences lie. These differences draw from the ethnomethodological approaches discussed below as well as Goffman's work on face-to-face interactions.

A key concept developed by the symbolic interactionists is that of 'definition of the situation', initially described in Thomas as the "stage of examination and deliberation" (1923, p.42) that occurs before action. There has been a great deal of research into describing the definition of the situation within the classroom, which has resulted in descriptions such as 'teacher-centred or learner-centred', 'direct or indirect' and 'traditional or progressive' (Boaler, 1997; Flanders, 1970). However, these descriptions do not reflect the fact that the definition of the situation is continuously changing and undergoing negotiation. Each individual will have their own personal definition of the situation and will influence the definitions of those around him.

The idea of 'definition of the situation' is in some respects similar to Goffman's (1974) notion of frame. Frame is the definition that participants give to the current social interaction. It includes the roles participants adopt, the actions of participants as well as the wider situational context. However, Goffman emphasises the dynamic nature of a frame as it is modified and refined through the interaction. This dynamic view of context links to the hermeneutic ideas of indexicality and reflexivity discussed below and the implications on the analysis of interactional data is central to this thesis.

In a mathematics classroom, the teacher's and pupils' definitions of the situations during whole class interactions will be influenced by the local context and the broader context in which the interaction occurs. Local

context includes the other participants' individual actions, the interpretation of these actions as well as the nature of the mathematical activities. The broader context includes the participants' beliefs about the nature of mathematics and what it means to learn mathematics as well as beliefs about the roles of teachers and pupils in the interactions (see chapter 5 for a discussion of the place of context from a conversation analytic perspective). In particular, the role of a teacher includes significant power and authority over his pupils (Edwards and Westgate, 1987). Interaction in classrooms often involves a negotiation between the different definitions of the situation and it is an alignment of these definitions that enables a classroom to function. This alignment links to Yackel and Cobb's research (1995; 1996) on the development of classroom norms and in particular sociomathematical norms discussed in the previous chapter. The authority that the role of teacher has over pupils influences the roles that pupils adopt during classroom interactions, with pupils adapting to the roles which the teacher supports (see also chapter 10). Pupils are also compelled to be in the classroom, and are not there of their own free-will. This will also impact on the definitions of the situations that pupils have, which can make some pupils reluctant to participate and they may demonstrate this reluctance in a variety of ways (e.g. Houssart, 2001).

Another key idea is that of 'taking the role of the other'. Mead defines this as an integral part of human action. It is through taking the role of the other that we develop our 'self' and how we control our own reactions (Blumer, 1994). Before we act, we interpret the situation both from our own point of view but also from the point of view of the other participants. We are able to make

predictions about the reactions from others when you act by taking their role. For example, when a teacher asks a question, he will have in mind the response(s) he expects and which pupils will respond, whether that is by answering the question, avoiding the question, or some other behaviour.

The idea of taking the role of the other also has a significant impact on my role as the researcher. Dilthey (1988) argued that the researcher needs to get inside the head of the participant and grasp the subjective consciousness or intent from inside, and separate themselves from their personal, cultural, historical and social background in order to understand the meaning of the participants' actions. This is done through observation and discussion with the individuals and the researcher remains unaffected by this process (Schwandt, 2000). The phenomenologist Alfred Schutz also proposed that the social science researcher should interpret actions from the perspective of the actors whilst taking the role of a disinterested observer (Schutz, 1962). This idea focuses on the fact that the researcher has only a cognitive interest in the participants' social actions, they are not actively taking part and consequently their focus may be on different aspects of the action than those of the participant. For example, the research may be focused on rules that affect the interactions which the participant takes for granted.

Whilst this study focuses on participants' social actions and the interpretations and predictions the participants make of each others' social actions, this is done through an analysis of the social actions themselves, not by 'getting inside the head' of the participants. This aspect of the perspective is drawn from Garfinkel's ethnomethodology discussed below.

Hermeneutics

The other significant ideas that have influenced this study are from the discipline of Hermeneutics, in particular the idea of the hermeneutic circle, indexicality and reflexivity.

Hermeneutics was originally developed for the purpose of interpreting texts, in particular biblical texts, but later Schleiermacher, Gadamer and others extended it to include all experiences that were to be interpreted. The hermeneutic circle is the idea that neither the whole experience nor any individual part of the experience can be understood without reference to one another (Rhoads, 1991). Understanding is a continual process of assessing each new experience within its context but also the context itself needs to be reassessed in light of the new experience. Schleiermacher talks about understanding the grammatical aspects of a statement first, and this in turn helps us to understand the statement as a whole in terms of its psychological aspects, which again change our understanding of the grammatical aspects. Consequently, interpretation is an on-going process over time, cycling between the parts and the whole until our interpretation is both coherent and consistent understanding of the whole. This process forms a type of critical testing of interpretations.

Indexicality refers to the idea that the meaning of a word or sentence is dependent on its context of use. For example, two teachers could ask exactly the same question, using exactly the same words and receive entirely different responses depending on the context in which they were asked. This context is not just the mathematical context, but also the social context.

It is therefore important, when analysing interactions, that this context is taken into account.

There are a number of mathematical questions that I have used in my own teaching that exemplify this relationship: 'what is the smallest number greater than 0' often relies on the types of number that the students have had experience of. A child who only has experience of the natural numbers is likely to give the answer of 1, introducing decimals and fractions results in a wider range of responses and only some of the people familiar with fractions will realise that such a number does not exist! Responses to 'is a square a rectangle' often rely on the pupils' experiences of squares and rectangles, whether they have memorised a list of properties of have developed a more relational understanding (Skemp, 1976) of quadrilaterals.

This idea of indexicality is again central to any ethnomethodological approach, with a focus on the contexts that the participants themselves orient to. Garfinkel's initial definition of the term ethnomethodology includes the investigation of indexical actions (Garfinkel, 1967, p.11). The sense individuals make of a particular activity is constituted by themselves as individuals. How we interpret actions and activities is bound by the context in which the activity takes place. We understand actions in relation to the context in which they are performed. Classroom talk contains many indexical words and expressions that only make sense in the context of a particular discussion. For example, 'it' and 'that' only have meaning when what they are referring to is commonly known to all participants (Rowland, 1999). Ethnomethodologists expand the domain of indexical expressions.

Not only diectic expressions such as I, that, it, can only be understood in relation to the context in which they are spoken, but all expressions can only be understood in relation to the context in which they are spoken.

Reflexivity in one sense concerns the ways in which a researcher influences and changes the interpretation and meaning of the focus of the research. For example, in the classroom observation situation the presence of the observer or video recorder may alter the teacher's and the pupils' behaviour. Also, an interview will encourage the teacher to reflect upon his own actions and thought processes and this may affect the behaviour of the teacher during subsequent observations and interviews. The researcher needs to reflect upon how their own personal beliefs, interests, experiences etc. have shaped the research. Also, the researcher needs to reflect upon how their research questions, design and methods have impacted on both the data collected but also the interpretation of that data. This includes reflecting on the assumptions the researcher has made concerning the nature of knowledge and the structure of the external world. Bourdieu (Grenfell and James, 2010) argues that it is by being aware of these influences that the researcher can free themselves from them and come close to an objective interpretation of the data. Also, returning to Gadamer's view (1989), the researcher needs to reflect on how the research may have affected and changed them.

Reflexivity is also apparent in the development of the classroom community and its practices. The practices within the classroom are constrained by the knowledge, beliefs, attitudes and goals of both the teacher and his pupils. At

the same time, these beliefs, attitudes and goal are influenced by the practices in the classroom. In particular, the teacher's understanding and appreciation of pupils' understandings develop partly through pupils' responses and explanations (Cobb and Yackel, 1996). Yackel and Cobb's exemplification of sociomathematical norms draws on many of the indexical and reflexive ideas within ethnomethodology, demonstrating how these norms are constituted by the interactions within the classroom, rather than being predefined external criteria for how to interact.

Reflexivity has a central role but a subtly different meaning in any ethnomethodological approach to the analysis of activities and practices, in particular in the way that it applies the accountability of these activities and practices. Garfinkel argues that "the activities whereby members produce and manage settings of organised everyday affairs are identical with members' procedures for making those settings 'account-able." (1967, p.1).

In the context of identity, who you are is constituted by how others interact with you, and how they express their understanding of who you are by what they do. A great deal of research in mathematics education focuses on unidirectional influences, for example how the actions of the teacher influences learning (Lau, *et al.*, 2009) or focusing on how learners are continually interpreting these actions (Stone, 1993) but these relationships are reflexive. How the teacher acts is responsive to how learners act and vice versa.

Ethnomethodology

One key assumption that the symbolic interactionists make is that the underlying social world is ordered. Ethnomethodologists, on the other hand, reject this assumption and maintain that there is no external order or set of rules that the individuals follow. Instead the order is constituted by individuals through their actions. Garfinkel's experiments illustrate the range of activities through which individuals seek and find order in everyday activities, but also how this order is then used by individuals to sustain and develop activities or to initiate new ones (Heritage, 1984). These activities are constituted through the "reflexive processes of the documentary method of interpretation" (p.103).

Ethnomethodologists argue that when we observe or participate in social interactions, we select only certain pieces of information and we try to organise this information into some sort of underlying pattern so that it makes sense to us. This underlying pattern can change in this process, but most importantly it is through these patterns that we interpret the world. Garfinkel called this the documentary method of interpretation. "Not only is the underlying pattern derived from its individual documentary evidences, but the individual documentary evidences, in their turn, are interpreted on the basis of 'what is known' about the underlying pattern. Each is used to elaborate the other" (Garfinkel, 1967, p.78).

Garfinkel demonstrated this method in his "student counselling experiment" (Garfinkel, 1967, p.79). In this experiment, students were offered advice about their personal problems in the form of yes and no answers from a 'counsellor' who was concealed behind a screen. In reality, each student

was given exactly the same randomly generated sequence of yes and no responses irrespective of the questions they had asked. Garfinkel found that the students were interpreting the counsellors' responses within the context of the problem but also reshaping their analysis of the problem in light of the counsellor's response, even when it was contradictory to a previous response. This has huge ramifications on how we respond to each other.

So, for example, when a teacher asks a specific pupil a question, most people argue that there is a (classroom) norm that states that the pupil must now answer the question. However, the moment that the teacher asks the question, the situational context for both the teacher and the pupil has changed. The teacher has now initiated an interaction and is expecting some form of answer from the pupil, and the pupil must now choose how, if at all, to answer the question. This change in situation occurs regardless of how the pupil chooses to respond, the situation has been reflexively reconstituted by the teacher asking the question (Heritage, 1984, p.106). The teacher and the pupil use the norm to interpret the responses and actions that follow. The participants are both creating and interpreting the interaction through reference to this norm.

Where ethnomethodology differs from most discussions of the roles of norms is in the way that they view norms as reflexively constituted in the situation in which the interaction occurs. The participants are not recognising the situation as some predetermined situation in which the norm applies, where the interaction is guided or regulated by the norms of the situation.

Garfinkel (1967) focused on studying everyday activities and the social order of participants' actions. He introduced the term ethnomethodology to "refer to the investigation of the rational properties of indexical expressions and other practical actions as contingent on-going accomplishments of organized artful practices of everyday life" (p.11). He argued that actions are produced from common sets of procedures that participants orient to both to produce their own actions but also to interpret the actions of others. The interpretations that participants make cannot be directly observed and therefore the research can only speculate on the content of these interpretations. Garfinkel argued that actions and interactions are socially ordered, and that this order is observable by the research and to participants in any interaction. Garfinkel's focus on common-sense knowledge and everyday activities is in stark contrast to previous sociological research. Previously, research had focused on how social norms were internalised but ethnomethodology instead focused on how people accounted for the actions of themselves and others (Hutchby and Wooffitt, 1998, p.31).

As a consequence of the constitutive reflexive relationship between actions and context, it is not actually possible to separate ourselves completely from our background influences. They are part of our personal perspectives, they form the underlying patterns in which we select what is important and ignore everything else. Our personal perspectives dictate the ways in which we interpret all our experiences. Gadamer takes this view one step further and questions whether it is even desirable to detach ourselves from our background. He argues that the researcher's prejudices (by which he means pre-judgements of any kind) need to be suspended but the researcher needs

to remain aware of their influences in order to understand others' perspectives: "the important thing is to be aware of biases so that the text can present itself in all its otherness" (Gadamer, 1989, p.269). Gadamer views these prejudices as the basis of all understanding. Additionally, he argues that the researcher is shaped by this experience, and it is important that the researcher reflects upon this two-way influence of personal preconceptions and beliefs.

Additionally, there is a great deal 'taken-for-granted' in any interaction. These 'taken-for-granted' aspects are what enables participants, and researchers, to make sense of any interaction. Many aspects of classroom interaction would seem strange and abnormal for anyone who did not know the 'taken-for-granted' rules or norms that enable successful interaction, yet it is impossible to explicitly identify what these 'taken-for-granted' rules are. Garfinkel illustrates this through asking us to write down the rules of tic-tactoe. Most descriptions include that it is a game for two people, but then you can ask whether these two people need to be able to see each other, or whether these people need to speak the same language or even whether they need to be alive. No matter how much detail you include in the description, more questions can be raised because we always take something for granted.

Ten Have (2007) argues that the researcher having membership knowledge and skills enables them to understand the practices being studied. It is this membership knowledge that enables researchers to recognise similarities and differences between actions. In order to analyse actions in interactions,

the researcher needs to "understand it as a participant" (ten Have, 2007, p.44). In fact, the researcher's knowledge of the institutional context is vital in order to analyse the relevance that the actions themselves have for the participants as such knowledge may be 'taken-for-granted' but not known by outsiders (Arminen, 2000). An ethnomethodological study, or one using a conversation analytic approach like this one, circumvents the debates around the influences of the researcher by insisting that all interpretations of the data must be demonstrated in the data themselves. All evidence for claims must be found in the interactions themselves, and not drawn from a wider membership knowledge of the context.

Summary

The theoretical ideas outlined in this chapter, particularly those from ethnomethodology, set out the assumptions that underpin a conversation analytic approach to analysing data. In particular, the role and interpretation of context is in stark contrast to the majority of methodologies used to research classroom interaction. These differences in assumptions are examined further in chapter 5 where conversation analysis is outlined and contrasted with other discourse analysis methodologies but they have also influenced the research questions presented in the next chapter. In a CA approach, the analysis is focused on what participants do in their interactions through their utterances, and how this is accomplished in the talk. Consequently, the research questions focus on the activity in the interactions and their accomplishment.

Chapter 4: Research Questions

The main aim of this study is to develop a description of the interactional organisation of the secondary mathematics classroom. The key research questions underpinning the discussions in this study focus on general patterns of interaction that indicate that actions performed in whole class discussions in mathematics are organised. What are the organising structures that the teachers and pupils use to co-produce talk that enables certain actions to be performed? These patterns are observable through noticing similarities and differences across different lessons and between different teachers. Whilst it is the similarities across the different lessons that indicate an underlying organisational structure, the differences may reveal 'deviant cases' (see below) which can lead to a deeper understanding of how these structures, and the similarities or differences between lessons and teachers for the learning and teaching of mathematics, can then be considered.

These questions were not inspired by my own experiences as a mathematics teacher or a mathematics education teacher, or by my extensive reading of the literature, but instead arose from initial encounters with data. The interest in whole class interactions involving the teacher in secondary maths classrooms has clearly developed out of my own roles. This interest did then develop into the formulation of some research questions that drove the initial analysis of the data collected in the pilot study. These questions focused on the nature of questions asked both by teachers and pupils, the intentions

behind these questions and the nature of the response to them. However, this initial analysis combined with extensive reading of the literature concerning the analysis of classroom interactions led to the rejection of these research questions and a return to the pilot study data with a more 'open' view. This 'open' view enabled me to notice patterns or features of the interactions which prompted further investigation.

The collection of further data occurred with this continued aim, and not preformed ideas of what to look for or defined research questions. This new strategy is consistent with a CA study and is often referred to as 'unmotivated looking' (see below). Having said this, I am unable to view any classroom data as a 'detached observer' and therefore with a completely open mind. I have 'membership knowledge' of the context and was becoming more and more familiar with the conversation analytic literature. However, it is this same 'membership knowledge' that enables me to identify some of the subtle similarities and differences in the transcripts of the lessons observed.

The CA approach finally adopted leads to some more general questions that structured the data analysis. These include:

- What is the teacher or pupil doing in their turn?
- How are the other participants in the classroom understanding this action?
- What is the form of the action and what alternative forms are available?

- In what ways does the action and the form of the action influence or constrain subsequent turns?
- How do the sequences of actions influence the learning and teaching of mathematics and the role identities of the participants in the classroom.

The focus on the interactional organisation results in the identification of sequences of observable actions within whole class interactions. Observable actions largely consist of the utterances or speech acts that occur, but gestures including writing or presenting on the whiteboard, also influence the interactions. Therefore, whilst the focus of this study is on spoken utterances, other observable actions are included where they are necessary in the analysis. For example, where a hand movement gesturing the shape of a curve or an image drawn on a whiteboard take the role of a turn and are treated as such by the teacher and pupils.

Many studies into discourse and communication in classrooms have focused on the role of the teacher, both in terms of the management and content of interactions (Walshaw and Anthony, 2008). Others have focused on the interactions of a small group of pupils during tasks, in particular where they have been working at a computer (Kumpulainen and Mutanen, 1999). In this study, it is the interactions between a teacher and a large group (15-32) of pupils that is of interest.

The view is taken that whole class interactions are *jointly* constructed and locally managed by the teacher and the pupils. What a teacher is doing in their turn influences and constrains what a pupil can do in a subsequent turn.

One cannot occur without the other. This relationship holds in interactions between pupils working in pairs or small groups, and these interactions in turn are influenced by the presence of the teacher whether they take a turn or not. However, the focus in this study is necessarily limited to this relationship in whole class interactions.

Chapter 5: Methodology

There are a variety of methodological approaches to the study of classroom interaction, such as critical discourse analysis, speech act theory or discursive psychology. In this chapter, I examine a range of approaches for the analysis of discourse that have informed my choice of a conversation analytic approach for this study. Before the pilot study, the intention was to use a discourse analysis approach based on the work of Kumpulainen and Wray (2002), but the early attempts at analysing the data from the pilot study led to the change to a conversation analytic approach. Discourse analysis is a field of research methodologies and methods that investigate language in use and in social contexts (Wetherell, *et al.*, 2001). Discourse analysts are looking for patterns either within the language itself or in the patterns of activity in interactions.

There is some debate as to whether conversation analysis is a form of discourse analysis or not, I begin with a brief description of discourse analytic approaches in general, including those features that give rise to this debate, before focusing in particular on speech act theory. Whilst very different in their theoretical assumptions and approaches to the analysis of discourse, speech act theory and conversation analysis do share some features, such as the view of utterance as actions. The assumptions underlying speech act theory are also common to a great deal of current research in mathematics education focusing on interaction and communication. This main part of this chapter contains a detailed description of conversation analysis, building on many of the theoretical ideas discussed in the chapter 3. The chapter ends

with a brief discussion of discursive psychology, an approach that builds on a conversation analytic methodology and has been useful in the analysis in chapter 10.

Discourse Analysis (DA)

The term discourse analysis is used to describe a wide variety of research methodologies. The emphases and assumptions of different disciplines, such as anthropology or sociology, that have analysed discourse have led to the development of a variety of analytic approaches, each of which has offered some insight into interactions, and each of which has some limitations to the approach. Discourse analytic approaches adopt the stance that our descriptions are not determined by objective properties of features and therefore descriptions can be constructed in a variety of ways (Wooffitt, 2005). Conversation analysis is an approach to the analysis of interaction that some authors (Mey, 1993; Rowland, 1999; Taylor, 2001) include as a discourse analysis approach whilst others (Levinson, 1983; Seedhouse, 1996) view them as distinct approaches to the analysis of classroom interaction. A number of studies of classroom interaction adopt a conversation analytic approach and this approach is particularly common in research into language learning (Seedhouse, 2004; Waring, 2009) however few using it to study mathematics classroom interactions (Barwell, 2003; Forrester and Pike, 1998).

Many discourse analysis approaches categorise the naturally occurring patterns of interaction in the classroom, in particular the structural-functional linguistic approach. The classroom data are analysed according to both their

structural patterns and their functions. Structuring this analysis is usually a discourse hierarchy, for example Sinclair and Coulthard (1975) used lesson, transaction, exchange, move and act, where lesson is the largest and act is the smallest discourse unit. Each part of the IRF sequence is a speech act. However, it can be challenging for an observer to identify precisely the function of a particular speech act. Many teachers' questions could be a request for information, an instruction or command or an admonishment. In fact, speech acts can perform a multitude of functions, particularly in complex interactional settings such as classrooms.

The aim of discourse analytic research is to offer an interpretation of the meaning and significance of language in use. The complex nature of the situated use of language means that it is not possible to claim that findings reflect an absolute truth of reality (Banister, *et al.*, 1994, p. 3). As Barwell (2009) argues, discourse is not a 'window on the mind' and any analysis involves interpretation. Discourse analysis cannot make claims about the underlying meanings, attitudes, or beliefs of the participants. Instead, analysis focuses on the discourse itself and not the participants who produced the discourse (Taylor, 2001, p.19).

Discourse analysis approaches involve a simplification and reduction of the data through the categorisation of patterns and hence are open to the criticisms that they cannot account for the complex and dynamic nature of classroom interaction. They often also do not consider many of the contextual forces in play, such as norms and role relationships (Wooffitt, 2005).

Speech Act Theory

Speech act theory (Austin, 1975; Searle, 1969) holds the view that utterances can be usefully analysed as social actions such as a declaration, request, or assessment. In other words, utterances '*do*' something. It often involves the analysis of isolated utterances, focussing on syntactic and semantic features of these utterances. Speech-act theorists ask what action is a participant performing in an individual utterance and then examines the context in which it was uttered to explain how different people respond to this utterance. In particular, speech-act theory focuses on the rules and contexts through which participants understand an utterance as an act.

Austin identifies three types of 'force' of speech acts: locutionary: illocutionary and perlocutionary. Locutionary force refers to the actual act of speaking and includes features such as the grammatical form and intonation used. Illocutionary is the direct action an utterance is performing and perlocutionary refers to an indirect consequence of the act. The idea of illocutionary force is relevant to the CA sequential unit of adjacency pairs, which are discussed in more depth below. CA analysis of data involves the "analytic integration of ... the 'illocutionary' dimension of a current utterance with the 'perlocutionary' dimension of its prior" (Drew and Heritage, 1992b), extending the focus of analysis to sequences of utterances rather than the individual utterance usually considered by speech act theorists.

Austin's original development of speech-act theory restricted the utterances that could be viewed as acts, but this evolved to include all utterances (Searle, 1969), and acknowledged that utterances could perform indirect

acts. For example, a teacher saying 'can you sit down' is not a request for information but an order to sit down.

The Birmingham discourse group developed a speech-act based approach for analysing interaction. Sinclair and Coulthard's (1975) analysis of classroom interactions, whilst still focused on the actions performed by an individual utterance, began to look at the patterns of acts. They only examined classroom interactions in the development of their framework, and their findings have been used to argue how features of interaction constitute the teaching and learning function of the exchanges. Yet their analysis focuses on the social context of the classroom and the influences of the institutional roles of participants in instruction, and other authors (Cameron, 2001b; Drew and Heritage, 1992a) would argue that the model and consequent linguistic rules relate to the nature of the task and not the institutional setting.

Speech-act theory has since been criticised for the number and complexity of the rules relating the context and the speech act to explain how the act is understood differently by others (Drew and Heritage, 1992a; Levinson, 1983). The difficulty of verifying speech-act analyses of intention or understandings also became an increasing problem for researchers, partly as a result of the focus on analysing isolated and often invented utterances. There was also some difficulty in identifying certain utterances as speech acts. In particular, the answer to a question can only be defined in relation to the questions, "there is no proposed illocutionary force of answering" (Levinson, 1983, p.293). Austin's Speech Act Theory was developed at the

same time as Sack's work developing Conversation Analysis and whilst there are some similar ideas and focuses between the two, there are also many differences which are explored below.

Conversation Analysis (CA)

The origins of conversation analysis approaches lie in the lectures given by Harvey Sacks between 1964 and 1972. They are influenced by the work of Goffman and Garfinkel, in particular ethnomethodology with the assumption that talk is highly organised and socially ordered (Hutchby and Wooffitt, 2008). This orderliness of talk is not determined by "innate cognitive structures of language" but instead reflects a "socially organised order of interpersonal action" (Wooffitt, 2005, p.59). Conversation obeys certain rules, procedures, or methods that organise and structure the sequencing of turns, who can speak and what they can say and CA analyses how particular utterances perform particular activities at the particular place in the interaction where they occur (Wooffitt, 2005). CA investigates these normative rules or patterns of use and how participants jointly construct the interaction and their shared understanding of this, which indicate how participants co-ordinate their interactions by drawing on their membership knowledge or *communicative competences* (Wooffitt, 2001, p.49). Participants are assumed to know these rules for interaction and they design their turns for the other participants in the interaction, and can therefore recognise when these rules are deviated from. Participants' intentions, motives, or interests are not part of any analysis; the analysis of interactions is of interest in its own right.

Focussing on the actual language used in interaction, the language is treated as "containing everything relevant for analysis" (Cameron, 2001a, p.88) This contrasts with many discourse analysis or pragmatic approaches where contextual features are often used to explain the meaning participants ascribe to utterances. In a CA approach, the context is only drawn upon in the analysis if the participants themselves orient to it through their interactions.

Conversation analysis is an *emic* analysis of discourse. Research involves "working within the conceptual framework of those studied" (Silverman, 1993, p.24), the participants in an interaction jointly create the meanings and activities of the interaction, and it is how these participants orient to these meanings that is of interest. "It is important to investigate *their* (participants) interpretations of what is happening in the interaction rather than to impose somewhat arbitrarily a set of assumptions and relevancies, which might in fact, have no bearing on the details of participants' actual conduct" (Wooffitt, 2001, p.42). The alternative approach, *etic* analysis, draws upon the researcher's own conceptual framework to interpret the meanings and activities of the interaction.

Video and Audio recordings and the transcription of naturally occurring talk are used as the main source of data. The analysis of naturally occurring data is key to a CA approach, it can often appear grammatically disorganised but features such as false starts, hesitations, and overlaps can tell us a great deal about the actions being performed in the interaction (Wooffitt, 2005). Many DA approaches can include data sources such as constructed texts,

field notes, or interview transcripts. Most CA approaches only use these sources of data if the focus of the study is on the production of those texts, i.e. the sequential structure of news interviews. Spoken interactions are analysed, searching for patterns that are normally unapparent and in theory reveals the complexity that structures the 'conversation'. In other words, the data are analysed to find out how the participants understand and respond to each other in talk. As such, conversation analysis approaches are inductive; there are no pre-determined categories that are applied to the data, instead themes are drawn from the data. Consequently, in any presentation of an analysis, recordings and detailed transcriptions are also used.

Conversation analysis uses naturally occurring interactions, but not necessarily a conversation. Many researchers now generally use the term talk-in-interaction instead of conversation (Drew and Heritage, 1992b; Schegloff, 1987), to reflect the data considered first by Sacks as he developed the approach. Sack's initial work focused on phone calls to a suicide helpline, not naturally occurring conversations.

The classroom context is viewed as being dynamic in conversation analytic approaches. In order to examine what language is doing, we need to consider its situated use (Taylor, 2001). It is shaped by the participants through their interactions, and in the case of the institutional setting of the classroom, through the institutional and pedagogic goals. Drew and Heritage (1992a) describe talk as both 'context shaped', where it is affected by both the local context such as the current activity or the previous turn and more global contexts such as an institution, and 'context renewing' in that any talk

provides a context for future utterances (p.18). Classroom interaction is considered in relation to meaning and context and the sequence of events is central to the analysis, "the meaning of an action is heavily shaped by the sequence of previous actions from which it emerges" (Heritage, 2005, p.105). Conversation analysis approaches focus on the interactional patterns that emerge from the data. Individual utterances are considered within the broader interactional context and their position within the sequence of utterances. The act an utterance performs depends on its sequential position, in contrast to the isolated analysis that many DA approaches take. Participants' understanding of each other develops as the sequence of turns develop, an utterance displays the speaker's understanding of the previous turn and subsequent turns either build on this mutual understanding or the original speaker takes steps to repair the situation. This relationship between these turns indicates their situatedness and is referred to as the next-turn proof procedure (Hutchby and Wooffitt, 2008; Sacks, *et al.*, 1974).

Conversation analysts recognise that context is important, but in any interaction, there are a wide variety of contexts that may affect the interaction, such as the gender of the participants, the physical context in which the interaction takes place, the time at which the interaction takes place and the nature of the participants. These contexts will also be viewed differently by different individuals, such as the participants and the researcher. In a conversation analytic approach, only the contexts that the participants demonstrate as being relevant through what they say and how they say it, are considered. Contextual information can distort an analysis and choices about which information may be relevant influence the

interpretation of the data. For example, labelling participants as male or female often leads to an interpretation (not necessarily conscious) that gender is relevant to the interaction. "Thus CA offers an alternative of the view ... that our conduct automatically reflects the context in which it occurs" (Wooffitt, 2005, p.69) often adopted in other discourse analytic approaches.

As with all ethnomethodological approaches, the object of study is "the set of techniques that the members of a society themselves utilize to interpret and act within their own social worlds" (Levinson, 1983, p.295). Or, as Mey (1993) describes it, the rules of discourse which belong to the people and are used by them for social activity. Consequently, role identities of participants are not included in transcriptions until these identities are 'proved' in the interaction in the way that the participant both produces and interprets the interaction. This can help to prevent prejudgments being made about the content of interactions based on these identities as well as ensuring only those contexts that are directly relevant are considered.

As such, throughout this thesis, participants are given names and not roles such as teacher and pupil. Also, the names chosen for the pupils are used both to indicate male and female roles, such as Sam, Charlie, Ashley and so forth. Whilst it is clear in the majority of transcripts which participant is the teacher as is outlined later in this chapter, the pseudonyms of the four teachers are used consistently throughout, the background information on each of these teachers is only discussed where it is made relevant through the interactions themselves.

Another key device in the analysis of data using a conversation analytic approach is the analysis of 'deviant cases'. If an interactional sequence does not fit with a pattern in the analysis, it is not considered as irrelevant or uninteresting, but instead as highly informative. The detailed analysis of any sequence of turns that seems to differ from rules or principles that have so far been formulated during the data analysis is undertaken in order to support an explanation for the patterns and structures apparent so far. This analysis of a deviant case may serve to confirm the rules or principles developed so far by revealing more detail as to how participants are orienting to these rules or principles. "the violation of the rules results in incoherent discourse which is noticed and attended to by interlocutors, and ... the violation of these rules can usually be accounted for" (Tsui, 1991, p.111) or it may result in a reformulation of these rules or principles in such a way as to include the deviant case as a standard example. So whilst a deviant case at first glance may undermine a conversation analysts claims about structures and patterns of interactions, may ultimately be used to demonstrate the participant's orientation to these patterns or structures (Wooffitt, 2005).

CA also approaches analysis of transcripts initially through 'unmotivated looking. Whilst no analysis is truly unmotivated (Psathas, 1995) the term means beginning the analysis of data without expectations of what might be found. This is in contrast to many discursive approaches to research where the data are purposively sampled in light of researchers' interests. In this study, the design of the research questions consequently needed to reflect this 'unmotivated looking'.

Themes

Ten Have (2007) proposes four themes within CA approaches to analysing talk-in-interaction. These are: turn-taking organisation; sequence organisation; repair organisation; and finally the organisation of turn-design. Each of these themes are discussed in more detail below as they form the basis of the analyses in chapters 8 and 9.

Sequence Organisation

The theme of sequence organisation has been discussed extensively by Schegloff (2007), and ten Have describes it as "any utterance in interaction is considered to have been produced for the place in the progression of the talk where it occurs, especially just after the preceding one, while at the same time it creates a context for its own 'next utterance' (ten Have, 2007, p.130). Sequence organisation describes the shape of sequences of utterances which enable something to be 'done' through the interaction. Schegloff and Sacks (1973, p.299) use the much cited phrase "why that now" to highlight the importance of sequencing in the participants 'doing' actions through their talk.

One key idea within sequence organisation is that of adjacency pairs, which are discussed again in chapter 8. Briefly, an adjacency pair consists of two parts with a normative relationship in that after a speaker says a first-pairpart (FPP) the second-pair-part (SPP) becomes conditionally relevant. It is the illocutionary 'intention' of the FPP that characterise the type of adjacency pair. For example, following a question, an answer is relevant. Following an offer, an acceptance is relevant. This relationship is also apparent in

Mehan's analysis (1979, p.50) where choice elicitations are followed by choice replies and so forth. Once an FPP has been uttered then the type constraints what SPP are possible. If the SPP is missing, it is 'noticeably absent' (see chapter 8 for more detail) and the interaction usually continues as if the original FPP was not uttered. However, characterising the type of adjacency pair based on the illocutionary force of the FPP is often not possible until the SPP and the wider context of the sequence are considered. A FPP as a speech act could represent a wide variety of things, such as a request or a question. Furthermore, CA often use subsequent turns to characterise utterances as the FPP of an adjacency pair or as a presequence (Schegloff, 2007). Utterances that might appear when considered on their own to be requests for information may in fact be a pre-request for something else. It is only through the interactional work by all the participants in an interaction that the nature and type of an utterance can be determined (Mey, 1993, p.252). The normative relationship between FPPs and SPPs enables participants to find meaning to the interactions and produce the next turn.

Mehan (1979, p.63) argues that this reflexive relationship between first pair parts and second pair parts indicates that the acts that utterances are classified as are the 'social acts' defined by Mead, rather than the 'speech acts' in speech act theory.

Another relevant sequence type is "telling sequences" (Schegloff, 2007, pp.41-44) where a story or joke is being produced. These are often prefaced, which prepares for the telling sequence. These pre-sequences

can check if the hearer has heard it before but can also prepare the participants for the type of responses that are expected during the 'telling'. Schegloff's (2007) detailed discussion of sequence organisation includes details of pre-expansions, insert expansions and post-expansions as organisational structures that vary the sequential organisation of adjacency pairs, but these are only briefly discussed in the analysis of data in this thesis and consequently are not expanded on here.

Turn taking

A large number of CA studies have focused on the organisation of turntaking, including this one, and the aspects most relevant to this study are discussed in more depth in chapter 8. However, the importance of this idea and its relationship to the other themes means that it is worth exploring here too. Sacks et al. observed that overwhelmingly only one person speaks at a time and that the speaker changes frequently with minimal gap or overlap. It is this observation that led to a detailed analysis of the systematic of turntaking organisation by Sacks et al. in 1973 (cf. chapter 8). Sacks et al.'s analysis identified the key features of turn-taking organisation. The size of turn and the ordering of turns are locally managed by the participants themselves but also through their construction of turns, participants are demonstrating an orientation to the other participants (Sacks, *et al.*, 1974).

Sacks et al. examine 'turn constructional units' (TCU) and the rules of turntaking at 'transition relevant places' (TRP). The TCU is relevant to the other themes within CA, but is a somewhat subjective idea. Schegloff (2007, pp.3-4) describes three resources for recognising TCUs. The first two, grammar

and intonation, are used extensively by other DA approaches as a means of distinguishing between units for analysis. The third, more subjective criteria, is that of it being a recognisable action by the participant. In other words, when a speaker is perceived as having done something like making a request or answering a question. As a speaker completes a TCU, a TRP occurs in that the transition to a next speaker becomes relevant. This does not necessarily mean that a change of speaker does occur, just that it is a place where a change in speaker is relevant. The existence of TRPs is evident through the success that people have with taking the next turn in an appropriate position, i.e. with minimal overlap or gap. This is particularly apparent in situations where there are a large number of participants hearing an utterance, such as in a classroom or at a public speech where the hearers can collectively take the next turn, for example by applauding.

Repair Organisation

Repair organisation is another key theme of this study and is discussed in more depth in chapter 9. Repair describes the ways in which interactional trouble is dealt with, for example, problems of mishearing or understanding. A repair is split into three parts, the trouble source, the repair initiation, and the performance of the repair. Any utterance can be considered as a trouble source and is potentially repairable. A further distinction between whose turn the trouble source, repair initiation and repair performance is also made. Self-repair initiation and/or performance is when the same person as whose turn contained the trouble source initiates and/or performs the repair whilst other-repair initiation and/or performance is when someone other than the participant in whose turn the trouble occurs initiates or performs the repair.

The organisation of turn-design.

The final of ten Have's themes is the organisation of turn-design, which he uses to summarise some other key ideas in conversation analysis. These include recipient design and preference organisation. Recipient design refers to the idea that a speaker "builds an utterance in such a way that it fits its recipient". Preference organisation is discussed more extensively in this study (see chapter 9), and refers to the idea that when there are a range of possible actions following a previous turn, one action may be 'preferred' over another, and this preference is demonstrated through features of the sequence of turns in which the action occurs.

Conversation Analysis and Institutional Talk

One thing that marks institutional talk, such as that in classrooms, from ordinary talk is that institutional talk is usually goal oriented and task oriented. Each lesson will have an overriding goal that controls the classroom interaction, and in the case of the mathematics lesson, the most apparent goal is to learn mathematics, but other goals may also be influencing the interaction, such as goals concerning wider issues of behaviour and social interaction skills. Different individuals within a particular lesson may have different goals, and these goals influence the jointly constructed discourse. Drew and Heritage (1992a) describe two other primary features of institutional interactions. The interactions may also be additionally constrained by institutional norms that are special or particular to the institution. The talk may be "associated with inferential frameworks and procedures that are specific to the institution" (1992a, p.22). Finally, the

activities of the institution will shape the meanings and understandings that participants give to interactions. Heritage and Greatbatch (1991) call these features of institutional interaction, the 'fingerprint' of the patterns in the interactions.

Institutional interactions are also often asymmetrical in that many have a preestablished system of turn allocation, such as classrooms or courtrooms. This asymmetry has led to discussions associated with the moral, social or political impact of the constraints on institutional interaction (Walsh, 2006). The pre-allocation of turns offers the 'questioner' the right to the questioning turn which can easily be built into longer turns including many TCU (see chapter 8 for some examples). The answerer, on the other hand only has the right to the turn until they have produced a recognisable 'answer'.

However, it is important to distinguish between interaction that occurs in particular institutional contexts and interactions that occur in activities that are common to the institutional context. A key theme in conversation analysis and other ethnomethodological approaches is that context is shaped by the interactions. The institutional context is dynamic and locally produced and it is possible that the interactions cease to be constrained by the institutional rules or principles if the participants deviate from these. If participants organise their turn-taking so that it is different from ordinary conversation then this offers evidence that they are orienting to the institution in organising their interaction (Drew and Heritage, 1992a). Sinclair and Coulthard's (1975) characteristic three part sequence (IRF) also occurs in other instructional situations that are not within a classroom, such as parent-

child instruction (Seedhouse, 2004) which indicates that this pattern is characteristic because of the institutional activities of teaching and learning. When examining the sequential organisation of institutional talk, the question arises as to whether it is the institutional context that influences the structure of interactions or the activities that are taking place.

The vast majority of educational research using conversation analytic approaches relate to language classrooms (Seedhouse, 2004), in particular classrooms where pupils are learning English as an additional language. Whilst there are many features of language classrooms that are similar to mathematics classrooms, such as the number of participants and the institutional goals of learning and teaching, language classrooms have the additional feature that language is not only the medium through which teaching and learning take place but is also the object of that teaching and learning. Notable examples of a conversation analytic approach to mathematics education include an article by Forrester and Pike and more recent articles that take a discursive psychology approach (Barwell, 2003). Forrester and Pike (1998, p.335) suggest that adopting a conversation analytic approach offers the opportunity to examine the relationship between 'emerging intersubjectivity' in the mathematics classroom and mathematical ability:

"by examining how teachers and pupils as participants themselves orient to, and understand, what is going on, we may be able to gain insights into (a) the implicit models and metaphors of the mathematical activity shared by those involved (b) the techniques and strategies they collaboratively employ

to conduct the "business-in-hand" and (c) how intersubjective meanings are coproduced and represented within the ongoing interaction" (p.337).

Limitations

One limitation of conversation analysis approaches is the inability to generalise any findings to other contexts because of the central role of the particular context under study. Yet the aim of research using these approaches is an in-depth analysis of data in that particular context, not the extension of the findings to other contexts. Many features specific to mathematics classrooms may be extendable to other lessons but equally many will be specific to only mathematics lessons and even particular mathematics teachers

Discursive Psychology (DP)

One approach to the analysis of discourse that has developed relatively recently is discursive psychology (DP) where researchers have attempted to integrate ethnomethodological approaches, including conversation analysis, and psychology. Drawing from speech-act theory and CA, DP focuses on the actions performed by utterances. Similarly to CA, DP also considers the indexical nature of discourse, including the sequential context and a wider, possibly institutional context. Discursive psychologists draw from ethnomethodology in their focus on the analysis of talk-in-interaction. Finally, DP views discourse as both constructed and constructive (Potter and Edwards, 2003).

The focus of DP research are mental states such as knowing, remembering or feeling that are the focus of much psychological research, however

discursive psychologists focus on the discursive interactions which enable these mental states to have meaning to the participants (Wetherell, 2007). In contrast to cognitive psychology, analysis focuses on how people can construct and use descriptions of mental states, rather than what is going on in a participant's mind. It also looks at how participants use references to or descriptions of mental states, such as thinking or believing, to perform social actions (Wooffitt, 2005). Discursive psychologists argue that these descriptions are shaped by the participants' interests and interactions, so, for example, mathematical thinking is discursively constructed by participants. Similarly to CA, DP assumes that participants share knowledge and understanding of the rules of interacting, but additionally they assume that this shared knowledge includes alternative meanings.

In recent years, DP has diversified and Wetherell distinguishes between two types of discursive psychologist. The first of these includes those who follow the methodological principles of conversation analysis and who restrict their analysis to talk-in-interaction and do not attempt to extend this analysis to the character or personalities of the participants (Barwell, 2009; Potter and Edwards, 2003). The focus of this group of discursive psychologists is on how the participants themselves interpret the interactions and the mental states that are referred to in the interaction. They argue that we do not have access to participants' mental states and therefore we cannot infer the nature of these from analysis of participants' interactions. This does not, however, mean that we cannot examine how participants do 'remembering' or 'thinking' through their utterances. Barwell's research into pupils with English as an additional language working with mathematics investigates how the

pupils construct their own accounts or versions of events and the psychological states of the participants.

However, other discursive psychologists argue that this takes a very narrow analytic approach to the analysis of discourse (Wetherell, 2007). The conversation analytic approach to data is that features such as gender, social class and so forth do not have a bearing on the interactions unless it can be shown that participants themselves are orienting to these features. Critical discursive psychologists do not only analyse interactions on the basis of what the participants themselves orient to, but also consider the wider historical and cultural language context (Edley, 2001). These historical and cultural language contexts involve a wide range of ways of talking about things that participants choose from when interacting. However, there are culturally and historically dominant ways of talking that means that not all options are equal. Each choice involves assumptions about the status of facts and what is an accurate description of the world. Potter and Wetherell (1987) use the phrase "interpretative repertoires" to describe the historically and culturally developed collection of words and metaphors that participants use to describe and evaluate actions. Participants' interactions "develop together as opposing positions in an unfolding, historical, argumentative exchange" (Edley, 2001). Discursive psychologists include identity as partly constituted through language and that this identity is also expressed using interpretative repertoires.

Summary

This chapter has outlined the conversation analytic approach adopted in this study, and compared and contrasted it with other methodological approaches

including discourse analysis, speech act theory and discursive psychology. These comparisons serve to highlight the key differences in the approach to analysing interaction taken in this thesis when compared to other studies within mathematics education and build on the theoretical underpinnings outlined in chapter 3.

The next chapter describes the methods used both in data collection and in the analysis of the data. In the chapters that follow, the analysis focuses on two of the key themes identified by ten Have, those of turn-taking and sequence organisation and preference organisation in particular. The sequential organisation of each of the extracts presented in chapter 7 is then used to examine the discursive construction of teacher and pupil and then mathematics in chapter 10. This final chapter explores the identities of the participants as they are dynamically constructed in the interaction, drawing upon some of the key features of discursive psychology.

Chapter 6: Methods

In this chapter I will outline the methods I chose for the collection and analysis of data then finally describe the presentation of my findings

Pilot Study

For the initial pilot study, a single teacher was selected on the basis that he offered an environment where pupils were encouraged to discuss mathematics and learn mathematics through investigation. This pilot study was designed to refine the research questions, the data collection methods and the analytical tools used prior to the larger study. All lessons over a two week period, with the exception of two, were observed and videoed and both the teacher and a small group of pupils were interviewed following each lesson. The video proved invaluable both in its use in the stimulated recall interviews but also it allowed me to review the lessons repeatedly, which was vital in the development of my analytic framework. The number of pupils interviewed following the lessons varied in terms of perceived ability and number. This enabled me to see the impact on both perceived ability and number of pupils on the quantity and quality of response. Groups of three or more pupils often gave more detailed responses and used each other for support and comparison in their interpretations of the whole class interactions that had occurred in the lesson.

The exploration of the data collected in this pilot study led to the earlier mentioned revision of my research questions and my methods. As I became more interested and intrigued by the implicit content of interactions, the role of the interviews became less fundamental and offered little to the

conversation analysis approach I came to adopt. Combined with the difficulties in gaining consent and access to interview participants and technological constraints to using stimulated recall interviews, the decision was taken to not collect this data in the main study.

The shifts in my research questions and research methods following the pilot study led also to a change in the sampling. The class for the pilot study was purposively chosen because of the potential that it offered something different and interesting worth studying. The teacher was known to encourage discussion and a problem solving approach in his mathematics lessons. However, it became clear that the data collected could not, by themselves, address my research questions, particularly how the interactions influence the learning and teaching of mathematics.

Main Study

The sample for the main study is essentially a volunteer sample, though with some restrictions on the volunteers. As my research was focussing on interactions, I wanted teachers with at least a few years teaching experience so that the structure of the interactions in their classrooms are likely to be more established and routine. I also did not want to use teachers with whom I had a long-term professional relationship in my role as mathematics education tutor and initial teacher educator because of the ethical issues that might arise both during and following my research.

The number of cases was not predetermined. A minimum of two offers a greater potential for any findings to be extended to a wider range of classrooms, but too many cases would result in an unmanageable quantity of

data. Three suitable teachers from three different schools volunteered, which combined with the data from the pilot study, resulted in a total of seventeen lessons which were video-recorded, three from Edward, four from Tim and Simon and six from Richard. The vast majority of videos were made with the researcher present, in one case in an observation room, with one lesson being videoed without the researcher present. In total twenty-four lessons had been organised to be video recorded but school trips, a flu pandemic, internal exams and participant teacher health meant that only seventeen were video-recorded.

It is usual at this point for detailed descriptions of each of the teachers and each of the schools to be given. For example, Andrew has been teaching for seven years. His first degree is in Engineering. The school is an inner city comprehensive with a large number of pupils receiving free school means and an above average number of pupils with SEN. 37% of pupils gained a grade C in GCSE mathematics last year. However, the sharing of this information influences how the data is interpreted by the reader. A central tenet of an ethnomethodological or conversation analytic approach is that contextual features, such as gender or the nature of the school, are only considered relevant to the analysis if it is evident in the data collected that the participants themselves treat these features as relevant. Consequently, these features are not shared here, though these features will have still affected my own interpretations of the data.

Whilst the pseudonyms I have chosen for the teachers are all recognisably male names, one of the teachers was in fact female. The conversation

analytic approach that I adopted for the main part of this study and in particular in the analyses described in chapter 10 emphasises the need to only draw upon those identities that are orientated to in some way during the interactions. Towards the end of the analysis process, it became clear that there were no significant differences between the teachers that related to the gender of the particular teacher. In only one of the classes was gender specifically mentioned during a whole-class interaction, but also the structures of turn-taking, preference organisation and the discursive construction of identities and mathematics did not appear to relate to the gender of the teacher. Consequently, the original female pseudonym was changed to a male one.

The schools vary from an independent fee-paying school to an inner city comprehensive school with high levels of social deprivation. The teachers have a wide range of differing experiences of teachers, and, in some cases, of other careers before teaching, as well as contrasting academic routes into teaching. These contrasting contextual features will undoubtedly influence the structure of whole class interactions, but similarities in these interactions across the four cases are likely to offer some insight into whole class interactions in secondary mathematics classrooms in general. Having said this, the small number of cases inevitably limits the generalisability of any findings. It is also not the intent of this study to make any such generalisations, instead the focus is on developing a detailed description and analysis of the interactions in the data collected.

Data Collection

This study aimed to research mathematics lessons as they naturally occurred. The teachers know that the focus of the research was on wholeclass interactions, with a particular focus on questioning. They were not given any specific details of the research questions or what topics to teach or how to teach. There were two main purposes to this, firstly to reduce the influence of my presence on the content and structure of the interactions as it was important that these were naturally occurring, and secondly a balance needed to be struck in terms of informed consent. Whilst the teachers, pupils and parents need sufficient information or complex terminology may confuse rather than inform participants, but this information may also influence them to behave in different ways and the data cease to be naturally occurring. Issues around informed consent are discussed in more detail later in the section on ethical considerations.

Video

The analytic approach chosen means that it is essential to video record lessons. Videos enable a discussion to be repeatedly replayed and transcribed to enable the analysis to be firmly based in the data. This also allows for a finer analysis of the interactions themselves, as seemingly simple utterances are often in fact far more complex and their temporal position is central to the analysis. Although gestures, facial expressions, and direction of gaze were not collected, where appropriate significant relevant actions carried out by participants were noted; for example, the teacher nominating the next speaker by gesture, a participant writing on the

whiteboard as part of the turn, or demonstrating the shape of a curve with gestures.

Videoing lessons, however, can be intrusive and may distort the behaviour of both the teacher and the pupils. It is not possible to eliminate the effect of the presence of the video or the researcher on the participants, though some steps were taken to minimise this effect. The videoing of lessons as a professional development tool is becoming more commonplace in schools, and two of the classes in this study had been videoed on several occasions before with their teacher, so in theory the presence of another adult with a video camera was less of a novelty. The choice of around six lessons with each teacher over a period of time was also an attempt for my presence and the camera to become more familiar and less noticeable to the students. In the pilot study, the recorded lessons were intended to be consecutive, however this was altered to weekly in the main study in an attempt to minimise the effect further. Not all the verbal and non-verbal activity within the classroom was accessible through the videos, so whilst aspects such as eye gaze, hand raising and other gestures are analytically interesting, they have not been included in the analysis as the data are insufficient to reach any meaningful conclusions. Whilst additional cameras in one of the classrooms made this data accessible, in the other three classrooms additional recording equipment may have exacerbated the observer effect (Mori and Zuengler, 2008).

Field notes were kept of each lesson observed. These served many purposes but primarily they outlined the context in which the whole class

discussion occurred. Notes of the mathematical tasks that were undertaken during the lesson were taken, including snippets of responses from pupils that occurred outside of the whole class discussion under study. Also evidence of relationships between pupils and pupils and the teacher were noted, such as which pupils appeared to be working together and which pupils the teacher worked with individually or in small groups. "since so much more is understood than is ever said, how is that observer to know what the participants are taking for granted about, or reading into, the interaction" (Edwards and Westgate, 1987, p.14).

Transcription

Transcripts need to be authentic in that they not only preserve the information needed by the researcher but that they do this in line with the nature of the original interaction, but also it needs to be useable; easy to read and adapt in response to new data (Johansson, 1995). Choices need to be made about the layout of the transcript and the descriptive categories used, such as distinguishing between long and short pauses.

The layout of a transcript can be vertical, column or partitur (a musical score style of transcription designed to visualise temporal sequencing and simultaneity between utterances of different speakers and between verbal and non-verbal behaviour). Arranging turns in a vertical manner is the most common form of transcript and is easy to use when there are multiple speakers, in contrast to the column form where a new column is required for each speaker. However, some authors argue that this format can bias the reader towards seeing the speakers as having equal roles in the discussion

(Ochs, *et al.*, 1979). The column format does allow simultaneous speaking to be more clearly displayed by aligning them horizontally. The partitur format is more complex and is ideally suited to short interactions where there are many simultaneous utterances and addresses many of the disadvantages of the vertical and column system, such as emphasising turn-taking whilst preserving time. Yet it is very difficult to construct transcripts of this type without specialist computer software. The position of researcher comments also needs to be considered. They could be included within the utterances and are usually distinguished from the data by placing them in brackets. Alternatively, they could be placed on a separate line. The use of formatting options, such as bold, italic, and underlined can be used to give something visual prominence. It is also important that the reader can easily distinguish between the spoken words, researcher comments and the codes used.

The most well-known transcription system was developed by Jefferson (2004) and was designed with the intention of capturing speech as it is heard by the participants in such a way as any claims made about the data could also be checked by other researchers. The level of detail incorporated in Jefferson's transcription system has increased as researchers' needs have developed. For example, there are now detailed categories for the transcribing of laughter. This detail is not intended as a means to classify the semantics of utterances, but rather it is needed to help CA researchers identify the ways in which participants construct and constitute the rules of interaction (Mey, 1993). The level of detail in Jefferson's transcription system is an indication of the CA assumption that no data are irrelevant.

Hence, CA transcripts include details such as false starts to words, pauses both short and longer, and in and out breaths which all might influence the interpretation of an interaction (Wooffitt, 2005).

Transcription became easier as I became familiar with the class as I was able to recognise voices and identify pupils accordingly. My field notes proved invaluable in this identification process, particularly in the early stages of the data collection. At each iteration of the transcriptions, I had to make decisions about structure of sentences, place of emphasis, information that needed to be included or not, and so forth. Originally transcriptions were made to be as literal as possible as it was felt that in the initial stages the data needed to be as complete as possible. This included detailed transcriptions of pauses and overlaps in speech, rising and falling intonation and quieter or louder speech. In the early phases of the analysis, role identities and names or pseudonyms were not included as these can convey information about the participants that may or may not be salient. In later iterations, the roles of teacher and pupil were added as the participants were clearly orienting to these roles in their interactions.

However, this level of detail can make the transcripts difficult to follow and would have involved considerable effort from a reader to make sense of the text. During the process of data analysis, these transcriptions were reworked to make the data more accessible whilst still referring to the original audio and video recordings to check they remained an accurate representation of the data. Some features were also removed to preserve the anonymity of the participants. It is these re-worked transcripts that are

included in this thesis. Details of the transcription notation used in this study can be found in Appendix A and an example of the fuller version of the transcription can be found in the extract from Edward's lesson in Appendix B.

Data analysis methods

In this study, a conversation analysis approach is taken. The key assumption underlying this approach is that discussion in mathematics classroom is ordered and the challenge in this study is to discover, describe, and analyse this order. However, traditional conversation analysis approaches place no emphasis on the nature of the participants and the context in which a discussion takes place. In this study, it is recognised that the language in classrooms reflects wider influences, in particular beliefs about the nature of mathematics, about teaching and about learning, and are therefore inseparable. Teachers and pupils draw on their personal background knowledge, respond to the constraints of particular types of discourse at various stages in the lesson and they regularly reinterpret the meaning of what was said in the light of what was then said after it, or make provisional interpretations while waiting for further 'evidence' (Edwards and Westgate, 1987). The analysis of the data is not based on these influences, but does acknowledge them where they are apparent in the discussion. Interactions are constructed both through the participants' interpretation of many factors not easily accessible to an outsider, and in ways which are influenced by the structure of the discourse itself.

One challenge to adopting a conversation analysis approach is that it requires naturally occurring data, This primarily means that the discussion is

not occurring specifically for the purpose of analysis. This naturally occurring data can occur in structured settings, such as courtrooms (Drew, 1992) and classrooms (Seedhouse, 2004), and naturalness refers to the presence of the recorder or observer not influencing the interaction (Taylor, 2001). Yet the presence of the researcher will always, however unintentionally, affect the content and structure of the discussion. This was apparent in many of the video recordings used in this study as both the pupils and the teachers referred to the video camera at some point in most lessons, either directly talking about its presence or by the pupils 'acting' in front of it when the teacher was not present in the classroom.

The pilot study was a very rich source of data, and repeated viewing of the video and reading of the accompanying transcript led to the development of a coding system focussing on the function of each contribution. This began with a simple structure based on the IRF sequence, was the teacher's utterance a question, a statement, or some form of feedback to a pupil's response? Likewise, were the pupils offering a response to a teacher's question, asking their own question, or offering an explanation? Initially I was also interested in the relationships between pupils and between the pupils and the teacher as evidenced in the discussion. However, the extracts chosen for analysis because of their mathematical nature, offered little evidence of these interactions. This initial process of coding the data was then extended using a systemic functional linguistic approach adapted by Kumpulainen and Wray (2002) to include more social aspects of the interactions. However, this analysis not only had the difficulties commonly associated with categorisation, such as utterances serving multiple functions

or where to place utterances that did not quite 'fit' the categories, but also the results only gave an indication of what participants were doing in the lessons and now how they were doing this. Whilst this is an interesting research area in its own right, it was how the whole class interactions were constructed such that conversations about mathematics were successfully carried out that intrigued me and this method of coding was discarded.

Following the collection of further data, a more open-minded approach to the analysis of the data was taken. This is a common strategy in many conversation analysis studies. The analysis was iterative, involving the repeated watching of videos, listening to audio recordings and reading transcripts. At each point, I was looking for patterns in the data, but only a vague awareness developed through my reading of the literature, of what these patterns might be. In the earlier stages, several features of the interactions were interesting and the scope of this study started to expand considerably as I noted a number of patterns and interesting features. It was necessary to focus on some of these patterns and features, and ignore others, though these remain for further exploration later. These patterns and features are discussed in more depth in chapters 8, 9 and 10.

Presentation of findings

Choices also needed to be made about how the analysis of the data is presented in this study. The inductive nature of the conversation analysis finally adopted leads to the inclusion of detailed transcripts in the presentation of any findings. However, the limits on space placed by the awarding institution, and later journal editors, result in a careful consideration

of what extracts need to be included. Many authors circumvent these restrictions by including online databases of their transcriptions; however, the consent gained for the collection of the video recordings at the beginning of this study led to ethical dilemmas in the extensive publication of data in such a publically accessible way.

Any choices I make about which data are included in the presentation of findings again influences the interpretation of these data. The presentation of the data in full, as is common in CA studies, is not possible within the thesis space restrictions, yet this would enable the reader to see how the data was interpreted and acts as a form of reliability in that the reader can check any conclusions I make. By including short extracts with a beginning and an end, to illustrate points I am making, I am conveying significance on that particular extract and reducing the possibility of demonstrating that the pattern or feature occurs in the rest of the data. Consequently, I have chosen a combination of longer transcripts which are presented in Chapter 7 and are referred to in subsequent chapters. These transcripts begin and end with boundary exchanges (Coulthard, 1992) which mark the beginning and end of a topic of discussion. In one of the transcripts, taken from Richard's lessons, there is an earlier boundary exchange where Richard has changed the topic but one of his pupils later returns to the first topic that occurs in this particular interaction and the transcript ends when this second discussion of the topic ends. Additionally, there are short extracts in each of the analysis and discussion chapters that are not included in the longer transcripts and serve to illustrate the points I am making.

The lines in the transcripts presented in chapter 7 are numbered consecutively to enable them to be easily and uniquely referenced in the analysis. The turn numbers are also retained so that the reader is aware of the position of the extract in the overall lesson and the relative position in relation to the shorter extracts used in the discussion chapters.

Initially, analysis was conducted lesson-by–lesson and teacher-by-teacher, which might naturally lead to the presentation of findings on a teacher-byteacher basis. However, the conversation analytic approach of identifying rules that apply consistently in a range of classrooms made the theme-based presentation more appropriate. Therefore, in the chapters that follow, data from each teacher are included in the presentation of general rules that structure whole-class interactions in secondary mathematics classrooms. Towards the end of each chapter, illustrations of 'deviant cases' are given and only the extracts where these 'deviant cases' occur are included, which often involves only one of the teachers due to the rarity of these events.

Reliability and Validity

The issues surrounding the reliability and validity of qualitative research are widely discussed and disputed, but are rarely mentioned in conversation analytic research itself. Seedhouse (2007) explores many of the reasons for this, but many of threats to reliability and validity that other methodological approaches face do not apply to conversation analytic research. Firstly, any analysis is presented alongside the transcript of the data themselves. Whilst there is some interpretation involved in the transcription process itself, the process and conclusions of the analysis are made transparent to the reader,

and enable the reader to analyse the data themselves. Secondly, the way conversation analysis treats context means that only features that the participants themselves orient to in the interaction are used in the analysis. The analysis is not focused on the researcher's interpretations of the interactions, but the participants' interpretations, which they demonstrate through how they construct their turns at talk.

Ethical Considerations

This study followed the BERA ethical guidelines (BERA, 2004) but even within these choices needed to be made. Adolescents are in that delicate phase between being a child and becoming an adult, resulting in me making complex ethical decisions. The principle of Informed consent applies to all research involving human participants but is particularly complex when involving adolescents. On the one hand, they may not have sufficient understanding of the research, the processes used, and the implications of their participation, to give their informed consent. On the other hand, these pupils are approaching adulthood and many have a similar level of understanding to that of their parents, and possible a better understanding of the implications of the research for them because it is their classroom and their mathematics lessons, something their parents are not part of. I decided that it was important for the pupils to feel part of the research and therefore I sought their consent, but because of their vulnerable status consent from their parents was also sought. There was also the power relation between myself and the teachers (as well as the pupils) that needed to be considered. My own position as an academic and a teacher educator puts me in a powerful situation and I need to be careful about not abusing that power.

Each pupil, their parents and the teacher were given information sheets outlining the research and giving them the time to consider participation before consenting. The information sheet was designed to contain sufficient information in order to make the decision, in language that was accessible to all participants. However, there are limits to the amount of detail any researcher can offer participants but also there are limits to the amount of detail a participant may want. The consent form included different levels of consent, pupils could opt for not participating at all, appearing in the classroom videos, and being interviewed. All participants were offered the right to withdraw from the research at any time.

Confidentiality, and anonymity are extremely important in most research. They encourage objectivity, greater willingness to be honest. All names in this research have been changed. The information sheet details who will have access to the data and the ways in which confidentiality is ensured. However, the research design and the focus on interactions and not individuals mean that there should be few sensitive issues or emotional topics where participation, anonymity and confidentiality become an issue.

One participant, whilst willing to provide data for the research, expressed concerns about being identified by others and as such, I agreed to only share anonymised, transcribed data and not the video or audio recordings that I worked with. In order to further protect this participant's anonymity, I decided to only use transcriptions in the presentations of my findings so that the identity of the participant could not be identified through the process of elimination. Whilst this does restrict the amount of information available to

others interested in my research, the anonymity of participants was felt to be more important.

All whole-class interactions were transcribed using an adaptation of the Jefferson notation system (2004) which did not include rising and falling of intonation because of the differences in regional accents between the different participants. Whilst this is usually a methodological choice, in this study it is primarily an ethical decision as the regional accents and intonations would uniquely identify the teachers who participated in this study.

Summary

This first part of the thesis has set the context in which the analysis that follows occurred. Chapter 2 examined much of the existing literature in mathematics education relating to classroom interactions and many of the identified features in this literature occur in the data in this study, but the contrasting methodological approach of CA enables a different perspective on these features, and how they are locally managed by the teachers and pupils. This contrasting methodological approach is discussed and contrasted with other forms of discourse analysis in chapters 3 to 5.

Chapter 3 examines the theoretical background to a conversation analytic approach which underlies the different ways in which an ethnomethodological or CA approach considers both the collection and the analysis of data. In particular, there is a focus on analysing naturally occurring data and how the participants themselves structure their interactions in an orderly way. These theoretical underpinnings lead to

interpretation of context that is restricted to those aspects that the participants themselves orient to in the way that they structure their interactions. This notion of context is discussed in more depth in chapter 5 where the conversation analytic approach is outlined and contrasted with other discourse analytic approach.

The next part of this thesis begins with three extracts from the data set which are drawn upon extensively in the three chapters that follow. It is usual for a CA study to present the transcribed data alongside any analysis to enable the reader to see for themselves the basis upon which the analysis is made. However, restrictions on space mean that the quantity of data that can be included is very limited. The three extracts presented were chosen because of the contrasting nature of the mathematical activity that occurred in each extract, which forms the basis of chapter 10. However, this meant that extracts from only three of the four teachers was included so a fourth extract using the full Jefferson transcription is presented in the appendix but is not drawn upon in the analysis. Other extracts are included throughout the following three chapters from the wider data set to illustrate aspects of the interactions that may not appear in the three extracts in chapter 7.

Chapters 8 and 9 draw upon two of the key themes identified by ten Have (2007). Chapter 8 focuses on the structure of turn-taking in whole class discussions and the implications this may have on the teaching and learning of mathematics, while chapter 9 examines the sequential organisation of the interactions, in particular the preference organisation of both adjacency pairs and repair. Chapter 10 then builds on the findings from chapters 8 and 9 to

first examine the discursive construction of the identities of pupil and teacher and then the construction of mathematical activity in each of the extracts presented in chapter 7.

Part 2: The study

Chapter 7: Transcripts

Extract 1 taken from Tim's lesson 1.

Г <u>а</u> а .	T		
001	T13	Tim:	ok
002			(0.6)
003			your fir:st thing today, I've put a problem on the
004			board, I will have a problem on the board in about
			_
005			(0.3)
006			30 seconds, ok I want you to look at that. first
007			question is quite an easy one, the second question
008			we have to need to think about in terms of (.)
1			
009			what it actually means,
010			(1.3)
011			ok. and I want you to try your best and try and
012			understand (.) how far you can get it done, ok.
013			here is your problem. have a go at this. I've just
014			inherited twelve thousand pounds,
015			(0.4)
016			ok and being the generous man that I am I want to
017			donate (.) some of that to charity. but because
018			I'm not totally generous,
019			(1.2)
020			ok. I'm going to donate one quarter of the twelve
021			thousand pounds, then the following week I want to
021			
1			donate a quarter of that amount, following week a
023			quarter of that amount. ok. how much will I donate
024			in each of the first four weeks, the first few are
025			obviously easy. how much will you donate in total.
026			ok let's just do the first one together, in week
020			
			one how much have I donated?
028	T14		(0.8)
029	T15	в:	thre[e thousand]
030	т16	С:	[three thou]sand
031	T17		
	TT /	Tim:	three thousand pounds.
032			(3.3) ((writes on whiteboards))
033			wee:k two:, how much am I donating if I'm donating
034			a quarter of that. (.) Harry?
035	T18	Harry:	seven point s-, seven (.) point five, no seven
	110	narry.	
036			hundred and fifty
037	T19		(1.5) ((750 written on the whiteboard by Tim))
038	T20	Tim:	ok a quarter of that, seven hundred and fifty
039			pounds. ok. I want you to try and work out
1			(0.8)
040			
041			it has mant
10 4 0			the next
042			(0.5)
042 043			
043			(0.5) two weeks, and then I want you to think about
043 044			(0.5) two weeks, and then I want you to think about (0.3)
043 044 045			(0.5) two weeks, and then I want you to think about (0.3) how much are you going to end up donating in
043 044 045 046			(0.5) two weeks, and then I want you to think about (0.3) how much are you going to end up donating in total.
043 044 045			(0.5) two weeks, and then I want you to think about (0.3) how much are you going to end up donating in
043 044 045 046			(0.5) two weeks, and then I want you to think about (0.3) how much are you going to end up donating in total.
043 044 045 046 047 048			<pre>(0.5) two weeks, and then I want you to think about (0.3) how much are you going to end up donating in total. (0.9) ok. we'll talk about that more in a minute. so</pre>
043 044 045 046 047 048 049			<pre>(0.5) two weeks, and then I want you to think about (0.3) how much are you going to end up donating in total. (0.9) ok. we'll talk about that more in a minute. so (0.6)</pre>
043 044 045 046 047 048 049 050			<pre>(0.5) two weeks, and then I want you to think about (0.3) how much are you going to end up donating in total. (0.9) ok. we'll talk about that more in a minute. so (0.6) give you two minutes, how much are you going to</pre>
043 044 045 046 047 048 049 050 051			<pre>(0.5) two weeks, and then I want you to think about (0.3) how much are you going to end up donating in total. (0.9) ok. we'll talk about that more in a minute. so (0.6) give you two minutes, how much are you going to donate in the first four weeks, you've got two</pre>
043 044 045 046 047 048 049 050			<pre>(0.5) two weeks, and then I want you to think about (0.3) how much are you going to end up donating in total. (0.9) ok. we'll talk about that more in a minute. so (0.6) give you two minutes, how much are you going to</pre>
043 044 045 046 047 048 049 050 051 052			<pre>(0.5) two weeks, and then I want you to think about (0.3) how much are you going to end up donating in total. (0.9) ok. we'll talk about that more in a minute. so (0.6) give you two minutes, how much are you going to donate in the first four weeks, you've got two more to work out. talk amongst yourselves, how</pre>
043 044 045 046 047 048 049 050 051 052 053			<pre>(0.5) two weeks, and then I want you to think about (0.3) how much are you going to end up donating in total. (0.9) ok. we'll talk about that more in a minute. so (0.6) give you two minutes, how much are you going to donate in the first four weeks, you've got two more to work out. talk amongst yourselves, how much am I going to donate in to:tal.</pre>
043 044 045 046 047 048 049 050 051 052 053 054			<pre>(0.5) two weeks, and then I want you to think about (0.3) how much are you going to end up donating in total. (0.9) ok. we'll talk about that more in a minute. so (0.6) give you two minutes, how much are you going to donate in the first four weeks, you've got two more to work out. talk amongst yourselves, how much am I going to donate in to:tal. (0.7)</pre>
043 044 045 046 047 048 049 050 051 052 053			<pre>(0.5) two weeks, and then I want you to think about (0.3) how much are you going to end up donating in total. (0.9) ok. we'll talk about that more in a minute. so (0.6) give you two minutes, how much are you going to donate in the first four weeks, you've got two more to work out. talk amongst yourselves, how much am I going to donate in to:tal. (0.7) off you go</pre>
043 044 045 046 047 048 049 050 051 052 053 054	T21-'	531	<pre>(0.5) two weeks, and then I want you to think about (0.3) how much are you going to end up donating in total. (0.9) ok. we'll talk about that more in a minute. so (0.6) give you two minutes, how much are you going to donate in the first four weeks, you've got two more to work out. talk amongst yourselves, how much am I going to donate in to:tal. (0.7)</pre>
043 044 045 046 047 048 049 050 051 052 053 054	T21-:	F31	<pre>(0.5) two weeks, and then I want you to think about (0.3) how much are you going to end up donating in total. (0.9) ok. we'll talk about that more in a minute. so (0.6) give you two minutes, how much are you going to donate in the first four weeks, you've got two more to work out. talk amongst yourselves, how much am I going to donate in to:tal. (0.7) off you go</pre>

056	Т32	Tim:	okay. stop what you're doing.
057			(3.2)
058			some of you used calculators some of you didn't.
059			okay that's good. I don't mind either way.
060			(1.7)
061			I want you thinking about it. okay. the values you
062 063			got for the first three weeks were three thousand, (1.0)
063			seven hundred and fifty, (.) one eighty seven
065			fifty and forty six eighty eight?
066			[yep,]if you round it.
067	Т33	D:	[forty seven]
068	Т34	Tim:	ok
069			(0.7)
070			what I was wanting to think about is what
071			(0.3)
072			is actually happening. some of us talked about
073 074	mor	E:	when do you s:top, do you stop.
074		L: Tim:	nope why not. hands up.
076	130	± ±111 •	(1.7)
077			why not. Jamie?
078	Т37	Jamie:	because the number: (.) keeps getting smaller,
079			cause of ((inaudible))
080	Т38	Tim:	so it keeps getting smaller.
081		Jamie:	уер
082		Tim:	but will there be a point where we actually s:top?
083		PP:	yes/yes
084		Tim:	why
085		F:	because [it will get to]zero
086		G:	[you'll run out of money]
087		Tim:	because you would have run out of money?
088 089		H: I:	if you put a penny you'd run out of the twelve thousand
089		⊥. Tim:	will you?
091		H:	[((inaudible))]
092		Ι:	[yeah if] you keep on going then
093		Tim:	I'm only ever given a qua:rter. if I think about
094			my first week, I'm only giving three thousand
095			pounds, quite a lot (.) left over and I'm only
096			ever giving a small amount so will I actually run
097			out, will I actually give away the whole twelve
098			thousand
	T52	J:	yeah
	T53	PP:	[yeah]
	T54 T55	PP: Tim:	[no] weah ((nominating next pupil to speak))
i	T56	K:	yeah ((nominating next pupil to speak)) are we actually going (less than one p)
	T57	Tim:	less than 1p, so realistically, in terms of
105	1 U I		realism, we would have to stop because we wouldn't
106			be able to pay. ok. because we wouldn't have any
107			way of paying.
108			((Tim brings up picture of triangle on the board))
109			this was on the corner of the board.
110			(2.1)
111			ok.
112 113			(0.8) this was on the corner of the board because this
$113 \\ 114$			is actually a useful way
$114 \\ 115$			(0.3)
<u> </u>	.1	<u>i</u>	

	,		· · · · · · · · · · · · · · · · · · ·
116			of you actually looking at it. imagine that's my
117			money,
118			(1.7)
119			ok. a quarter of that I'm going to throw away and
120			donate. this is my quarter.
121			(2.1)((shading in middle triangle))
122			I've just given tha:t
123			(0.8)
124			away
125	T58	L:	why
126			(0.9)
÷		m	
127		Tim:	that's my three thousand p[ounds]
128	Т61	L:	[oh]
129	Т62		(0.7)
130	т63	Tim:	ok this is my (.) three thou:sand pounds
131			((writes £3000 in the middle triangle))
132			ok. each one of those triangles is three thousand
133			pounds isn't it.
134		L:	yeah
135	Т65	Tim:	yeh? so:: (.) are you happy then that to give away
136			a quarter of that amount
137			(0.3)
138			would be the same as me doing
139			-
1			(1.4) ((draw another triangle in the centre of the
140			top triangle))
141			that!
142			(0.9) ((shading it in))
143			giving away that.
144	Т66	PP:	yeah
145		Tim:	
	10/	1 1111 •	because I've given away another quarter of three
146			thousand. (0.5) yeah?
147	T68	PP:	°yeah°
148	Т69	Tim:	if I did that agai::n,
149			(4.1)
150			((draws another triangle above the second one and
151			shades it))
-			
152			and again
153			(0.4)
154			and again
155			(0.4)
156			and again, ok. if I zoomed in and zoomed in I
157			could keep drawing little triangles couldn't I,
158			[yeah?]
	m70		
159		PP:	[°yeah]yeah°
160	т71	Tim:	but what fraction,
161			(1.3)
162		1	what fraction of that triangle have I shaded.
163			(2.6)
164			what fraction of that triangle have I actually
165			sha:ded. Jamie?
		Tomi -	
166	Т72	Jamie	um
167			(0.6)
168			is it a half.
169	T73		(0.6)
170		Tim:	have I shaded a half?
171		•	
			(1.0)
172		Jamie	no
173	T77	Tim:	I haven't shaded a ha:lf.
174	1		(1.3) C[hris]
1/4			
175	т78	Chris	[a quar]ter

176	T79		(1.4)
	Т80	Tim:	I'm shading a quarter each time, but I'm shading a
178			quarter of a quarter, quarter of a quarter so
179			(0.4)
180			it's not going to be a quarter exactly. look at
181			it. look at it in rows.
182	Т81	M:	°six thousand pounds°
183	Т82	Tim:	look a rows of same triangles.
184	4	M:	a third
	Т84	Tim:	good.
186			(0.8)
187			ok. if I look at those, that row I've shaded a
188			third, that row I've shaded a third, that row I've
189			shaded a third, that row I've shaded a third that
190 191			row and from then on it is always shared, I'm
191			actually sharing, (0.3)
193			shading in a third. so in the end, how much am I
194			actually going to give away?
195	т85	PP:	a third
196		<u>+ + •</u>	(0.8)
197		Tim:	so how much is that
198			(1.7)
199		N :	four thousa[nd pounds]
200		Tim:	[four thou]sand pounds.[ok?]
201		0:	[oh yeah]
202	<u>.</u>	Tim:	some of you
203			(0.6)
204			were working it out
205			(0.5)
206			and hopefully if you were working it out properly,
207			were you getting closer and closer to four
208			thousand pounds?
209		Q:	no
210		R:	u:m yeah
211		S:	no
212		R:	yes?
213	T97	Tim:	well
214			(0.6)
215			Ashley here got to three thousand nine hundred and
216 217	m 0.0		ninety eight pounds seventy eight.
	198 T99	Tim:	((laughter from a few pupils)) and obviously there as he keeps going he adds a
210 219	1 2 2	± ±11(•	smaller and smaller bit on,
220			(1.6)
221			it's going to get closer and closer to that. yeah?
222			ok. this actually links into what we're going to
223			talk about, a limit of a sequence. this sequence,
224			when we added it up actually reached a limit it
225			reached a limit of four thousand ok. it's not
226			going to go any higher than four thousand because
227			that's what we're working out and that's what
228			we're talking about today, limits
229			(0.3)
230			of sequence. ok yesterday all our sequences has
231			nice, either nice easy rules, add four add five,
232			or quadratic rule, ok. what I want to look at is
233			something slightly different toda:y, ok. for my
234 235			rul:e
		1	(0.2)

236	for this, ok. I am going to divide my number by
237	five and then add
238	(0.3)
239	four. ok

Extract 2 taken from Simon's lesson 1.

240T67Simon:okay part two:. do you throw that little bit of paper I gave you yesterday with the table on and we filled in one side.243(1.2)244I'm going to a:sk you today to do some practice this and before we do that245I'm going to a:sk you today to do some practice this and before we do that246(0.4)247I just want to go through another example just remind everyone (0.3)250of um (1.5)251(1.5)252of how it's done. so can we just li-, the other side that we haven't filled in.253it's this one here you should have one, ocops, it's this one here you should have one, ocops, should have one that looks a little bit like th (0.8)258ok?259(1.1) now I'm going to be honest with you, I was talk to um261I was talking to Mrs Smith the other, yesterday and she thinks I'm being much too nice to you will did this table. do you know why.266Charlie:267Charlie:270T69271=272what instead of, ye- ye:s! because I gave you that extra column there. ((teacher points to the third column on the projected table)) because	e on and ractice on e just to e other ooops, you like this. as talking et and sterday o you when
242we filled in one side.243(1.2)244I'm going to a:sk you today to do some practice245this and before we do that246(0.4)247I just want to go through another example just248remind everyone249(0.3)250of um251(1.5)252of how it's done. so can we just li-, the other253side that we haven't filled in.254(7.2)255it's this one here you should have one, ooops, sould have one that looks a little bit like th258ok?259(1.1)260now I'm going to be honest with you, I was talk261to um262(1.8) ((teacher pulls down projector sheet and263this table. do you know why.264I did this table. do you know why.265charlie.266T68267(0.9)268charlie.270T69271what instead of, ye- ye:s! because I gave you271that extra column there.272((teacher points to the third column on the projected table))274because	ractice on e just to e other ooops, you like this. as talking et and sterday o you when
243(1.2)244I'm going to a:sk you today to do some practice245this and before we do that246(0.4)247I just want to go through another example just248remind everyone249(0.3)250of um251(1.5)252of how it's done. so can we just li-, the other253side that we haven't filled in.254(7.2)255it's this one here you should have one, coops, j256should have one that looks a little bit like th25(0.8)258ok?260now I'm going to be honest with you, I was talk261to um262(1.8) ((teacher pulls down projector sheet and263then up again))264I was talking to Mrs Smith the other, yesterday265and she thinks I'm being much too nice to you wi266(6.9)267(0.9)268charlie.269T68270T69271wat instead of, ye- ye:s! because I gave you271that extra column there.272(teacher points to the third column on the273projected table))274because	e just to e other ooops, you like this. as talking et and sterday o you when
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262 263(1.8) ((teacher pulls down projector sheet and then up again))264 265 265 266 266 267 268I was talking to Mrs Smith the other, yesterday and she thinks I'm being much too nice to you with I did this table. do you know why. (0.9) Charlie.269 268T68 Charlie:'cause you gave us the extra column?=270 271 272 273T69 274Simon:274= what instead of, ye- ye:s! because I gave you that extra column there. ((teacher points to the third column on the projected table)) because	sterday o you when
263then up again))264I was talking to Mrs Smith the other, yesterday265and she thinks I'm being much too nice to you will266I did this table. do you know why.267(0.9)268Charlie.269T68270T69271Simon:272what instead of, ye- ye:s! because I gave you273projected table))274because	sterday o you when
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<pre>270 T69 Simon: =what instead of, ye- ye:s! because I gave you 271 272 that extra column there. 273 ((teacher points to the third column on the projected table)) 274 because</pre>	
271that extra column there.272((teacher points to the third column on the273projected table))274because	
<pre>272 ((teacher points to the third column on the 273 projected table)) 274 because</pre>	ve you
273 projected table)) 274 because	_
274 because	the
275 (0.4)	
276 sometimes in the exam they won't give you that	11 that
	;
277 extra column they'll just give you these two, as	two, and
278 they'll expect you to know (.) that it might be	
279 useful (.) to put this extra column on, do you	
280 know what I mean. and in a minute, when you do	do you
281 some practice from the text book it's the same	do you
282 thing. they just give you this bit of the table	do you you do
and they expect you to use your initiative (.)	do you you do e same
	do you you do e same e table
284 draw in the extra column to do it. ok. well let	do you you do e same e table ve (.) to
go through these then, the mo:de, the median, the	do you you do e same e table ve (.) to ell let's
286 <u>mean and the range. I think we'll leave the mean</u>	do you you do e same e table ve (.) to ell let's dian, the
287 till last because it's a bit like the mean one.	do you you do e same e table ve (.) to ell let's dian, the the mean
Alex and (.) Chris, paying attention now	do you you do e same e table ve (.) to ell let's dian, the the mean
289 specially, right any offers anyone for telling i	do you you do e same e table ve (.) to ell let's dian, the the mean an one. um
	do you you do e same e table ve (.) to ell let's dian, the the mean <u>an one. um</u>
290 what, m-why of course we always want to know wh	do you you do e same e table ve (.) to ell let's dian, the the mean <u>an one. um</u> elling me
(.) what the \underline{mode} , the $\underline{me}dian$ the mean and the	do you you do e same e table ve (.) to ell let's dian, the the mean an one. um elling me know why
292 <u>ra</u> nge are.	do you you do e same e table ve (.) to ell let's dian, the the mean an one. um elling me know why
293 (1.7)	do you you do e same e table ve (.) to ell let's dian, the the mean an one. um elling me know why
	do you you do e same e table ve (.) to ell let's dian, the the mean an one. um elling me know why
	do you you do e same e table ve (.) to ell let's dian, the the mean <u>an one. um</u> elling me know <u>why</u> nd the
294 and e- I'm especially interested in people	do you you do e same e table ve (.) to ell let's dian, the the mean an one. um elling me know why nd the le
294and e- I'm especially interested in people295answering who haven't answered who haven't said	do you you do e same e table ve (.) to ell let's dian, the the mean <u>an one. um</u> elling me know <u>why</u> nd the le 't said
294and e- I'm especially interested in people295answering who haven't answered who haven't said296anything in class (.) you know for the last, la	do you you do e same e table ve (.) to ell let's dian, the the mean <u>an one. um</u> elling me know <u>why</u> nd the le 't said ast, last
294and e- I'm especially interested in people295answering who haven't answered who haven't said	do you you do e same e table ve (.) to ell let's dian, the the mean <u>an</u> one. um elling me know <u>why</u> nd the le 't said ast, last a bit

299	T	1	clearly have no understanding. some other people.
300			George?
	T70	<u> </u>	er um days absent three is the mode because it's
302	170	George:	the most common one.
303	т71	Simon:	right. the mode, so these are all days absent
304			there's some people won't never had a day absent,
305			some people have one day, some people had six
306			days, some people had seven days. the most common
307			number of days to have like absent the mode is
308			three because a hundred and twenty five people had
309			three days off. that beats any o- any other sort
310			of number of days off, so the mode is three. good
311			choice of where to start, well done. um go on then
312			Charlie
	m 70		
1	T72	Charlie:	is the range a hundred and seven- seventeen
:	T73	Simon:	range a hundred and seventeen. the <u>ran</u> ge is the
315			biggest number take away the smallest number. the
316			biggest number is a hundred and twenty five, the
317			smallest number is eight, a hundred and twenty
318			five take away eight. Drew.
319	Т74	Drew:	no because the (.) the range is going to be in
320			days absent so it'll be eight.
321	т75	Simon:	ah. remember Charlie. this table does not have any
322			numbers a hundred and twenty five in there. this
323			table only consists of days absent from zero (.)
324			up to eight. do you see that. you're saying the
325			most number of days people had absent is a hundred
326			and twenty five days but no one had a hundred and
327			twenty five days absence. what was the highest
328			number of days absence.
329	т76	A:	eight
330		Simon:	it was eight. and what was the lowest number of
331	± ′ ′	01110111	days absent.
332	T78	В:	zero or one I don't know
	<u>т</u> 79	Simon:	you don't know. ok someone else then, what's the
334	115	0111011.	lowest number of days pe- someone was absent.
335			George.
	т80		
		ר ר -	(2.5)
337		Alex:	
	Т82	~	(0.9)
	Т83	George:	zero.
340	Т84	Simon:	that was Alex talking I want to hear it from you.
341			look at the table, what was the lowest number of
342			days that someone had absent.
343	Т85	George:	zero.
344	Т86	Simon:	it is zero, because ↑twenty people had <u>no</u> days
345			off. the \uparrow highest number is eight, the lowest
346			number is zero, the median, the mo- the range
347			sorry is eight. ok um
348			(1.8)
349			remember I said to you yesterday about people
350			making that mistake of that doing
351			(0.3)
352			that take away that, you've got to make sure it's
353			not you (.) doing that. Kieran hopefully you will
354			remember that and not do that again. ok. um go on
355			then
:	1	1	
1			(0 5)
356			(0.5) Ashley
356 357	Т87	Ashley:	(0.5) Ashley. is the median um ta- add up all the

359	Ĩ		(0.3)
360			frequencies so [that] adds up to five hundred
361	T88	Simon:	[ok] have you done that alr[eady]
362	T89	Ashley:	[um]
363		-	ok yeah
364	Т90	Simon:	ok Ashley sounds about right doesn't it, it sounds
365			like too nice a number to turn down. she's added
366			up all these numbers, and it adds up to five
367			hundred. yeah
368			(0.5)
369			check that if you want to. remember sometimes in
370			the question they tell you at the start, you know
371			(.) five hundred children were surveyed or
372			something like that. ok . um
373			(0.5)
374			five hundred go on then Ashley.
375	Т91	Ashley:	and then find the (di.) middle, is that two
376			hundred and fifty?
377	Т92	Simon:	u::m what's the <u>mi</u> ddle number out of five
378			[hundred]
1	Т93	Ashley:	[is] it two hundred and fifty
380			[and two hundred and fifty one]
381	Т94	Simon:	[it's] an even number
382			[is]n't it
	Т95	Ashley:	[yeah]
	Т96	Simon:	so you do that trick,
385			(0.4)
386			five hundred plus one is five hundred and one,
387			halve it, it's two hundred and fifty
388			(0.6)
389 390			and a half ((writing calculation on the whiteboard
	Т97	F:	as it is spoken)) three
392	T98	Simon:	so we are looking for
393	1 20	5111011.	(0.8)
394			the two hundred and fiftieth and two hundred and
395			fifty first (.) person.
	Т99	Ashley:	and then
L		Simon:	it would make life easier if they were in the same
398			same band let's hope so go on then.
	T101	Ashley:	and then add up like
400		- 1 -	(0.4)
401			twenty, fifty five, sixty and a hundred and twenty
402			five 'cause that sort of comes up to about two
403			hundred and fifty when you add it it comes up to
404			two hundred and sixty so that means
405			(0.3)
406			the number falls in the ((inaudible))
1	T102	Simon:	so Ashley's worked out it's in the number fours.
408			Harry this is what we're doing ok. we we've we
409			know this represents fi- the threes you say
		Pupils:	yeah
1	T104	Simon:	we know this represents five hundred people, you
412			know the <u>me</u> dian is the middle person,
413			(0.6)
414			we're putting all these numbers fall these zeros
415			ones and twos and threes (.) in order up to eight,
416			there's five hundred of them
417			(0.4)
418		<u> </u>	and we want to know where is the middle person.

110	1	1	
419			the first twenty people (.) were absent for zero.
420			doing a running total here,
421			(0.3)
422			the first seventy five people were zero or one.
423			adding on that sixty, the first a hundred and
424			five, hundred and thirty five people,
425			(0.8)
426			were zero one or two. adding on a hundred and
427			twenty five that's two hundred and sixty, I can
428			see where you go that number from now,
429			(0.5)
430			two up to two hundred and sixty people
431			(0.3)
432			it goes up to zero, one two or three. so the
433			question now is what band is the two hundred and
434			fiftieth person in, it ↑just about creeps in (.)
435			at the end of that band there. ok.
436			(0.8)
430			does that make sense everyone?
	m1 0 F		
(T105		(0.8)
	T106	H:	yeah
440	T107	Simon:	three.
441			(0.9)
442			the median is three. the mode is three, the median
443			is three. um:
444			(1.1)
445			someone else then, what about the mean. that's the
446			
			last one, the tricky one, have you got something
447			to say Chris
		Chris:	I was going to say mean
449	T109	Simon:	ok go on then, go on
450	T110	Chris:	right er you have to, er you have to do the the
451			days absent times the frequency (.) part now so
452			it's zero times twenty is zero
453	TT111	Simon:	okay shall we just have (.) thirty seconds of
454			everyone doing that then. w- (.) we know that
455			twenty people were absent for no days, fifty five
456			
1			people were absent for <u>one</u> day, sixty people
457			absent for two days. we want to add up (.) all the
458			days that people were absent. I'll do that and you
459			do that and we'll see if we agree.
460			(3.2)
461			yesterday quite a lot of people thought that
462			nought times twenty was twenty so let's see if
463			that's the same today.
	T112	-	((writes on the board, mumbling arithmetic))
i		Simon:	this isn't as nice as the one yesterday because
465	1 1 1 1	S 111011 .	the numbers are a bit
1			
467			(0.8)
468			bigger
469			(0.5)
470			and there's more to add up I think. six times
471			thirty two u::m
472			(2.4)
473			yep seven times twenty seven,
	T114	Ι:	a hundred and eighty nine
		Simon:	a hundred and eighty nine. eight times eight
		J:	sixty four
1	T117	Simon:	sixty four, thank you. can someone add up all
478			those numbers there, have you done it, have you

479	1		got it.
480			(2.5)
481			
-			I ca- I ↑really can't believe how many people are
482			sitting there without a calculator. um I just find
483			it amazing. Alex
	T118		one seven six oh
485	T119	Simon:	one (.) seven (.) six (.) oh. ((teacher is writing
486			the digits as he says them))
487			(0.6)
488			ok.
489			(3.7)
490			I'm just gon- I'm just waiting ten twenty seconds
491			for people to catch up with that.
492			(6.7)
493			Drew
	T120	Drew:	um (.) now do you (.) divi:de um (.) one one
495		210	thousand seven hundred and sixty by five hundred?
	T121		(1.4)
į		Simon:	let's ask um
497		5111011:	
			(0.3)
499			Charlie in the corner. what does that number there
500		~	represent, this five hundred.
501	Т123	Charlie:	er::m how many (.) times,
502			(0.9)
503			um how people there was
	T124	Simon:	good how many people were surveyed. George. what
505			does that <u>one</u> thousand seven hundred and sixty
506			represent.
507	T125	George:	um the total (um number of days off)
508	T126	Simon:	if you add up everyone's days of absence it will
509			add up to one thousand seven hundred and sixty, so
510			
			as <u>Harry</u> said, what we're going to do now is one
510 511			as <u>Harry</u> said, what we're going to do now is one thousand seven hundred and sixty, divided by five
510 511 512			as <u>Harry</u> said, what we're going to do now is one thousand seven hundred and sixty, divided by five hundred it's going to give you
510 511 512 513			as <u>Harry</u> said, what we're going to do now is one thousand seven hundred and sixty, divided by five hundred it's going to give you (0.7)
510 511 512 513 514	π127	Harry:	as <u>Harry</u> said, what we're going to do now is one thousand seven hundred and sixty, divided by five hundred it's going to give you (0.7) what is it Harry
510 511 512 513 514 515		Harry:	as <u>Harry</u> said, what we're going to do now is one thousand seven hundred and sixty, divided by five hundred it's going to give you (0.7) what is it Harry three point five two?
510 511 512 513 514 515 516		Harry: Simon:	as <u>Harry</u> said, what we're going to do now is one thousand seven hundred and sixty, divided by five hundred it's going to give you (0.7) what is it Harry three point five two? three point five I'm going to call that. three (.)
510 511 512 513 514 515 516 517			as <u>Harry</u> said, what we're going to do now is one thousand seven hundred and sixty, divided by five hundred it's going to give you (0.7) what is it Harry three point five two? three point five I'm going to call that. three (.) point five. so the <u>average</u> , the <u>mean</u> average
510 511 512 513 514 515 516 517 518			as <u>Harry</u> said, what we're going to do now is one thousand seven hundred and sixty, divided by five hundred it's going to give you (0.7) what is it Harry three point five two? three point five I'm going to call that. three (.) point five. so the <u>average</u> , the <u>mean average</u> number of days absent (.) is about three and a
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510 511 512 513 514 515 516 517 518 519 520			as <u>Harry</u> said, what we're going to do now is one thousand seven hundred and sixty, divided by five hundred it's going to give you (0.7) what is it Harry three point five two? three point five I'm going to call that. three (.) point five. so the <u>average</u> , the <u>mean</u> average number of days absent (.) is about three and a half. um (0.5)
510 511 512 513 514 515 516 517 518 519 520 521			as <u>Harry</u> said, what we're going to do now is one thousand seven hundred and sixty, divided by five hundred it's going to give you (0.7) what is it Harry three point five two? three point five I'm going to call that. three (.) point five. so the <u>average</u> , the <u>mean</u> average number of days absent (.) is about three and a half. um (0.5) <u>ye</u> sterday someone did, um one or two people made a
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540	questions 2, 3, and 4 and then question 6, if you
541	do those, I can come over and mark them together
542	with you, and then you can have a go at these
543	slightly different questions 8 and 9. ok? where
544	are the um textbooks?

Extract 3 taken from Richard's lesson 3.

545T2Richard:right. um good afternoon. I think we'l (2.7)546(2.7)547um thank you very much for your effort on the t (.) total task I thought you extremely well. um and we sort of ran were beginning to talk about people's on. um we had some good thoughts from groups but um we di -didn't hear from people. some people worked on similar people worked on slightly different th we'll perhaps hear back from some othe	s yesterday did of time and ideas and so a couple of a lot of
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553people. some people worked on similar554people worked on slightly different th	
553people. some people worked on similar554people worked on slightly different th	
554 people worked on slightly different th	things, some
we it perhaps heat back from some othe	-
556 a minute. but I wanted to start by ask	
-	ing you a
557 question.	
558 (0.4)	
559 um	
560 (0.3)	
561 what do you understand by the idea of	(.) proof.
562 mathematical proof. p r double o f.	-
563 (0.9)	
564 what do you understand by that (.) con	cept. that
565 idea. maybe say one thing about it (.)	
somebody else say something else. um::	nands going
567 up. Alex.	
568 T3 Alex: you can't prove anything apart from ma	ths because
569 it's all point of view.	
570 T4 Richard: oh I see	
571 (0.7)	
572 um: can you give an example or somethi	.ng
573 T5 Alex: um (there's different kinds of things)	
574 eyes might be slightly different, you	
575 (.) because it's like, (.) you see dif	
576 shades ((inaudible)) the eye could be	
577 T6 Richard: oh so when you look at your red thing	
578 somebody might (.) see it differently.	
579 T7 Alex: yeah	
580 T8 Richard: I see, whereas mathematically? what ar	e you saying
about maths that's different?	
582 T9 Alex: it's because the maths deals with abso	lute
583 (substances) like numbers, you can't b	
584 ((inaudible)) can you.	
585 T10 Richard: ah: ok that's very interesting. very g	
586 hello ((pupil enters the room)). somet	
587 or-, related to that or different to d	lo with the
588 idea of proof, um Drew.	
589 T11 Drew: um, it's not really, because normal (.) things
590 outside in the real world can be prove	
is a set a s	-
591 (0.3)	UPES IT ON !
591(0.3)592if a tornado comes through and someone	e yets it on
591(0.3)592if a tornado comes through and someone593video	e gets it on
591(0.3)592if a tornado comes through and someone593video594T12(0.8)	_
591(0.3)592if a tornado comes through and someone593video594T12595T13Richard:right, so you could prove something by	' having some
591(0.3)592if a tornado comes through and someone593video594T12595T13Sichard:right, so you could prove something by596evidence of it, like in history maybe.	' having some
591(0.3)592if a tornado comes through and someone593video594T12595T13Richard:right, so you could prove something by	v having some
591(0.3)592if a tornado comes through and someone593video594T12595T13Second Comparisonright, so you could prove something by596evidence of it, like in history maybe.597(1.2)	v having some
591(0.3)592if a tornado comes through and someone593video594T12595T13Sichard:right, so you could prove something by596evidence of it, like in history maybe.597(1.2)598thank you, Fran?	7 having some
591(0.3)592if a tornado comes through and someone593video594T12595T13Sichard:right, so you could prove something by596evidence of it, like in history maybe.597(1.2)598thank you, Fran?599T14Fran:yo- you can have like (.) a belief tha	7 having some
591(0.3)592if a tornado comes through and someone593video594T12595T13Sp6right, so you could prove something by596evidence of it, like in history maybe.597(1.2)598thank you, Fran?599T14600(0.3)	7 having some
591(0.3)592if a tornado comes through and someone593video594T12595T13Sp6right, so you could prove something by596evidence of it, like in history maybe.597(1.2)598thank you, Fran?599T14600(0.3)901you can	7 having some
591(0.3)592if a tornado comes through and someone593video594T12595T13Sp6right, so you could prove something by596evidence of it, like in history maybe.597(1.2)598thank you, Fran?599T14600(0.3)	v having some

604	1		true.
L	T15		
606		Richard:	(0.5) right so you have (.) a reason for believing it or
607	110	RICHALU:	
			a reason that could convince somebody else, yes.
608			um Jamie?
609	111/	Jamie:	you can fake um proof um
610			(0.6)
611			about things (.) but you can't fake numbers.
612	T18		(1.4)
613	T19	Richard:	you can <u>fa</u> ke <u>proo</u> f
614	Т20	Jamie:	yes
615	T21	Richard:	right, what yo- what are you thinking of=
616	T22	Jamie:	=you can't fake it with numbers [((inaudible))]
617	т23	Richard:	[uhhah]
618		Richard:	is that the same as what you are saying Alex or
			different.
619	T25	Alex:	er yeah. (.) well it's kind of ((inaudible))
620		Richard:	yes (.) it does doesn't it. um
621	1 C V	mitchatu.	(0.3)
622			um. this is not a pairwise discussion, this is a
623			whole class ((direct at two particular pupils)).
624			
624 625		Dretti	um Drew.
	127	Drew:	um you (.) can like
626			(0.4)
627			um Konstruction
628			(0.5)
629			make illusions with numbers and stuff, like with
630			the (.) <u>fir</u> st thing you did with the two.
631			(1.5)
632			um with the
634			(0.6)
635			um
636			(0.4)
637			point nine nine recurring
638	Т28	Richard:	<u>oh</u> yes
639			(1.8)
640			yeah go on, what about that?
641	Т29	Drew:	that (.) makes a whole but it's not quite a whole
642	Т30	Richard:	oh yeah, we had a big argument about that didn't
643			we, do you remember, (.) about whether point nine
644			recurring is the is same as one or not. I don't
645			think we completely
646			(0.3)
647			we weren't all in agreement at the end of that
648			were we. um yes, so that is an example of what?
649			that's an example of what I said was (difficult
650			in) maths, but it's still
651			(0.4)
652			not completely certain. is that why you mentioned
653			that?
	Т31	Drow.	
		Drew: Dichard:	yeah thank you latte have one more commont then I am
655 656	Т32	Richard:	thank you. let's have one more comment then I am
	m o o		going to (.) tighten up the t-totals a bit.
657	T33	Taylor:	you have <u>in</u> ductive and <u>de</u> ductive proof
į	Т34	Richard:	go on
	Т35	Taylor:	well one of them, I can't remember which one is
660			which, one of them is saying <u>so</u> me will (.) be
661			(right if its right <u>e</u> very other day, what every
662			they)
663	Т36	Richard:	oh r[ight]

664	TT 37	Taylor:	[and] its saying that a dog has a nose,
665	107	iayioi.	because you can see (it has a nose) ((inaudible))
666	<u>т</u> 38	Richard:	a dog has a nose=
667	T39	Taylor:	=cause a (.) e- everybody knows that a dog has a
668	133	Idy101.	nose, you can't take that away. it's absolutely
669			proof.
670	m 1 0	Richard:	ok and how does everyone know that?
671	140	RICHAIU.	(0.9)
672			because of what?
673	m / 1		
		Taylor:	because you can see it=
674	142	Richard:	=because you've seen it.
675			(0.8)
676 677			this is really interesting, this inductive,
			deductive. was anyone else come across those
678			words? where have you- where've you come across
679	m 4 0		that?
	T43	Taylor:	um, my dad
		Richard:	your dad [ok] oh right, yes.
682		Taylor:	[at home]
	Т46	Richard:	um
684			(0.7)
685			if I have understood this correctly, tell me if
686			this is not right, but I think inductive is like
687			where you build up more and more evidence for
688			something by like looking at more and more dog's
689			noses, and if they all (.) seem to have a nose
690			then you start to get more and more sure that they
691			have a nose, whereas deductive is a bit more like
692			(0.5)
693			um reasoning it through and saying a dog must have
694			a nose because of this that and the other, and
695			maybe some biological reason or something, a bit
696			more like deductive. whereas inductive is more
697			like collecting evidence. um
698			
699			that's a really helpful distinction because I was
700			going to ask about
701			(0.7)
702			um (O_O)
703			(0.8)
704 705			about what we did yesterday. um. it's kind of two
705			things there. what I'm trying to do is um (0.3)
706			this is just trying to say this is one thing this
708			is another thing. and (.) I want to know what you
708			think is the same or what's different about those
709			two.
711			(3.1)
712			I mean we did all that yesterday but I just
713			thought it'd be nice if we (.) stood back and
714			thought about
715			(0.6)
716			what it meant and w-hat's the same and what's
717			different about the right side and the left side.
718			(3.7)
719			°a hard question°. have a think.
720			(4.1)
721			you can say something that's quite obvious and
721			that's fine. I'd just like people to make
723			(0.4)
724			observations about what's the same and what's
1 4			UNDELVALIOND ANOUL WHAL D LITE DAME AND WHAL D

725	1		different.
726			(0.5)
727			((small group are laughing)) what are you laughing
728			at?
729	Т47	A:	((inaudible))
730	Т48	Richard:	come on then Drew what do you want to say.=
731	Т49	Drew:	=there's five (.) numbers
732	Т50		(1.1)
733	Т51	Richard:	there's five numbers, what here? ((pointing to the
734			left hand side of the board))
735	т52		(1.1)
736	Т53	Drew:	yeah
737	Т54	Richard:	what about here? (pointing to the right hand side)
738		Drew:	there's only four
739		PP:	class laughs
740	•••••••••••••••••••••••••••••••••••••••	B:	there's five squares, five squar[es.]
741	т58	Richard:	[fi]ve squares,
742			ok
743			(1.2)
744			um e- what did you say Lesley?
745 746	Т59	Lesley:	there's only four numbers on the one on the
	т60	Richard:	righ[t] [ri]ght. you mean that, that, that and that
747	100	RICHAIU:	((pointing to the numerical values in the
740			algebraic T)). ok, but there's five
750			(0.4)
751			five squares, is there five anything elses, what
752			names would you give to these things that are in
753			the squares ((pointing to the algebraic
754			expressions)). I'm looking for the technical term
1	1		
755			for them. C?
755 756	Т61	С:	for them. C? formulas
756	T61 T62	C: Richard:	
756 757 758			formulas formulas, you could say. formula often has equals in it, doesn't it like the formula for <u>speed</u>
756 757 758 759			formulas formulas, you could say. formula often has equals in it, doesn't it like the formula for <u>speed</u> equals distance divided by time, that I was
756 757 758 759 760			formulas formulas, you could say. formula often has equals in it, doesn't it like the formula for <u>speed</u> equals distance divided by time, that I was getting muddled about a couple of lessons ago.
756 757 758 759 760 761			formulas formulas, you could say. formula often has equals in it, doesn't it like the formula for <u>speed</u> equals distance divided by time, that I was getting muddled about a couple of lessons ago. formula often has an equals, if it doesn't have an
756 757 758 759 760 761 762			formulas formulas, you could say. formula often has equals in it, doesn't it like the formula for <u>speed</u> equals distance divided by time, that I was getting muddled about a couple of lessons ago. formula often has an equals, if it doesn't have an equals it's just
756 757 758 759 760 761 762 763			formulas formulas, you could say. formula often has equals in it, doesn't it like the formula for <u>speed</u> equals distance divided by time, that I was getting muddled about a couple of lessons ago. formula often has an equals, if it doesn't have an equals it's just (0.5)
756 757 758 759 760 761 762 763 764			formulas formulas, you could say. formula often has equals in it, doesn't it like the formula for <u>speed</u> equals distance divided by time, that I was getting muddled about a couple of lessons ago. formula often has an equals, if it doesn't have an equals it's just (0.5) like one side of a formula, um does anyone know
756 757 758 759 760 761 762 763 764 765	Т62	Richard:	formulas formulas, you could say. formula often has equals in it, doesn't it like the formula for <u>speed</u> equals distance divided by time, that I was getting muddled about a couple of lessons ago. formula often has an equals, if it doesn't have an equals it's just (0.5) like one side of a formula, um does anyone know what that's called. tha- that sort of thing D?
756 757 758 759 760 761 762 763 764 765 766	T62 T63	Richard: D:	formulas formulas, you could say. formula often has equals in it, doesn't it like the formula for <u>speed</u> equals distance divided by time, that I was getting muddled about a couple of lessons ago. formula often has an equals, if it doesn't have an equals it's just (0.5) like one side of a formula, um does anyone know what that's called. tha- that sort of thing D? five things
756 757 758 759 760 761 762 763 764 765 766 766	Т62	Richard:	formulas formulas, you could say. formula often has equals in it, doesn't it like the formula for <u>speed</u> equals distance divided by time, that I was getting muddled about a couple of lessons ago. formula often has an equals, if it doesn't have an equals it's just (0.5) like one side of a formula, um does anyone know what that's called. tha- that sort of thing D? five things well I often call them things actually, because I
756 757 758 759 760 761 762 763 764 765 766 767 768	T62 T63	Richard: D:	formulas formulas, you could say. formula often has equals in it, doesn't it like the formula for <u>speed</u> equals distance divided by time, that I was getting muddled about a couple of lessons ago. formula often has an equals, if it doesn't have an equals it's just (0.5) like one side of a formula, um does anyone know what that's called. tha- that sort of thing D? five things well I often call them things actually, because I (.) avoid the technical term
756 757 758 759 760 761 762 763 764 765 766 766	T62 T63	Richard: D:	<pre>formulas formulas, you could say. formula often has equals in it, doesn't it like the formula for <u>speed</u> equals distance divided by time, that I was getting muddled about a couple of lessons ago. formula often has an equals, if it doesn't have an equals it's just (0.5) like one side of a formula, um does anyone know what that's called. tha- that sort of thing D? five things well I often call them things actually, because I (.) avoid the technical term (0.4)</pre>
756 757 758 759 760 761 762 763 764 765 766 767 768 769 770	T62 T63 T64	Richard: D: Richard:	<pre>formulas formulas, you could say. formula often has equals in it, doesn't it like the formula for <u>speed</u> equals distance divided by time, that I was getting muddled about a couple of lessons ago. formula often has an equals, if it doesn't have an equals it's just (0.5) like one side of a formula, um does anyone know what that's called. tha- that sort of thing D? five things well I often call them things actually, because I (.) avoid the technical term (0.4) um but that's not technical enough. um (.) yes</pre>
756 757 758 759 760 761 762 763 764 765 766 767 768 769 770	T62 T63	Richard: D:	<pre>formulas formulas, you could say. formula often has equals in it, doesn't it like the formula for <u>speed</u> equals distance divided by time, that I was getting muddled about a couple of lessons ago. formula often has an equals, if it doesn't have an equals it's just (0.5) like one side of a formula, um does anyone know what that's called. tha- that sort of thing D? five things well I often call them things actually, because I (.) avoid the technical term (0.4) um but that's not technical enough. um (.) yes is it that</pre>
756 757 758 759 760 761 762 763 764 765 766 766 767 768 769 770 771	T62 T63 T64	Richard: D: Richard:	<pre>formulas formulas, you could say. formula often has equals in it, doesn't it like the formula for <u>speed</u> equals distance divided by time, that I was getting muddled about a couple of lessons ago. formula often has an equals, if it doesn't have an equals it's just (0.5) like one side of a formula, um does anyone know what that's called. tha- that sort of thing D? five things well I often call them things actually, because I (.) avoid the technical term (0.4) um but that's not technical enough. um (.) yes</pre>
756 757 758 759 760 761 762 763 764 765 766 766 767 768 769 770 771 772	T62 T63 T64	Richard: D: Richard:	<pre>formulas formulas, you could say. formula often has equals in it, doesn't it like the formula for <u>speed</u> equals distance divided by time, that I was getting muddled about a couple of lessons ago. formula often has an equals, if it doesn't have an equals it's just (0.5) like one side of a formula, um does anyone know what that's called. tha- that sort of thing D? five things well I often call them things actually, because I (.) avoid the technical term (0.4) um but that's not technical enough. um (.) yes is it that (0.3)</pre>
756 757 758 759 760 761 762 763 764 765 766 766 767 768 769 770 771 772 773 774	T62 T63 T64	Richard: D: Richard:	<pre>formulas formulas, you could say. formula often has equals in it, doesn't it like the formula for speed equals distance divided by time, that I was getting muddled about a couple of lessons ago. formula often has an equals, if it doesn't have an equals it's just (0.5) like one side of a formula, um does anyone know what that's called. tha- that sort of thing D? five things well I often call them things actually, because I (.) avoid the technical term (0.4) um but that's not technical enough. um (.) yes is it that (0.3) the (.) one of the left is a proof of the one on</pre>
756 757 758 759 760 761 762 763 764 765 766 767 768 769 770 771 772 773 771 775 776	T62 T63 T64 T65	Richard: D: Richard: Drew:	<pre>formulas formulas, you could say. formula often has equals in it, doesn't it like the formula for speed equals distance divided by time, that I was getting muddled about a couple of lessons ago. formula often has an equals, if it doesn't have an equals it's just (0.5) like one side of a formula, um does anyone know what that's called. tha- that sort of thing D? five things well I often call them things actually, because I (.) avoid the technical term (0.4) um but that's not technical enough. um (.) yes is it that (0.3) the (.) one of the left is a proof of the one on the right?</pre>
756 757 758 759 760 761 762 763 764 765 766 767 768 767 768 769 770 771 772 773 774 775	T62 T63 T64 T65	Richard: D: Richard: Drew:	<pre>formulas formulas, you could say. formula often has equals in it, doesn't it like the formula for <u>speed</u> equals distance divided by time, that I was getting muddled about a couple of lessons ago. formula often has an equals, if it doesn't have an equals it's just (0.5) like one side of a formula, um does anyone know what that's called. tha- that sort of thing D? five things well I often call them things actually, because I (.) avoid the technical term (0.4) um but that's not technical enough. um (.) yes is it that (0.3) the (.) one of the left is a proof of the one on the right? ah ok, you're back to that. yes that's</pre>
756 757 758 759 760 761 762 763 764 765 766 767 768 769 770 771 772 773 774 775 776 777 778	T62 T63 T64 T65 T66 T66	Richard: D: Richard: Drew: Richard: PP:	<pre>formulas formulas, you could say. formula often has equals in it, doesn't it like the formula for speed equals distance divided by time, that I was getting muddled about a couple of lessons ago. formula often has an equals, if it doesn't have an equals it's just (0.5) like one side of a formula, um does anyone know what that's called. tha- that sort of thing D? five things well I often call them things actually, because I (.) avoid the technical term (0.4) um but that's not technical enough. um (.) yes is it that (0.3) the (.) one of the left is a proof of the one on the right? ah ok, you're back to that. yes that's interesting. um I think they're called expressions aren'[t they]. [ah:]</pre>
756 757 758 759 760 761 762 763 764 765 766 767 768 769 770 771 778 775 776 775 776 777 778 779	T62 T63 T64 T65 T66	Richard: D: Richard: Drew: Richard:	<pre>formulas formulas, you could say. formula often has equals in it, doesn't it like the formula for speed equals distance divided by time, that I was getting muddled about a couple of lessons ago. formula often has an equals, if it doesn't have an equals it's just (0.5) like one side of a formula, um does anyone know what that's called. tha- that sort of thing D? five things well I often call them things actually, because I (.) avoid the technical term (0.4) um but that's not technical enough. um (.) yes is it that (0.3) the (.) one of the left is a proof of the one on the right? ah ok, you're back to that. yes that's interesting. um I think they're called expressions aren'[t they].</pre>
756 757 758 759 760 761 762 763 764 765 766 767 768 769 770 771 772 773 774 775 776 777 778 779 780	T62 T63 T64 T65 T66 T66	Richard: D: Richard: Drew: Richard: PP:	formulas formulas, you could say. formula often has equals in it, doesn't it like the formula for <u>speed</u> equals distance divided by time, that I was getting muddled about a couple of lessons ago. formula often has an equals, if it doesn't have an equals it's just (0.5) like one side of a formula, um does anyone know what that's called. tha- that sort of thing D? five things well I often call them things actually, because I (.) avoid the technical term (0.4) um but that's not technical enough. um (.) yes is it that (0.3) the (.) one of the left is a proof of the one on the right? ah ok, you're back to that. yes that's interesting. um I think they're called expressions aren'[t they]. [ah:] that's what you were about to say. right they're called expressions aren't they, those things.
756 757 758 759 760 761 762 763 764 765 766 767 768 769 770 771 772 773 771 772 773 774 775 776 777 778 779 780 781	T62 T63 T64 T65 T66 T66	Richard: D: Richard: Drew: Richard: PP:	formulas formulas, you could say. formula often has equals in it, doesn't it like the formula for <u>speed</u> equals distance divided by time, that I was getting muddled about a couple of lessons ago. formula often has an equals, if it doesn't have an equals it's just (0.5) like one side of a formula, um does anyone know what that's called. tha- that sort of thing D? five things well I often call them things actually, because I (.) avoid the technical term (0.4) um but that's not technical enough. um (.) yes is it that (0.3) the (.) one of the left is a proof of the one on the right? ah ok, you're back to that. yes that's interesting. um I think they're called expressions aren'[t they]. [ah:] that's what you were about to say. right they're called expressions aren't they, those things. ((pointing)) so this one's got five <u>numbers</u>
756 757 758 759 760 761 762 763 764 765 766 767 768 769 770 771 772 773 774 775 776 777 778 779 780 781 782	T62 T63 T64 T65 T66 T66	Richard: D: Richard: Drew: Richard: PP:	formulas formulas, you could say. formula often has equals in it, doesn't it like the formula for <u>speed</u> equals distance divided by time, that I was getting muddled about a couple of lessons ago. formula often has an equals, if it doesn't have an equals it's just (0.5) like one side of a formula, um does anyone know what that's called. tha- that sort of thing D? five things well I often call them things actually, because I (.) avoid the technical term (0.4) um but that's not technical enough. um (.) yes is it that (0.3) the (.) one of the left is a proof of the one on the right? ah ok, you're back to that. yes that's interesting. um I think they're called expressions aren'[t they]. [ah:] that's what you were about to say. right they're called expressions aren't they, those things. ((pointing)) so this one's got five <u>numbers</u> (0.3)
756 757 758 759 760 761 762 763 764 765 766 767 768 769 770 771 772 773 771 772 773 774 775 776 777 778 779 780 781	T62 T63 T64 T65 T66 T66	Richard: D: Richard: Drew: Richard: PP:	formulas formulas, you could say. formula often has equals in it, doesn't it like the formula for <u>speed</u> equals distance divided by time, that I was getting muddled about a couple of lessons ago. formula often has an equals, if it doesn't have an equals it's just (0.5) like one side of a formula, um does anyone know what that's called. tha- that sort of thing D? five things well I often call them things actually, because I (.) avoid the technical term (0.4) um but that's not technical enough. um (.) yes is it that (0.3) the (.) one of the left is a proof of the one on the right? ah ok, you're back to that. yes that's interesting. um I think they're called expressions aren'[t they]. [ah:] that's what you were about to say. right they're called expressions aren't they, those things. ((pointing)) so this one's got five <u>numbers</u>

785	1	T	isn't it. and Drew said something about one of
786			them being a proof.
787			(1.2)
788			go on
789	т69	Drew:	yep, no that's it
	105 T70	Richard:	which which one do you think is the proof
791	T71	Drew:	the one on the left
5	T72	Richard:	this one's a proof
į	T73	Drew:	yeah
	T74	Richard:	why's that one the proof?
		Drew:	because the one on the right it says um t-total
796	1/5	DIEW.	equals five x plus thirty, but to prove it you
797			have to ((inaudible))
	T76	Richard:	ah so for you this one is proving it and this one
799	170	RICHALU.	is, wha- wha- (.) how would you rate this
800	T77	Drew:	that's sort of,
801	1 / /	DICW.	(0.9)
802			that's sort of the evidence and
803			[that's the proof.]
804	T78	Richard:	[ah: ok], does anyone think of it (.)
805			the other way around. (.) can anyone see a reason
806			why you might think of it (.) the opposite way
807			round to that. I am not saying that Drew is wrong
808			cause I think, I can see what he's saying. um
809			Steve.
L	T79	Steve:	um because if you get an answer like the hundred
811			and five there, um
812			(0.3)
813			then you've got to try and find out some (.) proof
814			to back up your answer and on the right hand side
815			that gives you the proof.
	Т80	Richard:	((inhale)) so you're saying it the other way
817			around.
818	T81	Steve:	yeah.
	Т82	Richard:	some people wrote out a few of these and said they
820			all seem to be multiples of five. I can't remember
821			which people, who came up with some sort of
822			observation like this, yes people over there did
823			and other people said they all seem to be
824			multiples of five, and I said will they always be
825 826			multiples of five, and so they did another one,
826 827			and they did another one, then they did another one and then they got bored of doing them and said
827 828			look there's always going to be a multiple of
o∠o 829			five. um
830			(0.3)
831			but I think that that's not completely convincing,
832			that doesn't completely convince me (.) when
833			people do lots and lots of examples, why do you
834			think maybe I'm not
835			(0.4)
836			totally convinced by that?
837			(3.8)
838			they did another one. I can't remember where it
839			was. they put it somewhere else on the grid it was
840			it was (.) a different number but it was still a
841			multiple of five. and then did another one,
842			another one erm (1.1) wh- why was I not totally
843			convinced by that do you think.
844			(0.8)

	1		F?
845 846	т83	F:	you could turn the (.) t like
847	100	± •	(0.3)
848			sideways or something and [try then]
849	T84	Richard:	[mm:] you could do,
850			but even if you did and even if you kept it that
851			way round, I <u>s</u> till wasn't totally convinced, <u>they</u>
852			would have been con <u>vi</u> nced and they could have, I
853			don't think they really see, saw what I was make
854		_	such a fuss about
855 856	Т85	G:	((inaudible)) and only <u>one</u> in the whole grid that's like
857	T86	Richard:	yes, only one in the whole grid might be
858	100	iti cilara.	(0.6)
859			different or s[omething]
860	T87	G:	[yeah]
861	T88	Richard:	that's what you're saying isn't it. um and that's
862			my problem, that life's full of exceptions isn't,
863			just because something happens a <u>lo</u> t doesn't mean
864			it happens always. yeh? um like
865			(0.3)
866			can you think of any examples of that. where you
867 868	т89	Н:	have a rule that doesn't always work. i before e ex-cept after [c]
869	109 T90	п: Richard:	[yeah] I was thinking of
870	190	RICHAIU.	that one as well actually. um i before e except
871			after c yeah.
872			(0.8)
873			um
874			(1.1)
875			yeah and er what else do you have. an:d
876	Т91	Ι:	((inaudible))
877	Т92	Richard:	that's another one yeah? so um
878			(0.9)
879 880			i before e
881			(1.2)
			it do- doesn't work does it you know and so
			it do-, doesn't work does it. you know. and so
882			most of the time it's quite a nice rule
			most of the time it's quite a nice rule (0.6)
882 883			most of the time it's quite a nice rule
882 883 884			most of the time it's quite a nice rule (0.6) but (.) it doesn't always work does it. and so
882 883 884 885 885 886 887			most of the time it's quite a nice rule (0.6) but (.) it doesn't always work does it. and so it's worth being <u>su</u> re that things (.) <u>always</u> work and I think this (.) does. can you see what I'm what I see when I see that is that will always
882 883 884 885 886 887 888			most of the time it's quite a nice rule (0.6) but (.) it doesn't always work does it. and so it's worth being <u>su</u> re that things (.) <u>always</u> work and I think this (.) does. can you see what I'm what I see when I see that is that will always work everywhere it will always be a multiple of
882 883 884 885 886 886 887 888 889			most of the time it's quite a nice rule (0.6) but (.) it doesn't always work does it. and so it's worth being <u>su</u> re that things (.) <u>always</u> work and I think this (.) does. can you see what I'm what I see when I see that is that will always work everywhere it will always be a multiple of five. why would I
882 883 884 885 886 887 888 889 889 890			<pre>most of the time it's quite a nice rule (0.6) but (.) it doesn't always work does it. and so it's worth being sure that things (.) always work and I think this (.) does. can you see what I'm what I see when I see that is that will always work everywhere it will always be a multiple of five. why would I (1.2)</pre>
882 883 884 885 886 887 888 889 890 891			<pre>most of the time it's quite a nice rule (0.6) but (.) it doesn't always work does it. and so it's worth being sure that things (.) always work and I think this (.) does. can you see what I'm what I see when I see that is that will always work everywhere it will always be a multiple of five. why would I (1.2) why might someone be convinced by that. you're</pre>
882 883 884 885 886 887 888 889 890 891 892			<pre>most of the time it's quite a nice rule (0.6) but (.) it doesn't always work does it. and so it's worth being sure that things (.) always work and I think this (.) does. can you see what I'm what I see when I see that is that will always work everywhere it will always be a multiple of five. why would I (1.2) why might someone be convinced by that. you're doing very well these are hard questions. I'm sure</pre>
882 883 884 885 886 887 888 889 890 891 892 893			<pre>most of the time it's quite a nice rule (0.6) but (.) it doesn't always work does it. and so it's worth being sure that things (.) always work and I think this (.) does. can you see what I'm what I see when I see that is that will always work everywhere it will always be a multiple of five. why would I (1.2) why might someone be convinced by that. you're doing very well these are hard questions. I'm sure (.) my year twelves would would struggle</pre>
882 883 884 885 886 887 888 889 890 891 892			<pre>most of the time it's quite a nice rule (0.6) but (.) it doesn't always work does it. and so it's worth being <u>su</u>re that things (.) <u>always</u> work and I think this (.) does. can you see what I'm what I see when I see that is that will always work everywhere it will always be a multiple of five. why would I (1.2) why might someone be convinced by that. you're doing very well these are hard questions. I'm sure (.) my year twelves would would struggle (0.3)</pre>
882 883 884 885 886 887 888 889 890 891 892 893 894 895	Т93	J:	<pre>most of the time it's quite a nice rule (0.6) but (.) it doesn't always work does it. and so it's worth being sure that things (.) always work and I think this (.) does. can you see what I'm what I see when I see that is that will always work everywhere it will always be a multiple of five. why would I (1.2) why might someone be convinced by that. you're doing very well these are hard questions. I'm sure (.) my year twelves would would struggle (0.3) with some of these ideas as well. J?</pre>
882 883 884 885 886 887 888 889 890 891 892 893 894 895	Т93	J:	<pre>most of the time it's quite a nice rule (0.6) but (.) it doesn't always work does it. and so it's worth being <u>su</u>re that things (.) <u>always</u> work and I think this (.) does. can you see what I'm what I see when I see that is that will always work everywhere it will always be a multiple of five. why would I (1.2) why might someone be convinced by that. you're doing very well these are hard questions. I'm sure (.) my year twelves would would struggle (0.3)</pre>
882 883 884 885 886 887 888 889 890 891 892 893 894 895 896	т93	J:	<pre>most of the time it's quite a nice rule (0.6) but (.) it doesn't always work does it. and so it's worth being sure that things (.) always work and I think this (.) does. can you see what I'm what I see when I see that is that will always work everywhere it will always be a multiple of five. why would I (1.2) why might someone be convinced by that. you're doing very well these are hard questions. I'm sure (.) my year twelves would would struggle (0.3) with some of these ideas as well. J? because there's no chance of anything changing</pre>
882 883 884 885 886 887 888 889 890 891 892 893 894 895 896 897 898 899		J: Richard:	<pre>most of the time it's quite a nice rule (0.6) but (.) it doesn't always work does it. and so it's worth being <u>su</u>re that things (.) <u>always</u> work and I think this (.) does. can you see what I'm what I see when I see that is that will always work everywhere it will always be a multiple of five. why would I (1.2) why might someone be convinced by that. you're doing very well these are hard questions. I'm sure (.) my year twelves would would struggle (0.3) with some of these ideas as well. J? because there's no chance of anything changing because there's nothing like an illusion, like ((inaudible)) (by themselves) yes I see what you mean, <u>but I still</u> have a worry</pre>
882 883 884 885 886 887 888 889 890 891 892 893 894 895 896 897 898 899 900			<pre>most of the time it's quite a nice rule (0.6) but (.) it doesn't always work does it. and so it's worth being <u>su</u>re that things (.) <u>always</u> work and I think this (.) does. can you see what I'm what I see when I see that is that will always work everywhere it will always be a multiple of five. why would I (1.2) why might someone be convinced by that. you're doing very well these are hard questions. I'm sure (.) my year twelves would would struggle (0.3) with some of these ideas as well. J? because there's no chance of anything changing because there's nothing like an illusion, like ((inaudible)) (by themselves) yes I see what you mean, <u>but I still</u> have a worry when I do it this way that <u>may</u>be there's a place</pre>
882 883 884 885 886 887 888 890 891 892 893 894 895 894 895 896 897 898 899 900 901			<pre>most of the time it's quite a nice rule (0.6) but (.) it doesn't always work does it. and so it's worth being <u>su</u>re that things (.) <u>always</u> work and I think this (.) does. can you see what I'm what I see when I see that is that will always work everywhere it will always be a multiple of five. why would I (1.2) why might someone be convinced by that. you're doing very well these are hard questions. I'm sure (.) my year twelves would would struggle (0.3) with some of these ideas as well. J? because there's no chance of anything changing because there's nothing like an illusion, like ((inaudible))(by themselves) yes I see what you mean, <u>but I still</u> have a worry when I do it this way that <u>may</u>be there's a place near the edge or something, where it's not quite</pre>
882 883 884 885 886 887 888 890 890 891 892 893 894 895 896 897 898 899 900 901 902			<pre>most of the time it's quite a nice rule (0.6) but (.) it doesn't always work does it. and so it's worth being <u>su</u>re that things (.) <u>always</u> work and I think this (.) does. can you see what I'm what I see when I see that is that will always work everywhere it will always be a multiple of five. why would I (1.2) why might someone be convinced by that. you're doing very well these are hard questions. I'm sure (.) my year twelves would would struggle (0.3) with some of these ideas as well. J? because there's no chance of anything changing because there's nothing like an illusion, like ((inaudible)) (by themselves) yes I see what you mean, <u>but I still have a worry</u> when I do it this way that <u>may</u>be there's a place near the edge or something, where it's not quite going to work or <u>may</u>be if one is one of the</pre>
882 883 884 885 886 887 888 889 890 891 892 893 894 895 894 895 896 897 898 899 900 901			<pre>most of the time it's quite a nice rule (0.6) but (.) it doesn't always work does it. and so it's worth being <u>su</u>re that things (.) <u>always</u> work and I think this (.) does. can you see what I'm what I see when I see that is that will always work everywhere it will always be a multiple of five. why would I (1.2) why might someone be convinced by that. you're doing very well these are hard questions. I'm sure (.) my year twelves would would struggle (0.3) with some of these ideas as well. J? because there's no chance of anything changing because there's nothing like an illusion, like ((inaudible))(by themselves) yes I see what you mean, <u>but I still</u> have a worry when I do it this way that <u>may</u>be there's a place near the edge or something, where it's not quite</pre>

905			it will make a difference. whereas if I do <u>this</u> I
906			don't have that worry K:.
907	Т95	к:	is it because there's five squares in the t and
908			(.) so you're <u>alw</u> ays timesing (.) \underline{x} by five (.) so
909			it's always
910			[gonna be a multiple of five]
911	Т96	Richard:	[yea:h: wha]tever this x is
912			you're always gonna end up with this total down
913			here with <u>five</u> x and you know that five goes into
914			five x, and you know five goes into thirty and no
915			matter what five x is, no matter what x is. five x
916			could be all sorts of things. we know that five
917			goes into all of them, and we know that five goes
918			into thirty so five must go into that no matter
919			what x is. so <u>tha</u> t's what I'd regard as a proof.
920			because it doesn't depend upon the <u>par</u> ticular (.)
921			value of fifteen or x it could work no matter
922			where you went on the grid.
923			(0.8)
924			um go on last comment.

Chapter 8: Turn-Taking

The contrast between ordinary conversation and classroom interaction is particularly evident to most observers in the organisation of turns. These differences have been explored extensively by many researchers, including key studies by Sinclair and Couthard (1975), Mehan (1979) and, from a CA perspective, McHoul (1978). More recently, this work has been extended by Seedhouse (2004) in his exploration of language classrooms and Maroni et al. (2008) in their examination of Italian primary classrooms. In all interactions, there are tacit rules governing who speaks when, how long for and what can be said. In formal interactions, such as those that occur in classrooms, there are often additional constraints on who can speak when. These tacit rules can be revealed by the actions of participants, demonstrating their orientation to such rules and the sanction of participants when these rules are violated. The most widely discussed pattern of turn-taking in the classroom is the IRF pattern and this illustrates the orientation of pupils and teachers to the rules of interactions in the classroom.

In this chapter, I examine the structure and local management of turn-taking which constrain the content of interactions with the aim of offering further insight into the relationship between whole class interactions and the learning of mathematics. I will argue that particular features of turn-taking in formal classrooms have many pedagogic advantages such as supporting wait time between pupil and teacher turns, and the inclusion of a wide range of pupils. I will also argue, however, that it is not until we alter the structures of turn-taking that pupils can become really (emotionally) engaged in the

mathematics. Sacks et al. (1974) discuss the adaptation of turn-taking systems to the type of activities that are being undertaken and this is evident in the research into the different structures of turn-taking systems in institutional settings such as classrooms (Mchoul, 1978), courtrooms (Drew, 1992) and news interviews (Greatbatch, 1992). Furthermore, the types of activity or the nature of the mathematics that is the focus of an interaction are constrained by the turn-taking system that is the norm in mathematics classrooms as is revealed through an analysis of the orientation to and deviations from the rules of turn-taking.

Throughout this chapter, the word *rule* is used to describe the structural features of turn-taking. The use of this term reflects the use in CA literature, in particular Sacks et al. (1974), and does not mean "a set of determinate rules the application of which governs turn-taking", instead it refers to normative conventions or procedures that structure the changes in speaker as well as other architectural features of interactions (Wooffitt, 2005, p.29). The term *expectations* could also describe the structures rather than the term *rule*, but these expectations oblige participants to design their turns in particular ways.

The Rules of Turn-Taking

"a turn ... refers to an opportunity to hold the floor, not what is said while holding it" (Goffman, 1981). Sacks et al. (1974) outline a model for the organisation of turn-taking in ordinary conversation, emphasising the local management of turn-taking. The authors note that generally only one person speaks at a time, overlaps between speakers are short and there are no gaps when the speaker

changes. Their analysis of naturally occurring data leads to a set of 'rules' governing the transfer of a turn from one speaker to the next. Firstly, if the current speaker nominates another participant to speak next, then that participant is obliged to take the next turn and no other has the right to this next turn. If the next speaker has not been nominated by the current speaker, then another participant can self-select as next speaker with the participant speaking first having the right to the turn. If neither of these first two scenarios occur, then the current speaker can continue and keep the turn. These rules then apply recursively at each point in a turn where transition to a different speaker is relevant (referred to as a transition-relevance place (TRP) by Sacks et al., see chapter 5 for more detail).

However, turn taking does not necessarily follow the rules of ordinary conversation in formal classrooms (Mchoul, 1978; van Lier, 1984). McHoul's analysis of discourse in geography classrooms leads him to develop an adaptation of Sack's et al.'s 'rules', which function in formal classroom contexts. These 'rules' highlight the different roles of the teacher and their pupils as they provide a normative structure which supports and enables different kinds of turns from teachers and pupils (Wooffitt, 2005).

If the teacher is the current speaker then, as before, the teacher can nominate the next speaker. The pupil that has been nominated is obliged to take the next turn and no other pupil has the right to speak. However, if the teacher has not nominated a pupil to take the next turn, then the teacher is then obliged to continue the turn. If it is a pupil who has the current turn, then that pupil can select the next speaker. At this point, it is worth noting

that there are no occasions in McHoul's data where a pupil selects anyone other than the teacher as next speaker, and perhaps consequently, McHoul states that it is the teacher that has the right to the next turn. Otherwise, another participant can self-select as next speaker, with the teacher being first speaker. Finally, if neither of these two scenarios occurs then the pupil who is currently speaking can continue.

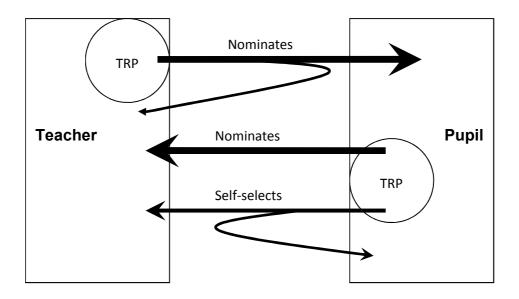


Figure 1: McHoul's rules of turn-taking in the formal classroom

These adapted 'rules' illustrate the constraints on the roles in the local management of turn-taking in the classroom. For example, these rules do not allow pupils to self-select as next speaker if the teacher is the current speaker. Instead, they allow the teacher to pause during their current turn without risking 'interruption' by a pupil. They also allow for gaps between the speakers changing from pupil to teacher, when the pupil has not nominated the next speaker, as it is the teacher who has the right to first start. This scenario also restricts the possibility of pupils self-selecting following another

pupil's turn, as whilst the option for them to do so is there, it is the teacher who has the right as first speaker. Furthermore, the situation where a pupil who has the current turn selects another pupil as the next speaker is not considered.

These restrictions on pupils self-selecting minimise the possibility of overlap in classroom interactions, whilst increasing the opportunities for gaps between turns compared to ordinary conversation. Yet it is the teacher that controls these gaps. If the teacher has nominated the next speaker then only that pupil has the right to the next turn and as such any pause between the teacher's and the pupil's turn belongs to the pupil and no other pupil has the right to self-select in this pause. If the teacher has not nominated the next speaker then the teacher has the right to continue the turn, again with no pupil having the right to self-select in any pause at this point. Finally, if the current speaker is a pupil, the right to the next turn returns to the teacher, whether they are nominated by the pupil or self-select as first starter, so the teacher has control over the length of the pause before they speak, though if this pause is too long the pupil can continue. In either case, it is the teacher who is controlling who can speak, when they can speak and how long they can speak for.

Overlap in ordinary conversation generally occurs when two participants selfselect as next speaker. When this occurs it is the first to speak who has the right to the turn and generally the other participant finishes their turn promptly, without completing what they had to stay. Some authors differentiate between overlapping speech and interrupting (Maroni, *et al.*,

2008) yet this distinction can be difficult to make without referring to participants' intentions, which are not always evident in the data themselves. Consequently, the term overlapping is used in this study to avoid the negative connotation often associated with the term interruption.

In McHoul's rules there is no opportunity for multiple participants to selfselect and consequently the potential for overlaps is minimised. Whilst McHoul's rules do not allow for multiple pupils to self-select as next speaker he does provide an example where this occurs, and describes it as a violation of the rules. In his example, the teacher has solicited a response by asking a question, but has not nominated a pupil to answer the question. Many pupils self-select to answer the teacher's question and, to use McHoul's description, 'chaos' ensues. However, as soon as the teacher nominates a pupil to speak next the other pupils stop talking and the normal structure of turn-taking resumes. McHoul describes this as using "renormalizing acts as a reparative technique" (1978).

Mehan's (1979) analysis of a primary classroom results in a detailed description of the use of turn allocation strategies, focusing on the teacher's strategies for nominating the next speaker. These include the nomination of a particular pupil by name or some form of gesture, inviting the pupils to bid for the turn, usually by raising their hands; and finally inviting or soliciting a response, where the response normally involves pupils answering a question or completing a sentence in unison. Mehan's work deliberately only focuses on the strategies used by the teacher, but does allow for pupils self-selecting,

albeit normally in unison, as next speaker and the examples offered cannot be described as 'chaotic'.

Mehan also observed that the place where pupils can self-select is after the completion of an IRF sequence, rather than after any specific turn (1979, p.140). As an IRF sequences makes another IRF conditionally relevant (Waring, 2009) and the initiation usually occurs in the same turn as the previous feedback or evaluation, the opportunities for pupils to self-select are rare.

The differences in turn-taking structure are not only between formal classrooms and other contextual settings. Maroni, Gnisci and Pontecorvo (2008) found differences in the turn-taking strategies in pupils of different ages with older pupils taking more turns than those in younger years. Cazden (2001) also offers a variety of examples from a variety of contextual situations indicating that differences in turn-taking strategies may also relate to cultural differences, the number of participants in the interaction and the form of the interaction itself.

Procedural Relevance of the Classroom Context

Before examining the turn-taking systems in the transcripts taken from secondary mathematics classrooms in this study, I demonstrate that the classroom context is procedurally relevant (Schegloff, 1992, p.110) to the talk-in-interaction in each of the classrooms in this study. That is, I demonstrate that the teachers and their pupils orient to their respective identities and roles in a classroom context through their language use. I shall do this by demonstrating how the turn-taking system in each classroom

differs from ordinary conversation and is instead structured in the way McHoul describes.

I shall consider each of the extracts presented in Chapter 7, beginning with Simon's lesson and ending with Tim's lesson. This order of analysis is significant as it is in Simon's lessons that deviations from the rules of turntaking occur the least, whilst in Tim's lessons these are more frequent.

The first noticeable feature of Simon's lessons is the difference in size of turns between Simon and his pupils. Simon's turns vary in length from over a minute to only one second in this extract, with many of his turns lasting several seconds. On the other hand, the longest turn taken by a pupil in this extract is 14.9 seconds and many of the turns last less than a second. Both the teacher and the pupils are orienting to their respective institutional roles: the teacher takes the first turns and has the right to speak for as long as he wishes whilst the pupils generally only speak when addressed.

In Simon's first turn in lines 240-268, there are seven noticeable pauses. In an informal context, any participant could self-select during such a pause. Most of these pauses occur at a TRP, yet even at the last pause of 0.9 seconds in line 267, no other participants self-select as the next speaker despite a question being asked immediately before. Instead, several pupils have raised their hands and wait until the teacher selects someone to be the next speaker in line 268, Charlie in this case. By not self-selecting as next speaker and by raising their hands to bid for the next turn, the pupils are orienting to their institutional roles. In ordinary conversation, participants do not normally raise their hands to indicate they will answer a question.

The interaction continues with Simon selecting each pupil who takes a turn by name. Following each pupil's utterance, the turn returns to Simon. Until line 335, there are no gaps and no overlapping speech.

In line 335, Simon selects George to answer his question. The pause that follows then belongs to George as he is expected to take the next turn. Another pupil, Alex, self-selects in line 337 to give the answer 'zero', in effect demonstrating his orientation to the institutional context by showing his understanding that Simon, as teacher, requires an answer of his pupils. Alex is indirectly reprimanded for self-selecting in line 340 where Simon states that it was George that needed to answer, not Alex. However, Simon does not take the turn until George has spoken, following an additional pause of 0.9 seconds in line 338. Whilst it is clear from Simon's reprimand that he has heard Alex's answer, he ignores it until George has responded. Simon is not asking the question because he does not himself know the answer but to check that George does. By repeating the question despite having a correct answer from Alex and reprimanding Alex, Simon is indicating that he has control of the turn-taking and is orienting to his role as teacher but also that the purpose of the question was to check George's understanding.

In lines 360-362, Simon and Ashley's turns overlap. Simon starts speaking at the point where Ashley has answered the question but begins to extend her answer to include the method she used. By starting his turn at this point, Simon is asserting himself as controlling the turns, both in terms of their length and their content, as in this context Ashley's turn was constrained to contain the answer to Simon's question and no more. These lines also

demonstrate the way in which the turn returns to Simon specifically in his role as teacher. Simon stops talking to allow Ashley to continue to the next TRP, despite the fact that this does not allow Ashley to give a complete answer. Simon returns the turn to Ashley in line 375 but not until he has expanded on Ashley's previous turn and consequently asserted his role in managing and controlling the topic.

In lines 378-382, Simon and Ashley's turns overlap on three separate occasions. Ashley has asked a question in lines 375-376, which offers an answer to the earlier problem of finding the middle number but by phrasing it as a question Ashley is both indicating uncertainty (Rowland, 1999) about the correctness of her answer and mitigating the effect of any repair that might follow (see chapter 9).

365 like too nice a number to turn down. she's	
366 up all these numbers, and it adds up to fi	ive
367 hundred. yeah	
368 (0.5)	
369 check that if you want to. remember sometime	imes in
370 the question they tell you at the start, y	you know
371 (.) five hundred children were surveyed or	r
372 something like that. ok . um	
373 (0.5)	
374 five hundred go on then Ashley.	
375 T91 Ashley: and then find the (di.) middle, is that tw	NO
376 hundred and fifty?	

Extract 1 - Ashley phrases her response as a question

373			(0.5)
374			five hundred go on then Ashley.
375 T	91	Ashley:	and then find the (di.) middle, is that two
376			hundred and fifty?
377 T	92	Simon:	u::m what's the middle number out of five
378			[hundred]
379 T	93	Ashley:	[is] it two hundred and fifty
380			[and two hundred and fifty one]
381 T	94	Simon:	[it's] an even number
382			[is]n't it
383 T	95	Ashley:	[yeah]
384 T	96	Simon:	so you do that trick,
385			(0.4)
386			five hundred plus one is five hundred and one,
387			halve it, it's two hundred and fifty
388			(0.6)
389			and a half ((writing calculation on the whiteboard
390			as it is spoken))

Simon lesson 1

Simon repeats Ashley's question and Ashley overlaps this by offering an answer, again phrased as a question but at the same time revealing that the difficulty lies in the meaning of 'the middle number' and not with the calculation itself. Simon overlaps Ashley's turn and indicates in this turn that he has understood what the difficulty is. Ashley begins her turn, overlapping Simon's previous turn, to agree with Simon but it is Simon who continues the turn in which he gives the answer to the question originally asked by Ashley in lines 375-376.

In line 437, Simon asks the whole class if that makes sense and does not nominate a specific pupil to answer. This turn is followed by a pause of 0.8 seconds in line 438, before a pupil self-selects to agree and Simon immediately takes the next turn. However, Simon begins this turn by repeating the answer and follows this with a pause of 0.9 seconds in line 441, and then later a pause of 1.1 seconds in line 444, allowing the pupils opportunities to self-select to take the next turn. No pupil takes that opportunity.

In line 493, Simon asks Drew to continue a calculation and Drew responds hesitantly in lines 494-495, and phrases her answer as a question. Simon does not immediately answer the question but instead pauses for 1.4 seconds, offering Drew the opportunity to expand on her turn or another student to self-select. He asks related questions to two other pupils before indicating that Drew's answer was appropriate in lines 509-512 and indicating that she is to complete her answer in line 514. By handling Drew's

response in this way, Simon is asserting himself as controlling the turns but is also orienting to his role as questioner and evaluator and the pupils' roles as answerers. It is also possible that Simon is using the two inserted questions and the pauses to ensure that the pupils' attention is focused on the task and that other pupils understand what Ashley is doing, orienting to his role of classroom manager and assessor.

With the exception of the turns in lines 337, 391, 474, and 476, only pupils nominated take turns in this extract. In line 337, a pupil self-selects after the nominated pupil pauses for 2.5 seconds and is indirectly reprimanded for doing so. In line 391, the pupil self-selects to give the final answer to the calculation Simon and F are working through but the turn is ignored until line 409, where Simon reaches a point where he needs the final answer. Whilst the pupil is not reprimanded for speaking when the turn is not hers, the teacher does assert his control over the topic by ignoring the turn until it is relevant to his own turns. The other two occasions where pupils self-select are in lines 473-477 when Simon is completing a table on the whiteboard which requires mental multiplications and by voicing the calculations and pausing after the first calculation, Simon is in effect inviting pupils to selfselect to give the answers, which he accepts. This is similar to Mehan's (1979) third turn-allocation strategy of inviting a response through soliciting the completion of a sentence, or in this case an arithmetical calculation.

At no point does a pupil select anyone except the teacher, Simon, as next speaker following her turn. Finally, whilst there are numerous pauses at TRPs, both when a change of speaker occurs and where it does not, no pupil

self-selects as next speaker, despite many of these pauses being noticeably long.

The analysis above clearly shows that Simon tightly controls who can speak, when they speak, what they can say when they speak and how long they can speak for. Simon and his pupils orient to the rules of turn-taking in formal classrooms as outlined by McHoul (1978), and deviations from these rules are sanctioned.

Now I examine the extract from Richard's lesson also presented in chapter 7. In a similar way to Simon, the disparity in the lengths of turns taken by Richard and his pupils is again clear, with Richard's turns longer than most of the pupils' turns, but here the pupils' turns are longer than in the extract from Simon's lesson. The extract begins with Richard controlling the allocation of turns and the turns returning to Richard following a pupil speaking. The pattern of teacher followed by pupil returning to teacher continues throughout the extract with the exception of one occasion where a pupil self-selects in line 740. Here there is trouble in the interaction and Richard initiates a repair in line 744 (see chapter 9).

Again, in Richard's turns there are noticeable pauses. For example in line 561, Richard asks the question "what do you understand by the idea of proof" and follows this with a pause of 0.9 seconds, where a few pupils raise their hands to bid for the turn before the teacher repeats the question and nominates Alex. In this extract, there are several occasions where pauses also occur during pupils' turns (lines 600, 640 and 625-637 for example), and pauses occur between the pupil's and the teacher's turn (lines 594, 605 and

612 for example). In each of these pauses, no other pupil selects as next speaker, the interaction remains between the nominated pupil and the teacher.

As in Simon's lessons, overlapping speech is rare in Richard's lessons. In the extract, overlaps occur in three distinct situations. Firstly, where a pupil has the turn and Richard overlaps to indicate that he has understood what the pupil is saying, as in line 617. Vice versa, there are also occasions where Richard is speaking and either one or many pupils overlap to indicate that they understand. These types of overlap are common in story telling (Liddicoat, 2011) where participants use them to indicate involvement and that they are listening to the story teller. In line 849, Richard starts speaking before F has finished but F's answer is not offering an explanation as to why Richard is not convinced that it will always be a multiple of five, and instead is returning to the earlier task where possible variations of the task are suggested. In effect, F's turn is returning the topic to an earlier discussion and is consequently altering the direction of the lesson and Richard's overlap ends F's turn and returns to the topic of being convinced (see chapter 10 for a more detailed analysis). Additionally, F's turn includes 'or something' shortly before Richard begins speaking which can be used by speakers to project the end of the turn. In the wider transcripts of Richard's lessons, there are also examples of overlap where a nominated pupil begins their turn before Richard has completed his. Each of these occurs at a TRP where the pupil has the right to the next turn as Richard has nominated them as next speaker.

Extract 2 - Pupil overlaps teacher's turn

 001
 T55
 Richard: so what would the mistake be [(.) to do it

 002
 ((inaudible))

 003
 T56
 Chris:

 004
 migh]t (.) think that the ((inaudible))

 Richard Lesson 2

Both Richard and his pupils are orienting to their institutional roles in that Richard has control over who speaks when, what they can say and for how long the turn can last. In addition, pauses are handled differently to ordinary conversational contexts in that other participants are not self-selecting in these pauses. In comparison to Simon's lessons, Richard's turns are shorter ranging from 0.5 seconds to 81.4 seconds with an average length of 9 seconds, and his pupils' turns are longer with an average length of 4 seconds (where the maximum length in Simon's transcript is 3 seconds), but the imbalance between teacher and pupils remains, with Richard having longer turns overall than his pupils, again indicating an orientation by the participants to their institutional roles.

Finally, it is noticeable that turns oscillate consecutively between the teacher and a particular pupil far more in Richard's lessons than in Simon's lessons. For example, in the transcript from Simon's lesson there is an extended exchange between Simon and Ashley in lines 357-409 as Ashley gives more details about the procedures she followed in calculating the mean. Whereas in the transcript from Richard's lesson there are extended exchanges between Richard and Alex in lines 567-585 which Richard also returns to in line 618, between Richard and Jamie in lines 608-618 and between Richard and Drew in lines 624-655 in just the first few pages of transcript. However, this may be a feature of the activity being done in the interactions, rather

than a feature of the particular turn-taking structure of the teacher's classroom.

In Tim's lessons there are some examples where the turn-taking varies from the structure described by McHoul, however the majority of the interactions are similar to those of Simon and Richard. Again, in Tim's lessons, including the extract in chapter 7, the length of turns differs between Tim and his pupils. Tim's turns are often longer, up to almost a minute long, whilst his pupil's turns are often only one or two words long with none of the pupils' turns in the extract above lasting longer than 3 seconds.

Similarly to the other teachers, when Tim pauses during a turn pupils do not self-select to speak unless the pause follows a solicitation. There is one deviation from this in the extract where in line 125 a pupil asks a question relating to the connection Tim is making between the task the pupils have just completed and the image Tim has projected onto the whiteboard. This pupil has self-selected at a TRP and Tim answers the question in lines 127 and 130-133. However, as is consistent with the existing literature on classroom discourse, self-initiated questions by pupils are rare in all the teachers' lessons (Cazden, 2001; Mercer, 1995) and I explore the implications of self-selecting to ask a question by pupils later in this chapter.

However, where Tim differs from the other teachers is in the way pupils are selected to speak. In Tim's lessons, as can be seen in the extract, pupils are nominated to take the next turn, either by name or by some form of gesture, relatively rarely. In all of Tim's lessons at least 50% of pupils' turns result from a pupil self-selecting as next speaker. Tim still has control of the turn-

taking in his classroom. He speaks first and he indicates a change in speaker. Whilst the other teachers predominantly do this by nominating the next speaker, Tim solicits a response from his pupils, often by asking a question, but does not direct this question towards a specific pupil. Neither does he direct his questions to the class as a whole by using 'everyone' or 'anyone' as Simon and Richard both do.

In Tim's lessons, many pupils do bid for turns by raising their hands and Tim himself explicitly asks pupils to do this in line 75. Where a pupil has been nominated as next speaker, it is that pupil that takes the next turn, no other pupil self-selects. For example, in line 165 Tim explicitly nominates Jamie as the next speaker. Jamie begins her turn guite hesitantly with a pause of 0.6 seconds, but no other participant self-selects to speak in this pause and Jamie completes her turn by offering an answer phrased as a question. This turn is followed by a 0.6 second pause before the turn returns to Tim in line 170. In ordinary conversation, any participant would have the right to selfselect as next speaker in this pause but in the classroom context, the turn usually returns to the teacher as it does in this example. By allowing this pause, Tim is offering Jamie the opportunity to alter or add to her turn, indicating that Jamie's answer is not the expected one (see the next chapter for more discussion of this). Tim's question in line 170 also indicates that Jamie's answer is not the required one and the pause that follows offers Jamie a further opportunity to self-repair which she does not take up, but does indicate that she has understood Tim's handling of the turns to indicate that her answer is not appropriate. Whilst this second pause does offer an opportunity for other pupils to self-select, they do not do so.

When pupils do self-select as next speaker, this is almost exclusively in response to an undirected solicitation from Tim, and often results in more than one pupil taking the next turn. For example, in lines 29 and 30, two pupils self-select to give the same answer, overlapping as they do so. Similarly, in lines 85 and 86, two pupils overlap in taking the next turn, but this time offering different answers. Neither pupil stops their turn to allow the other to finish theirs, which is what usually happens in ordinary conversation. The pupils are not sanctioned for self-selecting and Tim often accepts and uses these turns in his own turns. So whilst Tim is orienting to his institutional role of teacher by controlling the change of speaker and the topic of interaction, and by asking questions, and the pupils are orienting to their roles of pupil by self-selecting only when another has not been nominated and by answering questions, Tim is not controlling who takes the next turn as tightly as the other teachers. He does not nominate specific pupils to take turns and pupils are rarely sanctioned for self-selecting.

Looking specifically at situations where speakers overlap in Tim's lessons, whilst there are several occasions where pupils overlap each other, almost exclusively following an undirected solicitation by Tim, there are few occasions where Tim himself is overlapped. The only occasion in the extract above is in lines 199 and 200. Here the pupil overlapping has done so at a TRP and it is Tim who retains the turn. Elsewhere in the data, where Tim's turns do overlap, it is when Tim is responding to one particular pupil's turn whilst other pupils are also taking the turn. The extract below is taken from Tim's second lesson:

	001	Т55	Tim:	generally, shoe size and height. they're not
	002			necessarily linked in as tightly as we're
	003			talking about in direct proportion but they do
	004			have positive correlation, they do increase as
	005			one increases
	006	Т56	Chris:	a man's nose and ears
	007	т57	Drew:	(giggle) yeah
	008	Т58	Chris:	no because they never sto[p growing do they]
	009	Т59	Ashley:	[they never stop grow]
	010			ing
	011	т60	Jamie:	age in [ears]
\rightarrow	012	Т61	Tim:	[is that] true?
		Tim I	esson 2	

Extract 3 – Example of Teacher's turn overlapping a Pupil's turn.

In line 12, Tim's turn is overlapping the previous pupil's utterance but is responding directly to the pupil who spoke in line 8 and he is not overlapping this particular turn. When several pupils are self-selecting as next speaker and this results in pupils speaking concurrently, these turns stop as soon as Tim speaks. This indicates that the pupils are orienting to both their own roles as pupils and to Tim's role as teacher. When Tim speaks it is a pupil's role to listen, yet no such relationship, beyond those that exist in ordinary conversation, exists between pupils.

I have now shown in the analysis of all three transcripts presented in chapter 7, that the structure of turn-taking makes the institutional setting of the wholeclass interactions procedurally relevant. In each case, the teacher controls the turn-taking. All the teachers control who can speak when, largely through the specific nomination of the next speaker, though in Tim's case this may be a generic pupil rather than a specific individual. All the teachers control what can be said in subsequent turns, largely through the use of consecutive adjacency pairs. Finally, there are pauses throughout both the teachers' turns and their pupils' turns, in which no one self-selects as next speaker.

Consequences of the Rules of Turn-Taking in Formal Classrooms

The control that a teacher has and the asymmetric roles of the teacher and pupils have in formal whole class discussions affords and constrains several pedagogic strategies. These include the control of who takes a turn, control of the topic of discussion and the potential for wait time.

In contrast to ordinary conversation, the rules of turn-taking in formal classrooms allow for pauses between turns. If the teacher has the current turn, then either the teacher nominates the next speaker or the teacher continues the turn. There is no option for another participant, a pupil, to self-select as the next speaker. Consequently, the teacher can safely pause during their turn without the risk of being interrupted, as can be seen in lines 240 to 268 of the extract from Simon's lesson. This ability to pause without interruption includes the slot following a First Pair Part (FPP), such as a question, and just before the teacher nominates the next speaker. In other words, wait time between a teacher question and a pupil answer is structurally built in to the rules of turn-taking in formal classrooms.

Furthermore, a nominated pupil can pause or hesitate at the beginning of their nominated turn for longer than would be possible in ordinary conversation, as the nomination secures the turn as theirs and no other has the right to speak. In the extract below, after a turn-initial filler Alex pauses for 2.6 seconds before giving her answer.

Extract 4 - An example of hesitation and pausing at the beginning of a pupil's turn

001	Т8	Richard:	what does that (.) produce. ((clears
002			throat)). what does that produce Alex?
→ 003	Т9	Alex:	um: (2.6) three hundred and <u>fi</u> ve over two
			hundred and fifty.

Richard lesson 5

Whilst a long pause or delay may indicate trouble and result in the initiation of a repair (see chapter 9), as it would in ordinary conversation, the length of pause and amount of hesitation that a pupil can use is longer than in ordinary conversation.

Finally, if a pupil has the current turn the rules of turn-taking in formal classrooms also structurally enable wait time at the end of the pupil's turn. Three options following a pupil's turn are possible. The teacher is nominated to take the next turn, the teacher has the right to self-select as next speaker or the turn returns to the pupil who is currently speaking. The pupil who is currently speaking needs to leave a considerable (compared to ordinary conversation) pause to ensure that the teacher is not going to self-select as next speaker before they can continue the turn. Alternatively, these pauses enable the pupil to expand their answer without fear of another pupils self-selecting as happens in the extract from Richard's lesson in lines 625 to 637. Finally, this considerable gap offers the teacher the opportunity to pause before taking the next turn.

Rowe (2003) argues that one second appears to be the threshold or default maximum length of turn that both teachers and pupils will allow during interactions. Jefferson's (1988) analysis of ordinary conversations also revealed a 'standard maximum silence' of around one second, after which point participants in the interaction begin to treat the silence as a source of trouble (see chapter 9). So whilst the structure of turn taking in formal classrooms does allow longer pauses, it may be the interpretation of longer

pauses in ordinary conversation as sources of trouble that teachers and pupils are orienting to in their interactions.

Longer pauses do occur in the data, but predominantly during a teacher's turn and they often involve interactions with resources, such as the whiteboard, or textbook. However, there are exceptions where pauses of longer than a second occur. The first of these involves what Rowe describes as wait time 2, a pause between a pupil giving an answer and the teacher taking the next turn. There are four examples of this in the extracts in chapter 7, in all of which the teacher interprets the pupil's turn as a source of trouble and these are discussed in chapter 9 (Tim line 176, Richard lines 612 and 732, Simon line 338). The other exception can be categorised as part of Rowe's wait time 1, in that it occurs between a question asked by a teacher and a pupil answering, but with the pause of interest itself occurring during the teacher's turn and in between the asking of the question and the nominating of a pupil to answer.

It is the rules of turn taking in formal classrooms that allows these pauses to occur. In each case, a TRP occurs at the end of the teacher asking the question, yet because pupils do not have the option of self-selecting as next speaker following the teacher's turn, only the teacher has the right to continue the turn. Whilst the question does solicit an answer, the pupils wait until the teacher provides the answer by continuing the turn, or the teacher nominates a pupil to give the answer. For example, in Tim's lesson, lines 160-165, Tim asks the question three times, with pauses of 1.3 and 2.6 seconds between each reformulation of the question, but no pupil self-

selects to speak during these pauses and an answer is not given until a pupil is specifically nominated. These reformulations imply that Tim interprets no answers as trouble in the question that is being asked (supported by the small number of hands being raised to bid for the next turn). It might be that the different rules of turn-taking may be in conflict, with Tim orienting to the rules for ordinary conversation and waiting for a pupil to self-select, and the pupils orienting to the rules for the classroom.

Rowe identified ten student outcome variables that are affected by teachers increasing the pauses between turns. Many of these changes in outcome variables can be explained through the structure of turn-taking and the preference organisation of repair in classrooms. For example, Rowe found that the length of pupils' responses increase, as did the nature of the content of these responses. The presence of a pause following a pupil's turn where other pupils cannot self-select enables the pupil to continue their turn, providing the teacher does not speak during this pause. With the strong preference organisation associated with talk that results in the interpretation of pauses as sources of trouble, the pupil continuing the turn is preferred to the pause continuing beyond a certain tolerance level and hence the pupil will act by speaking to avoid the dispreferred silence. Rowe also found that the number of unsolicited but appropriate responses increased and also that the failure to respond or to give speculative responses decreased when wait time was increased. These again relate to the increase in pauses leading to pupils speaking in order to avoid the dispreferred silence.

A number of studies and government initiatives (Black, *et al.*, 2003) have attempted to encourage teachers to increase the wait time both between their turn and a pupil's, and also following the pupil's turn. However, these studies and initiatives have been limited in their success (Rowe, 2003). Teachers talk of uncomfortable silences but also observations of lessons reveal a continued prevalence of less than 1 second pauses. The 'standard maximum silence' in ordinary conversations (Jefferson, 1988) may account for the uncomfortableness felt by teachers in that pauses of longer duration usually indicate trouble in the interaction. Therefore, whilst the differences in the structure of turn-taking in the classrooms enable longer pauses to occur structurally, without necessarily being interpreted as trouble sources, the preference organisation within ordinary conversation seems to continue to be a barrier to increase wait times.

The ability to control who speaks and what they can say have other pedagogic advantages that are evident in the data, but are beyond the scope of this dissertation. Firstly, a comparison between Simon who almost exclusively nominates pupils to take the next turn, either by name or gesture, and Tim who often solicits answers from the class as a whole leaving pupils to need to self-select as next speaker, reveals a difference in the number of different pupils who participate in the whole class interactions. In Simon's lessons (and Richard's) the majority of the class are called upon to participate at some point during the whole class interactions, whilst in Tim's the minority participate. However, participation rates do not necessarily relate to the quality of the discussion or the learning of mathematics (Mendez, *et al.*, 2007).

Secondly, in teaching and learning interactions the institutional goal involves the pupils learning something and it is the institutional role of the teacher to specify what this is and to enable this learning to take place. The teacher usually has some preformed idea of the topic of whole class interactions and the rules of turn-taking in formal classrooms enable the teacher to maintain control of the topic (see van Lier, 1988 for a further discussion of topic control). One of the mechanisms that enables this control of topic is the ability for a teacher to keep a turn, even with significant pauses, without risking a pupil self-selecting as next speaker. The teacher can introduce new topics, open up topics for discussion or close them down all within a single turn.

Deviations from the Rules of Turn-Taking in Formal Classrooms

Now that I have demonstrated that in the classrooms in this study both the teacher and the pupils are orienting themselves to the institutional context of a formal classroom, and have discussed some of the pedagogical advantages to these structures, I shall examine in more detail where the interactions deviate from the rules proposed by McHoul (1978).

Mehan (1979) identifies two situations where interactions deviate from the normal rules of turn-taking in the classroom. Firstly, where a pupil violates the rules and is not sanctioned, and where the rules are not violated but a pupil is sanctioned. He then identifies four strategies that the teacher uses to handle these situations: doing nothing; getting through; accepting the unexpected; and opening the floor (p. 108). Doing nothing occurs when a pupil replies before the nominated pupil replies, or between the nominated

pupil and the teacher taking the next turn, and this unnominated reply is not sanctioned, evaluated, or acknowledged. Mehan argues that this is a form of sanction as it tells the pupil that their reply is not acceptable even if it is appropriate. Getting through occurs when the teacher has attempted to get a response using a variety of strategies and a pupil who has not been nominated gives the response the teacher recognises as appropriate. In these situations, the teacher positively evaluates the response, even when it violates the rules of turn-taking. The third strategy is used again when a pupil takes a turn even when not nominated, but this time it is when the pupil has taken the turn between the nominated pupil and the teacher's next turn but after there is trouble in the nominated pupil's turn. Mehan argues that because the nominated pupil's reply is not accepted by the teacher, the floor is open to other pupils to self-select. Mehan argues that this is the pupils creatively creating opportunities to take the next turn. The final strategy occurs when a pupil's reply 'provides more than expected' (1979, p.118), for example it includes an explanation or description of a method of the response given.

There are no occasions in my data where a pupil is sanctioned for taking a turn, when the rules have not been violated. Consequently in what follows, I specifically examine the occasions where pupils self-select as next speaker and are not sanctioned for doing so.

Pupils self-selecting to ask a question

Pupils rarely self-select to ask questions and the majority of these questions that are asked are seeking clarification of what they need to do in relation to a task. In the data in this study, the questions that pupils ask can be

categorised into four types. The most frequently occurring type is questions seeking clarification of the task. Pupils also ask questions as a mechanism for indicating that they have understood the previous turn, to initiate repairs (see chapter 9) and finally to ask to clarify their understanding of mathematics. It is this last category that I explore further next.

Pupils asking mathematically related questions are rare in all the lessons in this study, but in the majority of cases the pupil has raised their hand to indicate they have a question to ask, and the teacher nominates them as next speaker. Pupils self-selecting to ask mathematically related questions is rarer still, with only three instances in the data.

In the extract from Tim's lesson, a pupil asks "why" in line 125 following Tim's introduction of the image. This question is followed by a pause of 0.9 seconds in line 126 before Tim answers the question. On the other two occasions, similar pauses occur:

	001 002 003 004	Т234	Tim:	one in eight. ok. if I cancel them down, that and that cancels. that and that cancels I'm left with (0.7) a tenth. so-
→	005 006 007 008	T235 T236 T237 T238	Chris: Tim: Chris:	how do you know that cancels with that how do you know what this cancels down
	009 010 011 012	Т239	Tim:	
	013	Т240	Chris:	5
	014 015	T241	Tim:	and I know that because there's just an eight on the top and we're timesing them.
	016	Т242	Chris:	1 2
	017 018 019 020 021 022 023	T243	Tim:	<pre>so I can just cancel them down straight away. so, despite what you thi:nk (0.4) it doesn't matter when you go. you still have the same (.) probability if y- if you chose before now which position to go in, you would have the same probability of winning</pre>
	020			probability of winning

In line 5, Chris asks a question which Tim follows with a repeat of the question seeking clarification as to what numbers Chris is referring to. Once what the question is asking has been established there is a pause of 1.1 seconds in line 8 before Tim begins his answer to the question. The pupil has self-selected to speak but has not overlapped Tim's turn, there is a minimal gap between the speakers and the intonation of Tim's turn does not indicate a TRP. The pupil twice responds to Tim's explanation using a 'change-of-state' token, "oh", indicating that Tim's explanation is resulting in a change in her understanding of the mathematics.

In Simon's third lesson, the class are calculating averages including the mean, mode and median.

	001	T215 Simon:	if you do my little trick, twenty two plus one
	002		divided by
	003		(1.1)
	004		um
	005		(0.6)
	006		two, twenty two plus one is twenty three, divided
	007		by two you get um eleven and a half.
\rightarrow	800	T216 Chris:	why do you add one
	009	T217 Simon:	um
	010		(0.4)
	011		that is a very (.) good question. think about this
	012		yeah. Say you've got three people
		Simon lesson 2.	

Extract 6 - Another example of a pupil self-selecting to ask a mathematically related question

In line 8, a pupil self-selects to ask a question. Simon begins his turn with a filler then pauses for 0.4 seconds before evaluating the question. He then begins answering the question. In this extract, Chris does self-select at a TRP.

These pauses are not present when the pupil has been nominated to take the next turn. By self-selecting to ask the question, the pupils have deviated from the rules of turn-taking. Whilst it could be argued that the pauses occur because by asking a question the pupil has both changed the topic and who has the roles of questioner and answerer, these two aspects of the interaction also occur when a pupil has been nominated to ask the question. The differences lie instead in the expectation of the turn. In each of the occasions where a pupil self-selects to ask a question, the change in speaker is not expected. On no occasion is a pupil sanctioned or ignored when self-selecting to ask a question, irrespective of the nature of the question. Simon also positively evaluates the act of asking a question in the turns that follow the majority of questions asked by pupils, as in the extract in chapter 7.

Whilst it appears that self-selecting to ask questions is not a sanctionable act, the majority of questions are asked following a nomination by the teacher. In all three extracts offered above where the pupil has self-selected to ask the question, the teacher is writing on the whiteboard at the time of the question and is providing an explanation of the mathematics. The videos of the lessons do not enable us to know whether in each case the pupil has raised their hand before asking the question, but in all three cases the possibility of being nominated by the teacher is not there. Presumably, at the end of the teacher's explanation the intention is that pupils will have understood the explanation, therefore unsolicited pupils' questions that seek clarification or further explanation of the mathematics help to maximise the effectiveness of the explanation and are consequently allowable.

The rarity of pupils' self-selected mathematically related questions can be accounted for by the rules of turn-taking in formal classrooms to some extent; however, the data show that deviations from the rules in this case are not sanctionable and are in fact rewarded in some instances. Other factors, therefore, are likely to influence the asking of mathematical questions, such as issues of face, or sufficient knowledge and competence to ask such questions.

Multiple Pupils self-selecting following Teacher solicitation

There are noticeable differences between the teachers in their use of questions that are not directed at particular pupils. Neither Edward nor Richard ask questions where they have not indicated which pupil should answer the question. Simon does use this strategy but only in a similar way to Mehan's (1979) findings where pupils complete a sentence in unison, or as Schegloff (2000) puts it, chordally. In Simon's case these are all arithmetic calculations that are not directly related to the objective of the current task.

However, Tim frequently asks undirected questions that solicit an answer (van Lier, 1988) but where pupils need to self-select in order to answer. A teacher may ask an undirected question when they are confident that whilst there may be multiple starters, and consequently overlapping speech, these will all (or largely) offer the same answer, therefore minimising the time for which many pupils speak at once (Mchoul, 1978). For example in Tim's extract in chapter 7, Tim asks an undirected question in lines 20-27 to which two pupils offer the same answer in lines 29 and 30. Tim has no difficulty taking the next turn without any overlap with both answers being the same

and also with them being the answer Tim is expecting. In lines 85 and 86, two pupils offer answers that overlap, which whilst different in form, offer support for the previous unison 'yes'. These answers are not in unison and both pupils complete their turn despite overlapping. It might appear that the rules of turn-taking that specify only one participant can speak at a time are being violated. However, as Schegloff puts it, "the chordal production is done and heard as convergent and consensual, not competitive" (2000, p.6).

If we consider the other occasions where pupils overlap when answering an undirected question from Tim, the structure of unison responses and overlapping agreeing responses are the same. Appropriate responses are given simultaneously (Schegloff, 2000). In effect, Tim is interacting with the class as a whole. The interaction is between Tim and his class, not a collection of individual pupils. This is what Rowe describes as a two-player game conceptualisation of the classroom (1974). Consequently, if we analyse the interaction as taking place between two participants, Tim and his class, the rules of turn-taking are not violated. This is also the case when Simon asks undirected questions, in that the responses are simultaneous answers to arithmetical calculations.

However, in Tim's lessons there are several occasions where arguments start when multiple pupils self-select as next speaker with different answers. Whilst it could be argued that the teacher was confident that the pupils that self-selected would give the same answer, this is not evident in the data.

In lines 93-98, Tim asks the pupils if they ever will actually give away the whole twelve thousand pounds. The title of the lesson and the learning

objective of the lesson that were shared with the pupils at the very beginning of the lesson is limits of sequences. The task in the extract given is the first task of the lesson following a brief exchange recapping the previous lesson on calculating the nth term of quadratic sequences. At this stage of the lesson, the pupils have not been introduced to the notion of a limit.

Additionally, the sequence the task is based on is infinite and does have a limit that is not a term of that sequence. However, the context in which the sequence is placed does require you to stop donating a quarter as there is a minimal unit of 1p. In the turns that precede Tim's question in lines 93-98, the class have discussed the issue of whether the sequence stops and a pupil has put forward an explanation that you must stop because "you'd run out of the twelve thousand". In his turn, Tim is asking whether this is actually true. Consequently, it seems unlikely that Tim was expecting the same answer from multiple self-selecting pupils. On the one hand at least one pupil is arguing that you do give away the twelve thousand pounds and an affirmative response could be expected. On the other hand, Tim's turn in lines 93-98 could be interpreted as the initiation of a repair (see chapter 9) and a negative response would be expected. In lines 99-100 and 101, both answers are given. In line 102, Tim nominates a specific pupil as next speaker and the turn-taking returns to the system described by McHoul (1978), an example of a renormalizing act.

Whilst this return to the turn-taking system offers further evidence that both the pupils and the teacher are orienting themselves to the rules for formal classrooms, McHoul's account for pupils self-selecting as next speaker following a teacher's solicitation is inadequate, as is Mehan's discussion of

unison responses. In Tim's lessons, undirected questions are not only asked when the potential for overlap is minimized, there are occasions where Tim asks undirected questions where multiple speakers offering contradictory response are very likely. In the extract below, I offer one such occasion.

Extract 7 - Example of an undirected question followed by multiple speakers offering contradictory responses

\rightarrow	001	T268	Tim:	two: ok? which is the first prime number.
	002	T269		one
	003	т270	Drew:	tw[o
	004	т271	Jamie:	[zero
	005	т272	Drew:	two
	006	т273	Chris:	one
	007	т274	Tim:	one[two
	008	Т275	Ashley:	[two
	009	Т276	George:	is seven one sir?
	010	т277	Tim:	okay we might discuss it tomorrow
	011	Т278	Jo:	one
	012	Т279	Tim:	which one's the first prime number?
	013	T280	Pupil:	um
	014	T281	Tim:	why one?
	015	T282		(2.1)
	016	T283	Tim:	why is it not a prime [number]
	017	T284	Chris:	[because] you can only divide it by, one
	018	T285	Ashley:	yeah but you can divide it by itself, because it
	019			divides by itself
	020	T286	George:	yeah but you can't, you can only divide it by one
	021			though
	022	T287	Ashley:	yeah and that's dividing by itself
	023	T288	George:	yeah but one's ((inaudible) number)
	024	T289	Tim:	wind them up and let them go ((directed at
	025			camera)). so what is it, one or two, one or two.
	026	T290	Pupil	t[wo
	027	T291	Pupil	[one
	028	T292	Pupil	two
	029	T293	Pupil	one
	030	T294	Pupil	two
	Tim le	sson 1		

In line 1, Tim asks which is the smallest prime number. There is no indication in the data whether Tim believes that the potential for overlap is minimised or not. However, in lines 7, 14, 16, and 25 Tim is asking a question where it is clear from the preceding turns that different pupils will give different and contradictory answers. What is particularly striking in this extract in lines 17 to 23 there is in fact very little overlap between speakers despite pupils self-selecting with different answers. The fact that these

answers are only one word in length makes this more likely, which may be a particular feature of mathematics classrooms that enables this type of argument to occur. However, there is a notable difference in the nature of the interaction at this point. In the vast majority of the data in this study, pupils' answers are directed towards the teacher. Where there is overlap between pupils, it often results from multiple pupils self-selecting as next speaker following an undirected solicitation from the teacher. In these few lines however, the pupils have turned to face each other and the explanations are given as justifications for why their answer is correct. A point of contention (Gellert, 2011) has arisen between different pupils, and the interactions cease to be between teacher and pupils, but are now between disagreeing pupils. As Egbert (1997) describes it, there has been a schisming of the whole-class interaction into multiple interactions.

I would argue that at this point, where the interaction is between pupils and not between teacher and pupil, the pupils are no longer orienting themselves to the formal classroom context and instead the rules that govern ordinary conversation apply. Tim's comment to the camera "wind them up and let them go" also indicates that the nature of the interaction has changed.

This strategy of encouraging self-selection to develop a point of contention is supported further by an example from Tim's lessons where the point of contention does not naturally arise:

Extract 8 - Example where a point of contention does not arise

001	T173	Tim:	never ending. good. never ending number, infinite
002			number of decimal places, will you every actually
003			hit five.
004	T174	A:	no=
005	T175	D:	=no

006 T176 (0.8) 007 T177 Tim: why Tim lesson 1

Here Tim has asked a question requiring a yes or no answer. The two pupils that offer answers in lines 4 and 5 offer the same answer of no. There is then a pause of 0.8 seconds before Tim asks why. This pause offers other pupils the chance to self-select as next speaker and to give either the answer 'no' or 'yes'. However, no pupil does offer the contradictory answer of 'yes' and the point of contention does not arise. Tim makes no evaluation of the given answer in line 7, but instead asks for an explanation, a strategy Tim often uses to indicate that there was trouble with the previous responses.

There is also one occasion in the data taken from Tim's lessons where the turn-taking differs further from the teacher-pupil oscillation:

001	T182	Tim:	
002			six shirts take two hours to dry on a washing line,
003			how long will it take to dry three shirts. should be
004			a question mark at the end.
005	T183	A:	what?
006	T184	в:	what?
007	T185	С:	the:y'd be the same wouldn't they
800	T186	D:	oh that is easy
009	T187	Е:	another trick que[stion]
010	T188	F:	[one] hour
011	T189	Tim:	one hour
012	T190	G:	no [that's a trick [question]
013	T191	F:	[((inaudible))]
014	T192	Н:	[that's a] trick question
015	T193	I:	it'll take two hours
016	T194	J:	it'll take two hours
017	T195	К:	oh
018	T196	L:	(it [will take one hour.)]
019	T197	M:	[it'll take one hour because]
020	T198	N:	it will take one hour.
021	T199	J:	<pre>no it wouldn't [it would take]two (.) [because (.)</pre>
022			they've all got to dry ((inaudible))]
023	T200	I:	[no it wouldn't]
024	T201	0:	[((inaudible))]
025	T202	I:	[there all t-shirts aren't they]
026	T203	J:	it doesn't matter how many's on the [line.]
027	T204	P:	[yeh]
028	T205	Q:	it's all gonna take two
029	T206	Tim:	[good]. [ok? you've got to (be aware of the)]

Extract 9 - Example where the turn-taking varies from Teacher-Pupil oscillation

Tim lesson 2

Again, Tim has asked a question to the whole class in a lesson on direct and indirect proportion. In line 7, a pupil offers the answer that "they'd be the same", presumably referring to the two hours being the same, whilst in line 10 another pupil offers the contradictory answer of one hour. Tim repeats this answer in line 11 before the argument continues. What is noticeably different in this extract is that the pupils offer justifications for their answers without prompting from Tim. In line 19 a pupil ends a turn with "because", indicating that she was about to justify or explain why. Then in line 21, a student offers a justification for her answer of two hours, which is supported by another pupil in line 25. In the other two cases presented earlier, Tim prompted for explanations by asking why and this is also the case in another example from Tim's fourth lesson not included here. In the example above, the need for an explanation or justification occurs naturally as a means to 'win' the argument. However, whether Tim prompts for this explanation or not, the pupils are directing their comments at each other, and experience a need to explain and justify their position mathematically, and are often emotionally engaged in the argument as is indicated by the emphasis placed on particular words by the pupils but also by the fact that the turns are made loudly and clearly enough to be transcribed despite the fact that there are often multiple speakers during these exchanges.

Pupils self-selecting to establish common knowledge.

Tim also invites pupils to self-select when in the interaction they are recalling prior knowledge or performing calculations that are needed for but are not part of, the main focus of the lesson. The extract offered below comes from

Tim's lesson on probability. In the first part of the lesson, the class has played a game involving paper cups and following the extract the class examine the probabilities of each move in the game. In this extract, the interaction focuses on the most likely outcome when rolling two dice.

001 002	T176	Tim:	ok. what number would be most likely do you think. what total would be most likely.
003	Т269	A:	[six]
004	т270	D:	[eight]
005	Т271	Tim:	what total will go
006			(5.4) ((draws diagonal boxes in the two-way table
007			on the whiteboard))
008	т272	C:	seve[n]
009	Т273	B:	[si]x
010	Т274	Tim:	what total would go diagonally across the [board]
011	т275	Е:	[six]
012	Т276	F:	seven
013	T277	Tim:	seven (.) good. seven would be the most likely and
014			there'd be six of those (.) out of (.) a total of
015			thirty six, so. how do we ge:t
016			(1.8)
017			one out of thirty six quickly if we know (.) the
018			probability of getting a six (.) is one sixth.
 ·			

Tim lesson 4

The first two responses in lines 3 and 4 are given simultaneously but neither is the answer required by Tim. Tim rephrases the question, offering a visual hint in lines 5-7 and line 10, before accepting the answer given in line 12. There is no negative evaluation of answers given but more noticeably, no explanation follows the acknowledgement of the answer "seven". The idea of the most likely outcome is not used and is not directly relevant to the game that follows. Once the 'correct' answer has been established Tim moves on to calculating the probability of two events occurring, which is then used in the analysis of the game.

Summary

In this chapter I have demonstrated how McHoul's rules of turn-taking in formal classrooms are oriented to in all the lessons in the data within this study and how this contrasts to turn-taking in informal settings. All the teachers control who can speak when and what can be said during pupils' turns. The turn-taking is locally managed by the teachers and deviations from the rules are both rare and are usually sanctioned. The institutional context of the formal classroom is procedurally relevant in each of the teachers' classrooms. In each of the transcripts, the teacher has longer turns than the pupils and controls who talks when. Whilst there are some differences in how each of the teachers manages the change of speaker, the management of turn-taking demonstrates the participants' orientations to the roles of teacher and pupil.

Orienting to these rules enables an orderliness of classroom interaction that enables the teacher to maintain control of the topic and minimises the potential for overlap (or interruption). The nomination of next speaker adopted in most classrooms also enables a wide variety of pupils to be called upon to take the next turn. The opportunities and constraints of pauses between and during turns have then been explored, particularly in relation to the notion of 'wait time'. I have demonstrated that wait time is structurally built in to the turn-taking system in formal classrooms. The rules of turntaking enable considerable pauses between a teacher and a pupil's turn (wait time 1) and between a pupil's turn and the teacher's next turn (wait time 2) as well as during both a teacher's and a pupil's turns. However it is the teacher who has control of these pauses and there is a tension between

preference organisation in ordinary conversation where silence is dispreferred, and the structure of turn-taking in classroom interaction. I have also argued that the structure of turn-taking can account for many of the previous findings relating to wait time in the literature, which are related to the dispreference for silence.

Finally, I have examined the deviations from the rules of turn-taking, in particular where pupils self-select but are not sanctioned for doing so. The first of these instances is where pupils ask mathematically related questions. In these situations, the teachers pause before answering but do not sanction the asking of the question, implying that these questions are allowable and possibly pedagogically advantageous.

The second type of deviation is where multiple pupils self-select, sometimes in unison. In this instance, there are differences between the teachers and the case of Tim is particularly interesting. Tim encourages unison or chordal answers, as do the other teachers, but he also establishes arguments over points of contention where the pupils become emotionally engaged by the mathematics through enabling multiple pupils to self-select and offer contrasting responses. In a similar way to the examples offered by Mendez et al. (2007) the pupils are agreeing and disagreeing with each other and offering reasons for their responses. The need to explain or justify responses occurs naturally. In some instances, the class reaches agreement either through pupils convincing each other or Tim's evaluations, but there are also occasions where the disagreement is not resolved.

Tim also uses pupil self-selection to establish common knowledge that can be drawn upon in the interaction that follows. In these interactions, incorrect responses are often ignored or baldly negatively evaluated and the correct response is baldly accepted or positively evaluated without explanations.

The pedagogical implications of the analysis in this chapter need careful consideration. The turn-taking structure, which enables to IRF pattern of interaction, has both pedagogical advantages and disadvantages. The structure of turn-taking needs to be appropriate for the pedagogic purpose of the whole-class interaction. The structure enables teacher control of who speaks and the topic of the interaction, and different levels of control are appropriate for different types of interaction. Tight control where the purpose of the interaction is to establish common knowledge that can be built on enables a fast pace and the wider focus of the lesson to remain. This tight control also enables the teacher to alter and adapt the direction of the interaction in light of pupil responses. It also allows the teacher to control the amount of time pupils have to think and respond to questions. However, turn-taking structures that more closely resemble ordinary conversation enable argument and debate, a natural need for explanation and justification and additional opportunities for pupils to ask questions.

One initiative proposed by Black et al (2003) is the use of random name generators in the nomination of pupils to answer questions. This strategy randomly allocates turns amongst the pupils, often leading to a wider range of pupils taking the turn and more useful assessment information for the teacher. However, control over the nomination of which pupil takes the turn

or the deliberate use of self-selecting enable other types of interaction, such as an argument or debate. The structure of turn-taking and the teacher's control of this structure need to reflect the pedagogic goal of the interaction, and this structure needs to be locally managed as the goal changes on a turn-by-turn basis.

Many of the features discussed in this chapter are not only related to the structure of turn-taking. Preference organisation and in particular the preference organisation of repairs in interaction have also featured throughout this chapter, particularly in relation to wait time, and it is in the next chapter that preference organisation is explored in greater depth.

Chapter 9: Preference Organisation

Introduction

In this chapter, I outline the notion of preference in the CA literature before examine the preference organisation of whole-class interactions in the specific context of the secondary mathematics classroom. I begin by introducing the structures of adjacency pairs and repair within the conversation analytic approach. I then outline the preference organisation of both adjacency pairs and repair as they occur in analyses of ordinary conversations. The main part of this chapter examines how this preference organisation applies to interactions in classroom context. I end by discussing the implications the preference organisation of sequences of turns might have on the teaching and learning of mathematics.

Adjacency Pairs

Adjacency pairs are an important unit of conversational organisation consisting of two paired parts, such as question-answer, assessmentagreement, or request-acceptance. These two parts are sequential, ordered, and involve more than one speaker but also the utterance of the first part governs the range of expectable second parts; for example, a question requires an answer, not an acceptance or an agreement. The rule of operation is that "given the recognizable production of a first pair part, on its first possible completion its speaker should stop, a next speaker should start and should produce a second pair part of the same pair type" (Schegloff, 2007, p.14). The production of this second pair part, also reflects the hearer's understanding of the first pair part. So for example, the utterance

"can you shut the window" could be question if followed by "yes" or could be understood as a request if followed by the closing of the window (though if the first pair part was actually a request and the second pair part was an answer this would be a source of trouble and the subsequent turns would deal with this). The first pair part and the second pair part are reflexively related. Usually, an adjacency pair takes place over two turns, which is the minimal form, though they can be extended through inserting sequences between the first pair part and the second pair part. Most adjacency pair types have various but restricted types of second parts, an invitation or an offer can be accepted or declined, a request can be granted or rejected.

Adjacency pairs in the classroom are most frequently in the form of questionanswer, though many questions function as indirect requests for the pupils to display knowledge.

Repair

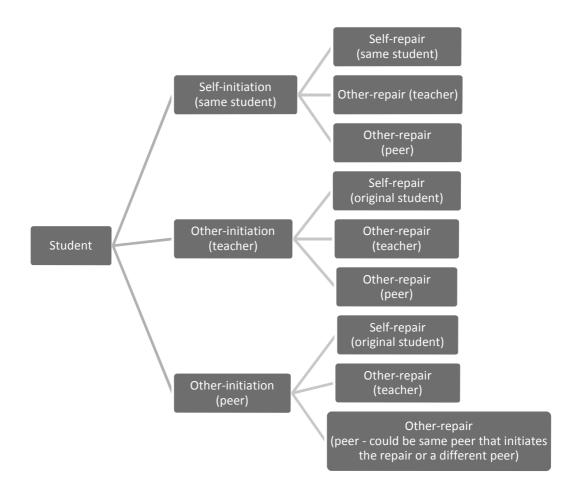
Repair is defined as a mechanism used to deal with trouble in speaking, hearing, or understanding (Schegloff, *et al.*, 1977). The terms repair and trouble extend the domain of the correction of mistakes or errors. Trouble can take the form of a mistake or error, but in this chapter, it is defined more broadly to include any difficulties occurring in the interaction under consideration (Seedhouse, 1996). Repairs involve resolving a source of trouble to enable the interaction to continue successfully whilst correction only applies to the replacement of something 'incorrect' with the 'correct' form. Participants are often faced with troubles in speaking, hearing, or understanding. Whilst many of these are not addressed, there are also

occasions where there is no apparent trouble but the utterance is still 'corrected'. As a result, everything is a possible source of trouble and consequently is repairable.

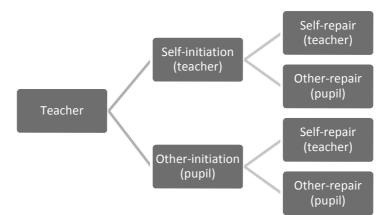
Repair consists of three parts: the trouble source, the initiation, and the outcome. These three parts are distinguishable because of the occurrence of trouble where no repair is attempted so there is no initiation of a repair, and unsuccessful repairs where the repair has been initiated but not performed (Schegloff, *et al.*, 1977). These distinctions lead to the realisation that the person in whose turn the trouble occurs, the person who initiates the repair and the person who performs the repair may or may not be the same person.

The range of troubles that are considered by Schegloff et al. (1977) under the heading of repair is broad, including word recovery trouble, self-editing even when no hearable trouble has occurred, pauses, and corrections. They argue that the same structural systems apply to the handling of the repair of all of these types of trouble.

A distinction between self and other is central to much research on social interactions. In studies on the organisation of repair (Schegloff, 2007; Schegloff, *et al.*, 1977), distinctions are made between self-initiated self-repair, self-initiated other-repair, other-initiated self-repair and other-initiated other-repair. In this chapter, a further distinction within the category of other is made between other-teacher and other-peer. If the trouble source occurs in a pupil's turn then nine repair trajectories are now possible.



However, when the trouble source occurs in the turn of the teacher, the original four repair trajectories are possible.



These different trajectories are illustrated below using extracts from the data collected in the study. This serves two purposes: they illustrate the differences between the different trajectories but also by using data from this

study, I am confirming that the trajectory occurs in whole class interactions in secondary mathematics classrooms.

A self-initiated self-repair occurs when the person in whose turn the trouble occurred also indicates that there is trouble that warrants repair and performs the repair.

→	001 002 003 004 005 006 007 008 009	T146	Tim:	<pre>no? what wh- (0.3) if you rolled (0.9) two sixes and you got about ten you'd get five each. two sixes on two dice. it's very unlik it's not (0.6) unlikely if you know what I mean. yep.</pre>
	009	Tim less	son 4.	uniikely II you know what I mean. yep.

Extract 11 - Example of a self-initiated self-repair where the self is the teacher

Here Tim cuts himself off "very unlik-", initiating a self-repair which he then performs immediately "it's not (0.6) unlikely". The trouble source, repair initiation and the performance of the repair have all occurred in the same turn in this example.

Extract 12 - Example of a self-initiated self-repair where the self is a pupil

001	T161 Georg	e: er cause (.) I think Mo- Moscow is minus ei	ght
002		degrees and London minus five degrees. I me	an
003		minus ten degrees Moscow yeah	
	Edward lesson	1	

George initially offers an answer of minus eight degrees for the temperature in Moscow. Later in the same turn, he offers a different answer of minus ten degrees that serves both as the initiation of the repair by indicating that minus eight degrees is not what he meant, and the performance of the repair, the offering of a different answer. Again this has all occurred in the same turn but is at the second opportunity as the repair occurs at a TRP rather than where the trouble source appears.

Self-initiated other-repairs occur when the person in whose turn the trouble occurs indicates there is trouble, but a different person performs the actual repair.

Extract 13 - Example of self-initiated other-repair where the other is the teacher

001	T216	Sam:	is it because
002			(0.6)
003			um
004			(0.4)
005			what's the that ce-
006	Т217	Edward:	Verhoinsk
	Edward	lesson 1.	

Finally, other-initiated repairs are where the indication that a repairable trouble has occurred happens in a different person's turn to the one in which the trouble occurred. These repairs can then be performed by the person in whose turn the trouble occurred (self-repair) or a different person's (otherrepair), or not at all.

Extract 14 - Example of an other-teacher-initiated self-repair

001 002	т276	Edward:	why, where are you getting those numbers from though
003	Т277	Alex:	because it's (.) in the middle of ten
004	Т278	Edward:	ok so that is basically (.) what we're
005			going to do but <u>hol</u> d ona second. I don't
006			agree with you that five is the number
007			between one and te[n]
800	Т279	Alex:	[an]d a half
	Edward	lesson 3	

Extract 15 - Example of other-teacher-initiated other-teacher-repair

001	т136	Edward:	can anyone remember why we call this one a
002			prism. what's special about this one.
003	T137	George:	a prison ((many shouting out))
004	T138	Edward:	no not a prison. a prism.
	Edward	lesson 2.	

Extract 16 - Example of other-teacher-initiated other-peer-repair

001	т255	Jamie:	fif- no sixty
002	T256	Edward:	no: (.) not quite
003	T257	Chris:	nearer fifty
004			(1.1)
005			fifty five=
006	T258	Edward:	=fifty five
	Edward	lesson 1	

Preference Organisation

The term preference refers to the structural features of sequence organisation and turn organisation in interactions. There is, however, considerable variation in the meaning of preference in the conversation analytic literature. Most authors focus on describing the common features of preferred and dispreferred responses, with few attempting to define preference explicitly. These features include the markedness of responses, the frequency of types of responses, and issues relating to face. The definition offered by Bilmes (1988) and extended by Boyle (2000), which both draw on the original lectures of Sacks, will be outlined first before the features of preferred and dispreferred responses are discussed in more detail.

With any FPP, there are at least three possible outcomes, the preferred response(s), the dispreferred response(s) or no response. The term preferred response refers to the response that is 'noticeably absent' if it is not given. For example if you create a piece of artwork for someone, the preferred response is some form of praise. If no response is given then it is the praise that is 'noticeably absent' and it is assumed that the artwork is not worthy of praise. It is not assumed with a non-response that it is a criticism that is missing. This noticeably absent response then needs to be accounted

for. Boyle (2000) makes a further distinction between dispreferred responses that are noticeable and accountable as either sanctionable or not sanctionable depending on the nature of the account given. The interactional context within which the FPP and SPP occur will also significantly influence what responses are preferred or dispreferred.

According to Schegloff (2007) an action is dispreferred if its occurrence is delayed, modulated or mitigated in some way (through hedging or the offering of an excuse for example). It does not refer to the liking or disliking of the participants to the particular response but rather relates to the markedness of the response. This distinction between the psychological meaning of preference and the conversation analytic structural meaning of preferences is complex and interpreted differently by different authors. Schegloff's explanation of dispreferred responses in relation to their markedness is the basis of most discussions of preference organisation (Drew and Heritage, 1992b).

Delays and pauses in turns are often interpreted as marking a dispreferred turn. Jefferson (1988) suggests that pauses between 0.8 and 1.2 seconds are treated as a sign of trouble in an interaction, as repairs are often initiated following pauses of this length.

In adjacency pairs, such as the question-answer pair, the ranges of potential second parts are not structurally equal, or as Schegloff and Sacks put it they are not "symmetrical alternatives" (1973, p.314). Some possible second parts, such as possible answers are preferred whilst others, such as non-answers are dispreferred.

In ordinary conversation, preferred second parts are generally unmarked. They are usually structurally simple, occurring in the next-turn, without delay, hesitation, or mitigation. Dispreferred second parts are structurally more complex. They typically occur after a delay. This can be by pausing before speaking, prefacing the second part with markers such as 'well' or 'er' or by displacing the second part over several turns. Dispreferred seconds are also marked or hedged in some way that could include a hesitant delivery, restarting the turn, or self-editing during the turn. Finally, they often include an indication as to why the preferred answer is not performed. Consequently dispreferred SPP are generally longer, and require more effort from the speakers. In classrooms, the preference organisation of question-answer adjacency pairs is often more noticeable. As teachers predominantly ask questions to which they know the answer, this known answer is usually the preferred SPP, with other answers and no responses as dispreferred SPPs. There are occasions where this is not the case and these occasions are examined later in this chapter.

Bilmes (1988) takes a slightly different view on the meaning of preferred and dispreferred responses. He accepts that whilst there is an association between the frequency of markers in dispreferred responses and the number of dispreferred responses, he does not accept these as a defining feature. He argues instead that they are markers of reluctance to give the response. By incorporating a statistical aspect to preference organisation, he argues that we are looking at what people are doing rather than at what they are inferring from the sequence of interaction.

The location of markers in responses remains an indication that the response is likely to be a dispreferred response and as such, Schegloff's description is used in this study, though there are situations discussed later in this chapter where the issues Bilmes raises are relevant and these are discussed in more depth on those occasions.

Many of the issues relating to the interpretation of preference relate to the distinction between the psychological meaning and the structural use by Sacks. In particular, many authors discuss avoiding dispreferred responses (Levinson, 1983; Mey, 1993) whilst Boyle (2000) argues that using the term avoidance implies a psychological interpretation of preference. In this chapter, I use the term avoid in the sense of reducing the probability of a dispreferred response occurring and not to imply the avoidance of something that is disliked. Instead, the avoidance of dispreferred responses refers to the structural devices used by participants that increase the likelihood of a preferred response.

Preference Organisation of Repair in Ordinary Conversation

Schegloff et al.'s (1977) examination of ordinary conversations reveals a clear preference for self-repair. Firstly, self-initiated self-repairs in the same turn occur most frequently, followed by self-initiated repair in the turn transition space, then other-initiated self-repair and finally other-initiated other-repair. Other-initiated other-repair occurs very rarely in ordinary conversation. A preference for self-initiated over other-initiated self-repair was also apparent. This preference for self-initiated self-repair is also a consequence of the trouble source, self-initiation and self-repair all occurring

within the same turn or within the turn's transition space. The opportunities for self-initiated self-repair occur before those for other-initiation. In fact, Schegloff et al. (1977) found that other-initiations of repairs were regularly preceded by a delay, which offers a further opportunity for a self-initiation and self-repair.

The preference of self-repair over other-repair following an other-initiation is further supported by the propensity of others to initiate a repair even when they could perform the repair themselves. On the rare occasions where an other-initiated other-repair is performed, these are usually marked or modulated in some way, through hesitation, being phrased as a question or include modulators such as 'I think'.

Schegloff et al. (1977) outlined different techniques employed in self-initiated or other-initiated repairs. For example, cut-offs or hesitations in selfinitiations which occur in the same turn; what? huh? who? and partial repeats in other-initiations. Additionally self-initiated repairs are usually completed successfully within the same turn or in the turn's transition space of the utterance that includes the trouble source, whilst other-initiated repairs were often completed over several turns.

The trouble in self-initiated self-repairs within ordinary conversation is predominately combined with the repair itself:

Schegloff, Jefferson and Sacks, 1977 p.364.

Extract 17 - Example of self-initiated self-repair in the same turn and the location of the trouble is identified in the repair.

N: she was givin' me a:ll the people that were go:ne this yea:r I
 mean this quarter y'//know
J: yeah

Here N initiates and performs the repair by changing 'this yea:r' to 'this quarter' which both indicates the location of the trouble in the original choice of words 'this year' and performs the repair.

In other-initiated repairs, the other-initiation usually includes a technique for locating the trouble source, offering the speaker in whose turn the trouble occurred, another opportunity for self-repair:

Extract 18 - Example of an other-initiated repair including a location of the trouble source.

```
A: Hey the first time they stopped me from sellin' cigarettes was this morning. (1.0)
B: From selling cigarettes?
A: From buying cigarettes.
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Schegloff, Jefferson and Sacks, 1977 p. 370

The trouble occurs in A's turn. B initiates a repair and locates the trouble by repeating the trouble source 'from selling cigarettes', emphasising the word that is causing the trouble. A then self-repairs in the following turn, replacing 'selling' with 'buying'. The pause between A's first turn and B's turn offers an opportunity for A to self-repair before B initiates the repair.

Schegloff et al. (1977) note that other-repairs are more common in adultchild interactions and is "a device for dealing with those who are still learning or being taught to operate with a system which requires, for its routine operation, that they be adequate self-monitors and self-correctors as a condition of competence" (p.381). It is then reasonable to expect that otherrepair will be more frequent in teacher-pupil interactions. However, the more frequent occurrence of other-repair should be transitional with pupils moving towards self-repair as they become more competent and, in the case of the mathematics classroom, as they develop as mathematicians.

Preference Organisation in Classrooms

Greenleaf and Freedman (1993) analyse a short extract from a secondary English class to examine the relationship between the structure of the wholeclass interactions and the pupils' learning. They define a preferred response to a teachers' question as one that is used or taken up by the teacher. Hence, these responses become resources in the lesson and therefore contribute to the teacher's goal in the lesson. This in turn leads to a distinction between the evaluation of these turns and preferred or dispreferred responses. They offer examples of a preferred response, which is negatively evaluated, and a dispreferred response, which is positively evaluated to support this distinction.

Whilst there is evidence in the data in this study that there is a distinction between teacher evaluations of responses and whether the response is preferred or dispreferred, Greenleaf and Freedman's definition of preference to some extent differs from the original discussion of preference by Sacks (1973) and emphasises the role of both the FPP and the SPP in identifying the preference organisation of the SPPs. Greenleaf and Freedman do define preference in relation to the sequence of interactions that follow, but not to the FPP. Their use of preference specifically relates to problem solving activities in the classroom, yet I would argue that even in these contexts there are responses that are preferred but not explicitly used or taken up by the teacher, though they might implicitly be. In addition, there can be responses that are taken up by the teacher which are dispreferred responses to the original FPP. For example, if a pupil answers a question that demonstrates a common misconception the teacher may want to use this as

a teaching point in the moment, but it remains a dispreferred response to the original teacher's turn as it does not actually contribute to the goal of the lesson, unless the teacher's goal included an exploration of this misconception.

McHoul's (1990) research on repair in geography classrooms revealed that other-initiated self-repair is more common than self-initiated self-repair. Liebscher and Dailey-O'Cain's (2003) analysis of language classroom interactions explores the different uses of turn-constructional devices by pupils and teachers. They build upon Schegloff et al.'s analysis of otherinitiated repairs in ordinary conversation focusing in particular on the degree to which the device specifies the trouble source. The least specific type includes words such as 'pardon?', 'what?', or 'uh?', which give no indication of the location of the trouble. Next are individual question words such as 'who?', 'when?', or 'what?' and then partial or full repeats of the troublesource turn, possibly followed by a question word. Finally, there are candidate understandings where the turn includes a possible understanding of the trouble-source turn. In ordinary conversation, participants usually start with less-specific devices moving to more specific when necessary. Liebscher and Dailey-O'Cain identify three other more specific devices particular to classroom interactions: the unspecified understanding check, 'yes?' or 'no?'; requests for repetition; and finally requests for definition, translation or explanation. Their analysis revealed that the pupils used more specific repair initiation techniques when interacting with the teacher than in other interactions.

Mehan (1979, p.55) identified three teacher strategies for troubles that arise from teacher questions, such as no answer, partially complete answers and incorrect answers as well as answers that do not match the question type, for example in Mehan's terms, a choice elicitation followed by a product response. These strategies are: prompting; repeating or simplifying the question until the 'noticeably absent' response is produced.

In the remainder of this chapter, I examine the preference organisation of adjacency pairs and then repairs in each of the classrooms in this study. Whilst there are many similarities in the structure of preference organisation to earlier studies, there are some deviations and I explore the implications these structures may have on the teaching and learning of mathematics.

Preference Organisation in the Mathematics Classroom

The question-answer adjacency Pair

The most frequently occurring conversational unit in the classroom is the question-answer adjacency pair. Following the utterance of the First Pair Part (FPP) (the question), usually by the teacher, different Second Pair Parts (SPP) are available to the second speaker, of which the most relevant is an answer. Some questions types add a further restraint on what can be considered a relevant SPP. FPPs, which include 'who', 'what' and so forth make only answers containing a person or a location etc. respectively relevant. A relevant answer must also conform in type to the question asked (Schegloff, 2007, p.78). As in ordinary conversation, a relevant answer is the preferred response. Answers that do not conform in type, non-answers such as 'I don't know' or repair initiations are dispreferred. However, the structural

features of both preferred and dispreferred SPPs differ subtly in the mathematics classroom.

Questions in mathematics lessons are predominantly asked by the teacher. These are usually 'known answer' questions and their role is usually to check knowledge or understanding and Alpert argues that this "explains the frequent silence or reluctant participation" of pupils in classroom interactions (1987, p.37). Consequently, the question-answer adjacency pairs that occur in the data generally involve the teacher asking the question and a pupil responding with an answer. On the few occasions where a pupil does ask a question, these questions are usually task oriented; they are requests for clarification or for information. In all the extracts the teachers and pupils orient to the obligations to produce explanations, questions or answers, but also orient to how these actions should be performed (Heritage and Greatbatch, 1991).

Pupil answers are often marked in some way, irrespective of the content or nature of the answer. Both answers that are appropriate and correct, as well as incorrect answers and non-answers are often hesitant, particularly at the start of the turn, and are often hedged or marked in some other way. The frequency of SPPs prefixed with hesitation, hedging or discourse markers are far higher in whole-class interactions than in ordinary conversation which is perhaps unsurprising as hedging and hesitating is typical of novice talk (Atwood, *et al.*, 2010).

Within the ordinary conversation context, hesitation or hedging usually indicates a dispreferred response. In the classroom setting, a dispreferred

response would be an answer that differs from what the teacher is expecting, a non-answer such as 'I don't know' or the absence of any answer at all. If markedness indicates a dispreferred response, the question thus arises as to why pupils' answers are marked more frequently than would be expected in ordinary conversations. Why are pupils giving dispreferred SPPs more frequently than in ordinary conversation?

One explanation for the more frequent use of markers in answers could be that the pupils are treating their answer as the dispreferred answer of differing from what the teacher is expecting. The IRF sequence occurs so frequently that pupils expect a third turn following their answer which contains an evaluation of that answer. By hesitating or hedging their answers, and consequently identifying them as dispreferred, they are in turn mitigating any possible negative evaluation in the teacher's next turn. Consequently, the pupil is orienting to the institutional role of the teacher as evaluator. There are several possible reasons a pupil might mark their answers as dispreferred in this way that relate to the evaluation that often follows in the teacher's next turn. It could indicate that pupils are not able to distinguish themselves whether their answer will be a preferred or dispreferred response. In particularly, they are not able (or willing) to make an evaluation of their own answer.

This has implications for the mathematics that pupils are experiencing in whole-class interactions. Being able to check your own answers is an important aspect of working mathematically but pupils also need to develop an awareness of the reasonableness and appropriateness of their answers

even before they check them. By marking their answers, combined with the prevalence of the IRF, the responsibility for making evaluations and assessments rests with the teacher.

Alternatively, the pupil might believe that their answer is correct but may not want to appear arrogant to the rest of the class and thus marking their answer as dispreferred is a face-preserving move. This raises an interesting issue relating to the meaning of preference in different authors' articles. Whilst Schegloff (2007) and Pomerantz (1984), for example, use the prevalence of hesitation and accounts to 'define' a dispreferred response. Bilmes (1988), on the other hand, describes the pauses that often occur with dispreferred responses as 'reluctance markers' (p. 173) and argues that whilst these markers are associated with dispreferred responses, it is possible to have a marked preferred response and consequently they cannot define a dispreferred response. These markers instead of indicating that a dispreferred response is to follow, in fact mark the speakers reluctance to give the response that follows. In the mathematics classroom, preferred responses may be given reluctantly for reasons relating to how this preferred response may make them appear to their classmates. By looking at the frequency of different sequences of interaction, Bilmes argues that we are looking at what participants do rather than the inferences that they make in the sequence. Consequently, the frequency of marked dispreferred responses tells us that participants frequently mark their dispreferred responses, and they do not in themselves infer a dispreferred response.

Another explanation specifically relating to hesitation at the start of the SPP, relates to the structure of turn-taking in the classroom context. Whole class interactions usually start by the teacher asking the class as a whole a question, this is a one-to-many situation. However, at the end of this FPP the teacher usually nominates the next speaker by name or in some other way such as using a gesture. It is not usually until the end of the teacher's turn that the nominated pupil knows that the next turn is theirs. Using hesitation markers at the start of their turn does two things. Firstly, it avoids the dispreferred response of offering no answer at all (silence) by indicating that the pupil has accepted the next turn as theirs and an answer might be forthcoming (Wooffitt, 2005). Secondly, it gives the pupil some time to think about and formulate their response. When pupils self-select, preliminarily bids for the next turn or the interaction is between the teacher and a pupil, not the whole class, these hesitation markers are far less common.

As mentioned earlier, teachers' questions are frequently 'known-answer' questions, in that the teacher themselves knows the answer. Another way of looking at these teacher questions is as indirect requests for information; it is not the answer to the question that the teacher is interested in but the information of whether the pupils know the answer to the question. In this respect, both correct and incorrect answers as well as inappropriate answers (answers that do not match in type) all serve the purpose of giving information to the teacher of whether the pupil(s) knows the answer. Any response which gives the teacher the information they require would be a preferred response which links to Greenleaf and Freedman's (1993)

approach to defining preference to refer to whether the teacher makes use of the SPP.

The more frequent occurrence of marked and hesitant responses by pupils therefore, is not an indication of a dispreferred response in mathematics classrooms. This relates directly to the sequential relationship between the FPP and the SPP in defining preference. Whether a response is preferred or dispreferred depends both on the FPP and on the SPP. By looking purely for the presence of markers, hedging, or hesitation, we are solely looking at the SPP when defining preference. It is by viewing teachers' questions as indirect requests and the subsequent turns including the answer, that an understanding of preferred responses to teachers' guestions is reached.

There is also a relationship between the pedagogic nature of the interaction and the prevalence of marking or hesitation in pupils' turns. For example, in the transcript from Tim's lesson very few of the pupils' turns in the first part are marked or hedged in any way, whereas in the second part several of the responses are hesitant or marked. In the first part of the extract from Tim's lesson, the pupils are reporting findings from the recent pair work or group work, whereas in the second part the pupils are being introduced to a new representation and are being asked to make connections. If we take the argument that pupils mark their responses to oblige the teacher to make the evaluations, then in unmarked responses the pupil is making that evaluation. When reporting back from small group work that the teacher has possibly been involved in, responses have often already been evaluated or agreed to by either the teacher or the peers in the group.

These explanations indicate that the organisation of turn-taking and the nature of adjacency pairs are both constrained by the institutional setting within which they take part but also the nature of the mathematical activity. However, they also raise the issue of what a dispreferred response to a teacher's question can be defined to be.

Two other features that characterise dispreferred responses in ordinary conversation that are clearly evident in the current dataset are the use of preemptive reformations and the offering of an explanation or account before or after the dispreferred response.

Dispreferred responses in both settings are often preceded by a significant delay. This offers the first speaker the opportunity to reformulate their FPP. In the classroom this often involves the teacher (first speaker) reformulating the question to either make the question easier or to broaden the range of answers which will be considered acceptable.

In the extract below, Richard and Drew are continuing a one-to-one interaction where Drew had provided the answer (SPP) to a question asked by Richard (FPP).

001 002 003	T20 T21	Richard: Drew:	so what did you do next then Drew. um m [(.) I worked it out on] a calculator
003 004 005 006 007 008 009 010 011 012	T22 T23	Richard: Richard:	[(.) I worked it out on [] a calculator [how did you get those numbers from] some people wrote that sort of thing. they wrote their answer it's (.) longer because I worked it out and it's longer. um what I really wanted was the details of how you worked it out, of what you did (0.5) um
012 013 014			(0.3)

Extract 19 - Teacher reformulates question after a pupil hesitation.

015	who (.) can pick up the thread there.
016	(2.2)
017	a lot of people are throwing numbers around and I
018	think they sort of (.) work their way round the
019 020	class without people necessarily knowing where
020	they came from.

Richard lesson 1

Here Richard attempts to reformulate the question to be more specific in asking Drew for more details about the answer given in a previous turn. Drew's previous turn included the steps that he took to get to a final answer but no explanation of why he took these steps. The initial question posed by the teacher might have caused Drew some difficulty as he had already presented his final answer in a previous turn. Richard's reformulation overlaps Drew, resulting in them speaking simultaneously. Drew's overlapped response could be interpreted as an answer to the question as originally posed, though not to the reformulated one, and Richard treats it as such by taking the next turn. He does this with no transition space following the completion of Drew's 'calculator', consequently not offering Drew the interactional space to answer the new reformulated question.

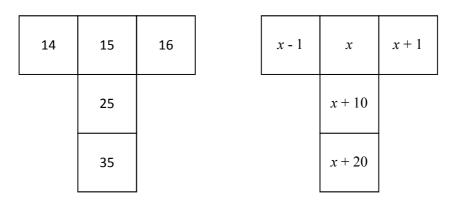
In the next extract taken from chapter 7, Richard is addressing the whole class and does not nominate the next speaker.

708 T40 Richard 709	: and (.) I want to know what you think is the same or what's different about those
710	two.
711	(3.1)
712	I mean we did all that yesterday but I just
713	thought it'd be nice if we (.) stood back and
714	thought about
715	(0.6)
716	what it meant and w-hat's the same and what's
717	different about the right side and the left side.
718	(3.7)
719	°a hard question°. have a think.
720	(4.1)

721	you can say something that's quite obvious and
722	that's fine. I'd just like people to make
723	(0.4)
724	observations about what's the same and what's
725	different.
726	(0.5)
727	((small group are laughing)) what are you laughing
728	at?

Richard Lesson 1

In Extract 20, Richard is referring to two diagrams on the board, one on the left representing a numerical example and the other an algebraic representation of the relationships between the numbers given in the first diagram (see Figure 2). Richard initially asks what is the same and what is different about the two diagrams. After a long pause of 3.7 seconds in line 718, no answer is forthcoming so Richard offers an account for why that might be 'a hard question' before clarifying (and possibly broadening) the range of acceptable answers.



T-total = 105

T-total = 5x + 30

Figure 2: T-Totals images from whiteboard in Richard's lesson

The two extracts above have similar features to those in ordinary conversation where a delay follows a FPP. The first speaker has "an opportunity to change the first pair part to a form which will allow the response which is apparently 'in the works' to be delivered as a preferred response, rather than a dispreferred one" (Schegloff, 2007, p.70). In both these extracts, the teacher has altered their original FPP in some way to support the pupils in giving some answer, rather than the dispreferred silence that had followed the original question. These alterations are in effect funnelling or focusing patterns described by Wood (1994).

The second feature of whole-class interaction that is similar to ordinary conversation is the giving of an account or explanation or part-answer before a non-answer.

Extract 21 - Pupil gives two possible answers before giving a non-answer.

001 002	Т76	Simon:	it was eight. and what was the lowest number of days absent.
003	T77	Ashley:	zero or one I don't know
	Simor	n lesson 1	

The pupil here is unsure whether the minimum is zero or one so she offers both before claiming that she does not know. Alternative responses could have included any part of this SPP, so the question arises as to why the pupil chose this particular response. There are other occurrences in the data, and in several cases, the first half of the response, which includes the account, explanation or part-answer, is significantly longer:

```
Extract 22 - Pupil offers an account of what she did before saying she doesn't know.
```

001	Т40	Chris:	um. (.) it um at first we did, (.) we j- we
002			just did (.) well what I've got written down
003			is one hundred thousand divided by sixty
004			divided by sixty divided by ten but
005	T41	Richard:	right
006	Т42	Chris	I don't know what I did ((laughs))
	Richa	rd lesson 1.	

In Extract 22, Chris' first turn offers an account of what she did to answer the question before in her second turn she claims she does not know what she

did. Although her calculations are correct and give her the correct answer the question on the worksheet, the second turn indicates that it is an explanation of where the numbers have come from that is expected. This is not apparent in the question asked by the teacher immediately preceding this interaction but has occurred in earlier interactions with other pupils (see Extract 19). Here the dispreferred response of 'I don't know' has been delayed across three turns. Chris' first turn is also hesitant and marked which often indicates that a dispreferred response is to follow.

The discussions above lead to a preference organisation of question-answer adjacency pairs that differs from the preference organisation of questionanswer adjacency pairs in ordinary conversation. The differences lie not in the order of the preferred and dispreferred responses but in the structure of those responses. In both mathematics classrooms and ordinary conversation, an answer is preferred, whilst accounts and non-answers such as 'I don't know' or 'I can't remember' are dispreferred and no answer at all is dispreferred further still. However, preferred responses in mathematics classrooms are frequently marked, given hesitantly and are hedged in some way. These features are more commonly associated with dispreferred responses in ordinary conversation.

Interestingly, both correct and incorrect answers are treated as preferred responses within the question-answer adjacency pair structure, though incorrect answers may be subject to repair-initiation, which is examined below. Treating teachers' known-answer questions as indirect requests for information offers an explanation as to the preference for both correct and

incorrect answers. Both types of answer give the teacher the information they were requesting, whether or not the pupils know the answer. 'I don't know' and other responses of this type remain dispreferred answers. Whilst they do give the teacher the information they were requesting, these types of answer do not contain any additional information, which the teacher can use in subsequent turns. They are minimal responses which, if genuine, give the teacher little to work with, but also they might be used to avoid answering questions as a face-preserving move and consequently may not indicate a lack of knowledge on the part of the pupil. However, 'I don't know' does not necessarily indicate the pupil's cognitive state, it can also be used as an interaction device. In the majority of cases where a pupil say's "I don't know" in their turn, the teacher either offers the turn to another pupil or changes the topic of the question.

Avoiding dispreferred answers

In classrooms, there are other devices available to both the teachers and the pupils in avoiding the dispreferred response. Three of these result from the fact that in whole-class interactions there are more than two speakers present. The first two are techniques that can be used by teachers to avoid a dispreferred response, whilst the third is available to both the teacher and the pupils.

Firstly, the teacher can offer the question to whole class, by not nominating the next speaker verbally or gesturally.

Extract 23 - Example of a question asked to the whole class.

001 T136 Edward: Tom had (.) 002 ((picks up pyramid and holds it in the air))

003	this, which is a pyramid.	so it's slightly
004	different, to these ones.	these ones are
005	prisms (.) these ones are	pyramids. now why
006	does, can anyone remember	why we call this one
007	a prism. what's special at	pout this one.

Edward lesson 2.

Practically, now around thirty possible next speakers can self-select. Those that do not know the answer do not need to speak, and hence 'I don't know' is avoided. The probability that a pupil in the class can offer a preferred response is far higher than asking one particular individual, and hence the dispreferred response of no answer is avoided. In these situations, the next speaker can be selected in several different ways. A pupil can self-select to be the next speaker or a pupil can be selected by the teacher following a preliminary bidding for nomination by the pupils raising their hands. In each of these cases, a pupil is unlikely to self-select or bid for a turn as next speaker unless they can offer (or believe they can offer) a preferred response.

There are also occasions where a teacher might know which pupils are able to give the required answer when the discussion concerns tasks the pupils have done individually or in small groups beforehand which the teacher has seen, as in the extract from Tim's lesson in chapter 7 lines 72-73, and a pupil can be nominated by the teacher following a significant pause. Alternatively, a teacher might ask a pupil that, based on previous experience, is likely to offer a preferred response.

Secondly, the teacher has the option of asking pupils to talk in pairs first (or groups) before offering (or bidding for) a response.

001 T2 002 003 004 005 006 007	4 Tim:	<pre> ok what do I mean by that, what do I mean, if I said to you two quantities are in direct proportion what do I mean by that. (2.0) if I said two things are in direct proportion, what do I mean. have a guess.</pre>
800		(3.1)
009		ok in fact.
010		(0.7)
011		talk to the people on your table first
т:.		

Extract 24 - Pupils are asked to discuss in groups before giving an answer

Tim lesson 2.

In the extract above, there are two delays of 2 seconds and 3.1 seconds where pupils have the opportunity to bid for the next turn, but only one pupil's hand is raised. Although the option exists to ask pupils who do not have their hand up or to ask the one pupil who does have their hand raised, offering the pupils the chance to discuss possible responses should increase the number of pupils either able or willing to offer a response while also offering the teacher an opportunity to overhear which pupils might give a preferred response.

In addition, when the question is a 'known answer' question and the purpose of asking is to check for understanding, the teacher gathers this information to some extent by observing the number of raised hands. In the situation where the teacher feels that not enough hands have been raised to indicate understanding, discussion in pairs or groups offers a further option of pupils to share what they do know and do understand with the possibility that through collaboration with their peers they may arrive at a preferred answer. The teacher does not actually require the answer itself, just an indication of how many pupils are willing to answer the question.

Finally, a pupil can self-select even though another pupil has been nominated.

Extract 25 - Pupil self-selects after the nominated pupil does not answer.

```
001
   T78 Simon:
                     you don't know. ok someone else then,
002
                      what's the lowest number of days pe-
003
                     someone was absent. Chris.
004 T79
                      (3.8)
005 Т80
         George:
                     zero
     Simon lesson 1.
```

The long delay between Simon and George's turns indicates that Chris is unable or unwilling to offer a preferred response. George offers the correct answer even though by speaking when he is not the nominated speaker is a breach of the rules that govern turn-taking in the classroom and is sanctionable. However, as mentioned in the previous chapter, this option can result in sanctions from the teacher.

It is worth noting that a 'don't know' response is preferred over no answer.

Extract 26 - Pupil offers a non-answer				
001	T147	Edward:	(1.2)	
002			right there is a ten gap between forty and	
003			thirty, but if I add forty and thirty I get	
004			seventy (.) so why (.) why don't I just	
005			stick the minus at the front.	
006			(2.3)	
007			Ashley?	

just 008 T148 Ashley: I can't remember Edward lesson 1

Not all questions asked by teachers are known-answer questions. Many of the questions Richard asks, as can be seen in the transcript in chapter 7, are often about what the pupils have done in their pair work, what they have found difficult or easy, or what they have learnt. In many of these cases the answers given cannot be judged as right or wrong and are not evaluated as

such by Richard (Alpert, 1987). Instead, the answers are evaluated as interesting if they are evaluated at all. In these answers, the hesitations and markers often occur shortly into the turn rather than right at the start, as in Richard's transcript lines 574-576 or lines 599-604. The exception is where the pupil's turn is making a negative assessment of another pupil's turn as in Richard lines 589-593. The preference organisation of question-answer adjacency pairs in these situations is the same as ordinary conversation.

Summary of the preference organisation of question-answer adjacency pairs in secondary mathematics classrooms

The frequency of SPPs prefixed with hesitation, hedging or discourse markers is far higher in whole-class classroom interactions than in ordinary conversation, to the extent that pupils' answers are marked more frequently than not. In ordinary conversation, these markers are used to indicate a dispreferred response. In mathematics' classroom interactions, a wider variety of pupil answers includes these markers, whether they are treated as appropriate or not by the teacher in the following turn.

Pupils may use hesitation, hedging, or discourse markers to pre-empt a negative evaluation. When they answer a question, they do not necessarily know whether their answer is correct or not. Whether they offer their response because they believe it is the correct response, they do not know the correct response but think their response is likely to be accepted, or because they have different interpretations of the expected response is to some extent irrelevant as in all situations it is the teacher with both the knowledge and the authority to make the judgement about the appropriateness of the response. However, the use of markers does

demonstrate orientation to the institutional roles of teacher and pupil and this aspect is discussed in more depth in the next chapter. Since both incorrect and correct answers are treated as preferred, it is the relationship between the FPP, SPP and the teacher's subsequent assessment turn that is key here.

Pupils may also use hesitation markers to indicate that they intend to take the next turn whilst giving themselves time to construct their response. This indicates avoidance of a dispreferred SPP, a non-answer. This is as a consequence of the classroom context where there are several possible next speakers and often the nomination of the next speaker occurs immediately before a TRP.

There are also several strategies that enable dispreferred responses to be avoided that are available to teachers and pupils that are not available in ordinary conversation. The teacher can offer a question to the whole class and oblige pupils to self-select or bid for the turn, increasing the probability that the next turn will include a preferred response. The teacher can also offer pupils opportunities to discuss their responses in small groups where they can be evaluated and accepted by their peers before offering a response to the teacher. However, the use of each of these strategies have different pedagogical consequences. Each strategy offers different assessment information, and involves a different range of pupils and the strategy used needs to reflect the purpose(s) of the FPP.

There are also several structural features of adjacency pairs in the secondary mathematics classroom that reflect the preference organisation of

adjacency pairs in ordinary conversation. Dispreferred SPPs can include accounts and explanations before the delivery of the dispreferred response but also the FPP can be altered to avoid possible dispreferred responses, a common occurrence in many classrooms. Also where the questions are not known-answer questions, the structural features of the preference organisation is the same as that in ordinary conversation.

Repair in Secondary Mathematics Classrooms

As with ordinary conversations, self-initiated self-repairs are the most frequently occurring type of repair trajectory in the secondary mathematics classroom. This is perhaps unsurprising given that they often occur in the same turn as the trouble source so that the very structure of interactions can explain the predominance of self-initiated self-repairs. This preference for self-initiated self-repairs applies to both pupils and teachers.

Other-initiated repairs, on the other hand, are far more common in classrooms than in ordinary conversations. These generally occur in two distinct contexts. The first context relates to the rules that govern turn-taking in the classroom and any breach of these rules is sanctionable. In the data, the majority of these occasions resulted from a pupil self-selecting as next speaker, when, either another pupil had been nominated to speak or when other pupils are bidding for the next term. On each of these occasions, it is the teacher who initiated the repair.

Extract 27 - Pupil is sanctioned for answering a question without putting their hand up.

001	T88	Chris:	because it's in the middle of the er
002	т89	Simon:	shall we do like hands up and stuff Chris
	Simon lesson 3		

However, it is worth noting that the teacher makes a decision when initiating this type of repair and does not always choose to do so. There are several occasions in the data where a pupil has breached the rules that govern turntaking and the teacher has not initiated a repair on this turn and some of these are discussed in chapter 8.

The second context relates to the mathematical context in which the trouble occurred. Mathematical errors or incomplete answers are more likely to be followed by an other-initiated repair than by a self-initiated repair.

When it is the teacher initiating the repair this is most commonly in the form of an insertion sequence, such as another question-answer adjacency pair, which breaks the question down into one more likely to receive an expected answer. This can be either by simplifying the original question, limiting the possible answers or rephrasing the question to be more specific about what an acceptable answer might be.

Extract 28 - Example of a teac	her-initiated repair o	ver several turns	resulting in a pupil self-
repair.	-		

001	T153	Ashley:	er is it minus three degrees
002	T154	Edward:	minus
003	T155	Ashley:	three degrees
004	T156	Edward:	<pre>minus three, why are you say[ing it's minus]</pre>
005	T157	Ashley:	[no it will be]
006			minus seven
007	T158	Edward:	why is it minus seven
008	T159	Ashley:	because Moscow, is it minus ten Moscow?
009			Mosco::w
010	T160	Edward:	ok (.) Ashley carry on
011	T161	Ashley:	er cause I think Mo- Moscow is minus eight
012			degrees and London minus five degrees. I
013			mean minus ten degrees Moscow yeah.
014	T162	Edward:	right. Moscow is minus ten, (.) what's
015			London.
016	T163	Ashley:	minus five
017	T164	Edward:	minus five
018	T165	Ashley:	so that means it's five degrees exactly.
019	T166	Edward:	so the gap is five degrees…

Edward lesson 1.

In these situations, the repair of the initial trouble is most often performed by the original pupil in whose turn the trouble initially occurred, though often after an extended sequence of alternating turns with the teacher. This technique links quite closely to Wood's ideas of funnelling and focusing (1998). Occasionally, the teacher offers the question to another pupil resulting in an other-teacher initiated other-peer repair trajectory. There is an example of this in the extract from Simon's lesson in chapter 7 in lines 333-335, where A has given two possible answers in line 332 and Simon then invites the rest of the class to bid for the term before nominating George to take the next turn.

In the vast majority of the teacher-initiated repairs, the teacher pauses before initiating the repair thus offering a further opportunity for self-initiated selfrepair or the teacher asks for more detail or information, which often results in a self-initiated self-repair. When another pupil initiates a repair, this is often done with no gap or only a short pause between the turn in which the trouble occurred and the initiation of the repair, and the peer usually performs the repair in the same turn.

Extract 29 - Example of a peer-initiated and performed repair.

001	T260	Tim:	what is a pri:me number. Drew.
002	Т261	Drew:	an number that can only be like divided by
003			itself.
004	Т262	Chris:	and one
005	Т263	Drew:	and one.
006	T264	Tim:	and one. good. ok
	Tim le	sson 1.	

In these situations, the original pupil in whose turn the trouble occurred has the option to accept or reject the repair. If the repair is accepted this is done by the original pupil repeating the repair performed by their peer as in Extract 30, also indicating that perhaps if the peer had only initiated the repair and not performed the repair the original pupil would have performed a repair (i.e. suggesting a preference for self-repair).

Some aspects of other-initiated repairs appear far more complex. Not only do the frequencies of repair trajectories vary between teachers, but they also vary between different types of mathematical trouble. These specific differences are discussed later in this chapter.

Self-Initiated Repairs

In all cases where a repair was self-initiated by a pupil giving an incorrect response, the repair was performed in the same turn by that pupil, although the repair is not always performed successfully. For example in the extract from Tim's lesson lines 35-36 the pupil changes her answer from seven point five to seven hundred and fifty.

Extract 30 - Other-teacher-initiated other-peer-repair

001	Т255	George:	fif- no sixty		
002	Т256	Edward:	no: (.) not quite		
003	т257	Chris:	nearer fifty.		
004			(1.1)		
005			fifty five=		
006	T258	Edward:	=fifty five		
	Edward lesson 1.				

In Extract 12 from Tim's lessons, the pupil self-initiates and self-repairs in the turn transition space. In this second extract (Extract 30), George self-initiates and performs a repair, however the answer is still not correct and the teacher initiates a further repair, which is successfully performed by Chris in line 5.

The remaining cases where a repair is self-initiated by a pupil, the initiation is of the form of "I don't know". The following turn is always taken by the teacher, but the repair is not performed in this turn. Either the turn is returned to the pupil who initiated the repair, mostly with a modification of the original question, or it is taken by another pupil.

	001 002 003	T166	Edward:	so the gap is five degrees. is it plus five degrees (0.4)
	004			or minus five degrees.
	005	T167	Ashley:	er
	006			(1.9)
\rightarrow	007			don't know. I really don't know
	800	T168		(3.2)
	009	T169	Edward:	it is five. you're right
	010			(0.6)
	011			but is it plus five or minus five.
	012	T170	Chris:	°minus°
	013	T171	Alex:	er is it plus five
	014	т172	Edward:	you've got a fifty fifty shot Ashley
	015	т173	Ashley:	is it minus?
	016	T174	Edward:	why is it minus.
		Edward	lesson 1	

Extract 31 - Pupil self-initiates a	a repair by saying	g "I don't know"
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In Extract 31, Edward restates the question after Ashley has initiated the repair. The two pupils who take the following turns are ignored by Edward, with his gaze remaining directed towards Ashley. He further prompts Ashley for a response in line 14, (encouraging Ashley to guess, and thus indicating a preference for an answer over a non-answer) and Ashley's response in the next turn is an acceptable answer.

Teacher differences in other-initiation of repairs.

Edward and Simon

Edward and Simon's lessons show a preference organisation similar to that in ordinary conversation following other-initiations. When the teacher initiates the repair, Edward and Simon usually do not perform the repair in the same turn. The turn is primarily returned to the pupil whose response was not what was required, then to other pupils in the class, and the teacher does not perform the repair until both the original pupil and the other pupils have failed to perform the repair.

However, there are exceptions. When the trouble is with an answer that involves an explanation the occurrence of other-teacher-initiated otherteacher-repair is far more frequent than both in ordinary conversation and in the data from the other teachers.

001 002	T15	Alex:	is it minus ten because (0.6)
003			when, when it's in minus numbers you take away
004			get (.) like a higher number so (.) if it's
005			minus ten then you take away minus thirty then
006			it gets to minus forty which is the
007			temperature range, (verhoinshk)
008	T16	Edward:	almost
009			(0.4)
010			almost it needs a little bit. you are thinking
011			about going backwards which is good.
012			(1.6)
013			no?
014			(1.8)
015 016			the reason (.) is because (.) if you think
018			about your number line, if you're starting of at minus forty, (.) if you're thinking of
017			it as a temperature scale, if you're thinking of
010			of it as a thermometer, if you're starting at
020			minus forty and its getting thirty degrees
021			warmer, so you're going up thirty, you're
022			going up
023			(0.4)
024			your scale
025			(0.6)
026			thirty. now if you're reading the temperature,
027			if you're using the thermometer, it's going
028			up the scale
029			(0.4)
030			ok. so you're adding, so you're going up the
031			scale.

Extract 32 - Example where the repa	air is performed by the teacher
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Edward lesson 1

Here Edward indicates that Alex's answer is not sufficient to answer the question in the way that Edward is expecting. His initial reaction is couched

in vague terms and contains several significant pauses before he offers his own explanation of why -40 - -30 is -10. The nature of the trouble here is in communicating the method to the class as a whole. Alex is hesitant when offering her own explanation but also starts with the answer and works back to the question. It is also possible that the 'rule' Alex is using could also be a source of trouble if this is not a rule that the teacher wishes the rest of the class to adopt. Edward's own response focuses on using different contexts such as the number line, thermometer scales, and the original context of the task itself, temperature changes between locations. This response is also directed at the class as a whole,

There are two other occasions where a repair is both initiated and performed by the teacher in the turn immediately following the trouble source. Both these occasions occur in Simon's lessons and both involve a pupil attempting to apply a mathematical term to a situation where it does not apply:

Extract 33 - Other-teacher-initiated	and performed repair
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001	T140	Chris:	(inaudible) line of best fit
002	T141	Simon:	it's not actually a line of best fit. we are
003			going to draw a trend line. we are going to,
004			because a line of best fit would be a straight
005			line going like that ok. that might be another
006			way of doing it. we are going to draw a trend
007			line
		_	

Simon lesson 3.

In Extract 32 and Extract 33, the trouble source can be seen as a rule or description that a pupil has made relevant to the interaction, but is either not relevant to the mathematical situation or is interpreted as a potential source of mathematical difficulty. Whilst in many classrooms 'incorrect' ideas and

descriptions can be used to develop an argument or support the teacher's design of topic progression, this is not always appropriate in mathematics. Mathematics is a hierarchical subject where pupils build upon prior knowledge and skills, and consequently 'incorrect' rules and description can often be detrimental to the future work and development of the mathematics.

Both Edward and Simon use their turns following a trouble source predominantly to initiate a repair but support the pupils in performing a selfrepair. They do this by locating the trouble source,:

Extract 34 - Locating the trouble source in a repair initiation.

	T149 T150		twenty eight twenty eight what
003	Т151	A:	um er (.) degrees Celsius
	Edward	lesson 1	

Edward lesson 1.

repeating the question by directing it back to the pupil as in Extract 31 above,

or following up with focussing or funnelling questions or an insertion sequence as in Extract 28. When self-repair does not occur, they then

encourage peer repair:

Extract 35 - Offering the performance of a repair to a peer.

001	Т55	Harry:	even chance
002	Т56	Edward:	an even chance. Ok … Harry why do you think
003			it's an even chance.
004	т57	Harry:	you could win in ((inaudible))
005	т58	Edward:	okay right. does anybody want to take that a
006			little bit further and explain to Harry a
007			little bit more, why we might need to er think
			about it in a little more detail. Ashley?

Edward lesson 3.

However, some sources of trouble are repaired instantly as in Extract 33,

Extract 15 and:

Extract 36 - Immediate repair performed by the teacher.

001 T138 B: it's an octagon 002 T139 Edward: ok this one's actually a hexagon. so they've 003 both got, they've both got a hexagon on the 004 bottom

Edward lesson 2.

Notably, in each of these cases the trouble resulted from a problem with the use of terminology rather than an issue directly relating to the topic being discussed. These are the only cases in the data where the trouble is of this nature, though there are cases where the trouble source is the terminology but the topic of discussion is the use of terminology and here the preference for self-repair remains.

Tim and Richard

In both Tim's and Richard's lessons, the number of other-teacher-initiated other-teacher repairs are very rare. In Tim's lessons, peer-initiated repairs and peer-repairs are far more common than in Edward's and Simon's lessons. Whilst in Richard's lessons very few repairs related to the correctness of given answers are initiated or performed.

Tim

Two situations occur in the data that frequently result in peer-repairs. Firstly, where Tim nominates the next speaker to answer and this answer is incorrect, if there is no self-repair and no other pupil self-selects, Tim nominates a different pupil to take the next turn.

001 002 003 004 005 006 007 008 009 010	T140	Tim:	<pre>what would be the probability if I had two dices of rolling two sixes. let's, we're talking about a total of twelve aren't we.((writes 12 on the board)) if I had two dice, (0.5) what would be the probability (0.5) of (2.2)</pre>
011 012	T141	Ashley:	rolling two: sixes on two dice. Ashley. one in twelve

013	T142	Tim:	one in twe:lve.
014			(2.7)
015			no::. yes.
016	T143	Chris:	two in twelve.
017	T144	Tim:	no.
018	T145	George:	is it like a half
019	T146	Tim:	no? what wh-
020			(0.3)
021			if you rolled
022			(0.9)
023			two sixes and you got about ten you'd get
024			five each. two sixes on two dice. it's very
025			unlik it's not
026			(0.6)
027			unlikely if you know what I mean. yep.
028	T147	Drew:	two sixes, two sixths
029	T148	Tim:	no. anybody else
030	T149	Sam:	one in thirty six
031	T150	Tim:	good. thank you Sam.
	Tim le	sson 4	

Here, the first pupil nominated by Tim offers an answer. Tim repeats this answer in the following turn. This is followed by a long pause of 2.7 seconds in line 14. This pause offers the opportunity for the first pupil to self-repair or for other pupils to self-select as next speaker. When this does not happen, Tim continues as current speaker with a hesitant negative evaluation of Ashley's answer before nominating the next speaker. (Here the word yes is directed at a pupil who is bidding for a turn by raising their hand and indicates that this pupil has the next turn, it is not a positive evaluation of the previous turn). The turns then alternate until a correct response is given. Other-peer-repairs in this form also occur in all the other teachers' lessons but very rarely compared to Tim's lessons. In Edward's and Simon's lessons the turn usually returns to the pupil in whose turn the trouble was, rather than being redirected to another pupil.

The second situation or pattern of interaction only occurs in Tim's lessons. Here, Tim offers the question to the whole class. He does not nominate the next speaker. Pupils then need to self-select in order to answer the question. In many cases, more than one pupil self-selects and this results in pupils' turns overlapping, and often more than one answer is offered. Occasionally, pupils make evaluations (initiate repairs) on other pupil's answers. The other teachers do ask questions to the class as a whole, but only in Tim's lessons do multiple pupils self-select to answer these questions and give contrasting responses.

Extract 38 - Multiple s	peakers self-selecting	to answer a question.

001 T192	Tim:	ok. six shirts take two hours to dry on a
002		washing line, how long will it take to dry
003		three shirts. should be a question mark at the
004		end.
005 T193	- <u>1</u>	what?
006 T194	Pupil:	oh that is easy
007 T195	Pupil:	another trick que[stion]
008 T196	Pupil:	[one hour]
009 т197	Tim:	one hour
010 Т198	Pupil:	no that's a trick [question]
011 Т199	Pupil:	[that's a] trick question
012 т200	-	it'll take two hours
013 т201	Pupil:	it'll take two hours
014 т202	=	Oh
015 т203	1	it'll take one hour because
016 т204	-	it will take one hour.
017 T205		no it wouldn't [it would take]two because
018	raptr.	they
019 T206	Pupil:	[no it wouldn't]
020 T207	=	
	- <u>1</u>	there all t-shirts aren't they
021 T208	1	it doesn't matter how many's on the [line]
022 T209	-	[yes]
023 T210	I	it's all gonna take two [hours]
024 T211	Tim:	[good]. ok? you've
025		got to know whether (inaudible)
Tim I	esson 2	

In the Extract 38 above, four different pupils self-select as next speaker, with the fourth one interrupting the previous speaker. Only the fourth speaker offers an answer to the question. Tim then repeats this answer in line 9 before another pupil self-selects as next speaker, negatively evaluating the answer given and offers an explanation as to why the answer is incorrect. In essence this pupil is initiating a repair but does not actually perform the repair themselves, offering an opportunity for self-repair. A different pupil again offers another answer in line 12 and the turns alternate with several pupils attempting to offer explanations for why their answer is correct. In this example, the explanation that is finally heard by the whole class is the correct answer and Tim positively evaluates this answer in line 24. This is one of the few examples where a pupil has initiated a repair on another pupil's answer to a teacher question and has not performed the repair themselves in the same turn.

In the extract below, a pupil initiates and performs a repair on a peer's answer:

001	Т236	Tim:	what is a pr i:me number. Drew.
002	Т237	Alex:	an number that can only be like divided by
003			itself.
004	T238	Chris:	and one.
005	Т239	Alex:	and one.
006	T240	Tim:	and one. good. ok.
	Tim le	sson 1	

Extract 39 - Pupil initiates and performs a repair on a peer's answer.

Here Alex's answer is incomplete and Chris indicates this by self-selecting as the next speaker and completing the answer. There is a very short pause between these two turns so Chris has prevented Alex from self-repairing both by taking the turn immediately in the turn transition space and by performing the repair at the same time as initiating the repair. Alex then validates this repair immediately in the following turn by accepting the addition before Tim offers a positive evaluation of this repair in line 6.

There are several examples in Tim's lessons where another pupil prevents a peer from self-repairing by self-selecting as next speaker before the pupil in

whose turn the trouble occurred has the opportunity to take up the turn, often by interrupting them before their turn has completed. In addition, other-peerrepairs are usually performed at the same time as the repair initiation. This is in stark contrast to the preference organisation of repair both in ordinary conversations but also in teacher-pupil interactions.

Tim also handles negative evaluations differently from the other teachers. As with all the teachers, Tim rarely makes negative evaluations of his pupils' turns. He will explain why answers are not correct or effectively ignore incorrect answers as in Extract 37. When Tim does make a negative evaluation it is usually unmarked as in Extract 37 lines 17 and 29, though is occasionally given hesitantly as in line 15.

Richard

In Richard's lessons, the repair of mathematical trouble is very rare and this can be explained by nature of the questions asked and the answers given. Richard's lessons contain very few examples of the traditional IRF sequence and more closely resemble the question-answer adjacency pair that occurs naturally in ordinary conversation. In the IRF sequences that dominate the other three teachers' interactions, the third turn usually consists of an assessment following the adjacency question-answer pair. In Richard's questions generally focus on the process of doing mathematics and not on the product of right or wrong answers. He frequently asks how a particular pupil did something, what their method was, and where numbers they have given him have come from. Frequently, Richard is searching for more than one method and appears interested in what a pupil has done and why they

have done that and consequently often asks for more information in the third turn or rephrases the answer to check his own understanding of what the pupil has said. Consequently, there are very few examples where an otherinitiated repair occurs when the trouble is mathematical in nature as there are very examples of mathematical trouble in the whole-class interactions. One example is:

001 002 003 004 005 006 007	Τ18	Richard:	<pre>ten to put it back (like) or dividing by a hundred to put it back again. do you have a comment on that. (3.2) I think that's right Jack you were saying multiply by a hundred but then don't forget</pre>
008 009	т19	Chris:	to divide by a hundred again later isn't it by a thousand or no ten thousand.
010	T20	Richard:	because you've done a hundred and another
011			hundred
012			(1.4)
013			um: what about this equals sign here is this
014			(1.5)
015 016			what's the- er is that correct
016			(0.4) or should I not have done that. George?
017	Т21	George:	cause you've timesed both of- timesed both
019	101	000190.	of them so the value of the fractions
020			they're not changed, they're the same so you
021			don't need to divide again at the end.
022	Т22	Richard:	who agrees with George.
023			(1.2)
024 025			people always agree with George don't they.
025 026			alright um th- it is equal isn't it and it's really equal so these two (.) divisions.
020			doesn't matter which one we do they both
028			give the same answer. this one might be
029			easier like Alex says but we don't have to
030			put it back again later. are you happy that
031			(.) we don't need the ten thousand, well not
032			ten thousand times that?
	Dicho	rd loogon 1	

Extract 40 - Example where Richard initiates a repair on a pupil's turn.

Richard lesson 4

Alex has previously described how she multiplied the numerator and denominator of a fraction by one hundred to make both integers (the denominator is currently a decimal with two decimal places). She completed her turn by saying that you needed to divide by one hundred at the end. In the immediately preceding turns, the interaction has focused on simplifying the resulting fraction and in line 1 Richard is returning to Alex's suggestion of dividing by a hundred at the end. Richard often asks the class if they agree with an answer or if they want to comment on one, irrespective of whether the answer is correct or not, though he does this far more frequently when the answer is incorrect. Chris gives a vague response in line 9, offering an alternative answer but also indicating either that he believes the original answer to be incorrect or that he has interpreted Richard's turn as a repairinitiation. In line 17, Richard nominates George who offers an answer that Richard positively evaluates. In Richard's next turn in lines 10-17, there are several pauses each offering Chris and the other pupils the opportunity to self-select as next speaker, and consequently perform a repair on the incorrect answers.

Implications of Preference Organisation on Whole-Class Questioning. In the previous chapter, the relationship between the structures of turn-taking and wait time was explored and the implications for teaching and learning mathematics were discussed. Wait time is also relevant to discussions on preference organisation, both of question-answer adjacency pairs and repairs. Pauses during and between turns are used often in ordinary conversations to offer opportunities for self-initiated repairs. Delays in the taking up of the next turn either indicate some form of trouble, or anticipate a dispreferred response. In the mathematics classroom, the data have shown that any answer to a teacher's question is preferred to a non-answer (such as silence). By lengthening the pause following the teacher asking a question, further opportunities are offered for pupils to take the next turn,

whether that is after being nominated or by self-selecting. This avoidance of a dispreferred response relates to the findings that increased wait time reduces the number of teacher questions that are not responded to (Rowe, 1974).

Implications for question types.

We have seen from the data that interactional work is done to avoid dispreferred answers to teacher questions. One way in which the teacher can avoid such interactional work is to ask questions that are likely to be answered and likely to be answered correctly. This also relates to the dispreference of other-initiated other-repair (whether by teacher or peer).

In ordinary conversations and in classrooms, interactional work is done to avoid other-initiated and other-performed repairs. This might be through offering further opportunities for self-initiated repairs or through insertion sequences in the question-answer adjacency pair that alter the question to support pupils in reaching a preferred answer. We know both from previous research and this research that these techniques for avoiding dispreferred actions are common in many classrooms. However, another option is also available, which is the avoidance of questions that might result in a dispreferred response. This would offer an explanation for the dominance of closed, low-level questions in whole-class interactions.

Closed and low-level questions not only limit the range of possible answers that a pupil can select from and often rely on the recall of known information, but also reduce the risk of a dispreferred response. Asking a higher- level question might require cognitive processing before an answer is offered,

which often results in a longer pause between the question and the answer. Asking open questions where the quality of the response is also dependent on an ability to communicate clearly as well as the mathematical content, can give rise to more trouble types. An answer to an open question or a question that requires an explanation may be mathematically what the teacher is expecting but may be communicated in such a way that the other pupils (and possibly the teacher) may have trouble understanding the response given. Alternatively, the answer may involve an 'incorrect' procedure or definition that in themselves may cause interactional trouble.

The order of preference over self- and other- initiated and performed repairs, whether we make a distinction between peer and teacher or not, is the same as in ordinary conversation. The noticeable difference is the more frequent occurrence of other-initiated and other-repairs, though they are still structurally dispreferred to self-initiated self-repairs. Similarly, the rules that govern turn-taking have the same structure. The noticeable difference here is the frequency with which the teacher nominates the next speaker or self-selects as next speaker.

Transferring the rules that govern turn-taking and the preference organisation of repair in ordinary conversation to a classroom context offers an explanation for the dominance of closed low-level questions, and the frequent use of teacher 'scaffolds' in whole class interactions. Consequently, either this dominance will continue, as in many of the lessons in this study, or work will need to be done to alter and adapt these rules and the preference

organisation of repair to enable different question types to be used successfully.

The preference organisation and the handling of repairs does not only affect the nature of questions asked. In all the lessons from all four teachers, there is still a clear dispreference for other-initiated other-teacher repairs despite their more frequent occurrence when compared to ordinary conversation (McHoul, 1990). Seedhouse (1996) argues that this dispreference combined with the dispreference for negative evaluations gives learners the message that mistakes should be avoided, even when the teacher explicitly claims that making mistakes is part of the learning process. This may be the case in Simon's and Edward's lessons, where there is a clear preference for selfrepair and negative evaluations are often mitigated and marked in some way. In Tim's lessons, repairs are also frequently made by a pupil's peers and many of Tim's negative evaluations are bald. Tim also frequently initiates a repair by explaining why a response is incorrect, without any evaluation and before offering the next turn to a peer. There is a difference in the ways that mistakes are handled in these classrooms that give different impressions on the role of mistakes in learning mathematics. By baldly negatively evaluating mistakes and initiating repairs by explaining the possible cause or explanation for the trouble, errors or mistakes are constructed as part of the learning process. A key feature of working mathematically is making conjectures that can be tested and rejected, modified or accepted as a result. Not only do pupils need to feel they can make mistakes and conjectures, however, they also need to be able to make their own judgements about accuracy or appropriateness. The mere presence of an evaluative move in

the IRF pattern of interaction reinforces the notion of right or wrong and the teacher's role in making this distinction. This also leads to pupils adjusting their responses to meet teacher's expectations rather than in response to their own awareness of the mathematics (Sfard, 2001).

However, it is not solely the teacher that constructs this IRF pattern, it is a joint production by the teacher and the pupils. In this chapter I have demonstrated that pupils often mark their responses in some way and hesitation or phrasing turns as questions, for example line 358 in the transcript from Simon's lessons, require an assessment or evaluation from the person taking the next turn, in this case the teacher. The teacher is consequently obliged to make an assessment in their turn. The teacher can do interactional work to avoid making this assessment, such as by explicitly asking another pupil to make the assessment, but the preferred response to pupils' responses phrased as questions is an assessment.

If we want our pupils to make conjectures then 'I don't know' may still be dispreferred but incomplete or partial answers should not result in the otherinitiation of a repair or an assessment as we would want pupils to make their own judgements about whether a conjecture needs accepting, modifying or rejection. In addition, if we want argument and debate then question-answer adjacency pairs might not be the most useful adjacency pair to encourage this.

So what should the role of whole class question and answer sessions be? Both this chapter and the previous one have demonstrated that the structures of turn-taking and preference organisation in the mathematics

classroom support and enable knowledge sharing, the inclusion of a number of pupils, and the assessment of pupils, and can be used to increase the wait time between turns and to encourage pupils to explain their answers. The extracts from Richard's lesson also show that question and answer adjacency pairs can be used to report on pair and group work, which might include conjectures being made. However, whole-class question and answer sessions may not be suitable for supporting pupils in making conjectures, generating argument and debate or seeing making mistakes as part of the learning process.

Summary

The preference organisation of question-answer adjacency pairs differs in the mathematics classroom from ordinary conversation. Pupils' answers are often marked, irrespective of their context. This could be because pupils are treating their answers as dispreferred to mitigate a possible negative assessment in the following turn, or because they are anticipating that their answer will not match that of the teacher. In these situations, the judgement about the appropriateness or correctness of the answer is made by the teacher through the IRF structure. However, pupils may also mark their answers as a face-preserving move as in the whole-class situation answers are given in front of an audience of their peers. Finally, the markedness may also be a consequence of the turn-taking structure. In all these situations, the markedness is not an indication that it *is* a dispreferred response.

There are also differences in the ways that the teachers in this study handle repairs. The preference for self-initiated self-repairs that exists in ordinary conversations is also prominent in all the lessons. However, following a

teacher's initiation of a repair there are differences in the prevalence of selfrepair, teacher repair and peer repair between the teachers. There are also differences in how the different teachers initiate the repairs and how sources of mathematical trouble are negatively evaluated. The combination of these differences leads to different messages about the role of mistakes in the learning of mathematics.

The preference organisation of repair in Edward's and Simon's classes is similar to that in ordinary conversation. There is a clear preference for self-repair, but also peer-repair is more frequent than teacher repair in both classrooms. There are three exceptions where Edward or Simon initiate and perform a repair without offering opportunities for self-repair or peer-repair first. The first is when the trouble source is within a pupil's explanation rather than the original answer. The second is when pupils' have used mathematical terminology inappropriately. The final exception is where the mistake is not directly relevant to the focus of the lesson. Both Simon and Edward frequently support pupils to self-repair through locating the trouble source, repeating the question or through use of insertion sequences such as funnelling or focusing questions. So both teachers predominantly use mistakes, but through their handling of these sources of trouble they are indicating that mistakes are something to avoid.

Peer initiated and peer performed repairs are far more common in Tim's lessons than in the other teachers'. These peer-initiated repairs are often unmitigated and do not always offer an opportunity for self-repair. Negative

evaluations are also rare in Tim's lessons, but where Tim does make them, they are given baldly. Tim will often offer explanations as to why a response is a source of trouble or will ignore the response. Again, Tim uses sources of trouble as teaching opportunities but through his handling of these sources of trouble he is not indicating that they are to be avoided. Tim often uses sources of trouble to develop points of contention between different pupils, which enables the pupils to handle the trouble.

In conclusion, the preference organisation of both adjacency pairs and repair have an effect on the handling of the mathematics. They affect the role of questions and the nature of questions asked and the nature of the responses given. They also affect the way that mathematical mistakes are viewed within the classroom. In the next chapter, the structure of turn-taking and preference organisation are combined with other discursive features of classroom interactions to examine further how these differences have consequences for the nature of mathematics that the teacher and pupils are doing in whole-class interactions.

Chapter 10: Discursively Constructing Learning Mathematics and Mathematics Classroom Identities

"by saying things in different ways, different things are achieved"

(Barwell, 2003)

The previous chapters have focused on the sequential organisation of whole class interactions using a conversation analytic approach. Particular features of interactions, such as the structure of turn-taking and the prevalence of the IRF pattern have been examined and have been shown to be characteristic of secondary mathematics classrooms. In this chapter, the focus of analysis extends to a more detailed examination of the identities of teacher and pupils in each of the extracts. The same method of analysis is then used to examine the nature of the mathematics that is discursively constructed in each of the extracts.

The conversation analytic approach to the analysis of identity is through a turn-by-turn analysis of the identities that participants orient to. Any assumptions about the relevance of particular identities, such as female, white, married, are ignored unless an analysis of the sequential context demonstrates that the participants themselves draw upon these identities. Schegloff (1997) argues that any approach that begins with such assumptions is imposing the analysi's own perspectives onto the analysis. The analysis in this chapter starts with the identification of different participants within the interaction and the classroom identities of teacher and pupil are identifiable through the structure of turn-taking. Characteristics

such as gender and ethnicity are not included until they are demonstrated to be relevant to the participants themselves through their interactions.

The identity of a participant can be characterised in many different ways and any ethnomethodological approach, such as conversation analysis or discursive psychology, would argue that it is the participants themselves that demonstrate what characterisations are relevant. When an analyst imposes their own categories of identity it is the analyst who is deciding which categories are relevant, and consequently which identities are not relevant, not the participants. For example, the inclusion of descriptions of the gender of individual participants immediately makes this categorisation a factor to consider in any analysis, whether consciously or not. This does not mean that at the macro level there are no differences between the participation of males or females, but if the structure and content of the turns does not reveal a difference in a turn-by-turn analysis, then claims about the effect of gender would not be supported by the transcripts when using a CA approach.

Ethnomethodological approaches do not enter into the debate as to ontology of identity or to the nature of reality (Wowk, 2007). The truth or validity of identity categories is not considered, equally any cognitive analysis about the relationship between identity and what participants think, feel or do is also not something that the conversation analytic approach takes a position on. Instead, their analyses focus on how people display and orient to identity and the consequences this has on the interactional activities (Benwell and Stokoe, 2006). Discursive psychologists take the view that discourse constitutes identity and ask the question how are the identities discursively

produced? In this chapter, I examine how the participants' identities are discursively constituted during whole class interactions using the tools of CA, but also using the same analytic techniques, I examine what it means to do mathematics during these interactions. Identity is something that you 'do' in interaction, all utterances are doing something and this chapter focuses on the doing of teacher, pupil and the doing of mathematics.

The conversation analytic approach to identity has largely developed from Sacks' membership categorisation devices (Sacks, 1995). We impose order on the world so that it has meaning to us, and one of the ways we do this is by categorisation. Categories enable us to infer particular features that are associated with the category. For example, the category of teacher infers features such as expert, authority, professional, caring and so forth. Any person can be a member of any number of categories, each implying a range of characteristics. Additionally, a person who displays certain features can be treated as a member of an associated category; "not only do categories imply features, but features imply categories" (Antaki and Widdicombe, 1998). A teacher, for example, will control the turns, topic and will ask questions to which they know the answer, demonstrating this by evaluating the answer given. However, a pupil can also orient to the category of teacher if they control the turns, topic and use the IRF pattern in their interactions.

An analysis that focuses on what identities participants themselves orient to involves refraining from describing the participants in classroom interactions using the standard relational pair of teacher or pupil unless the social actions

of the participants indicate that these identities are consequential in the interaction; the identities are having a visible effect on the interactions. This also means that identity, or the construction of identity, is indexical to the interaction in which it is oriented to. Furthermore, during interactions, individual utterances are reflexively related to the utterances that came both before and after (Green, *et al.*, 1988, p.19). Consequently, identities are fluid and dynamic; developing, altering and adapting through these interactions.

Zimmerman distinguishes between three different types of identity that are oriented to in interactions; discourse identities, situated identities and transportable identities (1998). Discourse identities are assumed by participants in interactions in a turn-by-turn basis, and include identities such as current speaker, listener, questioner or answerer. By assuming certain discourse identities, other discourse identities may be consequently assumed by other participants. For example by assuming the discourse identity of current speaker, other participants are required to assume the discourse identity of listener (though they may not actually do this). Situated identities relate to the contextual situation in which the interaction is taking place, so in a classroom interaction the situated identities might include teacher and pupil. Finally, transportable identities are carried by participants across interactions in different settings, but are not necessarily made relevant in the interactions. Such identities include for example, white, female, mother, and daughter and are often physically or culturally 'visible' in the interaction, but not necessarily invoked or oriented to in an interaction (Zimmerman, 1998, p.91).

Richards (2006) adds to Zimmerman's categorisation of identities in interactions including a 'default' identity that "derives entirely from the context in which the talk is produced and applies where there is a generally recognized set of interactional expectations associated with that context" (p. 60) such as teacher and pupil, with the relevant discourse identities of questioner and answerer. These are identities that the participants would be expected to orient to in the interaction because of the context in which it was taking place. Situated identities are associated with the activities that are being done in an interaction, but during a classroom discussion the situated identities of teacher and pupil may cease to be relevant as an interaction develops. However, the interaction returns to a structure that contains the features of a teacher-pupil interaction, and consequently the teacher pupil identities are default identities because of the situational context in which the interaction occurs.

The identity categories of teacher and pupil are not predetermined but are demonstrated in the interactions. The identities of teacher and pupil are apparent through who controls turns and topics, asks questions, answers questions, gives instructions and makes evaluations. However, it is not only these organisational features that makes these default or situated identities apparent. Language is constitutive, the words that are used to describe concepts or activities create the meaning that these have. Consequently, the words used to describe participants constitute their identities, and the words used to describe mathematics or the activities of mathematics constitute the nature of the mathematics or mathematical activity. It is not only the words that constitute the identities, however, but also what is done with those

words. It is this constitutive nature of language and interaction that is drawn upon in the second half of this chapter, when the nature of mathematical activity is examined in each of the extracts.

Boaler (2002) argues that the nature of mathematical activities pupils participate in during their mathematics lessons not only affects their relationship with mathematical knowledge, but also their own developing identities as learners and as people. In this chapter, I examine how the teachers and pupils talk about mathematics and mathematical tasks and activities, rather than examining the nature of the particular tasks themselves. Whilst Boaler's research clearly demonstrates a relationship between the nature of tasks and pupils' developing identities, I will argue that the way these tasks and activities are talked about and talked into being may also affect pupils' developing identities.

In the next section of this chapter, I will also argue that the situated identities of teacher and pupil that are oriented to in the whole class interactions are different for the different teachers. That is to say the teacher and pupil identities not only include different discourse identities, but also different situated identities. This is done through a turn-by-turn sequential analysis of the interaction in each of the extracts.

Throughout this chapter, I shall describe participants as 'orienting to', 'making relevant' or 'treating as' particular identities to reinforce the idea that by orienting to, making relevant or treating as an identity, participants are doing something.

Doing Teacher and Pupil

Tim's Lesson and the Identities of Teacher and Pupil

In the first line of the extract from Tim's lesson, line 1, Tim's orientation to the situational identity of teacher and his pupils' orientation to their identities as pupils is immediately clear. The discourse marker 'ok' here indicates a change in topic, and this is followed by a pause of 0.6 seconds. In Sinclair and Coulthard's terms, this forms a framing move (Coulthard, 1992, p.22). In this pause, there is no attempt by any of the participants to self-select as next speaker, despite the long length of the pause. So this pause demonstrates the participants' orientations to their situated and default identities.

When Tim continues the turn with a focusing move by saying "your first thing today", placing emphasis on the word first, Tim is indicating that he is about to give his pupils a task, and that this task is only the first in a series of tasks that he is going to set. Another indication of Tim orienting to his default and situated identity of teacher.

He describes the task as a 'problem', identifying himself as a problem-poser. He then asks his pupils to look at the problem, before indicating that he is about to ask two questions about the problem. Here Tim is reinforcing his role of designer, setter and controller of tasks, making it clear that his pupils will need to assume the identities of observer and reader of the problem and then answerer of questions. Tim is explicitly identifying the identities that his pupils will need to assume, and what activities they will need to do.

Next Tim asks his pupils to 'try your best' and 'try and understand' in lines 11-12. Again, Tim's identity as teacher is apparent. He is the one telling the other participants what to do. The emphasis here is on 'trying', 'having a go' and 'understanding' which together with 'how far you can' indicates that the focus of the task is on the process of doing mathematics and not the product.

Tim then outlines the question, mostly reading from the whiteboard, adding 'being the generous man that I am' and 'because I'm not totally generous'. By posing the problem in the first person, Tim is personalising the problem. The problem is about him, and at the same time is assuming a transportable identity as a 'generous man'. Tim then asks the two questions he referred to earlier in his turn, assuming the discourse identity of questioner. His turn ends with him re-asking the first question, modifying it to be more specific. In lines 23 to 27, where Tim poses the two questions, the first question asks "how much will I donate" whereas the second question is phrased as "how much will you donate". This first question is both phrased in the first person and is described as "obviously easy" in line 25. In contrast, the second question is initially phrased as 'you' donating and the responsibility for answering the question changes from 'you' to us 'together' in lines 26-27. This change in pronouns emphasises the change in state of the problem being about someone in particular to become one that the pupils own, which Mason et al. identify as a feature of the transition from entering the problem to attacking the problem (Mason, et al., 2010, p.35).

After a long pause, two speakers offer answers. Neither speaker has been nominated as the next speaker, but this self-selection by both pupils

indicates that they are orienting to Tim's utterance to "do the first one together". Tim takes the next turn, repeating exactly the answers given by the pupils, assessing or evaluating the answer as correct, recording the answer on the whiteboard and ending his turn with another question for which he nominates a pupil following a minimal pause. Each of these activities demonstrates Tim's orientation to the situated identity of teacher. It is a teacher who makes assessments and evaluations of answers, decides what should be recorded and nominates the next speaker.

The next turn is taken by the nominated speaker, Harry, in lines 35-36 who gives a minimal answer in that it only contains the information needed to answer the question and no more, though the turn is lengthened by a self-initiated self-repair. This answer is written on the whiteboard by Tim and then repeated in line 38, indicating that Harry's answer is appropriate and correct.

Tim continues the turn by re-directing the doing of the task to the pupils, using 'I want you' twice during the turn and the problem is about how much the pupils donate. Tim also makes a distinction between what his pupils will do individually, 'you', and what they will do as a whole class, 'we'. In lines 50-53, the problem is again divided into two parts: the calculation of how much will be donated in the first four weeks, where it is the pupils doing the donating, and how much is donated in total, where it is Tim that is doing the donation. The use of 'I' and 'you' in lines 39-52 emphasise the identities of teacher and pupil, where Tim is orienting to the identity of teacher through his controlling of both what needs to be achieved and what the other

participants, the pupils, need to do. These lines focus on the doing of the task. In line 53, Tim shifts to asking "how much am I going to donate", consequently returning to the original problem the class are solving.

There are several noticeable pauses during this turn, during which no other speaker self-selects as next speaker. Tim also indicates how much time they will have to undertake the task and explicitly says 'talk amongst yourselves', making interaction relevant to the task. Each of these features continues to demonstrate that both Tim and the rest of the participants are orienting to the situated and default identities of teacher and pupils.

In line 73, Tim asks the pupils 'do you stop'. One pupil immediately responds 'nope' to which Tim responds 'why not, hands up'. The why not on its own would have returned the turn to the pupil who responded in the previous turn but Tim adds 'hands up', which requires this pupil and others to bid for the next turn. Also, by making the rules of interaction explicit, Tim is orienting to the situated identity of teacher. After a long pause of 1.7 seconds in line 76, Tim repeats the question 'why not' before nominating Jamie to take the next turn.

Line 73 also marks a change in the activity being undertaken in the interaction. At this point, Tim asks the question 'do you stop', which initially results in another participant self-selecting as next speaker with the answer 'nope'. Whilst the pupil is orienting to his discourse identity of answerer of Tim's question, by self-selecting with no gap between turns, he is not orienting to a situated identity of pupil. However, Tim is orienting to his situated identity by explicitly asking for hands up at the beginning of his

subsequent turn. The activity that Tim is asking his pupils to do has changed from remembering facts or doing arithmetical calculations to explaining and justifying. This transition has been constructed through Tim's discussion of the calculations and the emphasis placed on thinking before Tim asks the question in line 73 as discussed in the paragraph below. In each of the pupils' turns between lines 78 and 92, the speakers are explaining or justifying whether or not the sequence is finite. Tim's turns during this interaction are repeating either the question as in line 82, or the previous explanation or justification, as in lines 80 and 87. Tim asks the question 'why' the sequence is finite or infinite a total of five times in response to answers that it does not stop (e.g. lines 74 and 101) or that it does stop (e.g. lines 83, 99 and 100). So whilst the discourse identities of questioner and answerer are oriented to throughout the interaction, the pupils who speak take on the additional discourse identities of explainer and justifier and Tim's insertions encourage and support these identities.

In line 104, Tim starts his turn by repeating the end of the previous speaker's turn, before assessing it as an appropriate answer 'in terms of realism'. He is agreeing with those that said you must stop, but at the same time indicating that another perspective may also be appropriate and explaining why the answer is appropriate.

In line 125, the speaker assumes the discourse identity of questioner whilst Tim, in line 127, acts as answerer. However, in Tim's next turn he rephrases his answer as a question by adding "isn't it" in line 133, reorienting to the situated identity of teacher and questioner. The extract ends with Tim

summarising what they have found, defining the term 'limit', remembering what was done yesterday and comparing this with what he is planning on doing this lesson. Here Tim is managing the plan for the lesson and situating it within the topic the class is currently working on.

Throughout the extract, Tim orients to the role of teacher and the other participants are both treated as pupils and orient to the role of pupil. The role of teacher includes posing problems, asking questions and initiating repairs. Tim uses his turns to shift the focus of attention and to develop points of contention. He also constrains the turns of his pupils who consequently explain and justify their answers through the joint construction of a point of contention.

Simon's Lesson and the Identities of Teacher and Pupil

The extract from Simon's lesson begins with Simon introducing the task, which he describes as doing some practice, orienting to the situated identity of teacher. He begins by reminding the other participants about a sheet they had worked on in a previous lesson, which he 'gave' them and which they 'filled in'. This reference to a previous lesson "inducts students back into this specific classroom collective with a group who shares an intellectual and social past, present and future" (Atwood, *et al.*, 2010, p.21). This is followed by Simon outlining how he intends them to use this same sheet today. In this first part of his turn, Simon is orienting to identity of teacher, controlling the tasks, and the resources. He is describing the (mathematical) activities that he wants to do as 'filling in' tables, doing practice, going through examples, and remembering. He identifies himself as reminder of

procedures by describing the going through examples as reminding (lines 247-8), and consequently expert.

Towards the end of this turn, Simon identifies himself as an honest person and a nice teacher, attributing the description of 'nice' to another teacher, consequently adding authority to the transportable identity. This first turn ends by Simon assuming the discourse identity of questioner and is followed by a pupil assuming the role of answerer. There are several pauses of considerable length during this first turn, including one of 7.2 seconds after a TRP, yet no other participant self-selects to take the turn, even following statements that take the grammatical form of a question. In particular, at the end of the turn, there is a pause of 0.9 seconds in line 267 between Simon asking a question and him nominating Charlie to take the next turn. Here, all the participants are orienting to the situated identities of teacher or pupil, by adhering to the rules of turn-taking (see chapter 8) in the formal classroom rather than the rules of ordinary conversation.

In line 270, Simon evaluates the pupil's turn, orienting to the identity of teacher, before reinforcing his identity as a 'nice' teacher by contrasting his actions with those of the examiners and then the textbooks, consequently identifying the examiners and authors of textbooks as not nice. He builds up a description of the examiners and textbook authors as people who "expect you to know" and "expect you to use your initiative".

The turn continues and Simon introduces the topic of averages (of grouped data) where he identifies the mean as 'the mean one' before asking a question. Finally, the turn ends with Simon identifying himself as the

controller of turns, whilst his pupils identify themselves as bidders for turns by raising their hands. Again, in this turn Simon describes the activities as doing practice and filling in tables, and the question the turn ends with is asking pupils to remember definitions of the key terms, 'mode', 'median' and 'mean'. In the asking of this question Simon self-repairs to insert the phrase "why of course we always want to know why", emphasising the word why on both occurrences. However, this phrase is preceded and immediately followed by 'what' questions, and it is these questions that the pupils answer in their subsequent turns. Again, there is a significant pause during Simon's turn, immediately following a question during which the pupils are bidding for the next turn by raising their hands, and not self-selecting as next speaker.

In lines 301-2, George is assuming the discourse identity of answerer and responds to Simon's request for information and a definition. Simon's next turn evaluates George's answer and expands on the explanation before positively evaluating George's choice and answer. Simon does not give any indication of what aspect of George's choice was 'good'. Whilst Charlie's turn in line 313 is grammatically constructed as a question, it does not include the intonation associated with a question and is in fact an answer to Simon's earlier question in lines 289-292. Simon's following turn includes an explanation of Charlie's answer but no evaluation. It is in line 319 that the evaluation occurs, and here Drew is assuming the discourse identity of evaluator and answerer. Simon continues in his role as explainer and then questioner in lines 321-328 and the pupils return to the discourse identities of answerers.

Following Simon's question in lines 333-334, there is a significant pause of 2.5 seconds following the nomination of George as next speaker. The next turn is taken by Alex, who has self-selected as next speaker. Simon reinforces his situated identity of controller of turns by addressing his next turn to George, indirectly sanctioning Alex for taking the turn and directly sanctioning George for not taking the turn, before re-asking the question. Alex has assumed the discourse identity of answerer but Simon has not ratified this identity, insisting that George assumes the identity of answerer, which he does in line 343. As George repeats Alex's answer, the utterances has only interactional relevance as it offers no new information, demonstrating orientation to the rules for classroom talk by both Simon and George. In the next turn, Simon assumes the discourse identities of evaluator, explainer, and reminder before ending the turn as questioner.

Ashley's turn in lines 358-360 begins with Ashley assuming the identity of answerer, first offering the answer to Simon's question then offering the method for how they got their answer. At the point where Ashley can be interpreted as completing an answer, Simon overlaps Ashley's turn, orienting to the situated identity of teacher and hence controller of topic. At the point where Ashley and Simon speak concurrently, Ashley has defined the term median, but has not applied the definition to the question. In Simon's subsequent turn he evaluates Ashley's full answer, and repeats the answer. He then invites the class to 'check that if you want to' before asking them to remember and then asking Ashley to continue her application of the median to the task.

In lines 375-376, Ashley's turn ends with a question and Simon's subsequent turn restates the same question before another pupil answers. Ashley assumes the discourse identity of questioner, but interestingly Simon retains his identity of questioner and does not change to that of answerer. Simon's turn in line 381 is a reminder of the procedure for calculating the median by emphasising that the number is even, before completing the explanation in lines 384-390, which he describes as a 'trick'.

In lines 392-395, Simon continues his explanation, and consequently begins to assume the role of answerer to his own question. Ashley attempts to resume the role of answerer in line 396, but does not succeed until Simon ratifies this identity at the end of his turn in line 398. From the point in which Ashley answers Simon's question in lines 358-360 until Simon's turn ending in line 409, Ashley and Simon are both orienting to the discourse identity of explainer, whilst also orienting to the other's identity of explainer. This can be interpreted as a conflict between Simon's situated identity of teacher and consequently expert and explainer, and the discourse identity of Ashley as expert as she is explaining her own method for answering the question. Simon repeats the entire explanation jointly constructed by himself and Ashley in lines 411-437, firmly establishing his discourse identity of explainer and reporter of knowledge.

Similarly, Chris attempts to assume the role of answerer and explainer in lines 450-452, but Simon takes the next turn and assumes this role himself, with the pupils only offering answers to arithmetical calculations needed by Simon in his explanation. Drew then assumes the role of explainer in lines

494-5, but phrases the turn as a question, consequently assuming the role of questioner. Again, Simon does not assume the role of answerer in the next turn, but reformulates the question for another pupil to answer in the following turns. Thus, Simon maintains his situated identity of teacher, which he continues for the remainder of the extract.

Throughout the extract, Simon is orienting to his situated identity of teacher. The discourse identities that he orients to are those associated with the role of teacher, questioner, explainer, evaluator, reminder, controller of turns etc. The pupils also orient to many of the discourse identities associated with the situated identity of pupil. These include answerer of questions and listener. On the other hand, they do assume discourse identities not commonly associated with the pupil situated identity, such as explainer and questioner. However, Simon does not ratify these identities by assuming the role of listener and answerer and instead retains his own discourse identity of explainer and questioner while retaining control of who can speak when and what they can say.

Richard's Lesson and the Identities of Teacher and Pupil

In Richard's first turn, he assumes the situated identity of teacher by controlling the time, task, turns and evaluating his pupils' efforts. In this turn, he describes his pupils as having ideas and good thoughts before assuming the discourse identity of questioner. However, Richard's question is not an indirect request for information requiring a pupil to remember or explain, but instead is asking for their own opinion. In lines 568-569, Alex responds by offering her own opinion and an explanation for this opinion. In the next turn, Richard assumes the discourse identity of story-receiver using a continuation

marker, 'oh I see', to encourage Alex to assume the role of story-teller (described as 'Passive Recipiency' by Jefferson, 1985), which is followed by a pause of 0.7 seconds in line 571, offering Alex the opportunity to continue her previous turn. This attempt is not successful and Richard gives a more direct request for Alex to continue her story, which Alex does in her subsequent turns, though not without further prompting from Richard. Richard then evaluates Alex's story in line 585, again orienting to his situated identity of teacher, before requesting another story from a different pupil.

Drew evaluates and disagrees with Alex's story in line 589 and follows this with an example which is offered as an example which is not considered by Alex's own account of proof. There is a pause of 0.8 seconds in line 594 before Richard again orients to the role of story-receiver and summarises Drew's turn. This summary could be intended to encourage Drew to continue her explanation and the subsequent pause of 1.2 seconds in line 597 offers Drew the opportunity to take the turn, but instead Drew does not ratify the proposed identity of story-teller by continuing, instead nodding her head to indicate that she has finished her turn and therefore interpreting Richard's turn as a summary of her own position.

Richard continues to assume the role of story-receiver, with the exception of a brief comment in lines 622-623 to deal with two pupils who were talking to each other, and consequently were not orienting to their roles as pupils, until line 642. Drew has introduced a topic that was discussed in a previous lesson some time ago, and Richard adds a description of the original discussion in his turn in lines 642-652, altering his role as co-participant in

the telling of the story and not just receiver. In lines 655-656, Richard orients to his situated identity by controlling the turns and the tasks and requesting one more comment.

Taylor offers another description of proof in line 657 and Richard again assumes a story-receiving role, which Taylor orients to by continuing her story in lines 659-662, 664-665 and lines 667-669 following Richard's turns. Richard's turn in line 663 includes a change-of-state token and is overlapped by Taylor and his turn in line 666 repeats what Taylor has said, with no evaluation. Both of these types of turn are common in story-telling situations and are used to indicate that the story receiver has heard and understood the story and encourage its continuation. Richard returns to his situated identity of teacher in lines 685-697 where he restates Taylor's description before evaluating it and then changing the topic in lines 704-716, and asking a question in lines 716-728. At this point, the interaction continues with the pupils assuming the discourse identities of answerers and Richard assuming the discourse identities of questioner and evaluator. Richard attempts to get Drew to continue her answer turn in lines 783-788, pausing for 1.2 seconds and using a continuation marker, encouraging Drew to assume the role of story-teller, but Drew declines, and Richard reformulates his attempt into a question which Drew then answers in line 791.

In line 819, Richard then commences his own story, assuming the role of story-teller and the identity of someone who needs convincing before ending his turn with a question.

In line 837, Richard pauses for 3.8 seconds immediately following the point where he has asked the question "why do you think maybe I'm not totally convinced by that?". This pause offers the pupils the opportunity to either bid for the turn or self-select as next speaker. Richard then reformulates his story before re-asking the question and finally nominating a pupil to take the next turn.

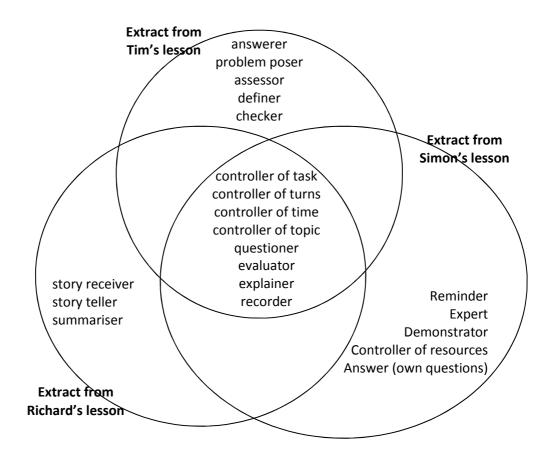
In lines 846, F offers an answer to Richard's question by suggesting a possible variation of the task. In the pair work in the previous lesson on the T-totals task, the pupils have been invited to alter and vary the task in any way they liked to see what happened. Richard's turn in lines 849-854 though focuses the attention away from varying the orientation of the T, onto what is needed for Richard to be convinced. G answers Richard's original question in lines 855-856 and Richard positively evaluates G's answer but this evaluation is interpreted as an assessment by G when he offers an agreement in line 860 (Pomerantz, 1984).

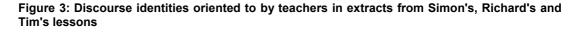
In lines 896-898, J offers a summary of Richard's turn that Richard positively evaluates but reformulates emphasising the differences in the accounts before K offers an answer in lines 907-910, which offers a way of convincing Richard as he indicates in his subsequent turn.

In the extract from Richard's lessons, the situated and discourse identities oriented to by the participants are more closely related to the nature of the mathematical tasks and activities than in the other extracts. Whilst the roles of teacher and pupil are still evident in the structure of turn-taking, the control of the topic and the evaluations of turns, these roles are subtly different from

those in Tim's and Simon's lessons. The nature of the interactions bears a lot of similarities to story-telling interactions, with both the pupils and Richard orienting to the roles of story-teller and story receiver. Though the utterances themselves are not actually stories, the similarities in the interactional actions are interesting in the way that they support and encourage pupils to offer their ideas.

Differences and similarities in the Discourse Identities of Teacher and Pupil There are many similarities in the ways the participants orient to the situated identities of teacher and pupil. It is clear in each of the extracts that one participant is controlling the turns and this same participant is predominantly asking the questions, making evaluations of the other participants' turns and also largely controls the order, topic, and timing of the activities within the interactions. The rest of the participants predominantly listen and answer questions. However, whilst the situated identities of teacher and pupil are clearly identifiable the discourse identities that contribute to these situated identities differ between the extracts.





The identity of expert is commonly associated with the identity of teacher and in each of the extracts the teacher orients to the identity of expert but in different ways. In the extract from Simon's lesson, the identity of expert is made relevant through the demonstration of examples, the reminding of facts and procedures, and the evaluator of pupils' answers. Simon does not, however, transmit knowledge or procedures in the extract itself. Rather, through question and answer sequences, Simon's pupils remember the procedures and perform the necessarily calculations and Simon's role is in selecting which pupils to take the turns at remembering and evaluating those turns. This role of expert is also oriented to by Simon's pupils whose turns are usually marked in some way, such as being phrased as a question or containing other marks of uncertainty. By doing this, Simon's pupils are obliging Simon to make evaluations of their turns, making relevant both Simon's expertise in making these judgements and their own positions of a lack of expertise to make these judgements themselves. The identity of expert in the extract from Simon's lesson is about knowledge, where Simon has the knowledge and can use it to make evaluations and the questionanswer adjacency pairs are focused on checking whether the pupils have acquired this knowledge.

In contrast, in the extracts from both Tim's and Richard's lesson, the identity of expert is oriented to through the modelling of mathematical behaviour. Tim models a problem solving process through his personalisation of the problem itself and his structuring of the task. The task is structured so that initially pupils are trying out the first few examples (specialising), before making conjectures about what happens if they keep going, before making connections between an image and the original numerical problem and finally linking the problem to the mathematical focus of the lesson. The identity of expert in the extract from Tim's lesson is about doing mathematics. Tim orients to the identity of expert through his modelling of solving problems and through the question-answer adjacency pairs Tim initiates, his pupils are obliged to perform many of these problem solving processes.

Richard models a different aspect of mathematical activity in the latter section of the extract from his lesson. He has set the topic for the lesson as mathematical proof through the earlier discussions of what his pupils understand by that phrase, and one of his pupils makes the connection

between this discussion and the investigation the pupils have been working on in a previous lesson. In the first part of the extract, the identity of expert is oriented to by the majority of the participants. Richard asks questions which make the pupils' own understandings relevant and Richard does not evaluate these understandings and consequently does not orient to the identity of expert. Towards the end of the extract, Richard models the importance of being convinced and asks his pupils to convince him that the T-total will always be a multiple of 5. In a similar way to the extract from Tim's lesson, the identity of expert is again about doing mathematics, in this case convincing and justifying. Richard is modelling a mathematical need for proof and uses the question-answer adjacency pairs to create this need for proof in his pupils, and obliging them to offer justifications and Richard evaluates his pupils' turns in relation to the appropriateness of their justification.

So, whilst in all three extracts the teacher orients to the situated identity of expert, the nature of this expertise is different. In the case of the extract from Simon's lesson, being an expert is about being knowledgeable about mathematics. In the cases of the extracts from Tim's and Richard's lessons, being an expert is about behaving and acting like a mathematician.

The discourse identities of the pupils in the extracts also differ. The discussions above talk about the differences in relation to expertise, but there are also differences in how pupils' questions are handled that have consequences for the nature of the identity of pupil. In both Tim's and Simon's lesson, pupils ask questions and in the extract from Tim's lesson,

Tim answers this question, orienting to the question as a request for clarification. In the extract from Simon's lesson, Simon repeats or rephrases the question and offers it to other pupils in the classroom. Many of the pupils in Simon's lesson mark their answers to Simon's question to display uncertainty. This includes phrasing answers as questions. In the majority of these instances, Simon orients to the identity of expert and evaluates the answer given. There are also instances where Simon returns the question to another pupil in the class. The first type of these instances is in lines 321-328 where Simon's rephrasing of the question initiates a repair which is performed by another pupil. The second type is in lines 497-500 where Simon breaks the question down into smaller steps, relating the numbers given to the context and in doing so supports the pupils in checking the calculation given in the original turn in lines 494-495.

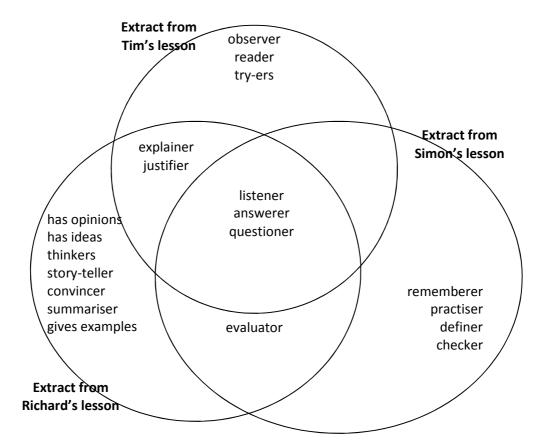


Figure 4: Discourse identities oriented to by pupils in the extracts from Simon's, Richard's and Tim's lessons

There are also other interactional differences in what teachers and pupils do in their turns in each of the extracts. In the extracts from Tim's and Richard's lesson, the pupils offer their own thoughts and ideas. In the extract from Richard's lesson this is in response to a request for their understanding of the meaning of the phrase mathematical proof, whilst in the extract from Tim's lesson it is in response to what is the smallest prime number. In both scenarios, there is a 'correct' response to have in that there is a definition of mathematical proof and an answer to the questions what is the smallest prime number. However, in the extracts the pupils offer their own views and answers and these are discussed or debated by other pupils.

One final difference is the discourse identity of explainer. Predominantly in the extract from Simon's lesson, it is the pupils that describe the procedures for calculating measures of central tendency while it is Simon that offers explanations for the calculations and procedures that arise in the interaction in the extract from his lesson. In the extracts from Tim's and Richard's lesson, the pupils frequently offer explanations to support their answers.

The next section of this chapter explores how these differences in the situated identities of teacher and pupil combine with how tasks and activities are described to constitute the nature of mathematics in each lesson.

The discursive construction of mathematics and mathematical activity.

Tim's lesson and the discursive construction of mathematical activity In the earlier analysis of the extract from Tim's lesson, I have offered an analysis that firstly shows that Tim and his pupils are orienting to and constructing the situated identities of teacher and pupil. The discourse identities oriented to by the participants relate to how Tim constructed the overall activity within the extract, constructing it as solving problems that involve 'thinking', 'understanding' and 'having a go'. These discourse identities included problem solver and explainer or justifier. Next, I take a closer look at the turn-by-turn interactions in parts of the extract in order to look more closely at what Tim and his pupils are doing, focusing in particular on the nature of the mathematical activities and actions that they are doing.

Tim starts by asking his pupils to 'look' at the problem in line 6. The problem is given in words, yet Tim asks his pupils to 'look' at the problem and not 'read' it. By doing this, Tim is asking his pupils to think about the problem, going beyond reading the question. Tim then continues to describe the second question as something that "we have to need to think about in terms

of (.) what it actually means" in lines 8-9. Here Tim is aligning himself with his pupils as a problem-solver through his choice of the pronoun 'we'. The process of thinking about the problem and working out what it means and what you will need to do is one of the first stages of solving any problem (Mason, *et al.*, 2010). Presumably, Tim knows what the problem is asking them to do because of its positioning within the topics of the lesson and the relationship between the solution of the problem and the other activities that are undertaken in the lesson. Therefore, by using 'we' instead of 'you', Tim is referring to the generic processes that a problem-solver goes through when encountering a problem.

In line 38, when Tim repeats Harry's answer and is consequently indicating that Harry's answer is appropriate and correct, Tim adds 'pounds' on to Harry's answer. The adding of the units in Tim's revoicing of Harry's answer is often discussed in the literature as a device that teachers use to encourage pupils to give complete and mathematical answers. However, in this interaction it also serves the purpose of focusing attention on the particular problem of donating money, rather than the generic calculation of a quarter of three thousand.

In line 58, Tim refers to his pupils using calculators, noting that some were using them whilst others were not. This is something that Tim has noticed and by mentioning it, he is making it relevant. He continues the turn by stating that it is good and that he does not 'mind either way'. In line 61, Tim then emphasises the he wants his pupils 'thinking' about 'it', before he lists the values for the earlier calculations. The way in which he handled the use

of calculators to perform the calculations and then the answers to these calculations is by contrasting these calculations with what he wants the pupils to do, which is thinking, as he indicates in lines 61 and 70. He mentions that it does not matter whether a calculator was used and there is no discussion of how the values of three thousand, seven hundred and fifty and so on were calculated. The results are listed by Tim himself and he has not asked any of the pupils to offer these. Instead the emphasis is on thinking about what is going on, leading to the question 'do you stop' in line 73.

In line 73 when Tim first raises the issue of whether you stop, he introduces it by referring to conversations he has had with some of the pupils whilst they have been working on the task as individuals or in small groups. By mentioning these conversations he is making them relevant to the current interaction and also he is indicating that the question is something that is important enough to discuss, and that the answer needs to be thought about. Consequently, Tim is indicating that whether you stop or don't is not immediately clear. Then in lines 75-77, Tim asks 'why not' twice. At this point, asking why would have also required an explanation in the turn that follows, but the inclusion of the word 'not' constrains this response further by requiring it to be an explanation as to why you do not stop. Here again by constraining the next turn to explaining why not. Tim is also indicating that he does not want an explanation for why you do stop. Jamie offers an explanation in his turn that Tim repeats in the following turn, interpreted by Jamie as a checking of what he has said through his agreement to Tim's repeat in line 81. In the next turn, Tim rephrases the original question,

starting with the word 'but. This in effect initiates a repair on the previous responses, indicating that the original answer of 'nope' is incorrect, as is demonstrated by the multiple pupils self-selecting in the next turn with an affirmative answer. No pupil self-selects here to agree with the previous answer by answering 'no'.

In just these few turns, Tim and his pupils have constructed an argument over whether the sequence has a limit or not. Through the construction of his turns, Tim has indicated that there are two possible sides to this argument and has constrained his pupils into constructing both sides of the argument. This argument continues until line 107, and develops as a difference between the amount of money being given away each time and the total amount being given away. The amount of money being given away each time is 'getting smaller' until eventually you give away a penny, at which point there is no smaller monetary unit that can be given away, as is argued in line 103. On the other hand, the total amount of money available is also decreasing and some pupils are arguing that this twelve thousand pounds will eventually all be given away, argued in line 89. However, many of the pupils' and the teacher's turns use pronouns to describe what is getting smaller; 'it keeps getting smaller', 'it will get to zero'. In these cases the 'it' could refer to either the amount being given away or the amount of money left. It is not until lines 88 and 89 where the distinction is first made, with H focusing on the penny and I focussing on the twelve thousand. Tim's response in line 90 is interpreted by both pupils as a request for an explanation which, unfortunately, is inaudible in H's case. It is I's focus on twelve thousand that Tim chooses to respond to in lines 93-98. In this turn,

Tim is rephrasing the question to specifically focus on whether they will "give away the <u>wh</u>ole twelve thousand".

In lines 100 and 101, multiple students give the two possible, but contradictory, answers. Tim repeats one of the answers before nominating an individual student, who has their hand raised, to take the next turn. This student responds with a question in line 103, which returns the topic to the amount that is being given away in each iteration. Tim takes the next turn, repeating the end of the previous turn, before expanding the explanation that you cannot give away part of a penny. Tim emphasises the reality aspect of this explanation by describing it as 'realistically' but also by explaining that there is no 'way' to pay the money.

In lines 74 to 107, a point of contention has arisen (Gellert, 2011), but the analysis above shows how this point of contention has been constructed and developed by Tim. Tim's turns in lines 82, 87, 90 and 93-98 can all be interpreted as an initiation of a repair (see chapter 9). Some of these initiations are following a turn where a pupil has said that you do stop, whilst others follow turns where a pupil has said that you do not stop. Combining these with the contextualisation following earlier conversations, Tim has introduced the idea of a sequence ending, and hence a connection to finite and infinite sequences, as a point of contention. Tim closes down the discussion in line 104, beginning his explanation with the word 'so', indicating that the explanation that follows is a conclusion and introducing an image with no pause following the explanation, therefore preventing a student from self-selecting to take a turn as the topic changes.

The image that Tim displays is a large triangle:

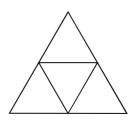


Figure 5: - Unshaded Triangle projected onto Tim's whiteboard Following the displaying of this image, there are two pauses of 2.1 seconds in line 110, and another of 0.8 seconds in line 112 before Tim talks about the image. Tim introduces the triangle as 'useful' for 'looking' at it, before asking the pupils to make the connection between the image and the donating problem. This is followed by a long pause of 1.7 seconds in line 119. Immediately before that pause, Tim has asked his pupils to 'imagine' the triangle as representing the £12 000. The pause gives the pupils the opportunity to begin to make the connection between the image and the previous problem. Tim follows this by describing the donation of a quarter before connecting this quarter to the centre triangle of the image.

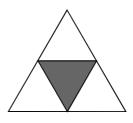


Figure 6 - Image of triangle following Tim's first shading

When Tim first introduced the image he described it as 'that's my money', without attending to which aspects of the image relate to which aspects of the problem. The shading of the centre is more explicitly linked to the quarter that is donated, but the whole is not specified. Tim does not specify

what it is a quarter of or that the large triangle represents the £12 000. The next turn is taken by a pupil who self-selects to ask 'why'. This 'why' initiates a repair on Tim's previous turn but does not locate the source of the trouble (see chapter 9). There is a pause of 0.9 seconds before Tim responds, and he chooses to clarify the relevance of the shaded triangle, and emphasises the relationship between the shaded triangle and the £3 000 donated. Whilst the pupil who asked the question in line 125 indicates that he understands in line 128 with a change-of-state token (Heritage and Clayman, 2010), Tim continues to emphasise the three thousand pounds in the following two turns. Now, instead of asking the pupils to 'imagine' and make the connection between the image and the problem, Tim is checking that the pupils are following what he is doing and the connections he is making. In line 133, the turn ends with a tag question 'isn't it' meaning the preferred (see chapter 9) response is an agreement from the pupils in the next turn. The next stage, where Tim shades the next triangle, is phrased as a question, again with a preferred response of agreement and similarly Tim's turns in lines 145-146, and 148-158 are designed with a preference for agreement from the pupils in the next term.

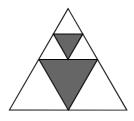


Figure 7 -Image of triangle following Tim's second shading

In line 160, Tim changes the topic of the interaction to focus solely on the image of the triangle and to ask what fraction of the triangle has been shaded:

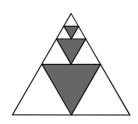


Figure 8 - Image of triangle following Tim's third shading

Tim starts the turn by saying "but what <u>fraction</u>". At the moment, Tim has not given sufficient information to enable the pupils to answer the question as it is not clear which fraction Tim is referring to, and Tim also emphasises the word fraction. Tim is indicating that there is a transition in the focus and following a pause of 1.3 seconds in line 161, Tim asks the question "what fraction of that triangle have I shaded". In the interactions between lines 118 and 159, the attention has been focused on the shaded triangles. By asking what fraction has been shaded, Tim is shifting the attention to the image as a whole.

In the turn that follows, Jamie offers a hesitant and hedged answer as to what fractions have been shaded. A pause follows this answer in line 169 before Tim rephrases Jamie's answer as another question. Tim is initiating a repair on Jamie's answer, and the pause in line 171 followed by Jamie's response in line 172 indicate that Jamie recognises this as a repair initiation but is unable to perform the repair by offering a different answer. Similarly, Chris' answer of a quarter is also followed by an initiation of a repair, but this initiation is more specific as the process of shading the triangle involves

shading a quarter each time and therefore the answer of a quarter is not unreasonable, but Tim changes from focusing on the image as a whole to individual rows of the image in lines 181 and 183. When directing the focus onto the rows, Tim also emphasises that the triangles on these rows are each the same size. This locates the trouble as an issue of calculating the fraction of the whole image that is shaded when the image is made up of successively smaller triangles. The expected answer of a third is then given in line 184.

Tim then expands the answer given in lines 187 to 193, focusing the attention onto each row in turn and the shading of the single triangle in each row before linking back to the original problem of donating the money in lines 193-194. Multiple pupils self-select to give the answer of a third in line 195, which remains focused on the image and the representation of the shaded triangles as being given away. The pause of 0.8 seconds in line 196 indicates that this is a source of trouble and Tim rephrases the question in line 197 to indicate that he had returned to the topic of the money and the appropriate response is given in line 199. Tim's repeat of the answer in line 200 overlaps the pupil's answer and is followed by an 'ok?' which serves to check that other pupils have made the connection between the image and the original problem of donating money and the earlier calculations that the pupils performed. These connections are reinforced by Tim in lines 202-208, 213-216 and 218-221. Tim then makes a reframing move in line 222 using the discourse marker 'ok' and making the connection between the specific problem the class have just been working on and the overall topic of the lesson.

In lines 160-165, 170, 173, 177, and 187-194, the questions all focus on the actions of Tim, what fraction has he shaded. Whilst this personalises the problem as before, it also focuses the attention on the process of the shading. Instead of asking what is shaded, Tim asks what have I shaded. Tim is making his actions relevant to the question through his use of 'I' in 'have I shaded' in line 162, 'I haven't shaded' in line 173 and 'I'm shading' in line 177, rather than using the passive form commonly found in mathematical questions. This makes the image dynamic as it is the product of the process of individual shadings and attention can move between the recall of the actions of shading and the final image presented. If Tim had asked what is shaded, the image would be static with the emphasis on the final product. This distinction is particularly important for the topic in question, limits of infinite sequences, where the limit itself, the static image in this case, is not reached.

In lines 222-232, Tim begins to use the pronoun 'we' again to talk about both what the class are about to do and about what they have already done, emphasising the collaborative nature of the activities undertaken in this lesson. Tim then refocuses onto a new specific task in line 232, changing to using 'what I want', and again the task is phrased in the first person; it is Tim that is dividing by five and then adding four.

In this extract, the activities are firstly about solving problems and then making connections. Calculations are part of the activity but the attention is on their use in solving the problem rather than the calculations themselves. Tim makes this distinction through his contrasting of 'working it out' and

'thinking' (for example in line 39 then line 43), though both involve active engagement with the problem, which he instantiates through his personalisation of the problem. Tim uses his turns to shift and focus attention of different aspects of first the problem and later the image of the triangle. He also uses his turns to encourage and support discussions about the mathematics, whether that is as a whole-class or in small groups. Doing mathematics is also about thinking and meaning, and through the construction of a point of contention, doing mathematics includes arguing, explaining and justifying.

Simon's Lesson and the Discursive Construction of Mathematical Activity

The earlier analysis of Simon's extract in this chapter examines how Simon and his pupils are orienting to the identities of teacher and pupil, but does not explore what they are teaching and learning. In the extract, Simon is contextualising the activities of teaching and learning mathematics within a wider context of doing school mathematics.

Firstly, Simon positions the activities within a time-frame and plan for this lesson. In turn 67, lines 240-242 he does this by explicitly referring to a previous lesson, which he does again in lines 294-298, 349-350, 461-463, 465-470 and 521-524. In lines 252 and 253 the reference is more implicit when he refers to the sheet that was partially completed last lesson. In lines 264-266, and then continued in turn 69, lines 270-271, Simon refers to the preparations that he has made for the task they are about to do by mentioning the preparation of the table with the 'extra column'.

In lines 244-255, Simon describes the time line for this lesson by describing what he has planned for the lesson and contrasting this with the next activity of going 'through another example', which begins in line 284. Similarly, in lines 280-281, Simon mentions that the pupils will be doing some practice from the textbooks shortly. Simon mentions the time line of the lesson again in lines 535-544 as the class transition from 'going through an example' to 'doing some practice from the textbook'. All the explicit references to the time-frame in which the activities occur are from this lesson or the lesson yesterday. There are no references to earlier or future lessons. However, the references to the examiners point to the GCSE modular examinations the pupils will be taking next term.

Simon also positions these tasks and activities within the wider mathematics community in the school by his description of the conversation with Mrs Smith in lines 264-266. He then positions this conversation and subsequently today's tasks and activities within the context of the wider educational examination system, in this case the GCSE examination system. In turn 69, lines 276-284, Simon describes how the examiners will present the task but also describes the expectations the examiners will have for the pupils, expecting them 'to know' and 'using their initiative' and 'adding the extra column'. This final reference includes the textbook authors who have the same expectations of pupils in terms of using their initiative and adding the extra column. The authors of questions, whether textbook authors or examiners, are referred to again in lines 369-372 when Simon is describing the usual presentation of questions. These references to Mrs Smith, the

examiners and textbook authors are also part of how Simon is 'accounting' for him including the extra column.

In the extract from Simon's lessons, the mathematical tasks and activities have been constructed by Simon as something that can be described as school mathematics. There is a plan for the lesson that relates to the activities done in the previous lesson. This plan is discussed within the context of supporting and preparing Simon's pupils for their GCSE examinations. The tasks and activities of the lesson are part of a school mathematics curriculum, which are endorsed by other mathematics teachers in the department as well as examiners and textbook authors.

So what does doing school mathematics involve in Simon's extract? Simon describes the tasks and activities as 'doing' practice and remembering and this is often what the pupils do in their turns. In lines 301-2, George is remembering the definition and procedure for finding the mode and in lines 358-360, Ashley is remembering the definition and procedure for finding the median. The interaction in lines 313-347 is all about remembering and applying the procedure for calculating the range. The focus on remembering and carrying out procedures continues throughout the extract. Even in line 499, where the interaction changes briefly to a focus on what the numbers mean, this change lies within the context of the procedure for calculating the mean. The shift in the focus of attention to what the numbers represents also serves as a check for the procedure for calculating the mean.

Richard's Lesson and the Discursive Construction of Mathematics.

Richard begins the lesson by talking about "ideas" in lines 550, 551, 561 and 565, but also emphasises the role of discussing and talking about mathematics in lines 552-556 and 565-567. When asking his question about the understanding of the term proof, Richard makes it explicit that he wants several pupils to offer their understanding, and therefore that it is something worth discussing. Alex responds first and makes a distinction between mathematical proof and other proofs. Richard has in effect asked for his pupils' opinions and his subsequent turns in lines 570-572, 577-578 and 580-581 all encourage Alex to extend his description or to check Richard's understanding of what Alex has said.

Whilst Richard positively evaluates Alex's contribution in line 585, Drew disagrees with Alex in lines 589-593, mitigating her disagreement by hesitating and not using a bald 'no'. Drew then follows this with her own account of what proof means. The interaction continues with other pupils offering their own opinions, which Richard encourages and praises through revoicing and positive evaluations and using a story-telling style of interaction.

In line 642, Richard describes a 'big argument' the class had had over whether 0.9 recurring is the same as 1. Whilst the current interaction over the meaning of the term proof bears some similarity to an argument in that opposing views are put forward and are agreed with or disagreed with, the interaction is noticeably different from the arguments in Tim's lesson. In Richard's lesson, all the contributions are treated positively by Richard through revoicing and evaluation and Richard describes many of these as

"interesting". The interaction is not focussed on 'winning' the argument but is more about sharing and discussing different views.

Richard changes the focus of the discussion in line 704, returning to a discussion of yesterday's lesson. Again, Richard emphasises thinking in lines 709 and 714 before asking the pupils to look at what is the same and what is different between the two representations on the board.

14	15	16		<i>x</i> - 1	x	<i>x</i> + 1
	25		-		<i>x</i> + 10	
	35				<i>x</i> + 20	

T-total = 105 T-total = 5x + 30

Figure 9 - T-totals images from whiteboard in Richard's lesson

Again, Richard encourages a number of pupils to respond and make observations about the similarities and differences and Drew, B and Lesley are all nominated to offer their observations.

There is a topic insertion in lines 751-780 where Richard asks for the mathematical vocabulary used to describe 'x' and 'x + 1', though Richard downplays the importance of using the mathematical term in lines 767-768, and also by referring to it as the 'technical term'.

During this inserted topic change, Drew returns to Richard's earlier question on the similarities and differences between the two images, but also to the earlier discussion on proof in lines 773-774. Richard initially acknowledges Drew's turn and returns to give the 'technical term' expression, before changing the topic back to the two T-totals images on the board by directing attention to Drew's observation. The following discussion then revolves around the relationship between evidence and proof. Richard encourages an answer that contradicts Drew's answer in lines 804-809 by explicitly asking for one, but he does not evaluate either point of view, clarifying his own understanding of what has been said or revoicing what has been said.

Richard then introduces the relationship between using examples and being convinced in his turn in lines 819-843, saying that he is not "totally convinced" by lots and lots of examples. He also makes the distinction between a rule being 'nice' in line 882 and being convinced in lines 884-891.

Throughout the extract, Richard talks about what the class is doing as being about 'thinking' and discussing. He encourages pupils to give a range of perspectives on a variety of questions, rarely evaluating a contribution as correct or incorrect, rather evaluating them as interesting or not. Mathematics is seen as something that is about debate leading to being convinced about something. Richard also shares his own thinking with his pupils, so similarly to the extract from Tim's lesson there is a sense of personal involvement with the mathematics. Both Tim and Richard are 'doing' mathematics in the interactions.

Similarities and differences in the discursive construction of mathematics and mathematical activity

The similarities and differences in the situated identities of the teachers and pupils in the extracts combined with how the activities are discursively

constructed also have consequences on the nature of the mathematics in the extracts.

Simon and his pupils orient to the role of teacher as expert as the person who has the knowledge and experience to help pupils to 'remember' facts and procedures. He 'demonstrates' how to perform procedures and frequently asks his pupils to 'remember' or 'practice'. Evaluating pupil answers is also the role of the teacher. In contrast, Tim and Richard's pupils orient to the roles of explainer, discusser, debater and justifier, though the roles that Tim and Richard orient to are different. Tim and Richard also personalise the mathematics, though in different ways, and the turns are about doing mathematics rather than remembering mathematics.

The mathematical activities and tasks that are either being done through the interactions or are talked about during the interactions also offer some contrasts:

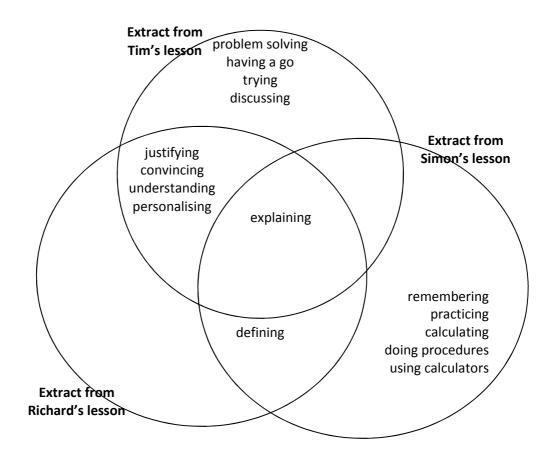


Figure 10: the mathematical activities in the extracts in chapter 7

The data in this study only offer a small glimpse of the teachers' discursive actions and the extracts analysed here and presented in chapter 7 make this window even smaller. In previous chapters, the structure of interactions has been shown to alter as the nature of the mathematics changes. The extract from Tim's lesson is about solving a problem and making connections between different representations, while Richard's extract is about the meaning of proof and an application of proof to an investigative task the pupils have been working on in pairs. The extract from Tim's lesson is introducing a new topic, whilst in the extract from Richard's lesson they are building on work they did in the previous lesson. The extract from Simon's lesson is about practicing and applying procedures learnt in the previous two

lessons. It may be that it is the nature of these tasks and activities that means that the interactions include justifications and explanations, and Simon's task on remembering and applying definitions and procedures for measures of central tendency involves remembering and calculating.

To examine this further, the full data set was examined looking specifically at the mathematical tasks and activities that occur. The majority of Richard's lessons involve pupils reporting on their work as individuals or pairs, and the majority of Richard's lessons are spent with pupils working in this way. There is very little activity as a whole class. The turn-taking and preference organisation across all the lessons are largely consistent with those in the extract presented in chapter 7. There are also occasions where pupils are making conjectures or performing calculations.

Extract 41 - Example where pupils are making conjectures.

001 002 003 004 005	Т22	Richard:	so t-total is the title of this task and the idea is to think about what the numbers add up to. what do those what's the total of the t. what are the numbers inside the t. (4.8)
006 007			some people are using calculators and some
007	т23	Chris:	people not that's interesting um Chris. er is it a hundred and five
009 010 011 012	T24	Richard:	well done. a hundred and <u>five.</u> very good um now then if I was to put the t somewhere else, (.) would it still be a hundred and five what do you think would it <u>change</u> , what
013			it be the same. Ashley
014 015 016 017 018	T25 T26	Ashley: Richard:	<pre>it would <u>change</u> it would <u>change</u>. can you say more about that change. (0.6) Charlie</pre>
019 020	Т27	Charlie:	um if you move the t (.) lower down the t- board then the total will be higher
021	T28	Richard:	if we move the t <u>low</u> er down the total will be <u>high</u> er. who agrees with that. a few people. maybe. um these are the sorts of things I thought we could investigate

Richard lesson 2

In the first part of this extract, Richard is asking his pupils to calculate the total of the five numbers inside the T-shape on the grid (see Figure 9). He begins by talking about "the <u>idea</u>" and asks his pupils to "think about" the task as a whole, before asking "what's the total" and "what are the numbers inside the t". He has not directly asked the pupils to 'add up the numbers inside the t' but it is a calculation they will need to do in order to "think about what the <u>numbers</u> add up to". Whilst the pupils are needing to perform an arithmetic calculation, they are still being asked to think. Richard's turn in line 9 begins with a positive evaluation of Chris' response, then a repeat of the answer before another positive evaluation.

In lines 10-13, Richard is now asking if the total will change if you move the T around the grid. Richard echoes Ashley's response in line 14 before asking for more detail, but there is no evaluation of Ashley's response. There is a pause of 0.6 seconds in line 17 before Richard nominates Charlie to take the next turn. This pause offers an opportunity for Ashley to continue her turn and to "say more" but Richard's gaze is moving around the classroom and other pupils have their hand raised and Richard nominates Charlie to "say more". In lines19-20, Charlie conjectures about the effect moving the T will have on the total. Richard repeats Charlie's response, emphasising "lower" and "higher" but does not positively evaluate this response either.

What is different about this extract from the rest of the transcripts from Richard's lesson is that here the pupils are directly interacting with the task for the first time and are not reporting back on work they have done as individuals or pairs.

The majority of Simon's lessons involve calculating and performing procedures and much of the work is done as a whole class. There are occasions where the activities differ:

001 T10 Simon: ... there's only one thing you need to think 002 about at the moment. cumulative frequency. 003 do you know what the word 004 (0.4)005 cumulative mi- might refer to? does that mean 006 anything to you, (are you new) to that word. 007 Sam a collection of erm numbers and um stuff 008 T11 Sam: Simon: 009 T12 collecting 010 (0.3)011 yeh sounds a bit like Ashley 013 T13 Ashley: like um if you (.) accumulate you like gather 014 and (.) collect 015 T14 Simon: yeh if you <u>a</u>ccumulate you <u>a</u>dd things up, you 016 gather things up and that is exactly what 017 we're going to do. ok. here's a table ... Simon lesson 4

Extract 42 - Example from Simon's lesson where the discussion is not about a procedure.

In Extract 42, the focus is on the meaning of the phrase cumulative frequency. In line 1, Simon asks his pupils to "think" and in line 5 emphasises that the question is about meaning. Simon positively evaluates both Sam's and Ashley's responses, though after a pause of 0.3 seconds in the case of Sam's response. Simon revoices Sam's response in line 9 and Ashley's response in lines 15-16.

Whilst this chapter has focused on a comparison of the extracts from the three teachers and the ways that the mathematical tasks and activities are done in the interactions between the teacher and pupils, the discursive construction of these tasks is indexical to the immediate context within the extract. The construction is dynamic and fluid, and changes with each turn. Therefore, although generalisations as to the mathematics that the pupils experience in their lessons cannot be made, the purpose of this chapter was

to illustrate how classroom interactions and discourse can affect the nature of activity. The choice of words, the role of pronouns, the pauses between and during turns, the emphasis placed, and the reflexive relationship of the adjacency pairs within talk all interact to construct different experiences of the tasks.

There are also examples of where the choice of words does not reflect the acts the turns are doing, for example in the extract from Simon's lesson, line 290, where he says "why of course we always want to know why" but the questioning is about what. This serves to highlight that it is not just what is said, but how it is said and what the teachers and pupils are doing with their turns at talk that makes the context for the interaction.

If we look at the overall choice of words by the three teachers, a raw count of particular words, such as 'think', 'remember', 'understand' as well as the proportion of total teacher words to each of these words reveals noticeable differences between the individual teachers. However, the use of each of these words is reflexively related to the activity being done or talked about in the interaction.

The word 'think' appears repeatedly in all four of the teachers' lessons, predominantly when they are each talking about the plan of the lesson, but it is used similarly by each teacher when they are indicating or describing what the pupils are or should be doing. In the discussions above, I have discussed the ways in which Tim uses the word 'think' to describe what he wants his pupils to do, and the way he contrasts it with 'work out' and 'calculate'. All the teachers use the word 'think' when the task they have set

is a problem requiring problem solving skills. They also all use it when they are asking pupils for their opinions.

One difference between the teachers is that Simon and Richard also use 'think' to indicate uncertainty in relation to the mathematics:

	001 002 003	т13	Simon:	I don't want to go on this for <u>too</u> long because it's only a bit of background so I'll do another one, this is one (.) you haven't
\rightarrow	004			maybe seen before. this big, I think they
	005			call it a big <u>sig</u> ma sign, <u>tha</u> t means add up.
	006			yeah. so when you see that, that means
	007			(0.3)
	008			um the <u>sum</u> , we won't say add up, we say the
	009			sum, the <u>sum</u> of x. ok. I guess actually
\rightarrow	010			because tha-, I think that might be a capital
	011			Greek s, so I think maybe it stands for sum
	012			or sum of or something like that, so that
	013			just means like <u>add</u> ing up all the xs. ok? so
	014			in this case
	015			(0.3) ((writing on the board))
	016			the sum of x, what is it? add them up?
	Simon	lesson 2		

Extract 43 - Simon using 'think' to indicate uncertainty

The purpose of this chapter is emphasise that the way we describe and talk about mathematical tasks and activities affects the meaning these have for pupils. Ainley et al. (2006) suggest something similar in their planning paradox, where a teacher can present pupils with a task such as designing a bedroom, but the teacher can often find it difficult to take advantage of the mathematical opportunities, assess learning or monitor mathematical thinking. However, the situation is more than this as the opportunities offered by the task are also constrained by the interactional norms for the class .

Implications

Mathematics classrooms need to offer a range of ways of working mathematically (Watson, 2008). These ways need to include activities such

as practising techniques and remembering mathematical facts, but also those activities that can be considered as ways of working as a mathematician. Burton's (2004) descriptions of mathematical behaviour include asking questions about the mathematics, making mistakes and using them, describing, explaining and discussing ideas as well as looking for patterns and developing conjectures out of these patterns, and making connections between mathematical ideas or representations. Each of these activities occurs at some point during the transcripts in this study.

However, the design of a task focusing on providing pupils with opportunities to work in different ways is not sufficient for enabling these ways of working to actually occur. The discursive construction of the task affects which features of the task attention is focused on, and which mathematical activities are emphasised and performed by the teacher or the pupils.

The notion of scaffolding (Wood, *et al.*, 1976) to describe the transition of support to independence in learning is frequently cited in both professional and academic literature, but largely it is discussed in relation to tasks and the ways teachers structure these tasks and intervene when the pupils are working on the tasks. I would argue that the metaphor of scaffolding includes the use of particular structures of interaction to model doing mathematics and initially support pupils in doing mathematics. Whilst some features of interactions that focus on the influence of the content of turns, such as asking why or revoicing, have received considerable attention in the research literature, I would argue that what a turn is doing as well as what a turn is saying is a form of scaffolding. For example, in the extract from Tim's lesson, lines 70-104, Tim is asking why in lines 75-55 and line 84 which

obliges the pupils to offer explanations in the subsequent turns. But Tim is also constructing a debate or argument through the ways in which he constructs his turns. He constructs the interaction so that his pupils offer explanations for both sides of the argument, and this is done through Tim both explicitly asking for explanations but also through a turn that could be interpreted as an initiation of a repair in line 82. The pupils' subsequent turns indicate that they interpret Tim's turn as the initiation of a repair and this leads to the point of contention.

Also, the structure of interactions differs depending on the nature of the mathematical activity and it is often the teacher that initiates the shifts in the structures of interactions. In lines 109-117 in the extract from Tim's lesson, Tim explicitly makes a connection between the problem and the image. He structures his turn to include long pauses of 1.7 seconds, 2.1 seconds and 0.8 seconds, offering time for the pupils to make the connection. The structure of the interaction changes in line 125 when a pupil self-selects to ask a question. Tim's subsequent turns in lines 130-159 are structured as question answer adjacency pairs, though with Tim's questions restricting pupils' answers to agreements (or dispreferred disagreements). Whilst it is the pupil's turn in line 125 that marks the shift in the structure of the interaction, it is Tim who controls and manages the subsequent structure.

The structure of interactions changes as pupils begin to do explanations, conjecturing or justifying without teachers structuring the interactions in ways that oblige pupils to do these activities, such as in the examples where Tim has constructed a point of contention and the need for explanations and justifications naturally arises. In the extract from Richard's lesson in lines

773-774 a pupil makes a connection between the earlier discussions on the nature of proof and the investigation they have been working on. Richard has not explicitly structured the interaction to support the pupils in making this connection, but it is Richard who allows the shift in interaction in his following turns in lines 775-777 and 779-788. The structure of the interaction then changes to a question and answer dialogue between Richard and Drew to draw out the connection that Drew has made before Richard invites other pupils to offer a contrasting connection in his turn in lines 804-809.

However, this study has only begun to reveal the relationship between interactions and the doing of different mathematical activities. There are too few examples of conjecturing, for example, to identify the features of the interaction that are reflexively related to the act of doing conjecturing through a single case analysis (Hutchby and Wooffitt, 1998). In extract 40, a pupil makes a conjecture in lines 19-20 and Richard revoices this conjecture in lines 21-22, emphasising the relationship 'lower' and 'higher', but there is no evaluation of the content of the conjecture, just an evaluation of the turn itself. As this is the only extract where a pupil makes a conjecture, we cannot say whether the structure of Richard's turn is related to the mathematical activity of conjecturing or not but a single case analysis of pupils conjecturing during whole-class discussions would enable us to see how the structure of the third turn relates to conjecturing.

The shifts in attention need to be managed carefully as ambiguity introduced through the use of pronouns such as 'it' or vague language can lead to trouble in the interaction and subsequently result in a change in the structure of the interaction in order to repair the trouble source, as occurs in Tim lines

160-185. Here Tim is shifting the attention to the fraction of the whole image that has been shaded in the previous turns. However, he asks "what fraction of that triangle have I shaded" without making it explicit which triangle he is describing. In the turns that follow he also shifts the attention between the diagram and the original numeric problem, again using vague language in lines 193-194 which result in pupils answering the question about the diagram rather than in the original context of the numeric problem.

This chapter has also examined how the discursive construction of the task can support the shifting of focus of attention to a variety of features of the task and consequently support pupils in making connections both between different representations and different topics within mathematics. Each of the extracts presented in chapter 7 focuses on different mathematical activities and the discursive construction of the tasks and these different constructions result in pupils doing very different mathematics. Mathematics is constructed as a school-based activity oriented around a curriculum and examination system in one extract, and is constructed as solving problems and making connections in another, and exploration and investigation leading to convincing and proof in the third. The attention of the pupils is managed by each of the teachers in different ways. In the extract from Simon's lesson, attention is focused on the procedures for calculating measures of central tendency and spread, and this is situated within the context of answering examination questions. In the extract from Tim's lesson, the mathematics is about solving a problem that is personal to Tim. Solving this problem involves 'doing' explaining, justifying and making connections between different representations. The extract from Richard's

lesson similarly involves making connections, but this time between the need for proof and the investigation that the pupils are working on. The problem belongs to the pupils in this extract in that they are exploring their own conjectures and 'doing' the investigation involves specialising, generalising and convincing. Mathematics involves discussing in both the extracts from Tim's and Richard's lessons, but it also involves sharing opinions in Richard's extract.

Summary

In this chapter, I have analysed each of the extracts in chapter 7 in turn, focusing firstly on the discursive construction of the roles of teacher and pupil, before examining the discursive construction of the mathematics that is being done in the interaction.

In each extract, the roles of teacher and pupil are easily identified through the control of turns and topics, and by who asks the questions and who answers them. Yet the roles of teacher and pupil are also different in each extract. The teacher role can include problem poser, story teller, story receiver or expert but though the role of teacher is often associated with at least that of problem poser or expert, these aspects do not always feature in the talk. Similarly, the roles of pupils not only alter between the extracts but also within each extract.

How the tasks and activities in the lesson are described as well as how these activities are done through the interaction also varies between the extracts. Tim's extract is about thinking through a problem and making connections between the numerical problem and a visual representation. Richard's extract is about meaning and being convincing, whilst Simon's extract is

about remembering and practising. The mathematics experience in each of the brief extracts differs considerably and serves to illustrate how wholeclass interactions can profoundly affect the mathematics that is being done.

Chapter 11: Conclusions and Implications

In this final chapter, the main findings discussed in chapters 8, 9 and 10 are summarised. A number of limitations and issues related to this study are then discussed before the implications that this study may have both on the teaching and learning of mathematics and on mathematics education research are explored. Suggestions for further research arise both from the limitations of this study and the implications the findings may have.

This thesis has examined the structures of whole-class interactions in transcripts from four secondary mathematics teachers, and how these structures offer opportunities and constraints in the teaching and learning of mathematics. This has been done using a conversation analytic approach focusing on a micro-analysis of the sequential organisation of the interactions. Chapter 8 focused on the turn-taking of each extract and the rules that teachers and pupils orient to in whole-class interactions. The IRF pattern is often used to describe the structure of turn-taking that predominates in classrooms (Lee, 2008; Mehan, 1979; Sinclair and Coulthard, 1975) and this structure or pattern was also a key feature of each of the extracts presented in chapter 7 and the data set as a whole. However, by adopting a conversation analytic approach, an alternative structure developed by McHoul (1978) is explored and the opportunities as well as the constraints that this structure offers for mathematical activity have been exemplified.

All of the teachers in this study controlled the turn-taking in whole-class interactions in that they controlled both who can speak and what could be

said during these turns. This control is managed locally on a turn-by-turn basis and enables the teacher to control the topic on a turn-by-turn basis. The structure of the turn-taking in classrooms has built-in mechanisms to support orderliness of interaction, and a great deal of mathematical activity is possible. The rules of turn-taking limit the possibility of multiple speakers speaking at once to those situations where the teacher specifically invites pupils to answer questions in unison. The rules also structure who can speak when and for how long, enabling a smooth transfer of turns to the next speaker. The structure of turn-taking gives control over the turns to the teacher, enabling them to control the topic and the nature of the interaction. The control a teacher has also enables the teacher to include a variety of pupils in the interaction. Despite these constraints on the turn-taking, pupils can make conjectures, provide justifications, make connections, perform calculations and procedures and so forth during their turns, though this largely dependent upon the design of the teacher's turn that initiates the interaction. Each of these types of mathematical activity occurs in the transcripts within this thesis.

In particular, wait time is structurally built in to the rules of turn-taking in whole-class discussions, as it enables pauses both between and during turns. It is the teacher that controls these pauses, but there is a tension between the rules for ordinary conversation and formal classrooms, where, in the former, silence is dispreferred. Consequently, whilst wait time is supported structurally the dispreference of silence offers an explanation for why attempts to increase wait time have had limited success (Black, *et al.*, 2003; Rowe, 2003). However, it is also this dispreference for silence that

results in pupils expanding upon their own turns, self-selecting to take turns and building upon other pupils' turns. This relationship explored in this thesis between the structure of turn-taking and the opportunities for wait time has implications on the professional development and education of teachers.

In each of the classrooms in this study pupils self-selected as next speaker in order to ask mathematically related questions, however in order to ask these questions the pupils need to deviate from the rules of classroom interaction. When these questions are mathematically related, however, none of the teachers sanctioned this deviation from the rules of turn-taking.

The rules of turn-taking also support the pupils in describing and explaining their mathematics. The data also offer examples where the structure deviates from the usual turn-taking rules and in many of these examples the pupils are arguing with their peers over a point of contention which leads them to justify their responses and to become personally engaged with the mathematics. However, just because the structure of turn-taking enables these things to occur, does not mean that they do and this issue was partly explored in chapters 9 and 10.

Chapter 9 focused on the preference organisation of adjacency pairs and repair, in particular, the role of markers and hesitations in mathematics classrooms. Pupils frequently mark their answers in some way in whole class interactions, though the nature and positioning of hesitations varies depending on the contexts. There are many possible explanations for this, such as mitigating a possible negative assessment by the teacher in the next turn or as a face-preserving move. The markers at the start of a pupil's turn

can also indicate that the pupil intends to take the turn, and therefore avoid the wait time between the teacher's nomination and the pupil speaking being interpreted as a source of trouble. This offers an explanation for why, despite the structure of turn-taking in classrooms enabling wait time between turns, the wait time between a teacher speaking and a pupil taking the next turn remains short (Rowe, 2003).

The preference organisation of repair differs slightly from that in ordinary conversation, but possibly as a consequence of the number of participants involved in the interaction. There is a clear preference for self-initiated self-repairs in all the classrooms, followed by peer-repairs then teacher-repairs, but the trajectories of these repairs are handled differently by different teachers and these differences have consequences for the mathematics. In particular, the handling of trouble that consists of mathematical mistakes has implications on the role of these in the teaching and learning of mathematics.

Whilst all the teachers in this study used mistakes in their teaching, the way that these were handled interactionally differed between the teachers. The key difference was whether interactionally mistakes are to be avoided or are something that can be built on. Consequently, whilst teachers may explicitly argue that making mistakes is part of the learning process, they may implicitly be treating these mistakes as something to be avoided.

There are a few situations where there is a preference for teacher-initiated teacher repair. These include where the trouble source is an explanation that a pupil is offering for an earlier response, inappropriate use of

mathematical terminology or a mistake that is not directly relevant to the topic of the interaction.

In chapter 10, the structures of turn-taking and preference organisation were brought together along with a more detailed turn-by-turn analysis of the three extracts in chapter 7 to examine what it means to 'do' teacher and pupil in each of the extracts, and then what it means to 'do' mathematics. The similarities and differences between the three teachers revealed differences in the roles of expertise and the personal involvement of both the teacher and the pupils. In one case, 'doing' expert focused on knowledge and consequently who could make evaluations, but also what doing mathematics involved. In the other two cases, doing expert involved modelling mathematical behaviours.

The differences in the ways that the different teachers discursively constructed the tasks and activities also has implications on the mathematics that the pupils are doing. One extract from Simon's lesson situates the activities within the context of school mathematics, emphasising the place of the tasks within the planned sequence of lessons and orienting to the examinations that the pupils will take in the future, both through explicitly mentioning them, but also through the style and structure of the tasks the students undertake. Answering questions is about practising procedures, recalling facts and rules and presenting these in a way that will satisfy the examiners.

The extract from Tim's lesson is about solving problems and working with multiple representations. These problems are personal to Tim and solving

problems is a collective activity involving 'doing' explaining, justifying and making connections between different representations. In the extract from Richard's lesson, the problem is personal to the pupils in that they have made conjectures and are exploring these conjectures. The mathematics is about exploring a problem and convincing each other, and in particular Richard, of patterns that are found.

Limitations

The limitations facing this study are perhaps most revealed in chapter 10, but run through the study as a whole. This study focused on teacher-pupil interaction and the relationship between talk and the mathematics being done in the interaction. The study takes a conversation analytic approach, which treats turns as designed and situated within the immediate context of the preceding and subsequent turns. How pupils make sense of the mathematics and the tasks done in the lesson is explored through a microanalysis of the interaction. This approach does not draw upon wider contextual information, including readily apparent features such as the gender of the participants. The turn-by-turn analysis does not reveal any differences between the talk of the different genders, both in the case of the teachers and the pupils. However, a raw count of the number of turns taken by pupils shows a large difference in the participation rates of boys and girls (similar to those found by Aukrust, 2008). It would appear that gender is relevant to classroom interaction, but this relevance is not revealed by a CA approach (Wetherell, 1998). Similarly, Fairclough (1995) offers an example where participants in job interviews do not explicitly orient to the gender of other participants but this does not mean that gender was not relevant in the

way it shaped the expectations that underpinned the interactions. Other contextual features such as the socio-economic backgrounds of the participants or the biographies of the teachers may also be revealed by a more critical approach.

In this study I have taken a CA approach but Edley and Wetherell (1997) argue that we need to merge CDA and CA in order to establish satisfactory accounts of interactions. CA and CDA are, however, incommensurable due to the differences in the ways in which they each approach the analysis of data. For CA, the analysis focuses exclusively on the turn-by-turn interactions and features such as the gender or participants, or issues of power etc. are only drawn upon in the analysis if the participants themselves orient to them in the interaction. CA approaches the data with questions focusing on what the participants are doing in the interaction. CDA on the other hand approaches the analysis of data with particular questions or a particular focus, such as how are power relationships managed in the interaction (Schegloff, 1997; Wetherell, 1998). Having said this, this does not mean that the same data could not be used separately by a CA researcher and a CDA researcher with both findings contributing to our developing understanding of what is going on in particular interactions, just that these approaches cannot be done together.

Silverman argues that CA can be used initially to analyse how participants structure the interactions, whilst other methodologies, ethnography in particular, can be used to answer questions such as why the talk is structured in this way (2010, p.239). The majority of studies of mathematics

classroom interaction have used either a discourse analysis approach or a critical discourse analysis approach. This study contributes a different perspective on the nature of these interactions that can complement and contribute to studies drawing from these other methodologies.

Whilst differences between teachers' constructions of mathematics have been identified, the possible sources or causes of these differences have not been addressed. This focus has studied how the participants construct the activities within whole-class discussion, not why they are constructed in this way. It is possible that the beliefs of the teachers about the nature of mathematics, the nature of teaching mathematics and the nature of learning mathematics affect how they construct the mathematics in their classrooms. It is also possible that the experiences of the teachers themselves as learners of mathematics and doers of mathematics are limited in some way, and the teachers themselves may not have had experience of justifying or convincing in a mathematical way for example. In the extracts from Tim's and Richard's lessons, the interactions involved a range of mathematical behaviours which the teachers themselves modelled. If, as a teacher, you have not had experience of behaving in this way, it is unlikely that you will be able to model these behaviours through the ways in which you interact with your pupils. This has significant implications on the initial training of teachers and the role of subject knowledge within teaching.

The analysis in this study also considers pupils' turns as if the whole-class is an individual and does not differentiate between those pupils who participate in the interactions and those that do not, whether this is through choice or

not. The focus of analysis was in the structure of the interactions rather than on the nature of participation. Other researchers have argued that participation plays an important role in the development of pupils' mathematical identities (Boaler, *et al.*, 2000a) drawing upon self-reporting by pupils through interviews. Whilst this study has not examined the nature of participation, it does offer tools that can be used to analyse how pupils and teachers do mathematics and the nature of this mathematics that they do.

Finally, there are limitations within the data collected. There are extracts within the data that are analytically interesting, but CA requires that the analyst returns to the wider data set collected or other available data to explore whether similar extracts with similar properties occur in order to develop a detailed account of the structural features of these sequences. Whilst the data set collected in this study is substantial, some features of the interactions appeared on only one or two occasions and therefore conclusions about the relationship between these structures and the teaching and learning of mathematics cannot be made.

In this study, a detailed analysis of instances where pupils made conjectures, made a generalisation or gave a justification would have enabled a greater understanding of the relationship between the interactions and the mathematical activity. Unfortunately, these instances are rare in the data collected or only appear in the transcripts from one teacher. The data that appears in the majority of publications on whole-class interaction does not include sufficient detail in the transcripts, or transcripts of sufficient length for a CA informed analysis. A single case analysis (Hutchby and Wooffitt, 1998)

of particular mathematical behaviours would enable this analysis to occur. CA research often involves the fine-grained analysis of a collection of instances (Schegloff 1987) to develop a richer understanding of particular phenomenon. Whilst recognising the relatively limited scope of the analysis and findings of this study of the relationship between patterns and structures of interaction and the teaching and learning of mathematics, this study has demonstrated that the structure of interaction and the discursive construction of mathematical activity can have an impact on the nature of mathematics that both teachers and pupils experience

Implications

The main implication of this study is that the structure of interactions needs to reflect the pedagogic purpose. Different structures of turn-taking and preference organisation have consequences on the nature of the activity that can be done in the whole-class interaction.

The phrase wait time is often found in the current literature both on classroom interaction and on assessment (Black, *et al.*, 2003; Lee, 2006; Rowe, 2003; Tincani and Crozier, 2008). This study has demonstrated that the turn-taking structure of formal classrooms structurally enables wait time to occur and that it is the teacher that controls this wait time. This study also offered explanations for many of the research findings relating to the increase of wait time both between the teacher and pupil's turn and following the pupil's turn. However, it also offered an explanation for the difficulties in implementing an increased wait time in classrooms. These explanations may help teachers to understand both the role of wait time particularly in

relation to pedagogical purposes, and how to build it in to their own wholeclass discussions.

The tight control over interactions can be interpreted as having a negative impact on pupil learning, particularly in discussion relating to the asymmetries of power and expertise in the classroom. However, this control also has pedagogical advantages, such as when establishing common knowledge as a prerequisite for the main focus of the lesson, but control of the topic does not just mean a pre-defined focus from which participants cannot deviate. In fact, it enables the teacher to introduce new topics, open topics up for discussion or close them down on a turn-by-turn basis. The teacher can also relinguish this control and change the structure of turntaking whilst retaining sufficient control to alter and change these patterns of interaction to fit the pedagogical purpose. If teachers are aware of the choices they can make in the design of these interactions, then they can react dynamically as the focus or topic of the interaction develops. Thus where a shift from describing to explaining, or from explaining to justifying is needed by the teacher, an awareness of how the structure of interaction can affect the mathematical activity of the pupils can enable the teacher to alter the structure to support the pupils further.

The role of mistakes in the teaching and learning of mathematics has also been widely discussed in recent years, particularly with the introduction of the National Strategies in the UK. Teachers are encouraged to use mistakes as teaching and learning opportunities. Each of the teachers in this study used mathematical mistakes, but the differences in the ways these were

handled in the interactions have implications on the role these have in the teaching and learning of mathematics. Firstly, the preference organisation of repair in both Simon's and Edward's lessons treated mistakes as something to avoid despite both teachers supporting pupils to see and self-repair these mistakes. In terms of mathematics, this has consequences on the role of conjecturing and justification. Part of learning mathematics is learning to make, test, adjust, accept or reject conjectures and justification is key to this process of developing conjectures. The preference organisation of repair can affect who has the responsibility for testing, evaluating and accepting or rejecting these conjectures and for providing the justifications for this accepting or rejecting, but can also have consequences on the making of conjectures.

Chapter 10 focused on the construction of the situated identities of teacher and pupil and the discourse identities within these, before focusing on the discursive construction of mathematics in each of the extracts in chapter 7. This chapter highlights that it is not only the resources or tasks used that effects the mathematics that is done, but how these tasks or resources are talked into being. The mathematics actually done during the interaction is dependent upon the way the teacher describes the tasks, structures their questions, controls the turn-taking as well as how the pupils structure their own turns. This variation in the teachers' discursive constructions of the activities found in this study is in itself and indicator of the complexity of the process of teaching and learning mathematics and the careful balancing between different tensions that teachers consider. An awareness of these differences may support teachers in developing their approach to

mathematics in whole class discussions and help them to appreciate the opportunities and constraints that different approaches may offer.

Conclusion

This study has focused on the structures of whole-class discussions and the relationship between these structures and the learning of mathematics. It has demonstrated that the teacher and the pupils jointly construct these discussions, yet the teacher retains a great deal of control, which they can use for particular pedagogical purposes. The findings of this study have implications on the professional development and initial education of mathematics teachers by highlighting the role that different structures of interactions have but also by highlighting the impact of some these structures on the learning of mathematics. These structures of talk imply that whole-class discussions are not necessarily the best place for particular activities (Myhill and Warren, 2005) though further research is needed to explore the nature of structures that support different mathematical activities such as conjecturing or justifying.

This thesis makes three key contributions to the field of mathematics education and mathematics education research. Firstly, it has demonstrated that conversation analysis can be used in mathematics education research to both reveal features of interaction that may influence the teaching and learning of mathematics. More recent studies using CA within sociology and psychology have shown that CA can be used to explores issues, such as power, that are usually left to other methodologies to examine. Further research is needed to explore how the structural organisation of mathematics

classrooms affects pupils' learning of mathematics, but also pupils' developing identities in relation to mathematics.

Secondly, it also offers explanations for research findings from studies using different methodologies. Conversation analysis reveals how things are achieved interactionally, in particular how teachers and pupils do things on a turn-by-turn basis. Many discourse analysis based approaches have revealed relationships between particular types of turns and subsequent pupil behaviour. This study reveals how these turns are constructed and in some cases why the subsequent behaviour occurs, for example pupils expanding their turns following a period of silence or the relationship between revoicing and subsequent pupils' turns.

Finally, this study reveals how the discursive construction of mathematical tasks influences the nature of mathematical activity and the mathematics that pupils do. The differences in how tasks are constructed discursively influence the mathematics that pupils experience but will also affect how they behave mathematically, view mathematics and their own developing identities in relation to mathematics.

Appendices

Appendix A – Transcription conventions

The transcription conventions used are drawn from the system developed by Gail Jefferson (Jefferson, 2004). It includes details about the delivery of talk such as overlaps, delays, and emphasis. All spoken utterances have been transcribed verbatim wherever possible, with grammatical errors or other linguistic errors uncorrected. Many passages are marked inaudible. The lessons were recorded under normal classroom conditions, which meant that background noise was inevitable.

A non-proportional font (Courier new) has been used for all transcriptions to enable clear indications of overlaps. The normal written uses of punctuation are not followed. Standard punctuation marks such as commas, full stops, and question marks indicate intonation rather than syntax.

- []]Square brackets mark the start and end of overlapping speech.They are aligned to mark the precise position of overlap.
- \rightarrow Side arrows are used to draw attention to features of talk that are relevant to the current analysis.
- Underlining indicates emphasis; the extent of underlining within individual words locates emphasis and also indicates how heavy it is.
- CAPITALS mark speech that is hearably louder than surrounding speech. This is beyond the increase in volume that comes as a byproduct of emphasis.

°I know it,° 'degree' signs enclose hearably quieter speech.

- (0.4) Numbers in round brackets measure pauses in seconds (in this case, 4 tenths of a second).
- (.) A micropause, hearable but too short to measure.
- (()) Additional comments from the transcriber, e.g. about features of context or delivery.
- wa::nted Colons show degrees of elongation of the prior sound; the more colons, the more elongation.
 - 'Continuation' marker, speaker has not finished; marked by fallrise or weak rising intonation, as when delivering a list.
- Question marks signal stronger, 'questioning' intonation,irrespective of grammar.
 - Full stops mark falling, stopping intonation ('final contour'), irrespective of grammar, and not necessarily followed by a pause.
- bu- hyphens mark a cut-off of the preceding sound.
- = 'Equals' signs mark the immediate 'latching' of successive talk,
 whether of one or more speakers, with no interval.

Appendix B – Extract from Edward's lesson

001	Edward:	Ri:gh t ,
002		(0.6)
003		which pla:ce is the co::ldest then, (.) hands $$ up,
004		(0.6)
005		↑Charlie,
006	Charlie:	E:r Montrea:l
007	Edward:	Montrea::1 (.) good (.) a::nd which place in the
008		Twarme:st?
009		(0.4)
010		e:r Jamie,
011		(.)
012	Jamie:	Madrid,
013	Edward:	Madrid. (.) Good (.) oka:y so ↑sa:y you're on
013	24.424.	holida:y, (.) if you're >going on holiday and you
014		get in your aeroplane start off in< London, (.) and
015		you're, (.) going to Madrid
010		(0.4)
018		ofkay?
019		(0.6)
020		Does (.) it get
021		$\frac{10000}{(0.4)}$ (1) 10 get
022		Thotter (.) or colder
023		(0.4)
024		and by >how much<
025		(0.6)
026		Harry
027	Harry:	Hotter (.) by::: seven,
028	marry.	$\frac{10}{(0.8)}$
029		centigra::de, >(I mean) (degrees)<
	Edward:	
030	Edward:	Seven degrees (.) o:1 <u>kay</u> (.) <u>lov</u> ely
030 031	Edward:	Seven degrees (.) o: tay (.) lovely (0.4)
030 031 032	Edward:	Seven degrees (.) o: <u>thay</u> (.) <u>lov</u> ely (0.4) tu::m what atbou::t >if you were< going on holiday
030 031 032 033	Edward:	Seven degrees (.) o: <u>tray</u> (.) <u>lov</u> ely (0.4) tu::m what atbou::t >if you were< going on holiday again, (.) from Ma <u>drid</u> (.) you're, (.) >you're
030 031 032 033 034	Edward:	Seven degrees (.) o: <u>thay</u> (.) <u>lov</u> ely (0.4) tu::m what atbou::t >if you were< going on holiday again, (.) from Madrid (.) you're, (.) >you're going on a little bit< of a <u>to</u> u::r
030 031 032 033 034 035	Edward:	Seven degrees (.) o: <u>thay</u> (.) <u>lov</u> ely (0.4) tu::m what atbou::t >if you were< going on holiday again, (.) from Madrid (.) you're, (.) >you're going on a little bit< of a <u>to</u> u::r (0.4)
030 031 032 033 034	Edward:	Seven degrees (.) o: <u>kay</u> (.) <u>lov</u> ely (0.4) ¹ u::m what a ¹ bou::t >if you were< going on holiday again, (.) from Madrid (.) you're, (.) >you're going on a little bit< of a <u>to</u> u::r (0.4) .shih (.) and you're in Mad <u>rid</u>
030 031 032 033 034 035 036	Edward:	Seven degrees (.) o: <u>kay</u> (.) <u>lov</u> ely (0.4) ^u::m what a [^] bou::t >if you were< going on holiday again, (.) from Madrid (.) you're, (.) >you're going on a little bit< of a <u>to</u> u::r (0.4) .shih (.) and you're in Mad <u>rid</u> (0.4)
030 031 032 033 034 035 036 037	Edward:	Seven degrees (.) o: <u>kay</u> (.) <u>lov</u> ely (0.4) tu::m what a bou::t >if you were< going on holiday again, (.) from Madrid (.) you're, (.) >you're going on a little bit< of a <u>to</u> u::r (0.4) .shih (.) and you're in Mad <u>rid</u> (0.4) and then you go onto <u>Mo</u> scow what happens the:n
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030 031 032 033 034 035 036 037 038 039	Edward:	Seven degrees (.) o: <u>kay</u> (.) <u>lov</u> ely (0.4) tu::m what a bou::t >if you were< going on holiday again, (.) from Madrid (.) you're, (.) >you're going on a little bit< of a <u>to</u> u::r (0.4) .shih (.) and you're in Mad <u>rid</u> (0.4) and then you go onto <u>Mo</u> scow what happens the:n
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030 031 032 033 034 035 036 037 038 039 040 041		Seven degrees (.) o: <u>kay</u> (.) <u>lov</u> ely (0.4) tu::m what a bou::t > if you were< going on holiday again, (.) from Madrid (.) you're, (.) > you're going on a little bit< of a <u>tou</u> ::r (0.4) .shih (.) and you're in Mad <u>rid</u> (0.4) and then you go onto <u>Mo</u> scow what happens the:n (0.4) hotter or colder and by how much (1.4)
030 031 032 033 034 035 036 037 038 039 040 041 042		<pre>Seven degrees (.) o: îkay (.) lovely (0.4) îu::m what aîbou::t >if you were< going on holiday again, (.) from Madrid (.) you're, (.) >you're going on a little bit< of a tou::r (0.4) .shih (.) and you're in Madrid (0.4) and then you go onto Moscow what happens the:n (0.4) hotter or colder and by how much (1.4) (Twenty eight) (.)</pre>
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030 031 032 033 034 035 036 037 038 039 040 041 042 043 044	A: Edward:	<pre>Seven degrees (.) o: îkay (.) lovely (0.4) îu::m what aîbou::t >if you were< going on holiday again, (.) from Madrid (.) you're, (.) >you're going on a little bit< of a tou::r (0.4) .shih (.) and you're in Madrid (0.4) and then you go onto Moscow what happens the:n (0.4) hotter or colder and by how much (1.4) (Twenty eight) (.)</pre>
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057		(0.4)
058		Ashley
059		-
060		(0.4)
061	Ashley:	[E::r]
062	Edward:	[Does] it get hotter or colder if you go from
063	Edward.	
		[Moscow]
064	Ashley:	
065		[Colder]
066	Edward:	=to Montreal
067	Ashley:	Colder
068	Edward:	By how many degrees
069	Ashley:	Three
070	Edward:	↑ Three
071		(0.4)
072		oka::y so ni::ce and easy
073		(0.4)
074		now
075		(0.4)
076		ÎCai::ro
077		$(\overline{0.8})$
078		another city,
079		(0.6)
080		Cai::ro (.) is thi::rty degrees wa:rmer
081		(0.4)
082		
083		than Montreal
		(2.8)
084		so we're in Montrea::1 (.) that's where we've ended
085		up >we're gonna go< to <u>C</u> airo: (.) <u>we</u> <u>kn</u> o::w that
086		when we get there it's going to be <u>thir</u> ty degrees
087		wa::rmer (.) than Montreal
088		(0.4)
089		Drew
090	Drew:	
091	В:	No:::
092	C:	.hhhh
093	D:	[(U::m)]
	Edward:	
094		[Mea::n]i:ng,
095	E:	Nineteen
096	F:	Nineteen
097	Edward:	Nineteen
098		(0.4)
099		okay 🕇 good (.) how'd you work it out Jamie
100	Jamie:	U:::m
101		(0.8)
102		(know about eleve:n)
102		(0.6)
104		an the::n
105		(0.4)
106		(about ten) (.) (divide) eleven and then (.) take
107		away (.) no (.) I had $\underline{th}i:$ rty (.) >and then took
108		away< eleven
109	Edward:	So you did <u>th</u> i:rty,
110		(0.8)
111		take away eleven
112		(0.4)
113		(0^{kay})
ттЭ		(UIRAY)

Appendix C – Ethical Forms

Information Sheet

The development of mathematical learning in whole class discussions.

This study aims to look at the relationship between a mathematics teacher's use of whole class discussion to develop mathematical understanding and the development of this understanding in their pupils as demonstrated in their use of language.

A sequence of lessons with one particular class will be videoed and observed, then a small group of pupils and the teacher will be interviewed about the whole class discussions focussed on mathematics in that lesson, using the video of the lesson to support recall of the context.

The videos will not be viewed by anyone other than those within the Mathematics Education group at the University of Warwick. The transcripts of the interviews will be anonymised before they are used for analysis and in any presentations of the findings. All information shared with the researcher will remain confidential unless there is a potential risk to the safety of an individual. You will have the right to withdraw from this study at any time. Neither individuals nor the school will be identifiable from any published materials resulting from this study.

All data from the study will be stored securely, either in a locked filing cabinet or in password protected electronic files. Copies of any findings resulting from this research will also be made available to all volunteers if they wish.

Thank you for your interest in this study.

Jenni Ingram

Assistant Professor of Mathematics Education

Institute of Education

University of Warwick

Coventry

CV4 7AL

Consent Form

Project Title: The development of mathematical learning in whole class discussions

Name of Researcher:

(to be completed by participant)

I confirm that I have read and understood the information sheet dated..... For the above project which I may keep for my records and have had the opportunity to ask any questions I may have.

I agree to take part in the above study and am willing to:

Be videoed in my Mathematics Lessons

Be interviewed individually following the lesson.

Be interviewed as part of a small group following the lesson.

I understand that my information will be held and processed for the following purposes:

Analysis by Jenni Ingram and her colleagues of all data collected in the research.

Publication of the findings of this research.

I understand that my participation is voluntary and that I am free to withdraw at any time without giving any reason without being penalised or disadvantaged in any way.

Name of Participant	Date	Signature
Parental Consent	Date	Signature
Name of person taking consent if different from Researcher	Date	Signature
Researcher	Date	Signature

Ethical Approval

Research degrees: Application for Ethical Approval

About you

Name Jenni Ingram Project title Queshoning and Mathematical Thereing Supervisor Peter Johnston - Wilder and Mary Briggs Degree (please circle): MA by research EdD (MPHil / PhD)

Please ensure you have read the Guidance for the Ethical Conduct of Research available in the handbook.

About the participants

Please specify all participants in the research including ages of children and young people where appropriate. Also specify if any participants are vulnerable e.g. as a result of learning disability.

Participants include I teacher and the class of their choice including up to 32 pupils below the age of 18

Respect for participants' rights and dignity

How will the fundamental rights and dignity of participants be respected, e.g. confidentiality, respect of cultural and religious values?

All data will be treated conjudentially; importantly data collected from pupils will not be shared with the teacher with it has

Privacy and Confidentiality been aronymused (or not at all)

How will confidentiality be assured? Please address all aspects of research including protection of data records, thesis, reports/papers that might arise from the study.

All data will be stored securely in either hard-copy or softwapy form. All data will be anonymised by are appearing papers or reports or the thesis Consent - will prior informed consent be obtained from participants? Yes NO

- from others?

Head Teacher, parents

Yes No

- explain how this will be obtained. If prior informed consent is not to be obtained, give reason:

Letters to all parents concerning violeoing of lessons plus additional consent forms for those pupils being interviewed, for both the pupils and their parents. - will participants be explicitly informed of the student's status?

Competence

How will you ensure that all methods used are undertaken with the necessary competence?

Triangulation of data through video and interviewing with the possibility of participants viewing their transcripts Responsibility

Well-being i)

How will participants' safety and well-being be safeguarded?

Parhapants have the right to withdraw at any shage.

Addressing dilemmas ii)

Even well planned research can produce ethical dilemmas. How will you address any ethical dilemmas that may arise in your research?

Any delemmas regarding a child's safety will be subject to the confidence agreement

iii) Misuse of research

How will you ensure that the research and the evidence resulting from it are not misused?

Data will only be accessible to others once anonymised and only through the publication of papers and the thesis. Integrity

How will you ensure that your research and its reporting are honest, fair and respectful to others?

Parkapants can view transcripts. Data will be brangerlated - Mysely, Teacher and pupils. Have you and your supervisor discussed and agreed the basis for determining

authorship of published work other than your thesis?

Other issues?

Please specify other issues not discussed above, if any, and how you will address them.

Signed

Research student Supervisor Action

Date 6¹⁶ Febrary Date GHL Feb 2008

Please submit to the Research Office (Louisa Hopkins, room WE132)

Action taken

Approved

4 LINDS Dr

Approved with modification or conditions - see below

Action deferred. Please supply additional information or clarification - see below

Name

Signature

Date 5/1/07

Stamped

notes of Action This is well produe. Please who well "As lowing dilemans" the other night arise - disins nell separism - allegt y have by highly the main some uff, 39

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