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### **Valuing Voluntary Disclosure using a Real Options Approach**

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# Valuing Voluntary Disclosure using a Real Options Approach\*

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April 30, 2012

## Abstract

This paper outlines a real options approach to valuing those announcements which are made by firms outside of their legal requirements. From the firm's perspective, information is disclosed only if the manager of the firm is sufficiently certain that the market response to the announcement will have a positive impact on the value of the firm.

When debt financing is possible we find that the manager adopts a more transparent disclosure policy, thus violating the Modigliani-Miller theorem on irrelevance of capital structure.

*Keywords:* Voluntary Disclosure, Real Options, Modigliani-Miller Theorem.

**JEL Classification Numbers:** C61, D81, M41.

## 1 Introduction

Corporate voluntary disclosure has become an important element of capital market dynamics in that it conveys value-relevant information for market pricing (Wen [25]). As well as this, it typically contains information related to a firm's activities which may not be immediately stated in accounting reports.

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According to Dempster [4], the issue has become increasingly topical and important in the aftermath of some major corporate scandals such as Enron, WorldCom and others. Such events have raised concerns over the transparency of U.S. firms and, in particular, the quality of their financial reporting and disclosures. This paper demonstrates how a real options approach to valuation can contribute to our understanding of corporate disclosure, and in particular, *voluntary* corporate disclosure which is concerned with those announcements willingly made by firms which are outside of their legal and regulatory requirements.

One of the earliest findings in the disclosure literature, provided by Grossman and Hart [12] and Grossman [11], has become known as the “unraveling result”. If the managers of firms, holding private information, choose not to disclose their information to outside investors, then the investors will discount the value of the firm down to the lowest possible value consistent with whatever voluntary disclosure is made. Once the managers realize this, they will have an incentive to make full disclosure. Dye [7], however, challenges this result, and provides a reasoning for why it may not always hold. He shows that the qualitative features of an optimal disclosure policy for management may take the form of a policy dependent cutoff in which management disclose only if the information is sufficiently good, otherwise they withhold disclosure. His reasoning is due to the uncertainty of investors about the firm’s information endowment; that is, investors may not be able to distinguish between managers holding undisclosed information from managers being uninformed. In such a setting, investors seeing non-disclosure must temper their inferences concerning the likelihood of a manager having observed bad news and opting not to disclose by the fact that non-disclosure may have arisen due to managers being uninformed. Since these early seminal contributions by Grossman [11] and Dye [7], a large body of work has emerged on corporate voluntary disclosure. Verrecchia [24] provides an extensive survey on such disclosure models.

The economic analysis of disclosures at its fundamental level investigates voluntary disclosures. Even though provision of information, such as a publicly traded company’s financial statements, is mandatory, the economic approach is motivated by the observation that we can only assess the effect of mandatory disclosures relative to the disclosures that would have arisen in the absence of such regulation. Indeed Wen [25] demonstrates that the efficiency effect of compulsory disclosure is contingent on the information required and the economic environment.

One interpretation of voluntary announcements, says Dempster [4], “is that they provide an opportunity for managers to communicate to the marketplace that they are aware of, and up to date with, current investor demands and interests”. She cites an example from Subramani and Walden [20], who, in their study of the market impact of e-commerce announcements, argue that “the reason for the significant positive abnormal returns that they found were

in part because investors viewed announcement of such initiatives as favorable signals of certain firm attributes”. Furthermore, Graham et al. [10] argue that companies voluntarily disclose information to provide clarity to investors.

Importantly, across various streams of research investigating corporate disclosure, there is a growing recognition that the various announcements that firms make have an inherent strategic value in their power to influence external perceptions directly and firm performance indirectly (see Dempster [4]).

In principle, a firm can make an announcement about anything it chooses, and thus, there is an infinite number of announcement applications. Examples of such announcements include competitive pricing strategies, new product introductions, various mergers, acquisitions and other alliances, and a range of detailed structural changes within the firm. However, in practice, firms tend to make announcements only about key strategic and organizational events that could impact substantially on their value and success (see Dempster [4] and Bayus et al. [2]).

In this paper we view voluntary disclosure of information relating to the state of the firm to the marketplace as a (real) option held by the firm’s manager. Exercising the option to disclose information is a strategic decision on the part of the firm, which implies that the manager will only do so if he is sufficiently certain that the payoffs are positive, i.e. that the option is deep enough in the money. We measure the payoff to the disclosure option by the impact of market response to the information on the value of the firm. This reflects the standard corporate practise to (partly) remunerate managers based on the firm’s stock market performance. This, effectively, aligns the manager’s incentives with potential sellers of the firm’s equity. They, after all, are interested in firm value to be as high as possible.

We also adapt the model to show that it provides an example of a violation to the Modigliani–Miller theorem on irrelevance of capital structure on firm value. With respect to the current setting, we show that when some of the disclosure cost is financed with debt, the limited liability aspect of debt dominates the loss to the firm from compensating the lender for expected default losses, and consequently, the optimal disclosure threshold for the manager is lower. As such, this is an interesting example where excess risk taking by managers due to limited liability protection of debt financing actually has a positive effect.

From a modelling point of view, our paper is most closely related to Thijssen et al. [23] and Sabarwal [17]. The model of the arrival of imperfect signals over time follows that of Thijssen et al. [23]. That paper analyzes the problem of a firm with the opportunity to invest in a project which has an uncertain profitability. They assume that there is no negative impact on firm value through exercising their investment option. However, making such an assumption in our set-up is not realistic. If the manager makes an announce-

ment, the shareholders may react negatively and respond by selling off some of their investment in the firm. This implies that disclosure may, indeed, have a direct negative impact on firm value. This results in a lower threshold than the threshold under their set-up. Sabarwal [17] shows how the ideas regarding the value of the option to wait provide a violation to the Modigliani-Miller theorem, but he deals with uncertainty using the standard framework of the real options literature (see Dixit and Pindyck [5]), whereas in our model, uncertainty is resolved over time and thus, standard stochastic calculus tools cannot be used.

The paper is organized as follows; the benchmark model is described and the optimal stopping problem for the disclosure threshold is solved in the next section. Section 3 discusses some of the important features inherent in the manager’s optimal disclosure policy. Section 4 analyses the model from another dimension, namely if some of the disclosure costs are financed with debt, and Section 5 finally concludes. All proofs appear in the Appendix.

## 2 The Model

### 2.1 Background and Motivation

Consider a firm that has invested in a new project and that the manager receives private information over time pertaining to the project’s performance (like, for example, sales figures). We assume that the firm’s management is not legally obliged to disclose this information. Thus, the manager can exercise discretion over whether to share his private information with the market or to withhold it. His objective is to adopt a disclosure policy such that his own current expected (discounted) utility from wealth is maximized.

It is assumed that the manager of the firm is uncertain about how the private information he holds will be perceived by the market. The more positive are the signals he obtains, the more likely the market will interpret the information favourably. Hence, each time a signal is obtained, the manager updates his belief as to the likely market response in a Bayesian way. This source of uncertainty is the same as that in Suijs [21], but differs with Dye [7] in the sense that he assumes the uncertainty arises because the market is unsure what, if any, information the manager has obtained. According to Suijs [21], “the assumption of response uncertainty is necessary to prevent (extremely) high returns from being disclosed, an act that would initiate the unravelling process” described by Grossman and Hart [12].

Suijs [21] discusses several reasons why such response uncertainty may arise. One such reason is that “the market can interpret the disclosed information in different ways.” For example, he points out, in Dutta and Trueman [6],

response uncertainty arises because firms do not know how investors will interpret the firm's private information. The example they refer to in their paper is on the disclosure of order backlog. On one hand, investors can interpret a high-order backlog positively if they believe that it signals that the demand for the firm's product is high. On the other hand, a high-order backlog can be interpreted negatively by investors if they believe that it "signals problems with the firm's production facilities or a manager's lack of control over operations". In terms of the current story, the disclosure of the signals (that is, the disclosure that the firm has invested in a new product) may be interpreted favorably by the market in that it signals growth and innovation within the firm through newer and more improved products. Alternatively, such news may be interpreted unfavorably as the market views the investment as a costly and risky venture with little chance of success.

We further assume that all disclosures are (ex post) verifiable; that is, a manager will not issue misleading information in an attempt to alter the market's perception of his firm's prospects. Stocken [19] examines in detail the credibility of a manager's disclosure of privately observed nonverifiable information. His main finding is that a manager will almost always endogenously truthfully disclose his private information because if the market perceives lack of credibility in the disclosure, it will ignore it and this can lead to deeper problems for the firm in the future. The credibility could also be interpreted from the manager's reputation perspective. According to a study conducted by Graham et al. [10], executives believe that a reputation for not consistently providing precise and accurate information can lead to the under-pricing of the firm's stock.

In this model, disclosure is costly and this cost cannot be recouped once the disclosure option has been exercised. For example, there may be some direct costs associated with producing and disseminating information; that is, information may need to be disclosed or certified by a third party such as an accounting firm. Note that these costs are direct and do not relate to the (indirect) proprietary costs that are typically referred to in the disclosure literature such as the cost of revealing firm sensitive information to competitors. There also exists an exogenous opportunity cost of waiting for more, and possibly better, information signals to arrive. By waiting for further signals, the manager can be more certain of the overall profitability of the firm (owing to the new investment), which will reduce the likelihood of misinforming the market and thereby damaging his reputation. These costs of announcing the information, and therefore exercising the disclosure option, could greatly outweigh the benefit of disclosure.

We assume throughout that a fraction of the firm is owned by the manager. Therefore, the manager's compensation depends upon the firm's activities, and as such, he is compensated with a fraction of the option to disclose. Note that if the manager's compensation does not depend on the disclosure option

itself, the manager should not have any preference for the timing of disclosure. Assuming that the manager's preferences are quasi-linear in his share in the firm's value we can assume the manager to maximize firm value. In this way, the incentives of the manager are aligned with those of a shareholder whose only aim is to maximise firm value. This is consistent with the typical principal-agent set-up of Mas-Colell et al. [15].

## 2.2 Model Set-up

We assume that the firm has invested in a project and receives (private) information regarding the project's profitability. The manager has, at any time, the option to voluntarily disclose the information at a sunk cost  $I \geq 0$ . The manager is uncertain about market reaction to the disclosure. The market reaction to the disclosed information can be either good ( $\gamma = 1$ ) or bad ( $\gamma = 0$ ) resulting in a change in firm value of  $V^P > I$  or  $V^N < 0$ , respectively. Over time, the manager receives information, the arrivals of which follow a Poisson process with parameter  $\mu > 0$ . Information is interpreted by the manager as either increasing the likelihood of a positive market response or decreasing it. Each batch of information, however, is an imperfect signal which reflects the true market reaction with probability  $\theta \in (1/2, 1)$ .<sup>1</sup> In this set-up, the number of signals indicating a positive market reaction net of the number of signals indicating a negative market reaction is a sufficient statistic for the manager's optimal disclosure policy. At time  $t$  this number of signals is denoted by  $s_t \in \mathbb{Z}$ . Under the assumptions regarding the arrival and precision of information it can be shown that  $s_t$  evolves over time according to (cf. Thijssen et al. [23])

$$ds_t = \begin{cases} 1 & \text{w.p. } [1_{(\gamma=1)}\theta + 1_{(\gamma=0)}(1-\theta)]\mu dt \\ 0 & \text{w.p. } 1 - \mu dt \\ -1 & \text{w.p. } [1_{(\gamma=1)}(1-\theta) + 1_{(\gamma=0)}\theta]\mu dt. \end{cases} \quad (1)$$

Suppose that the manager has a prior over the probability of a positive market reaction equal to  $p_0 \in (0, 1)$ . If, at time  $t \geq 0$ , the manager observes  $s_t$ , then his posterior probability of a favorable market response follows from an application of Bayes' rule (see Thijssen et al. [23]):

$$p_t := p(s_t) = \frac{\theta^{s_t}}{\theta^{s_t} + \zeta(1-\theta)^{s_t}}, \quad (2)$$

where  $\zeta = (1-p_0)/p_0$  is the prior odds ratio. Note that  $p_t$  is a monotonically increasing function in  $s_t$ , and that the inverse function is given by

$$s_t := s(p_t) = \frac{\log\left(\frac{1-p_t}{p_t}\right) - \log(\zeta)}{\log\left(\frac{1-\theta}{\theta}\right)}. \quad (3)$$

---

<sup>1</sup>This assumption is made without loss of generality. A choice of  $\theta = \frac{1}{2}$  implies that the signal is pure noise, since the initial prior is not revised. Furthermore, a choice of  $\theta = 0.2$  is as informative as a choice of  $\theta = 0.8$  since the same analysis may be carried out for  $1 - \theta$ .



This implies that we can either work with the number of net signals or the posterior belief. In the following we use both approaches intermittently, depending on analytical convenience.

If the manager discloses the information at time  $t \geq 0$ , then, conditional on the prior  $p_0$ , the expected change in the firm's value equals

$$U(s_t) := p(s_t)V^P + (1 - p(s_t))V^N - I. \quad (4)$$

Assuming that the manager discounts future payoffs at a constant rate  $r > 0$ , his problem can be formulated as an optimal stopping problem,

$$U^*(s_t) = \sup_{\tau \geq t} E_t [e^{-r\tau} U(s_\tau)], \quad (5)$$

where  $E_t$  denotes the expectation conditional on all information available up to and including time  $t$ , and the supremum is taken over stopping times.

Problem (5) has an analytical solution, which takes the form of a threshold policy: the manager should disclose the information as soon as the posterior belief exceeds a certain threshold belief  $p^*$ . Adapting the arguments in Thijssen et al. [23] to our setting, this threshold can be shown to be given by (see Appendix A for details)

$$p^* = \left[ \frac{V^P - I}{I - V^N} \Pi + 1 \right]^{-1}, \quad (6)$$

where

$$\Pi = \frac{\left( \beta_1(r + \mu) - \mu\theta(1 - \theta) \right) \left( \frac{r}{\mu} + 1 - \theta \right) - \mu\theta(1 - \theta)\beta_1}{\left( \beta_1(r + \mu) - \mu\theta(1 - \theta) \right) \left( \frac{r}{\mu} + \theta \right) - \mu\theta(1 - \theta)\beta_1} \quad (7)$$

and  $\beta_1 > \theta$  is the larger (real) root of the quadratic equation

$$\Psi(\beta) \equiv \beta^2 - \left( \frac{r}{\mu} + 1 \right) \beta + \theta(1 - \theta) = 0. \quad (8)$$

Note that there is no guarantee that there exists an integer  $s$  such that  $p^* = p(s)$ . In other words, the optimal disclosure threshold in terms of net signals can be any real number. Since signals are integers this implies that the manager should wait until the posterior probability, driven by (2) exceeds  $p^*$ . In other words, the disclosure threshold in terms of net signals is  $s^* = \lceil s(p^*) \rceil$ .<sup>2</sup>

### 3 Properties of the Optimal Disclosure Policy

In this section we provide an insight into the main features that emerge from the manager's disclosure policy. The following proposition shows that the policy under the current (real options) approach gives a more stringent criterion

<sup>2</sup>For  $s \in \mathbb{R}$ ,  $\lceil s(p^*) \rceil := \min\{k \in \mathbb{Z}_+ | k \geq s\}$ .

on disclosure than the classical NPV approach demands; that is, the manager will wait longer before disclosing than under the NPV approach. The more stringent criteria is supported by Dixit and Pindyck [5], and the reasoning is that the classical NPV approach does not incorporate the opportunity cost of waiting for more informative signals to arrive through exercising the option immediately.

The manager's belief threshold under the classical NPV rule (disclose at the first time when the PV exceeds the cost of disclosure) is given by

$$p_{NPV} = \frac{I - V^N}{V^P - V^N}. \quad (9)$$

The proof that  $p^* > p_{NPV}$  proof can be found in Appendix B.

**Proposition 1.** *The real options approach leads to a well-defined threshold probability,  $p^*$ , and requires a more stringent criteria on the timing of disclosure than the classical NPV approach would demand.*

The result of a comparative static analysis of the threshold,  $p^*$ , with respect to the model's key economic variables is given in the following proposition, the proof of which is obtained through simple calculus and is, therefore, omitted.

**Proposition 2.** *The threshold belief in a positive market response,  $p^*$ , decreases with  $V^P$  and  $V^N$ , and increases with  $I$ .*

These results are intuitive and are driven by the manager's policy to maximize firm value.

Similar to our paper, Suijs [21] shows that the unraveling argument leading to full disclosure need not apply when the firm is uncertain about investor response. In that paper, the firm's objective is to acquire as much of the investor's capital as possible. He assumes that the investor can invest in the firm, a risk-free asset, or some alternative risky investment project. While we do not make this assumption directly, implicit in our set-up is that any profit from disclosure is obtained through acquiring capital investment when the response to the disclosed information is positive. A lack of information may induce investors to opt for alternative investment opportunities. Therefore, the greater the impact an announcement will have on the positive value of the firm, that is, the higher  $V^P$ , the less time the manager will wait before he exercises his option to disclose the information. This is consistent with Suijs [21] who finds that a stronger positive response makes disclosure a more attractive option (relative to non-disclosure) and, therefore, the firm can be less certain about the market reaction being positive for disclosure to be optimal.

Conversely, the greater the impact an announcement will have on a negative trading response, that is, the lower  $V^N$ , the longer the firm will wait

before making a disclosure. Unlike Thijssen et al. [23],  $V^N \neq 0$ , and therefore, if the investors learn of bad news about the firm's stock, they will sell off some of their shareholdings and this lowers the valuation of the firm. Thus, it is straightforward to see why the firm conceals information that is likely to have a strong negative impact on the stock price. This is again corroborated by Suijs [21] who points out that a stronger negative response makes disclosure less attractive compared with nondisclosure, and thus, the firm must be more certain about being a good firm for disclosure to be optimal. However, if the news is unlikely to have a very strong negative impact, the manager will be more likely to disclose the information to prevent the market from inferring that the firm is in a worse state than it actually is. Graham et al. [10] conduct a comprehensive survey that asks CFOs to describe their choices related to reporting accounting numbers and voluntary disclosure and find that one advantage for releasing bad news is that it can help a firm to develop a reputation for providing timely and accurate information. CFOs place a great deal of importance on acquiring such a reputation: 92% of their survey respondents believe that developing a reputation for transparent reporting is a key factor motivating voluntary disclosures.

The greater the cost of making an announcement, the longer the manager chooses to wait before making an announcement. This is owing to the fact that if the (direct) costs of, say, preparing or disseminating information are high, the manager requires more time to confirm the accuracy of the information signals. By so doing, he obtains a stronger conviction about how the market will react to the news and the likelihood of making a wrong disclosure decision is reduced.

The comparative static result with respect to signal quality suggests that the more informative are the signals, the longer the manager will wait before disclosing his information; that is, the higher  $\theta$ , the higher  $p^*$ . The proof of the following proposition can be found in Appendix C.

**Proposition 3.** *The threshold belief in a positive market response,  $p^*$ , increases with  $\theta$ .*

In the context of our model, the more informative are the signals, the less uncertainty the manager has regarding the impact from his disclosure choice. However, the comparative static result with respect to  $\theta$  appears to be at odds with the intuitive and, indeed, widely accepted result in real options literature, that an increase in uncertainty should have an inhibiting effect on disclosure. In other words, standard results (Dixit and Pindyck [5], McDonald and Siegel [16], etc.) imply that we would expect that the more informative are the signals, the earlier disclosure will occur. However, in the standard framework, all of the uncertainty inherent in the model is captured by one parameter, namely the variance, whereas in our model, the uncertainty arises not only through the quality of the information signal,  $\theta$ , but also through the (random) arrival times

of the information signals,  $\mu$ , and more specifically, the manager's uncertainty regarding how the market will interpret the information if disclosed. This latter effect is a latent variable, and thus cannot be measured. While a higher quality signal will reduce the manager's uncertainty as to the likely market response, for reasons discussed in Section 2.1, it will never be eliminated entirely. It is the combination of these effects that drive the uncertainty in this model and thus, obtaining an unambiguous conclusion on what the uncertainty effect should be is not trivial.

In order to understand why a negative relationship exists between the information quality of signals and the optimal disclosure threshold,  $p^*$ , we examine the probability that disclosure will take place (i) when the true state of the world is a negative market response and (ii) when the true state of the world is a positive market response. An increase in this probability corresponds with a lower disclosure threshold. Our reasoning for examining the probability of disclosure is motivated by Sarkar [18] who suggests that in order to gauge the overall effect of uncertainty on the level of investment, one can look at the probability that investment will take place within a specified time period. We note, however, that Sarkar [18] examines the investment-uncertainty relationship for the standard real options model where uncertainty is constant over time.

Firstly, we assume that the true state of the world is a negative market response ( $\gamma = 0$ ). The probability that the threshold,  $s^*$ , is reached, and thus that the manager will disclose is given by

$$P^{(s^*)}(s_t) := \left( \frac{\theta}{1-\theta} \right)^{s_t - s^*}, \quad (10)$$

where  $s_t < s^*$ . The derivation of this result is outlined in Thijssen et al. [23]. Since  $s_t < s^*$ , the probability of disclosure decreases when the quality of the signals increases. Intuitively this is sensible: if the true state of the world is a negative market response and if the signal quality is high, implying the manager is obtaining accurate, but negative signals, the likelihood he will disclose quickly decreases and the disclosure threshold,  $p^*$ , will be higher.

On the other hand, if the true state of the world is a positive market response ( $\gamma = 1$ ), the probability that the manager will disclose before a finite time  $T$  is given by

$$\tilde{P}^{(s^*)}(s_t) := \int_0^T f_{s^*}(t) dt, \quad (11)$$

where  $f_s(t)$  is the *unconditional* density of first passage times and is given by

$$f_s(t) = \left( \frac{\theta}{1-\theta} \right)^{-\frac{s_t}{2}} \frac{s_t}{t} I_{s_t} \left( 2\mu\sqrt{\theta(1-\theta)t} \right) e^{-\mu t}. \quad (12)$$

$I_{s_t}(\cdot)$  denotes the modified Bessel function with parameter  $s_t$  (see Thijssen et al. [23] and Feller [9]).

We demonstrate in Figure 1 that this probability is an increasing function of  $\theta$ .<sup>3</sup> Hence, when the true state of the world is a positive market response, and the information quality is high, the probability that disclosure will occur increases, or equivalently, fewer positive (over negative) signals are required to make disclosure an attractive option. This is intuitive.

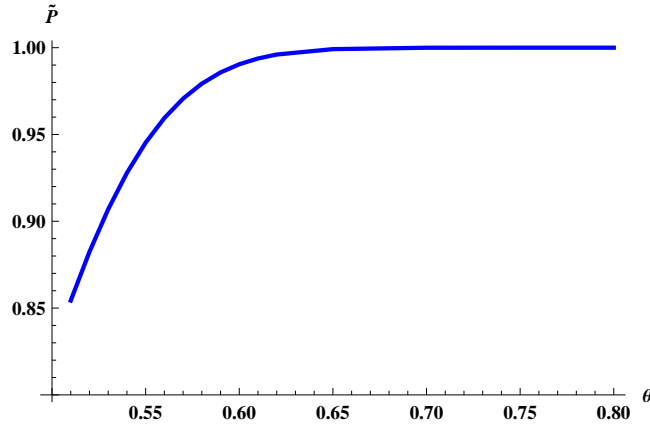


Figure 1: Probability of disclosure for  $\gamma = 1$ .

However, concerning the comparative statics with respect to  $\theta$ , Proposition 3 asserts that  $p^*$  increases in  $\theta$ . This arises from the fact that for certain combinations of  $\theta$  with the other parameter values, a low value of  $s^*$  can be associated with a high value of  $p^*$  (see Figures 2 and 3), that is, the threshold belief is reached after fewer positive signals have been obtained. This can occur if, for example, one highly accurate and very positive signal is obtained.  $s_t$  will only change by +1, but the likely impact of such information on the firm's value through a positive market response may be so strong that the manager's belief variable  $p_t$  will “jump” upwards by an amount such that  $p^*$  is reached.

Finally, it is possible to say something about the frequency of information arrival on the optimal disclosure policy. Unfortunately, it is not possible to determine the direction of the relationship between  $p^*$  and the arrival rate,  $\mu$ , unambiguously. Therefore, we use numerical simulation results to ascertain the direction of the relationship. From Figure 4, we see that  $p^*$  is increasing in the arrival rate (for one set of parameter values). We repeatedly carried out these computations for a wide range of parameter values, and can confirm that this result is robust to a wide choice of values. Hence, for reasonable parameter values, we can conclude that  $p^*$  increases in  $\mu$  which, in turn, implies that it decreases in the expected time between signal arrivals.<sup>4</sup>

<sup>3</sup>The parameterisation is as follows;  $V^P = 15$ ,  $V^N = -10$ ,  $I = 5$ ,  $r = 0.04$  and  $\mu = 4$ . This parameterisation is used for all figures in this chapter, unless otherwise stated.

<sup>4</sup>Dixit and Pindyck [5] show that  $\mathbb{E}[T] = \int_0^\infty \mu T e^{-\mu T} dT = \frac{1}{\mu}$ .

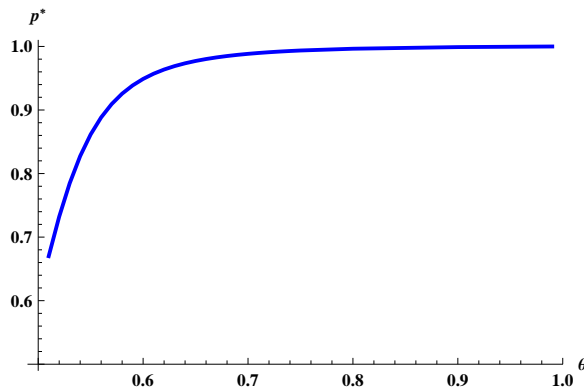


Figure 2: Comparative statics for  $p^*$  with respect to  $\theta$

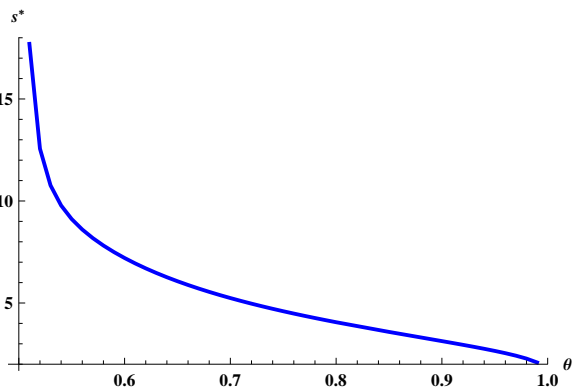


Figure 3: Comparative statics for  $s^*$  with respect to  $\theta$

Einhorn and Ziv [8] examine corporate voluntary disclosures in a multi-period setting. They conclude that inter-temporal dynamics occur because a firm's use of their private information is assumed to be history dependent.

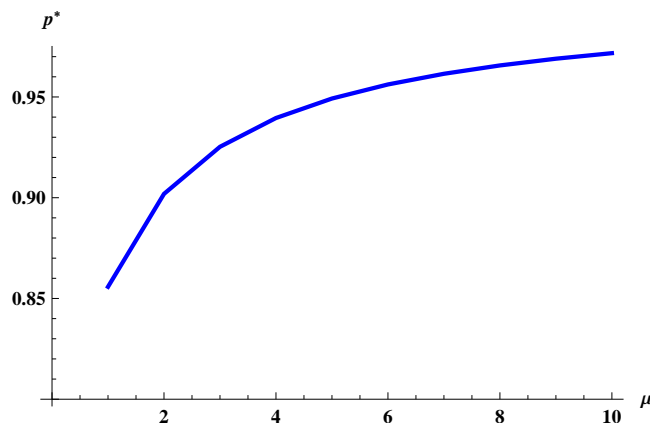


Figure 4: Comparative statics of  $\mu$  with  $\theta = 0.6$  and  $r = 0.04$ , fixed.

Their multi-period model demonstrates that by providing current disclosure, the manager increases the firm's implicit commitment to provide similar disclosures in the future. Thus, in the absence of disclosure, the market is likely to infer that the manager possesses negative information which they consider too unfavourable to disclose and will consequently revise down their expected valuation of the firm accordingly. Our model supports their result, and goes a step further by showing that in the absence of disclosure as a direct consequence of receiving few, or no, signals (as opposed to receiving bad information), the manager will opt to disclose at a lower threshold in an attempt to temper the market's uncertainty and to prevent it from inferring that the firm must be withholding some negative information. The result is further supported by anecdotal evidence. Graham et al. [10] find that executives believe

that lack of clarity, or a reputation for not consistently providing timely and accurate information, can lead to under-pricing of a firm's stock. Their survey evidence suggests that 48.8% of CFOs use voluntary disclosures to correct an undervalued price. Moreover, Healy and Palepu [13] observe that managers use corporate disclosures to reduce the likelihood of undervaluation.

## 4 Debt and Voluntary Disclosure

In this section we present an example of a situation where the Modigliani–Miller theorem on the irrelevance of capital structure is violated. In practice, exceptions to the theorem are widely observed in many areas of finance. With respect to the current setting of disclosure theory, Ahmed and Courtis [1] conduct an empirical study on factors affecting the level of voluntary disclosure using leverage as an explanatory variable. In particular, they show that companies with capitalisation structures showing higher proportions of fixed interest securities relative to equity are significantly associated with the release of higher quantities of informational disclosure.

Similar to Sabarwal [17], we adopt ideas regarding value of waiting from real options theory, and show that if a disclosure option resides with the manager, his optimal disclosure strategy is affected by the form of capital structure. In particular, our main finding corroborates with Ahmed and Courtis [1] in that the amount of information disclosed is positively related to leverage. While they assert that this leverage-based association may be related to corporate size, in that larger companies tend to use proportionally higher amounts of fixed interest securities as a financing technique because of the tax advantages, our reasoning is owing to the lower downside risk afforded by limited liability. However, their assertion of corporate size is only one explanation for the observed link, and this is owing to their choice of explanatory variables. However, adopting a broader interpretation of their result, there is nothing to suggest that limited liability does not play a part in establishing this causation in their setting, and thus, my model may provide an adequate theoretical explanation for their empirical result.

In terms of adapting the benchmark model, we consider how the manager's disclosure policy is affected when some of the sunk costs associated with disclosure are financed with debt. As in Section 2, we assume that the manager is compensated via stock options and that he is given complete discretion about the disclosure policy he adopts.

Typically, it may not be realistic to assume that some of the disclosure costs are financed with debt since the relative magnitude of such costs are too small to warrant such an assumption as being plausible. However, the disclosure aspect of whether to reveal the news about the project is still a contributor to the overall project's profitability through  $V^P$ ,  $V^N$ , and  $I$ , and thus disclosure is

simply a compound option (an option problem within a bigger option problem) which is beyond the scope of this section. If there is an injection of debt which generally funds the project's sunk costs, some of which are obviously made up of the sunk disclosure costs, then in that sense, debt is, at least partially, funding the disclosure. The idea for the coupon rate is similar; that is, part of the payoff from disclosure, if impact is  $V^P$ , is used to repay the debt via the coupon payment and if impact is  $V^N$  then none of the disclosure payoff goes toward the debt obligation.

The main reason for the association between debt and disclosure relates to the manager-lender conflict. Without debt, in an all equity firm, the manager incurs both upside and downside risk from making a disclosure. However, with debt, the manager's downside risk is limited, in that some of the sunk costs of disclosure are covered, and the lender is assumed to have first claim on revenues obtained from disclosure up to a fixed coupon,  $C$ , which the lender determines (this assumption is consistent with Sabarwal [17]). This may provide the manager with *ex post* incentives to make decisions that are not in the lender's best interest. For example, the manager may opt to disclose some negative information about the prospects of the firm if he wishes to discourage other players from entering the market. Recognising this possibility, the lender demands a higher interest rate on the loan which implies a higher coupon payment.

The manager's objective is still to maximise his (discounted) expected utility from wealth, which is equivalent to maximising firm value owing to the compensation assumption, and the lender adopts a zero profit condition. The disclosure problem is then to determine an equilibrium belief level,  $p_d^{**}$ , such that, simultaneously, the manager's and the lender's objectives are satisfied, for a given level of debt. We further show how the disclosure policy changes as the debt level changes.

## 4.1 Manager's Problem

The manager gains when the firm's payoff from disclosure exceeds its debt obligation, and suffers a loss otherwise. However, he suffers far less damage if his payoff falls just short, or way short, of the debt obligation, than if all of the disclosure cost was financed with equity.

Denote by  $0 < D \leq I$  the firm's only debt payment. We assume that if the response to disclosure is negative, and consequently, the impact on firm value is  $V^N$ , the firm defaults on its loan; that is,  $V^N - (I - D) < 0$ . In the event of a default, the payment to the lender is 0 and the manager suffers by the amount  $V^N - (I - D) > V^N - I$ . On the other hand, if the response is positive, the impact on firm value is  $V^P$  and we assume that the manager can meet his debt obligation. This implies that  $V^P - (I - D) - C \geq 0$  and the lender is paid  $C$  while the manager retains the residual  $V^P - I + D - C \geq 0$ .



The expected change in value from disclosure at time  $t$ , conditional on  $p_0$ , is given by

$$\begin{aligned} U^D(s_t) &= p(s_t)(V^P - I + D - C) + (1 - p(s_t))(V^N - (I - D)) \\ &= p(s_t)(V^P - C) + (1 - p(s_t))V^N - (I - D), \end{aligned} \quad (13)$$

where  $p(s_t)$  is given by equation (2).

Solving for the optimal threshold, via an optimal stopping approach (see Appendix A) yields the optimal threshold, when some of the cost is financed with debt, and this is given by

$$p_d^* = \left[ \frac{V^P - C - (I - D)}{(I - D) - V^N} \Pi + 1 \right]^{-1} \quad (14)$$

where  $\Pi$  is given by equation (7). Moreover, (14) is a well-defined probability, if, and only if,  $(I - D) \leq (V^P - C)$ , which is satisfied.

## 4.2 Lender's Problem

In this subsection, the problem is outlined from the lender's perspective. Similar to Sabarwal [17] (who adapts the more standard model of real options, (see Dixit and Pindyck [5]; McDonald and Siegel [16]) to include a competitive lending sector), we assume that the lender adopts a zero profit condition. Hence, he gets  $C$  with probability  $p_t$  and zero otherwise. Thus, his profit is given by

$$\pi_t^l = p_t C - D. \quad (15)$$

Note that  $p_t$  denotes the lender's belief, at time  $t$ , about the state of the firm. The lender acts to attain the zero profit condition implying

$$p_t^* = \frac{D}{C} \quad (16)$$

which is a well-defined probability if, and only if,  $D \leq C$ . This implies that he will only lend to the firm, to help them finance their disclosure policy, if he is sufficiently well compensated for the likelihood that the firm will default on its debt obligation if they adopt a very transparent disclosure policy; that is, he will only lend to the manager if he is prepared to pay a coupon which exceeds the amount of debt he is given.

## 4.3 Equilibrium

For a given level of debt, we want to find a coupon,  $C^*$ , such that  $p_d^* = p_l^*$ ; that is, the manager's belief threshold about when to disclose is equal to the

lender's belief threshold about when to lend. Equating equations (14) and (16) and solving for the coupon level  $C^*$  yields

$$C^* = \frac{D \left( (V^P - I + D)\Pi + (I - D - V^N) \right)}{D\Pi + (I - D - V^N)} \quad (17)$$

implying that the equilibrium belief threshold for the manager, and indeed the lender, is given by

$$p_d^{**} = \left[ \frac{V^P - C^* - (I - D)}{(I - D) - V^N} \Pi + 1 \right]^{-1}. \quad (18)$$

The main findings from an analysis of this equilibrium threshold are given in Proposition 4 and Proposition 5 below. The proofs are outlined in Appendices D and E, respectively.

**Proposition 4.** *The equilibrium belief,  $p_d^{**}$ , is a well-defined probability.*

**Proposition 5.** *In equilibrium, the manager will disclose earlier, when some of the financing comes from debt, than if all of the disclosure was financed with equity; that is  $p_d^{**} < p^*$ . Moreover, the greater the level of debt obtained, the lower the threshold above which the manager will disclose, in equilibrium.*

Firstly considering the manager's optimal disclosure policy, it has been shown in Proposition 5 that to the extent that debt reduces the manager's disclosure cost the disclosure threshold is lower. This is owing to the fact that, with debt, the manager is likely to prefer a riskier and more transparent disclosure policy because his downside risk is limited; that is, the loss he may incur from a negative response reduces with the level of debt he obtains.

However, on the other hand, to the extent that the lender anticipates the likelihood that the response to disclosure will be negative, and thus, the manager defaults on his debt, he demands a coupon that compensates him for this risk. Thus, the manager's payoff from disclosure decreases in the coupon payment demanded. Hence, the higher the coupon payment, *ceteris paribus*, the longer the manager waits before disclosing as he requires greater conviction that a positive response will ensue.

Overall, in equilibrium, one might intuitively expect that after compensating the lender for expected default losses, the net effect of such debt financing on the optimal disclosure threshold is zero. However, we find that, in fact, the net effect is negative; that is, the coupon payment demanded is not so high that the manager requires even greater conviction before disclosing that the response will be positive than if all his financing arose from equity.

Our reasoning for this result corresponds with that of Sabarwal [17] and is the following: The disclosure threshold is affected by three main components;

the manager's share of the disclosure cost,  $I - D$ , the value that goes to the lender that arises directly from disclosure, and an additional impact of limited liability. As in Sabarwal [17], "the lender's zero profit condition implies that the reduction in the manager's share of the cost is exactly offset by the value obtained by the lender". Therefore, the net effect on the disclosure policy is the impact of limited liability. With limited liability, some of the downside risk (that is, the risk of a loss in firm value owing to a negative response to disclosure) is transferred to the lender, and from the manager's perspective, his own lower tail of risk curtailed. According to Sabarwal [17], the non-neutrality of debt can be motivated in term of the "bad news principle" proposed by Bernanke [3]. With respect to the current setting, this implies that in the presence of limited liability debt financing, waiting for more favourable signals is valuable, but not as much as it is in the standard case, essentially because adverse realisations to firm value after disclosure (owing to a negative market response) are marginally less costly for the firm. Hence the manager adopts a more transparent disclosure policy; that is, the optimal threshold is lower.

## 5 Conclusion

This research shows how adopting a real options approach can aid our understanding of corporate voluntary disclosure. The concept of a disclosure option is discussed and in this way the corporate disclosure literature is linked together with the real options literature. The decision to disclose, or withhold, information is strategic on the part of the firm. This implies that the manager will only announce the information if he is sufficiently certain that the market response to the information will have a positive impact on the value of the firm, and thus, on his own utility from wealth. An analytical expression for the manager's threshold belief in a positive market response to the disclosed information is derived and analyzed using a real options framework. We show that the approach taken in this paper demands a higher threshold belief in a positive market response than under the classical NPV approach.

An extension to the model shows that the Modigliani-Miller theorem of investment financing is violated in the instance of corporate disclosure. When some of the disclosure cost is financed with debt, the manager adopts a lower disclosure threshold owing to the limited liability aspect of debt which dominates the loss incurred by the manager through compensating the lender for expected default losses.

To conclude, there are two points worth noting with regard to relevant issues which are absent in the analysis. The first is that the market for voluntary disclosure is assumed to be complete; that is, the payoff to the manager from making a disclosure voluntarily may be perfectly replicated through trading with existing marketed securities. However, this assumption is at odds with

reality, and therefore, an examination of the same problem, but under the assumption of incomplete markets, could have an interesting effect on the current results. The problem in incomplete markets is that there is no unique way to value the option. A possible way forward here would be to adopt the approach taken in Thijssen [22]. He views market incompleteness as a case of ambiguity over the correct way to discount future payoffs. A multiple prior model together with the assumption of ambiguity aversion leads to a well-defined option value. In a standard real options setting Thijssen [22] shows that the effect of market incompleteness is not trivial. It is to be expected that similar results hold in the case of voluntary disclosure. The second aspect worth noting is that the manager does not face any competitive pressure whilst deciding on an optimal disclosure policy. Once again, this assumption is an abstraction from reality, the examination of which ought to be conducted in further research.

## References

- [1] Ahmed, K. and J. Courtis (1999). Associations between corporate characteristic and disclosure levels in annual reports: A meta-analysis. *British Accounting Review* 31, 35–61.
- [2] Bayus, B., S. Jain, and A. Rao (2001). Truth or consequences: An analysis of Vaporware and new product announcements. *Journal of Marketing Research* 38(1), 3–14.
- [3] Bernanke, B. (1983). Irreversibility, uncertainty, and cyclical investment. *Quarterly Journal of Economics* 98, 85–106.
- [4] Dempster, A. (2006). Valuing announcement options. *European Management Journal* 24(2-3), 117–127.
- [5] Dixit, A. and R. Pindyck (1994). *Investment under uncertainty*. Princeton, NJ: Princeton University Press.
- [6] Dutta, S. and B. Trueman (2002). The interpretation of information and corporate disclosure strategies. *Review of Accounting Studies* 7, 75–96.
- [7] Dye, R. (1985). Disclosure of non-proprietary information. *Journal of Accounting Research* 23, 123–145.
- [8] Einhorn, E. and A. Ziv (2008). Inter-temporal dynamics of voluntary corporate disclosures. *Journal of Accounting Research* 46(3), 567–589.
- [9] Feller, W. (1971). *An introduction to probability theory and its applications*. New York: John Wiley and Sons.

- [10] Graham, J., C. Harvey, and S. Rajgopal (2005). The economic implications of corporate financial reporting. *Journal of Accounting and Economics* 40, 3–73.
- [11] Grossman, S. (1981). The informational law of warranties and private disclosure about product quality. *Journal of Law and Economics* 24, 461–483.
- [12] Grossman, S. and O. Hart (1980). Disclosure laws and take-over bids. *Journal of Accounting Research* 23, 123–145.
- [13] Healy, P. and K. Palepu (2001). Information asymmetry, corporate disclosure, and the capital markets: A review of the empirical disclosure literature. *Journal of Accounting and Economics* 31, 405–440.
- [14] Jensen, R. (1982). Adoption and innovation of uncertain probability. *Journal of Economic Theory* 27, 182–193.
- [15] Mas-Colell, A., M. Whinston, and J. Green (1995). *Microeconomic theory*. New York: Oxford University Press.
- [16] McDonald, R. and D. Siegel (1986). The value of waiting to invest. *Quarterly Journal of Economics* 101, 707–728.
- [17] Sabarwal, T. (2005). The non-neutrality of debt in investment timing: a new NPV rule. *Annals of Finance* 1, 433–445.
- [18] Sarkar, S. (2000). On the investment-uncertainty relationship in a real options model. *Journal of Economic Dynamics and Control* 24(2), 219–225.
- [19] Stocken, P. (2000). The credibility of voluntary disclosure. *RAND Journal of Economics* 31(2), 359–374.
- [20] Subramani, M. and E. Walden (2001). The impact of e-commerce announcements on the market value of firms. *Information Systems Research* 12(2), 135–154.
- [21] Suijs, J. (2007). Voluntary disclosure of information when firms are uncertain of investor response. *Journal of Accounting and Economics* 43, 391–410.
- [22] Thijssen, J. (2011). Incomplete markets, knightian uncertainty, and irreversible investment. *Journal of Economic Dynamics and Control* 35, 909–921.
- [23] Thijssen, J., K. Huisman, and P. Kort (2004). The effect of information streams on capital budgeting decisions. *European Journal of Operational Research* 157, 759–774.

- [24] Verrecchia, R. (2001). Essays on disclosure. *Journal of Accounting and Economics* 32, 97–180.
- [25] Wen, X. (2008). Voluntary disclosure and investment. preprint (2011), available at <http://ssrn.com/abstract=1262382>.

# Appendix

## A Derivation of the Optimal Disclosure Policy

The critical value of the conditional belief in a positive market response to an announcement, denoted  $p^* = p(s^*)$ , is the point such that the manager is indifferent between disclosing the information and withholding it. That is, if  $p_t > p^*$ , the manager is confident that there will be a positive trading response to the announcement if disclosed. On the other hand, if  $p_t < p^*$ , the manager is not confident enough in a positive response and waits for more information to arrive.

In order to solve for  $p^*$ , the approach taken is to solve the optimal stopping problem (5) by examining two scenarios. This solution approach is similar to the approach taken by Jensen [14] and Thijssen et al. [23]. The *stopping value*, denoted by  $U(s)$  and given by (4), is the expected return to the firm from disclosing the information to the market immediately. This is the first scenario examined. The alternative scenario is that it is optimal not to disclose immediately, but to wait for more signals to arrive. The value of the option, known as the *continuation value*, denoted by  $C(s)$ , represents the discounted expected value of the next piece of information.

Since there are no cash-flows accruing from the disclosure option,  $C(\cdot)$  should satisfy the Bellman equation over a small interval of time  $dt$ , i.e.

$$C(s_t) = e^{-rdt} E_t[C(s_{t+dt})]. \quad (\text{A.1})$$

This equation says that the value of the option at time  $t$  should equal its discounted expected value at time  $t + dt$ , where the time interval  $dt$  becomes infinitesimally small. In a small time interval  $dt$ , no information is received by the manager with probability,  $1 - \mu dt$ . On the other hand, information arrives with probability  $\mu dt$ . If information arrives, the value of the option jumps, either to  $C(s_t + 1)$  if the information is deemed to signal a positive market reaction, or  $C(s_t - 1)$  otherwise. Assuming that the current number of net signals is  $s_t$  (and, hence, that the current posterior belief in a positive market reaction is  $p(s_t)$ ), this implies that (A.1) becomes

$$C(s_t) = (1 - rdt) \left\{ (1 - \mu dt)C(s_t) + \mu dt \left[ p(s_t)[\theta C(s_t + 1) + (1 - \theta)C(s_t - 1)] + (1 - p(s_t))[\theta C(s_t - 1) + (1 - \theta)C(s_t + 1)] \right] \right\} + o(dt). \quad (\text{A.2})$$

Substituting for  $p(s_t)$  using (2), dividing by  $dt$  and taking the limit  $dt \downarrow 0$ , we obtain the following difference equation:

$$\hat{C}(s_t + 1) - \frac{r + \mu}{\mu} \hat{C}(s_t) + \theta(1 - \theta) \hat{C}(s_t - 1) = 0, \quad (\text{A.3})$$

where

$$\hat{C}(s_t) := (\theta^{s_t} + \zeta(1 - \theta)^{s_t})C(s_t),$$

Equation (A.3) has a general solution given by

$$\hat{C}(s_t) = A_1\beta_1^{s_t} + A_2\beta_2^{s_t},$$

where  $A_1$  and  $A_2$  are constants and  $0 < \beta_2 < 1 - \theta < \theta < \beta_1$  are the solutions to the fundamental quadratic

$$\Psi(\beta) \equiv \beta^2 - \left(\frac{r}{\mu} + 1\right)\beta + \theta(1 - \theta) = 0. \quad (\text{A.4})$$

So, the value of the disclosure option equals

$$C(s_t) = \frac{A_1\beta_1^{s_t} + A_2\beta_2^{s_t}}{\theta^{s_t} + \zeta(1 - \theta)^{s_t}},$$

Imposing several boundary conditions then leads to a solution for the unknowns  $s^*$ ,  $A_1$ , and  $A_2$ . First of all, if  $s_t \rightarrow -\infty$ , the probability of the posterior belief ever reaching  $p^*$  goes to zero and, hence, the option becomes worthless. So, it should hold that  $\lim_{s_t \rightarrow -\infty} C(s_t) = 0$ . Since  $0 < \beta_2 < 1 - \theta$  this implies that  $A_2 = 0$ . A second condition is that the value of the option should be continuous at  $s^*$ . The third boundary condition is another continuity condition that stems from the realization that the point  $s^* - 1$  is special. In deriving  $C(\cdot)$  it was (implicitly) assumed that after receiving the next signal disclosure still does not take place. But, for  $s_t \in [s^* - 1, s^*)$ , the manager knows that if the next signal indicates a positive market reaction, then disclosure should take place. Denoting the option value in the range  $[s^* - 1, s^*)$  by  $CU$ , it can be shown to be given by

$$\begin{aligned} CU(s_t) = \frac{\mu}{r + \mu} \Big\{ & [p(s_t)\theta + (1 - p(s_t))(1 - \theta)]U(s_t + 1) \\ & + [(1 - p(s_t))\theta + (1 - \theta)p(s_t)]C(s_t - 1) \Big\}. \end{aligned} \quad (\text{A.5})$$

So, the value of the disclosure option is

$$U^*(s_t) = 1_{(s_t < s^* - 1)}C(s_t) + 1_{(s^* - 1 \leq s_t < s^*)}CU(s_t) + 1_{(s_t \geq s^*)}U(s_t).$$

This is a *free-boundary problem*, for which the constant  $A_1$  and threshold  $s^*$  can be found by the continuity conditions  $C(s^* - 1) = CU(s^* - 1)$  and  $CU(s^*) = U(s^*)$ . Solving in terms of  $p^* := p(s^*)$  gives

$$p^* = \left[ \frac{V^P - I}{I - V^N} \Pi + 1 \right]^{-1}, \quad (\text{A.6})$$

where

$$\Pi = \frac{\left(\beta_1(r + \mu) - \mu\theta(1 - \theta)\right) \left(\frac{r}{\mu} + 1 - \theta\right) - \mu\theta(1 - \theta)\beta_1}{\left(\beta_1(r + \mu) - \mu\theta(1 - \theta)\right) \left(\frac{r}{\mu} + \theta\right) - \mu\theta(1 - \theta)\beta_1} \quad (\text{A.7})$$

and  $\beta_1 > \theta$  is the larger (real) root of the quadratic equation (A.4).



## B Proof of Proposition 1

First, we show that  $p^*$ , given by (6), is a well-defined probability.

$p^* > 0$  if, and only if,  $\Pi > 0$ , where  $\Pi$  is given by equation (7). If  $r = 0$ , from equation (8),  $\beta_1 = \theta$ , and  $\Pi = 0$ ; i.e. the numerator of (7) is zero. Hence  $p^* = 1 > 0$ .

Finding the total derivative of the numerator of  $\Pi$ , denoted  $n(\Pi)$ , with respect to  $r$  yields:

$$\begin{aligned} \frac{\partial n(\Pi)}{\partial r} &= \frac{\partial n(\Pi)}{\partial r} + \frac{\partial n(\Pi)}{\partial \beta_1} \frac{\partial \beta_1}{\partial r} \\ &= \frac{1}{\mu} \left( \beta_1(r + \mu) - \mu\theta(1 - \theta) \right) + \beta_1 \left( \frac{r}{\mu} + 1 - \theta \right) \\ &\quad + \frac{\partial \beta_1}{\partial r} \left( (r + \mu) \left( \frac{r}{\mu} + 1 - \theta \right) - \mu\theta(1 - \theta) \right) \end{aligned}$$

This expression is positive since  $r > 0$ ,  $\beta_1 > \theta$  and, trivially,  $\frac{\partial \beta_1}{\partial r} > 0$ .

Therefore  $n(\Pi) > 0$ .

On the other hand, when  $r = 0$ , the denominator of  $\Pi$ , denoted  $d(\Pi)$ , is  $\theta\mu^2(2\theta - 1) > 0$ , since  $\theta > \frac{1}{2}$  by assumption. Furthermore

$$\begin{aligned} \frac{\partial d(\Pi)}{\partial r} &= \frac{\partial d(\Pi)}{\partial r} + \frac{\partial d(\Pi)}{\partial \beta_1} \frac{\partial \beta_1}{\partial r} \\ &= \frac{1}{\mu} \left( \beta_1(r + \mu) - \mu\theta(1 - \theta) \right) + \beta_1 \left( \frac{r}{\mu} + \theta \right) \\ &\quad + \frac{\partial \beta_1}{\partial r} \left( (r + \mu) \left( \frac{r}{\mu} + \theta \right) - \mu\theta(1 - \theta) \right) > 0. \end{aligned}$$

Therefore  $d(\Pi) > 0$ .

This proves that  $\Pi > 0$  and  $p^* > 0$ .

$p^* \leq 1$  if, and only if,  $\Pi \geq 0$ . Indeed,  $\Pi \geq 0$ , since  $r \geq 0$ , and thus  $p^* \leq 1$ .

Hence,  $p^*$ , given by equation (6), is well-defined.

Moreover,  $p^* > p_{NPV}$ , where  $p_{NPV}$  denotes the belief threshold when the benefits from disclosure are exactly equal to the (direct) costs incurred. Thus it is obtained by solving for  $p_t$  when  $U(s_t) = 0$ , such that  $U(s_t)$  is given by (4). Hence

$$p_{NPV} = \frac{I - V^N}{V^P - V^N}. \quad (\text{B.1})$$

An algebraic manipulation shows that  $p^* > p_{NPV}$  if, and only if,

$$\Pi < 1.$$

Again, an algebraic manipulation shows that

$$\begin{aligned} \Pi &< 1 \\ \iff 1 - \theta &< \theta, \end{aligned}$$

which is satisfied, since  $\theta > \frac{1}{2}$ . ■

## C Proof of Proposition 3

From equation (6), it is easily obtained that  $\frac{\partial p^*}{\partial \theta} > 0$  if, and only if,  $\frac{\partial \Pi}{\partial \theta} < 0$ , where  $\Pi$  is given by (7).

To determine the sign of  $\frac{\partial \Pi}{\partial \theta}$ , one only needs to compare  $\frac{\partial}{\partial \theta}(\frac{r}{\mu} + 1 - \theta)$  with  $\frac{\partial}{\partial \theta}(\frac{r}{\mu} + \theta)$ . Since these derivatives have opposite signs, and  $\beta_1(r + \mu) - \mu\theta(1 - \theta) > 0$ , it is indeed the case that  $\frac{\partial \Pi}{\partial \theta} < 0$ .

Therefore,  $\frac{\partial p^*}{\partial \theta} > 0$ . ■

## D Proof of Proposition 4

It is easily established that  $p_d^{**}$ , given by equation (18), is well-defined if, and only if,  $C^* \leq V^P - I + D$ , where  $C^*$  is given by equation (17).

This condition is adhered to when

$$\frac{D\left((V^P - I + D)\Pi + (I - D - V^N)\right)}{D\Pi + (I - D - V^N)} \leq V^P - I + D. \quad (\text{D.1})$$

Algebraic manipulation reduces the expression (D.1) and  $C^* \leq V^P - I + D$  holds once

$$-(I - D - V^N)(V^P - I) \leq 0.$$

This is satisfied since  $V^P \geq I$  and  $I - D - V^N > 0$ , by assumption. ■

## E Proof of Proposition 5

$p_d^{**} < p^*$  if, and only if,

$$\frac{I - V^N}{V^P - V^N} < \frac{D}{C^*}, \quad (\text{E.1})$$

where  $C^*$  is given by equation (17).

After substituting for  $C^*$ , an algebraic manipulation reduces this expression to the condition that (E.1) holds if

$$\Pi > \frac{V^N - (I - D)}{V^P - V^N + D},$$

where  $\Pi$  is given by equation (7).

As shown in Proposition 1,  $\Pi > 0$ . Additionally,  $V^N - (I - D) < 0$  and  $V^P - V^N + D > 0$ , by assumption. Hence, the condition is satisfied. Thus,  $p_d^{**} < p^*$ .

It is, therefore, trivially satisfied that  $\frac{\partial p_d^{**}}{\partial D} < 0$ . If  $D = 0$ ,  $p_d^{**} = p^*$ . For  $D > 0$ , it has just been shown that  $p_d^{**} < p^*$ . Hence,  $p^{**}$  decreases in  $D$ . ■