

Eötvös Loránd University Faculty of Education and Psychology
Doctoral School of Education

Árpád Bánhalmi

Application of the knowledge space theory extended via the
conjunctive Bayesian networks in addressing specific problems
in the teaching of mathematics for high-level courses

Theses

2015

ADATLAP

a doktori értekezés nyilvánosságra hozatalához

I. A doktori értekezés adatai

A szerző neve: **Bánhalmi Árpád**

A doktori értekezés címe és alcíme: **Konjunktív Bayes-hálókkal kiterjesztett tudástérelmélet és alkalmazása a felsőfokú matematikaoktatás néhány problémájára**

A doktori iskola neve: **ELTE PPK Neveléstudományi Doktori Iskola**

A doktori iskolán belüli doktori program neve: **Neveléstudományi Kutatások**

A témavezető neve és tudományos fokozata: **dr. Nahalka István CSc, PhD**

A témavezető munkahelye: **nyugdíjas, ELTE PPK óraadó**

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II. Nyilatkozatok

1. A doktori értekezés szerzőjeként hozzájárulok, hogy a doktori fokozat megszerzését követően a doktori értekezésem és a tézisek nyilvánosságra kerüljenek az ELTE Digitális Intézményi Tudástárban. Felhatalmazom a Neveléstudományi Doktori Iskola hivatalának ügyintézőjét, Madar Veronikát, hogy az értekezést és a téziseket feltöltse az ELTE Digitális Intézményi Tudástárba, és ennek során kitöltse a feltöltéshez szükséges nyilatkozatokat.

2. A doktori értekezés szerzőjeként kijelentem, hogy

a) az ELTE Digitális Intézményi Tudástárba feltöltendő doktori értekezés és a tézisek saját eredeti, önálló szellemi munkám és legjobb tudomásom szerint nem sértem vele senki szerzői jogait;

b) a doktori értekezés és a tézisek nyomtatott változatai és az elektronikus adathordozón benyújtott tartalmak (szöveg és ábrák) mindenben megegyeznek.

3. A doktori értekezés szerzőjeként hozzájárulok a doktori értekezés és a tézisek szövegének plágiumkereső adatbázisba helyezéséhez és plágiumellenőrző vizsgálatok lefuttatásához.

Budapest, 2015. április 19.

Bánhalmi Árpád

The genesis of the so-called knowledge space theory can be traced back to the 1980s, its foundations having been created by *Jean-Paul Falmagne* and *Jean-Claude Doignon* (Doignon and Falmagne 1985). Similarly, the 1980s saw the emergence of the theory of Bayesian networks as a popular line of thinking – the origins of which go as far back as the 1920s (Wright 1921) – but it was *Judea Pearl* who transformed it into a distinctly individual theory (Pearl 1988). While the birth of what is known as the knowledge space theory was clearly motivated by pedagogical considerations (Falmagne and Doignon 2011), there is ample evidence in a variety of disciplines of the application of the Bayesian networks (Russell and Norvig 2005). The endeavour for a simultaneous use of these two theories for pedagogical purposes is manifested in various works of both *Desmarais and his colleagues* and those of *VanLehn and his colleagues*. Attempts by *Desmarais and his colleagues* to use the knowledge space theory and the Bayesian networks concurrently were based on erroneous interpretations and incorrect mathematical models. Their endeavour is, therefore, considered a failure. (Desmarais, Maluf and Liu 1996; Desmarais and Pu 2005; Desmarais, Pu and Blais 2007). As for *VahLehn* and his colleagues, the notional context of the knowledge space theory was, with the exception of the notion of knowledge state, disregarded in their Bayesian network models and their findings were not meant to be linked to other notions comprised in the knowledge space theory (VanLehn 1988; Martin and VanLehn 1995; Conati and VanLehn 1996). As it was subsequently referred to by Ünlü and his colleagues in a paper published on the issue of the correlation between the knowledge space theory and other theories, the merging of the knowledge space theory with the theory on Bayesian networks continues to be seen as an unresolved problem open to further research (Ünlü, Schrepp, Heller, Hockemeyer, Wesiak and Albert 2013).

The findings of our research paper have brought to a successful conclusion efforts to solve a hitherto unresolved problem with respect to special models related to the knowledge space theory and the Bayesian networks, the principal outcome of that research effort being a model on knowledge space theory the scope of which has been extended by conjunctive Bayesian networks, one that is capable of representing the *probability distribution of ordinal knowledge space*. In the paper, statistical procedures to be used in connection with the model in question, applicable in empirical research, have been provided. To achieve this aim, statistical tests suitable for the purpose of verifying both the validity and the reliability of the model itself have been selected.

The viability of the recommended model has been demonstrated by its use in a specific line of knowledge, whereas its actual application has been demonstrated through a set of descriptive analyses based on a cross-sectional examination. According to the findings of this research study, performance levels pertaining to the field of Budapest Business School-related knowledge under scrutiny were mostly affected by performance levels stemming from the study period during the primary school and secondary school years. An analysis regarding the reasons thereof has been performed, while we have also examined to what extent relevant knowledge space characteristics are determined by performance levels associated with the primary school and secondary school years.

With the help of the recommended model we have performed an analysis on the issue pertaining to the teaching pathway with the highest probability, labelled as a specific research area in the literature (Tóth 2012), and we have also examined various teaching strategies from the vantage point of both unilateral and complex development. By doing so, we have placed our model in the context of a broader system corresponding to definitions in the literature.

As a consequence of the model itself, special relationships take shape in the models built upon the knowledge space theory, the scope of which is by now extended by conjunctive Bayesian networks.

In the function of the parameters of the Bayesian networks these relationships pave the way for a higher performance level to be achieved at maximum speed. Parameter values assigned to a maximum speed performance level and the performance level together constitute an *ideal teaching pathway*. It is possible for the parameters of the Bayesian networks to be modified during the learning and teaching processes, therefore the specification and analysis of the ideal teaching pathway resulted in *new methods* during the planning of course units, tasks and methods designed with the aim of improving certain skills.

We have also provided details regarding a further widening of the scope of the conjunctive Bayesian networks and the knowledge space theory in cases when those theories are applied together, which makes it possible for the efficiency of diverse methods of development to be assessed in the context of an entire group of students by applying decision trees. Moreover, we have pointed out that the effectiveness of the methods of development under scrutiny can be increased via differentiation, while we have also identified a method based on a formal concept analysis capable of splitting an entire group of students into sub-groups by using an appropriate method with the aim of enhancing efficiency.

The findings of our research are structured in such a manner that they constitute individual theses.

Thesis No. 1

The conjunctive models stemming from the knowledge space theory and the theory of Bayesian networks are compatible if applied in the context of ordinal knowledge spaces.

When talking about the knowledge space theory, a distinction needs to be made between conjunctive and disjunctive models. The conjunctive model is comprised of the ordinal and quasi ordinal knowledge space; in our research the focus is on the ordinal knowledge space as part of the conjunctive model of the knowledge space theory. One characteristic feature of the knowledge spaces that we exploited here stems from the fact that ordinal knowledge spaces can, uniquely, be traced back to their corresponding surmise relations (Falmagne and Doignon 2011). Surmise relations associated with ordinal knowledge spaces are weak partial orders (Falmagne and Doignon 2011) which, in the context of domain, include both the direct and indirect surmise connections. As the structure of conjunctive Bayesian networks that describe the probability distribution of ordinal knowledge spaces provides information on direct prevention alone, surmise relations associated with ordinal knowledge spaces can simultaneously be assigned to more than one direct surmise relation. One vital realization in our research was the fact that the probability description of knowledge spaces included in a specific empirical examination cannot be traced back to the surmise relations of the knowledge space theory without some degree of ambiguity. The notion of direct surmise relation will, instead, need to be introduced, upon which both the construction of knowledge spaces and the structure of Bayesian networks can be based.

As during the empirical research the elements of domain were identified with correct items of knowledge provided in response to test questions, rather than with test items or test questions, the domain of knowledge space theory has been redefined as the domain for correct answers to test questions; consequently, direct surmise relations have been renamed as a direct preliminary knowledge relation.

The local conditional probability models associated with the conjunctive Bayesian networks have been defined along with the conjunctive local probability models, thereby providing probability-based ordinal knowledge spaces.

Based on the numerical semantics of Bayesian networks, fundamental correlations thought to be vital for descriptive pedagogical analyses have been provided; the method used in connection with the calculation of optional knowledge performance level has been described, a formula has been set up for the probability distribution of the knowledge state of knowledge space used as a model; we have determined the calculation methods for conditional and unconditional probabilities interpretable for variables in the model and have provided a special formula for linear probability models interpretable in the context of the conjunctive Bayesian network. For all indicators derived from the model a general form of their interpretation has been provided.

Correlations exploiting numerical semantics may be considered valid in the course of an empirical examination if this has been corroborated statistically. To do so, the equivalence between numerical and topological semantics has been exploited. Based on the equivalence between these two types of semantics we have devised statistical instruments testing the validity of the model. For examinations based on a small number of samples taken, we have recommended the two-tailed Fisher's exact test.

Thesis No. 2

In some fields of knowledge the knowledge space theory – with its scope now extended by the Bayesian networks – can, with a considerable degree of reliability and validity, describe the knowledge space characteristic of a specific field.

The conjunctive Bayesian network describing the knowledge space in the field of “*Solution to tasks of derivation on the basis of rules of derivation*” – a topic comprised in the course unit entitled *Mathematics of Economics No. 1* – is valid and reliable.

By linking the knowledge space theory to the theory of the Bayesian networks we have developed a model. By justifying the validity of this thesis we acknowledge the applicability of the recommended model in at least one specific area; in fact, there exists a knowledge space with its corresponding fields of knowledge, whose linkage is, on the basis of probability, described by a conjunctive Bayesian network.

The population under scrutiny for examination during the sampling process comprised a total of 386 students enrolled during the 2013/2014 academic year in *International Business and Economics* and *Commerce and Marketing* courses offered by the College of International Management and Business of the Budapest Business School. 63 students were selected from that population by using the method of simple random sampling. We have examined the performance parameters related to a standard task entitled “*Solution to tasks of derivation on the basis of rules of derivation*”, the task itself having been examined from the point of view of five categories of preliminary knowledge. While direct connections arising from these five categories of preliminary knowledge were classified on the basis of the role they played in solving the problem posed by the task in question (surmise relations based on

total association), we also discovered a stochastic surmise relation between two categories of preliminary knowledge, this relation being unrelated to the process during which the task in question was to be solved. This stochastic surmise relation was exposed to a one-tailed Fisher's exact test and was found acceptable for the $\alpha > 0.15\%$ significance level. The structure of the conjunctive Bayesian network applied in this case was defined by the combination of the surmise relations based on total association along with the stochastic surmise relations.

Sub-thesis No. 2.1

The recommended model is valid if applied in connection with the task type examined.

The validity of the recommended model was examined via a test performed on the Markov property. The conditions of independence pertaining to the Markov property were established via seventeen independence-based assertions. These assertions were tested individually by performing a two-tailed Fisher's exact test and each assertion was found acceptable for the $\alpha < 4.34\%$ significance level. The validity of the model has thus been verified for the $\alpha < 4.34\%$ significance level and it is most probable that the recommended conjunctive Bayesian network describes a knowledge space characteristic of the population under scrutiny.

Sub-thesis No. 2.2

The recommended model is considered reliable if applied in connection with the task type examined.

The validity of the model was examined by using the two-tailed binominal test. We investigated the viability of a value – assigned to the estimated parameters of the model – being capable, on the basis of the sample, of characterizing the population in the context of total knowledge state. We found that the probability under scrutiny is a characteristic feature of the population for the $\alpha < 34.84\%$ significance level, i.e. the model is to be considered reliable.

Thesis No. 3

In the context of the parameters of the conjunctive Bayesian network the maximum growth level for probability pertaining to the total knowledge state will be achieved through the ideal teaching pathway.

Modelled with the help of the conjunctive Bayesian network, the probability of the total knowledge state will be a product of the parameters of the network. Based on this multiplication-based correlation, we have determined, by means of differential geometry, the corresponding values of the network that lead to a maximum level for an increased probability in relation to an achievable total knowledge state. The ideal teaching pathway will be defined by a change in the value of these corresponding parameters. We have come to the conclusion that optional initial parameter values will be assigned to exactly one ideal teaching pathway. Furthermore, we have also specified the geometrical

features, considered significant from the point of view of subsequent interpretations, which an ideal teaching pathway should have. One of the most significant outcomes of the examination performed during the overall scrutiny of the ideal teaching pathway is that – with the exception of a few special cases – the maximum growth rate for an increased probability regarding an achievable total knowledge state can be achieved if a complex development procedure is applied. In connection with the ideal teaching pathway we noted that it is a tool applicable even for practical pedagogy due to its approximation based on reference points.

To illustrate the approach based on reference points, we have used the sample and developed a set of thirty-four specific, practical test items designed in accordance with the ideal teaching pathway. In designing the construction of the test items, specific parameter values for the conjunctive Bayesian network of the task type “*Derivation on the basis of rules of derivation*” were taken into account along with the objective of aiming for the highest achievable growth level for performance in completing the task type in question. These test items, based on the principle of an ideal teaching pathway, are recommended as a supplementary tool in the teaching process applied for the task type in question.

Thesis No. 4

As for certain fields of knowledge, conclusions become available via the theory of knowledge space, with its scope now extended by the Bayesian networks, and can be both interpreted and applied for everyday pedagogical considerations.

The recommended model is a suitable method for useful conclusions to be drawn in the subject of “*Solution to tasks of derivation on the basis of rules of derivation*”, a topic comprised in the course unit called *Mathematics of Economics No. 1*.

The following conclusions, pertaining to students in the year referred to earlier on and based on our findings in respect of the recommended model and sample, demonstrate descriptive conclusions that can be provided by cross-section examinations:

A 24.5 % performance level for standard test items is affected mostly by the testee’s lowest performance level, i.e. a preliminary knowledge acquired during his or her primary and secondary school years. In resolving tasks in standard test items which pertain to a preliminary knowledge acquired during one’s primary and secondary education, a medium association is revealed ($\varphi = 0.5803$), which can be put down to two factors. First, one such factor, responsible for the correct solution of a task in a test item, has a positive effect on the preliminary knowledge acquired during one’s primary and secondary education, given the fact that among those given the task of doing a particular test item, the number of those who could cope successfully with a task based on preliminary knowledge stemming from primary and secondary education was 67.93 percentage points higher than in the case of those who failed to cope successfully with the test item in question. Second, completing a task pertaining to a preliminary knowledge stemming from primary and secondary education will have a positive effect on the probability of a test item being dealt with successfully, given the fact that percentage points in the case of those giving a correct answer in knowledge areas pertaining to a preliminary knowledge acquired during primary and secondary education is 49.57 higher than among those who fail to give a correct answer to a question in a test item. The resultant, as it were, of these

two factors, is the association, neither too strong, nor too weak, between the completion of a test item and that of a testee's knowledge acquired during primary and secondary education. Closely connected with this is the fact that the percentage figure for uncertainty in completing a test item is a figure 33.67% lower for those who can cope successfully with tasks requiring a preliminary knowledge derived from primary and secondary education than for all students in the same year – a number which is significant, albeit not too high. This is an indication of the fact that the successful completion of a test item is the result of additional factors, i.e. the knowledge acquired during one's studies at the Budapest Business School, which contribute significantly to this outcome. Our analysis reveals that the measure to which tasks in test items are resolved successfully is determined almost to the same degree by (all) knowledge acquired during a testee's years in higher education as by one's knowledge acquired prior to that. This has also been confirmed by a partial sensitivity test. If approximately all students in the same year were to give correct answers based on knowledge acquired in higher education, the performance level for test items would be almost twice as high. The same is true of the students' preliminary knowledge acquired in primary and secondary education: if approximately all students in the same year were to use that knowledge competently, the performance level for test items would, by and large, be twice as high.

Looking beyond the scope of individual types of preliminary knowledge, we have explored the characteristics pertaining to the probability distribution of knowledge states. We have found that the knowledge space linked to a particular test item splits into two separate teaching pathways. One is characteristic of those who became part of the teaching process without being able to competently use whatever knowledge had been acquired during primary and secondary education in coping with test items. The other describes those with an adequate preliminary knowledge acquired prior to their studies in higher education. The probabilities of knowledge states associated with the two teaching pathways reveal a symmetry, which means that those with an identical BBS-based preliminary knowledge represent an identical proportion of students in the same year. In the context of the test items examined, the preliminary knowledge of students acquired during their primary and secondary education is, basically, indifferent from the point of view of the learning/teaching process they became part of, only the probability distribution of knowledge states associated with BBS-based preliminary knowledge is identical in the two categories. The explanation behind this phenomenon is the effort made by students, irrespective of their preliminary knowledge, to acquire the material taught at the BBS. A comparison between the probability of latest knowledge state of the two teaching pathways we have found that 24.15% among those with a good preliminary knowledge from primary and secondary education have managed to acquire the material taught at the BBS, while among those with an unsatisfactory preliminary knowledge from primary and secondary education, the percentage figure for those who have acquired the material taught at the BBS is 25.42%. This means that 25.42% of students in the same year failed to cope with a test item *because* they lacked the preliminary knowledge from primary and secondary education required for the test item to be successfully tackled.

As an addition to the learning/teaching process that has been carried through, we have recommended the ideal teaching pathway for the test items in question.

Thesis No. 5

Some theoretical problems referred to in the literature specialized in the knowledge space theory can be tackled successfully by means of the knowledge space theory extended by the Bayesian networks.

The method according to which the modal teaching pathway is to be carried through has led to contradictions in the knowledge space theory extended by the Bayesian networks.

One viable application of the knowledge space theory extended by the Bayesian networks is to examine whether assertions in the literature are correct (Tóth 2012). Within the model itself we have defined the modal teaching pathway and, based on this, we have scrutinized changes in the probability changes of knowledge states belonging to a specific knowledge space by using one example. Changes in probability changes have resulted in conclusions pertaining to probability transitions and we have found that students may experience a knowledge state without having previously acquired the knowledge resulting from the teaching pathway and essential for that state to be achieved. Thus we have acknowledged that in this model the hypothesis based on the modal teaching pathway being carried through is no longer feasible, i.e. broadly speaking, it cannot be implemented optionally for all kinds of knowledge space. Naturally, though, the same method will not necessarily pose a contradiction if examined in a different model.

Thesis No. 6

As for the model of the knowledge space theory extended by the conjunctive Bayesian networks, it is possible to draw conclusions from the efficiency of teaching strategies at various performance levels.

In the case of knowledge spaces describable via Bayesian networks, teaching strategies with a one-sided development capability put an upper limit on the probability of the total knowledge state and, in function of the network parameters, probability will not increase at a maximum pace.

Having defined the notion of teaching strategy (Falus 2003) as part of the knowledge space theory, an examination – aimed at a specific category of teaching strategies – of the consequences of the knowledge space theory extended by the conjunctive Bayesian networks, was performed. Our examination proved that teaching strategies with a one-sided development capability fall short of achieving an optimal performance level for groups of students under scrutiny in cases where the door is open for complex development.

Sub-thesis No. 6.1

One-sided development options put a clear upper limit on the achievable performance level of the total knowledge state, which will be higher or, at best, identical with the performance level.

According to the proposition formulated in the sub-thesis, the probability of the total knowledge state will not be higher than a clearly definable value if it is the result of one-sided development options. This has been verified by the findings of a partial sensitivity test performed in connection with specific performance levels. A detailed description of the sensitivity test has led to the realization that the probability of an optional knowledge state in knowledge spaces describable via conjunctive Bayesian networks will increase to a degree identical with the measure in which the probability of any of its partial states will increase. We have exploited the opportunity arising from the fact that the probability of partial states will never be lower than the knowledge state therein, and that the probability value of a partial state is not higher than 1, from which fact the proposition itself directly follows.

Sub-thesis No. 6.2

As for one-sided development options, a corresponding development method which, depending on the parameters of the network, will lead to an increased growth rate in performance can, in fact, always be provided.

The proposition formulated in the sub-thesis refers to the growth rate of the probability of the total knowledge state occurring in the context of one-sided development options. Among the features characterising the ideal teaching pathway was the fact that the method of ideal teaching pathway is always accompanied by the practice of complex development. The proposition formulated in the sub-thesis is a consequence thereof.

Thesis No. 7

A modified version of the model of the knowledge space theory extended by the conjunctive Bayesian networks is capable of testing the effectiveness in specific cases of decisions of a pedagogical nature.

The knowledge space theory extended by the conjunctive Bayesian networks, along with the implementation of the decision tree theory, are capable of modelling the effectiveness of the methods of development; moreover, a procedure – designed to lead to an optimal decision regarding the implementation of a specific method of development – can be provided. Decisions considered optimal from the point of view of an entire group of students will not necessarily be advantageous for specific sub-groups of students. These sub-groups can be separated by way of a formal concept analysis and, by implementing the methods of a formal concept analysis, a curriculum designed for the differentiated development of these sub-groups can be provided.

This thesis gives an account of the manner in which the knowledge space theory with its scope now extended by the conjunctive Bayesian networks can be further improved. A justification is provided for new options in widening the scope of methodological opportunities, which help implement the recommended model in designing test items for placement tests as well as courses corresponding to test results.

Sub-thesis No. 7.1

By implementing the recommended model and the decision trees together it becomes possible to identify the student to whom a specific development method is recommended in order to reach a maximum increase in performance for all students in the same year.

A model based on the knowledge space theory and on probability trees has been described to provide information on specific knowledge states and their corresponding probabilities, to be achieved by a specific development method for students in a particular year. The effectiveness of development methods has been linked with changes in knowledge state and, in accordance with the notional context of the knowledge space theory, the development of a particular student has been defined in terms of the widening of the scope of knowledge states. We have looked into the problem of how our decisions, based on pedagogical considerations, can be identified with the modification of the parameters of the probability tree. This modification of probabilities has led to an optimization issue which, when solved, resulted in the finding that without an alternative optimum the parameter values assigned to decisions will be either 0 or 1. Due to the fact that these parameters represent conditional probabilities, a probability tree will no longer be useful in describing the problem itself, therefore we used an appropriate decision tree during the process. The initial probabilities of the decision model intended for the problem at hand have been provided via the conjunctive Bayesian networks; also, we have elaborated a decision-making criterion capable of testing the efficiency of development methods.

Sub-thesis No. 7.2

A procedure based on decision trees and interpreted at the level of a particular year of students – the implementation of which can help decide which development method should be recommended to a particular student or students – can also be used in connection with specific groups of students in the same year. These student groups can be determined by implementing a formal concept analysis.

By embracing methods used in the formal concept analysis (Takács 2000; Fatalin 2008; Belohlavek 2013) the model has been fine-tuned. We have come to the conclusion that our domain-related decisions can be accompanied by our partial decisions pertaining to formal concepts which pave the way for a differentiated education. We have provided a model in which – by using formal concepts – we can separate sub-groups of students to whom, with a view to the decision-making criterion used for the domain, the development method which we have tested will not be recommended, although these sub-groups are in need of development in a certain area. Formal concepts are a direct source to provide information regarding the curricular aspects of that area.

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