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ITERATIVE ARRAYS FOR CODE CONVERSIONS

by

M. COMBET

1972

Joint Nuclear Research Centre
Ispra Establishment - Italy

Technology

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Application to the conversion of codes is made with examples for the linary to decimal conversion and vice-versa.

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ABSTRACT

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KEYWORDS

ITERATIVE METHODS
MATHEMATICS
MATRICES
NUMBER CODES
MATHEMATICAL OPERATORS
DIGITAL COMPUTERS
DIGITAL CIRCUITS

TABLE OF CONTENTS

I. INTRODUCTION *)

Iterative arrays offer lot of new possibilities for the logical design.
Such circuits, which will be realised in a next future are very usefull for parallel arithmetical operations in the computers.
This paper describes a division matrix which can be used for the conversion of numbers written in base a in their equivalent in base b.
The description of the elementary cell and the matrix is made with their mathematical expressions.
Application to the conversion of codes is made with examples for the binary to decimal conversion and vice-versa.

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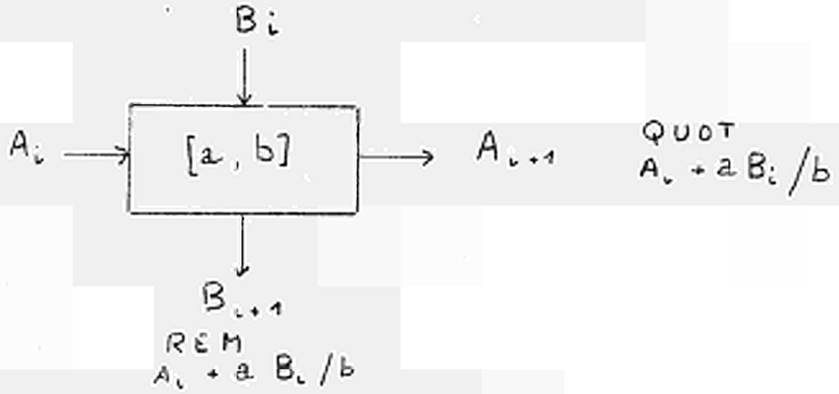
*) Manuscript received on September 28, 1972

2. DIVISION MATRIX

The division matrix and its elementary cell are now described.

2.I Elementary Cell

Elementary cell is a mathematical operator which computes quotient and remainder of a division .



Fig; I. Elementary Cell

This cell has 2 inputs: an horizontal one on which it receives digit A_i in base a and a vertical one B_i in base b.

Such a cell computes the division of $A_i + a.B_i$ by b and gives the quotient on A_{i+1} output (in base a) and the remainder on B_{i+1} output (in base b).

Mathematical expressions of such a division may be done by using modulo operations.

The remainder can be expressed in the following way

$$B_{i+1} = \left(A_i + a \cdot B_i\right) \bmod b$$

and the quotient

$$A_{i+1} = \frac{1}{b}\left[A_i + a \cdot B_i - \left(A_i + a \cdot B_i\right) \bmod b\right]$$

$$= \frac{1}{b}\left[A_i + a B_i - B_{i+1}\right]$$

An elementary cell may be completely defined by 2 parameters (a and b). Its internal configuration will be dependant of the code with which digit are written, as we can see in the following paragraphs.

Inputs and outputs are actually made of several wires when number is expressed with bits I and 0 (for instance 4 wires at least will be necessary for representing decimal digits). But relations between inputs A_i and B_i and the outputs A_{i+1} and B_{i+1} will be always the same.

2.2 Division Matrix

A matrix can be now buildt with m rows of n identical cells and connected as indicated in Fig. 2

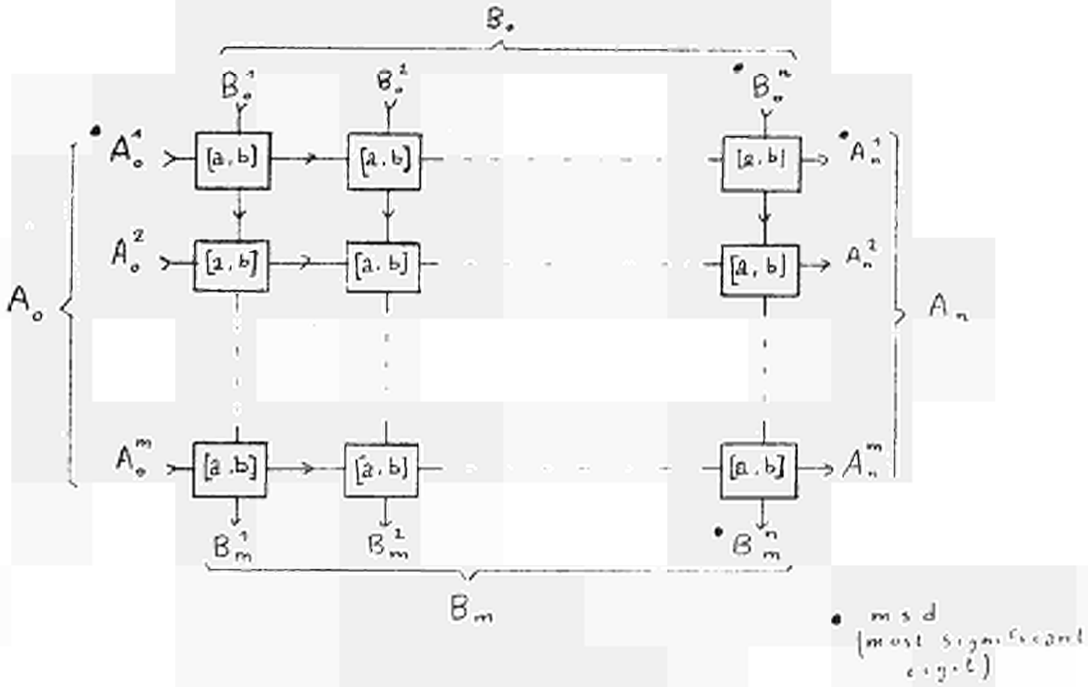


Fig. 2 - Division Matrix

Now, let us consider the first comumn of the matrix (Fig. 3) . It is obvious that this column will divide by b a number of which each digit is entered in the A_i inputs successively, most sigificant digit in the top cell.

If $B_0^1 = 0$, most significant digit is divided by b in the first cell where it sends the remainder to the second cell . The remainder is added to the second digit after multiplication by a and divided by b, etc..

This method is identical to that we use when we divide by pencil - and - paper method.

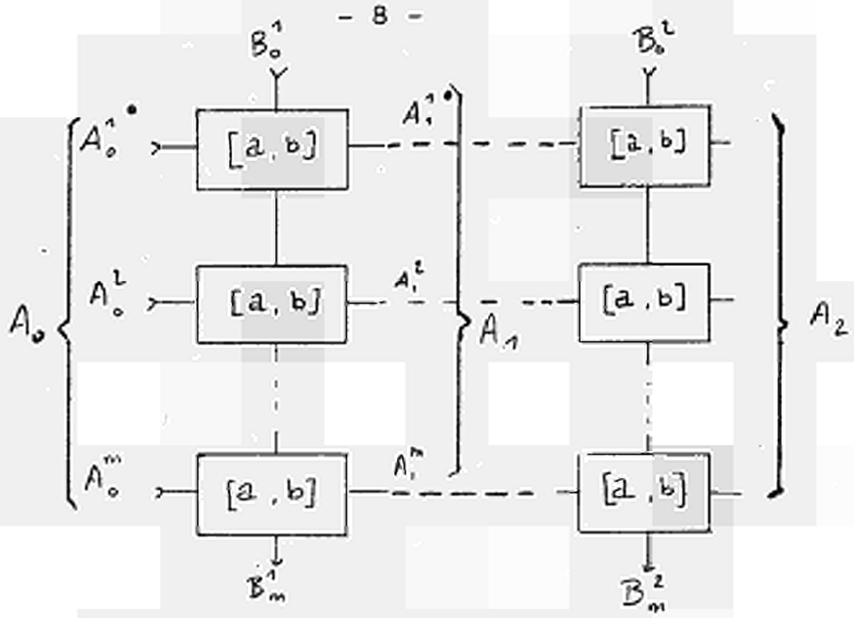


Fig. 3 - Column division.

When $B_0^1 \neq 0$, the top cell divides $A_0^1 + a.B_0^1$ and we must correct the preceding results by adding $a.B_0^1$ to A_0 .

But $A_0 = A_0^1.a^{m-1} + A_0^2.a^{m-2} + \ldots \; A_0^m.a^0$ where A_0^1 has a coefficient a^{m-1} ; then, we must add $a^{m-1}.a.B_0^1 = a^m.B_0^1$ to A_0 and say that the first column divides $A_0 + a^m.B_0^1$ by b, the quotient is A_1 and the remainder B_m^1 .

$$B_m^1 = (A_0 + a^m.B_0^1) \bmod b$$

$$A_1 = \frac{1}{b}(A_0 + a^m.B_0^1 - B_m^1)$$

When two columns of cells are involved, we have two successives division, but term B_0^2 enters only in the second division.

We can write for every division :

$$A_0 + a^m.B_0^1 = A_1.b + B_m^1$$
$$A_1 + a^m.B_0^2 = A_2.b + B_m^2$$

Eleminating A_1 , it yields :

$$A_0 + a^m.B_0^1 = (A_2.b + B_m^2 - a^m.B_0^2).b + B_m^1$$
$$A_0 + a^m.B_0^1 + a^m.B_0^2.b = A_2.b^2 + B_m^1 + B_m^2.b$$

A_2 is the quotient and $B_m^1 + B_m^2.b$ the remainder of the division of $A_0 + a^m.B_0^1 + a^m.B_0^2.b$ by b

When the n columns of the complete matrix are involved, we can write the expressions for every division and replace succesively every quotient in the following expression; finally, we get :

$$A_o + a^m.B_o^1 + a^m.B_o^2 b + \ \dots + a^m B_o^n b^{n-1} =$$
$$= A_n.b^n + B_m^1 + B_m^2 b + \dots + B_m^n b^{n-1}$$

$$A_o + a^m(B_o^1 b^0 + B_o^2 b^1 + \quad + B_o^n b^{n-1}) = A_n . b^n + (B_m^1 b^0 + B_m^2 b^1 + \quad + B_m^n b^{n-1})$$

Now, the first parenthesis is B_o, the second B_m

$$A_o + a^m . B_o = A_n . b^n + B_m$$

A_n is the quotient and B_m the remainder of the division of $A_o + a^m.B_o$ by b^n

$$B_m = (A_o + a^m.B_o) \bmod b^n$$
$$A_n = \frac{1}{b^n}(A_o + a^m.B_o - B_m)$$

2.3 Example

An example is now given when a = IO and b = 2; A_i are decimal digits 0,I,2,..,9 and B_i is 0 or I.

Twenty cases are possible of which we give the list with the corresponding outputs for the cell.

A_i	B_i	A_{i+1}	B_{i+1}
0	0	0	0
I	0	0	I
2	0	I	0
3	0	I	I
4	0	2	0
5	0	2	I
6	0	3	0
7	0	3	I
8	0	4	0
9	0	4	I
0	I	5	0
I	I	5	I
2	I	6	0
3	I	6	I
4	I	7	0

A_i	B_i	A_{i+1}	B_{i+1}
5	I	7	I
6	I	8	O
7	I	8	I
8	I	9	O
9	I	9	I

Indeed, the cell [IO,2] divides A_i + IO. B_i by 2
A_{i+1} is the quotient and B_{i+1} the remainder.

Using tle matrix Fig. 4, on which we put

A_0 = 237 base IO B_0= IIO base 2 (= 6_{IO})

we can write the various values on the different points of the matrix, using the preceding table, and get the quotient 779 and the remainder IOI (= 5_{IO}).

This matrix has divided 237 + IO^3.6 = 6237 by 2^3 = 8 the quotient is 779, and the remainder 5.

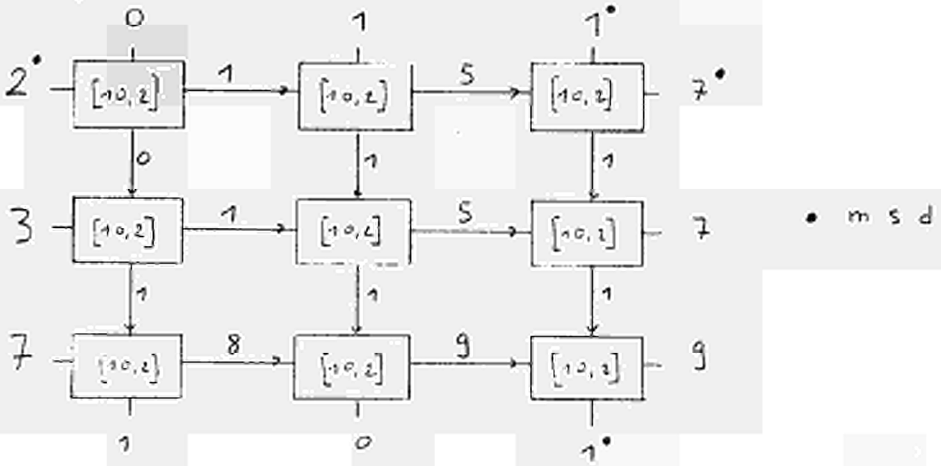


Fig. 4 - Matrix [IO,2]

3. CODE CONVERSION

A division matrix may be very usefull for the conversion of a number written in code a into its equivalent in code b. Integers and number less than I have to be considered separately.

3.I Integers

For this purpose, we put the number to be converted at input A_o and we take $B_o = 0$.

We can notice that the successive quotients A_1, A_2, .. will be more and more small, and if the number of columns is large enough, the last quotients will be 0.

Making $B_o = 0$ and $A_n = 0$, in the expression giving A_n, it yields :

$$0 = \frac{1}{b^n}(A_o + a^n.0 - B_m)$$

$$A_o = B_m$$

B_m has the same value as A_o, but written in code b

The number n of columns necessary wil be given by noticing in your preceding example, that the largest decimal number with 3 digits may be 999; this number divided by $2^{IO} = I024$ will give 0, as a quotient, and IO columns are necessary.

In the general case, we must have

$$a^m - I \geqslant b^n$$

$$n \geqslant \frac{Log(a^m - 1)}{Log\, b}$$

3.2 Fractionnal numbers

The same matrix may be used for converting numbers less than I, written in base b in their equivalent in base a

If the matrix has an adequate number of rows, we feed the number B to be converted in the inputs B_o, and taking $A_o = 0$, the expression of A_n gives :

$$A_n = \frac{1}{b^n} (0 + a^m . B_o - 0)$$

$$\frac{1}{a^m} A_n = \frac{1}{b^n} B_o$$

$\frac{1}{a^m} A_n$ is a number of which the point is just before the most significant digit.

In the case of fractionnal numbers, it is not always possible to have an adequate number of rows, because the numbers of digit may be infinite, if m rows are involved, the preceding formula becomes :

$$\frac{1}{a^m} A_n = \frac{1}{b^n} (B_o - \frac{B_m}{a^m})$$

m must be large enough to give the error $\frac{B_m}{a^m}$ less than the required precision.

Therefore, an [a,b] matrix converts integers from base a into base b and fractionnal numbers from base b into base a.

If, now, we need convert integers from base b into base a and fractionnal numbers from base a into base b,we have to design a new matrix of which th elementary cell will have parameters permutated.

3.3 Examples

We will designed the two matrices for the conversion of numbers, integers and fractionnals, from binary into decimal and vice-versa.

Fig. 5 shows the cell [IO,2] , its table (Fig. 6) and the matrix [IO,2] converting I3 (base IO) and 0,OII (base 2) (Fig. 7)

Fig. 8 shows the cell [2,IO] , its table (Fig. 9) and the matrix [2,IO] converting IIOI (base 2) and 0,375 (base IO).

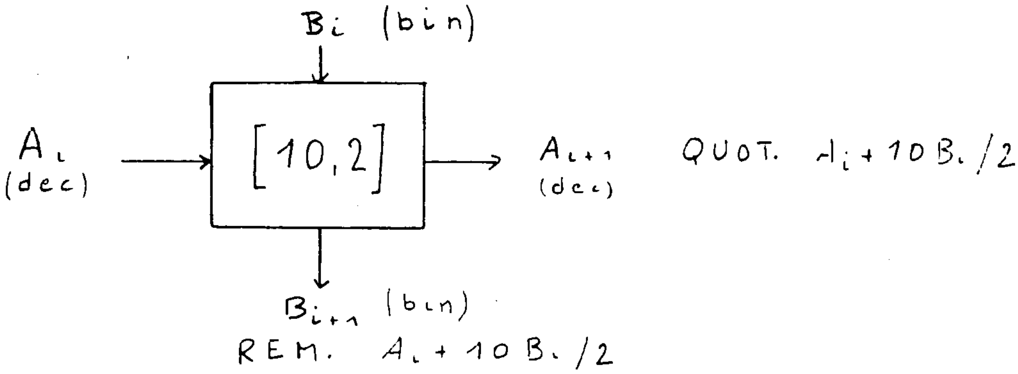
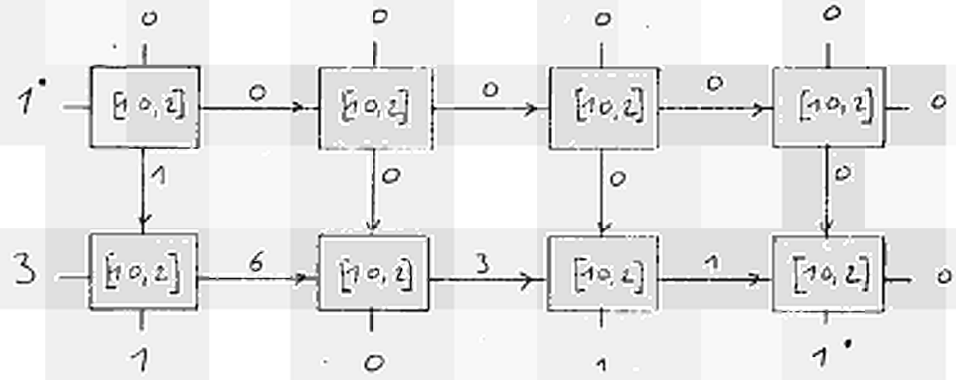


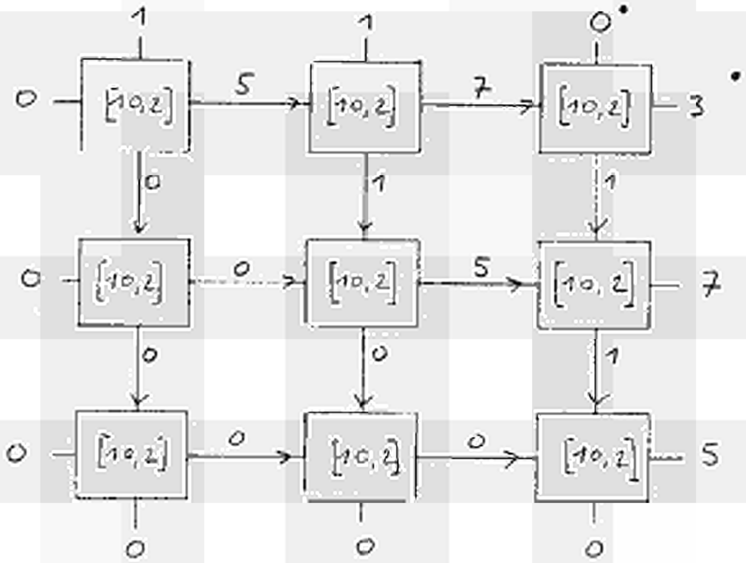
Fig. 5 Cell [IO,2]

A_i	B_i	A_{i+1}	B_{i+1}
0	0	0	0
I	0	0	I
2	0	I	0
3	0	I	I
4	0	2	0
5	0	2	I
6	0	3	0
7	0	3	I
8	0	4	0
9	0	4	I
0	I	5	0
I	I	5	I
2	I	6	0
3	I	6	I
4	I	7	0
5	I	7	I
6	I	8	0
7	I	8	I
8	I	9	0
9	I	9	I

Fig. 6 - Table for cell [IO,2]



$13_{10} \longrightarrow 1101_2$



$0,011_2 \longrightarrow 0,375_{10}$

Fig. 7 - Conversions with [IO,2] matrix

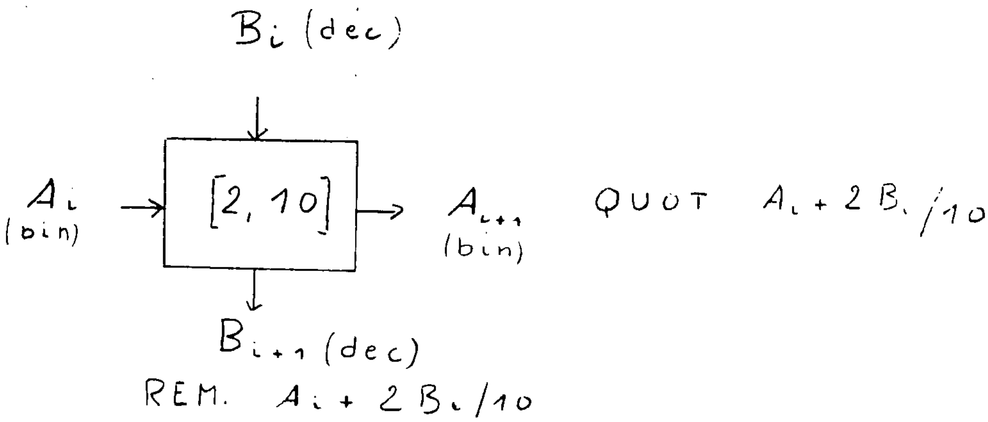
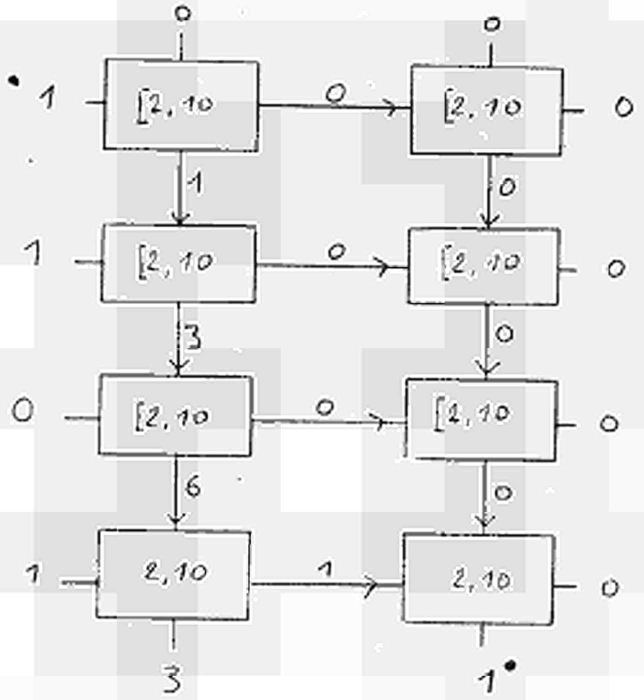


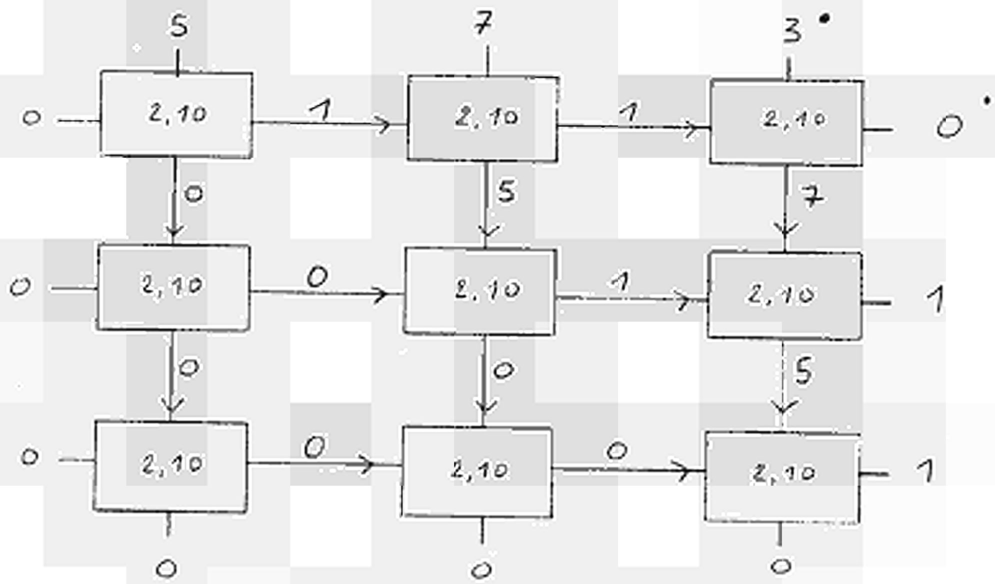
Fig. 8 - Cell [2,IO]

A_i	B_i	A_{i+1}	B_{i+1}
0	0	0	0
0	I	0	2
0	2	0	4
0	3	0	6
0	4	0	8
0	5	I	0
0	6	I	2
0	7	I	4
0	8	I	6
0	9	I	8
I	0	0	I
I	I	0	3
I	2	0	5
I	3	0	7
I	4	0	9
I	5	I	I
I	6	I	3
I	7	I	5
I	8	I	7
I	9	I	9

Fig. 9 - Table for [2,IO] cell



$1101_2 \longrightarrow 13_{10}$



$0,375_{10} \longrightarrow 0,011_2$

Fig.IO - Conversions with 2,IO matrix

4. EXAMPLE FOR BCD CODE

Now, we must show how to realise the elementary cell with electronic components.

As we must use a representation of numbers with bits, inputs and outputs of the cell will have more than I wire when numbers have a base more than 2.

As an example, we shal compute the circuitry of a [IO,2] cell when I,2,4,8 BCD code is used.

We shall write the table of this cell in this code :

A_i				B_i	A_{i+1}				B_{i+1}
t	z	y	x		T	Z	Y	X	
0	0	0	0	0	0	0	0	0	0
0	0	0	I	0	0	0	0	0	I
0	0	I	0	0	0	0	0	I	0
0	0	I	I	0	0	0	0	I	I
0	I	0	0	0	0	0	I	0	0
0	I	0	I	0	0	0	I	0	I
0	I	I	0	0	0	0	I	I	0
0	I	I	I	0	0	0	I	I	I
I	0	0	0	0	0	I	0	0	0
I	0	0	I	0	0	I	0	0	I
0	0	0	0	I	0	I	0	I	0
0	0	0	I	I	0	I	0	I	I
0	0	I	0	I	0	I	I	0	0
0	0	I	I	I	0	I	I	0	I
0	I	0	0	I	0	I	I	I	0
0	I	0	I	I	0	I	I	I	I
0	I	I	0	I	I	0	0	0	0
0	I	I	I	I	I	0	0	0	I
I	0	0	0	I	I	0	0	I	0
I	0	0	I	I	I	0	0	I	I

Fig. II - IO,2 cell table in I,2,4,8 code

We can consider X,Y,Z,T and B_{i+1} as booleean functions of the 5 variables x,y,z,t and B_i

4.I Minimisation of the functions

Before beginning the minimisation, we will notice that X,Y,Z and T don't depend of variable x because when considering the preceding table by group of 2 rows we see they are identical but x; thus, X,Y,Z and T are function of y,z,t and B_i only.

We see also immediately, that:

$$B_{i+1} = x$$

Now, X,Y,Z and T are minimised with Karnaugh's tables.

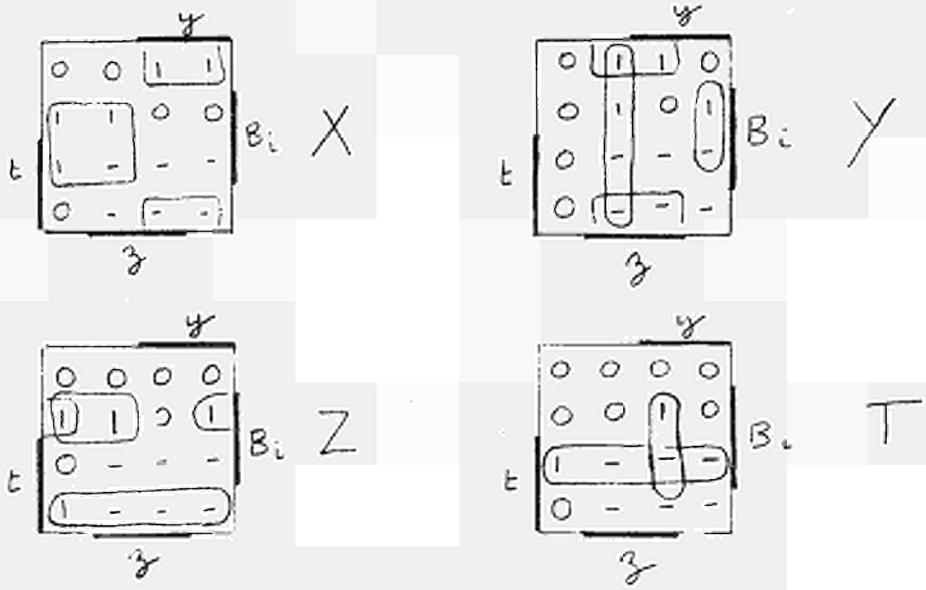


Fig. I2 - Minimisation of X,Y,Z and T.

The Boolean expressions of these functions are

$$X = B_i.\overline{y} + \overline{B_i}.y$$

$$Y = \overline{B_i}.z + B_i.y.\overline{z} + \overline{y}.z$$

$$Z = \overline{B_i}.t + B_i.\overline{z}.\overline{t} + B_i.\overline{y}.\overline{t}$$

$$T = B_i.t + B_i.y.z$$

$$B_{i+1} = x$$

4.2 - Cell and matrix in I,2,4,8 code

These functions can now be constructed with NAND gates as we can see the complete schema in Fig. I3

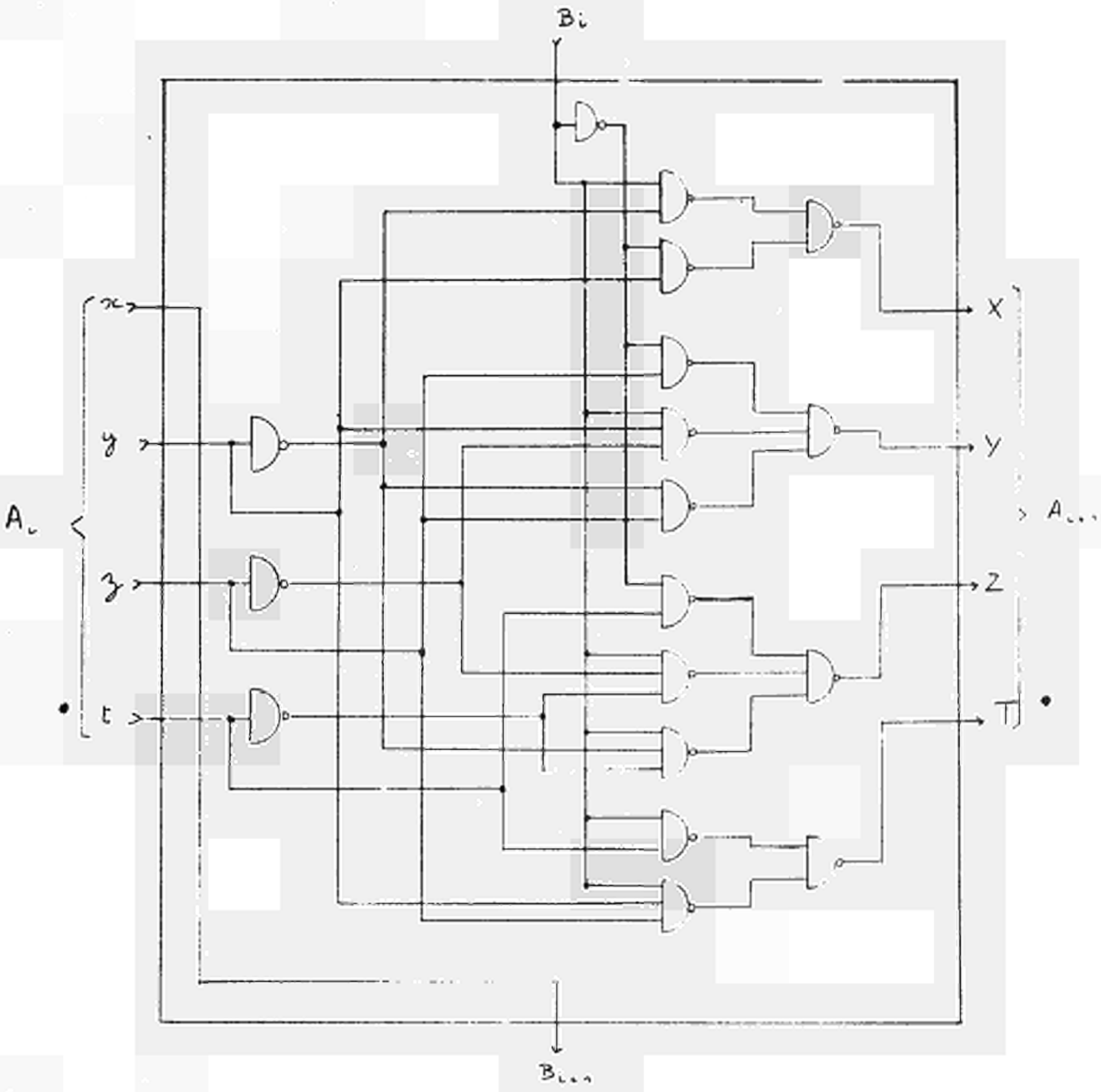


Fig. I3 - Cell [IO,2] in I,2,4,8 code

With 9 cells of this type we can now make a matrix on which the example seen on Fig. 4 is :

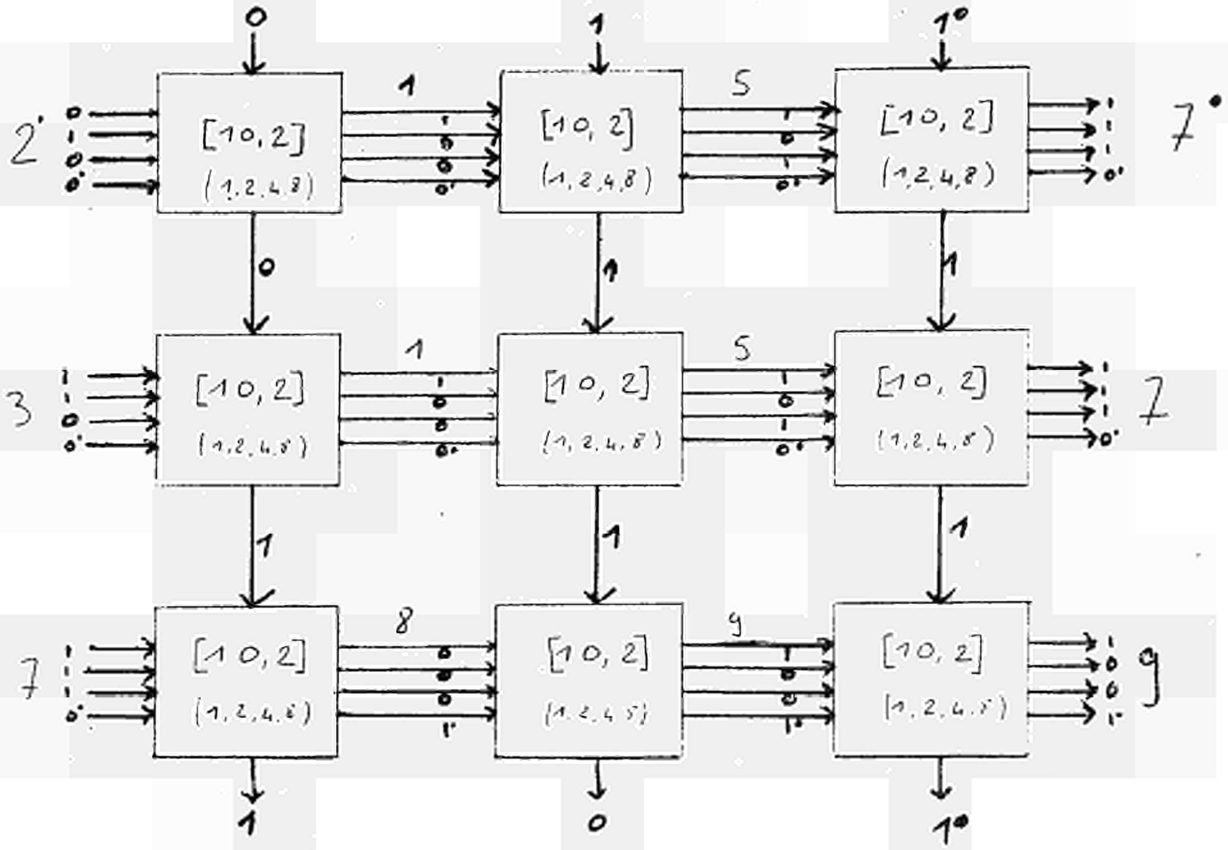


Fig. I4 - Example of Fig.4 in I,2,4,8 code

5. ADVANTAGES OF DIVISION MATRICES

The principal advantages of division matrices are now summarised

5.I Conversion of integers and fractionnal numbers

Generally,converters can only convert integer or fractionnals numbers, but non both.

5.2 Unique circuit for conversion and reverse conversion

Two matrices are needed for converting from base a in base b, and the same two matrices may be used for the reverse conversion

5.3 Various codes possible

Cells may be easily designed for various representation of digits A and B with bits (I,2,4,8 - I,2,4,2 - excess 3 2 out of 5 - etc..)

5.4 Expansibility

When numbers with more digits are to be converted, several matrices may be tied together for making a larger one. Fig. I5

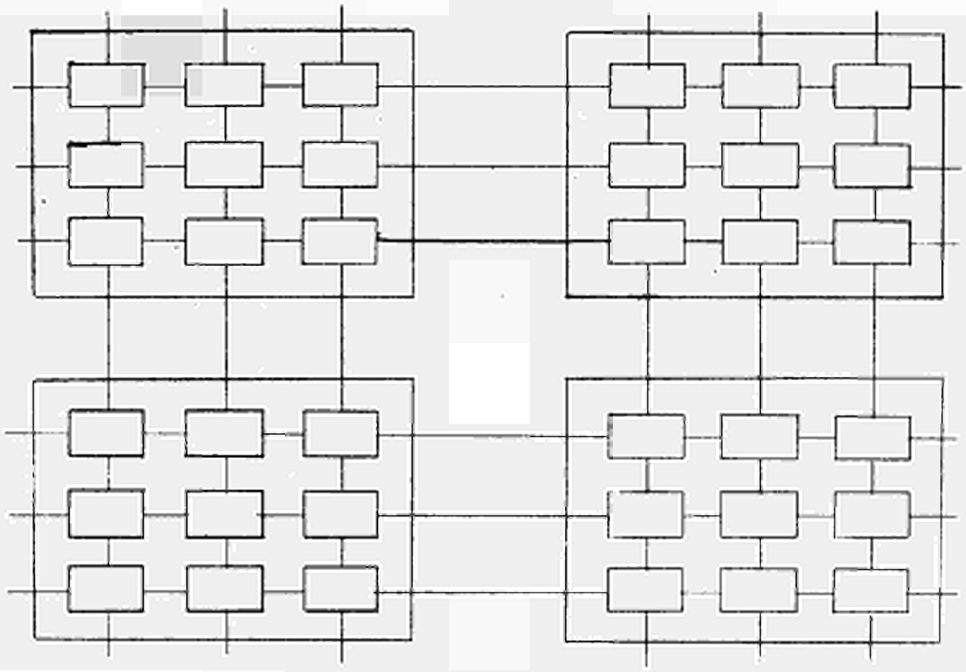


Fig. I5 - Expansibility of matrices

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JULY 1972

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Alfred Nobel

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