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# MEASUREMENT OF CORRELATION PROPERTIES OF THE BUBBLE FIELD IN TWO-PHASE FLOW

by

W. MATTHES

1972

Joint Nuclear Research Centre
Ispra Establishment - Italy

Nuclear Study

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## ABSTRACT

The bubble field in a two-phase flow shows inherent statistical properties which can be described by properly defined correlation functions. These correlation functions represent the whole physics of the two-phase flow in the sense that (a) the space-time flow pattern and (b) the physical interactions between the bubbles, which may be partly induced by the carrier fluid, show up in the detailed structure of this function. In this paper we develop general expressions relating experimental results (measured in an experiment suggested in [1]) to the correlation function and to the individual items (a), (b) mentioned above.

### KEYWORDS

TWO-PHASE FLOW
BUBBLES
HYDRODYNAMICS
CORRELATION FUNCTION
SPACE DEPENDENCE
TIME DEPENDENCE

## Introduction

In 1970 we suggested in a short note (see [1] ) an experimental method for the measurement of the velocity of gas bubbles in a two phase flow. A comparison of the experimental curves reported in [1] with the expected behaviour shows discrepancies which are also found in all measurements made in the meantime. The expected behaviour was based on the assumption that the bubbles in a two-phase flow are in no way correlated and perform their motion statistically independent from each other. The discrepancies mentioned above however show that this assumption cannot be true and asked for a detailed investigation of this point. This investigation led to the introduction of the statistical point of view in the description of the properties of the bubble field and to a general theory for the experiment in question.

## General Theory

To simplify the investigation we consider a stationary two-phase flow with an embedded bubble field of point-like bubbles. The motion of the two-phase mixture in space and time will be described relative to a space fixed coordinate system. Due to the assumption of the stationarity of the two-phase flow we may describe the statistical properties of the bubble field by the density $G(\mathfrak{r},\mathfrak{v})$ and the correlation function $G(\mathfrak{r}\,\mathfrak{v}\,\tau \;/\mathfrak{r}'\mathfrak{v}'\,0\,)$

where:

$G(\mathfrak{r},\mathfrak{v})d\mathfrak{r}\,d\mathfrak{v}$ is the mean number of bubbles in $d\mathfrak{r}$ at $\mathfrak{r}$ with velocities in the range $d\mathfrak{v}$ around $\mathfrak{v}$ , and

$G(\mathfrak{r}\,\mathfrak{v}\,\tau\,/\mathfrak{r}'\mathfrak{v}'\,0\,)d\mathfrak{r}\,d\mathfrak{v}$ is the mean number of bubbles in $d\mathfrak{r}\,d\mathfrak{v}$ at $\{\mathfrak{r},\mathfrak{v}\}$ at time $\tau$ if there was a bubble at $\{\mathfrak{r}',\mathfrak{v}'\}$ at time 0.

Derived quantities are for instance:

a) the density $G(\mathfrak{r})$, given by:

$$G(\mathbf{r}) = \int G(\mathbf{r}, \mathbf{v})\, d\mathbf{v} \tag{1}$$

($G(\mathbf{r})d\mathbf{r}$ is the mean number of bubbles in $d\mathbf{r}$ at $\mathbf{r}$ ).

b) the mean velocity $\langle \mathbf{v}(\mathbf{r}) \rangle$ at $\mathbf{r}$ defined by:

$$\langle \mathbf{v}(\mathbf{r}) \rangle = \frac{\int \mathbf{v}\, G(\mathbf{r}, \mathbf{v})\, d\mathbf{v}}{\int G(\mathbf{r}, \mathbf{v})\, d\mathbf{v}} \tag{2}$$

c) the correlation function $G(\mathbf{r}\tau / \mathbf{r}'0)$ defined by:

$$G(\mathbf{r}\tau / \mathbf{r}'0) = \frac{\int d\mathbf{v} \int d\mathbf{v}'\, G(\mathbf{r}\mathbf{v}\tau / \mathbf{r}'\mathbf{v}'0)\, G(\mathbf{r}'\mathbf{v}')}{\int d\mathbf{v}'\, G(\mathbf{r}'\mathbf{v}')} \tag{3}$$

$G(\mathbf{r}\tau / \mathbf{r}'0)d\mathbf{r}$ is the mean number of bubbles in $d\mathbf{r}$ at $\mathbf{r}$ and time $\tau$ if there was a bubble at $\mathbf{r}'$ at time 0.

d) the velocity correlation function $\langle v_i(\mathbf{r}\tau) v_k(\mathbf{r}'0) \rangle$ defined by:

$$\langle v_i(\mathbf{r}\tau) v_k(\mathbf{r}'0) \rangle = \frac{\int d\mathbf{v} \int d\mathbf{v}'\, v_i v_k'\, G(\mathbf{r}\mathbf{v}\tau / \mathbf{r}'\mathbf{v}'0)\, G(\mathbf{r}'\mathbf{v}')}{\int d\mathbf{v} \int d\mathbf{v}'\, G(\mathbf{r}\mathbf{v}\tau / \mathbf{r}'\mathbf{v}'0)\, G(\mathbf{r}'\mathbf{v}')} \tag{4}$$

where $v_i(\mathbf{r}\tau)$ ($v_k(\mathbf{r}'0)$) is the component of the velocity of a bubble at $\{\mathbf{r}, \tau\}$ ($\{\mathbf{r}', 0\}$) in an arbitrarily chosen direction i (k).

The correlation function $G(\mathbf{r}\mathbf{v}\tau / \mathbf{r}'\mathbf{v}'0)$ may be broken into two parts (see [2] ), a self part $G_s(\mathbf{r}\mathbf{v}\tau / \mathbf{r}'\mathbf{v}'0)$ which denotes the probability that the bubble to be found at time $\tau$ at $\mathbf{r}$ with velocity $\mathbf{v}$ is the same bubble that was at time 0 at $\mathbf{r}'$ with $\mathbf{v}'$, and a distinct part $G_d(\mathbf{r}\mathbf{v}\tau / \mathbf{r}'\mathbf{v}'0)$ which specifies that the bubble at $\{\mathbf{r}\mathbf{v}\tau\}$ is different from the bubble at $\{\mathbf{r}'\mathbf{v}'0\}$ . Since the two events are mutually exclusive, we have :

$$G(\mathbf{r}\omega\tau/\mathbf{r}'\omega'0) = G_s(\mathbf{r}\omega\tau/\mathbf{r}'\omega'0) + G_d(\mathbf{r}\omega\tau/\mathbf{r}'\omega'0) \tag{5}$$

The same separation can be made for $G(\mathbf{r}\tau/\mathbf{r}'0)$:

$$G(\mathbf{r}\tau/\mathbf{r}'0) = G_s(\mathbf{r}\tau/\mathbf{r}'0) + G_d(\mathbf{r}\tau/\mathbf{r}'0) \tag{6}$$

where for instance:

$$G_s(\mathbf{r}\tau/\mathbf{r}'0) = \frac{\int d\omega \int d\omega' \, G_s(\mathbf{r}\omega\tau/\mathbf{r}'\omega'0)\, G(\mathbf{r}'\omega')}{\int d\omega' \, G(\mathbf{r}'\omega')} \tag{7}$$

Analog to (4) we define:

$$\langle v_i(\mathbf{r}\tau) v_k(\mathbf{r}'0)\rangle_s = \frac{\int d\omega \int d\omega' \, v_i v_k' \, G_s(\mathbf{r}\omega\tau/\mathbf{r}'\omega'0)\, G(\mathbf{r}'\omega')}{\int d\omega \int d\omega' \, G_s(\mathbf{r}\omega\tau/\mathbf{r}'\omega'0)\, G(\mathbf{r}'\omega')} \tag{8}$$

and similar for $\langle v_i(\mathbf{r}\tau) v_k(\mathbf{r}'0)\rangle_d$, replacing $G_s$ by $G_d$ in (8).
Combining (3), (4), (5), (7) and (8) we obtain

$$\langle v_i(\mathbf{r}\tau) v_k(\mathbf{r}'0)\rangle G(\mathbf{r}\tau/\mathbf{r}'0) = \langle v_i(\mathbf{r}\tau) v_k(\mathbf{r}'0)\rangle_s G_s(\mathbf{r}\tau/\mathbf{r}'0) + \langle v_i(\mathbf{r}\tau) v_k(\mathbf{r}'0)\rangle_d G_d(\mathbf{r}\tau/\mathbf{r}'0) \tag{9}$$

The correlation functions $G(\mathbf{r}\omega\tau/\mathbf{r}'\omega'0)$ and $G(\mathbf{r}\tau/\mathbf{r}'0)$ have to satisfy some boundary conditions.

For $\tau = 0$ we have obviously:

$$G(\mathbf{r}\omega 0/\mathbf{r}'\omega'0) = \delta(\mathbf{r}-\mathbf{r}')\,\delta(\omega-\omega') + G_d(\mathbf{r}\omega/\mathbf{r}'\omega') \tag{10}$$

where

$G_d(\mathbf{r}\omega/\mathbf{r}'\omega')$ is the space pair correlation function which gives the bubble density at $\{\mathbf{r},\omega\}$ with respect to an arbitrarily chosen bubble at $\{\mathbf{r}'\omega'\}$

For large $\tau$ on the other hand we expect that the bubbles at $\{\mathbf{r}, \mathbf{v}\}$ are statistically independent of the bubble at $\{\mathbf{r}', \mathbf{v}'\}$ at time zero and put

$$\lim_{\tau \to \infty} G(\mathbf{r}\mathbf{v}\tau/\mathbf{r}'\mathbf{v}'0) = G(\mathbf{r}, \mathbf{v}) \tag{11}$$

where $G(\mathbf{r}\mathbf{v})$ is the time independent bubble density at $\{\mathbf{r}, \mathbf{v}\}$.

Similarly we assume statistical independence between bubbles separated by large distances from each other, considered at the same time, that means:

$$\lim_{|\mathbf{r}-\mathbf{r}'| \to \infty} G_d(\mathbf{r}\mathbf{v}/\mathbf{r}'\mathbf{v}') = G(\mathbf{r}, \mathbf{v}) \tag{12}$$

Now we turn to the description of our experiment. The underlying principle is this: Consider two arbitrary (space-fixed) surfaces $F_T$ and $F_D$ crossing the bubble field (see Fig. 1). If a bubble crosses $F_T$ or $F_D$ an electric pulse will be produced. The pulse from a bubble crossing $F_T$ (trigger-surface) is used to trigger a multichannel time analyzer. The pulses from $F_D$ (detector-surface), due to bubbles crossing $F_D$, are fed into the time channels and stored corresponding to their time delay relative to the trigger pulse. At the end of an analyzing cycle the analyzer stops and a new triggering count from $F_T$ is necessary to start the next cycle. The measurements performed over some Time T, after which we have N trigger-counts from $F_T$ and $N_i$ counts stored in the i-th channel which is characterized by its width $\Delta\tau$ and delay time $\tau_i$.

If $P(\tau)\Delta\tau$ is the probability for a pulse from $F_D$ within $\Delta\tau$ at $\tau$ after a trigger pulse from $F_T$ at $\tau = 0$ we have

$$N_i = N\, P(\tau_i)\Delta\tau \tag{13}$$

and the problem consits in constructing an expression for $P(\tau)$. This can be done in the following steps:

1)

$$d\mathbf{v}\,(\mathbf{v}\cdot d\mathbf{f})\, G(\mathbf{r}\mathbf{v}\tau/\mathbf{r}'\mathbf{v}'0) \tag{14}$$

is the mean number of bubbles with a velocity in the range $d\mathfrak{v}$ around $\mathfrak{v}$ which cross at time $\tau$ the area element $d\mathfrak{f}$ at $\mathfrak{r}$ per unit time if there was a bubble at $\{\mathfrak{r}',\mathfrak{v}'\}$ at time 0.

2)

$$\int_{F_D}\int d\mathfrak{v}\,(\mathfrak{v}\cdot d\mathfrak{f})\,G(\mathfrak{r}\mathfrak{v}\tau/\mathfrak{r}'\mathfrak{v}'0) \tag{15}$$

is the mean number of bubbles crossing the detector surface $F_D$ at time $\tau$ per unit time (mean crossing rate or counting rate) if there was a bubble at $\{\mathfrak{r}',\mathfrak{v}'\}$ at time 0.

3)

$$d\mathfrak{v}'\,(\mathfrak{v}'\cdot d\mathfrak{f}')\;G(\mathfrak{r}'\mathfrak{v}') \quad \text{is the} \tag{16}$$

number of bubbles with velocity in $d\mathfrak{v}'$ around $\mathfrak{v}'$ which cross $d\mathfrak{f}'$ at $\mathfrak{r}'$ per unit time.

4)

$$C_T=\int_{F_T}\int d\mathfrak{v}'\,(\mathfrak{v}'\cdot d\mathfrak{f}')\,G(\mathfrak{r}',\mathfrak{v}') \tag{17}$$

is the mean number of bubbles crossing $F_T$ per unit time.

5)

$$\frac{d\mathfrak{v}'\,(\mathfrak{v}'\cdot d\mathfrak{f}')\,G(\mathfrak{r}'\mathfrak{v}')}{C_T} \tag{18}$$

is the probability that a bubble crossing $F_T$ crossed this surface through $d\mathfrak{f}'$ at $\mathfrak{r}'$ with $\mathfrak{v}'$ in $d\mathfrak{v}'$.

Combining (15) with (18) we obtain finally:

$$P(\tau)=\frac{1}{C_T}\int_{F_D}\int_{F_T}\int d\mathfrak{v}\int d\mathfrak{v}'\,(\mathfrak{v}\cdot d\mathfrak{f})(\mathfrak{v}'\cdot d\mathfrak{f}')\,G(\mathfrak{r}\mathfrak{v}\tau/\mathfrak{r}'\mathfrak{v}'0)\,G(\mathfrak{r}'\mathfrak{v}') \tag{19}$$

If we denote by

$v_i$ the component of $\mathfrak{v}$ on df, and by
$v'_k$ the component of $\mathfrak{v}'$ on df'

we can write for (19):

$$P(\tau) = \frac{1}{C_T} \int_{F_D} df \int_{F_T} df' \langle v_i(\mathfrak{x}\tau) v_k(\mathfrak{x}'0) \rangle \, G(\mathfrak{x}\tau|\mathfrak{x}'0)\, G(\mathfrak{x}') \tag{20}$$

Inserting the decomposition (5) in (19) we obtain

$$P(\tau) = P_s(\tau) + P_d(\tau) \tag{21}$$

with

$$P_s(\tau) = \frac{1}{C_T} \int_{F_D} \int_{F_T} \int d\mathfrak{v} \int d\mathfrak{v}' (\mathfrak{v}\, df)(\mathfrak{v}'\, df')\, G_s(\mathfrak{x}\mathfrak{v}\tau|\mathfrak{x}'\mathfrak{v}'0)\, G(\mathfrak{x}'\mathfrak{v}') \tag{22}$$

and a similar expression for $P_d(\tau)$ with $G_s$ replaced by $G_d$.

To prepare the general theory for a practical situation we make some simplifications. At first we assume the two-phase flow to move in one direction $\mathbf{k}$ such that a bubble which has the velocity $\mathbf{v}' = v'\mathbf{k}$ at $\mathbf{r}'$ keeps this velocity and moves within the time interval $\tau$ from $\mathbf{r}'$ to the point $\mathbf{r}' + \tau\mathbf{v}'$
The self correlation function $G_s$ has in this case the form:

$$G_s(\mathbf{r}\mathbf{v}\tau/\mathbf{r}'\mathbf{v}'0) = \delta(\mathbf{v}-\mathbf{v}')\,\delta(\mathbf{r}-\mathbf{r}'-\tau\mathbf{v}') \tag{23}$$

From (22) we obtain:

$$P_s(\tau) = \frac{1}{C_T}\,\frac{1}{\tau^5}\int_{F_D} df \int_{F_T} df'\, X_i\, X_k\, G(\mathbf{r}', \frac{X}{\tau}) \tag{24}$$

with $X = \mathbf{r} - \mathbf{r}'$

$X_i = (\mathbf{r}-\mathbf{r}')n$; n = normal direction of df at $\mathbf{r}$

$X_k = (\mathbf{r}-\mathbf{r}')n'$; n'= normal direction of df' at $\mathbf{r}'$

Secondly we choose the flow direction $\mathbf{k}$ to be the z-direction and take the two surfaces $F_D$ and $F_T$ as parallel planes at a distance D, and vertically to the z-direction (see Fig. 2).
The density $C(\mathbf{r}', \mathbf{v}')$ is then given by

$$G(\mathbf{r}', \mathbf{v}') = G(\mathbf{r}', v_z)\,\delta(v_x)\,\delta(v_y) \tag{25}$$

and (24) gives us:

$$P_s(\tau) = \frac{1}{C_T}\,\frac{D^2}{\tau^3}\int_B df'\, G(\mathbf{r}', \frac{D}{\tau}) \tag{26}$$

where the integration extends only over the overlap part B of the two surfaces $F_T$ and $F_D$ (see Fig. 2). $P_s(\tau)$ can be made to disappear if B is made equal to zero (no overlap between $F_T$ and $F_D$).

If we plot $P_s(\tau)$ not as a function of $\tau$ but directly as a function of the velocity $v = D/\tau$ and use the relation

$$P_s(\tau)d\tau = P_s(v)dv \tag{27}$$

we obtain from (26):

$$P_s(v) = \frac{1}{C_T} \cdot v \cdot \int_B df \; G(x', v) \tag{28}$$

For a very small overlap region $\Delta B$ around the point $\varphi$ this gives:

$$P_s(v) = \frac{\Delta B}{C_T} \cdot v \cdot G(\varphi, v) \tag{29}$$

In Fig. 3 we sketch for this special case the general structure of $P_s(\tau)$ as derived from $P_s(v)$. $P_s(\tau)$ will be different from zero only within a very small interval $\Delta\tau$ around $\tau_o$ which corresponds to the average velocity of bubbles at $\varphi$ . The sharp peak of $P_s(\tau)$ within $\Delta\tau$ reflects the fluctuations of the velocity around its mean value $v_o$ (= $D/\tau_o$) as described by $G(\varphi, v)$. Recall that for our model we made the assumption that a bubble crossing $F_T$ at $\varphi$ with velocity v continues its straight-line motion with this constant velocity. A fluctuation of the velocity implies therefore that the individual bubbles crossing $F_T$ at $\varphi$ have different velocities. This distribution of velocities among different bubbles is described by $G(\varphi, v)$.

The $P_s$ part of the correlation function contains therefore the complete information about the velocity distribution of the flow pattern of the two-phase flow.

The next point we have to investigate is the $P_d(\tau)$ part of (21). To simplify the consideration we introduce some further approximations. At first we make the convolution approximation (see [3] ):

$$G_d(\mathbf{r}\mathbf{v}\tau/\mathbf{r}'\mathbf{v}'0) = \int d\boldsymbol{\rho}\int d\mathbf{u}\; G_s(\mathbf{r}\mathbf{v}\tau/\boldsymbol{\rho}\,\mathbf{u}\,0)\,G_d(\boldsymbol{\rho}\,\mathbf{u}/\mathbf{r}'\mathbf{v}') \tag{30}$$

This reduces the investigation of $G_d(\mathbf{r}\mathbf{v}\tau/\mathbf{r}'\mathbf{v}'0)$ to the discussion of the space pair correlation function $G_d(\mathbf{r}\mathbf{v}/\mathbf{r}'\mathbf{v}')$. Applying the model (23) for the bubble motion we obtain from (30)

$$G_d(\mathbf{r}\mathbf{v}\tau/\mathbf{r}'\mathbf{v}'0) = G_d(\mathbf{r}-\tau\mathbf{v},\mathbf{v}/\mathbf{r}'\mathbf{v}') \tag{31}$$

The meaning is obvious: a particle which moves with constant velocity $\mathbf{v}$ and is at time $\tau$ at position $\mathbf{r}$ was at time 0 at position $\mathbf{r}-\tau\mathbf{v}$.

We insert this result in (19) and write the expression for the above mentioned experimental conditions:

$$P_d(\tau) = \frac{1}{C_T}\int_{F_D} df\int_{F_T} df'\int d\mathrm{v}\int d\mathrm{v}'\;\mathrm{v}\,\mathrm{v}'\,G_d(\mathbf{r}-\tau\mathrm{v},\mathrm{v}/\mathbf{r}'\mathrm{v}')\,G(\mathbf{r}'\mathrm{v}') \tag{32}$$

As a second approximation we neglect the fluctuations in the velocity from bubble to bubble (at the same place) and assume a deterministic velocity field $\mathrm{v}(\mathbf{r})$. This assumption means

$$G(\mathbf{r}'\mathrm{v}') = G(\mathbf{r}')\,\delta(\mathrm{v}-\mathrm{v}(\mathbf{r})) \tag{33}$$

and a similar relation for $G_d$ so that we arrive finally at:

$$P_d(\tau) = \frac{1}{C_T}\int_{F_T} df'\int_{F_D} df\;\mathrm{v}(\mathbf{r})\,\mathrm{v}(\mathbf{r}')\,G_d(\mathbf{r}-\tau\mathrm{v}(\mathbf{r})/\mathbf{r}')\,G(\mathbf{r}') \tag{34}$$

For a statistically <u>homogeneous</u> bubble field this $G_d(\mathbf{r}/\mathbf{r}')$ function will

a) be a function of the distance $r = |\mathbf{r}-\mathbf{r}'|$ only, and

b) will tend to the average density G of the homogeneous field for large r.

The general appearance of $G_d(r)$ will be as sketched in Fig. 4. The "hole" in the pair-distribution function is due: first to the finite size of a bubble and second to physical forces ( see [4] ) which act between the bubbles and which may be partly induced by the carrier-fluid. The peaks at larger radii represent the local ordering of the bubbles established under the action of these forces. This ordering disappears with increasing distance.

Introducing the variable

$$\mathbf{t} = \mathbf{r} - \tau \mathbf{v}(\mathbf{r}) \tag{35}$$

we write for (34):

$$P_d(\tau) = \frac{1}{G_T} \int_{F_T} df' \int_{D(\tau)} df \; v(\mathbf{t})\, v(\mathbf{r}')\, G_d(\mathbf{t}/\mathbf{r}')\, G(\mathbf{r}') \tag{36}$$

where the integration over $F_D$ is transformed in an integration over the surface $D(\tau)$ which is obtained from $F_D$ by shifting each point $\mathbf{r}$ of $F_D$ to $\mathbf{r} - \tau\, v(\mathbf{r})$ (See Fig. 5).

For small $\tau$ , that is for large distances between the points of $F_T$ and $D(\tau)$, the pair correlation function $G_d$ can be replaced by $G(\mathbf{t})$ and we obtain from (36):

$$P_d(\tau) = \int_{D(\tau)} df \; v(\mathbf{t})\, G(\mathbf{t}) \tag{37}$$

= mean crossing rate of bubbles through $D(\tau)$ (counting rate) which is independent of $\tau$ .

$P_d(\tau)$ is therefore for small $\tau$ (the same holds for large $\tau$) equal to the average constant counting rate of $F_D$ (which is just the constant background of $P(\tau)$. As soon as the moving integration surface $D(\tau)$ comes near enough to $F_T$ to feel the correlation with the trigger bubble (at $\tau = \tau_1$ lets say), $P_d(\tau)$ will obtain a structure as suggested by the behaviour of $G_d(r)$ in Fig. 4. This fact is also indicated in Fig. 5.

If we restrict the detector surface $F_D$ to within a region with constant velocity $v_o$, then all $D(\tau)$-surfaces are parallel (see Fig. 6) and (36) reads:

$$P_d(\tau) = \frac{v_o}{c_T} \int_{F_T} df(\kappa') \int_{D(\tau)} df(\tau)\, v(\kappa')\, G_d(\tau \mid \kappa')\, G(\kappa') \tag{38}$$

The general behaviour of $P_d(\tau)$ in this case is also indicated in Fig. 5. A remarkable feature of $P_d(\tau)$ is the symmetry around the value $\tau_o = D/v_o$. The actually measured distribution $P(\tau)$ is finally the superposition of $P_s(\tau)$ and $P_d(\tau)$ as given in (21).

Experiment

In our actual experiment (see Fig. 7 and 1) use for $F_T$ and $F_D$ two small rectangles of width H and length 2L ($H \ll L$) parallel to each other, separated by a distance D and vertical to the flow velocity. These rectangular shapes for $F_T$ and $F_D$ are realized in the experiment by two thin light beams crossing the two phase flow. A bubble crossing a light beam leads to an intensity variation of the light beam due to scattering and refraction and gives rise to an electric pulse in a photo-diode which is irradiated by the light beam. Measurements were done mainly for two special arrangements (see Figs. 8a and 8b).

Case a) $F_T$ parallel to the y-axis, $F_D$ parallel to the x-axis (Fig. 8a),

Case b) $F_T$ and $F_D$ both parallel to the y-axis (Fig. 8b).

For case a) we have the simplest case with a small overlap region $\Delta F = H^2$. The $P_s(\tau)$ part for this situation is given by (29) and has the general structure shown in Fig. 3. The $P_d(\tau)$ part has the structure shown in Fig. 6 if $F_D$ covers a region of the two-phase flow in which the velocity is constant. This was achieved by using a two-phase flow with a rectangular cross-section with sides R and S such that $R \ll S$ and taking $F_T$ //S and $F_D$//R. The measured $P_s(\tau)$ and $P_d(\tau)$ will have a general shape as shown in Fig. 9.

To discuss the case b) above ($F_T$ and $F_D$ both //S) we assume a complete overlap of $F_T$ and $F_D$ and use expression (26) for $P_s(\tau)$. As H is very small we may consider G(xy,v) as a function of y only and have:

$$P_s(\tau) = \frac{1}{c_T} \frac{D^2 H}{\tau^3} \int_{-L}^{+L} dy \; G\left(y, \frac{D}{\tau}\right) \tag{39}$$

Neglecting for the moment the fluctuations of the velocity at a fixed point and assuming a deterministic space-dependent velocity field we have:

$$G(y,v) = G(y)\,\delta(v - v(y)) \tag{40}$$

If we recall the general relation for the $\delta$ -function:

$$\delta(\varphi(x)) = \sum \frac{1}{|\varphi'(x_i)|} \delta(x-x_i) \tag{41}$$

where the $x_i$ are the roots of $\varphi(x) = 0$ we obtain from (39) and (40):

$$P_s(\tau) = \frac{2}{c_T} \frac{D^2 H}{\tau^3} \frac{G(y_0)}{\left|\frac{dv}{dy}\right|_{y_0}} \tag{42}$$

where $y_0$ is the solution of

$$D = \tau\, v(y_0) \tag{43}$$

The construction of the general shape of $P_s(\tau)$ is sketched in Fig. 10. The sharp peak of $P_s(\tau)$ at $\tau_1$ defined by $D = \tau_1 v(0)$ will of course be rounded off due to the fluctuations of velocity which we have neglected. As $P_d(\tau)$ in this case will behave as shown in Fig. 5 we find for $P(\tau)$ the expected shape in 11.

## Conclusion

In this paper we presented a general theory for an experiment to measure the statistical properties of a bubble field in stationary two-phase flow. Correlation functions were introduced which have a simple physical meaning and which describe the different aspects of the two-phase flow in a proper way. The overall flow-pattern (denstity-velocity distribution in space) of the two-phase mixture is represented by the self-correlation function. The internal structure of the bubble field, due to physical interactions between the bubbles and the carrier fluid, is described by the $G_d$-correlation function. General expressions are derived relating experimental results to these correlation functions.

Only part of the information stored in the self-correlation function was used up to now in actually performed measurements (determination of the average velocity in [1] ). Physical models for the interactions between neighbouring bubbles have to be constructed to make full use of the power of this correlation experiment.

A first step in this direction will be the investigation of the influence on the space-pair correlation function due to the hydrodynamic forces acting between bubbles. The theory of these forces is presented in detail for instance in the book by L.M. Milne-Thomson (see [4] ).

## Literature

1 MATTHES, W. et al., The Review of Scientific Instruments; Vol. 41, No. 6, 843 (1970).

2 R. PECORA, Journ. of Chem. Physics; Vol. 40, No. 6, 1604 (1964).

3 R.J. GLAUBER, Scattering of Neutrons by Statistical Media; Boulder Lectures in Theoretical Physics, Vol. IV (1961).

4 L.M. MILNE-THOMSON, Theoretical Hydrodynamics, Mac Millan & Co. LTD, London-New York (1960), page 504.

## Figure Captions

Fig. 1: General arrangement for the trigger-surface $F_T$ and detector-surface $F_D$ in the two-phase flow.

Fig. 2: Trigger-surface $F_T$ and detector-surface $F_D$ are both plane areas parallel to each other and vertical to the flow direction.

Fig. 3: Construction of the $P_s(\tau)$-Peak out of the $P_s(v)$ distribution.

Fig. 4: General shape for the space-pair correlation function $G_d(\tau)$ for a statistically homogeneous bubble field.

Fig. 5: Expected behaviour of $P_d(\tau)$ due to the moving integration surface $D(\tau)$ scanning the $G_d(r/r')$ function ( $r'$ on $F_T$, $r$ on $D(\tau)$).

Fig. 6: Expected behaviour of $P_d(\tau)$ if $D(\tau)$ covers a region of the two-phase flow within which the flow velocity is constant.

Fig. 7: Special choice for $F_T$ and $F_D$ in the actual experiment. $F_T$ and $F_D$ are small rectangles of width H and length 2L ($H \ll L$) parallel to each other, separated by a distance D and both vertical to the flow velocity.

Fig. 8: Orientations of $F_T$ and $F_D$ for most of the experiments.
a) $F_T$ and $F_D$ both parallel to the $\mathbf{y}$-axis
b) $F_T$ parallel to the y-axis, $F_D$ parallel to the x-axis

Fig. 9: Behaviour of the $P(\tau)$ distribution for the situation of Fig. 8a.

Fig. 10 Construction of the general shape of $P_s(\tau)$ for the situation of Fig. 8b under the assumption of a deterministic space dependence of the velocity: $\mathbf{v} = v(y)$.

Fig. 11: Behaviour of the $P(\tau)$ distribution for the situation of Fig. 8b.

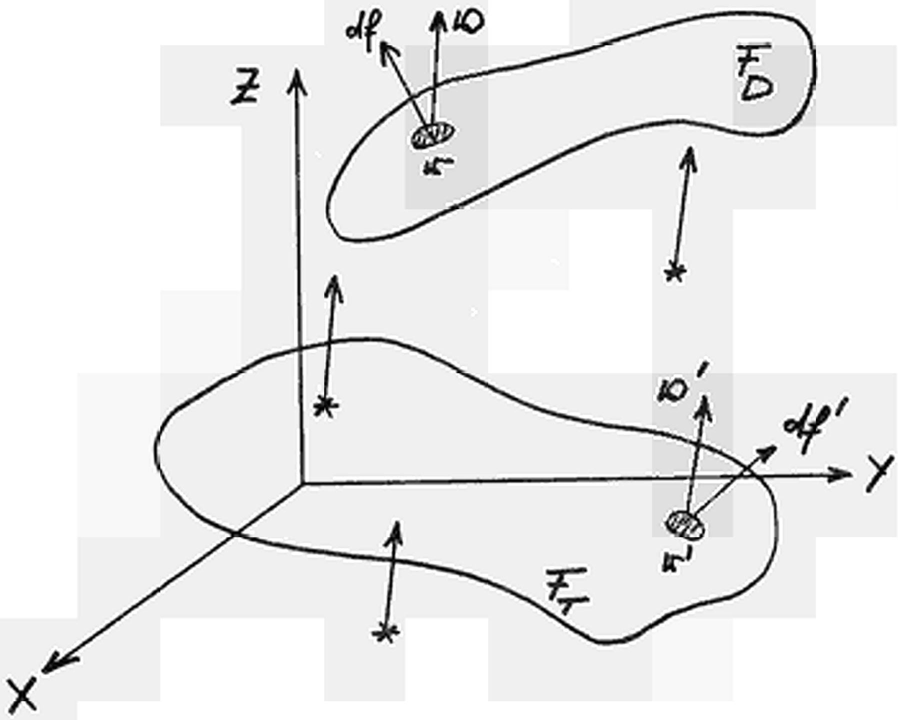



Fig. 1

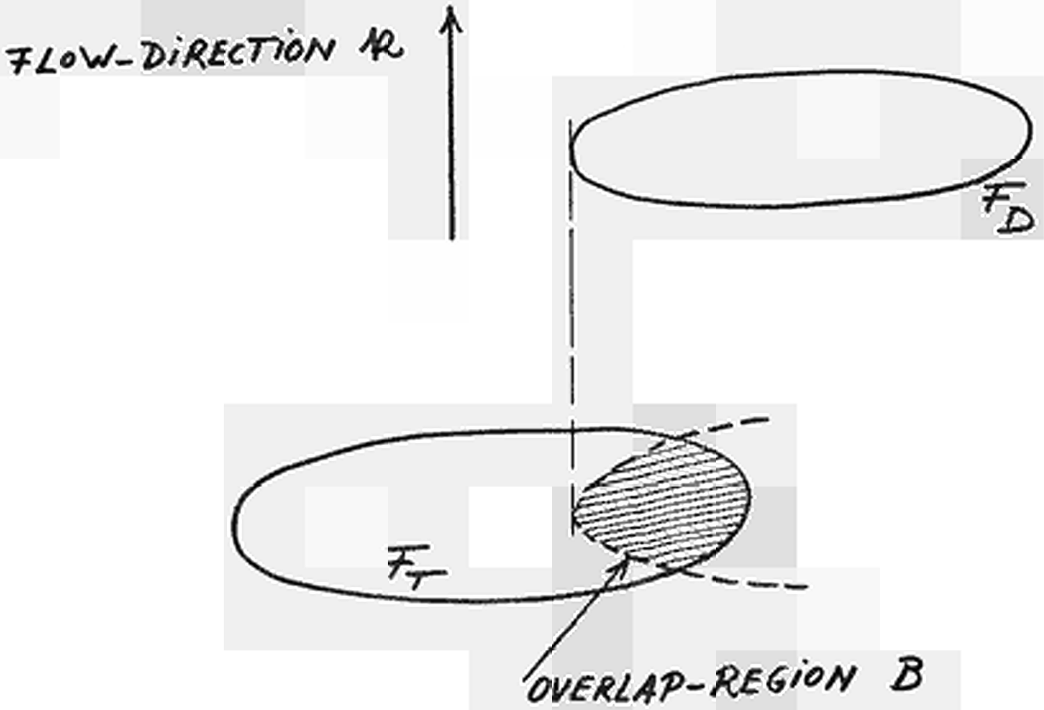



Fig. 2

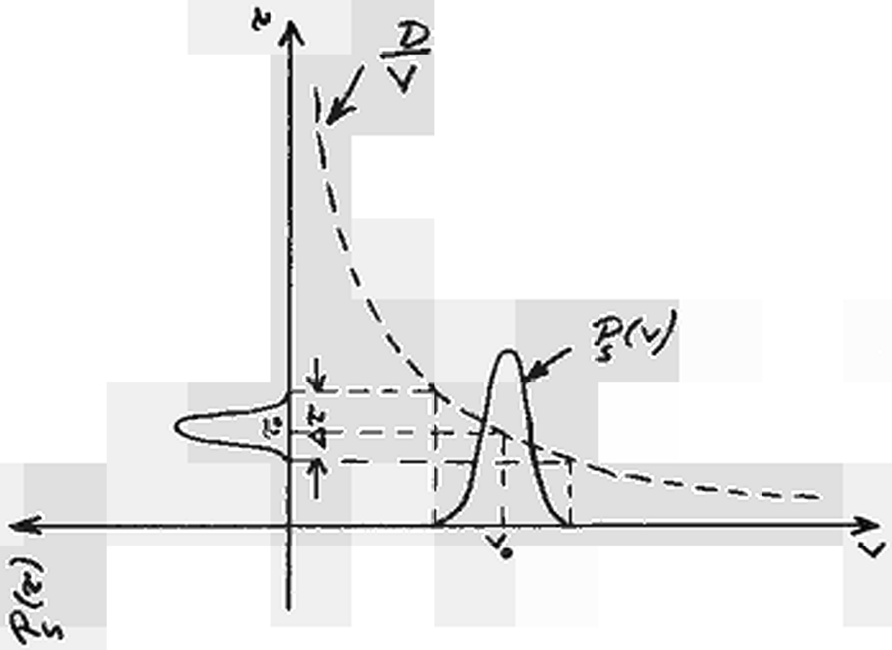



Fig. 3

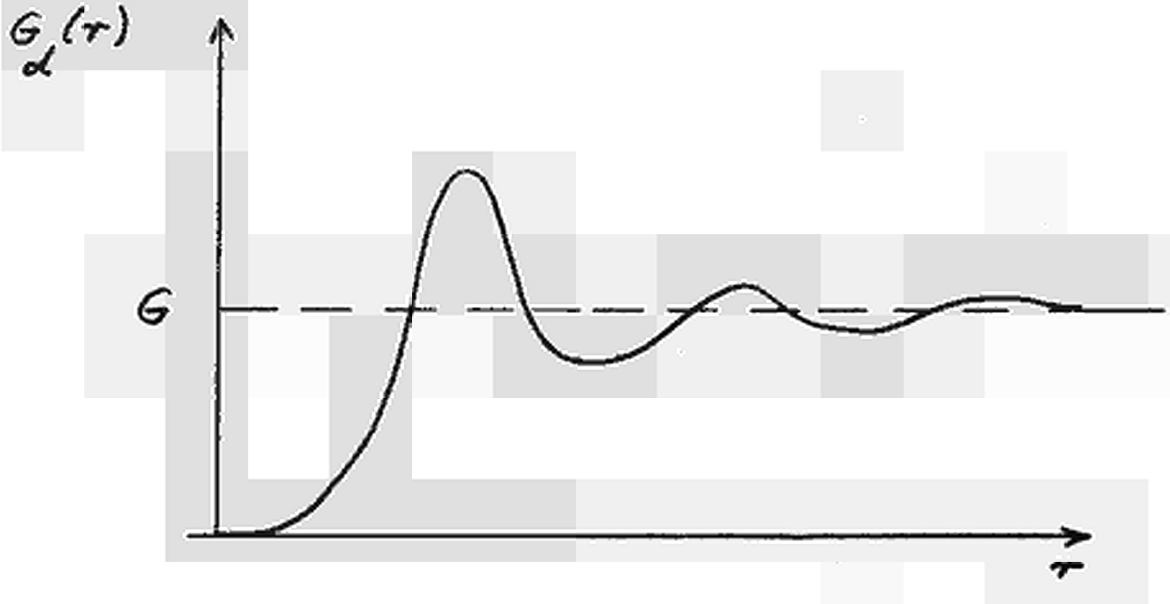



Fig. 4

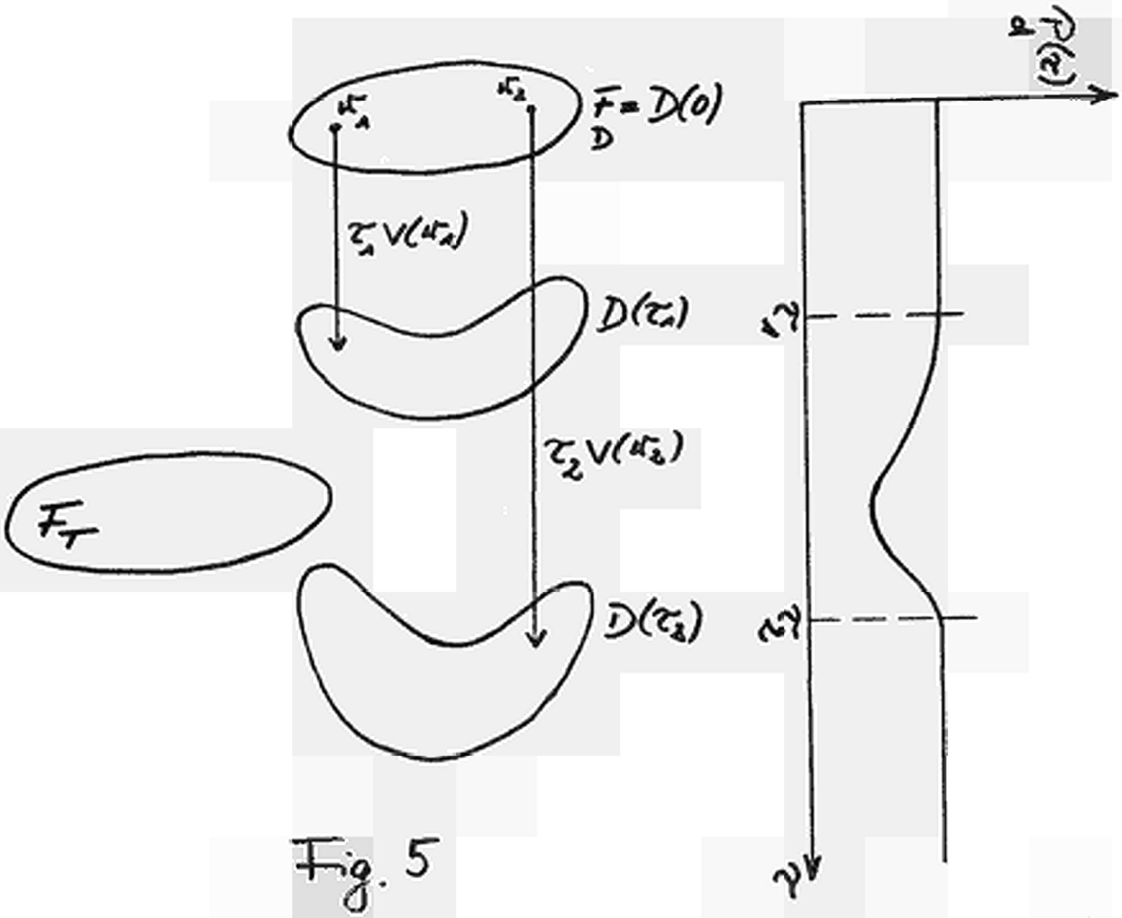



Fig. 5

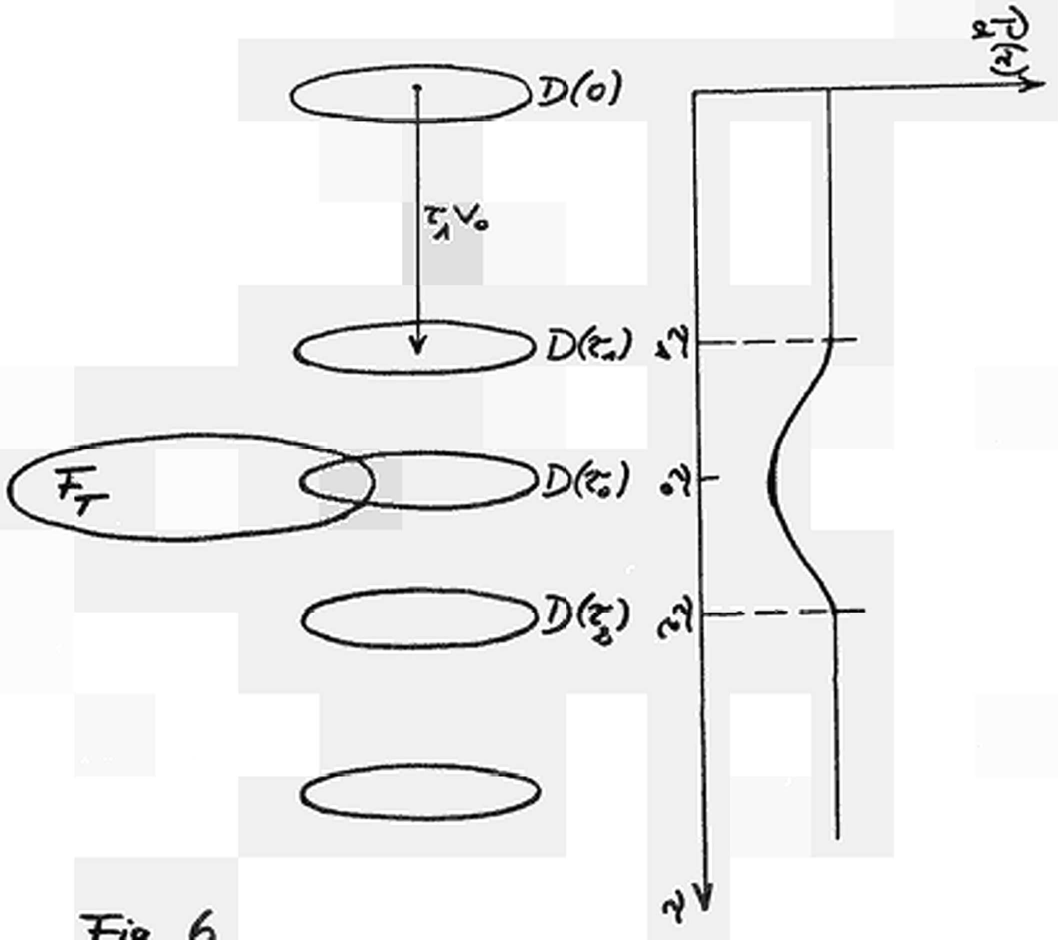



Fig. 6

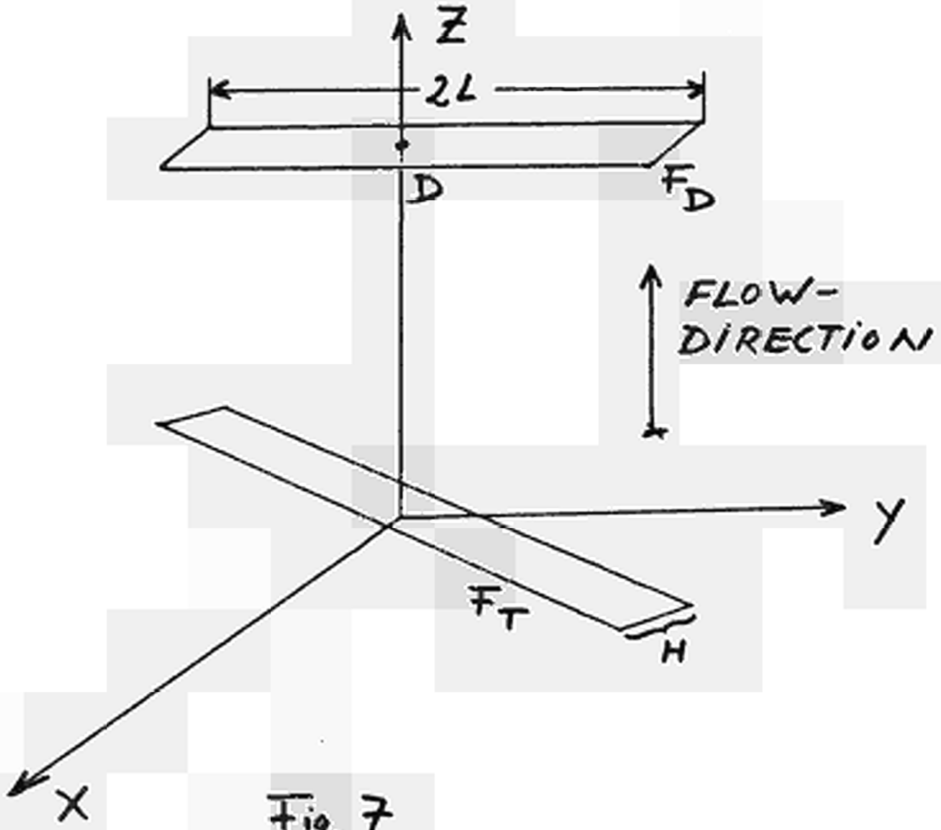



Fig. 7

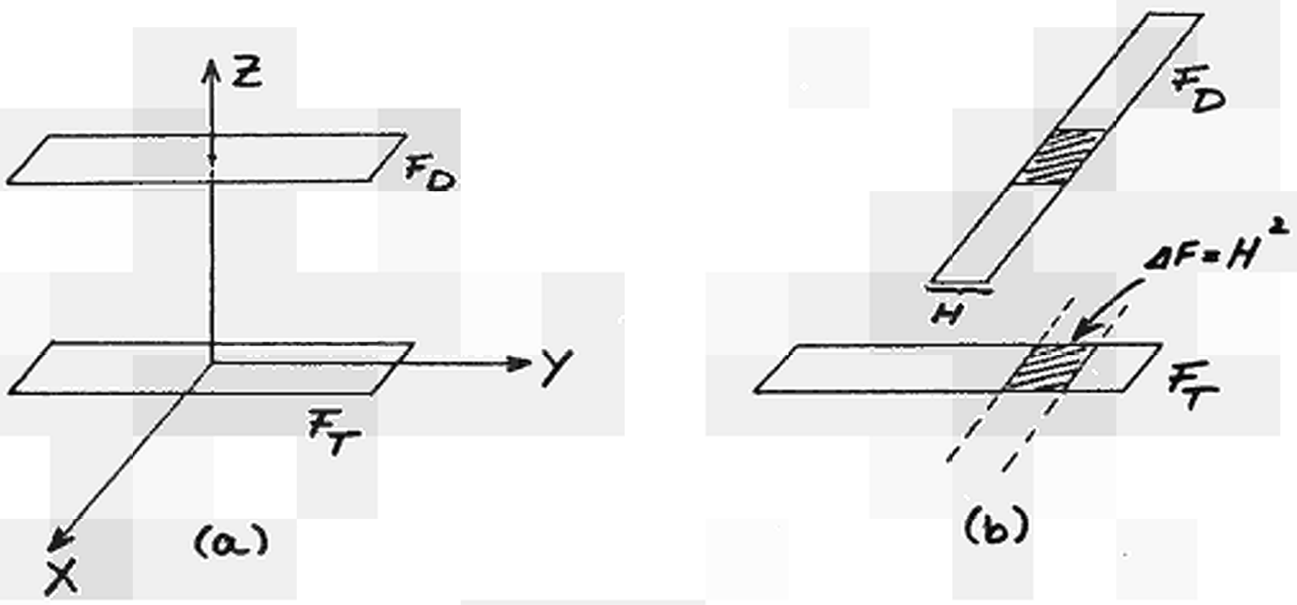



Fig. 8.

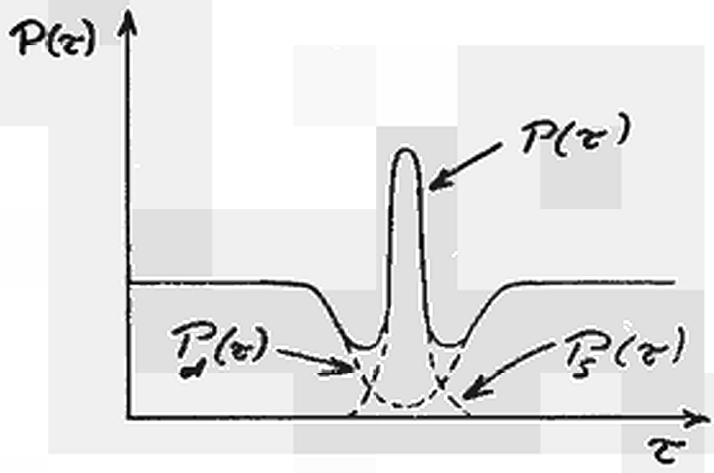



Fig. 9

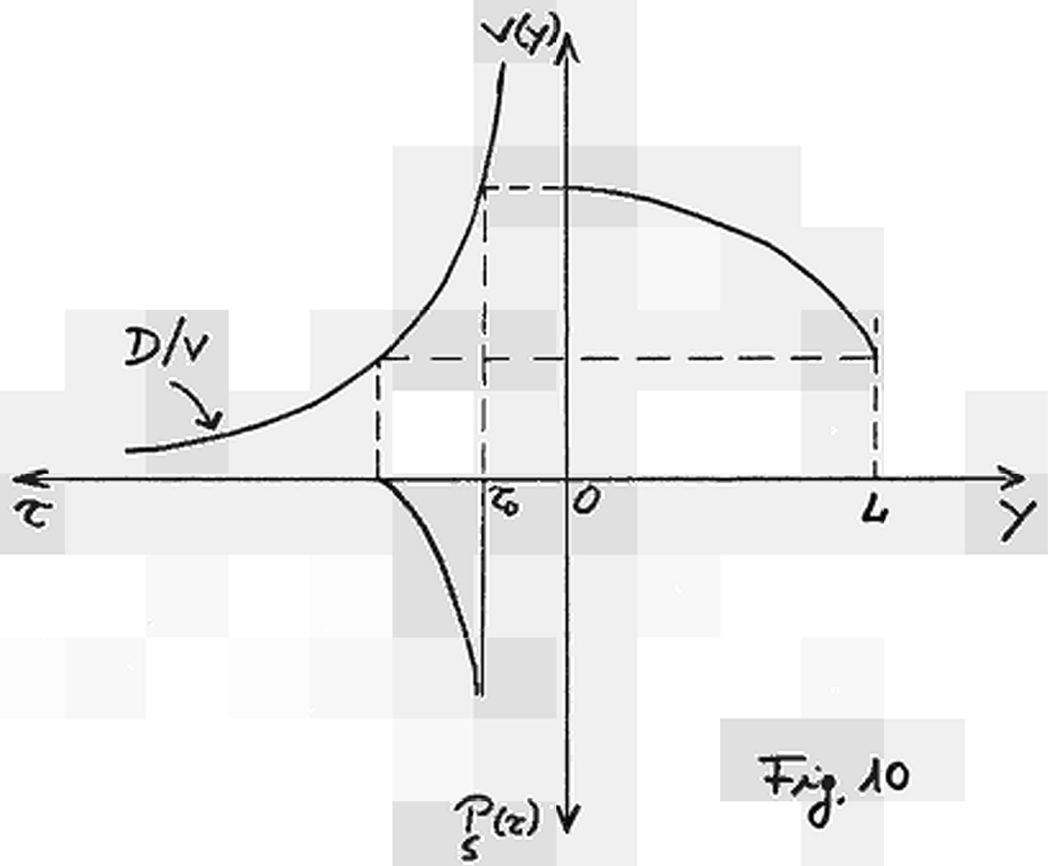
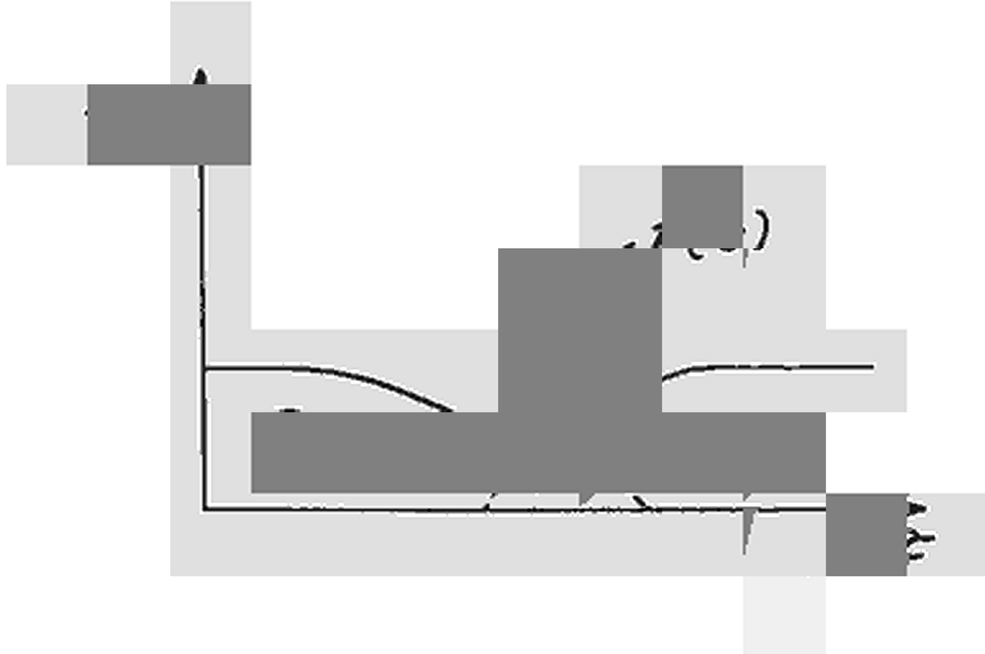



Fig. 10



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Alfred Nobel

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