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CALCULATIONS FOR FISSION NEUTRON AND GAMMA MULTIPLICITY MEASUREMENTS

by

J.P. THEOBALD, J.A. WARTENA and W. KOLAR

1972

Joint Nuclear Research Centre
Geel Establishment - Belgium

Central Bureau for Nuclear Measurements - CBNM

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ABSTRACT

A set of detectors can be used to measure multiplicities of neutron or gamma quanta emitted in fission. Subsets of these detectors can be linked with coincidence units. In this paper probabilities for x-fold coincidences are calculated for a given number of emitted particles or quanta. Methods of combinational analysis have been applied.

KEYWORDS

FISSION NEUTRONS
GAMMA RADIATION
FISSION
NEUTRON DETECTION
GAMMA DETECTION
COINCIDENCE METHODS
PROBABILITY

CALCULATIONS FOR FISSION NEUTRON AND GAMMA MULTIPLICITY MEASUREMENTS

J.P. Theobald, J.A. Wartena and W. Kolar

Central Bureau for Nuclear Measurements, EURATOM, Geel, Belgium

1. INTRODUCTION

The de-excitation of the fissioning nucleus by neutron emission shows variations from resonance to resonance in neutron induced fission of ^{235}U and ^{239}Pu. The assignment of these variations to the nuclear channel spin associated with these resonances has been attempted.[1 2 3] Possibly the neutron multiplicity is not constant.

At the C.B.N.M. the fission cross sections of ^{235}U and ^{239}Pu have been measured with four liquid scintillator neutron detectors arranged in a cylindrical geometry around the sample and linked with six double and four triple coincidence units. The ratio of fission yields measured with the double to those registered with the triple coincidence condition is dependent on the above mentioned variations. As these liquid scintillator detectors are in principle also sensitive for the about 7 to 9 fission gamma quanta, the following question is important for multiplicity measurements:
What is the probability $P(n,k,s|\varepsilon)$ for a s-fold coincidence in k detectors with a total efficiency ε per particle, when n particles are impinging upon these detectors?

The answer to this question could be interesting for similar experiments.

2. FORMALISM

Because of its simplicity the case is treated first that the efficiency ε of the set of k detectors for the detection of one particle (neutron or gamma quantum) equals unity.

Case $\varepsilon_0 = 1$

The probabilities $P(n,k,s=j|1)$ are given by two factors: $C(n,j)$ and $\frac{k^{[j]}}{k^n}$ with

$$C(n,j) = \frac{1}{(j-1)!} \sum_{i=0}^{j-1} (-1)^i \binom{j-1}{i} (j-i)^{n-1} \tag{1}$$

and
$$k^{[j]} = k(k-1)(k-2)\cdots(k-j+1) = \binom{k}{j} j!$$

$C(n,j)$ stands for the number of possible combinations to select n particles into j detectors. $\frac{k^{[j]}}{k^n}$ represents the probability for the detection of n particles in j out of k different detectors. (n-j) particles are simultaneously detected in these j detectors and not contributing to the order j of the coincidence. In conclusion

$$P(n,k,j|1) = C(n,j) \cdot \frac{k^{[j]}}{k^n} \tag{2}$$

$j \leq k$

In table 1, 2 and 3 the possible combinations for two, three and four coincidences with five particles labelled 12345 are displayed. Table 4 shows $C(n,j)$ values for $1 \leq j \leq 4$ and $j \leq n \leq 10$. The formula (1) for the $C(n,j)$ is derived in the appendix to this paper.

TABLE 1

C (5,2)

Det. I	Det. II
1	2345
12	345
13	245
14	235
15	234
123	45
124	35
125	34
1234	5
1235	4
1245	3
1345	2
145	23
135	24
124	25

TABLE 2

C (5,3)

Det. I	Det. II	Det. III
1	2	345
1	3	245
1	4	235
1	5	234
1	23	45
1	24	35
1	25	34
12	34	5
12	35	4
12	45	3
13	24	5
13	25	4
13	45	2
14	23	5
14	25	3
14	35	2
15	23	4
15	24	3
15	34	2
123	4	5
124	3	5
125	3	4
134	2	5
135	2	4
145	2	3

TABLE 3

C (5,4)

Det. I	Det. II	Det. III	Det. IV
1	2	3	45
1	2	4	35
1	2	5	34
1	23	4	5
1	24	3	5
1	25	3	4
12	3	4	5
13	2	4	5
14	2	3	5
15	2	3	4

n \ j	1	2	3	4	5	6	7	8	9	10	11	12
1	1.											
2	1.	1.										
3	1.	3.	1.									
4	1.	7.	6.	1.								
5	1.	15.	25.	10.	1.							
6	1.	31.	90.	65.	15.	1.						
7	1.	63.	301.	350.	140.	21.	1.					
8	1.	127.	966.	1701.	1050.	266.	28.	1.				
9	1.	255.	3025.	7770.	6951.	2646.	462.	36.	1.			
10	1.	511.	9330.	34105.	42525.	22827.	5880.	750.	45.	1.		
11	1.	1023.	28501.	145750.	246730.	179487.	63987.*	11880.*	1155.*	55.	1.	
12	1.	2047.	86526.	611501.	1379399.*	1323653.*	627396.*	159027.*	22275.*	1705.*	66.*	1.

Table: 4 C(n,j) values for $1 \leq n \leq 12$ The values with an asterisk can have small rounding errors.

Case $0 < \varepsilon < 1$

In this case the efficiency per detector is $\frac{\varepsilon}{k}$. In order to make equation (2) again applicable a number $\varkappa$ of virtual detectors is introduced which increase the total efficiency to unity ($\varepsilon_0 = 1$). Then $\varkappa = \frac{k}{\varepsilon}$ and includes the real detectors k.

For this detector assembly equation (2) in the form

$$P(n, \varkappa, j \mid \varepsilon_0) = C(n, j) \, \frac{\varkappa^{[j]}}{\varkappa^{n}} \qquad (3)$$

($j \leq \varkappa$) is valid. j represents here the number of virtual coincidences that is to say detection events in virtual detectors. Then the probability S(j,s,k) for selecting s real or observable coincidences out of j virtual ones, when k is the number of real detectors, is identical with the probability to select with k trials s red balls out of a black box containing $\varkappa$ balls, from which (j) are red. This probability is given by :

$$S(j, s, k) = \binom{k}{s} \frac{j^{[s]} (\varkappa - j)^{[k-s]}}{\varkappa^{[k]}} \qquad (4)$$

with $\varkappa = \frac{k}{\varepsilon}$ and $S(j,\ s<j,\ k = \varkappa) \equiv 0$

From equations (3) and (4) an expression (5) can be deduced, which gives for an efficiency ε the probability $P(n,k,s/\varepsilon)$ for an s-fold coincidence ($s = 1$ is a single detection event) caused by n particles impinging upon k detectors :

$$P(n, k, s \mid \varepsilon) = \sum_{j=s}^{n} S(j, s, k) \, P(n, \varkappa, j \mid \varepsilon_0) \qquad (5)$$

For practical calculations (5) has the following explicit form : $P(n, k, s \mid \varepsilon) =$

$$= \frac{k!}{s!(k-s)!} \sum_{j=s}^{n} \frac{j!(\varkappa - k)!}{(j-s)!(\varkappa - j - k + s)!} \, \frac{1}{\varkappa^{n}} \sum_{i=0}^{j-1} \frac{(-1)^{i} (j-i)^{n-1}}{i!(j-1-i)!} \qquad (6)$$

For this formula only simple relations like

$$\binom{n}{m} = \frac{n!}{m!(n-m)!} \quad \text{and} \quad n^{[m]} = \binom{n}{m} m!$$

have been used.

The probabilities $P\ (n,k,s/\varepsilon)$ have the property :

$$\sum_{s=0}^{n} P(n,k,s|\varepsilon) = 1 \qquad (7)$$

where

$$P(n,k,0|\varepsilon) = (1-\varepsilon)^{n} \qquad (8)$$

is the probability to detect not one of the n particles, The latter relation is also discussed in the appendix.

3. NUMERICAL CALCULATIONS

J. Terrell published the probabilities $P_{\nu}(\nu-\bar{\nu})$ for the emission of ν neutrons as functions of $(\nu-\bar{\nu})$, when $\bar{\nu}$ is the average number of neutron emitted per fission event,[4] With the energy spectrum[4] of the neutrons one can in principle calculate the efficiency of a given detector assembly. For the fission gamma radiation the situation is not as clear. R.L. Van Hemert et al. have used a Poisson distribution for the distribution of gamma quanta numbers around their average.[5] About 4-5 quanta with an average energy of about 1 MeV are emitted from each of the fragments [5]. The energy spectrum of the prompt gamma rays has been investigated by H.R. Bowman et al.[7]. However, it remains difficult to determine without calibration measurements the efficiency for gamma radiation of a given set of detectors. In the tables at the end of this paper the probabilities $P(n,k,s/\varepsilon$ are tabulated for

$$1 \leq n \leq 12$$
$$k = 4$$
$$0 \leq s \leq k$$

and several efficiencies ε.

For numerical calculations of $P(n,k,s/\varepsilon)$ it can be convenient to introduce Γ functions into equation (6) instead of factorials.

Without the introduction of virtual detectors $P(n,k,s/\varepsilon)$ can also be derived in the following way :

The probability to detect s = 1 particle out of n = 1 with k detectors is

$$P\ (1,k,1/\varepsilon) = k \left(\frac{\varepsilon}{k}\right)^{n}$$

to detect s = 1 out of n = 2 is :

$$P\ (2,k,1/\varepsilon) = k\cdot\left(\frac{\varepsilon}{k}\right)^{n-1}\cdot\left(\frac{\varepsilon}{k}\right) + 2k\left(\frac{\varepsilon}{k}\right)(1-\varepsilon)$$

and s = 2 out of n = 2 :

$$P(2,k,2/\varepsilon) = k\cdot\left(\frac{\varepsilon}{k}\right)^{n-1}(k-1)\left(\frac{\varepsilon}{k}\right).$$

Going on in this way

$$P(3,k,1/\varepsilon) = k\cdot\left(\frac{\varepsilon}{k}\right)^{n} + 3k\left(\frac{\varepsilon}{k}\right)^{n-1}(1-\varepsilon) + 3k\cdot\left(\frac{\varepsilon}{k}\right)^{n-2}(1-\varepsilon)^{2}$$

$$P(3,k,2/\partial = 3k\left(\frac{\varepsilon}{k}\right)^{n-1}(k-1)\left(\frac{\varepsilon}{k}\right) + 3k\left(\frac{\varepsilon}{k}\right)^{n-2}(k-1)\left(\frac{\varepsilon}{k}\right)(1-\varepsilon)$$

$$P(3,k,3/\varepsilon) = k(k-1)(k-2)\left(\frac{\varepsilon}{k}\right)^{n}.$$

For the authors this way seems to be rather tedious. On the other hand it is of course possible to split formula (5), (6) in terms of the last calculation, e.g. :

$$P(3,k,2|\varepsilon) = \frac{k!}{2!(k-2)!}\sum_{j=2}^{3}\frac{j!\left(\frac{k}{\varepsilon}-k\right)!}{(j-2)!\left(\frac{k}{\varepsilon}-k-j+2\right)!}\left(\frac{\varepsilon}{k}\right)^{3}C(3,j)$$

$$= \frac{k(k-1)}{2}\left[6\cdot\left(\frac{\varepsilon}{k}\right)^{3} + 6\left(\frac{k}{\varepsilon}-k\right)\left(\frac{\varepsilon}{k}\right)^{3}\right]$$

$$= 3k(k-1)\left(\frac{\varepsilon}{k}\right)^{3} + 3k(k-1)\left(\frac{\varepsilon}{k}\right)^{2}(1-\varepsilon).$$

4. APPENDIX

A. Derivation of the equation (1) representing the C(n,j) coefficients.

The probability to detect $n = 1$ particles in one out of k detectors with the efficiency $\varepsilon = 1$ is

$$C\,(1,1) \times \frac{k}{k^n} = 1,$$

from which follows : $C\,(1,1) = 1^{n-1}$

The probability for $n = 2$ particles is

$$C\,(2,1)\,\frac{k}{k^n} + C\,(2,2)\,\frac{k\,(k-1)}{k^n} = 1$$

The first term is the probability to detect the two particles simultaneously in one detector, the second to detect them in two different detectors. With $k = 1$ one finds :

$$C\,(2,1) = 1^{n-1}$$

with $k = 2$

$$C\,(2,2) = 2^{n-1} - 1^{n-1}$$

With $n = 3$ particles the equation

$$C(3,1)k + C(3,2)\,k(k-1) + C(3,3)\,k(k-1)(k-2) = k^n$$

yields for k=1,2 and 3

$$C(3,1) = 1^{n-1}$$

$$C(3,2) = 2^{n-1} - 1^{n-1}$$

$$C(3,3) = \frac{1}{2}\left(3^{n-1} - 2 \times 2^{n-1} + 1^{n-1}\right)$$

respectively.

Going on in this way, one finds

$$C(n,4) = \frac{1}{6}\left(4^{n-1} - 3\cdot 3^{n-1} + 3\cdot 2^{n-1} - 1^{n-1}\right)$$

and in general

$$C\,(n,j) = \frac{1}{(j-1)!}\left[\binom{j-1}{0} j^{n-1} - \binom{j-1}{1}(j-1)^{n-1} + \ldots (-1)^{j-1}\binom{j-1}{j-1} 1^{n-1}\right]$$

$$= \frac{1}{(j-1)!}\sum_{i=0}^{j-1}(-1)^i\binom{j-1}{i}(j-i)^{n-1}$$

If C(n,j) is known, also C(n + 1,j) is known.
C (n+1, j+1) can be calculated from

$$C(n+1,1)k + C(n+1,2)k(k-1) + \dots + C(n+1,j)k(k-1)(k-2)\cdots(k-j+1) + C(n+1,j+1)k(k-1)(k-2)\cdots(k-j) = k^{n+1}$$

with k = j + 1
Using $\varkappa$ instead of k, the coefficients C(n,j) are defined by the equation :

$$\sum_{j=0}^{n} \varkappa^{[j]} C(n,j) = \varkappa^{n} \qquad (9)$$ for all $\varkappa$.

It is perhaps interesting to note that this equation is the reversal of the relation

$$\varkappa^{[n]} = \sum_{m=0}^{n} \mathfrak{S}_{n}^{[m]} \varkappa^{m}$$

which defines the "Stirling Numbers of the First Kind."[8]

B. Discussion of the case s = o
In the case s = o equation (5) or (6) takes the form

$$P(n,k,o\,|\,\varepsilon) = \frac{1}{\varkappa^{n}} \sum_{j=0}^{n} (\varkappa - k)^{[j]} C(n,j)$$

The sum equals $(\varkappa-k)^n$ because relation (9) is valid for all $\varkappa$.

$$\left(\frac{\varkappa - k}{\varkappa}\right)^{n} = (1-\varepsilon)^{n}$$

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LISTING OF

$P(N, 4, S/\varepsilon)$

VALUES

P(N,4,S,/ 0.10)

S N	0	1	2	3	4
12	98.81	1.189	0.491D-02	0.820D-05	0.462D-08
11	98.91	1.090	0.410D-02	0.616D-05	0.308D-08
10	99.00	0.992	0.335D-02	0.448D-05	0.196D-08
9	99.10	0.894	0.269D-02	0.314D-05	0.118D-08
8	99.20	0.795	0.209D-02	0.209D-05	0.655D-09
7	99.30	0.696	0.157D-02	0.131D-05	0.328D-09
6	99.40	0.597	0.112D-02	0.749D-06	0.140D-09
5	99.50	0.498	0.748D-03	0.375D-06	0.469D-10
4	99.60	0.399	0.449D-03	0.150D-06	0.937D-11
3	99.70	0.299	0.225D-03	0.375D-07	
2	99.80	0.200	0.750D-04		
1	99.90	0.100			

P(N,4,S,/ 1.00)

S N	0	1	2	3	4
12	88.64	10.895	0.459D 00	0.780D-02	0.446D-04
11	89.53	10.075	0.385D 00	0.589D-02	0.299D-04
10	90.44	9.240	0.318D 00	0.431D-02	0.191D-04
9	91.35	8.389	0.256D 00	0.303D-02	0.115D-04
8	92.27	7.523	0.201D 00	0.204D-02	0.643D-05
7	93.21	6.640	0.152D 00	0.128D-02	0.323D-05
6	94.15	5.742	0.109D 00	0.736D-03	0.139D-05
5	95.10	4.827	0.733D-01	0.370D-03	0.466D-06
4	96.06	3.896	0.443D-01	0.149D-03	0.937D-07
3	97.03	2.948	0.223D-01	0.375D-04	
2	98.01	1.983	0.750D-02		
1	99.00	1.000			

P(N,4,S,/ 2.00)

S N	0	1	2	3	4
12	78.46	19.76	1.702	0.589E-01	0.685E-03
11	80.06	18.44	1.440	0.447E-01	0.461E-03
10	81.70	17.06	1.196	0.329E-01	0.296E-03
9	83.37	15.63	0.971	0.233E-01	0.179E-03
8	85.07	14.14	0.767	0.157E-01	0.100E-03
7	86.81	12.59	0.584	0.998E-02	0.509E-04
6	88.58	10.98	0.423	0.577E-02	0.220E-04
5	90.39	9.31	0.286	0.292E-02	0.742E-05
4	92.23	7.58	0.174	0.118E-02	0.150E-05
3	94.11	5.79	0.088	0.300E-03	
2	96.04	3.93	0.030		
1	98.00	2.00			

P(N,4,S,/ 5.00)

S N	0	1	2	3	4
12	54.03	36.70	8.454	0.776E 00	0.237E-01
11	56.87	35.18	7.318	0.600E 00	0.162E-01
10	59.87	33.44	6.219	0.450E 00	0.105E-01
9	63.02	31.47	5.168	0.325E 00	0.650E-02
8	66.34	29.25	4.175	0.224E 00	0.370E-02
7	69.83	26.76	3.253	0.144E 00	0.190E-02
6	73.50	23.99	2.414	0.852E-01	0.835E-03
5	77.37	20.90	1.672	0.439E-01	0.285E-03
4	81.45	17.48	1.042	0.181E-01	0.585E-04
3	85.73	13.71	0.541	0.468E-02	
2	90.25	9.56	0.187		
1	95.00	5.00			

P(N,4,S,/ 6.66)

S / N	0	1	2	3	4
12	43.69	41.35	13.202	1.671	0.07008
11	46.81	40.25	11.575	1.307	0.04829
10	50.16	38.85	9.965	0.991	0.03177
9	53.74	37.12	8.389	0.723	0.01971
8	57.58	35.03	6.866	0.503	0.01132
7	61.69	32.55	5.419	0.328	0.00585
6	66.10	29.62	4.073	0.195	0.00259
5	70.82	26.21	2.858	0.102	0.00089
4	75.88	22.26	1.805	0.042	0.00018
3	81.30	17.73	0.950	0.011	
2	87.11	12.55	0.333		
1	93.33	6.66			

P(N,4,S,/ 8.00)

S / N	0	1	2	3	4
12	36.76	43.30	17.120	2.672	0.13767
11	39.96	42.66	15.167	2.108	0.09550
10	43.43	41.69	13.194	1.612	0.06325
9	47.21	40.33	11.223	1.187	0.03950
8	51.32	38.54	9.282	0.832	0.02285
7	55.78	36.25	7.403	0.547	0.01189
6	60.63	33.40	5.623	0.329	0.00530
5	65.90	29.92	3.987	0.173	0.00184
4	71.63	25.74	2.544	0.072	0.00038
3	77.86	20.75	1.353	0.019	
2	84.64	14.88	0.480		
1	92.00	8.00			

P(N,4,S,/10.00)

S / N	0	1	2	3	4
12	28.24	43.97	22.823	4.644	0.30985
11	31.38	44.15	20.539	3.710	0.21709
10	34.86	43.96	18.150	2.874	0.14522
9	38.74	43.33	15.684	2.144	0.09161
8	43.04	42.19	13.178	1.523	0.05352
7	47.82	40.44	10.678	1.014	0.02815
6	53.14	37.98	8.242	0.618	0.01269
5	59.04	34.67	5.938	0.329	0.00445
4	65.61	30.39	3.850	0.140	0.00093
3	72.90	24.98	2.081	0.037	
2	81.00	18.25	0.750		
1	90.00	10.00			

P(N,4,S,/13.33)

S / N	0	1	2	3	4
12	17.95	41.14	30.999	9.043	0.8546
11	20.71	42.64	28.644	7.379	0.6086
10	23.90	43.84	25.993	5.841	0.4139
9	27.58	44.62	23.068	4.451	0.2655
8	31.82	44.87	19.908	3.232	0.1577
7	36.72	44.41	16.571	2.201	0.0844
6	42.37	43.07	13.140	1.370	0.0387
5	48.89	40.61	9.726	0.747	0.0138
4	56.41	36.77	6.481	0.325	0.0029
3	65.09	31.21	3.600	0.088	
2	75.11	23.55	1.333		
1	86.66	13.33			

P(N,4,S,/20.00)

S N	0	1	2	3	4
12	6.87	29.40	39.999	20.430	3.2889
11	8.58	32.57	39.013	17.399	2.4189
10	10.73	35.80	37.382	14.380	1.6999
9	13.42	38.95	35.042	11.448	1.1275
8	16.77	41.88	31.955	8.687	0.6931
7	20.97	44.34	28.114	6.185	0.3839
6	26.21	46.00	23.571	4.029	0.1824
5	32.76	46.41	18.455	2.298	0.0674
4	40.96	44.96	13.012	1.050	0.0150
3	51.20	40.85	7.650	0.300	
2	64.00	33.00	3.000		
1	80.00	20.00			

P(N,4,S,/40.00)

S N	0	1	2	3	4
12	0.21	4.66	25.928	46.247	22.9410
11	0.36	6.45	29.988	44.721	18.4688
10	0.60	8.88	34.155	42.100	14.2587
9	1.00	12.11	38.152	38.300	10.4287
8	1.67	16.34	41.563	33.319	7.0968
7	2.79	21.74	43.800	27.288	4.3679
6	4.66	28.39	44.101	20.520	2.3160
5	7.77	36.12	41.579	13.559	0.9599
4	12.96	44.20	35.400	7.200	0.2400
3	21.60	50.80	25.200	2.400	
2	36.00	52.00	12.000		
1	60.00	40.00			

COMPUTER PROGRAM FOR SIMILAR LISTINGS.

```
C************************************************************************
C                                                                       *
C             PROGRAM DESCRIPTION  P(N,K,S,/$)                          *
C                                                                       *
C     THE VARIABLE  NPAR  CORRESPONDS TO  N                             *
C     THE VARIABLE  NKOIN CORRESPONDS TO  S                             *
C     THE VARIABLE  NDE   CORRESPONDS TO  K                             *
C     THE VARIABLE  KAPPA  IS THE RATIO OF  K/$       (SEE PUBLICATION) *
C                                                                       *
C     LIMITATIONS                                                       *
C     THE PROGRAM IS LIMITED TO VALUES OF N=12 AND S=4 (MAX.) .         *
C                                                                       *
C     INPUT CARD SEQUENCE                                               *
C     FOR NKOIN > 0 ALL THE POSSIBLE CASES FROM S=1 TO S=NKOIN          *
C     ARE CALCULATED.                                                   *
C     THE CASE NKOIN=0 NEEDS A SEPERATE CARD.                           *
C     THIS CARD HAS TO BE PRIOR THE CARD NKOIN > 0 .                    *
C     A BLANK CARD INDICATES THAT PRINT-OUT IS ASKED.                   *
C     THE LAST CARD OF THE WHOLE JOB MUST CONTAIN A NEGATIVE NUMBER     *
C     IN COLUMNS 1-5 .                                                  *
C                                                                       *
C************************************************************************
C
C
C          SET PARAMETER , CLEAR MEMORY
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION AMAT(12,5),NPART(12)
  500 DO 2 KI=1,5
      DO 3 JI=1,12
    3 AMAT(JI,KI)=0
    2 CONTINUE
      DO 4 JI=1,12
    4 NPART(JI)=0
      KRO=0
   50 READ(5,1) NPAR,NKOIN,KAPPA,NDE
    1 FORMAT(4I5)
      NCHEC=1
      PAR1=1.0
      KK=1
      KJ=0
      IF(NPAR)70,60,51
   51 IF(NKOIN) 41,41,42
C
C          START CALCULATION
C
   42 DO 30 KK=1,NKOIN
      NCHEC=2
      VAL1=1.0
      NLOW=NDE-KK+1
      NUP=NDE
      DO 10 I=NLOW,NUP
   10 VAL1=VAL1*DFLOAT(I)
      VAL2=FAK(KK)
      PAR1=VAL1/VAL2
      KJ=KK
   41 PROBA=0.0
      DO 20 J=KK,NPAR
      PAR2=0.0
      A1=FAK(J)
      A2=1.0
      NLOW=KAPPA-NDE-J+KJ+1
      NUP=KAPPA-NDE
```

```
      IF(NLOW) 20,20,27
   27 IF(NLOW-NUP) 23,23,24
   23 DO 21 I=NLOW,NUP
   21 A2=A2*DFLOAT(I)
   24 JS=J-KJ
      A3=FAK(JS)
      A14=DFLOAT(KAPPA)
      A4=(1./A14)**NPAR
      PAR2=PAR2+A1*A2*A4/A3
      BRES=0.0
      PAR3=0.0
      B11=DFLOAT(J)
      B1=B11**(NPAR-1)
      IB2=J-1
      B2=FAK(IB2)
      PAR3=PAR3+B1/B2
      IEND=J-1
      IF(IEND) 25,25,22
   22 DO 25 I=1,IEND
      IB12=J-I
      B22=DFLOAT(IB12)
      B23=B22**(NPAR-1)
      B2=FAK(I)
      IB3=J-1-I
      B3=FAK(IB3)
      ISIG=(-1)**I
      BRES=DFLOAT(ISIG)*B23/(B2* B3)
   25 PAR3=PAR3+BRES
      PARAM=PAR2*PAR3
      PROBA=PROBA+PAR1*PARAM
   20 CONTINUE
      IF(NCHEC-1)53,53,52
   53 KCO=1
      KRO=KRO+1
      GO TO 54
   52 KCO=KCO+1
   54 AMAT(KRO,KCO)=PROBA
      NPART(KRO)=NPAR
      IF(NCHEC-1) 50,50,30
   30 CONTINUE
      GO TO 50
C
C            END CALCULATION
C
   60 DO 210 KI=1,5
      DO 211 JI=1,12
  211 AMAT(JI,KI)=AMAT(JI,KI)*100.
  210 CONTINUE
      WRITE(6,5)
      WRITE(6,101)
      WRITE(6,110)
      WRITE(6,102) AMAT(12,2)
      WRITE(6,110)
      WRITE(6,101)
      WRITE(6,111)
      WRITE(6,103)
      WRITE(6,104)
      WRITE(6,101)
      WRITE(6,110)
      WRITE(6,110)
      DO 300 I=1,12
      JMAK=0
      DO 301 JL=1,5
```

```
      IF(AMAT(I,JL))307,307,301
  301 JMAK=JMAK+1
  307 IF(JMAK-4) 304,303,302
  304 IF(JMAK-3) 306,305,305
  302 WRITE(6,108) NPART(I),(AMAT(I,J),J=1,JMAK)
      GO TO 310
  303 WRITE(6,109) NPART(I),(AMAT(I,J),J=1,JMAK)
      GO TO 310
  305 WRITE(6,107) NPART(I),(AMAT(I,J),J=1,JMAK)
      GO TO 310
  306 WRITE(6,106) NPART(I),(AMAT(I,J),J=1,JMAK)
  310 WRITE(6,110)
  300 CONTINUE
      WRITE(6,110)
      WRITE(6,101)
      GO TO 500
   70 CALL EXIT
    5 FORMAT(1H1////)
  101 FORMAT(T40,'------------------------------------------------------
     1----------------------')
  111 FORMAT(T40,'I  S I          I          I               I
     1     I               I')
  110 FORMAT(T40,'I',T115,'I')
  102 FORMAT(T40,'I',29X,'P(N,4,S,/',F5.2,')',T115,'I')
  103 FORMAT(T40,'I    I    0     I    1     I       2       I      3
     1     I       4       I')
  104 FORMAT(T40,'I N  I          I          I               I
     1     I               I')
  108 FORMAT(T40,'I',I3,1X,'I',F7.2,3X,'I',F8.3,2X,'I',E12.3,3X,'I',E12.
     13,3X,'I',E12.3,3X,'I')
  109 FORMAT(T40,'I',I3,1X,'I',F7.2,3X,'I',F8.3,2X,'I',E12.3,3X,'I',E12.
     13,3X,'I',T115,'I')
  107 FORMAT(T40,'I',I3,1X,'I',F7.2,3X,'I',F8.3,2X,'I',E12.3,3X,'I',T99,
     1'I',T115,'I')
  106 FORMAT(T40,'I',I3,1X,'I',F7.2,3X,'I',F8.3,2X,'I',T83,'I',T99,'I',T
     1115,'I')
      END
```

```
      FUNCTION FAK(I)
      REAL*8 FAK
      IF (I)1,2,3
  1   WRITE(6,100)I
 100  FORMAT(1H0, ' NO FACTORIAL FOR NEG. NUMBER  I=',I5)
      RETURN
  2   FAK=1.
      RETURN
  3   FAK=1.
      DO 4 N=1,I
  4   FAK=FAK*DFLOAT(N)
      RETURN
      END
```

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>
> Alfred Nobel

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