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ALBEDO TECHNIQUES FOR CALCULATING THE RADIATION TRANSPORT THROUGH VOIDS

by

B. CHINAGLIA
(SORIN)

(Topical Report)

1972

Contract No. 008-61-12 PNII

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The mathematical solution of the integral equation giving the reflected current has been developed for some geometries of interest in practical cases: cylindrical ducts with or without reflecting ends, and annular gap between two concentric cylindrical surfaces.
Experimental tests on ducts in water medium have shown that in this case the space and energy distribution of the neutron flux may be adequately described with few neutron groups and the maximum absolute error was below a factor 2 for an attenuation of the order of 10^{-4}.

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ABSTRACT

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KEYWORDS

NEUTRONS
TRANSPORT THEORY
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NEUTRON FLUX
DIFFUSION
ANGULAR DISTRIBUTION
INTERFACES
ANALYTICAL SOLUTION
INTEGRAL EQUATIONS
CYLINDERS
ANNULAR SPACE
WATER
MEASURED VALUES
ENERGY
SPECTRA
GROUP THEORY
ERRORS
ATTENUATION

Index

1. Introduction *)

The problem of calculating the radiation transport in a complex system such as the shield of a reactor has two main difficulties: the first one is the description of the physical processes with a calculation procedure able to give correct answers also for deep penetration, the second one is the heterogeneity and complexity of the shield geometry. Actually no rigorous method can account at the same time for both effects: from one side the interaction of radiation with matter (absorption, energy degradation, scattering) and from the other side the influence of the complexity of the geometrical system.

It is therefore necessary to divide the calculation in two steps: firstly the shield geometry is simplified in such a way that a method equivalent to the solution of the transport equation can be applied. All the heterogeneities which cannot be described by the first method are then treated with some calculations which make use of somewhat simplified laws for the physical process but take into better account the geometrical effects. These heterogeneities are generally defined as irregularities.

In the present report some procedures for the evaluation of irregularities are described. They have been developed for a typical nuclear plant of a pressurized water reactor for ship propulsion.

The reference shield design is shown in Fig. 1 and we assume to use the following calculation codes for the regular shield:

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*) Manuscript received on January 24, 1972

a) Sabine /1/: monodimensional removal diffusion code for neutron transport

b) QAD /2/: point Kernel integration code, 3 dimensions, for γ and neutron transport.

Inspection of Fig. 1 shows that two kinds of heterogeneities are present:

a) heavy metal (iron, lead) and hydrogenous material (water, polyethylene) laminations

b) voids

and it is easy to verify that only the voids require a treatment separate from the regular shield calculation.

To prove this, at least in a qualitative way, one can consider a typical heterogeneous configuration, for instance the one shown in Fig. 2 a) or 2 b). In these cases Sabine can be used for calculating the flux arising from source S along the points lying on a symmetry axis r if:

- for case 2 a) the extension of S in a direction perpendicular to r is greater than the diffusion length in the various media A, B, C ....
- for case 2 b), if, in addition to the above condition, also the distance from the axis r to the surface between B and C is sufficiently great.

With QAD one could calculate everywhere and exactly the uncollided flux which however is not sufficient, or the total flux which is affected by errors arising from the difference between the actual geometry and the geometry implicit in the point kernel

attenuation function. This latter (both for γ and neutrons) is derived from the solution of the transport equation in a homogeneous reference medium M for a point isotropic source. Let us assume that a criterion exists for converting the paths in media A, B, C.... to equivalent paths in medium M (for instance based on the removal cross section); a strong difference may exist between the actual geometry and its equivalent used by QAD as shown in Fig. 2 c) and 2 d). In particular for a source

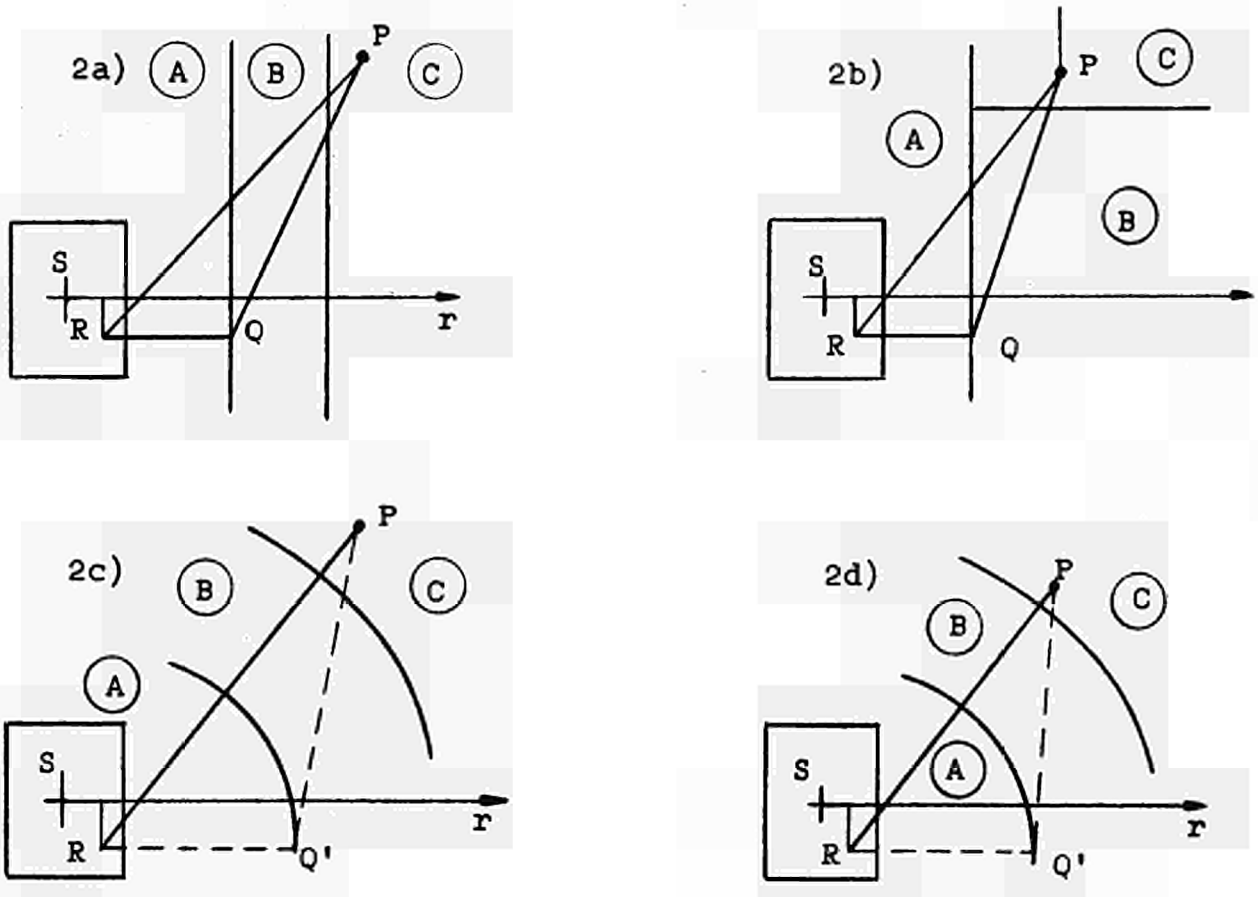

Fig. 2 - Examples of irregular shield geometries (2a and 2b) and of the geometry implicitely assumed in a point Kernel integration calculation (2c and 2d).
S: source; A,B,C: different material compositions;
r: symmetry axis.

element around R the importance of the collisions around point Q to the flux in P may be quite different. The difference however is negligible if the media A, B, C... have similar properties (scattering, absorption) and in this case the situation of Fig. 2 is considered "regular" and the use of Sabine or QAD is justified.

But if one medium, for instance B in Fig. 2, has a negligible density (air, thermal insulation) it may happen that the flux in P derives almost completely from contributions of the type RQP which are much greater than the contributions R Q' P implicitely assumed in the calculation. This situation is therefore an irregularity to be treated with some other method.

In the following a simple approach to the problem of radiation transport in voids is presented; this approach basically makes use of the concepts of "current" emerging from the medium into the void and "albedo" for the reflection on the walls.

Particular solutions are obtained for the cases of the reference design of Fig. 1 (voids enclosed by cylindrical surfaces) and for cases for which experimental data to be used for comparison exist (cylindrical ducts).

A computation method which takes into account the multiple reflections inside a cylinder has been developed in the program MRC-1 by P. Novario and is described in detail in a separate report [13].

With the albedo approach one has to determine:

a) the values of the albedo α

b) the initial current between energy E_1 and E_2 entering the void (the term initial means that this current is made up of particles which have never been reflected before within the considered energy group)

c) the total current which is obtained as the sum of the initial and reflected current

d) the flux in the wanted points in the void volume, obtained by integration of elementary contributions coming from the current entering the void at any point of the surface multiplied by geometrical factors.

These points are considered separately.

2. The albedo

The albedo is the ratio of a quantity R reflected from a surface to a quantity I incident on the surface. The quantities of interest are the current and the flux of particles and the corresponding albedo is called "number albedo". "Dose" and "energy" albedo are obtained by weighing the particle flow quantities by k(E) (the flux to dose conversion factor) or by E (the energy). It is assumed that both incidence and reflection occur at the same point; this is not strictly true, but the error arising from this assumption is negligible if one of the following conditions are fulfilled:

a) the diffusion length L in the medium is much smaller than the dimension of the void in a direction normal to the main propagation of particles in the void (for instance: the radius for a long cylinder having the source at its mouth, or the width perpendicular to the radiation streaming for a gap)

b) the flux in the void has a small variation along the main propagation direction in a distance L; this small variation may result from a uniform source distribution over the whole surface or from a high albedo value which causes the multiple reflections distribute uniformly the radiation.

A review of definitions and values of albedos is given by Selph /3/; from his report the following data of interest to this work are derived.

The symbols of Selph are used in reporting these albedoes:

α = differential albedo giving the reflected quantity per unit solid angle and/or per unit energy interval

A = total albedo (integral of α over all the reflection angles and the energies).
These are "number" albedoes; "dose albedoes" are indicated by index D.

Indexes 1, 2, 3 define:

α_1, A_1 = current out/incident flux

α_2, A_2 = current out/incident current

α_3, A_3 = flux out/incident flux.

The relationship correlating the three types of albedoes are:

$$\alpha_1 = \cos\vartheta_o \alpha_2 = \alpha_3 \cos\vartheta \qquad (2.1)$$

where ϑ_o and ϑ are the incident and reflection angles with respect to the normal to the surface.

Fast neutrons - Monte Carlo calculation

For fast neutrons the more detailed information is given by the doubly differential albedoes of Maerker and Muckentaler [4] for concrete. They contain all the information on the reflection properties: energy and direction of the reflected current for an incident current at a given energy and angle (a total of 5 variables). An expression fitting these data is given for $\alpha_{D2}(E_o,\vartheta_o,\vartheta,\Phi)$ where E_o = incident energy, Φ azimutal angle of reflection.

The data of Allen Futterer [5] refer to iron, water, concrete polyethylene and soil and are dose albedoes $\alpha_{D1}(E_o,\vartheta_o,\vartheta)$. A fitting is given by French and Wells [6] in the form:

$$\alpha_{D2}(E_o,\vartheta_o,\vartheta) = k(E_o)\cos^{\frac{1}{3}}\vartheta_o \cos\vartheta \qquad (2.2)$$

where $k(E_o)$ is a constant for a given material. This expression does not hold for water, in which case the dependence on ϑ_o is smaller.

Solutions for the monoenergetic case (thermal neutrons)

Many expressions have been derived for cases of isotropic scattering and constant cross section, which can be applied to thermal neutrons.

In the following $N = \Sigma_t/\Sigma_a$ and $p = \Sigma_s/\Sigma_t$; Σ_t, Σ_s and Σ_a are the total, scattering and absorption cross sections.

Fermi

$$A_2(\vartheta_o) = \frac{\sqrt{N}-1}{\sqrt{N}+\sqrt{3}\cos\vartheta_o} \tag{2.3}$$

for $N \gg 1$

Halpern

$$A_2 = \frac{\sqrt{N}-k}{\sqrt{N}} \tag{2.4}$$

for $N \gg 1$, where $k = 2.91$, 2.31 and 2.48 for normal, isotropic and cosine angular distribution of incidence.

Chandrasekhar

$$\alpha_2(\vartheta_o,\vartheta,p) = \frac{p}{2}\,\frac{\cos\vartheta}{\cos\vartheta+\cos\vartheta_o}\,x$$

$$x\ H(p,\cos\vartheta_o)H(p,\cos\vartheta) \tag{2.5}$$

where H = universal function tabulated for various values of p and $\cos\vartheta_o$ [7].

$$A_2(\vartheta_o,p) = 1-\sqrt{1-p}\cdot H(p,\cos\vartheta_o) \tag{2.6}$$

Pomraning

normal incidence:

$$A_2 = \frac{2}{(1+\nu)\ln(1-\nu^2)} \ln(1+\nu) - \nu \qquad (2.7)$$

isotrop. incidence:

$$A_2 = \frac{-4}{\nu^2 \ln(1-\nu^2)} \left[\ln(1+\nu) - \nu\right]^2$$

where ν (positive) satisfies:

$$\frac{2\nu}{p} = \ln\left(\frac{1+\nu}{1-\nu}\right)$$

Diffusion (Glasston and Edlund):

$$A_2 = \frac{i^-}{i^+} \qquad (2.8)$$

where $i^+ = \frac{\Phi}{4} - D\frac{\Phi'}{2}$

$$i^- = \frac{\Phi}{4} + D\frac{\Phi'}{2}$$

The above expressions refer to reflection from a semiinfinite half-space.

For strongly absorbing media and slabs of finite thickness x

Mockel

$$A_2 = \frac{1-e^{-2\alpha x}}{1-A\,e^{-2\beta x}}\left[\frac{1-f(p)}{1+f(p)} + g(p)\right] \qquad (2.9)$$

where $\alpha = 1.37 \cdot y^{0.44}$ $(y = 1-p)$ and the other parameters are given below for various distributions of the incident current:

Incidence	β	A	f(p)	g(p)
Normal	$1.37 \cdot y^{0.44}$	0.2775	$y^{0.388}$	$0.067\ p^{4.48}$
Isotr.	$1.33 \cdot y^{0.3675}$	0.640	$y^{0.5}$	0
Cos.	$1.37 \cdot y^{0.44}$	0.3882	$y^{0.41}$	$0.05\ p^{3.33}$

For the general treatment of the transport in a void an albedo of the type

$$\alpha_2(E_o, \vartheta_o;\ E, \vartheta, \Phi)$$

should be required. As seen in the above analysis of existing data for most of the albedo the dependence on one or more variables is not known. Furthermore the complexity of a function

of 5 variables would make impossible to solve by analytical methods the multiple reflection problem.

Some simplifications are therefore necessary and it is important to assess for any particular problem what simplifying assumptions can be made.

For our problem we assume that the reflected current:

a) has no dependence on the angle Φ

b) has always a $\cos\vartheta$ distribution

c) has a $\cos\vartheta_o$ distribution, so that the dependence of α on the true incidence angle must be taken into account only for the first reflection, whilst in the other cases a constant albedo evaluated for cosine incidence may be used.

Some justification for points a) and b) is derived from results of Ref. 5 and 6. The difference between the first and further reflections in the ϑ_o dependence can be better understood by the following considerations. The initial current giving rise to the first reflection may have any arbitrary distribution of incidence, whilst the current of particles which have already suffered one or more reflections with a $\cos\vartheta$ law of emission enters the medium with a distribution approaching a $\cos\vartheta_o$ law when the principal curvatures of the surface are constant (i.e. cylinders, spheres, parallel planes).

Furthermore a strong initial anisotropy is expected for some geometries only for high energy neutrons; since the albedo at high energies is usually low a wrong assumption for the second reflection has little importance. Low energy neutrons on the other hand derive from diffusion and previous reflections and

the $\cos\vartheta_o$ law is approached more rapidly.

The albedo needed is therefore of the form:

- $\alpha_2(E_o,E;\vartheta_o)\cos\vartheta$ for the first reflection, and
- $\alpha_2(E_o,E)\cos\vartheta$ for the second and further reflections, and the distribution of the incident current is close to $\cos\vartheta_o$.

We assume, following the procedure of Miller /8/, that $\alpha_2(E_o,E)$ can be derived from the diffusion theory.

In multigroup notation we write $\alpha_{g,j}$ the value of $A_2(E_o,E)$ giving the ratio of the current reflected in the energy group j at energy E to the incident current in the group g at energy E_o ($g<j$, being $E_o>E$).

Simple forms of $\alpha_{g,j}$ are:

Plane geometry:

$$\alpha_{1,1} = \frac{1-2k_1D_1}{1+2k_1D_1}$$

$$\alpha_{1,2} = \frac{4\Sigma_1}{(k_1+k_2)(1+2k_1D_1)(1+2k_2D_2)}$$

$$\alpha_{1,3} = \frac{\Sigma_2}{D_2}\cdot\frac{1+2D_2(k_1+k_2+k_3)}{(k_2+k_3)(k_1+k_3)(1+2k_3D_3)}\ \alpha_{1.2}$$

Cylindrical geometry (radius = a)

$$\alpha_{1,1} = \frac{K_o(k_1 a) - 2 k_1 D_1 \cdot K_1(k_1 a)}{K_o(k_1 a) + 2 k_1 D_1 \cdot K_1(k_1 a)} \qquad (2.11)$$

where Σ_g, k_g, D_g have the usual meanings in the diffusion theory and K_o and K_1 are the zero and first order modified Bessel functions.

In other cases a diffusion code for with the appropriate boundary conditions:

i_g^+ assigned

$i_j^+ = 0 \quad (j > g)$

can easily give the wanted values

$$\alpha_{g,j} = \frac{i_j^-}{i_g^+}$$

for a particular geometry and material composition.

According to Miller [8] this method produces values in good agreement with more elaborate (Monte Carlo) techniques.

The initial current

From the above definition the initial current in a given energy interval includes only those neutrons which have not been previously reflected with an incident energy belonging to the considered energy interval.

In a multigroup notation we can write:

$$i_g^o = i_g^* + \sum_{j=1}^{j=g-1} \alpha_{j,g}\, i_j^+ \qquad (3.1)$$

where:

i_g^o = initial current entering the void in group g (g=1 for the higher energy group)

i_g^* = current of neutrons of group g which enter the void and which have never crossed the void

$\alpha_{j,g}$ = albedo from group j to group g (current of neutrons emerging from medium in group g per unit current entering the medium in group j)

i_j^+ = total current entering the medium in group j.

The space dependence has been omitted in (3.1) where it is assumed that all the quantities refer to the same point.

The second term in the right hand side of (1) represents the contribution of those neutrons which have already crossed the void within a group j ($E_j > E_g$). This contribution is evaluated from the solution of the integral equation (4.1) for the

groups $j<g$, starting from the first group for which:

$$i_1^* = i_1^o$$

It is therefore necessary to define a criterion for calculating i_g^* for all the groups and all the points of the surface between the void and the medium. In addition it should be necessary to know the angular distribution of i_g^*. Both quantities depend on the properties of the medium and on the source spectrum and source position with respect to the void.

Only stochastic methods can give the solution to this problem in the general case, but they must be ruled out for practical calculations which require a simpler approach.

In the following some approximations for i_g^* are firstly reviewed and some refinements are proposed. The basis for obtaining the approximate formula is to consider that the void acts as a perfect absorber for the wanted current, since no reflected neutron must be included.

According to Channon and Seale [9] the initial current i_{th}^* of thermal neutrons entering the cylindrical surface of a duct in water having a plane source at the mouth is given by:

$$i_{th}^*(z) = \frac{1}{2} \Phi_{th}^o(z) \left(1 - \frac{k_o(k\,a)}{k_o(k\,a')}\right) \qquad (3.2)$$

where:

Φ^{o}_{th} = unperturbed flux in the medium (the flux which would exist at the considered point in the absence of the void and which can be calculated considering the void filled with the medium material)

K_o = modified Bessel functions

k = inverse of the diffusion length

a = duct radius

a' = extrapolated radius taken from transport theory results.

For the same geometrical configuration Paratte [10] gives:

$$i^{*}_{g}(z) = 0.5 \cdot \Phi^{o}_{g}(z) \cdot \left[1 - \frac{1}{\Phi^{o}_{g}(z)} \sum_{k=1}^{g} \alpha_{k,g} \Phi^{o}_{k}(z)\right] \qquad (3.3)$$

Miller [8] derives a similar expression as:

$$i^{*}_{g}(p) = \frac{\Phi^{o}_{g}(p)}{4} (1 - \alpha_{g,g}) + \frac{1}{2} J^{net}_{g}(p) \, (1+\alpha_{g,g}) \qquad (3.4)$$

where:

$\alpha_{g,g}$ = total current albedo describing the probability of group g neutrons being reflected within group g

Φ_g^o = unperturbed flux of group g

J_g^{net} = unperturbed net current of group g.

The two quantities Φ_g^o and J_g^{net} are obtained with a removal diffusion calculation. It is easy to verify that if $\alpha_{g,g}$ is obtained from the diffusion theory (equation 2.10) the relationship (4) is equivalent to:

$$i_g^* = J_g^{net} (1+\alpha_{g,g}) = \frac{\Phi_g^o}{2} (1 - \alpha_{g,g}) \qquad (3.5)$$

All the above expressions imply that:

a) the P1 approximation is valid for the neutron energy considered

b) the unperturbed flux Φ^o calculated in the absence of the void is a good approximation for the real flux existing in the medium and consisting of neutrons which have never crossed the void (these latter are already taken into account by the second term of equation (3.1).

The first assumption (a) is scarcely valid for high energy neutrons.

The second assumption (b) is justified for points which see the source volume through the medium under a solid angle much greater than through the void. If for instance the void is a cylindrical duct with the source at the mouth, the real cases will be between two extreme geometrical configurations: the first consisting of a source which extends to infinity in the radial direction, and the second of a point source on the duct

axis. In the first case the wanted flux approaches the unperturbed value far from the duct and also near the duct if its radius is small with respect to the diffusion length in the medium. In the second case the wanted flux vanishes unless near the mouth and the use of a unperturbed flux would be uncorrect.

Both restrictions may be however removed using a still simple approach based on the use of the removal diffusion theory.

For the first energy groups the flux has two components: removal and diffusion. The removal component is made of neutrons which have a straight line propagation: therefore an integration over the source volume gives the value and the angular distribution of the removal part of i^* with the condition of black body for the void. The remaining part (or the total initial current for the low energy groups) can be obtained with a removal diffusion calculation of the flux in the medium in which the void is a perfect absorber. The removal calculation must be performed with a multidimensional code; for the diffusion part a monodimensional code describing the neutron transport in a direction normal to the surface of interest will be sufficient in most cases. At the interface with the void the black body condition

$$\frac{d\Phi}{dx} \cdot \frac{1}{\Phi} = \frac{1}{l} \qquad (3.6)$$

must be applied (x is a coordinate normal to the surface and l the extrapolation length). The initial current entering

the void and coming from the diffusion flux is then given by:

$$i^* = -D \left(\frac{\partial \Phi}{dx}\right)_{x=o} \qquad (3.7)$$

and it is assumed that its angular distribution is proportional to $\cos\vartheta$.

As a first remark we observe that in the case of the cylindrical duct seen previously and a large source, in place of (3.2) from (3.6) and (3.7) one obtains:

$$i^* = \frac{\Phi^o \cdot kD}{K1 + K_o(ak)/K_1(ak)} \qquad (3.8)$$

The second remark concerns the removal component. As described later the total current is obtained by an iterative method whose first step is the integration of the initial current multiplied by a geometrical factor over the void surface. The angular distribution of the removal component generally is not simple and does not permit an easy evaluation of the integral. This difficulty is easily avoided since the result of the integration is simply the current entering the medium and this quantity can be calculated directly.

4. The total current

4.1. Radiation transport in the void

The equation for the transport in the void is given by Miller /8/ as:

$$i(E,P,\vec{\Omega}) = \int_{E}^{E_m} \int_{2\pi} \alpha(P,E' \to E,\vec{\Omega}' \to \vec{\Omega}) \cdot i^{+}(E',P,\vec{\Omega}') x dE' \, d\vec{\Omega}' + i^{*}(E,P,\vec{\Omega}) \tag{4.1}$$

where:

$i(E,P,\vec{\Omega})$ = current entering the void per unit energy interval at energy E and per unit solid angle about the direction $\vec{\Omega}$ at point P.

$i^{*}(E,P\vec{\Omega})$ = initial current defined in the previous section.

$i^{+}(E',P,\vec{\Omega}')$ = current entering the wall per unit energy interval at energy E' and per unit solid angle about the direction $\vec{\Omega}'$ at point P.

$\alpha(P,E' \to E,\vec{\Omega}' \to \vec{\Omega})$ = doubly differential current albedo (α_2 in the notation of Ref. 3) giving the current reflected at P per unit energy interval at energy E and per unit solid angle about the direction for a unit current of particles incident with energy E' and direction $\vec{\Omega}'$.

The above equation can be transformed in an integral equation containing only $i(E',P\Omega')$ by considering that (see Fig. 3):

$$i^{+}(E',P\vec{\Omega}')d\vec{\Omega}' = i(E',Q,\vec{\Omega}')\frac{\cos\vartheta_P}{|PQ|^2}dS_Q$$

where:

Q = intersection of the straight line through P in direction $\vec{\Omega}'$ with the void wall

ϑ_P = angle between the normal to the surface n(P) at P and the direction PQ

dS_Q = element of surface around Q

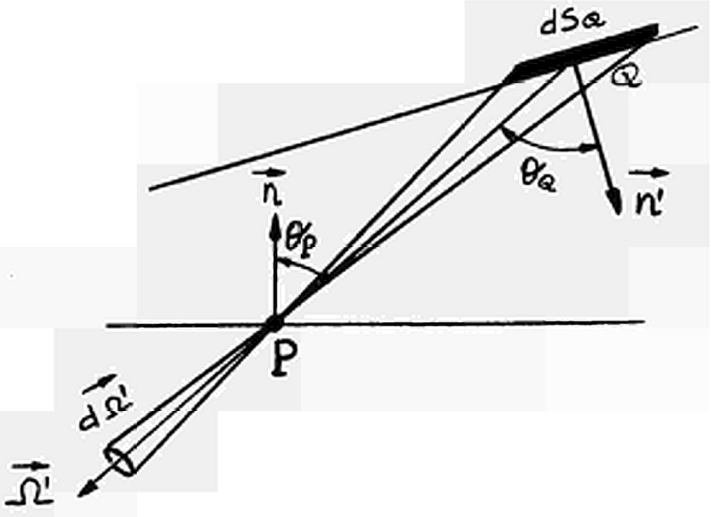

Fig. 3 - Geometrical relationship between i and i^{+}.

By substitution in (4.1) we obtain:

$$i(E,P,\vec{\Omega}) = \int_{E}^{E_m} \int_{S} \alpha(P,E \hookrightarrow E,\vec{\Omega} \hookrightarrow \vec{\Omega}) \cdot i(Q,E',\vec{\Omega}') x$$

$$x\ F(Q,P) dS_Q + i^*(E,P,\vec{\Omega}) \qquad (4.2)$$

where:

$$F(Q,P) = \frac{\cos\vartheta_P}{(|QP|)^2} \qquad (4.3)$$

and where the integral is now extended over the whole surface S between void and medium.

The spectrum is divided in energy groups and with obvious notation the wanted equations are obtained from (4.2) and (3.1):

$$i_g(P,\vec{\Omega}) = \int_S \alpha(P,g \to g,\vec{\Omega} \to \vec{\Omega}) \cdot i_g(Q,\vec{\Omega}')\, x$$

$$x\ F(Q,P) dS_Q + i_g^o(P,\vec{\Omega}) \qquad (4.4)$$

$$i_g^o(P,\Omega) = i_g^*(P,\Omega) + \sum_{j=1}^{j=g-1} \int \alpha(P,j \to g,\vec{\Omega} \hookrightarrow \vec{\Omega})\, x$$

$$x\ i_j(Q,\vec{\Omega}') \cdot F(Q,P) \cdot dS_Q \qquad (4.5)$$

4.2. The solution of the integral equation

The integral equation 4.4 has been firstly derived by Simon and Clifford [11] (SC) in the treatment of thermal neutron streaming in a cylindrical duct. In this case, assuming a cosϑ law of reflection, and constant reflection properties along the surface:

$$\alpha(P, g \to g, \vec{\Omega}' \to \vec{\Omega}) = \frac{\alpha}{\pi} \cdot \frac{|\vec{n}' \cdot \vec{PQ}|}{|\vec{PQ}|} \qquad (4.6)$$

where:

α = total (thermal) albedo (A_2 in the notation of Ref. 3)

$\vec{n}'$ = normal to the surface at Q.

If the radius of the cylinder is R, and z is a coordinate axis coincident with the cylinder axis, the SC formula is easily derived from 4.4

$$i(z) = \frac{\alpha}{2R} \int_0^H i(z')K(\rho)dz' + i^o(z) \qquad (4.7)$$

where:

$\rho = |z - z'|$

α = total albedo

H = total length of the duct

$K(\rho) = 1 - \rho(\rho^2 + 6a^2) \cdot (\rho^2 + 4a^2)^{-3/2}$

$i(z)$ = total current at z.

The SC approximation consists in considering:

$$\int i(z')K(\rho)dz' = i(z)\int K(\rho)dz' = i(z)G(z) \qquad (4.8)$$

and is justified by the shape of $K(\rho)$ which is strongly peaked at $\rho = o$ and by the expected small variation of $i(z)$.

Furthermore, since $G(z) \approx 2R$ along all the duct (if $H \gg a$) unless near $z = o$ and $z = H$, the well known expression

$$i(z) = \frac{1}{1-\alpha} i^{o}(z) \qquad (4.9)$$

is obtained.

Other more detailed analysis of radiation streaming in ducts [9] [12] use the same approximation, whilst Paratte [10] and Miller [8] solve the general equation (4.4)

Since it is not self-evident in what cases the approximation (4.8) may be valid, the solution of (4.4) is discussed briefly in the following. It will be shown that in the cases of interest in this work the integral equation admits always a solution as a convergent series; this series may be truncated and the remainder approximated by an expression of the type of (4.9) which corresponds to the particular case in which only the first term is retained.

Omitting for simplicity the group index g, the solution of (4.4) is given by:

$$i(P,\vec{\Omega}) = \sum_{n=0}^{\infty} T_n(P,\Omega) \qquad (4.10)$$

where:

$$T_o(P,\Omega) = i^o(P,\Omega)$$

$$T_n(P,\Omega) = \int T_{n-1}(Q,\vec{QP}) \cdot F(QP) \cdot \alpha(P,\vec{QP}\rightarrow\vec{\Omega}) dS_Q$$

It is outside the scope of the present discussion to look for the conditions of convergence of (4.10). It will be sufficient to observe that the iterative method used to derive T_n corresponds to the physical process: T_n is the current of particles which have suffered exactly n reflections. Therefore the series (4.10) gives always a finite solution in all the physical cases if the description of the albedo and of the initial current have a physical meaning.

With the hypotesis of sections 2 and 3 the integral equation and the series are greatly simplified and can be handled analytically in simple geometrical cases.

We consider firstly the case of low energy neutrons which have a $\cos\vartheta$ angular distribution as in the SC analysis (eq. 4.6) both for the initial and reflected current, according to the previous hypothesis. Furthermore the dependence of $\alpha(P,\vec{\Omega}'\rightarrow\vec{\Omega})$ on the direction of incidence Ω' is neglected.

Equation 4.4 may be rewritten as:

$$i(P) = i^*(P) + \alpha(P) \int i(Q) \cdot k(QP) \cdot dS_Q \qquad (4.11)$$

where $i(P)$ and $\alpha(P)$ are the total current and albedo and:

$$k(QP) = \frac{\cos\vartheta_P \cdot \cos\vartheta_Q}{|PQ|^2}$$

(ϑ_P and ϑ_Q defined in Fig. 3).

The solution is given by the series (4.10) where

$$T_n(P) = \alpha(P)\int T_{n-1}(Q)k(Q,P)dS_Q \tag{4.12}$$

or by:

$$i(P) = i^o(P)+\alpha(P)\int i^o(Q)\sum_{n=1}^{\infty} k_n(P,Q)dS_Q \tag{4.13}$$

which is obtained from (4.12) by changing the order of integration, where:

$$k_1(Q,P) = k(Q,P)$$

$$k_n(Q,P) = \int k_{n-1}(Q,R)\cdot k(R,P)dS_R$$

The proof for convergence of (4.12) is easily obtained; if α is the maximum value of $\alpha(P)$ and M the maximum value of $i^o(P)$:

$$T_1(P) < \alpha \cdot M \cdot \int k(Q,P)dS_Q \leq \alpha M \tag{4.14}$$

because

$$\int k(Q,P)dS_Q = \int k(P,Q)dS_Q =$$

$$= 1 \quad \text{for a close surface}$$

$$< 1 \quad \text{for an open surface}$$

The other terms $T_2, \ldots . T_n$ are always smaller than $\alpha^2 M, \ldots, \alpha^{n-1} . M$, and the series is always convergent since $\alpha < 1$.

The same reasoning holds for high energy neutrons also in the case of arbitrary distribution of the initial current and of an albedo dependent on the incidence angle, for $n > 2$; the term T_1 can be calculated by the exact expression (4.10) or, in a more approximate but simpler form, by

$$T_1(E,P,\vartheta) = \alpha(E,P,\bar{\vartheta}_o)\cos\vartheta \cdot i^+(E,P) \qquad (4.15)$$

where:

- $i^+(E,P)$ = total current entering the medium at P
- $\bar{\vartheta}_o$ = mean angle of incidence.

4.3. Solutions for particular geometrical cases

In the following the explicit forms of the equation (4.11) are given for the case of cylindrical surfaces and plane surfaces perpendicular to the cylinder axis. A cylindrical symmetry is assumed for the initial current and for the reflection properties.

a) Cylindrical ducts without ends

This is the configuration already considered (equation 4.7).

b) Cylindrical duct with reflecting medium at the entrance and exit ends

This configutarion is shown in Fig. 4. The equations for a single energy group are:

$$J_1(P_1) = J_1^o(P_1) + \lambda_1 \int_{S_2} J_2(P_2) \frac{\cos^2\vartheta_{1.2}}{r_{1.2}^2} dS_2 +$$

$$+ \lambda_1 \int_{S_3} J_3(P_3) \frac{\cos\vartheta_{1.3} \cdot \cos\vartheta_{3.1}}{r_{1.3}^2} dS_3$$

$$J_2(P_2) = J_2^o(P_2) + \lambda_2 \int_{S_1} J_1(P_1) \frac{\cos^2\vartheta_{1.2}}{r_{1.2}^2} dS_1$$

$$+ \lambda_2 \int_{S_3} J_3(P_3) \frac{\cos\vartheta_{2.3} \cdot \cos\vartheta_{3.2}}{r_{2.3}^2} dS_3$$

$$J_3(P_3) = J_3^o(P_3) + \lambda_3 \int_{S_1} J_1(P_1) \frac{\cos\vartheta_{1.3} \cdot \cos\vartheta_{3.1}}{r_{1.3}^2} dS_1$$

$$+ \lambda_3 \int_{S_2} J_2(P_2) \frac{\cos\vartheta_{2.3} \cdot \cos\vartheta_{3.2}}{r_{2.3}^2} dS_2$$

$$+ \lambda_3 \int_{S_3} J_3(P_3') \frac{\cos^2\vartheta_{3.3'}}{r_{3.3'}^2} dS_3 \qquad (4.16)$$

where:

$J_i^o(P_i), J(P_i)$ = initial and total current entering the void at a point on the i^{th} surface ($i = 1,2,3$); the group index has been omitted and $J_i = i_{j,g}$

P_i = point on the i^{th} surface

$\lambda_i = \alpha_i/\pi$

α_i = total albedo of the i^{th} surface

$\vartheta_{i,j}$ = angle between the normal to the surface at P_i and the direction $\vec{P_iP_j}$.

$r_{i,j}$ = distance between points P_i and P_j

i = surface index: $i = 1$ for the mouth, $i = 2$ for the end and $i = 3$ for the cylindrical surface.

After solution of the previous system, the flux $\Phi(P_4)$ at a point P_4 inside the duct may be calculated with the following formula:

$$\Phi(P_4) = \frac{1}{\pi}\int_{S_1} J_1(P_1)\,\frac{\cos\vartheta_{1.4}}{r^2_{1.4}}\,dS_1 + \frac{1}{\pi}\int_{S_2} J_2(P_2)\,\frac{\cos\vartheta_{2.4}}{r^2_{2.4}}\,dS_2 +$$

$$+ \frac{1}{\pi}\int_{S_3} J_3(P_3)\,\frac{\cos\vartheta_{3.4}}{r^2_{3.4}}\,dS_3 \qquad (4.17)$$

If the cylindrical coordinates (ρ, φ, z) shown in Fig. 4 are used, the expressions (4.16) and (4.17) may be transformed in:

$$J_1(\rho_1) = J_1^o(\rho_1) + \alpha_1\int_o^R J_2(\rho_2)F_1(\rho_1,\rho_2)d\rho_2 + \alpha_1\int_o^H J_3(z)F_2(\rho_1,z)dz$$

$$J_2(\rho_2) = J_2^o(\rho_2) + \alpha_2\int_o^R J_1(\rho_1)F_1(\rho_1,\rho_2)d\rho_1 + \alpha_2\int_o^H J_3(z)F_2(\rho_2,H-z)dz$$

$$J_3(z) = J_3^o(z) + \alpha_3\int_o^R J_1(\rho_1)F_3(\rho_1,z)d\rho_1 + \alpha_3\int_o^R J_2(\rho_2)F_3(\rho_2,H-z)d\rho_2 + \alpha_3\int_o^H J_3(z')F_4(z,z')dz' \qquad (4.18)$$

$(0\leq\rho_1\leq R; \quad 0\leq\rho_2\leq R; \quad 0\leq z\leq H)$

where:

$$F_1(\rho_1,\rho_2) = \frac{2\rho_2 H^2(\rho_1^2+\rho_2^2+H^2)}{\left[(\rho_1^2+\rho_2^2+H^2)^2-4\rho_1^2\rho^2\right]^{3/2}} \qquad (4.19\,a)$$

$$F_2(\rho_1,z) = \frac{2zR^2(R^2-\rho_1^2+z^2)}{\left[(R^2-\rho_1^2)^2+z^4+2z^2(R^2+\rho_1^2)\right]^{3/2}} \qquad (4.19\,b)$$

$$F_3(\rho_1,z) = \frac{\rho_1}{R}F_2(\rho_1,z) \qquad (4.19\,c)$$

$$F_4(z,z') = \frac{1}{2R}\left\{1-\frac{|z-z'|\left[(z-z')^2+6R^2\right]}{\left[(z-z')^2+4R^2\right]^{3/2}}\right\} \qquad (4.19\,d)$$

ρ_1 = radial coordinate on the mouth face

ρ_2 = radial coordinate on the end face

z = axial coordinate

The flux is given by:

$$\Phi(z,\rho) = \int_0^R J_1(\rho_1)\Psi_1(\rho_1,z)d\rho_1 + \int_0^R J_2(\rho_2)\Psi_1(\rho_2,H-z)d\rho_2$$

$$+\int_0^H J_3(z')\Psi_2(z,z')dz' \qquad (4.20)$$

where:

$$\Psi_1(\rho,\rho_1,z) = \frac{2z}{\pi}\,\rho_1 \int_0^{\pi} \frac{d\varphi}{\left[\rho^2+\rho_1^2-2\,\rho\rho_1\cos\varphi+(z^2)^2\right]^{3/2}} \qquad (4.21\ a)$$

$$\Psi_2(\rho,z,z') = \frac{2R}{\pi} \int_0^{\pi} \frac{(R-\rho\cos\varphi)\,d\varphi}{\left[R^2+\rho^2-2R\rho\cos\varphi+(z'-z)^2\right]^{3/2}} \qquad (4.21\ b)$$

The previous expressions Ψ_1 and Ψ_2 for the important case of points P_4 on the cylinder axis ($\rho = 0$) take the simple form:

$$\Psi_1(0,\rho_1,z) = \frac{2\,z\,\rho_1}{\left[\rho_1^2+z^2\right]^{3/2}} \qquad (4.22\ a)$$

$$\Psi_2(0,z,z') = \frac{2\,R^2}{\left[R^2+(z'-z)^2\right]^{3/2}} \qquad (4.22\ b)$$

The integral equations (4.18) are solved with the MRC-1 program which is described in Ref. [13]. The duct without ends previously seen is a particular case with $\alpha_1 = \alpha_2 = 0$.

c) Two concentric cylindrical surfaces

This configuration is shown in Fig. 5. It represents, for instance, the gap between the pressure vessel and the shield tank in a typical shield design. The current on the two surfaces is given by:

$$J_1(z) = J_1^o(z) + 2\lambda_1 \int_o^H J_2(z') \cdot F_5(z,z') \frac{dz'}{R_1}$$

$$J_2(z) = J_2^o(z) + 2\lambda_2 \int_o^H J_1(z') \cdot F_6(z,z') \frac{dz'}{R_1} +$$

$$+ \lambda_2 \int_o^H J_2(z') \cdot F_7(z,z') \frac{dz'}{R_1} \qquad (4.23)$$

where the same symbols of the previous case have been used with:

i = index of the surface:

i = 1: inner cylinder of radius R_1

i = 2: outer cylinder of radius R_2.

The dimensionless F function are given by:

$$F_5(z,z') = \frac{R_2}{R_1} \cdot F_6(z,z')$$

$$F_6(z,z') = \int_o^{\varphi_o} \frac{(R_2 \cos\varphi - R_1)(R_2 - R_1 \cos\varphi) \cdot R_1^2 \cdot d\varphi}{\left[R_1^2 + R_2^2 - 2 R_1 R_2 \cos\varphi + (z'-z)^2\right]^2}$$

$$F_7(z,z') = \int_o^{2\varphi_o} \frac{(1-\cos\varphi)^2 R_2^3 R_1 \cdot d\varphi}{\left[2 R_2^2 (1-\cos\varphi) + (z-z')^2\right]^{3/2}} \qquad (4.24)$$

where the limit φ_o is determined by:

$$\cos \varphi_o = \frac{R_1}{R_2}$$

The flux $\Phi_1(z)$ and $\Phi_2(z)$ on the wall surface is given by:

$$\varphi_1(z) = \frac{2 R_2}{\pi R_1} \int_o^H J_2(z') \cdot G_2(z,z') \frac{dz'}{R_1} \qquad (4.25\,a)$$

$$\varphi_2(z) = \frac{2}{\pi} \int_o^H J_1(z') \cdot G_2(z,z') \frac{dz'}{R_1} \qquad (4.25\,b)$$

$$+ \frac{2}{\pi} \int_o^H J_2(z') \cdot G_1(z,z') \frac{dz'}{R_1}$$

where the dimensionless G functions are:

$$G_1(z,z') = \int_o^{2\varphi_o} \frac{(1-\cos\varphi)\, R_2^2 \cdot R_1}{\left[2 R_2^2 (1-\cos\varphi) + (z'-z)^2\right]^{3/2}} \, d\varphi \qquad (4.26\,a)$$

$$G_2(z,z') = \int_o^{\varphi_o} \frac{(R_2 \cos\varphi - R_1) R_1^2 \, d\varphi}{\left[R_1^2 + R_2^2 - 2 R_1 R_2 \cos\varphi + (z'-z)^2\right]^{3/2}} \qquad (4.26\,b)$$

From the system (4.23) the integral equation for $J_2(z)$ is obtained:

$$J_2(z) = g(z) + \lambda \int_o^H J_2(z') \cdot F_8(z,z') dz' \qquad (4.27)$$

where:

$$g(z) = J_2^o(z) + \lambda \int_o^H J_1^o(z') \cdot F_6(z,z') \frac{dz'}{R_1} \tag{4.28}$$

$$F_8(z,z') = F_7(z,z') + \lambda \int_o^H F_5(z,z'') F_6(z'',z) \frac{dz''}{R_1} \tag{4.29}$$

The equation (4.27) may be solved by the iterative method (series 4.10 or 4.12) and $J_1(z)$, $\Phi_1(z)$ and $\Phi_2(z)$ calculated.

4.4. A remark on the SC approximation

The current in the cases a) and c) is given by an integral equation of the form:

$$J(z) = g(z) + \lambda \int_o^H J(z') K(|z-z'|) dz' \tag{4.30}$$

to which the SC approximation (4.8) could be applied.

In order to see the conditions of validity of (4.8), one can consider the terms given by (4.12) of the series

$$J(z) = \sum_{n=o}^{\infty} T_n(z)$$

in the form

$$T_n(z) = T_{n-1}(z) \cdot \int K(z,z') + \int \left[T_{n-1}(z') - T_{n-1}(z) \right] x$$

$$x\, K(|z-z'|)\, dz' \tag{4.31}$$

If $T_o(z) = g(z)$ is such that

$$\int \left[g(z') - g(z) \right] \cdot K(|z-z'|) dz' = 0 \qquad (4.32)$$

the second term of the right hand side of 4.31 vanishes for all the T_n and (4.8) is the exact solution. The condition (4.32) is rather restrictive and it is exactly true only at $z = H/2$ and when $g(z') - g(z)$ can be expanded in odd powers of $(z-z')$.

However the value of the integral (4.32) is negligible when:

- $\left[g(z')-g(z) \right]$ is slowly varying between $z-\varepsilon < z' < z+\varepsilon$ (ε is of the order of R for case a, and of R_2-R_1 for case c), so that it can be approximated linearly between these limits
- $g(z') \cdot K (|z-z'|) \ll g(z) \cdot K(o)$ outside the above limits;

this condition assures that the main contribution to the integral giving T_1 comes from the points z' near z.

Although in many cases of interest the above conditions are not met, the previous considerations are useful for evaluating the remainder of the series. To find an expression for the remainder we observe that T_{n+1} is always less varying than T_n.

This property assures that, for any $g(z)$, it will be always possible to find a value N such that $T_N(z)$ is sufficiently constant to satisfy the conditions given above for $g(z)$.

Therefore the series (4.10) can be written as:

$$J^{(N)}(z) = \sum_{0}^{N-1} T_n(z) + \frac{T_N(z)}{1 - \lambda \cdot \int K(z-z')dz'} \qquad (4.33)$$

The remainder can be evaluated at any iteration and the equation (4.33) is suited for computation since the iteration can be stopped at a value N which can be determined by the condition:

$$\frac{J^{(N+1)}(z) - J^{(N)}(z)}{J^{N}(z)} < \eta$$

where η is a positive number whose value depends on the required accuracy.

The expression (4.33) is particularly useful when $\alpha = \lambda \int K(z-z')\,dz'$ is close to unity. In this case it might happen that the integrals T_n are numerically overestimated with an error of the order of $1 - \alpha$ causing the series to diverge.

This error is avoided by the use of (4.33) where only few terms are calculated and, in addition, when the integral $\int K(z,z')dz'$ can be analytically solved, the value of the remainder is exact.

5. Experimental test

For testing the assumptions and the mathematical procedure we have compared the calculation results with some experimental data obtained in previous works. These data refer to cylindrical ducts of various size and wall composition in water.

5.1. Long ducts

A first series of experiments [14] has been performed with ducts having aluminum walls and a radius much smaller than the length. The source is a disk with the axis coincident with the cylinder axis as shown in Fig. 6.

The calculation has been performed with 3 energy groups:

$g = 1 \qquad E > 0.8$ MeV

$g = 2 \qquad 0.8$ MeV $> E > 0.4$ eV

$g = 3 \qquad 0.4$ eV $> E$

The details are given below.

5.1.1. Determination of the albedos

Some values for water are given in Tables 5-I, 5-II and 5-III.

Table 5-I refers to plane geometry and diffusion calculation and contains also the used K and D values. The effect of the geometry of the reflecting surface is shown in Table 5-III for cylinders of 5, 10 and 15 cm of radius. Table 5-II gives the dependence on the angle of incidence ϑ_o and is obtained from a MonteCarlo calculation*. The

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* Private communication by R. Nicks of CCR Euratom Ispra.

last column gives the values for a cosine distribution of the incident current.

Other values for thermal neutrons and water medium are:

Fermi:	normal incidence	0.806
	glazing incidence	0.92
Halpern:	normal incidence	0.766
	cosine incidence	0.815
Chandrasekhar:	normal incidence	0.776
	glazing incidence	0.92
	cosine incidence	0.82

A comparison of the above values indicates a good agreement in the cases of low energy neutrons where the albedo is important and some discrepancies between diffusion and MonteCarlo values for fast neutrons.

From the duct geometry it is seen that most of the reflections of the removal neutrons of group 1 occur at angles of incidence $\vartheta_o \geq 80°$ and therefore the values of Table 5-II for $\vartheta = 80°$ have been used for the first reflection. In all the other cases the data derived from the diffusion theory have been used.

5.1.2. Determination of the initial current.

For the neutrons belonging to the removal component the first term T_1 of the series is calculated directly by eq. 4.15 where.

$$i^{+}(z) = \frac{N}{2\pi} \cdot \int_{0}^{\rho_{max}} \frac{\rho^2 \sin\varphi(\rho)\, d\rho}{\left[\rho^2+(z+h)^2\right]^{3/2}} +$$

$$+ \frac{N}{2\pi} \int_{\varphi_1}^{\pi/2} \cos\varphi\, d\varphi \cdot \int_{\rho_1}^{\rho_2} \frac{\rho^2 \cdot e^{-\Sigma r_H(\rho)}\, d\rho}{\left[\rho^2+(z+h)^2\right]^{3/2}} \qquad (5.1)$$

The two terms of (5.1) represent the current of neutrons entering the medium at z through the void and through the medium and the void respectively; the symbols have the meaning reported in Fig. 6 or below:

$N = 1.28 \cdot 10^{8}$ n/cm^2·s (intensity of the source emission)

$\Sigma = 0.1$ cm^{-1} (removal cross section for water)

$$r_H = \left[\frac{z}{z+h} - 2R\cos\varphi\right]\left[1+\left(\frac{z+h}{\rho}\right)^2\right]^{\frac{1}{2}} \qquad \text{(path through water)}$$

$$\rho_1 = 2R \cdot \frac{z+h}{z} \cdot \cos\varphi$$

$$\rho_2 = R\cos\varphi + \sqrt{R_s^2 - R^2\sin^2\varphi}$$

$\varphi_1 = 0 \qquad (z > z^*)$

$$\varphi_1 = \cos^{-1}\left[\frac{R_s^2 - R^2}{4R^2 h}\,\frac{z^2}{z+h}\right]^{\frac{1}{2}} \qquad (z \leq z^*)$$

$$\rho_{max} = 2R\,\frac{z+h}{z} \qquad (z > z^*)$$

$$= R + a \qquad (z \leq z^*)$$

$$\sin\varphi(\rho) = \left[1 - \left(\frac{\rho z}{2R(z+h)}\right)^2\right]^{\frac{1}{2}} \qquad (z > z^*)$$

and $(z \leq z^*)$ if

$(0 < \rho < \rho')$

$$= \left[1 - \left(\frac{R^2+\rho^2-R_s^2}{2R\rho}\right)^2\right]^{\frac{1}{2}} \qquad (z < z^*) \text{ if}$$

$$(\rho' < \rho < R+a)$$

$$z^* = \frac{2Rh}{R_s-R}$$

$$\rho' = \left[\frac{(R_s^2 - R^2)(z+h)}{h}\right]^{\frac{1}{2}}$$

The diffusion component is evaluated with the procedure outlined in Section 3; however the rather simple geometry allows some simplifications.

The unperturbed diffusion and removal fluxes $\varphi_g^o(z)$ and $\varphi_R^o(z)$ are calculated by Sabine along the z axis (r=0) in water (without the duct). The subsequent step is the determination of the removal flux $\varphi_R(z,r)$ around the duct, which is considered as a perfect absorber.

Instead of performing the complete calculation of the diffusion fluxes deriving from $\varphi_R(z,r)$, an approximate evaluation is performed by considering $\varphi_R(z,r)$ constant with r, with a value corresponding to a distance L from the duct wall. L is of the order of the diffusion length and L = 5 cm has been used.

With the above hypothesis the diffusion flux in the medium to be used in eq. 3.8 is given by

$$\varphi_g^o(z) \cdot F(z) \qquad (5.2)$$

where:

$$F(z) = \frac{\varphi_R(z, a+L)}{\varphi_R^o(z,o)} \qquad (5.3)$$

TABLE 5-I

Albedos for water calculated by diffusion theory: plane and cylindrical geometry

Group g	Plane geometry			K (cm^{-1})	D (cm)
	$g_{i,i}$	$g_{i,i+1}$	$g_{i,i+2}$		
1	0.192	0.169	0.126	0.202	1.653
2	0.325	0.325	-	0.421	0.606
3	0.805	-	-	0.390	0.1375

TABLE 5-II

Dependence of the albedo for water on the angle of incidence φ_o for an incident fission spectrum - Monte Carlo calculation
Plane geometry

	Incidence angle ϑ_o				$\cos\vartheta_o$ distribution
	0°	30°	60°	80°	
$\alpha_{1.1}$	0.039	0.045	0.067	0.12	0.062
$\alpha_{1.2}$	0.120	0.130	0.192	0.23	0.161
$\alpha_{1.3}$	0.086	0.081	0.089	0.071	0.083
$\sum_g \alpha_{1,g}$	0.245	0.256	0.348	0.421	0.306

TABLE 5-III

Dependence of the albedo for water on the radius R of the cylinder - Diffusion calculation

	R = 5 cm	R = 10 cm	R = 15 cm	R = ∞
$\alpha_{3.3}$	0.768	0.785	0.792	0.805
$\alpha_{2.2}$	0.238	0.276	0.295	0.325
$\alpha_{1.1}$	0.0225	0.10	0.128	0.192

Observing that the total current i_g^+ can be written as

$$i_g^+ = (i_g - i_g^o)/\alpha_{g,g}$$

one obtains the initial currents (eq. 3.1):

$$i_1^o = \alpha'_{1,1} i_R^{*+} + \left(\frac{i^*}{\Phi^o}\right)_1 \cdot F \cdot \varphi_1^o$$

$$i_2^o = \alpha'_{1,2} \cdot i_R^{*+} + (i_1 - i_1^o) \cdot \frac{\alpha_{1.2}}{\alpha_{1.1}} + \left(\frac{i^*}{\Phi^o}\right)_2 \cdot F \cdot \varphi_2^o$$

$$i_3^o = \alpha'_{1.3} \cdot i_R^{*+} + (i_1 - i_1^o) \frac{\alpha_{1.3}}{\alpha_{1.1}} + (i_2 - i_2^o) \frac{\alpha_{1.2}}{\alpha_{2.2}} + \left(\frac{i^*}{\Phi^o}\right) \cdot F \cdot \varphi_3^o$$

(5.4)

where.

$\alpha'_{i,j}$: albedo from Table 5-II, $\vartheta = 80°$ (first reflection)

$\alpha_{g,j}$: albedo from Table 5-III

$\alpha_{g,g}$: albedo from Table 5-II

$(\frac{i^*}{\Phi^o})_g$: = 0.202 g = 1 (from eq. 3.8)
0.175 g = 2
0.050 g = 3

i_g = total current obtained by calculation with the MRC code.

5.1.3. Results

From the MRC code one obtains $\Phi_g(z)$ on the duct axis. For g = 1 one must still add the removal component of the flux coming from non reflected neutrons; this component has been directly calculated by QAD.

The experimental data to be used for comparison are threshold detector reaction rates, epicadmium reaction rate R_{epi} of $Au^{197}(n,\gamma)$ and thermal flux.

We consider firstly the low energy neutron results. The Au^{197} reaction rate has been transformed to an experimental $\Phi_2(z)$ flux with the hypothesis of 1/E spectrum by multiplying by the ratio between the total lethargy interval in group 2 and the Au^{197} resonance integral:

$$\Phi_2(z) = R_{epi}(z) \cdot \frac{14.5}{4.75}$$

The measured thermal flux can be compared directly with the calculated $\Phi_3(z)$.

The results are shown in Fig.s 7, 8, 9, 10 for different duct and source radii. The agreement is rather good both for the shape and the absolute value expecially for thermal neutrons. The epithermal group is always underestimated at large distances, but the discrepancies never exceed a factor 2.

One reason for this disagreement could be the spectral deformation (from the 1/E shape) far from the mouth; actually by the use of resonance detectors it has been verified that the spectrum is close to 1/E near the mouth, whilst no measurement has been made near the end since the fluxes are too low for an accurate determination.

It must be observed that in a configuration of the type of Fig. 6 the main contribution to the flux arises from the initial current entering the month face and streaming along the duct. This flux, sometimes called "direct flux" can be calculated with negligible error. In the present case however for $g = 2$ and $g = 3$ there is no initial current entering the mouth and the flux in these groups derives completely from reflections of fast neutrons or slowing down in the medium around the cylindrical surface. The test of the method is therefore particularly meaningful.

The fast neutron group $g = 1$ cannot be directly compared with the experimental data. This comparison is not strictly necessary since the correctness of the Φ_1, calculation is implicitely verified by the previous test on Φ_2 and Φ_3 which derive, by a large extent, from Φ_1. For completeness however we consider also the threshold detector reaction rates. These are sensitive only to neutrons belonging to

the higher energy part of group 1, say a group 0 which has a negligible albedo $\alpha_{0,0}$. Therefore it is not necessary to perform the multiple reflection calculation for obtaining the flux effective for the threshold reactions. In our case we have verified that satisfactory results are obtained simply by a QAD calculation, using the Moument Method Kernel for the attenuation in water. This appears from Figs. 7,8, 9, 10 for the $Ni^{58}(n,p)$ or $S^{32}(n,p)$ reaction rates.

5.2. A gap

Another experiment refers to a cylinder with the length much smaller than the radius, as shown in the insert if Fig. 11.

This cylinder approximates a gap in a plane shield.

In order to enhance the radial gradient of the initial current a collimator is placed between the source and the void as shown in Fig. 11.

The wall material is iron and in this case the epithermal group has a rather high albedo ($\alpha_{2.2}$=0.8).

The initial current along the radius has been derived from a measured flux inside the iron slabs without the void.

An added subroutine to the RMC permits the determination of the radial distribution of the flux, and Fig. 11 shows the comparison with the experimental data for the epithermal flux. The agreement is very satisfactory.

5.3. Applications

As an example of application of the method to a design problem we consider the gap existing around the pressure vessel in the reference shield of Fig. 1.

By applying the same procedure of the RMC with the geometrical Kernels of Sect.4.3 we have calculated the axial distribution of the epithermal flux inside the gap. The results are shown in Fig. 12 together with the flux calculated without reflections.

In this particular example the S.C. approximation would give a large underestimate of the flux far from the source.

6. Conclusions

The scope of this study was to set up some criteria and methods for solving the problem of neutron transport through voids for a particular shield design. The results are however quite general and could be applied to other experimental configurations.

With the proposed method elaborate techniques are not required, and in addition to a removal-diffusion and removal program a calculation code for solving the iterative process is needed. This code has been written by P. Novario /13/ for the important case of straight ducts with ends.

The problem of determining the initial current has been carefully analyzed and previous solutions revised; in particular the division in removal and diffusion components is expected to improve the precision still retaining a sufficient simplicity. In

some cases these components correspond to the so called "direct" and "leakage" currents. These terms have been avoided since they are somewhat misleading for low energy neutrons.

The correctness of the assumptions and of the whole procedure has been tested with experiments in rather severe conditions.

In fact it is found excellent agreement for low energy neutrons in ducts having only fast neutrons entering the mouth face. It has also been found that only 3 energy groups are necessary in this case.

The method has several application in a shield design; in addition to the problem of the gap already seen, the described techniques may be used to evaluate:

a) the neutron reflections inside the biological shield of Fig. 1; this effect increases the external doses in the upper part of the shield

b) the streaming along the gap around the nozzles

c) the neutron propagation outside the biological shield; in this case there is no reflection and one needs only to evaluate at a point external to the shield the contributions coming from the two components (removal and diffusion) of the initial current leaving the shield surface.

Improvements of the methos require only minor changes in the MRC code, such as the incorporation of different geometrical Kernels and of albedos of the type $\cos^n \vartheta$ with $n \neq 1$. These improvements will be the subject of a future work together with the approach to the bent duct problem.

Bibliography

/1/ C. Ponti, H. Preuseh, H. Schubart, "Sabine, a One Dimensional Bulk Shielding Program", EUR 3636.e, 1967

/2/ R.E. Malefant, "QAD: A series of Point Kernel general Purpose Shielding Programs", LA 3573, 1966

/3/ W.E. Selph, "Neutron and gamma-ray albedos", ORNL-RSIC 21 (DASA 1892-2), 1968

/4/ R.E. Maerker, F.J. Muckentaler, "Calculation and Measurement of the Fast-Neutron Differential Dose Albedo for Concrete", Nucl. Sci. Eng., 22, 455, 1965

/5/ F.J. Allen, A. Futterer, W. Wright, "Neutron reflection and flux versus Depth for Iron, ... Water", Ballistic Research Laboratories Reports, B.R.L. 1199, 1963, 1204, 1963

/6/ R.L. French, M.B. Wells, "An angular dependent Albedo for Fast Neutron Reflection Calculations", Radiation Research Associates Report RRA - M31, 1963

/7/ S. Chandrasekhar, "Radiative Transfer", Clarendon Press, Oxford, 1950

/8/ P.G. Miller, A. Packwood, J. Parker, "Techniques for calculating the penetration of radiation in ducts and voids", International Conference on the Physics Problems of Reactor Shielding, Harwell 1967, AERE-R-5773, 1968

/9/ F.R. Channon, R.L. Seale, "The Study of Thermal Neutrons in Shields containing Ducts by Source Separation Techniques", Nucl. Sci. Eng., 30, 242, 1967

[10] J.M. Paratte, "Study of the Neutron Distribution in Bent Air Ducts in a Water Shield", International Conference on the Physics Problems of Reactor Shielding, Harwell 1967, AERE-R 5773, 1968

[11] A. Simon, C.E. Clifford, "The Attenuation of Neutrons by Air Ducts in Shields", Nucl. Sci. Eng., 1, 156, 1956

[12] R. Artigas, H.E. Hungenford, "A numerical Treatment of the Attenuation of Neutrons by Air Ducts in Shields", Nucl. Sci. Eng., 36, 295, 1969

[13] P.G. Novario, "RMC-I: un programma di calcolo di flusso neutronico all'interno di un condotto cilindrico", SORIN Report S/626, 1970

[14] B. Chinaglia, D. Monti, "Studio delle irregolarità nelle schermature", Final Report of Euratom Contract 022-61-II ISPI, SORIN Report F/326, 1964.

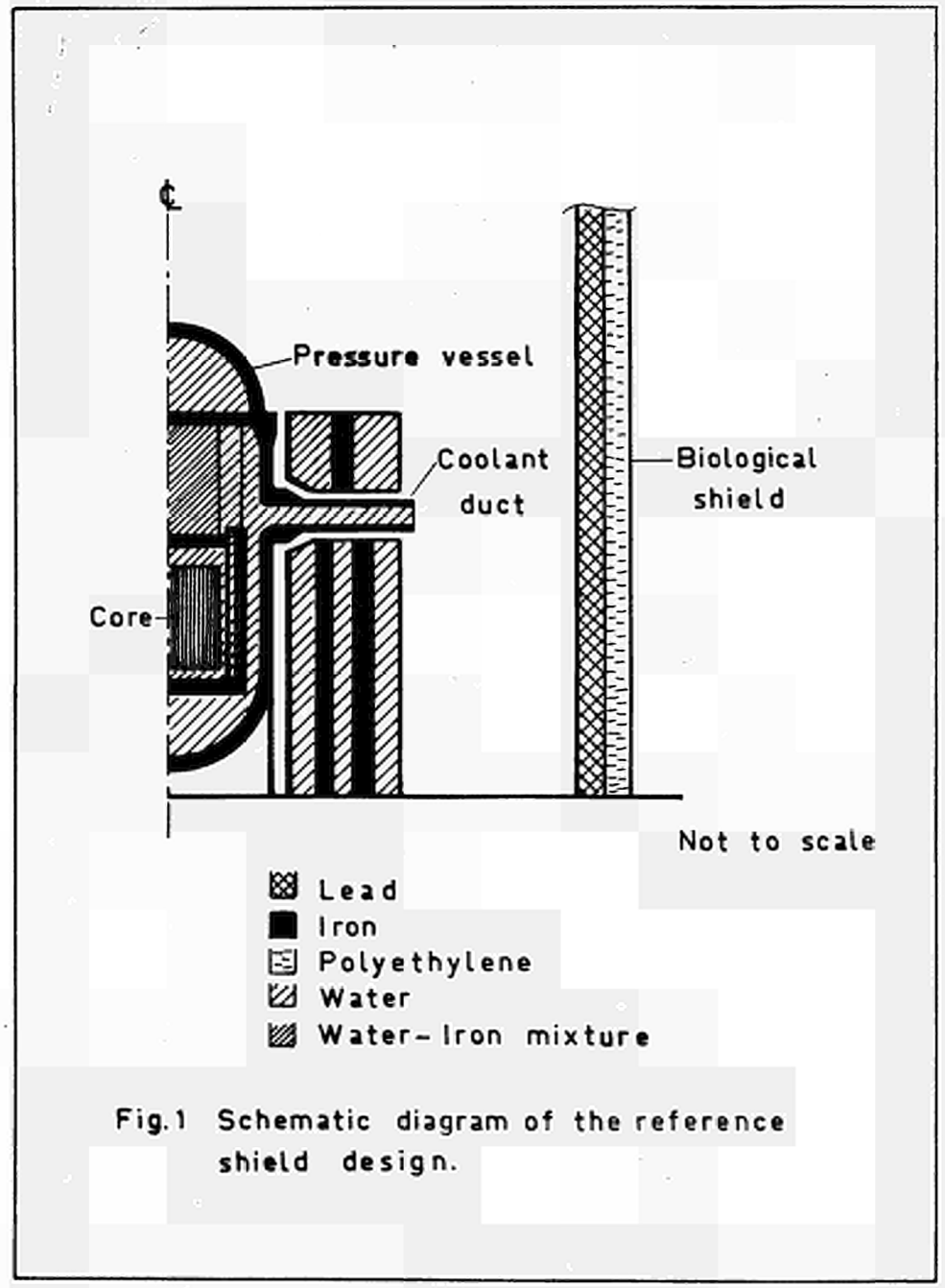


Fig. 1 Schematic diagram of the reference shield design.

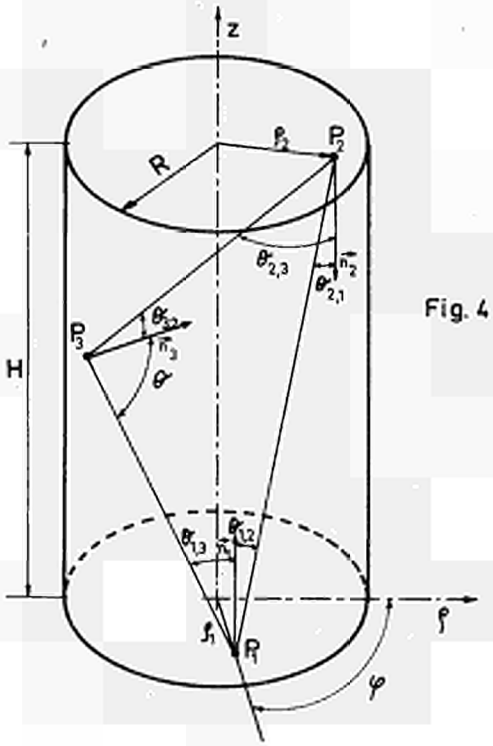


Fig. 4 Geometry of cylindrical duct with reflecting surfaces at the ends.

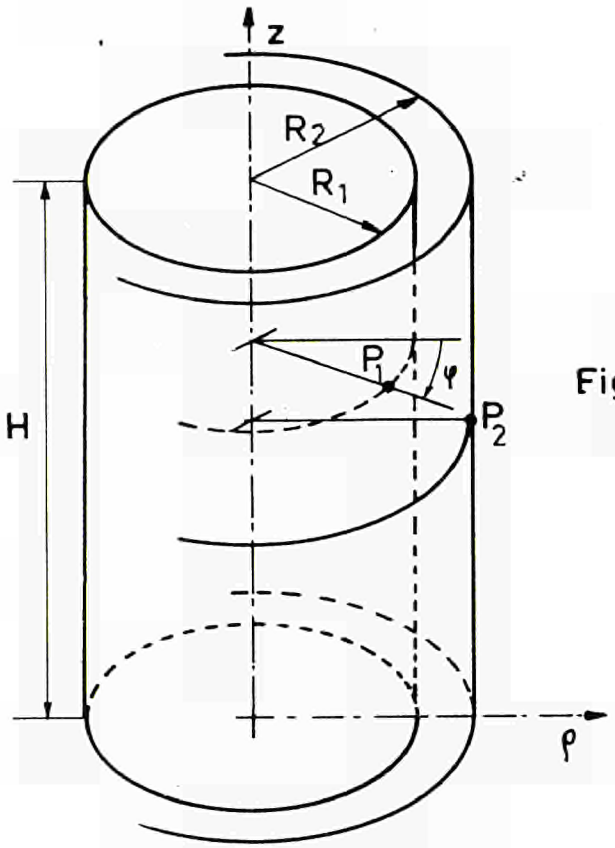
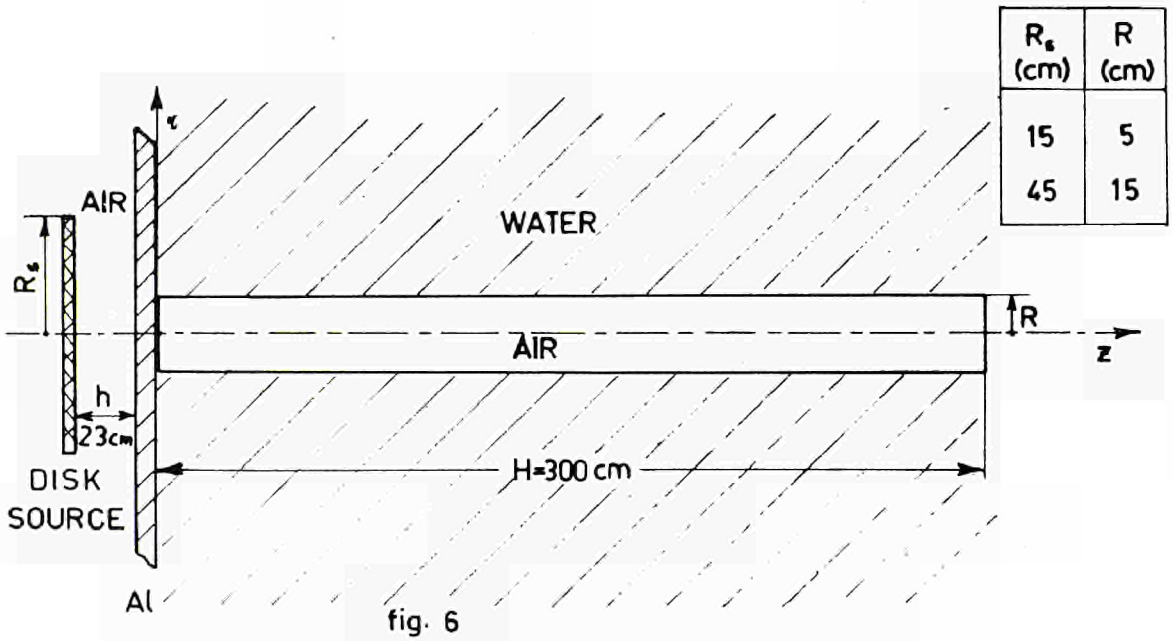


Fig. 5 Geometry of the void enclosed by two cylindrical surfaces.



R_s (cm)	R (cm)
15	5
45	15

fig. 6

Configuration for experiments with ducts in water

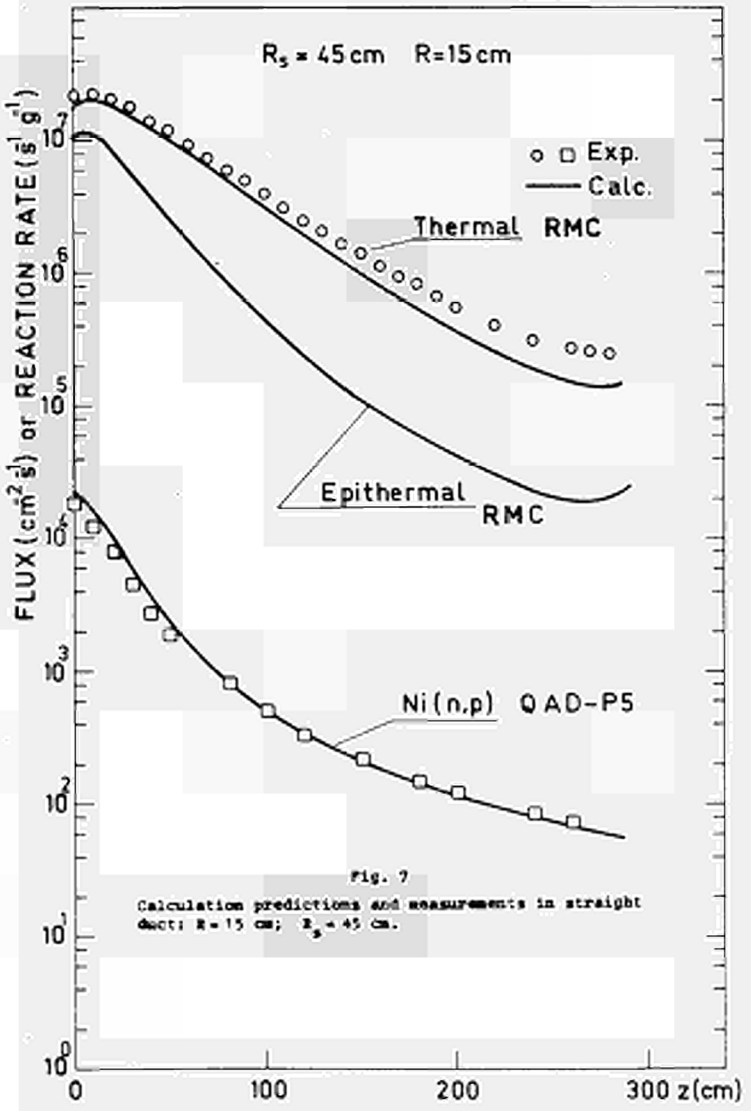


Fig. 7
Calculation predictions and measurements in straight duct: R = 15 cm; R_s = 45 cm.

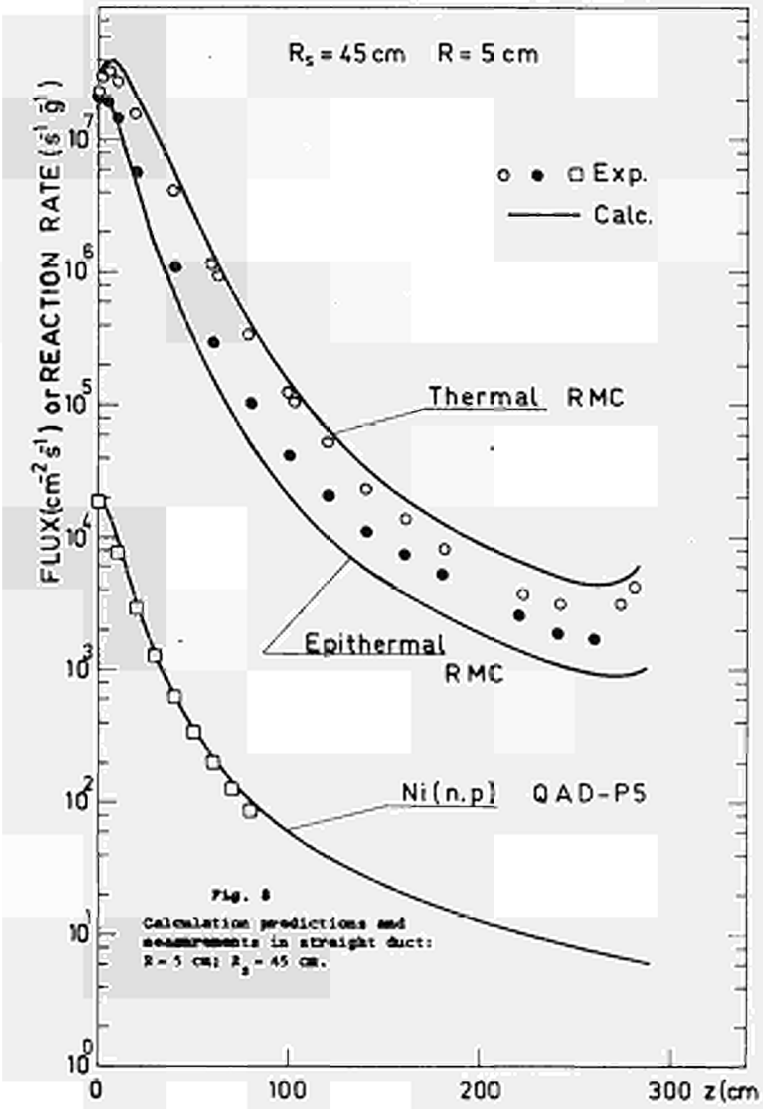


Fig. 8
Calculation predictions and measurements in straight duct: R = 5 cm; R_s = 45 cm.

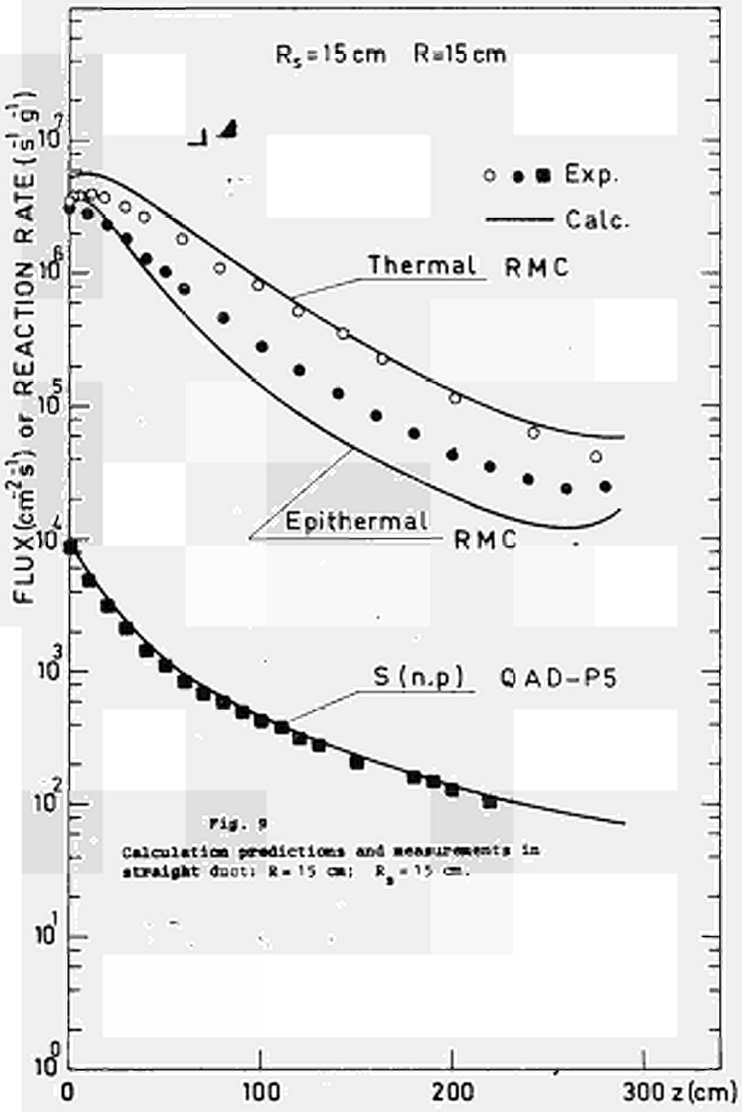


Fig. 9
Calculation predictions and measurements in straight duct: R = 15 cm; R_s = 15 cm.

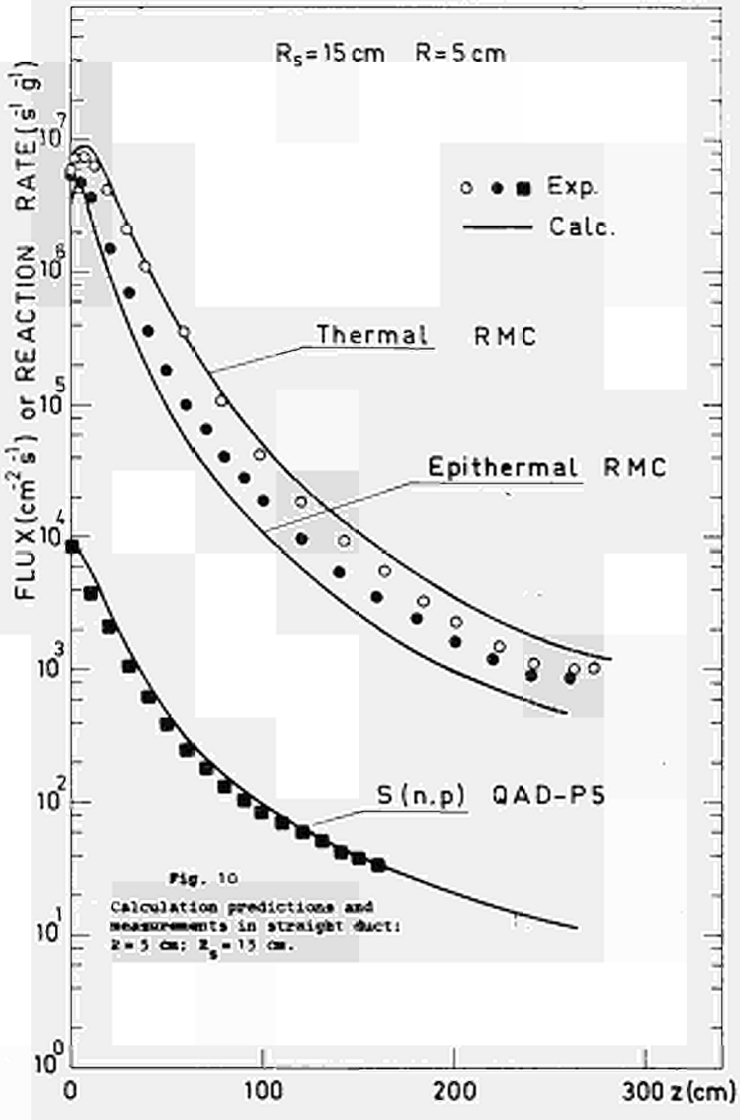


Fig. 10
Calculation predictions and measurements in straight duct: R = 5 cm; R_s = 15 cm.

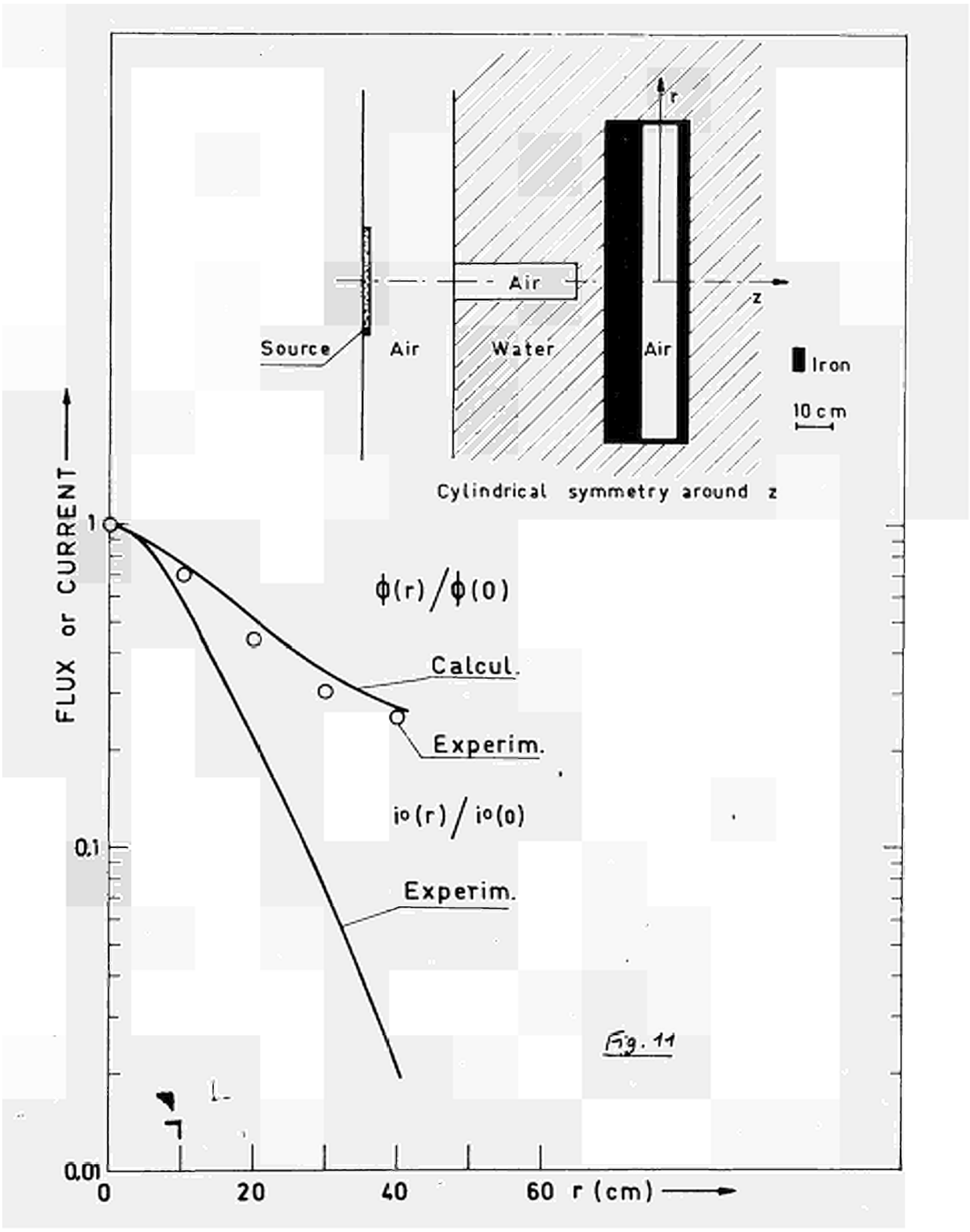

r
Air
z
Source
Air
Water
Air
Iron
10 cm
Cylindrical symmetry around z
FLUX or CURRENT
1
0.1
0.01
φ(r) / φ(0)
Calcul.
Experim.
i°(r) / i°(0)
Experim.
0
20
40
60 r (cm)

Fig. 11

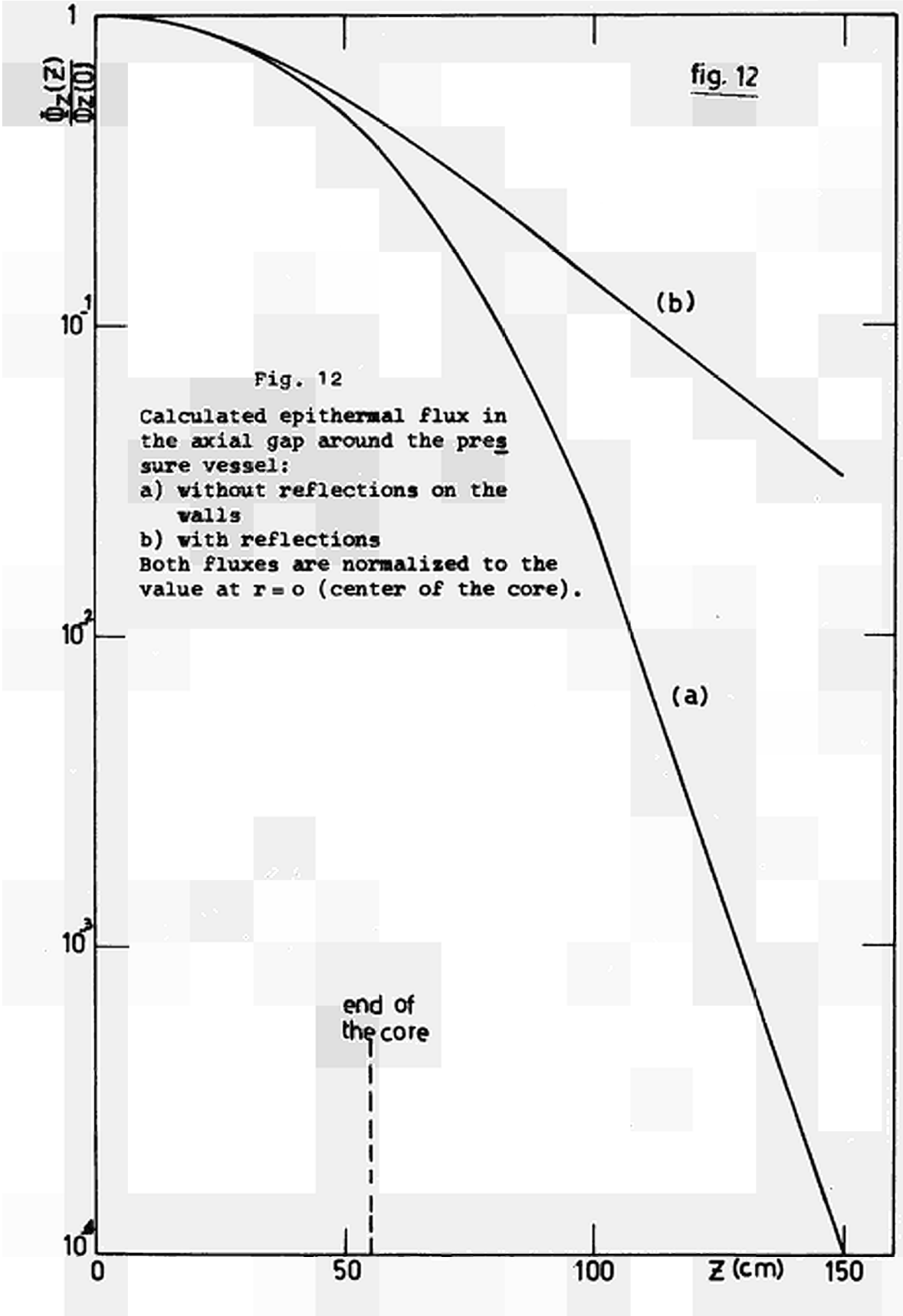


Fig. 12

Calculated epithermal flux in the axial gap around the pressure vessel:
a) without reflections on the walls
b) with reflections
Both fluxes are normalized to the value at r = o (center of the core).

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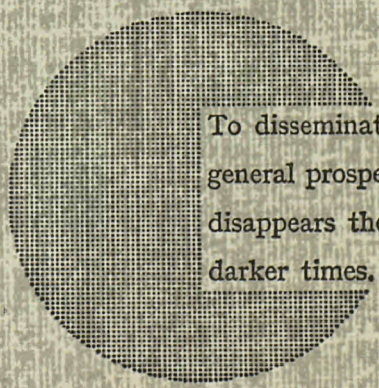

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Alfred Nobel

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