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PART I

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POWER PULSE FLUCTUATIONS IN THE SORA REACTOR

Theory

by

W. MATTHES

1971

Joint Nuclear Research Centre
Ispra Establishment - Italy

Nuclear Studies

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ABSTRACT

Fluctuations of the power pulses in periodically (reactivity-) pulsed reactors were confirmed experimentally. In this report we establish the necessary analytical methods to describe such phenomena. It turns out that the variance to mean ratio of some fluctuating quantities (e.g. the number of neutron counts in an absorption counter integrated over the power pulse) is power-independent and only a function of the criticality of the reactor. These quantities can therefore be used to determine the criticality of the reactor experimentally.

KEYWORDS

POWER
DISTURBANCES
SORA
REACTIVITY
STATISTICS
CRITICALITY
NEUTRONS
COUNTING RATES
INTEGRALS

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Power pulse fluctuations in the SORA reactor *)

Part I: Theory

A) General considerations (see [1], [2], 6)

We investigate the fluctuations of the power pulses in the Sora reactor in the point- and one group model. In this picture the state of the reactor is characterized at any time point t by the set of integral numbers $N = \{n_a\ n_b\ ..\}$ where n_a denotes the number of particles of type a. If we have only <u>one</u> a-type particle present we write simply N = a. Later we shall identify type-a particles for instance with neutrons, type-b particles with delayed neutron emitters of a certain decay constant, type-c particles with counts of fission processes etc.

To describe the time evolution of the particle field we introduce

a) P(tN) as the probability to find the system at time t in state $N\{n_a\ n_b\ \ldots\}$, and

b) P(sI,tN) as the probability to find the system at time t in the state $N\{n_a\ n_b\ \ldots\}$ if the system was at time s in the state $I\ \{i_a\ i_b\ \ldots\}$.

Obviously we have

$$P(t\,N) = \sum_{I,M} P(s\,I) \cdot P(s\,I,\,t\,N-M) \cdot Q(s,t\,M) \tag{1}$$

where Q(s,tM) is the probability that a source acting between the time points s and t produces the state M at time t.

To deal properly with equations such as equation (1) we refer to the standard methods (see for instance [1] and [2] and introduce the generating function

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*) Manuscript received on March 12, 1971

$$\phi(t, X) = \sum_N P(t\, N)\, e^{NX} \tag{2}$$

Where X is an abbreviation for the set $\{x_a\ x_b\ \ldots\}$ and XN for $(x_a n_a + x_b n_b + \ldots)$.

Performing the operations indicated in (2) on (1) we obtain, under the assumption that each particle produces its population independent of the presence of the other particles:

$$\phi(t, X) = \phi(s, F) \cdot Q(s, t\, X) \tag{3}$$

where

a) F is an abbreviation for the set $\{f(sa, tX),\ \ f(sb, tX)\ \ldots\}$

b) f(sa,tX) is the generating function of P(sa,tN) and

c) Q(s,tX) is the generating function of Q(s,tM)

Definition (2) gives us in

$$\left(\frac{\partial \ln \phi(t, X)}{\partial x_a} \right)_{X=0} = \langle n_a(t) \rangle = M(a\, t) \tag{4}$$

the mean number of a-type particles at time t, and in

$$\left(\frac{\partial^2 \ln \phi(t, X)}{\partial x_a\, \partial x_b} \right)_{X=0} = \langle n_a(t) n_b(t) \rangle - \langle n_a(t) \rangle \langle n_b(t) \rangle = D_{ab}(t) \tag{5}$$

the covariance between the a-type particles and b-type particles at time t.

Operation (4) and (5) applied on (3) leads to:

$$M(at) = \sum_{b} M(bs) \cdot G(bs, at) + Q(s, ta) \tag{6}$$

$$D_{ab}(t) = \sum_{b'} M(b's)\, G(b's, abt) + Q(s, abt) + \sum_{\nu\mu}' \left\{ D_{\nu\mu}(s) - \delta_{\nu\mu} M(\mu s) \right\} \times G(\nu s, at)\, G(\mu s, bt) \tag{7}$$

where

$$G(bs, at) = \langle n(bs, at) \rangle = \left(\frac{\partial f(bs, Xt)}{\partial x_a} \right)_{X=0} \tag{8}$$

is the mean number of a-type particles at time t due to the presence of <u>one</u> b-type particle at time s (Green's function).

Q(s,ta) is the mean number of a-type particles at time t due to the source.

$$G(\nu s, abt) = \langle n(\nu s, at)\, n(\nu s, bt) \rangle = \left(\frac{\partial^2 f(\nu s, Xt)}{\partial x_a \partial x_b} \right)_{X=0} \tag{9}$$

and Q(s,abt) is the covariance of a-type particles and b-type particles at time t due to the source acting between s and t.

To simplify (8) we make two assumptions:

a) At time point s we assume a poisson distribution for the particles

$$D_{\nu\mu}(s) = \delta_{\nu\mu} M(\nu s) \tag{10}$$

This assumption will be justified in our application (see chapter C).

b) To describe the action of the source we divide the time interval $\{s,t\}$ in K small time-intervals $\{\Delta t_1, \Delta t_2, \ldots \Delta t_K\}$ and introduce $w(t_1 L_1, t_2 L_2 \ldots t_K L_K)$ as the probability that the source produces the states $L_\nu(\ell_a \ell_b \cdots)$ in the time intervals Δt_ν around t_ν ($\nu = 1,2,\ldots K$).

The assumption we make now consists in:

$$w(t_1 L_1, t_2 L_2 \cdots t_K L_K) = \prod_{\nu=1}^{K} w(t_\nu L_\nu) \tag{11}$$

and we obtain

$$Q(s\, t\, X) = \prod_{\nu=1}^{K} w(t_\nu, F(t_\nu)) \tag{12}$$

where

$$F(t_\nu) = \{ \ln f(t_\nu a, t\, X), \ln f(t_\nu b, t\, X) \cdots \}$$

Let $s(t')dt'$ be the probability for a source event in dt and R(L) the probability that in this source event the state L is produced, then:

$$\begin{aligned} w(L=0, t') &= 1 - s(t')dt' \\ w(L \neq 0, t') &= s(t')dt' \cdot R(L) \end{aligned} \tag{13}$$

and

$$W(t', F) = (1 - s(t')dt') + s(t')dt' \, R(F(t'))$$

$$= e^{s(t')\{R(F(t')) - 1\}\,dt'} \tag{14}$$

Now we obtain:

$$\ln Q(s\,t\,X) = \int_s^t s(t')\{R(F(t')) - 1\}\,dt' \tag{15}$$

Under the further simplifying assumption that only <u>one</u> particle of some type will be emitted in each source event we obtain finally from (6) and (8) with (10) and (15):

$$M(at) = \sum_b \left\{ M(bs)\,G(bs, at) + \int_s^t s_b(t')\,G(bt', at)\,dt' \right\} \tag{16}$$

$$D_{ab}(t) = \sum_c \left\{ M(cs)\,G(cs, abt) + \int_s^t s_c(t')\,G(ct', abt)\,dt' \right\} \tag{17}$$

where $s_b(t')dt'$ is the mean number of b-type particles emitted from the source in dt'.

To proceed further we need an equation for f(sa,tX) to calculate the quantities G(b s,at) and G(c s,abt).

B) Kolmogoroff-equations

To obtain an equation for f(sa,tX) we consider the time-evolution of the particle field without a source.

Then we have

$$P(sI, tN) = \sum_{N'} P(sI, t'N')\, P(t'N', tN) \tag{18}$$

where we have to sum overall possible states N' at some arbitrary time point t' between s and t.

Assuming again that each particle produces its population independent of the presence of the other particles, we obtain:

$$f(sI, tX) = f(sI, t'\mathcal{F}') \tag{19}$$

where

$$\mathcal{F}' = \{\ln f(t'a, tX), \ln f(t'b, tX) \cdots\}$$

Now we write (19) in the form:

$$f(s-h\,a, tX) = f(s-h\,a, s\mathcal{F}') \tag{20}$$

$$\mathcal{F} = \{\ln f(as, tX), \ln f(bs, tX) \cdots\}$$

with the boundary condition:

$$f(ta, tX) = e^{X_a}$$

The mechanism for the change of the state of the system is controlled through the quantities:

$D_r(at)h$ probability that an a-type particle disappears in the time-element $(t, t+h)$ through channel r

$W_r(at, M)$ probability that the state $M\{m_a\ m_b\ \ldots\}$ will be simultaneously generated when an a-type particle disappears through channel r at time t. ($W(at, X)$ = generating function)

With this mechanism in mind it is meaningful to perform the limit operation $h \to 0$ in (20).

To perform this limit we consider first the transition probability $P(ta, t+h\ N)$.

$$P(ta, t+h\ N) = (1 - \sum_r D_r(at)h)\delta_{aN} + \sum_r D_r(at)h\ W_r(at, N) \tag{21}$$

or:

$$f(ta, t+h\ X) = e^{x_a} + g(at, X)\cdot h$$
$$g(at, X) = \sum_r D_r(at)\left\{W_r(at, X) - e^{x_a}\right\} \tag{22}$$

Using (22) we transform (20) into:

$$-\frac{\partial}{\partial s} f(sa, tX) = \sum_r D_r(as)\left\{W_r(as, F) - f(sa, tX)\right\} \tag{23}$$

$$F = \left\{\ln f(as, tX),\ \ln f(bs, tX), \cdots\right\}$$

We use now equation (23) to derive the equations for the moments of f and obtain for the

a) first moment:

$$-\frac{\partial}{\partial s} G(as,bt) = \sum_c H(as,c)\, G(cs,bt) \tag{24}$$

$$H(as,c) = \sum_r D_r(as)\left\{ \overline{m_r(as,c)} - \delta_{ac} \right\}$$

$$G(as,bs) = \delta_{ab}$$

b) second moment:

$$-\frac{\partial}{\partial s} G(as,mn\,t) = \sum H(as,c)\, G(cs,mn\,t) + \tag{25}$$

$$\sum_{bc} \left\{ H(as,bc) - H(as,c)\delta_{bc} \right\} G(bs,mt)\, G(cs,nt)$$

$$H(as,bc) = \sum_r D_r(as)\left\{ \overline{m_r(ab)\, m_r(ac)} - \delta_{ab}\delta_{ac} \right\}$$

$$G(as,mns) = \delta_{am}\,\delta_{an}$$

where m_r (at,b) is the actual number of b-particles generated if one a-type particle disappears at time t through channel r.

To obtain solutions of (35) and (36) we observe that we deal with two different kinds of particles, namely with particles which can disappear ($D \neq 0$, e.g. neutrons, delayed emitters etc.) and with particles which cannot disappear ($D = 0$, e.g. counts of fission processes, total number of a-type particles produced etc.).

To distinguish between these two different kinds we denote particles with $D \neq 0$ by greek indices $\{\nu, \mu, \rho \dots\}$ and particles with $D=0$ by capital latin indices $\{A, B \dots\}$. Small latin indices $\{a, b \dots\}$ shall denote any kind of particle.

Then we obtain from (24):

a) $$G(As, bt) = \delta_{Ab}$$

b) solve the system of equations

$$-\frac{\partial}{\partial s} G(\nu s, \mu t) = \sum_{\rho} H(\nu s, \rho)\, G(\rho s, \mu t)$$

$$G(\nu s, \mu s) = \delta_{\nu\mu}$$

c) $$G(\nu s, At) = \sum_{\mu} \int_{s}^{t} G(\nu s, \mu t')\, H(\mu t', A)\, dt' \tag{26}$$

and from (25):

a) $$G(As, a\, b\, t) = \delta_{Aa}\, \delta_{Ab}$$

b) $$G(\nu s, abt) = \sum_{\mu} \left\{ \int_{s}^{t} G(\nu s, \mu t') S(\mu t'; abt) + G(\nu s, \mu t) \delta_{\mu a} \delta_{\mu b} \right\} \quad (27)$$

where

$$S(\mu t'; abt) = \sum_{r} D_r(\mu t') \left\{ \sum_{A} \overline{m_r(\mu, A)}\, \delta_{Aa} \delta_{Ab} \right. \quad (28)$$

$$\left. + \sum_{ij} \overline{m_r(\mu i)[m_r(\mu j) - \delta_{ij}]}\, G(it'; at) G(jt'; bt) \right\}$$

Using (26) in (16) we find

$$M(\mu t) = \sum_{\nu} \left\{ M(\nu s) G(\nu s, \mu t) + \int_{s}^{t} S_\nu(t') G(\nu t'; \mu t) dt' \right\} \quad (29)$$

$$M(At) = \sum_{\nu} \int_{s}^{t} M_\nu(t') H(\nu t'; A) dt' \quad (30)$$

and (27) in (17) we get

$$D_{ab}(t) = \sum_{\mu} \left\{ \int_{s}^{t} dt' M(\mu t') S(\mu t'; abt) + M(\mu t) \delta_{\mu a} \delta_{\mu b} \right\} \quad (31)$$

C) Application to the Sora reactor (see [3], [4]).

Here we identify

a) the time point s with the beginning of the reactivity pulse

b) ν = 0 with neutrons

c) ν = 1,2 ... 6 with delayed emitters

and assume

a) a constant source emitting neutrons only, and

b) neutrons are the only particles with D≠0 within the reactivity pulse.

Then we have from (16):

$$M(at) = M(os)\,G(os, at) + \sum_{i=1}^{6} M(is)\,G(is, at) + S_o \int_s^t dt'\, G(ot', at) \tag{32}$$

If the time point t ranges at most up to the end of the power pulse we can put:

$$G(is, at) = \lambda_i \int_s^t G(ot', at) \tag{33}$$

Further we have for the mean number of neutrons M(os) at the beginning of the reactivity pulse

$$M(os) = S_{tot}(s) \cdot \tau \tag{34}$$

where

$$S_{tot}(s) = S_o + \sum_{i=1}^{6} \lambda_i\, M(is) \tag{35}$$

and τ is the mean neutron lifetime in the subcritical state between the pulses.

Then we obtain from (32):

$$M(at) = \sum_{to A}(s) \quad M^{(a)}(at) \tag{36}$$

where

$$M^{(a)}(at) = \tau\, G(os, at) + \int_{s}^{t} G(ot', at)\, dt' \tag{37}$$

As the neutron population developes within the reactivity pulse due to:

$$G(os, ot) = \exp \int_{s}^{t} H(ot', o)\, dt' \tag{38}$$

$$H(ot, o) = D_F(t)\,(\bar{\nu}_o - 1) - D_A = -\rho(t)/\tau \tag{39}$$

we have:

$$G(ot', ot) = G(os, ot) \,/\, G(os, ot') \tag{40}$$

and therefore

$$M^{(o)}(ot) = G(os, ot) \left\{ \tau + \int_{s}^{t} \frac{dt'}{G(os, ot')} \right\} \tag{41}$$

$$M^{(1)}(At) = \int_S^t M^{(1)}(ot')\,H(ot',A)\,dt' \tag{42}$$

$$G(ot';At) = \int_{t'}^{t} G(os,ot'')\,H(ot'';A)\,dt'' \Big/ G(os,ot') \tag{43}$$

$$H(ot,A) = \sum_r D_r(ot)\,\overline{m_r(A)}$$

For the covariance we find from (31) and (36):

$$D_{ab}(t) = S_{tot}(s) \quad M^{(2)}(abt) \tag{44}$$

where

$$M^{(2)}(abt) = \int_S^t dt'\,M^{(1)}(ot')\,S(ot';abt) + M^{(1)}(ot)\,\delta_{oa}\,\delta_{ob} \tag{45}$$

The foregoing formulae may now eaxily be applied to special situations. We give a fewexamples (see [5]).

a) <u>number of neutrons present at time A</u>

$$S(ot';oot) = D_F(ot')\,\overline{\nu_o(\nu_o-1)}\;G^2(ot';ot) \tag{46}$$

b) number of neutrons produced up to time t

$$S(0t';NNt) = \mathcal{D}_F(t')\left\{\overline{\nu_0} + \overline{\nu_0(\nu_0-1)} + \overline{\nu_0(\nu_0-1)}\; G^2(0t';Nt) + 2\,\overline{\nu^2}\, G(0t';Nt)\right\} \tag{47}$$

c) number of fissions produced up to time t

$$S(0t';FFt) = \mathcal{D}_F(t')\left\{1 + \overline{\nu_0(\nu_0-1)}\; G^2(0t';Ft) + 2\,\bar{\nu}_0\; G(0t';Ft)\right\} \tag{48}$$

The variance to mean ratio becomes

$$\frac{\mathcal{D}_{FF}(t)}{M(Ft)} = \overline{\nu_0(\nu_0-1)}\cdot A + 2\,\bar{\nu}_0\, B + 1 \tag{49}$$

$$A = \int_s^t M^{(1)}(0t')\,\mathcal{D}_F(t')\, G^2(0t';Ft)\,dt' \Big/ M^{(1)}(Ft) \tag{50}$$

$$B = \int_s^t M^{(1)}(0t')\,\mathcal{D}_F(t')\, G(0t';Ft)\,dt' \Big/ M^{(1)}(Ft) \tag{51}$$

d) number of neutron counts up to time t in an absorption counter

$$S(0t';CCt) = \mathcal{D}_C(t') + \mathcal{D}_F(t')\,\overline{\nu_0(\nu_0-1)}\; G^2(0t';Ct) \tag{52}$$

The variance to mean ratio becomes

$$\frac{D_{cc}(t)}{M(ct)} = \overline{\nu_0(\nu_0-1)}\;A + 1 \tag{53}$$

$$A = \int_s^t M^{(1)}(ot')\,D_F(t')\,G^2(ot';ct)\,dt' \Big/ M^{(1)}(ct) \tag{54}$$

For the numerical evaluation the reactivity variation $\rho(t)$ must be given, which is mainly due to a change in the fission cross-section. Thus we have

$$-\frac{\rho(t)}{\tau} = D_F(t)\,(\overline{\nu}_0 - 1) - D_A \tag{55}$$

with D_A assumed constant in time.

The main steps in the further development consist in the calculation of the following quantities

a) $$\exp\left\{-\int_s^t \rho(t')\,dt'/\tau\right\} = G(os,ot) \tag{56}$$

b) $$G(os,ot)\left\{\tau + \int_s^t \frac{dt'}{G(os,ot')}\right\} = M^{(1)}(ot) \tag{57}$$

c) $$\frac{1}{G(os,ot')}\int_{t'}^t G(os,ot'')\,D_F(t'')\,dt'' = G(ot';Ft) \tag{58}$$

d) $$\int_s^t M^{(1)}(ot')\,D_F(t')\,dt' \begin{cases} 1 \\ G(ot';At) \\ G^2(ot';At) \end{cases} \tag{59}$$

The numerical investigation of power pulse fluctuations and related quantities will be done in part II of this report.

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Alfred Nobel

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