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JN - METD1

A FORTRAN-IV PROGRAMME FOR SOLVING NEUTRON TRANSPORT PROBLEMS WITH ISOTROPIC SCATTERING IN BARE SPHERES AND HOMOGENEOUS SLABS BY THE j_N METHOD

by

T. ASAOKA

1971

Joint Nuclear Research Centre
Ispra Establishment - Italy

Reactor Physics Department
Reactor Theory and Analysis

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ABSTRACT

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KEYWORDS

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ISOTROPY
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MATHEMATICS
COMPUTERS
EIGENVALUES
EIGENFUNCTIONS
INTEGRAL EQUATIONS

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6. $F_n(\alpha_j, \xi, \mu, s)$, $\mu > 0$
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10. $C_n^b(\alpha_j, s)$
11. $C_n^b(\alpha_j, iy + \Sigma_j\nu_j - \Sigma_j\nu_j) \equiv \frac{\Sigma_j\nu_j}{\alpha_j y}(C_{n1}^b + iC_{n2}^b)$
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1. Introduction

The j_N method has been developed during the last ten years to achieve a simple but accurate analytical approach to neutron transport in a finite system. One of the essential points of the method lies in the expansion into spherical Bessel functions of the Laplace-Fourier transformed emission density of neutrons (or the distribution of secondary neutrons) and the kernel of the integral equation (resulting from the Laplace and Fourier transformations of an integral transport equation with respect to time and space, respectively). For spherical and plane geometries, the expansion of the transformed flux rather than the transformed emission density with respect to the Fourier transform variable is equivalent to an expansion of the original flux in Legendre polynomials with respect to space[1)2)]. Due to the expansion of the emission density, our final expression of the flux exactly satisfies the boundary conditions independently of the order of the j_N approximation (truncation order of the expansion).

The method has already been applied successfully to space-energy time-dependent transport problems in a bare spherical system[3)] as well as space-angle energy-time dependent problems in an infinite homogeneous slab with finite thickness[4)] (always assuming that the scattering of neutrons is spherically symmetric in the laboratory system). The neutron flux for a stationary state has also been obtained as a simple limiting case of time-dependent problems. A computer code for stationary problems in a homogeneous slab has been adapted for calculating also the first and second time moments of the flux due to an incident delta function source[5)].

An extention of this approach to take into account anisotropic scattering of neutrons as well as multilayer slab systems can easily be performed, as already shown by several authors[6)-9)]. Furthermore, the application of the method to convex geometries has recently been demonstrated for a homogeneous medium in which the neutron scattering is isotropic[1o)]. In this work, an expansion into ordinary Bessel functions of odd order was adopted for an infinite cylinder instead of the spherical Bessel functions for the slab and spherical geometries.

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*) Manuscript received on 16 September 1970

The present report is concerned mainly with the computer code JN-METD1 designed to solve neutron transport problems for bare spheres and infinite homogeneous slabs within the context of the multigroup and (up to) j_7 approximation (scattering being assumed spherically symmetric). The code can deal with the following problems:

(a) Stationary problems in bare spherical reactors to obtain the asymptotic time constant (decay constant of the fundamental mode), the value of the effective multiplication factor k_{eff} or the critical radius, and the flux distribution as a function of space and energy.

(b) Stationary problems in homogeneous slabs to obtain the space, angle and energy dependent flux due to a plane isotropic, point isotropic or monodirectional boundary source. Also the first and second time moments are calculated for the time-dependent flux in the slab with a point isotropic or monodirectional delta function source on one boundary.

(c) Time-dependent problems in a non-multiplying bare sphere without up-scattering of neutrons to evaluate the space, energy and time dependent flux resulting from the incidence of an external source at the centre, the time behaviour of the source being described by a delta-function or the Gaussian distribution.

(d) Time-dependent problems in a non-multiplying homogeneous slab without up-scattering of neutrons to evaluate the space, angle, energy and time dependent flux in the slab with a point isotropic or monodirectional source (described by a delta function or the Gaussian distribution in time) on one boundary.

2. Mathematical Formulae

Under the assumption of spherically symmetric scattering in the laboratory system, the j_N method has already been developed to deal with neutron transport in a bare sphere and an infinite homogeneous slab with finite thickness[3)4)]. We therefore only summarize the mathematical formulae here.

2.1 Time-dependent problems in a bare sphere

We consider first a bare sphere of radius R within the context of a multi-group (G energy groups) model and the j_N approximation (with an odd value of N). Let Σ_g and v_g be the macroscopic total cross section and speed of neutrons in the g-th group respectively and $c(g'\to g)$ the mean number of secondaryneutrons produced in the g-th group as a result of a collision in the g'-th group.

The number of neutrons at the radial co-ordinate r and at time t resulting from a neutron source $S_g\delta(t)\delta(r)/(4\pi r^2)$, in the case where R/v_g is finite, is written as[3] ([M] being the largest integer less than or equal to M)

$$v_g n_g(r,t) = S_g \exp(-\Sigma_g v_g t)\,\delta(t - r/v_g)/(4\pi r^2)$$
$$+ (1/r)\sum_{j=1}^{[(N+1)/2]G}\sum_{n=0}^{[N/2]} B_{2n+1}(g,s_j)\,G_{2n+1}(\Sigma_g R, r/R, \Sigma_1 v_1 s_j)\exp[\Sigma_1 v_1(s_j-1)t], \quad (1)$$

where

$$G_m(\alpha_g,\xi,s) \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{dk}{k}\exp(-i\alpha_g\xi k)\,j_m(\alpha_g k)\int_0^{\infty}\frac{dy}{y}\exp(-P_g y)\sin(ky), \quad (2)$$

in which $j_n(x)$ is the n-th order spherical Bessel function and $P_g \equiv 1-(\Sigma_1 v_1 - s)/(\Sigma_g v_g)$. The explicit expression for $G_m(\alpha_g,\xi,s)$ is shown in the Appendix 1, Section 4. In Eq. (1), $s = \Sigma_1 v_1 s_j$ and $B_m(g,s_j)$ stand for a real pole and the residue of $B_m(g,s)$ which satisfies the following linear equations (the notation $\sum_{n=0}^{N}{}'$ indicates a sum only over odd values of n):

$$\frac{1}{2m+1}B_m(g',s) = \sum_{g=1}^{G} c(g\to g')\sum_{n=0}^{N}{}' J_{mn}(\Sigma_g R, s)\,B_n(g,s)$$
$$+ \frac{i}{2\pi}\sum_{g=1}^{G} S_g\, c(g\to g')\,\Sigma_g A_m(\Sigma_g R, s), \quad m = 1, 3, \cdots, N, \quad (3)$$

where

$$J_{mn}(\alpha_g, s) \equiv \frac{\alpha_g}{\pi}\int_{-\infty}^{\infty}\frac{dz}{z}j_m(\alpha_g z)j_n(\alpha_g z)\int_0^{\infty}\frac{dy}{y}\exp(-P_g y)\sin(zy), \tag{4}$$

$$A_m(\alpha_g, s) \equiv \frac{\alpha_g}{\pi}\int_0^{\alpha_g}dy\,\exp(-P_g y)\int_{-\infty}^{\infty}dz\,z\,j_0(yz)j_m(\alpha_g z). \tag{5}$$

The expressions for $J_{mn}(\alpha_g, s)$ and $A_m(\alpha_g, s)$ are summarized respectively in the Appendix 1, Sections 1 and 8.

In the absence of up-scattering of neutrons in a non-multiplying bare sphere, Eq. (3) is reduced to

$$\Big[\frac{1}{2m+1} - c(g'\to g')J_{mm}(\Sigma_{g'}R, s)\Big]B_m(g', s) - c(g'\to g')\sum_{\substack{n=0\\ n\neq m}}^{N}{}' J_{mn}(\Sigma_{g'}R, s)B_n(g', s)$$
$$= \frac{i}{2\pi}\sum_{g=1}^{g'} S_g c(g\to g')\Sigma_g A_m(\Sigma_g R, s) + \sum_{g=1}^{g'-1} c(g\to g')\sum_{n=0}^{N}{}' J_{mn}(\Sigma_g R, s)B_n(g, s),$$
$$m = 1, 3, \cdots, N. \tag{6}$$

This equation indicates that the problem of finding the pole $s = \Sigma_1 v_1 s_j$ of $B_m(g, s)$ is the same as that in a one-group model:

$$\det\ \Big|\ \frac{\delta_{mn}}{2m+1} - c(g\to g)J_{mn}(\Sigma_g R, s)\ \Big| = 0, \quad m, n = 1, 3, 5, \cdots, N, \tag{7}$$

and the total number of poles for the g-th group is $[(N+1)/2]g$ instead of $[(N+1)/2]G$, including all poles of the higher groups due to the presence of the last term on the right-hand side of Eq. (6).

2.2 Stationary problems in a bare sphere

From Eq. (1), the asymptotic behaviour as $t\to\infty$ can be written as

$$v_g n_g(r, t) \sim (1/r)\sum_{n=0}^{[N/2]} B_{2n+1}(g, s_1)G_{2n+1}(\Sigma_g R, r/R, \Sigma_1 v_1 s_1)\exp[\Sigma_1 v_1(s_1 - 1)t], \tag{8}$$

where $s=\Sigma_1 v_1 s_1$ stands for the largest pole of $B_m(g,s)$ whose value is to be obtained by solving the determinantal equation:

$$\det \left| \frac{\delta_{gg'}\delta_{mn}}{2m+1} - c(g\to g')J_{mn}(\Sigma_g R, \Sigma_1 v_1 s_1) \right| = 0,$$
$$g, g' = 1, 2, \cdots, G \quad \text{for} \quad m, n = 1, 3, 5, \cdots, N, \tag{9}$$

which gives the asymptotic time constant $s_1 - 1$ as a function of the physical properties of a reactor and the geometrical dimension.

For a critical reactor, s_1 must be equal to unity and Eq. (9) with $s_1=1$ therefore gives the critical condition. In order to obtain the value of the effective multiplication factor k_{eff} for a given reactor, $c(g\to g')$ is divided into two parts. These are the scattering part $c_s(g\to g') = \Sigma_s(g\to g')/\Sigma_g$ and the fission part $c_f(g\to g') = \chi_{g'}(\nu\Sigma_f)_g/\Sigma_g$ where χ_g stands for the proportion of fission neutrons born in the g-th group. Using this separation, the value of k_{eff} is obtained by solving Eq. (9) with $s_1=1$ and

$$c(g\to g') = c_s(g\to g') + c_f(g\to g')/k_{eff}. \tag{10}$$

The ratios between the residues $B_m(g, s_1)$ can now be obtained by the use of Eq. (3) with $S_g=0$ and Eq. (9) for any of the above-mentioned three problems, that is, the evaluation of the time constant, critical condition or k_{eff}. Having thus obtained the residues, the flux distribution can be obtained from Eq. (8) for each problem.

2.3 Time-dependent problems in a homogeneous slab

We now consider here an infinite homogeneous slab with finite thickness a by assuming a neutron source $S_g(\mu,t)$ incident with the direction cosine μ upon the surface at the space co-ordinate $x=0$. The number of the g-th group neutrons is then written as[4)]

$$
\begin{aligned}
v_g n_g(x,\mu,t) &= (1/\mu)\, S_g(\mu, t - x/(v_g\mu)) \exp(-\Sigma_g x/\mu) \\
&+ \sum_j \sum_{n=0}^{N} B_n(g, s_j)\, F_n(\Sigma_g a/2, x/a, \mu, \Sigma_1 v_1 s_j) \exp[\Sigma_1 v_1 (s_j - 1)t] \\
&+ \frac{1}{2\pi}\int_{-\infty}^{\infty} dy \exp[-(\Sigma_g v_g - iy)t] \sum_{n=0}^{N} B_n(g, iy + \Sigma_1 v_1 - \Sigma_g v_g) \\
&\qquad \times F_n(\Sigma_g a/2, x/a, \mu, iy + \Sigma_1 v_1 - \Sigma_g v_g),
\end{aligned}
\tag{11}
$$

where

$$
F_n(\alpha_g, \xi, \mu, s) \equiv \frac{1}{4\pi}\int_{-\infty}^{\infty} dz \exp[i\alpha_g z(1-2\xi)]\, j_n(\alpha_g z) \int_0^{\infty} dy \exp[-(P_g - iz\mu)y], \tag{12}
$$

the explicit expressions for $F_n(\alpha_g, \xi, \mu, s)$ and $F_n(\alpha_g, \xi, \mu, iy + \Sigma_1 v_1 - \Sigma_g v_g)$ being given respectively in the Appendix 1, Sections 6 and 7. In the second term on the right-hand side of Eq. (11), the summation is performed only over the contribution coming from the poles of the g'-th group (g' = 1,2, ..,g,...,G) which satisfy the condition $s_{jg'} > 1 - \Sigma_g v_g/(\Sigma_1 v_1)$ [at most, j = 1,2,..., (N+1)G]. The function $B_n(g, s)$ satisfies the following equation:

$$
\begin{aligned}
\frac{1}{2n+1} B_n(g', s) &= \sum_{g=1}^{G} c(g \to g') \sum_{m=0}^{N} J_{nm}(\Sigma_g a/2, s)\, B_m(g, s) \\
&+ \sum_{g=1}^{G} c(g \to g')\, \Sigma_g a\, S_g C_n(\Sigma_g a/2, s), \quad n = 0, 1, 2, \cdots, N,
\end{aligned}
\tag{13}
$$

where

$$
\begin{aligned}
S_g C_n(\alpha_g, s) \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty} dz \exp(-i\alpha_g z)\, j_n(\alpha_g z) \int_0^1 d\mu \int_0^{\infty} dt\, S_g(\mu, t) \exp[-(s - \Sigma_1 v_1)t] \\
\times \int_0^{\infty} dy \exp[-(P_g - iz\mu)y].
\end{aligned}
\tag{14}
$$

For three cases where

(a) $S_g(\mu,t)=S_g\,\delta(t)$ (plane isotropic source),

(b) $S_g(\mu,t)=2S_g\,\mu\,\delta(t)$ (point isotropic source),

(c) $S_g(\mu,t)=S_g\,\delta(\mu-\mu_1)\,\delta(t)$ (monodirectional source),

the integral $C_n(\alpha_g,s)$ takes respectively the following forms:

$$C_n^a(\alpha_g,s)=\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{dz}{z}\exp(-i\alpha_g z)\,j_n(\alpha_g z)\int_0^{\infty}\frac{dy}{y}e^{-\beta_g y}(e^{izy}-1), \tag{15}$$

$$C_n^b(\alpha_g,s)=\frac{1}{\pi i}\int_{-\infty}^{\infty}\frac{dz}{z}\exp(-i\alpha_g z)\,j_n(\alpha_g z)\int_0^{\infty}\frac{dy}{y}e^{-\beta_g y}\left[\left(1+\frac{i}{zy}\right)e^{izy}-\frac{i}{zy}\right], \tag{16}$$

$$C_n^c(\alpha_g,s)=2F_n(\alpha_g,1,\mu_1,s), \tag{17}$$

the explicit expressions for $C_n^a(\alpha_g,s)$, $C_n^b(\alpha_g,s)$ and $C_n^c(\alpha_g,s)$ being shown in the Appendix 1, Sections 9, 10 and 12, respectively.

Since $J_{mn}(\alpha_g,s)=0$ when m+n = odd, a system of linear equations (13) can be split into two sets; one contains only the terms with even values of n and m and the other contains only those with odd values of n and m. Hence, for a non-multiplying slab in which there is no up-scattering of neutrons, the poles of $B_n(g,s)$ are to be obtained by solving two determinantal equations [see Eq. (7)] :

$$\det\left|\frac{\delta_{nm}}{2n+1}-c(g\to g)\,J_{nm}(\Sigma_g a/2,s)\right|=0, \tag{18}$$

$$n,m = 0,2,4,\ldots,N-1 \quad \text{or} \quad n,m = 1,3,5,\ldots,N,$$

under the condition that the value $s=\Sigma_1 v_1 s_j$ should be larger than $\Sigma_1 v_1-\Sigma_g v_g$. In this case, the maximum number of the poles for the g-th group is (N+1)g instead of (N+1)G.

Upon integrating Eq. (11) over μ from -1 to 1, the total flux is obtained in the form:

$$
\begin{aligned}
v_j n_j(x,t) = &\int_0^1 \frac{d\mu}{\mu} S_j(\mu, t-\frac{x}{v_j\mu}) \exp(-\frac{\Sigma_j x}{\mu}) \\
&+ \sum_j \sum_{n=0}^{N} B_n(j, s_j) G_n(\Sigma_j a/2, 2x/a-1, \Sigma_1 v_1 s_j) \exp[\Sigma_1 v_1 (s_j - 1)t] \\
&+ \frac{1}{2\pi}\int_{-\infty}^{\infty} dy \exp[-(\Sigma_j v_j - iy)t] \sum_{n=0}^{N} B_n(j, iy+\Sigma_1 v_1 - \Sigma_j v_j) \\
&\qquad \times G_n(\Sigma_j a/2, 2x/a-1, iy+\Sigma_1 v_1 - \Sigma_j v_j).
\end{aligned}
\tag{19}
$$

The expression for $G_n(\alpha_j, 2\xi-1, iy+\Sigma_1 v_1-\Sigma_j v_j)$ is shown in the Appendix 1, Section 5. Furthermore, the total number of neutrons reflected by or transmitted through the slab is obtained by integrating $|\mu| v_j n_j(x_0,\mu,t)$ ($x_0=0$ to observe neutrons reflected by the slab or $x_0=a$ for neutrons transmitted through it) over μ from -1 to zero or from zero to 1. This gives the form [see Appendix 1, Section 11 for the expression of $C_n^{\pm}(\alpha_j, iy+\Sigma_1 v_1-\Sigma_j v_j)$]:

$$
\begin{aligned}
\int d\mu |\mu| v_j n_j(x_0,\mu,t) = &\int_0^1 d\mu\, S_j(\mu, t-\frac{a}{v_j\mu}) \exp(-\frac{\Sigma_j a}{\mu})]_{x_0=a} \\
&+ \frac{1}{4}\sum_j \sum_{n=0}^{N} (2\frac{x_0}{a}-1)^n B_n(j, s_j) C_n^{\pm}(\frac{\Sigma_j a}{2}, \Sigma_1 v_1 s_j) \exp[\Sigma_1 v_1 (s_j-1)t] \\
&+ \frac{1}{8\pi}\int_{-\infty}^{\infty} dy \exp[-(\Sigma_j v_j - iy)t] \sum_{n=0}^{N} (2\frac{x_0}{a}-1)^n B_n(j, iy+\Sigma_1 v_1-\Sigma_j v_j) \\
&\qquad \times C_n^{\pm}(\Sigma_j a/2, iy+\Sigma_1 v_1-\Sigma_j v_j).
\end{aligned}
\tag{20}
$$

2.4 Stationary problems in a homogeneous slab

For a subcritical system with a stationary boundary source $S_j(\mu)$, only one largest pole $s=\Sigma_1 v_1$ of $B_m(j,s)$ is of importance. Hence, by multiplying $s-\Sigma_1 v_1$ on both sides of Eq. (13) and taking the limit $s \to \Sigma_1 v_1$, we get

$$\frac{1}{2n+1}B_n(g') = \sum_{g=1}^{G} c(g\to g')\sum_{m=0}^{N} J_{nm}(\Sigma_g a/2, \Sigma_1 v_1)B_m(g)$$
$$+\sum_{g=1}^{G} c(g\to g')\Sigma_g a S_g C_n(\Sigma_g a/2), \quad n=0,1,2,\cdots,N, \tag{21}$$

where $B_n(g) = \lim_{s\to\Sigma_1 v_1}(s-\Sigma_1 v_1)B_n(g,s)$ and $S_g C_n(\alpha_g) = \lim_{s\to\Sigma_1 v_1}(s-\Sigma_1 v_1)S_g C_n(\alpha_g, s)$ which takes the form given by Eq. (15), (16) or (17) with $s=\Sigma_1 v_1$ when the angular distribution of the source is plane isotropic, point isotropic or monodirectional.

The stationary vector flux, scalar flux and the total number of leakage neutrons can thus be written as follows:

$$v_g n_g(x,\mu) = \frac{1}{\mu}S_g(\mu)\exp(-\Sigma_g x/\mu) + \sum_{n=0}^{N} B_n(g)F_n(\Sigma_g a/2, x/a, \mu, \Sigma_1 v_1), \tag{22}$$

$$v_g n_g(x) = \int_0^1 \frac{d\mu}{\mu}S_g(\mu)\exp(-\Sigma_g x/\mu) + \sum_{n=0}^{N} B_n(g)G_n(\Sigma_g a/2, 2x/a-1, \Sigma_1 v_1), \tag{23}$$

$$\int d\mu|\mu| v_g n_g(x_0,\mu) = \int_0^1 d\mu\, S_g(\mu)\exp(-\Sigma_g a/\mu)\Big]_{x_0=a}$$
$$+\frac{1}{4}\sum_{n=0}^{N}\left(2\frac{x_0}{a}-1\right)^n B_n(g)C_n^{+}(\Sigma_g a/2, \Sigma_1 v_1). \tag{24}$$

In addition, it is also easy to obtain the time moments of the time-dependent flux. For example, the first three time moments of the angular flux (11) is written as[5)]

$$\int_0^\infty dt\, v_g n_g(x,\mu,t) = \int_0^\infty dt\,\frac{1}{\mu}S_g\left(\mu, t-\frac{x}{v_g\mu}\right)\exp\left(-\frac{\Sigma_g x}{\mu}\right)$$
$$+\sum_{n=0}^{N} B_n(g,\Sigma_1 v_1)F_n(\Sigma_g a/2, x/a, \mu, \Sigma_1 v_1), \tag{25}$$

$$\int_0^\infty dt\, t\, v_g n_g(x,\mu,t) = \int_0^\infty dt\, t\frac{1}{\mu}S_g\left(\mu, t-\frac{x}{v_g\mu}\right)\exp\left(-\frac{\Sigma_g x}{\mu}\right)$$
$$-\frac{d}{ds}\Big[\sum_{n=0}^{N} B_n(g,s)F_n(\Sigma_g a/2, x/a, \mu, s)\Big]_{s=\Sigma_1 v_1}, \tag{26}$$

$$\int_0^\infty dt\, t^2 v_g n_g(x,\mu,t) = \int_0^\infty dt\, t^2 \frac{1}{\mu} S_g(\mu, t - \frac{x}{v_g\mu}) \exp(-\frac{\Sigma_g x}{\mu})$$
$$+ \frac{d^2}{ds^2}[\sum_{n=0}^{N} B_n(g,s) F_n(\Sigma_g a/2, x/a, \mu, s)]_{s=\Sigma_1 v_1} . \qquad (27)$$

From a comparison between Eqs. (13) and (21), it is seen as expected that the zeroth moment of the flux due to an incident delta function source $S_g(\mu,t) = S_g(\mu)\delta(t)$ is equal to the stationary flux (22).

3. Procedures for Evaluating the Pole, Residue and Contribution of the Continuous Spectrum for a Non-Multiplying Medium without Up-scattering of Neutrons

3.1 Approximate values of the pole

We summarize here the procedure to find an approximate value of the pole, which is required for solving Eq. (7) or (18).

Figure 1 shows the curves giving 1/c as a function of Σa obtained by solving the determinantal equation (18) of the elements with even values of n and m by fixing the value $s = \Sigma v$, within the context of a one-group model and the j_3, j_5 or j_7 approximation (N = 3,5 or 7). (Note that the smallest c gives the value required to keep a slab of thickness Σa critical). In order to find the poles of the g-th group with $c(g \to g)$ and $\Sigma_g a$, the diagonal of a rectangle with sides $1/c(g \to g)$ and $\Sigma_g a$ is drawn as illustrated in Fig. 1. The points of intersection between the curves and the diagonal should have the abscissa $\Sigma_g a P_g$. In the example shown in Fig. 1 $[c(g \to g) = 1$ and $\Sigma_g a = 15]$, it is not clear if the lowest curve of the j_7 approximation intersects with the diagonal. In such cases, the asymptotic expression for the pole with a small absolute value of $\Sigma_g a P_g$ should be taken into consideration. The expressions in the j_3, j_5 and j_7 approximations are respectively written as follows $[c\alpha_g \equiv c(g \to g)\Sigma_g a/2$ and γ being the Euler Mascheroni constant $]$:

$$\Sigma_g a P_g \sim \exp\left[\frac{3}{2}-\gamma-\frac{1}{c\alpha_g}-\begin{cases}\dfrac{1}{12}\dfrac{1}{1-12/(5c\alpha_g)}\Bigg], \\[2ex] \dfrac{1}{10}\dfrac{1-144/(35c\alpha_g)}{[1-2.200260/(c\alpha_g)][1-5.342597/(c\alpha_g)]}\Bigg], \\[2ex] \dfrac{89}{840}\dfrac{[1-3.696539/(c\alpha_g)][1-7.795340/(c\alpha_g)]}{\left[1-\frac{2.198517_5}{c\alpha_g}\right]\left[1-\frac{4.279830_5}{c\alpha_g}\right]\left[1-\frac{9.265908}{c\alpha_g}\right]}\Bigg],\end{cases}\right. \tag{28}$$

which shows that, for example, in the j_7 approximation there are 4 poles for $c\alpha_g >$ 9.265908, 3 poles for 9.265908 $> c\alpha_g >$ 4.279830$_5$, 2 poles for 4.279830$_5 > c\alpha_g >$ 2.198517$_5$ and only 1 pole for 2.198517$_5 > c\alpha_g > 0$. On the other hand, when $\Sigma_g a P_g \gg 1$, the value of P_g is approximately equal to $c(g\to g)$.

The approximate value of the pole coming out of Eq. (18) with odd values of n and m or Eq. (7) can be found by following the same procedure as mentioned above with the help of Fig. 2. (Note that the negative values of P_g are applicable only for spherical geometry.) In this case, the asymptotic expressions for the pole with small $|\alpha_g P_g|$ ($\alpha_g = \Sigma_g a/2$ or $\Sigma_g R$) in the j_3, j_5 and j_7 approximations are respectively given by

$$\alpha_g P_g \sim \begin{cases}\dfrac{35}{24}\left(1-\dfrac{1.277171}{c\alpha_g}\right)\left(1-\dfrac{3.865686}{c\alpha_g}\right)\Big/\left(1-\dfrac{16}{5c\alpha_g}\right), \\[2ex] \dfrac{5005}{3856}\dfrac{[1-1.277148/(c\alpha_g)][1-3.281392/(c\alpha_g)][1-7.285614/(c\alpha_g)]}{[1-2.775481/(c\alpha_g)][1-6.590257/(c\alpha_g)]}, \\[2ex] 1.206287_5\dfrac{\left[1-\frac{1.277137}{c\alpha_g}\right]\left[1-\frac{3.263428}{c\alpha_g}\right]\left[1-\frac{5.423541}{c\alpha_g}\right]\left[1-\frac{11.748197}{c\alpha_g}\right]}{\left[1-\frac{2.741699}{c\alpha_g}\right]\left[1-\frac{4.994001}{c\alpha_g}\right]\left[1-\frac{11.010044}{c\alpha_g}\right]}.\end{cases} \tag{29}$$

From these expressions, it is seen that, for example, in the j_7 approximation there are 4 positive poles when $c\alpha_g >$ 11.748197, 3 when 11.748197 $> c\alpha_g >$ 5.423541, 2 when 5.423541 $> c\alpha_g >$ 3.263428, 1 when 3.263428 $> c\alpha_g >$ 1.277137 and none when 1.277137 $> c\alpha_g > 0$.

3.2 Evaluation of the residue

The procedure to evaluate the residue by solving Eq. (6) or (13) for a non-multiplying sphere or slab will be shown here by the use of Eq. (13) with odd values of n in the j_5 approximation $[cJ_{nm} \equiv c(g\to g)J_{nm}(\Sigma_g a/2, s)]$:

$$\begin{pmatrix} \frac{1}{3}-cJ_{11} & -cJ_{13} & -cJ_{15} \\ -cJ_{13} & \frac{1}{7}-cJ_{33} & -cJ_{35} \\ -cJ_{15} & -cJ_{35} & \frac{1}{11}-cJ_{55} \end{pmatrix} \begin{pmatrix} B_1(g,s) \\ B_3(g,s) \\ B_5(g,s) \end{pmatrix} = \begin{pmatrix} Z_1(g,s) \\ Z_3(g,s) \\ Z_5(g,s) \end{pmatrix}, \tag{30}$$

where

$$Z_m(g,s) \equiv \sum_{g'=1}^{g} S_{g'} c(g'\to g)\Sigma_{g'} a C_m(\Sigma_{g'}a/2, s) + \sum_{g'=1}^{g-1} c(g'\to g) \sum_{n=1,3,5} J_{mn}(\Sigma_{g'}a/2, s) B_n(g', s). \tag{31}$$

Equation (30) leads, for example, to

$$B_3(g,s_j) = \lim_{s\to\Sigma_1 V_1 s_j} \frac{s-\Sigma_1 V_1 s_j}{\Delta} \begin{vmatrix} 3cJ_{11}-1 & -3Z_1(g,s) & 3cJ_{15} \\ 7cJ_{13} & -7Z_3(g,s) & 7cJ_{35} \\ 11cJ_{15} & -11Z_5(g,s) & 11cJ_{55}-1 \end{vmatrix}, \tag{32}$$

where

$$\Delta \equiv \begin{vmatrix} 3cJ_{11}-1 & 3cJ_{13} & 3cJ_{15} \\ 7cJ_{13} & 7cJ_{33}-1 & 7cJ_{35} \\ 11cJ_{15} & 7cJ_{35} & 11cJ_{55}-1 \end{vmatrix}. \tag{33}$$

Hence, for the pole $s=\Sigma_1 V_1 s_{jg}$ of the g-th group obtained from the equation $\Delta = 0$,

$$B_3(g,\lambda_{jg}) = \left|\begin{matrix} 3cJ_{11}-1 & -3Z_1(g,\lambda) & 3cJ_{15} \\ 7cJ_{13} & -7Z_3(g,\lambda) & 7cJ_{35} \\ 11cJ_{15} & -11Z_5(g,\lambda) & 11cJ_{55}-1 \end{matrix}\right|_{\lambda=\Sigma_1 v_1 \lambda_{jg}} \Bigg/ \frac{d}{d\lambda}\left|\begin{matrix} 3cJ_{11}-1 & 3cJ_{13} & 3cJ_{15} \\ 7cJ_{13} & 7cJ_{33}-1 & 7cJ_{35} \\ 11cJ_{15} & 11cJ_{35} & 11cJ_{55}-1 \end{matrix}\right|_{\lambda=\Sigma_1 v_1 \lambda_{jg}}, \tag{34}$$

the explicit expression for $\frac{d}{d\lambda}J_{nm}(\alpha_g,\lambda)$ being given in Appendix 1, Section 3.

On the other hand, for the pole $\lambda=\Sigma_1 v_1 \lambda_{jg'}$ of the higher g'-th group,

$$B_3(g,\lambda_{jg'}) = \frac{1}{\Delta_{\lambda=\Sigma_1 v_1 \lambda_{jg'}}}\left|\begin{matrix} 3cJ_{11}-1 & -3\sum_{h=g'}^{g-1} c(h\to g)\sum_{n=1,3,5} J_{1n}(\Sigma_h a/2,\lambda)B_n(h,\lambda_{jg'}) & 3cJ_{15} \\ 7cJ_{13} & -7\sum_h c(h\to g)\sum_n J_{3n}(\Sigma_h a/2,\lambda)B_n(h,\lambda_{jg'}) & 7cJ_{35} \\ 11cJ_{15} & -11\sum_h c(h\to g)\sum_n J_{5n}(\Sigma_h a/2,\lambda)B_n(h,\lambda_{jg'}) & 11cJ_{55}-1 \end{matrix}\right|_{\lambda=\Sigma_1 v_1 \lambda_{jg'}}, \tag{35}$$

which shows that all poles of the higher g'-the group (g' = 1,2,...,g-1) are also the poles for the g-th group if all values of $c(g'\to g'+1)$ are not equal to zero (this condition means that there is the slowing down of neutrons from a certain group to the next group). For a homogeneous slab, however, the condition that $\lambda_{jg'} > 1-\Sigma_g v_g/(\Sigma_1 v_1)$ excludes some poles because the $\lambda_{jg'}$ has been obtained under the condition that $\lambda_{jg'} > 1-\Sigma_{g'}v_{g'}/(\Sigma_1 v_1)$ [this implies also the condition that $\lambda_{jg'} > 1-\Sigma_h v_h/(\Sigma_1 v_1)$ for h = 1,2,...,g'-1, as seen from the presence of the second term on the right-hand side of Eq. (31)] . In other words, some poles of the g'-th group for which $\Sigma_{g'}v_{g'} > \Sigma_g v_g$ may not be the poles of interest for the g-th group.

3.3 Contribution of the continuous spectrum

For time-dependent problems in a homogeneous slab, we have to evaluate the last term on the right-hand side of Eq. (11), (19) or (20). The contribution of the continuous spectrum represented by, for example, the last term of Eq. (20) can be rewritten in the j_5 approximation as follows:

$$\frac{v_g}{2\pi a}\exp(-\Sigma_g v_g t)\int_0^\infty \frac{dy}{y^2}\sum_{n=0}^{5}\left(2\frac{x_0}{a}-1\right)^n \Big[\left(B_{n1g}C_{n1g}^{b}-B_{n2g}C_{n2g}^{b}\right)\cos(yt)$$
$$-\left(B_{n2g}C_{n1g}^{b}+B_{n1g}C_{n2g}^{b}\right)\sin(yt)\Big], \tag{36}$$

where

$$B_n(g,\, iy+\Sigma_1 v_1-\Sigma_g v_g)\equiv\frac{1}{y}\left(B_{n1g}+iB_{n2g}\right),$$
$$C_n^{b}(\Sigma_g a/2,\, iy+\Sigma_1 v_1-\Sigma_g v_g)\equiv\frac{2v_g}{ay}\left(C_{n1g}^{b}+iC_{n2g}^{b}\right),$$

and use has been made of the fact that $B_{nmg}(y)$ and $C_{nmg}^{b}(y)$ are even or odd functions of y depending on whether n+m = even or odd.

Since $B_n(g,\, iy+\Sigma_1 v_1-\Sigma_g v_g)$ is written as [$cJ_{nm}\equiv c(g\to g)J_{nm}(\Sigma_g a/2,\, iy+\Sigma_1 v_1-\Sigma_g v_g)$; see Eqs. (30), (31) and (33)]

$$B_3(g,\, iy+\Sigma_1 v_1-\Sigma_g v_g)=\frac{1}{\Delta_{s=iy+\Sigma_1 v_1-\Sigma_g v_g}}\begin{vmatrix} 3cJ_{11}-1 & -3Z_1(g,\, iy+\Sigma_1 v_1-\Sigma_g v_g) & 3cJ_{15} \\ 7cJ_{13} & -7Z_3(g,\, iy+\Sigma_1 v_1-\Sigma_g v_g) & 7cJ_{35} \\ 11cJ_{15} & -11Z_5(g,\, iy+\Sigma_1 v_1-\Sigma_g v_g) & 11cJ_{55}-1 \end{vmatrix}, \tag{37}$$

B_{n1g} and B_{n2g} can be evaluated successively starting with g = 1 by using the expressions shown in the Appendix 1 [Section 2 for $J_{nm}(\alpha_g,\, iy+\Sigma_1 v_1-\Sigma_g v_g)$ and Section 11 or 13 for C_n^{b} or $C_n^{c}(\alpha_g,\, iy+\Sigma_1 v_1-\Sigma_g v_g)$ to obtain $Z_n(g,\, iy+\Sigma_1 v_1-\Sigma_g v_g)$ for the case where the angular distribution of the boundary source is point isotropic or monodirectional].

When $y\to\infty$, however, the value of the integrand of Eq. (36) changes very rapidly as a function of y. The evaluation of the integral over y from $y_0\gg 1$ to ∞ is therefore performed separately by the use of asymptotic expressions for the functions J_{nm}, C_n^{b}, C_n^{c}, G_n and/or F_n for large y [or for large $ay/(2v_g)$] , which can easily be obtained from the expressions shown in the Appendix 1, Sections 2, 11, 13, 5 and/or 7. In the case where the boundary source is monodirectional, this gives in the j_7 approximation the following forms ($\xi\equiv x/a$) :

(a) For the angular flux $v_g n_g(x,\mu,t)$ with $\mu>0$ written by Eq. (11);

$$
\begin{aligned}
&\int_{y_0}^{\infty}\frac{dy}{y^2}\sum_{n=0}^{7}\left[(B_{n1g}F_{n1g}-B_{n2g}F_{n2g})\cos(yt)-(B_{n2g}F_{n1g}+B_{n1g}F_{n2g})\sin(yt)\right]\\
&\sim\frac{1}{4}(3+7+11+15+1+5+9+13)\sum_{g'=1}^{g}c(g'\to g)\Sigma_{g'}v_{g'}S_{g'}\int_{y_0}^{\infty}\frac{dy}{y^2}\cos\left[\left(\frac{a\xi}{v_{g'}\mu}-t\right)y\right]\\
&-\frac{1}{4}(3X_2+7X_3+11X_4+15X_5+1+5X_6+9X_7+13X_8)\sum_{g'=1}^{g}c(g'\to g)\Sigma_{g'}v_{g'}S_{g'}\int_{y_0}^{\infty}\frac{dy}{y^2}\cos(yt)\\
&+\frac{1}{4}(3+7+11+15-1-5-9-13)\sum_{g'=1}^{g}c(g'\to g)\Sigma_{g'}v_{g'}S_{g'}\int_{y_0}^{\infty}\frac{dy}{y^2}\cos\left[\left(\frac{a}{v_{g'}\mu_1}+\frac{a\xi}{v_{g'}\mu}-t\right)y\right]\\
&-\frac{1}{4}(3X_2+7X_3+11X_4+15X_5-1-5X_6-9X_7-13X_8)\\
&\qquad\times\sum_{g'=1}^{g}c(g'\to g)\Sigma_{g'}v_{g'}S_{g'}\int_{y_0}^{\infty}\frac{dy}{y^2}\cos\left[\left(\frac{a}{v_{g'}\mu_1}-t\right)y\right],
\end{aligned}
\tag{38}
$$

where

$$
\begin{aligned}
&X_2\equiv 1-2\xi,\quad X_3\equiv(1-2\xi)[1-10\xi(1-\xi)],\quad X_4\equiv(1-2\xi)[1-28\xi(1-\xi)(1-\tfrac{9}{2}\xi(1-\xi))],\\
&X_5\equiv(1-2\xi)\{1-54\xi(1-\xi)[1-11\xi(1-\xi)(1-\tfrac{26}{9}\xi(1-\xi))]\},\quad X_6\equiv 1-6\xi(1-\xi),\\
&X_7\equiv 1-20\xi(1-\xi)[1-\tfrac{7}{2}\xi(1-\xi)],\quad X_8\equiv 1-42\xi(1-\xi)\{1-9\xi(1-\xi)[1-\tfrac{22}{9}\xi(1-\xi)]\}.
\end{aligned}
$$

Since $F_n(\alpha_g,\xi,\mu,s)=(-1)^n F_n(\alpha_g,1-\xi,-\mu,s)$, the value for the angle $-\mu$ and the space $a-x$ (or $1-\xi$) can be obtained by changing the sign of the first 4 terms in the parentheses of each term on the right-hand side of Eq. (38).

(b) For the total flux $v_g n_g(x,t)$ with $x\le a/2$ written by Eq. (19);

$$
\int_{y_0}^{\infty}\frac{dy}{y^2}\sum_{n=0}^{7}\left[(B_{n1g}G_{n1g}-B_{n2g}G_{n2g})\cos(yt)-(B_{n2g}G_{n1g}+B_{n1g}G_{n2g})\sin(yt)\right]
$$

$$\sim\begin{cases}-\frac{1}{4}(3+7+11+15+1+5+9+13)\sum_{g'=1}^{g}c(g'\to g)\Sigma_{g'}v_{g'}S_{g'}\int_{y_0}^{\infty}\frac{dy}{y^2}\cos(yt) \\ \quad -\frac{1}{4}(3+7+11+15-1-5-9-13)\sum_{g'=1}^{g}c(g'\to g)\Sigma_{g'}v_{g'}S_{g'}\int_{y_0}^{\infty}\frac{dy}{y^2}\cos\left[\left(\frac{a}{v_{g'}\mu_1}-t\right)y\right], \\ \qquad\qquad\qquad\text{for } \xi=0, \\ -\frac{1}{2}(3X_2+7X_3+11X_4+15X_5+1+5X_6+9X_7+13X_8)\sum_{g'=1}^{g}c(g'\to g)\Sigma_{g'}v_{g'}S_{g'}\int_{y_0}^{\infty}\frac{dy}{y^2}\cos(yt) \\ \quad -\frac{1}{2}(3X_2+7X_3+11X_4+15X_5-1-5X_6-9X_7-13X_8) \\ \quad \times\sum_{g'=1}^{g}c(g'\to g)\Sigma_{g'}v_{g'}S_{g'}\int_{y_0}^{\infty}\frac{dy}{y^2}\cos\left[\left(\frac{a}{v_{g'}\mu_1}-t\right)y\right], \quad \text{for } 0.5\geq\xi>0.\end{cases} \tag{39}$$

The value for $\xi > 0.5$ can be obtained by changing the sign of the first 4 terms in the parentheses of each term on the right-hand side because $G_n(\alpha_g,\xi,s)=(-1)^n G_n(\alpha_g,1-\xi,s)$.

(c) For the total number of leakage neutrons $\int d\mu|\mu|v_g n_g(x_0,\mu,t)$ written by Eq. (20);

$$\int_{y_0}^{\infty}\frac{dy}{y^2}\sum_{n=0}^{7}\left(2\frac{x_0}{a}-1\right)^n\left[\left(B_{n1g}C_{n1g}^{b}-B_{n2g}C_{n2g}^{b}\right)\cos(yt)-\left(B_{n2g}C_{n1g}^{b}+B_{n1g}C_{n2g}^{b}\right)\sin(yt)\right]$$

$$\sim\frac{1}{2}\left[\left(2\frac{x_0}{a}-1\right)(3+7+11+15)-(1+5+9+13)\right]\sum_{g'=1}^{g}c(g'\to g)\Sigma_{g'}v_{g'}S_{g'}\int_{y_0}^{\infty}\frac{dy}{y^2}\cos(yt)$$

$$+\frac{1}{2}\left[\left(2\frac{x_0}{a}-1\right)(3+7+11+15)+(1+5+9+13)\right]\sum_{g'=1}^{g}c(g'\to g)\Sigma_{g'}v_{g'}S_{g'}\int_{y_0}^{\infty}\frac{dy}{y^2}\cos\left[\left(\frac{a}{v_{g'}\mu_1}-t\right)y\right]. \tag{40}$$

In the j_N approximation, the first $[(N+1)/2]$ terms out of the first 4 and out of the last 4 terms remain in the parentheses of each term on the right-hand side of Eq. (38), (39) or (40). In addition, when the boundary source is point isotropic, the total number of terms on the right-hand side is reduced to be half, for example, only the first 2 terms remain in Eq. (38).

4. JN-METD1 Computer Code

4.1 Input data (see the Appendix 3)

After an ID card with a 20A4 format, 26 integers are read with a 26I3 format. These integers are defined as follows:

IJO	3, 5 or 7 for the j_3, j_5 or j_7 approximation (0 to stop the execution; see the Appendix 2, Section 1)
IIII	0 or 1 in the case where the boundary source is plane or point isotropic when NSPH = 0
NPRINT	1 or 0 when the intermediate results are required or not (1 for NSPH = 1 or NSTAT1 = 1)
NSTATY	1 or 2 for a stationary or time-dependent problem (2 when LLL = 1)
IGRP	Total number of energy groups
IHT IHS IHL	Arrangement of reaction type of the cross section (XSEC) for the g-th group; XSEC(IHT-2,g) = Σ_{ag} , XSEC(IHT-1,g) = Σ_{tg}, XSEC(IHT $\geq$ 3,g) = Σ_{trg} , XSEC(IHT+1,g) = Σ(g+IHS-IHT $\rightarrow$ g), ..., XSEC(IHS-1,g) = Σ(g+1 $\rightarrow$ g), XSEC(IHS $\geq$ IHT,g) = $\Sigma(g \rightarrow g)$, XSEC(IHS+1,g) = $\Sigma(g-1 \rightarrow g)$,..., XSEC(IHL $\geq$ IHS,g) = Σ(g-IHL +IHS $\rightarrow$ g) [Σ_{trg} is used instead of Σ_g (for taking into account the anisotropic scattering of neutrons) and Σ_{tg} is for calculating the values of $c(g \rightarrow g') = \Sigma(g \rightarrow g')/\Sigma_{tg}$. (Σ_{tg} may be equal to Σ_{trg})]
NSPH	0 or 1 for slab or sphere
JOD JJJ NKK N	When NSTATY = 1 and NSPH = 1, JJJ=2, NKK=2 or/and N=2 for critical radius, time constant or/and k_{eff} calculation (1 otherwise and JOD being not used), When NSTATY=1 and NSPH=0, JOD=1, JJJ=1 and NKK=0 (N being not used),

	When NSTATY=2, JOD=1 or 2 for NSPH= 0 or 1, JJJ=1 and NKK=0 for solving a new problem or for LLL=1, JOD, JJJ and NKK for restarting an unfinished problem for which punched cards for the poles and residues are available till the NKK($>$0)-th group of JJJ of JOD [see the subsection (d)], N for evaluating the contribution of the continuous spectrum by using 4N+1 values for NSPH=0, N$\gtrsim$70 being recommended (see the Appendix 2, Section 6), and N=0 for NSPH=1
LLL	0 (1 to obtain only the poles when NSTATY=2)
JNKK	The flux is to be calculated beginning with the JNKK-th group (1 when NSTATY=1 and NSPH=1)
NNN	The flux is to be calculated till the (JNKK+NNN)-th group (IGRP-1 when NSTATY=1 and NSPH = 1)
JJJJ	1/2/3 or 4 for calculating only the total number of leakage neutrons/total number of leakage neutrons and total flux/total flux and angular flux or total number of leakage neutrons, total flux and angular flux (3 when NSPH=1)
NOT	Total number of time points (1 when NSTATY=1)
NOM	0 [total number (odd$\gtrsim$3) of angle points when NSPH=0 and JJJJ$\gtrsim$3]
NOS	Total number ($\gtrsim$2) of space points (NOS$\gtrsim$3 when NSPH=1 and NSTATY=1)
NFLUXR NFLUXS NFLUXK	When NSTATY=1 and NSPH=1, NFLUXR=1, NFLUXS=1 or/and NFLUXK=1 to obtain the flux distribution for JJJ=2, NKK=2 or/and N=2 (0 otherwise), When NSTATY=2 and NKK$>$0, NFLUXR=NNNNN if no poles till the (NNNNN-1)-th group for JJJ of JOD (NFLUXR=1 when NKK=0), When NSTATY=2 and TPINT$>$0, NFLUXK=NOP; the total number of time points for pulse source

IAA	The total number of input cards for the present problem
NSTAT1	1 for obtaining the first and second time moments of the flux due to a $\delta(t)$ source, in addition to evaluating the stationary flux, when NSPH=0 (0 otherwise). [When NSPH=0, NSTATY=2 and LLL=0, NSTAT1=1 for solving a new problem or NSTAT1=0 for re-starting an unfinished problem for which punched cards for the stationary flux are available; see the subsection (c).]
NUPSAT	1 in the case where there is up-scattering of neutrons (or/and fission process) when NSTATY=1 and NSPH=0 (0 otherwise)

Next, 12 floating-point numbers are read with a 6E12.5 format. These are defined as follows:

A	Thickness of slab a or diameter of sphere 2R
DMU1	The value of $\mu_1 > 0$ of the monodirectional boundary source when NSPH=0 (0 otherwise)
EPSS	Required relative accuracy for the value of the pole when NSTATY=2 or for the time constant when NSTATY=1, NSPH=1 and NKK=2
TINT SCAL SCAL1	Time interval of NOT time points for the flux due to a $\delta(t)$ source when NSTATY=2; TINT*(SCAL)$^{g-JNKK+1}$ for the g-th group (g = JNKK,JNKK+1,..., JNKK+NNN-1) and TINT*(SCAL)^{NNN}*(SCAL1) for the (JNKK+NNN)-th group, TINT=SCAL=SCAL1=0 when NSTATY=1
TO TCON	Parameters describing the time behaviour of the pulse source, exp [-TCON*(t-TO)^2]
TSTAT TPINT	To obtain the flux due to the pulse source for the time points TSTAT, TSTAT+TPINT,...,TSTAT+NFLUXK*TPINT [TPINT=0 for obtaining only the flux due to a δ(t)-source or NSTATY=1]

T1 T2	The duration of the pulse source; exp $[-TCON*(t-TO)^2]$ from t=T1 to t=T2

In the subroutine JNMETD, the following data ordered respectively by energy-group beginning with g=1 are read with 8F10.6:

VG	Speed of neutrons $v_g > 0$ (it is recommended to chose the values which are around in the same order of magnitude as A when NSTATY=2 and NSPH=0)
BUCLG	Buckling $(B_y^2+B_z^2)g$ when NSPH=0 or fission spectrum χ_g when NSPH=1 and NSTATY=1
SOCE	Source intensity S_g at the slab boundary when NSPH=0 or at the centre of sphere when NSPH=1 and NSTATY=2, or $(\nu\Sigma_f)_g$ when NSPH=1 and NSTATY=1

The total number of these cards is therefore 3 [(IGRP+7)/8] . Next, the cross section XSEC is read with (8F9.6, F8.5) for all types of reactions arranged as mentioned above in the first group, then for those in the second group and so on, the total number of cards being [(IHL*IGRP+8)/9] .

The remaining input data depending on the input integers are:

(a) When NSTATY=1 and NSPH=1, one card is read with 8F10.6 in the subroutine JNMETD. These are

R1	Fixed radius of sphere for which the time constant (when NKK=2) or/and k_{eff} (when N=2) are calculated, or the first guess for the critical radius (when JJJ=2)
R2	The second guess for the critical radius if JJJ=2
S1 S2	The first and second guess for the time constant if NKK=2
CK1 CK2	The first and second guess for k_{eff} if N=2
EPSR	Required relative accuracy for the critical radius if JJJ=2
EPSK	Required relative accuracy for k_{eff} if N=2

If R2, S2 or CK2 is equal to zero, the input value R1, S1 or CK1 is regarded as the critical radius, time constant or k_{eff} without any iterations.

(b) When NSTATY=1, NSPH=0 and NUPSAT=1, one card is read with 7F10.7 in the subroutine RESCAL to evaluate the fundamental decay constant of neutrons in the system $\Sigma_1 v_1 (1-\lambda_1)$.

S1 S2	The first and second guess for the time constant $\lambda_1 - 1$
EPSS	Required relative accuracy for the time constant

(c) When NSTATY=2, NSPH=0, NSTAT1=0 and LLL=0, a punched card dump for the stationary flux with a (5E15.8) format is read in the JNMETD in the same order as in the punched output or in the output print [for each of (NNN+1) energy groups beginning with the JNKK-th group, NOM values for the angular flux for each space point (when JJJJ$\gtrsim$3) followed by NOS values for the total flux (when JJJJ $\gtrsim$ 2) and then 2 cards for the total number of leakage neutrons at x = a and x = 0 (when JJJJ$\neq$3)] . The total number of cards is 2(NNN+1) when JJJJ=1, (2+[(NOS+4)/5])x(NNN+1) when JJJJ=2, ([(NOS+4)/5]+[(NOM+4)/5]NOS)(NNN+1) when JJJJ=3 or (2+[(NOS+4)/5]+[(NOM+4)/5]NOS)(NNN+1) when JJJJ=4.

(d) When NSTATY=2, NKK >0 and LLL=0 [in case of NSPH=0 and NSTAT1=0, the present data follow the cards described in the Subsection (c)], a punched card dump with a (5E15.8) format for the pole and residue is read in the JNMETD beginning with K=1 and M=1 when NSPH=0 or with K=2 and M=1 when NSPH=1. [K=1 or 2 represents the fact that the values come from Eq. (13) with even or odd values of n and M=m stands for the m-th eigenvalue (or the pole) for each K and the associated residues.] Following the cards for (K,M), those for (K,M+1) are read if M+1$\leq$JHL $\equiv$[(IIO+1)/2] when K<JOD or M+1$\leq$JJJ when K=JOD. Then, the cards for K+1 (K+1$\leq$JOD$\leq$2) are read beginning with M=1 till M=JHL or JJJ. For each set of values of K and M, the cards should be ordered as follows [NN$\equiv$IGRP except for (K,M)=(JOD,JJJ) where NN$\equiv$NKK]:

SSPP	P_g, g=1,2,...,NN; [(NN+4)/5] cards only if K=1 [when these cards are not available because all values for $\Sigma_g a P_g$ are not very small (see the next subsection e), blank cards should be inserted]
SP	$1-\lambda_g$, g=1,2,....,NN; [(NN+4)/5] cards
RES	$B_n(g,\lambda_{g'})$, g'=1,2,...,g for each n (n=1,2,..,JHL) separately, beginning with g=g"+1, then g"+2 and so on till g=NN (in the same order as in the punched output), where no poles of interest exist or SP=0 for g=1,2,...,g" [also SSPP=0 and $B_n(g,\lambda_{g'})=0$] ; JHL* [(NP+1)*(NN-2.5*NP)-(NP'+1)*(g"-2.5*NP')] cards where NP ≡ [(NN-1)/5] and NP' ≡ [(g"-1)/5]

(e) When NSTATY=2 and NKK< IGRP, following the data mentioned in the Subsections (c) and (d) if any, the first and second guesses for the poles SP(1,g), g=1,2,..., IGRP, and SP(2,g), g=1,2,..., IGRP, for K=JOD and M=JJJ are read first with 7F10.7 in the RESCAL ([(2*IGRP+6)/7] cards). If no pole exists for the g-th group, SP(1,g)=SP(2,g)=0. If no iteration process is required for obtaining the value, SP(2,g)=0 [SP(1,g) is regarded as the pole] . Furthermore, if the value of $\Sigma_g a P_g$ is very small for the case where K=1 [see Eq. (28)], SP(1,g)=0, SP(2,g)≠0 (any value) and the first and second guess for the value of P_g is read next with 2E15.8. The total number of cards for P_g is therefore the same as the number of groups for which SP(1,g)=0 and SP(2,g)≠0 for each value of M for K=1. These input data are repeated in the same order as mentioned in the last subsection (d) till K=2 and M=JHL. If NKK=IGRP and JOD=1 or JJJ< JHL, the present input begins with the data for (K,M) next to (K=JOD, M=JJJ).

4.2 Computer programme

4.2.1 General

The JN-METD1 package consists of 24 programmes; MAIN, JNMETD, RESCAL, FLUXCA, INTCAL, PULSE, ITRTON, DET, SOLEQ, EP, F, FSML, DEROF, SDERF, CCALC, DEROC, GCAL, FMCAL, VARIAC, ADJPUL, FNCUT1, IFNCAL, GIMAG and FMIMG. In addition, the code makes use of the library subprogrammes, MAXO, EXP, DEXP, DLOG, DATAN, DSIN, DCOS and DSICI (see below).

Almost all input integers and floating-point numbers are transmitted through a COMMON where, in addition, all subscript variables and their dimension information are stored for the use of the adjustable dimensioning. The present size of the floating COMMON for all subscript variables is set to be 72,000 bytes so as to the programme requires the core storage less than 300 K bytes in the Fortran-IV, Version G on the IBM-360/65.

For altering the dimension of the floating COMMON to fit core storage, the following 27 statements should be adjusted (all 24 programmes are numbered respectively): In the MAIN programme, the 45th card (dimension of BCOM), 47th card (COMMON), 55th card (clear-COMMON), 318th card (available $\lesseqgtr$ required storage?), 321st card (available $\lesseqgtr$ required storage only for the stationary problem?) and 340th card (available $\lesseqgtr$ required storage for the time-dependent problem?). In the JNMETD, the 24th card (COMMON) and 32nd card (dimension of ECOM). In the INTCAL, the 17th card (COMMON) and 24th card (dimension of ECOM). Seventeen cards for COMMON: the 21st of RESCAL, 28th of FLUXCA, 13th of PULSE, 14th of EP, 5th of ITRTON, 18th of F, 14th of FSML, 16th of DEROF, 10th of SDERF, 18th of CCALC, 13th of DEROC, 17th of GCAL, 16th of FMCAL, 35th of FNCUT1, 14th of IFNCAL, 12th of GIMAG and 12th of FMIMG.

4.2.2 MAIN

In the main programme, sizes of the required arrays are computed based on input paramters and then first-word addresses are calculated for these arrays. The locations of these pointers and the associated arrays with their dummy dimensions are shown in Table I where the arrays which share the same storage locations are written in one block (for example, Real*4 subscript variables FNPOL, TFLUX and FFLUX share the same storage locations as for Real*8 arrays DELTA, E, ED and SS in the case where NSTATY=1.) The actual values for these integer variables specifying the sizes of arrays are summarized in Table II. The first-word addresses and the dimensions are transferred through a call statement and a part of a vector in COMMON is treated as a multi-dimensioned array in subprogrammes. The flow chart of the main programme is shown in the Appendix 2, Section 1.

4.2.3 JNMETD, DET, ITRTON, F, FSML and EP

The subroutine JNMETD (see the Appendix 2, Section 2) is devoted mainly to solve stationary problems in a bare sphere (NSTATY=1 and NSPH=1). As can be seen in the Section 2.1 of Appendix 2, for the time-constant calculation to obtain the value of S, for example, Eq. (9) is solved by using the two guesses S1 and S2, the function subprogramme F for evaluating J_{nm}, the subroutine DET to evaluate the determinant and the subroutine ITRTON to iterate the process for making the value of the determinant zero until the difference between two successive values of S becomes smaller than the product of the last value of S by EPSS. After having been obtained the value of S, the ratios between the residues are calculated by evaluating the cofactors of the determinant, then the subroutine FLUXCA (see below) is called for the calculation of the total flux Eq. (8) and in the end the neutron balance is calculated by normalizing the total number of fission neutrons produced in the reactor to the value of k_{eff}.

In the function subprogramme F(N, α, ...) for evaluating $\rho_q J_{nm}(\alpha_q, s)$, the series expansion shown in the Appendix 1, Section 1 is used if the absolute value of $\alpha = \alpha_q \rho_q$ is less than 2 (K=2 in the programme). The argument N stands for n and m [N = 1,2,3,...,20 corresponding to (n,m) = (0,0),(1,1),(0,2),(2,2),(1,3),(3,3),....,(7,7)] . As regards the control integers transmitted through the COMMON, LG=2 is for reducing execution time required for calculating $\rho_q J_{nm}(\alpha_q, s)$ with different indices n and m but with the same value of α (LG=1 otherwise). In addition, JOD=1 or 2 is for evaluating the function with even or odd values of the indices. When JOD=1 and $|\alpha|>2$, as is seen from the expression for $P_1 J_{00}$, it uses the function subprogramme $EP(n,x,b,\cdots)$ to evaluate the exponential integral $E_n(x)$. On the other hand, when $|\alpha|<2$, it uses the function subprogramme FSML(α,M,...) which evaluates $P_1 J_{nm}(\alpha_1,s)/\alpha$ with even values of n and m by the use of the series expansion [M=1,2,3,...,10 corresponding to (n,m)=(0,0),(0,2),(2,2),....,(6,6)]. The control integer LF=2 is for computing $P_1 J_{nm}/\alpha$ with the different indices but with the same α (LF=1 otherwise).

The EP($n,x,b,\cdots$) evaluates also the integral $\int_{b(1-x)}^{bx} dz\, e^{z}/z$ numerically based on the generalized Simpson's rule when $b>0$ [note that the integral is equal to $E_1[-b(1-x)]-E_1(-bx)$ if $b<0$]. For $b=0$, it uses the asymptotic expansion for large x or small x [11] depending on whether $x>17$ or $0<x<0.05$ for evaluating $E_n(x)$ with $0<n<10$. For $0.05<x<0.5$, $E_1(x)$ is calculated by using the asymptotic expansion for small x and then $E_n(x)$ with $n>1$ is evaluated by the use of the recurrence relation. Furthermore, for $0.5<x<17$, $E_1(x)$ is obtained through the numerical integration and then the recurrence relation is used for calculating the value of $E_n(x)$. The control integer LFF is fixed to be 2 for computing $E_n(x)$ with different n but with the same x (LFF=1 otherwise).

For other problems than the stationary neutron transport in a bare sphere, the JNMETD calls the subroutines RESCAL, FLUXCA, INTCAL and then PULSE. In the case where NSTATY=2, NSPH=0 and NSTAT1=1, the stationary problem is solved first by calling RESCAL and FLUXCA, and then the time-dependent problem is treated by putting NSTAT1=0 and by calling again these subroutines. When NUPSAT=1 and LLL=0, following the calculation of the neutron flux by going through RESCAL, FLUXCA and INTCAL, the fundamental decay constant $\Sigma_1 v_1 (1-s_1)$ is evaluated by fixing LLL=1 and by calling once more RESCAL.

4.2.4 RESCAL, SOLEQ, DEROF, SDERF, CCALC and DEROC

For time-dependent problems, the subroutine RESCAL (see the Appendix 2, Section 3) computes the pole on the basis of two guesses for its value [see Sections 3.1 and 4.1(e)]. The procedure for solving Eq. (7) or (18) to evaluate the pole is the same as that mentioned above for obtaining the time constant in the JNMETD. When, as an input, the guesses for the value of P_g have been read instead of $1-s_g$ (KKK=2 in this case and KKK=1 otherwise), the programme deals with, for example, Eq. (33) with the elements divided by $c(g\to g)\alpha_g$ (instead of the unmodified equation) by using the function subprogramme FSML directly (not via the function F).

After having been evaluated the pole ($1-s_g = 0$ for stationary problems), if required (LLL=0), the residues at this pole and at the poles of higher energy groups are calculated as mentioned in Section 3.2 with the help of the subprogrammes DEROF, DET, CCALC and F (see the Appendix 2, Section 3.1). In the case where NSTAT1=1, also the first and second derivatives of the residue with respect to the Laplace transform variable s at the point $s=\Sigma_1 V_1$ are evaluated by calling the subroutines DEROC, DEROF and SDERF to obtain the first and second time moments of the neutron flux (26) and (27). On the other hand, for problems with NUPSAT=1 (one cannot deal with only one energy group, successively, beginning with the highest group), it calls the subroutine SOLEQ to evaluate the residues (or their derivatives) by solving a system of simultaneous linear equations.

When NSTATY=2 and LLL=0, the RESCAL produces punched cards, with a 5E15.8 format, for the residue and the pole ordered by K and M in the same way as for the input mentioned in the Subsection (d) of Section 4.1. For each set of the values of K and M, the cards for the residue $B_n(g,s_{g'})$,g'=1,2,...,g, are punched in order of n (n=1,2,..., JHL, separately) for g=g''+1, then for g=g''+2 and so on till g= IGRP. When, as an input, the guesses for P_g have been read for g=g''+1,g''+2,...,g''' , the cards for P_g for g=1,2,...,g''' ($P_g \equiv 0$ for g=1,2,...,g'') are produced following those for the residue of the g''' -th group. All these cards are followed by those for the poles $1-s_g$ for g=1,2,...,IGRP ($1-s_g \equiv 0$ for g=1,2,...,g'').

The subroutine DEROF (α ,CAXV,KKK,....) evaluates CAXV* $(2n+1)\Sigma_g V_g P_g^2 \frac{d}{ds} J_{nm}(\alpha_g, s)$ when KKK=1, by making use of the explicit expression if $|\alpha|>2$ or the series expansion otherwise (see the Appendix 1, Section 3). The control integer JOD transmitted through the COMMON is fixed to be 1 for even values of n and m [the functions with (n,m)=(0,0),(0,2),(2,2),...,(IIO-1,IIO-1) being evaluated] or 2 for odd values of n and m [the values for (n,m)=(1,1),(1,3),(3,3),...., (IIO,IIO) being obtained] . On the other hand, when KKK=2, it calculates CAXV $*(2n+1)\Sigma_q \nu_q P_q^2 \frac{d}{ds} J_{nm}(\alpha_q, s)/\alpha$ with even values of n and m by the use of the series expansion.

The subroutine SDERF(α,....) calculates $(\Sigma_q \nu_q)^2 P_q^3 \frac{d^2}{ds^2} J_{nm}(\alpha_q, s)$ by using the explicit expression or the series expansion depending on whether the value $|\alpha|$ is larger or smaller than 3, for n=0 and 1 ($m \leq 7$) and for m=7 ($n \leq 7$). For other values of n and m, it adopts a linear relation among the functions:

$$\frac{d^2}{ds^2} J_{n+1,m} = \frac{2n+1}{2m+1} \left(\frac{d^2}{ds^2} J_{n,m-1} + \frac{d^2}{ds^2} J_{n,m+1} \right) - \frac{d^2}{ds^2} J_{n-1,m} ,$$

with the help of the symmetric relation $\frac{d^2}{ds^2} J_{nm} = \frac{d^2}{ds^2} J_{mn}$.

The subroutine CCALC(2α,....) evaluates $2\alpha C_{2n}^{a}(\alpha_q, s)$ (when JOD=1) or $2\alpha C_{2n+1}^{a}(\alpha_q, s)/i$ (when JOD=2), n=0,1,..., (IIO-1)/2, in the case where IIII=0, DMU1=0 and NSPH=0. In addition, it computes $2\alpha C_{2n}^{b}(\alpha_q, s)$ or $2\alpha C_{2n+1}^{b}(\alpha_q, s)/i$ in the case where IIII=1, DMU1=0 and NSPH=0. The explicit expressions are adopted for these calculations when $|2\alpha| > 2.5$, with the help of the function EP (the series expansions otherwise; see the Appendix 1, Sections 9 and 10). Furthermore, in the case where DMU1>0 and NSPH=0, it calculates $2\alpha C_{2n}^{c}(\alpha_q, s)$ or $2\alpha C_{2n+1}^{c}(\alpha_q, s)/i$, the series expansion shown in the Appendix 1, Section 12 being used when $|2\alpha/\mu_1| < 3$. It also computes $2\alpha A_{2n+1}(2\alpha_q, s)$ (Appendix 1, Section 8) in the case where NSPH=1 by using the explicit expression when $|2\alpha| > 0.3$ or the series expansion otherwise.

The first and second derivatives of the function $C_n^{b}(\alpha_q, s)$ or $C_n^{c}(\alpha_q, s)$ with respect to s are evaluated in the subroutine DEROC(α,...). In the case where DMU1=0, it computes $2\alpha \left(\Sigma_q \nu_q P_q \frac{d}{ds}\right)^m C_{2n}^{b}(\alpha_q, s)$ or $2\alpha \left(\Sigma_q \nu_q P_q \frac{d}{ds}\right)^m C_{2n+1}^{b}(\alpha_q, s)/i$ (m=1 and 2) depending on whether JOD=1 or 2, by the use of the series expansion (when $|2\alpha| < 4$) or the explicit expression with the help of the EP (when $|2\alpha| > 4$). On the other hand, in the case where DMU1>0, it calculates $2\alpha (\Sigma_q \nu_q P_q \frac{d}{ds})^m C_{2n}^{c}(\alpha_q, s)$ or $2\alpha (\Sigma_q \nu_q P_q \frac{d}{ds})^m C_{2n+1}^{c}(\alpha_q, s)/i$ by adopting the series expansion when $|2\alpha/\mu_1| < 4$ or the explicit expression otherwise.

4.2.5 FLUXCA, GCAL, FMCAL and VARIAC

The subroutine FLUXCA (see the Appendix 2, Section 4) calculates the contribution of the poles to the total number of neutrons leaking out of a slab [when JJJ (or JJJJ in the main programme) = 1,2 or 4], to the total flux in a sphere or slab (when JJJ=2, 3 or 4) or to the angular flux in a slab (when JJJ = 3 or 4).

The contribution to the total number of leakage neutrons is evaluated for two boundaries of a slab according to the second term on the right-hand side of Eq. (20) or (24) by calling therefore the subroutine CCALC for evaluating C_n^{b} . In addition, in the case where NSTAT1=1, the first and second time moments [multiplied respectively by $\Sigma_q\nu_q$ and $(\Sigma_q\nu_q)^2$] of the leakage neutrons due to a $\delta(t)$ source are calculated on the basis of the expressions similar to Eqs. (26) and (27) (the contribution of uncollided neutrons being excluded) by calling the DEROC for evaluating the derivatives of C_n^{b} .

For computing the contribution to the total flux (and the first and second time moments), that is the second term on the right-hand side of Eq. (1), (19), (23) or the right-hand side of Eq. (8), it calls the subroutine GCAL (α, ξ ,JIK,...,m) to calculate the values of $4\alpha(\Sigma_q\nu_q P_q \frac{d}{ds})^{m+1} G_{2n}(\alpha_q, 2\xi-1, s)$ when JOD=1 or $4\alpha i(\Sigma_q\nu_q P_q \frac{d}{ds})^{m+1} G_{2n+1}(\alpha_q, 2\xi-1, s)$ when JOD=2 [n=0,1,..., (IIO-1)/2] , the integer JIK=2 being for evaluating the functions with a different value of m (-1, 0 or 1) but with the same $\alpha=\alpha_q P_q$ and ξ (JIK=1 otherwise). [Note that $(\frac{d}{ds})^m G_{2n}(\alpha_q, 2\xi-1, s) = (\frac{d}{ds})^m G_{2n}(\alpha_q, 1-2\xi, s)$.] As is seen in the Appendix 1, Section 4, in the case where the explicit expression for $(\frac{d}{ds})^m G_n$ is used ($|2\alpha|>4$ in the programme), the integrals $E_1[2\alpha(1-\xi)]$ and $E_1(2\alpha\xi)$ for G_0 (only for $\alpha>0$, that is LLL=1) and $\int_{2(1-\xi)}^{2\xi} dz\, e^{-\alpha z}/z$ for G_n with a positive integer of n [α may be negative (LLL=2)] are evaluated first in the calling programme FLUXCA by using the function subprogramme EP. For $|2\alpha|<4$, the subroutine GCAL uses the series expansion for $(\frac{d}{ds})^m G_n$. In addition, when NSPH=1, it is required to evaluate $\lim_{\gamma\to 0} 4\alpha i\, G_{2n+1}(\alpha_q, \gamma, s)/\gamma$ as can be seen from Eqs. (1) and (8). Also these values are calculated in the GCAL according to the following forms (the series expansion being used when $|2\alpha|<2$):

$$\lim_{\gamma\to 0} \frac{4\alpha i}{\gamma} G_1(\alpha_g,\gamma,s) = 2\,(1-e^{-\alpha}) = -2\sum_{n=1}^{\infty}\frac{1}{n!}(-\alpha)^n,$$

$$\lim_{\gamma\to 0} \frac{4\alpha i}{\gamma} G_3 = 3-\frac{10}{\alpha^2}+2\left(1+\frac{5}{\alpha}+\frac{5}{\alpha^2}\right)e^{-\alpha} = 2\sum_{n=1}^{\infty}\frac{1}{(n+2)!}(n+1)(n-3)(-\alpha)^n,$$

$$\lim_{\gamma\to 0} \frac{4\alpha i}{\gamma} G_5 = \frac{15}{4}-\frac{35}{\alpha^2}+\frac{378}{\alpha^4}-2\left(1+\frac{14}{\alpha}+\frac{77}{\alpha^2}+\frac{189}{\alpha^3}+\frac{189}{\alpha^4}\right)e^{-\alpha}$$
$$= -2\sum_{n=1}^{\infty}\frac{1}{(n+4)!}(n+3)(n+1)(n-3)(n-5)(-\alpha)^n,$$

$$\lim_{\gamma\to 0} \frac{4\alpha i}{\gamma} G_7 = \frac{35}{8}-\frac{315}{4\alpha^2}+\frac{2079}{\alpha^4}-\frac{38610}{\alpha^6}+2\left(1+\frac{27}{\alpha}+\frac{324}{\alpha^2}+\frac{2178}{\alpha^3}+\frac{8613}{\alpha^4}+\frac{19305}{\alpha^5}+\frac{19305}{\alpha^6}\right)e^{-\alpha}$$
$$= 2\sum_{n=1}^{\infty}\frac{1}{(n+6)!}(n+5)(n+3)(n+1)(n-3)(n-5)(n-7)(-\alpha)^n.$$

The contribution to the angular flux (and to the first and second time moments), that is the second term on the right-hand side of Eq. (11) or (22) [and the second terms of Eqs. (26) and (27)] are calculated in the FLUXCA by calling the subroutine FMCAL(α, ξ, μ, JII, ..., m) which evaluates $4\alpha\,(\Sigma_g v_g P_g \frac{d}{ds})^{m+1} F_{2n}(\alpha_g,\xi,\mu,s)$ with $\mu>0$ when JOD=1 or $4\alpha i\,(\Sigma_g v_g P_g \frac{d}{ds})^{m+1} \times F_{2n+1}(\alpha_g,\xi,\mu,s)$ with $\mu>0$ when JOD=2 [m=-1,0,1 and n=0,1,...,(IIO-1)/2] . This subroutine uses the explicit expression or the series expansion shown in the Appendix 1, Section 6, depending on whether the value $|2\xi\alpha/\mu|$ is larger or smaller than 4. The integer JII=2 is for calculating the functions with different values of m and μ but with the same values of α and ξ (JII=1 otherwise). [Note that $(\frac{d}{ds})^m F_n(\alpha_g,\xi,\mu,s) = (-1)^n (\frac{d}{ds})^m F_n(\alpha_g,1-\xi,-\mu,s)$.]

In the case where NSTAT1=1, the above-mentioned calculations are followd by the evaluation of the mean emission time $\bar{t}$ and the variance σ^2 of the time-dependent flux due to a $\delta(t)$ source. For the angular flux, these are written as [see Eqs. (25), (26) and (27)]

$$\bar{t} = \int_0^{\infty} dt\, t\, v_g n_g(x,\mu,t) \Big/ \int_0^{\infty} dt\, v_g n_g(x,\mu,t), \tag{41}$$

$$\sigma^2 = \int_0^{\infty} dt\, t^2 v_g n_g(x,\mu,t) \Big/ \int_0^{\infty} dt\, v_g n_g(x,\mu,t) - (\bar{t})^2. \tag{42}$$

The $\bar{t}$ and σ^2 are evaluated in the subroutine VARIAC(....,K,NSTAT2) when K=2 and 3, respectively. When K=1 and NSTAT2=2, it produces punched cards for the stationary flux [see Section 4.1 (c)].

4.2.6 INTCAL, FNCUT1, IFNCAL, GIMAG, FMIMG and ADJPUL

The subroutine INTCAL evaluates the contribution of the continuous spectrum to the time-dependent flux when NSTATY (NCURVE in this subroutine)=2,NSPH=0 and LLL=0. It calculates also the contribution of uncollided neutrons to the flux. The flow chart of the INTCAL is shown in the Appendix 2, Section 5.

For the evaluation of the contribution of the continuous spectrum, it calls the function subprogramme FNCUT1 (see the Appendix 2, Section 5.1). The control integer KKKK=1, 3 or 5 is for evaluating the contribution to the total number of leakage neutrons, to the total flux or to the angular flux. Therefore, when JJJ (or JJJJ in the main programme) = 1, 2, 3 or 4, KKKK takes the value 1 only, 1 and 3, 3 and 5 or 1,3 and 5. For the calculation of the last term on the right-hand side of Eq. (20), it uses the fact that only the factor $(2x_0/a-1)^n$ depends on the value of x_0 . For the evaluation of the last terms of Eqs. (19) and (11), on the other hand, it makes use of the symmetric relations, $G_n(\alpha_g,\xi,s)=(-1)^n G_n(\alpha_g,1-\xi,s)$ and $F_n(\alpha_g,\xi,\mu,s)=(-1)^n F_n(\alpha_g,1-\xi,-\mu,s)$, respectively. The newly defined integer NCURVE (≠NSTATY only in the routine) is therefore put to be 1 immediately after the calculation with NCURVE=0 for evaluating the contribution for the mirrored point, for example, the contribution to $v_g n_g(a-x,-\mu,t)$ immediately after the calculation of that to $v_g n_g(x,\mu,t)$ for NOT time points.

As shown in the Appendix 2, Section 6, in the FNCUT1(g,t,ξ,....) the value of y_0 is first chosen to satisfy the conditions $y_0 \gg 1$ and $y_0 a/(2v_g) \gg 1$ for all groups g of interest [only at the first time when the subprogramme is called (LF=1)] . The numerical integration from $y=0$ to y_0 is divided into two parts at y = tan (0.9 $\tan^{-1} y_0$). The real and imaginary parts of the integrand are then evaluated for each value of $y = \tan[0.9(m/2N)\tan^{-1} y_0]$, m=0,1,,..,2N (JJJ=1), and $y = \tan[0.9\tan^{-1} y_0 + 0.1(m/2N)\tan^{-1} y_0]$, m=1,2,...,2N (JJJ=2). Since $B_n(g, iy+\Sigma_1 v_1-\Sigma_g v_g)$ depends only on g and y [see Eq. (37)] , the values for 4N+1 points of y are calculated only once for the group g (LOGCL=1 and IGRP=1, these integers being different from those defined in the main programme) by calling the subroutine IFNCAL and then the integrand is evaluated, by the use of these values of B_n , for different time, space or angle points.

In the course of evaluating $B_n(g, iy+\Sigma_1 v_1-\Sigma_g v_g)$, the values of $S_g, \Sigma_g, a \times C_n(\Sigma_g a/2, s)+\sum_m J_{nm}(\Sigma_g a/2, s)B_m(g', s)$, with $s=iy+\Sigma_1 v_1-\Sigma_g v_g$, are stored in the array Y1 or Y2 for g'=1,2,...,g-1, in order to use them for computing $\Sigma_n(j, iy+\Sigma_1 v_1-\Sigma_j v_j)$ and $B_n(j, iy+\Sigma_1 v_1-\Sigma_j v_j)$, j=g,g+1,...,IGRPP [see Eq. (31)]. This is the reason why the calculation begins always with g=1, the highest energy-group, when LF=1.] Since the arrays Y1 and Y2 require generally very big storage, it is recommended, in the case where these arrays are used (NSPH=0, NSTATY=2, LLL=0 and IGRPP =JNKK +NNN$>$0), to estimate the required size of the floating COMMON according to Tables I and II in advance of the execution so as to be sure that the size is less than the available storage.

After having been obtained the B_n's (B1 in the programme), the coefficients of cos(yt) and sin(yt) [see Eq. (36)] are evaluated by calling the subroutine GIMAG or FMIMG when KKKK=3 or 5 [for KKKK=1 , C_{n1}^{+} and C_{n2}^{+} have already been evaluated as a result of calling the IFNCAL (see below)]. The coefficients ZZA are then used to compute the integrand, and the integral is calculated on the basis of the generalized Simpson's rule [see Eq. (36)]. For evaluating the contribution for different time points [for the same energy-group, space-(and angle-) point and KKKK], it uses directly the coefficients (LOGCL=2 and IGRP=2). In addition, for the mirrored point (NCURVE=1), the sign of a part of the coefficients is changed due to the control integers, LOGCL=2 and IGRP=1, to obtain new values of the coefficients. For computing the integral for different space-(or/and angle-) points and/or different values of KKKK (but for the same energy-group), the calculated values of $B_n(g, iy+\Sigma_1 v_1-\Sigma_g v_g)$ are used to evaluate the coefficients for this problem (LOGCL=1 and IGRP=2). The last part of the FNCUT1 is devoted to the calculation of the integral over y from y_0 to ∞ according to Eq. (38), (39) or (40).

The subroutine IFNCAL ($ay/(2v_g)$,KKK,...) evaluates C_{n1}^{c} and C_{n2}^{c} (KKK=1 or 2 for odd or even n) shown in the Appendix 1, Section 13, when the angular distribution of the boundary source is monodirectional. When

$a y/(2v_1\mu_1) < 1.5$, the values are obtained by the use of the series expansion. In addition, this subroutine calculates J_{nm1} and J_{nm2} (KKK=1 for odd values of m and n) shown in the Appendix 1, Section 2, and C_{n1}^{δ} and C_{n2}^{δ} (KKK=1 for odd n) shown in the Section 11, the series expansions being used if $a y/(2v_1) < 1.5$.

The subroutine GIMAG(ξ, $a y/(2v_1)$,KKK,...) calculates the values of G_{n1} and G_{n2} shown in the Appendix 1, Section 5 (the series expansion is used for evaluating the function with $a y/v_1 < 2.5$) and the FMIMG(ξ, $a y/(v_1\mu)$, KKK,...) evaluates F_{n1} and F_{n2} with a positive value of μ shown in the Section 7, the series expansions being used if $|a y\xi/(v_1\mu)| < 4$. In these two subroutines, KKK=1 or 2 is for evaluating the functions with odd or even n. The control integer LFF transferred through COMMON is put to be 2, instead of 1, for computing the functions with different values of $a y/(2v_1)$ or $a y/(v_1\mu)$ but with the same value of ξ

The subprogrammes FNCUT1, IFNCAL and GIMAG make use of the library routine DSICI(SI,CI,x) to evaluate the sine and cosine integrals:

$$SI=\int_{\infty}^{x} dz\,\frac{\sin z}{z}=si(x) \quad \text{and} \quad CI=\int_{\infty}^{x} dz\,\frac{\cos z}{z}=Ci(x).$$

Following the evaluation of the continuous spectrum, the INTCAL adjusts the calculated values of the flux by calling the subroutine ADJPUL, because it has been found that the contribution of the continuous spectrum does not converge to the exact value so rapidly (as the order of the j_N approximation increases). The adjustment uses the fact that the integral of the time-dependent flux due to a delta function source, Eq. (25), is equal to the value of the stationary flux (22).

The ADJPUL therefore evaluates the integral of the time-dependent flux (the contribution of uncollided neutrons being not included) over time from 0 to ∞ under the assumption that the g-th group flux decays exponentially, after the time T(NOT), with the asymptotic decay constant $\Sigma_1 v_1(1-\lambda_{1g})$.
[The Simpson's rule is repeatedly adopted for the integration over time from T(NOT) to 0.] If the calculated value of the integral is larger than the stationary flux, the values of the time-dependent flux are put to be

zero beginning with the flux at t=0 until the integral becomes smaller than the stationary flux. Then, as for the case where the integral of the unmodified flux is smaller than the stationary flux, the first positive flux is adjusted so as to achieve the equality.

To show the measure of the accuracy of the adjusted time-dependent flux, the ADJPUL calculates also the mean emission time $\bar{t}$ (41) and the variance σ^2 (42) of the adjusted flux distribution, which should be equal to the values obtained previously from the stationary calculation (NSTAT1=1) in the FLUXCA.

As mentioned already, the INTCAL evaluates also the contribution of uncollided neutrons to the total number of leakage neutrons (for NSPH=0 and JJJ≠3), to the total flux (for JJJ≠1) or to the angular flux (for NSPH=0 and JJJ $\geq$ 3) according to the following forms [for NSPH=1, see the first term on the right-hand side of Eq. (1)]:

(a) When NSPH=0 and NSTATY=1 [see Eqs. (24), (23) and (22)],

$$\int_0^1 d\mu\, S_g(\mu)\, exp\left(-\frac{\Sigma_g a}{\mu}\right) = S_g E_2(\Sigma_g a),\ 2S_g E_3(\Sigma_g a)\ \text{ or }\ S_g\, exp\left(-\frac{\Sigma_g a}{\mu_1}\right), \tag{43}$$

$$\int_0^1 \frac{d\mu}{\mu} S_g(\mu)\, exp\left(-\frac{\Sigma_g x}{\mu}\right) = S_g E_1(\Sigma_g x),\ 2S_g E_2(\Sigma_g x)\ \text{ or }\ S_g\, exp\left(-\frac{\Sigma_g x}{\mu_1}\right)/\mu_1, \tag{44}$$

$$S_g(\mu)\, exp\left(-\frac{\Sigma_g x}{\mu}\right)/\mu = S_g\, exp\left(-\frac{\Sigma_g x}{\mu}\right)/\mu,\ 2S_g\, exp\left(-\frac{\Sigma_g x}{\mu}\right)\ \text{ or }\ S_g\, exp\left(-\frac{\Sigma_g x}{\mu_1}\right)\delta(\mu-\mu_1)/\mu_1, \tag{45}$$

depending on whether the boundary source is plane isotropic (IIII=0), point isotropic (IIII=1) or monodirectional (DMU1 > 0).

(b) When NSPH=0 and NSTATY=2 [see Eqs. (20), (19) and (11)],

$$\int_0^1 d\mu\, S_g\left(\mu, t-\frac{a}{v_g\mu}\right) exp\left(-\frac{\Sigma_g a}{\mu}\right) = \begin{cases} 2S_g\, exp(-\Sigma_g v_g t)\, a^2/(v_g^2 t^3) \\ \qquad \text{for }\ t > a/v_g \quad \text{(0 otherwise)}, & \text{(46b)} \\ S_g\, exp\left(-\frac{\Sigma_g a}{\mu_1}\right)\delta\left(t-\frac{a}{v_g\mu_1}\right), & \text{(46c)} \end{cases}$$

$$\int_0^1 \frac{d\mu}{\mu} S_j(\mu, t-\frac{x}{v_j\mu}) exp(-\frac{\Sigma_j x}{\mu}) = \begin{cases} 2S_j x\, exp(-\Sigma_j v_j t)/(v_j t^2) \text{ for } t > x/v_j > 0, \\ \qquad 2S_j \delta(t) \text{ for } x=0 \text{ (0 otherwise)}, & (47b) \\ S_j\, exp(-\Sigma_j x/\mu_1)\delta(t - x/(v_j\mu_1))/\mu_1, & (47c) \end{cases}$$

$$\frac{1}{\mu} S_j(\mu, t-\frac{x}{v_j\mu}) exp(-\frac{\Sigma_j x}{\mu}) = \begin{cases} 2S_j\, exp(-\Sigma_j x/\mu)\delta(t - x/(v_j\mu)), & (48b) \\ S_j\, exp(-\Sigma_j x/\mu_1)\delta(\mu-\mu_1)\delta(t - x/(v_j\mu_1))/\mu_1, & (48c) \end{cases}$$

depending on whether the boundary source is point isotropic or monodirectional.

As seen from Eqs. (43) and (44), the function subprogramme EP is used for evaluating the exponential integral $E_n(z)$. The flow diagramme of this routine is shown in the Appendix 2, Section 5.2.

4.2.7 PULSE

The subroutine PULSE calculates the neutron flux due to a pulse source of the Gaussian distribution in time, exp $[-\alpha(t-t_0)^2]$ for $T_1 < t < T_2$ (the value of α may be equal to zero), by performing the integration of the flux resulted from a $\delta(t)$-source computed in the INTCAL:

$$\phi_p(t) = \int_{T_1}^{T_2} dt'\, exp[-\alpha(t'-t_0)^2]\, \phi_\delta(t-t') = \int_{t-T_2}^{t-T_1} dt'\, exp[-\alpha(t-t_0-t')^2]\, \phi_\delta(t'). \qquad (49)$$

The integration over t' is performed numerically on the basis of the generalized Simpson's rule. However, for the case where the contribution of uncollided neutrons to $\phi_\delta(t)$ is written in the form which contains a delta function in time, the contribution to $\phi_p(t)$ is evaluated according to the following analytical expression:

(a) When NSPH=1 [see Eq. (1)],

$$\int_{t-T_2}^{t-T_1} dt' \exp[-\alpha(t-t_0-t')^2]\, S_g \exp(-\Sigma_g v_g t')\,\delta(t'-r/v_g)/(4\pi r^2)$$

$$= S_g \exp(-\Sigma_g r)\exp[-\alpha(t-t_0-r/v_g)^2]/(4\pi r^2)$$

$$\text{for } t-T_1 \geq r/v_g \geq t-T_2 \qquad \text{(0 otherwise)}, \qquad (50)$$

(b) When NSPH=0 and the boundary source is monodirectional [see Eqs. (46c), (47c) and (48c)],

$$\int_{t-T_2}^{t-T_1} dt' \exp[-\alpha(t-t_0-t')^2] \times \begin{cases} S_g \exp(-\Sigma_g a/\mu_1)\,\delta(t'-a/(v_g\mu_1)) \\ S_g \exp(-\Sigma_g x/\mu_1)\,\delta(t'-x/(v_g\mu_1))/\mu_1 \\ S_g \exp(-\Sigma_g x/\mu_1)\,\delta(\mu-\mu_1)\,\delta(t'-x/(v_g\mu_1))/\mu_1 \end{cases}$$

$$= \begin{cases} S_g \exp(-\Sigma_g a/\mu_1)\exp[-\alpha(t-t_0-\frac{a}{v_g\mu_1})^2] \text{ for } t-T_1 \geq \frac{a}{v_g\mu_1} \geq t-T_2 \text{ (0 otherwise)}, & (51c) \\ S_g \exp(-\frac{\Sigma_g x}{\mu_1})\exp[-\alpha(t-t_0-\frac{x}{v_g\mu_1})^2]/\mu_1, \quad (52c) & \text{for } t-T_1 \geq \frac{x}{v_g\mu_1} \geq t-T_2 \\ S_g \exp(-\frac{\Sigma_g x}{\mu_1})\delta(\mu-\mu_1)\exp[-\alpha(t-t_0-\frac{x}{v_g\mu_1})^2]/\mu_1, \quad (53c) & \text{(0 otherwise)}, \end{cases}$$

(c) For the total flux at $x=0$ and the angular flux in the case where NSPH=0 and the boundary source is point isotropic [see Eqs. (47b) and (48b)],

$$\int_{t-T_2}^{t-T_1} dt' \exp[-\alpha(t-t_0-t')^2] \times \begin{cases} 2S_g\,\delta(t') \\ 2S_g \exp(-\frac{\Sigma_g x}{\mu})\,\delta(t'-\frac{x}{v_g\mu}) \end{cases}$$

$$= 2S_g \exp(-\Sigma_g x/\mu)\exp[-\alpha(t-t_0-x/(v_g\mu))^2]$$

$$\text{for } t-T_1 \geq x/(v_g\mu) \geq t-T_2 \text{ (0 otherwise)}. \qquad (53b)$$

For these cases, as is shown in the Appendix 2, Section 7, the contribution of uncollided neutrons to $\phi_S(t)$ is first subtracted from $\phi_S(t)$, the numerical integration is performed by using the thus modified $\phi_S(t)$ and then the contribution of uncollided neutrons to $\phi_P(t)$ is added to the result. For other two cases where the total number of leakage neutrons and the total flux at $x>0$ in a slab with a point isotropic boundary source are calculated, Eq. (49) is used without any modification.

In addition, this subroutine calculates again the integral of $\phi_S(t)$ [not including the contribution of uncollided neutrons written in the form of a $\delta(t)$] over t from 0 to T(NOT), the mean emission time of $\phi_S(t)$ and the variance [the integration is performed only from t=0 to T(NOT)] . Furthermore, it computes the integral of $\phi_P(t)$ (including the contribution of uncollided neutrons) over t from W(1) to W(NOP), the maximum value of $\phi_P(t)$ for W(1) $\leq$ t $\leq$ W(NOP), the pulse width at half maximum, the mean emission time of $\phi_P(t)$ and the variance of the time distribution.

5. Remarks

It should be mentioned here first how the present computer code deals with neutron transport in a medium with highly anisotropic scattering by the use of the transport approximation. For such media, the values of $C(g\to g)$ are sometimes negative as seen, for example, in the hydrogen cross section of the LASL 16-group set[12]. The code JN-METD1 accepts also a negative value of $C(g\to g)$ but gives all values of the g-th group flux coming from the pole zero (for stationary problems, it makes the value zero to avoid the negative flux and for time-dependent problems there exists no pole). As a result, the calculated flux consists only of the contribution of uncollided neutrons which cannot be treated correctly in the transport approximation. The results for the g-th group with a negative value of $C(g\to g)$ are therefore not correct (as so the S_N calculation in the transport approximation) but those for other groups with positive values of $C(g\to g)$ have been found in a good agreement with the values obtained from the S_N calculation taking into account the linear anisotropic scattering[5]. This defect will be cleared up when our present work will be completed for dealing with multi-region slab systems with anisotropic scattering.

As already mentioned in the Section 4.2.6, the multigroup calculation of time-dependent problems in a homogeneous slab requires a big computer storage, so that the total number of integral points for evaluating the contribution of the continuous spectrum (the value of N in the input) is sometimes limited to be small ($N < 50$). Since the contribution is dominant only for thin slabs (and for times close to the moment when the wave front of the direct neutron beam arrives), it will be recommended in such cases to use the j_5 approximation instead of j_7 so as to adopt a larger number of $N > 70$. It saves also execution time of the computation by about 30%.

Typical running time on the IBM-360/65 is nearly 4 min. to obtain the time-dependent lowest group angular and total flux in a slab within the context of a 7-group j_7 approximation with 2 space, 3 angle and 56 time points, including the time required for obtaining the stationary flux as well as the time-dependent flux due to a pulse source. The calculation of the stationary angular, total and leakage flux in a slab takes 1 to 2 min. (depending on the slab thickness) in a 7-group j_7 approximation with 11 space and angle points. All four sample problems shown in the Appendix 3 take about 2 min.

References

1. H. KSCHWENDT, Nucl. Sci. Eng., 36, 447 (1969)

2. J. MIKA and R. STANKIEWICZ, Nucl. Sci. Eng., 36, 450 (1969)

3. T. ASAOKA, J. Nucl. Energy, 22, 99 (1968)

4. T. ASAOKA, Nucl. Sci. Eng., 34, 122 (1968)

5. T. ASAOKA and J.A. LARRIMORE, "Considerations on Pulsed Reactor Optimization", to be published in J. Nucl. Energy (1970)

6. H. KSCHWENDT, Trans. Am. Nucl. Soc., 12, 630 (1969)

7. T. ASAOKA and E. CAGLIOTI, Trans. Am. Nucl. Soc., 12, 635 (1969)

8. H. KSCHWENDT, "The SP_N-P_L-Method for Neutron Transport in Homogeneous Slabs with Anisotropic Scattering", submitted to Nucl. Sci. Eng. (1970)

9. H. HEMBD, "The Treatment of Neutron Transport in Multilayer Systems by the Integral Transform Method", submitted to J. Nucl. Energy (1970)

10. H. HEMBD, Nucl. Sci. Eng., 40, 224 (1970)

11. G. PLACZEK, in "Tables of Functions and Zeros of Functions", National Bureau of Standards, Applied Mathematics Series 37, p. 57, U.S. Department of Commerce (1954)

12. Argonne National Laboratory, "Reactor Physics Constants", ANL-5800, 2nd ed., p. 570 (1963)

Appendix 1. Analytical Expressions of Functions

The explicit expressions and series expansions of functions in the solution of the j_7 approximation are summarized here by introducing the abbreviations $\alpha \equiv \alpha_g P_g$, $P_g \equiv 1-(\Sigma_1 v_1 - s)/(\Sigma_g v_g)$, $\eta \equiv a y/(2 v_g)$ ($\alpha = i\eta$ when $s = iy + \Sigma_1 v_1 - \Sigma_g v_g$) and $\gamma \equiv$ Euler-Mascheroni constant.

1. $J_{nm}(\alpha_g, s)$

$$P_g J_{00} = 1 - \frac{1}{2} e^{-2\alpha} - \frac{1}{4\alpha}(1-e^{-2\alpha}) + \alpha \int_1^\infty \frac{dz}{z} e^{-2\alpha z}$$
$$= \alpha\left(-\ln 2\alpha - \gamma + \frac{3}{2}\right) + \sum_{n=2}^{\infty} \frac{1}{(n-1)(n+1)!}(-2\alpha)^n,$$

$$P_g J_{11} = \frac{1}{3} - \frac{1}{4\alpha} - \frac{1}{4\alpha^2} e^{-2\alpha} + \frac{1}{8\alpha^3}(1-e^{-2\alpha}) = -\sum_{n=1}^{\infty} \frac{n+2}{(n+3)!}(-2\alpha)^n,$$

$$P_g J_{02} = \frac{1}{4\alpha} - \frac{1}{2\alpha^2}\left(1+\frac{1}{2}e^{-2\alpha}\right) + \frac{3}{8\alpha^3}(1-e^{-2\alpha}) = -\sum_{n=1}^{\infty} \frac{n}{(n+3)!}(-2\alpha)^n,$$

$$P_g J_{22} = \frac{1}{5} - \frac{1}{4\alpha} + \frac{1}{4\alpha^2} e^{-2\alpha} + \frac{3}{8\alpha^3}(1+3e^{-2\alpha}) + \frac{3}{2\alpha^4} e^{-2\alpha} - \frac{3}{4\alpha^5}(1-e^{-2\alpha}) = \sum_{n=1}^{\infty} \frac{(n-3)(n+2)}{(n+5)(n+3)!}(-2\alpha)^n,$$

$$P_g J_{13} = \frac{1}{4\alpha} - \frac{5}{6\alpha^2}\left(1-\frac{3}{10}e^{-2\alpha}\right) + \frac{9}{8\alpha^3}\left(1+\frac{11}{9}e^{-2\alpha}\right) + \frac{5}{2\alpha^4} e^{-2\alpha} - \frac{5}{4\alpha^5}(1-e^{-2\alpha})$$
$$= \sum_{n=1}^{\infty} \frac{(n-3)n}{(n+5)(n+3)!}(-2\alpha)^n$$

$$P_g J_{33} = \frac{1}{7} - \frac{1}{4\alpha} - \frac{1}{4\alpha^2} e^{-2\alpha} + \frac{3}{4\alpha^3}\left(1-\frac{7}{2}e^{-2\alpha}\right) - \frac{45}{4\alpha^4} e^{-2\alpha} - \frac{15}{4\alpha^5}\left(1+\frac{13}{2}e^{-2\alpha}\right) - \frac{225}{8\alpha^6} e^{-2\alpha}$$
$$+ \frac{225}{16\alpha^7}(1-e^{-2\alpha}) = -\sum_{n=1}^{\infty} \frac{(n-5)(n-3)(n+2)}{(n+7)(n+5)(n+3)!}(-2\alpha)^n,$$

$$P_g J_{04} = -\frac{1}{4\alpha} + \frac{5}{3\alpha^2}\left(1+\frac{3}{20}e^{-2\alpha}\right) - \frac{45}{8\alpha^3}\left(1-\frac{17}{45}e^{-2\alpha}\right) + \frac{21}{2\alpha^4}\left(1+\frac{2}{3}e^{-2\alpha}\right) - \frac{35}{4\alpha^5}(1-e^{-2\alpha})$$
$$= \sum_{n=1}^{\infty} \frac{(n-3)(n-2)n}{(n+5)!}(-2\alpha)^n,$$

$$P_g J_{24} = \frac{1}{4\alpha} - \frac{7}{6\alpha^2}\left(1+\frac{3}{14}e^{-2\alpha}\right) + \frac{9}{4\alpha^3}\left(1-\frac{23}{18}e^{-2\alpha}\right) - \frac{55}{4\alpha^4} e^{-2\alpha} - \frac{25}{4\alpha^5}\left(1+\frac{53}{10}e^{-2\alpha}\right) - \frac{315}{8\alpha^6} e^{-2\alpha}$$
$$+ \frac{315}{16\alpha^7}(1-e^{-2\alpha}) = -\sum_{n=1}^{\infty} \frac{(n-5)(n-3)n}{(n+7)(n+5)(n+3)!}(-2\alpha)^n,$$

$$P_g J_{44} = \frac{1}{9} - \frac{1}{4\alpha} + \frac{1}{4\alpha^2} e^{-2\alpha} + \frac{5}{4\alpha^3}\left(1+\frac{37}{10}e^{-2\alpha}\right) + \frac{153}{4\alpha^4} e^{-2\alpha} - \frac{45}{4\alpha^5}\left(1-\frac{97}{6}e^{-2\alpha}\right) + \frac{4305}{8\alpha^6} e^{-2\alpha}$$
$$+ \frac{1575}{16\alpha^7}\left(1+\frac{51}{5}e^{-2\alpha}\right) + \frac{2205}{2\alpha^8} e^{-2\alpha} - \frac{2205}{4\alpha^9}(1-e^{-2\alpha}) = \sum_{n=1}^{\infty} \frac{(n-7)(n-5)(n-3)(n+2)}{(n+9)(n+7)(n+5)(n+3)!}(-2\alpha)^n,$$

$$P_9 J_{15} = -\frac{1}{4\alpha} + \frac{7}{3\alpha^2}\left(1-\frac{3}{28}e^{-2\alpha}\right) - \frac{45}{4\alpha^3}\left(1+\frac{29}{90}e^{-2\alpha}\right) + \frac{63}{2\alpha^4}\left(1-\frac{13}{18}e^{-2\alpha}\right) - \frac{175}{4\alpha^5}\left(1+\frac{17}{10}e^{-2\alpha}\right)$$
$$-\frac{945}{8\alpha^6}e^{-2\alpha} + \frac{945}{16\alpha^7}\left(1-e^{-2\alpha}\right) = -\sum_{n=1}^{\infty}\frac{(n-5)(n-3)(n-2)n}{(n+7)(n+5)!}(-2\alpha)^n,$$

$$P_9 J_{35} = \frac{1}{4\alpha} - \frac{3}{2\alpha^2}\left(1-\frac{1}{6}e^{-2\alpha}\right) + \frac{15}{4\alpha^3}\left(1+\frac{13}{10}e^{-2\alpha}\right) + \frac{171}{4\alpha^4}e^{-2\alpha} - \frac{75}{4\alpha^5}\left(1-\frac{23}{2}e^{-2\alpha}\right) + \frac{5355}{8\alpha^6}e^{-2\alpha}$$
$$+\frac{2205}{16\alpha^7}\left(1+\frac{65}{7}e^{-2\alpha}\right) + \frac{2835}{2\alpha^8}e^{-2\alpha} - \frac{2835}{4\alpha^9}\left(1-e^{-2\alpha}\right) = \sum_{n=1}^{\infty}\frac{(n-7)(n-5)(n-3)n}{(n+9)(n+7)(n+5)(n+3)!}(-2\alpha)^n,$$

$$P_9 J_{55} = \frac{1}{11} - \frac{1}{4\alpha} - \frac{1}{4\alpha^2}e^{-2\alpha} + \frac{15}{8\alpha^3}\left(1-\frac{19}{5}e^{-2\alpha}\right) - \frac{189}{2\alpha^4}e^{-2\alpha} - \frac{105}{4\alpha^5}\left(1+29e^{-2\alpha}\right) - \frac{4095}{\alpha^6}e^{-2\alpha} + \frac{1575}{4\alpha^7}\left(1-\frac{194}{5}e^{-2\alpha}\right)$$
$$-\frac{39690}{\alpha^8}e^{-2\alpha} - \frac{19845}{4\alpha^9}\left(1+14e^{-2\alpha}\right) - \frac{297675}{4\alpha^{10}}e^{-2\alpha} + \frac{297675}{8\alpha^{11}}\left(1-e^{-2\alpha}\right)$$
$$= -\sum_{n=1}^{\infty}\frac{(n-9)(n-7)(n-5)(n-3)(n+2)}{(n+11)(n+9)(n+7)(n+5)(n+3)!}(-2\alpha)^n,$$

$$P_9 J_{06} = \frac{1}{4\alpha} - \frac{7}{2\alpha^2}\left(1+\frac{1}{14}e^{-2\alpha}\right) + \frac{105}{4\alpha^3}\left(1-\frac{13}{70}e^{-2\alpha}\right) - \frac{126}{\alpha^4}\left(1+\frac{19}{56}e^{-2\alpha}\right) + \frac{1575}{4\alpha^5}\left(1-\frac{37}{70}e^{-2\alpha}\right)$$
$$-\frac{1485}{2\alpha^6}\left(1+\frac{3}{4}e^{-2\alpha}\right) + \frac{10395}{16\alpha^7}\left(1-e^{-2\alpha}\right) = -\sum_{n=1}^{\infty}\frac{(n-5)(n-4)(n-3)(n-2)n}{(n+7)!}(-2\alpha)^n,$$

$$P_9 J_{26} = -\frac{1}{4\alpha} + \frac{3}{\alpha^2}\left(1+\frac{1}{12}e^{-2\alpha}\right) - \frac{75}{4\alpha^3}\left(1-\frac{3}{10}e^{-2\alpha}\right) + \frac{693}{10\alpha^4}\left(1+\frac{5}{6}e^{-2\alpha}\right) - \frac{525}{4\alpha^5}\left(1-\frac{131}{50}e^{-2\alpha}\right) + \frac{10017}{8\alpha^6}e^{-2\alpha}$$
$$+\frac{6615}{16\alpha^7}\left(1+\frac{229}{35}e^{-2\alpha}\right) + \frac{6237}{2\alpha^8}e^{-2\alpha} - \frac{6237}{4\alpha^9}\left(1-e^{-2\alpha}\right) = \sum_{n=1}^{\infty}\frac{(n-7)(n-5)(n-3)(n-2)n}{(n+9)(n+7)(n+5)!}(-2\alpha)^n,$$

$$P_9 J_{46} = \frac{1}{4\alpha} - \frac{11}{6\alpha^2}\left(1+\frac{3}{22}e^{-2\alpha}\right) + \frac{45}{8\alpha^3}\left(1-\frac{59}{45}e^{-2\alpha}\right) - \frac{203}{2\alpha^4}e^{-2\alpha} - \frac{175}{4\alpha^5}\left(1+\frac{97}{5}e^{-2\alpha}\right) - \frac{4725}{\alpha^6}e^{-2\alpha}$$
$$+\frac{2205}{4\alpha^7}\left(1-\frac{230}{7}e^{-2\alpha}\right) - \frac{47880}{\alpha^8}e^{-2\alpha} - \frac{25515}{4\alpha^9}\left(1+\frac{358}{27}e^{-2\alpha}\right) - \frac{363825}{4\alpha^{10}}e^{-2\alpha} + \frac{363825}{8\alpha^{11}}\left(1-e^{-2\alpha}\right)$$
$$= -\sum_{n=1}^{\infty}\frac{(n-9)(n-7)(n-5)(n-3)n}{(n+11)(n+9)(n+7)(n+5)(n+3)!}(-2\alpha)^n,$$

$$P_9 J_{66} = \frac{1}{13} - \frac{1}{4\alpha} + \frac{1}{4\alpha^2}e^{-2\alpha} + \frac{21}{8\alpha^3}\left(1+\frac{27}{7}e^{-2\alpha}\right) + \frac{195}{\alpha^4}e^{-2\alpha} - \frac{105}{2\alpha^5}\left(1-\frac{313}{7}e^{-2\alpha}\right) + \frac{19575}{\alpha^6}e^{-2\alpha}$$
$$+\frac{4725}{4\alpha^7}\left(1+\frac{502}{5}e^{-2\alpha}\right) + \frac{532980}{\alpha^8}e^{-2\alpha} - \frac{99225}{4\alpha^9}\left(1-\frac{502}{7}e^{-2\alpha}\right) + \frac{17307675}{4\alpha^{10}}e^{-2\alpha} + \frac{3274425}{8\alpha^{11}}\left(1+\frac{125}{7}e^{-2\alpha}\right)$$
$$+\frac{15436575}{2\alpha^{12}}e^{-2\alpha} - \frac{15436575}{4\alpha^{13}}\left(1-e^{-2\alpha}\right) = \sum_{n=1}^{\infty}\frac{(n-11)(n-9)(n-7)(n-5)(n-3)(n+2)}{(n+13)(n+11)(n+9)(n+7)(n+5)(n+3)!}(-2\alpha)^n,$$

$$P_9 J_{17} = \frac{1}{4\alpha} - \frac{9}{2\alpha^2}\left(1-\frac{1}{18}e^{-2\alpha}\right) + \frac{175}{4\alpha^3}\left(1+\frac{11}{70}e^{-2\alpha}\right) - \frac{1386}{5\alpha^4}\left(1-\frac{195}{616}e^{-2\alpha}\right) + \frac{4725}{4\alpha^5}\left(1+\frac{589}{1050}e^{-2\alpha}\right)$$
$$-\frac{6435}{2\alpha^6}\left(1-\frac{253}{260}e^{-2\alpha}\right) + \frac{72765}{16\alpha^7}\left(1+\frac{69}{35}e^{-2\alpha}\right) + \frac{27027}{2\alpha^8}e^{-2\alpha} - \frac{27027}{4\alpha^9}\left(1-e^{-2\alpha}\right)$$
$$= \sum_{n=1}^{\infty}\frac{(n-7)(n-5)(n-4)(n-3)(n-2)n}{(n+9)(n+7)!}(-2\alpha)^n,$$

$$P_9 J_{37} = -\frac{1}{4\alpha} + \frac{11}{3\alpha^2}\left(1-\frac{3}{44}e^{-2\alpha}\right) - \frac{225}{8\alpha^3}\left(1+\frac{13}{45}e^{-2\alpha}\right) + \frac{1287}{10\alpha^4}\left(1-\frac{1240}{1287}e^{-2\alpha}\right) - \frac{1225}{4\alpha^5}\left(1+\frac{659}{175}e^{-2\alpha}\right)$$
$$-\frac{7119}{\alpha^6}e^{-2\alpha} + \frac{6615}{4\alpha^7}\left(1-\frac{1898}{105}e^{-2\alpha}\right) - \frac{84546}{\alpha^8}e^{-2\alpha} - \frac{56133}{4\alpha^9}\left(1+\frac{2086}{189}e^{-2\alpha}\right) - \frac{675675}{4\alpha^{10}}e^{-2\alpha}$$
$$+\frac{675675}{8\alpha^{11}}\left(1-e^{-2\alpha}\right) = -\sum_{n=1}^{\infty}\frac{(n-9)(n-7)(n-5)(n-3)(n-2)n}{(n+11)(n+9)(n+7)(n+5)!}(-2\alpha)^n,$$

$$P_g J_{57} = \frac{1}{4\alpha} - \frac{13}{6\alpha^2}\left(1 - \frac{3}{26}e^{-2\alpha}\right) + \frac{63}{8\alpha^3}\left(1 + \frac{83}{63}e^{-2\alpha}\right) + \frac{205}{\alpha^4}e^{-2\alpha} - \frac{175}{2\alpha^5}\left(1 - \frac{1013}{35}e^{-2\alpha}\right) + \frac{21645}{\alpha^6}e^{-2\alpha}$$
$$+ \frac{6615}{4\alpha^7}\left(1 + \frac{1702}{21}e^{-2\alpha}\right) + \frac{612990}{\alpha^8}e^{-2\alpha} - \frac{127575}{4\alpha^9}\left(1 - \frac{1754}{27}e^{-2\alpha}\right) + \frac{20322225}{4\alpha^{10}}e^{-2\alpha} + \frac{4002075}{8\alpha^{11}}\left(1 + \frac{1327}{77}e^{-2\alpha}\right)$$
$$+ \frac{18243225}{2\alpha^{12}}e^{-2\alpha} - \frac{18243225}{4\alpha^{13}}\left(1 - e^{-2\alpha}\right) = \sum_{n=1}^{\infty} \frac{(n-11)(n-9)(n-7)(n-5)(n-3)n}{(n+13)(n+11)(n+9)(n+7)(n+5)(n+3)!}(-2\alpha)^n,$$

$$P_g J_{77} = \frac{1}{15} - \frac{1}{4\alpha} - \frac{1}{4\alpha^2}e^{-2\alpha} + \frac{7}{2\alpha^3}\left(1 - \frac{109}{28}e^{-2\alpha}\right) - \frac{1431}{4\alpha^4}e^{-2\alpha} - \frac{189}{2\alpha^5}\left(1 + \frac{5309}{84}e^{-2\alpha}\right) - \frac{564525}{8\alpha^6}e^{-2\alpha}$$
$$+ \frac{23625}{8\alpha^7}\left(1 - \frac{10533}{50}e^{-2\alpha}\right) - \frac{16829505}{4\alpha^8}e^{-2\alpha} - \frac{363825}{4\alpha^9}\left(1 + \frac{17009}{70}e^{-2\alpha}\right) - \frac{721589715}{8\alpha^{10}}e^{-2\alpha}$$
$$+ \frac{9823275}{4\alpha^{11}}\left(1 - \frac{9665}{84}e^{-2\alpha}\right) - \frac{5284454175}{8\alpha^{12}}e^{-2\alpha} - \frac{200675475}{4\alpha^{13}}\left(1 + \frac{87}{4}e^{-2\alpha}\right) - \frac{18261468225}{16\alpha^{14}}e^{-2\alpha}$$
$$+ \frac{18261468225}{32\alpha^{15}}\left(1 - e^{-2\alpha}\right) = -\sum_{n=1}^{\infty} \frac{(n-13)(n-11)(n-9)(n-7)(n-5)(n-3)(n+2)}{(n+15)(n+13)(n+11)(n+9)(n+7)(n+5)(n+3)!}(-2\alpha)^n.$$

2. $J_{nm}(\alpha_g, iy + \varepsilon_1 v_1 - \varepsilon_g v_g) \equiv -\alpha_g (J_{nm1} + iJ_{nm2})$

$$J_{001} = -\int_1^{\infty}\frac{dz}{z}\cos(2\eta z) - \frac{1}{2\eta}\sin(2\eta) - \frac{1}{4\eta^2}[1 - \cos(2\eta)] = \ln(2\eta) + \gamma - \frac{3}{2} + \sum_{n=1}^{\infty}(-1)^n \frac{1}{n(2n+2)!}(2\eta)^{2n},$$

$$J_{002} = \int_1^{\infty}\frac{dz}{z}\sin(2\eta z) + \frac{1}{2\eta}[2 - \cos(2\eta)] - \frac{1}{4\eta^2}\sin(2\eta) = \frac{\pi}{2} + 2\sum_{n=1}^{\infty}(-1)^n \frac{1}{(2n-1)(2n+1)!}(2\eta)^{2n-1},$$

$$J_{111} = -\frac{1}{4\eta^2}\left\{1 - \frac{1}{\eta}\sin(2\eta) + \frac{1}{2\eta^2}[1 - \cos(2\eta)]\right\} = -\sum_{n=0}^{\infty}(-1)^n \frac{1}{(n+2)(2n+2)!}(2\eta)^{2n},$$

$$J_{112} = \frac{1}{\eta}\left\{\frac{1}{3} + \frac{1}{4\eta^2}\cos(2\eta) - \frac{1}{8\eta^3}\sin(2\eta)\right\} = -2\sum_{n=1}^{\infty}(-1)^n \frac{1}{(2n+3)(2n+1)!}(2\eta)^{2n-1},$$

$$J_{021} = \frac{1}{4\eta^2}\left\{1 + \frac{1}{\eta}\sin(2\eta) - \frac{3}{2\eta^2}[1 - \cos(2\eta)]\right\} = -2\sum_{n=0}^{\infty}(-1)^n \frac{2n+1}{(2n+4)!}(2\eta)^{2n},$$

$$J_{022} = \frac{1}{4\eta^3}\left\{2 + \cos(2\eta) - \frac{3}{2\eta}\sin(2\eta)\right\} = -4\sum_{n=1}^{\infty}(-1)^n \frac{n}{(2n+3)!}(2\eta)^{2n-1},$$

$$J_{221} = -\frac{1}{4\eta^2}\left\{1 + \frac{1}{\eta}\sin(2\eta) + \frac{3}{2\eta^2}[1 + 3\cos(2\eta)] - \frac{6}{\eta^3}\sin(2\eta) + \frac{3}{\eta^4}[1 - \cos(2\eta)]\right\}$$
$$= \sum_{n=0}^{\infty}(-1)^n \frac{n-1}{(n+3)(n+2)(2n+2)!}(2\eta)^{2n},$$

$$J_{222} = \frac{1}{\eta}\left\{\frac{1}{5} - \frac{1}{4\eta^2}\cos(2\eta) + \frac{9}{8\eta^3}\sin(2\eta) + \frac{3}{2\eta^4}\cos(2\eta) - \frac{3}{4\eta^5}\sin(2\eta)\right\}$$
$$= 2\sum_{n=1}^{\infty}(-1)^n \frac{2n-3}{(2n+5)(2n+3)(2n+1)!}(2\eta)^{2n-1},$$

$$J_{131} = \frac{1}{4\eta^2}\left\{1 - \frac{1}{\eta}\sin(2\eta) - \frac{1}{2\eta^2}[9 + 11\cos(2\eta)] + \frac{10}{\eta^3}\sin(2\eta) - \frac{5}{\eta^4}[1 - \cos(2\eta)]\right\}$$
$$= 2\sum_{n=0}^{\infty}(-1)^n \frac{(n-1)(2n+1)}{(n+3)(2n+4)!}(2\eta)^{2n},$$

$$J_{132} = \frac{1}{4\eta^3}\left\{\frac{10}{3} - \cos(2\eta) + \frac{11}{2\eta}\sin(2\eta) + \frac{10}{\eta^2}\cos(2\eta) - \frac{5}{\eta^3}\sin(2\eta)\right\} = 4\sum_{n=1}^{\infty}(-1)^n \frac{(2n-3)n}{(2n+5)(2n+3)!}(2\eta)^{2n+1},$$

$$J_{331}=-\frac{1}{4\eta^2}\left\{1-\frac{1}{\eta}\sin(2\eta)+\frac{1}{2\eta^2}[6-21\cos(2\eta)]+\frac{45}{\eta^3}\sin(2\eta)+\frac{15}{2\eta^4}[2+13\cos(2\eta)]-\frac{225}{2\eta^5}\sin(2\eta)\right.$$
$$\left.+\frac{225}{4\eta^6}[1-\cos(2\eta)]\right\}=-\sum_{n=0}^{\infty}(-1)^n\frac{(n-2)(n-1)}{(n+4)(n+3)(n+2)(2n+2)!}(2\eta)^{2n},$$

$$J_{332}=\frac{1}{\eta}\left\{\frac{1}{7}+\frac{1}{4\eta^2}\cos(2\eta)-\frac{21}{8\eta^3}\sin(2\eta)-\frac{45}{4\eta^3}\cos(2\eta)+\frac{195}{8\eta^5}\sin(2\eta)+\frac{225}{8\eta^6}\cos(2\eta)-\frac{225}{16\eta^7}\sin(2\eta)\right\}$$
$$=-2\sum_{n=1}^{\infty}(-1)^n\frac{(2n-5)(2n-3)}{(2n+7)(2n+5)(2n+3)(2n+1)!}(2\eta)^{2n-1},$$

$$J_{041}=-\frac{1}{4\eta^2}\left\{1+\frac{1}{\eta}\sin(2\eta)-\frac{1}{2\eta^2}[45-17\cos(2\eta)]-\frac{28}{\eta^3}\sin(2\eta)+\frac{35}{\eta^4}[1-\cos(2\eta)]\right\}$$
$$=4\sum_{n=0}^{\infty}(-1)^n\frac{(n-1)(2n-1)(2n+1)}{(2n+6)!}(2\eta)^{2n},$$

$$J_{042}=-\frac{1}{\eta^3}\left\{\frac{5}{3}+\frac{1}{4}\cos(2\eta)-\frac{17}{8\eta}\sin(2\eta)-\frac{7}{2\eta^2}[3+2\cos(2\eta)]+\frac{35}{4\eta^3}\sin(2\eta)\right\}$$
$$=8\sum_{n=2}^{\infty}(-1)^n\frac{(2n-3)(n-1)n}{(2n+5)!}(2\eta)^{2n-1},$$

$$J_{241}=\frac{1}{4\eta^2}\left\{1+\frac{1}{\eta}\sin(2\eta)-\frac{1}{2\eta^2}[18-23\cos(2\eta)]-\frac{55}{\eta^3}\sin(2\eta)-\frac{5}{2\eta^4}[10+53\cos(2\eta)]+\frac{315}{2\eta^5}\sin(2\eta)\right.$$
$$\left.-\frac{315}{4\eta^6}[1-\cos(2\eta)]\right\}=-2\sum_{n=0}^{\infty}(-1)^n\frac{(n-2)(n-1)(2n+1)}{(n+4)(n+3)(2n+4)!}(2\eta)^{2n},$$

$$J_{242}=\frac{1}{4\eta^3}\left\{\frac{14}{3}+\cos(2\eta)-\frac{23}{2\eta}\sin(2\eta)-\frac{55}{\eta^2}\cos(2\eta)+\frac{265}{2\eta^3}\sin(2\eta)+\frac{315}{2\eta^4}\cos(2\eta)-\frac{315}{4\eta^5}\sin(2\eta)\right\}$$
$$=-4\sum_{n=1}^{\infty}(-1)^n\frac{(2n-5)(2n-3)n}{(2n+7)(2n+5)(2n+3)!}(2\eta)^{2n-1},$$

$$J_{441}=-\frac{1}{4\eta^2}\left\{1+\frac{1}{\eta}\sin(2\eta)+\frac{1}{2\eta^2}[10+37\cos(2\eta)]-\frac{153}{\eta^3}\sin(2\eta)+\frac{15}{2\eta^4}[6-97\cos(2\eta)]\right.$$
$$\left.+\frac{4305}{2\eta^5}\sin(2\eta)+\frac{315}{4\eta^6}[5+51\cos(2\eta)]-\frac{4410}{\eta^7}\sin(2\eta)+\frac{2205}{\eta^8}[1-\cos(2\eta)]\right\}$$
$$=\sum_{n=0}^{\infty}(-1)^n\frac{(n-3)(n-2)(n-1)}{(n+5)(n+4)(n+3)(n+2)(2n+2)!}(2\eta)^{2n},$$

$$J_{442}=\frac{1}{\eta}\left\{\frac{1}{9}-\frac{1}{4\eta^2}\cos(2\eta)+\frac{37}{8\eta^3}\sin(2\eta)+\frac{153}{4\eta^4}\cos(2\eta)-\frac{1455}{8\eta^5}\sin(2\eta)-\frac{4305}{8\eta^6}\cos(2\eta)\right.$$
$$\left.+\frac{16065}{16\eta^7}\sin(2\eta)+\frac{2205}{2\eta^8}\cos(2\eta)-\frac{2205}{4\eta^9}\sin(2\eta)\right\}$$
$$=2\sum_{n=1}^{\infty}(-1)^n\frac{(2n-7)(2n-5)(2n-3)}{(2n+9)(2n+7)(2n+5)(2n+3)(2n+1)!}(2\eta)^{2n-1},$$

$$J_{151}=-\frac{1}{4\eta^2}\left\{1-\frac{1}{\eta}\sin(2\eta)-\frac{1}{2\eta^2}[90+29\cos(2\eta)]+\frac{91}{\eta^3}\sin(2\eta)+\frac{35}{2\eta^4}[10+17\cos(2\eta)]-\frac{945}{2\eta^5}\sin(2\eta)\right.$$
$$\left.+\frac{945}{4\eta^6}[1-\cos(2\eta)]\right\}=-4\sum_{n=0}^{\infty}(-1)^n\frac{(n-2)(n-1)(2n-1)(2n+1)}{(n+4)(2n+6)!}(2\eta)^{2n},$$

$$J_{152}=\frac{1}{4\eta^3}\left\{-\frac{28}{3}+\cos(2\eta)-\frac{29}{2\eta}\sin(2\eta)+\frac{7}{\eta^2}[18-13\cos(2\eta)]+\frac{595}{2\eta^3}\sin(2\eta)+\frac{945}{2\eta^4}\cos(2\eta)\right.$$
$$\left.-\frac{945}{4\eta^5}\sin(2\eta)\right\}=-8\sum_{n=2}^{\infty}(-1)^n\frac{(2n-5)(2n-3)(n-1)n}{(2n+7)(2n+5)!}(2\eta)^{2n-1},$$

$$J_{351}=\frac{1}{4\eta^2}\left\{1-\frac{1}{\eta}\sin(2\eta)-\frac{3}{2\eta^2}[10+13\cos(2\eta)]+\frac{171}{\eta^3}\sin(2\eta)-\frac{75}{2\eta^4}[2-23\cos(2\eta)]-\frac{5355}{2\eta^5}\sin(2\eta)\right.$$
$$\left.-\frac{315}{4\eta^6}[7+65\cos(2\eta)]+\frac{5670}{\eta^7}\sin(2\eta)-\frac{2835}{\eta^8}[1-\cos(2\eta)]\right\}$$
$$=2\sum_{n=0}^{\infty}(-1)^n\frac{(n-3)(n-2)(n-1)(2n+1)}{(n+5)(n+4)(n+3)(2n+4)!}(2\eta)^{2n},$$

$$J_{352}=\frac{1}{4\eta^3}\Big\{6-\cos(2\eta)+\frac{39}{2\eta}\sin(2\eta)+\frac{171}{\eta^2}\cos(2\eta)-\frac{1725}{2\eta^3}\sin(2\eta)-\frac{5355}{2\eta^4}\cos(2\eta)+\frac{20475}{4\eta^5}\sin(2\eta)$$
$$+\frac{5670}{\eta^6}\cos(2\eta)-\frac{2835}{\eta^7}\sin(2\eta)\Big\}=4\sum_{n=1}^{\infty}(-1)^n\frac{(2n-7)(2n-5)(2n-3)n}{(2n+9)(2n+7)(2n+5)(2n+3)!}(2\eta)^{2n-1},$$

$$J_{551}=-\frac{1}{4\eta^2}\Big\{1-\frac{1}{\eta}\sin(2\eta)+\frac{3}{2\eta^2}[5+19\cos(2\eta)]+\frac{378}{\eta^3}\sin(2\eta)+\frac{105}{\eta^4}[1+29\cos(2\eta)]-\frac{16380}{\eta^5}\sin(2\eta)$$
$$+\frac{315}{\eta^6}[5-194\cos(2\eta)]+\frac{158760}{\eta^7}\sin(2\eta)+\frac{19845}{\eta^8}[1+14\cos(2\eta)]-\frac{297675}{\eta^9}\sin(2\eta)$$
$$+\frac{297675}{2\eta^{10}}[1-\cos(2\eta)]\Big\}=-\sum_{n=0}^{\infty}(-1)^n\frac{(n-4)(n-3)(n-2)(n-1)}{(n+6)(n+5)(n+4)(n+3)(n+2)(2n+2)!}(2\eta)^{2n},$$

$$J_{552}=\frac{1}{\eta}\Big\{\frac{1}{11}+\frac{1}{4\eta^2}\cos(2\eta)-\frac{57}{8\eta^3}\sin(2\eta)-\frac{189}{2\eta^4}\cos(2\eta)+\frac{3045}{4\eta^5}\sin(2\eta)+\frac{4095}{\eta^6}\cos(2\eta)-\frac{30555}{2\eta^7}\sin(2\eta)$$
$$-\frac{39690}{\eta^8}\cos(2\eta)+\frac{138915}{2\eta^9}\sin(2\eta)+\frac{297675}{4\eta^{10}}\cos(2\eta)-\frac{297675}{8\eta^{11}}\sin(2\eta)\Big\}$$
$$=-2\sum_{n=1}^{\infty}(-1)^n\frac{(2n-9)(2n-7)(2n-5)(2n-3)}{(2n+11)(2n+9)(2n+7)(2n+5)(2n+3)(2n+1)!}(2\eta)^{2n-1},$$

$$J_{061}=\frac{1}{4\eta^2}\Big\{1+\frac{1}{\eta}\sin(2\eta)-\frac{3}{2\eta^2}[70-13\cos(2\eta)]-\frac{171}{\eta^3}\sin(2\eta)+\frac{45}{2\eta^4}[70-37\cos(2\eta)]+\frac{4455}{2\eta^5}\sin(2\eta)$$
$$-\frac{10395}{4\eta^6}[1-\cos(2\eta)]\Big\}=-8\sum_{n=0}^{\infty}(-1)^n\frac{(n-2)(2n-3)(n-1)(2n-1)(2n+1)}{(2n+8)!}(2\eta)^{2n},$$

$$J_{062}=\frac{1}{\eta^3}\Big\{\frac{7}{2}+\frac{1}{4}\cos(2\eta)-\frac{39}{8\eta}\sin(2\eta)-\frac{9}{4\eta^2}[56+19\cos(2\eta)]+\frac{1665}{8\eta^3}\sin(2\eta)+\frac{1485}{8\eta^4}[4+3\cos(2\eta)]$$
$$-\frac{10395}{16\eta^5}\sin(2\eta)\Big\}=-16\sum_{n=3}^{\infty}(-1)^n\frac{(2n-5)(n-2)(2n-3)(n-1)n}{(2n+7)!}(2\eta)^{2n-1},$$

$$J_{261}=-\frac{1}{4\eta^2}\Big\{1+\frac{1}{\eta}\sin(2\eta)-\frac{15}{2\eta^2}[10-3\cos(2\eta)]-\frac{231}{\eta^3}\sin(2\eta)+\frac{21}{2\eta^4}[50-131\cos(2\eta)]+\frac{10017}{2\eta^5}\sin(2\eta)$$
$$+\frac{189}{4\eta^6}[35+229\cos(2\eta)]-\frac{12474}{\eta^7}\sin(2\eta)+\frac{6237}{\eta^8}[1-\cos(2\eta)]\Big\}$$
$$=4\sum_{n=0}^{\infty}(-1)^n\frac{(n-3)(n-2)(n-1)(2n-1)(2n+1)}{(n+5)(n+4)(2n+6)!}(2\eta)^{2n},$$

$$J_{262}=-\frac{1}{\eta^3}\Big\{3+\frac{1}{4}\cos(2\eta)-\frac{45}{8\eta}\sin(2\eta)-\frac{231}{20\eta^2}[6+5\cos(2\eta)]+\frac{2751}{8\eta^3}\sin(2\eta)+\frac{10017}{8\eta^4}\cos(2\eta)$$
$$-\frac{43281}{16\eta^5}\sin(2\eta)-\frac{6237}{2\eta^6}\cos(2\eta)+\frac{6237}{4\eta^7}\sin(2\eta)\Big\}=8\sum_{n=2}^{\infty}(-1)^n\frac{(2n-7)(2n-5)(2n-3)(n-1)n}{(2n+9)(2n+7)(2n+5)!}(2\eta)^{2n-1},$$

$$J_{461}=\frac{1}{4\eta^2}\Big\{1+\frac{1}{\eta}\sin(2\eta)-\frac{1}{2\eta^2}[45-59\cos(2\eta)]-\frac{406}{\eta^3}\sin(2\eta)-\frac{5}{\eta^4}[35+679\cos(2\eta)]+\frac{18900}{\eta^5}\sin(2\eta)$$
$$-\frac{315}{\eta^6}[7-230\cos(2\eta)]-\frac{191520}{\eta^7}\sin(2\eta)-\frac{945}{\eta^8}[27+358\cos(2\eta)]+\frac{363825}{\eta^9}\sin(2\eta)$$
$$-\frac{363825}{2\eta^{10}}[1-\cos(2\eta)]\Big\}=-2\sum_{n=0}^{\infty}(-1)^n\frac{(n-4)(n-3)(n-2)(n-1)(2n+1)}{(n+6)(n+5)(n+4)(n+3)(2n+4)!}(2\eta)^{2n},$$

$$J_{462}=\frac{1}{\eta^3}\Big\{\frac{11}{6}+\frac{1}{4}\cos(2\eta)-\frac{59}{8\eta}\sin(2\eta)-\frac{203}{2\eta^2}\cos(2\eta)+\frac{3395}{4\eta^3}\sin(2\eta)+\frac{4725}{\eta^4}\cos(2\eta)-\frac{36225}{2\eta^5}\sin(2\eta)$$
$$-\frac{47880}{\eta^6}\cos(2\eta)+\frac{169155}{2\eta^7}\sin(2\eta)+\frac{363825}{4\eta^8}\cos(2\eta)-\frac{363825}{8\eta^9}\sin(2\eta)\Big\}$$
$$=-4\sum_{n=1}^{\infty}(-1)^n\frac{(2n-9)(2n-7)(2n-5)(2n-3)n}{(2n+11)(2n+9)(2n+7)(2n+5)(2n+3)!}(2\eta)^{2n-1},$$

$$J_{661}=-\frac{1}{4\eta^2}\Big\{1+\frac{1}{\eta}\sin(2\eta)+\frac{3}{2\eta^2}[7+27\cos(2\eta)]-\frac{780}{\eta^3}\sin(2\eta)+\frac{30}{\eta^4}[7-313\cos(2\eta)]+\frac{78300}{\eta^5}\sin(2\eta)$$
$$+\frac{135}{\eta^6}[35+3514\cos(2\eta)]-\frac{2131920}{\eta^7}\sin(2\eta)+\frac{14175}{\eta^8}[7-502\cos(2\eta)]+\frac{17307675}{\eta^9}\sin(2\eta)$$
$$+\frac{467775}{2\eta^{10}}[7+125\cos(2\eta)]-\frac{30873150}{\eta^{11}}\sin(2\eta)+\frac{15436575}{\eta^{12}}[1-\cos(2\eta)]\Big\}$$
$$=\sum_{n=0}^{\infty}(-1)^n\frac{(n-5)(n-4)(n-3)(n-2)(n-1)}{(n+7)(n+6)(n+5)(n+4)(n+3)(n+2)(2n+2)!}(2\eta)^{2n},$$

$$J_{662}=\frac{1}{\eta}\Big\{\frac{1}{13}-\frac{1}{4\eta^2}\cos(2\eta)+\frac{81}{8\eta^3}\sin(2\eta)+\frac{195}{\eta^4}\cos(2\eta)-\frac{4695}{2\eta^5}\sin(2\eta)-\frac{19575}{\eta^6}\cos(2\eta)$$
$$+\frac{237195}{2\eta^7}\sin(2\eta)+\frac{532980}{\eta^8}\cos(2\eta)-\frac{3557925}{2\eta^9}\sin(2\eta)-\frac{17307675}{4\eta^{10}}\cos(2\eta)+\frac{58471875}{8\eta^{11}}\sin(2\eta)$$
$$+\frac{15436575}{2\eta^{12}}\cos(2\eta)-\frac{15436575}{4\eta^{13}}\sin(2\eta)\Big\}$$
$$=2\sum_{n=1}^{\infty}(-1)^n\frac{(2n-11)(2n-9)(2n-7)(2n-5)(2n-3)}{(2n+13)(2n+11)(2n+9)(2n+7)(2n+5)(2n+3)(2n+1)!}(2\eta)^{2n-1},$$

$$J_{171}=\frac{1}{4\eta^2}\Big\{1-\frac{1}{\eta}\sin(2\eta)-\frac{5}{2\eta^2}[70+11\cos(2\eta)]+\frac{351}{\eta^3}\sin(2\eta)+\frac{9}{2\eta^4}[1050+589\cos(2\eta)]-\frac{25047}{2\eta^5}\sin(2\eta)$$
$$-\frac{2079}{4\eta^6}[35+69\cos(2\eta)]+\frac{54054}{\eta^7}\sin(2\eta)-\frac{27027}{\eta^8}[1-\cos(2\eta)]\Big\}$$
$$=8\sum_{n=0}^{\infty}(-1)^n\frac{(n-3)(n-2)(2n-3)(n-1)(2n-1)(2n+1)}{(n+5)(2n+8)!}(2\eta)^{2n},$$

$$J_{172}=\frac{1}{\eta^3}\Big\{\frac{9}{2}-\frac{1}{4}\cos(2\eta)+\frac{55}{8\eta}\sin(2\eta)-\frac{9}{20\eta^2}[616-195\cos(2\eta)]-\frac{5301}{8\eta^3}\sin(2\eta)+\frac{99}{8\eta^4}[260-253\cos(2\eta)]$$
$$+\frac{143451}{16\eta^5}\sin(2\eta)+\frac{27027}{2\eta^6}\cos(2\eta)-\frac{27027}{4\eta^7}\sin(2\eta)\Big\}=16\sum_{n=3}^{\infty}(-1)^n\frac{(2n-7)(2n-5)(n-2)(2n-3)(n-1)n}{(2n+9)(2n+7)!}(2\eta)^{2n-1},$$

$$J_{371}=-\frac{1}{4\eta^2}\Big\{1-\frac{1}{\eta}\sin(2\eta)-\frac{5}{2\eta^2}[45+13\cos(2\eta)]+\frac{496}{\eta^3}\sin(2\eta)+\frac{7}{\eta^4}[175+659\cos(2\eta)]-\frac{38476}{\eta^5}\sin(2\eta)$$
$$+\frac{63}{\eta^6}[105-1898\cos(2\eta)]+\frac{338184}{\eta^7}\sin(2\eta)+\frac{2079}{\eta^8}[27+298\cos(2\eta)]-\frac{675675}{\eta^9}\sin(2\eta)$$
$$+\frac{675675}{2\eta^{10}}[1-\cos(2\eta)]\Big\}=-4\sum_{n=0}^{\infty}(-1)^n\frac{(n-4)(n-3)(n-2)(n-1)(2n-1)(2n+1)}{(n+6)(n+5)(n+4)(2n+6)!}(2\eta)^{2n},$$

$$J_{372}=-\frac{1}{\eta^3}\Big\{\frac{11}{3}-\frac{1}{4}\cos(2\eta)+\frac{65}{8\eta}\sin(2\eta)-\frac{1}{10\eta^2}[1287-1240\cos(2\eta)]-\frac{4613}{4\eta^3}\sin(2\eta)-\frac{7119}{\eta^4}\cos(2\eta)$$
$$+\frac{59787}{2\eta^5}\sin(2\eta)+\frac{84546}{\eta^6}\cos(2\eta)-\frac{309771}{2\eta^7}\sin(2\eta)-\frac{675675}{4\eta^8}\cos(2\eta)+\frac{675675}{8\eta^9}\sin(2\eta)\Big\}$$
$$=-8\sum_{n=2}^{\infty}(-1)^n\frac{(2n-9)(2n-7)(2n-5)(2n-3)(n-1)n}{(2n+11)(2n+9)(2n+7)(2n+5)!}(2\eta)^{2n-1},$$

$$J_{571}=\frac{1}{4\eta^2}\Big\{1-\frac{1}{\eta}\sin(2\eta)-\frac{1}{2\eta^2}[63+83\cos(2\eta)]+\frac{820}{\eta^3}\sin(2\eta)-\frac{10}{\eta^4}[35-1013\cos(2\eta)]-\frac{86580}{\eta^5}\sin(2\eta)$$
$$-\frac{315}{\eta^6}[21+1702\cos(2\eta)]+\frac{2451960}{\eta^7}\sin(2\eta)-\frac{4725}{\eta^8}[27-1754\cos(2\eta)]-\frac{20322225}{\eta^9}\sin(2\eta)$$
$$-\frac{51975}{2\eta^{10}}[77+1327\cos(2\eta)]+\frac{36486450}{\eta^{11}}\sin(2\eta)-\frac{18243225}{\eta^{12}}[1-\cos(2\eta)]\Big\}$$
$$=2\sum_{n=0}^{\infty}(-1)^n\frac{(n-5)(n-4)(n-3)(n-2)(n-1)(2n+1)}{(n+7)(n+6)(n+5)(n+4)(n+3)(2n+4)!}(2\eta)^{2n},$$

$$J_{572}=\frac{1}{\eta^3}\Big\{\frac{13}{6}-\frac{1}{4}\cos(2\eta)+\frac{83}{8\eta}\sin(2\eta)+\frac{205}{\eta^2}\cos(2\eta)-\frac{5065}{2\eta^3}\sin(2\eta)-\frac{21645}{\eta^4}\cos(2\eta)+\frac{268065}{2\eta^5}\sin(2\eta)$$
$$+\frac{612990}{\eta^6}\cos(2\eta)-\frac{4143825}{2\eta^7}\sin(2\eta)-\frac{20322225}{4\eta^8}\cos(2\eta)+\frac{68970825}{8\eta^9}\sin(2\eta)+\frac{18243225}{2\eta^{10}}\cos(2\eta)$$
$$-\frac{18243225}{4\eta^{11}}\sin(2\eta)\Big\}=4\sum_{n=1}^{\infty}(-1)^n\frac{(2n-11)(2n-9)(2n-7)(2n-5)(2n-3)n}{(2n+13)(2n+11)(2n+9)(2n+7)(2n+5)(2n+3)!}(2\eta)^{2n-1},$$

$$J_{771}=-\frac{1}{4\eta^2}\Big\{1-\frac{1}{\eta}\sin(2\eta)+\frac{1}{2\eta^2}[28-109\cos(2\eta)]+\frac{1431}{\eta^3}\sin(2\eta)+\frac{9}{2\eta^4}[84+5309\cos(2\eta)]-\frac{564525}{2\eta^5}\sin(2\eta)$$
$$+\frac{945}{4\eta^6}[50-10533\cos(2\eta)]+\frac{16829505}{\eta^7}\sin(2\eta)+\frac{1485}{2\eta^8}[490+119063\cos(2\eta)]-\frac{721589715}{2\eta^9}\sin(2\eta)$$
$$+\frac{467775}{4\eta^{10}}[84-9665\cos(2\eta)]+\frac{52844541175}{2\eta^{11}}\sin(2\eta)+\frac{15436575}{4\eta^{12}}[52+1131\cos(2\eta)]$$
$$-\frac{18261468225}{4\eta^{13}}\sin(2\eta)+\frac{18261468225}{8\eta^{14}}[1-\cos(2\eta)]\Big\}$$
$$=-\sum_{n=0}^{\infty}(-1)^n\frac{(n-6)(n-5)(n-4)(n-3)(n-2)(n-1)}{(n+8)(n+7)(n+6)(n+5)(n+4)(n+3)(n+2)(2n+2)!}(2\eta)^{2n},$$

$$J_{772}=\frac{1}{\eta}\Big\{\frac{1}{15}+\frac{1}{4\eta^2}\cos(2\eta)-\frac{109}{8\eta^3}\sin(2\eta)-\frac{1431}{4\eta^4}\cos(2\eta)+\frac{47781}{8\eta^5}\sin(2\eta)+\frac{564525}{8\eta^6}\cos(2\eta)$$
$$-\frac{9953685}{16\eta^7}\sin(2\eta)-\frac{16829505}{4\eta^8}\cos(2\eta)+\frac{176808555}{8\eta^9}\sin(2\eta)+\frac{721589715}{8\eta^{10}}\cos(2\eta)$$
$$-\frac{4521045375}{16\eta^{11}}\sin(2\eta)-\frac{52844541175}{8\eta^{12}}\cos(2\eta)+\frac{17458766325}{16\eta^{13}}\sin(2\eta)+\frac{18261468225}{16\eta^{14}}\cos(2\eta)$$
$$-\frac{18261468225}{32\eta^{15}}\sin(2\eta)\Big\}=-2\sum_{n=1}^{\infty}(-1)^n\frac{n(2n-13)(2n-11)(2n-9)(2n-7)(2n-5)(2n-3)}{(2n+15)(2n+13)(2n+11)(2n+9)(2n+7)(2n+5)(2n+3)(2n+1)!}(2\eta)^{2n-1}.$$

3. $\frac{d}{ds}J_{nm}(\alpha_g, s)$

$$\Sigma_g v_g P_g^2\frac{d}{ds}J_{00}=-1+\frac{1}{2\alpha}(1-e^{-2\alpha})=\sum_{n=1}^{\infty}\frac{1}{(n+1)!}(-2\alpha)^n,$$

$$\Sigma_g v_g P_g^2\frac{d}{ds}J_{11}=-\frac{1}{3}+\frac{1}{2\alpha}\left(1+\frac{1}{\alpha}\right)\left[1-\frac{1}{\alpha}+\left(1+\frac{1}{\alpha}\right)e^{-2\alpha}\right]=-\sum_{n=2}^{\infty}\frac{(n-1)(n+2)}{(n+3)!}(-2\alpha)^n,$$

$$\Sigma_g v_g P_g^2\frac{d}{ds}J_{02}=-\frac{1}{2\alpha}\left[1-\frac{3}{\alpha}+\frac{3}{\alpha^2}-\left(1+\frac{3}{\alpha}+\frac{3}{\alpha^2}\right)e^{-2\alpha}\right]=-\sum_{n=2}^{\infty}\frac{(n-1)n}{(n+3)!}(-2\alpha)^n,$$

$$\Sigma_g v_g P_g^2\frac{d}{ds}J_{22}=-\frac{1}{5}+\frac{1}{2\alpha}\left(1+\frac{3}{\alpha}+\frac{3}{\alpha^2}\right)\left[-2\alpha\,\Sigma_g v_g P_g^2\frac{d}{ds}J_{02}\right]=\sum_{n=2}^{\infty}\frac{(n-3)(n-1)(n+2)}{(n+5)(n+3)!}(-2\alpha)^n,$$

$$\Sigma_g v_g P_g^2\frac{d}{ds}J_{13}=-\frac{1}{2\alpha}\left(1+\frac{1}{\alpha}\right)\left[1-\frac{6}{\alpha}+\frac{15}{\alpha^2}-\frac{15}{\alpha^3}+\left(1+\frac{6}{\alpha}+\frac{15}{\alpha^2}+\frac{15}{\alpha^3}\right)e^{-2\alpha}\right]=\sum_{n=2}^{\infty}\frac{(n-3)(n-1)n}{(n+5)(n+3)!}(-2\alpha)^n,$$

$$\Sigma_g v_g P_g^2\frac{d}{ds}J_{33}=-\frac{1}{7}+\frac{1}{2\alpha}\left(1+\frac{6}{\alpha}+\frac{15}{\alpha^2}+\frac{15}{\alpha^3}\right)\left[-2\alpha\,\Sigma_g v_g P_g^2\frac{d}{ds}J_{13}\Big/\left(1+\frac{1}{\alpha}\right)\right]$$
$$=-\sum_{n=2}^{\infty}\frac{(n-5)(n-3)(n-1)(n+2)}{(n+7)(n+5)(n+3)!}(-2\alpha)^n,$$

$$\Sigma_g v_g P_g^2\frac{d}{ds}J_{04}=\frac{1}{2\alpha}\left[1-\frac{10}{\alpha}+\frac{45}{\alpha^2}-\frac{105}{\alpha^3}+\frac{105}{\alpha^4}-\left(1+\frac{10}{\alpha}+\frac{45}{\alpha^2}+\frac{105}{\alpha^3}+\frac{105}{\alpha^4}\right)e^{-2\alpha}\right]$$
$$=\sum_{n=4}^{\infty}\frac{(n-3)(n-2)(n-1)n}{(n+5)!}(-2\alpha)^n,$$

$$\Sigma_g v_g P_g^2\frac{d}{ds}J_{24}=-\frac{1}{2\alpha}\left(1+\frac{3}{\alpha}+\frac{3}{\alpha^2}\right)\left[2\alpha\,\Sigma_g v_g P_g^2\frac{d}{ds}J_{04}\right]=-\sum_{n=2}^{\infty}\frac{(n-5)(n-3)(n-1)n}{(n+7)(n+5)(n+3)!}(-2\alpha)^n,$$

$$\Sigma_j v_j P_j^2 \frac{d}{d\lambda} J_{44} = -\frac{1}{9} + \frac{1}{2\alpha}\left(1+\frac{10}{\alpha}+\frac{45}{\alpha^2}+\frac{105}{\alpha^3}+\frac{105}{\alpha^4}\right)\left[2\alpha\Sigma_j v_j P_j^2 \frac{d}{d\lambda} J_{04}\right] = \sum_{n=2}^{\infty} \frac{(n-7)(n-5)(n-3)(n-1)(n+2)}{(n+9)(n+7)(n+5)(n+3)!}(-2\alpha)^n,$$

$$\Sigma_j v_j P_j^2 \frac{d}{d\lambda} J_{15} = \frac{1}{2\alpha}\left(1+\frac{1}{\alpha}\right)\left[1-\frac{15}{\alpha}+\frac{105}{\alpha^2}-\frac{420}{\alpha^3}+\frac{945}{\alpha^4}-\frac{945}{\alpha^5}+\left(1+\frac{15}{\alpha}+\frac{105}{\alpha^2}+\frac{420}{\alpha^3}+\frac{945}{\alpha^4}+\frac{945}{\alpha^5}\right)e^{-2\alpha}\right]$$
$$= -\sum_{n=4}^{\infty} \frac{(n-5)(n-3)(n-2)(n-1)n}{(n+7)(n+5)!}(-2\alpha)^n,$$

$$\Sigma_j v_j P_j^2 \frac{d}{d\lambda} J_{35} = -\frac{1}{2\alpha}\left(1+\frac{6}{\alpha}+\frac{15}{\alpha^2}+\frac{15}{\alpha^3}\right)\left[2\alpha\Sigma_j v_j P_j^2 \frac{d}{d\lambda} J_{15}/\left(1+\frac{1}{\alpha}\right)\right] = \sum_{n=2}^{\infty} \frac{(n-7)(n-5)(n-3)(n-1)n}{(n+9)(n+7)(n+5)(n+3)!}(-2\alpha)^n,$$

$$\Sigma_j v_j P_j^2 \frac{d}{d\lambda} J_{55} = -\frac{1}{11} + \frac{1}{2\alpha}\left(1+\frac{15}{\alpha}+\frac{105}{\alpha^2}+\frac{420}{\alpha^3}+\frac{945}{\alpha^4}+\frac{945}{\alpha^5}\right)\left[2\alpha\Sigma_j v_j P_j^2 \frac{d}{d\lambda} J_{15}/\left(1+\frac{1}{\alpha}\right)\right]$$
$$= -\sum_{n=2}^{\infty} \frac{(n-9)(n-7)(n-5)(n-3)(n-1)(n+2)}{(n+11)(n+9)(n+7)(n+5)(n+3)!}(-2\alpha)^n,$$

$$\Sigma_j v_j P_j^2 \frac{d}{d\lambda} J_{06} = -\frac{1}{2\alpha}\left[1-\frac{21}{\alpha}+\frac{210}{\alpha^2}-\frac{1260}{\alpha^3}+\frac{4725}{\alpha^4}-\frac{10395}{\alpha^5}+\frac{10395}{\alpha^6}-\left(1+\frac{21}{\alpha}+\frac{210}{\alpha^2}+\frac{1260}{\alpha^3}+\frac{4725}{\alpha^4}+\frac{10395}{\alpha^5}\right.\right.$$
$$\left.\left.+\frac{10395}{\alpha^6}\right)e^{-2\alpha}\right] = -\sum_{n=6}^{\infty} \frac{(n-5)(n-4)(n-3)(n-2)(n-1)n}{(n+7)!}(-2\alpha)^n,$$

$$\Sigma_j v_j P_j^2 \frac{d}{d\lambda} J_{26} = \frac{1}{2\alpha}\left(1+\frac{3}{\alpha}+\frac{3}{\alpha^2}\right)\left[-2\alpha\Sigma_j v_j P_j^2 \frac{d}{d\lambda} J_{06}\right] = \sum_{n=4}^{\infty} \frac{(n-7)(n-5)(n-3)(n-2)(n-1)n}{(n+9)(n+7)(n+5)!}(-2\alpha)^n,$$

$$\Sigma_j v_j P_j^2 \frac{d}{d\lambda} J_{46} = -\frac{1}{2\alpha}\left(1+\frac{10}{\alpha}+\frac{45}{\alpha^2}+\frac{105}{\alpha^3}+\frac{105}{\alpha^4}\right)\left[-2\alpha\Sigma_j v_j P_j^2 \frac{d}{d\lambda} J_{06}\right]$$
$$= -\sum_{n=2}^{\infty} \frac{(n-9)(n-7)(n-5)(n-3)(n-1)n}{(n+11)(n+9)(n+7)(n+5)(n+3)!}(-2\alpha)^n,$$

$$\Sigma_j v_j P_j^2 \frac{d}{d\lambda} J_{66} = -\frac{1}{13} + \frac{1}{2\alpha}\left(1+\frac{21}{\alpha}+\frac{210}{\alpha^2}+\frac{1260}{\alpha^3}+\frac{4725}{\alpha^4}+\frac{10395}{\alpha^5}+\frac{10395}{\alpha^6}\right)\left[-2\alpha\Sigma_j v_j P_j^2 \frac{d}{d\lambda} J_{06}\right]$$
$$= \sum_{n=2}^{\infty} \frac{(n-11)(n-9)(n-7)(n-5)(n-3)(n-1)(n+2)}{(n+13)(n+11)(n+9)(n+7)(n+5)(n+3)!}(-2\alpha)^n,$$

$$\Sigma_j v_j P_j^2 \frac{d}{d\lambda} J_{17} = -\frac{1}{2\alpha}\left(1+\frac{1}{\alpha}\right)\left[1-\frac{28}{\alpha}+\frac{378}{\alpha^2}-\frac{3150}{\alpha^3}+\frac{17325}{\alpha^4}-\frac{62370}{\alpha^5}+\frac{135135}{\alpha^6}-\frac{135135}{\alpha^7}\right.$$
$$\left.+\left(1+\frac{28}{\alpha}+\frac{378}{\alpha^2}+\frac{3150}{\alpha^3}+\frac{17325}{\alpha^4}+\frac{62370}{\alpha^5}+\frac{135135}{\alpha^6}+\frac{135135}{\alpha^7}\right)e^{-2\alpha}\right]$$
$$= \sum_{n=6}^{\infty} \frac{(n-7)(n-5)(n-4)(n-3)(n-2)(n-1)n}{(n+9)(n+7)!}(-2\alpha)^n,$$

$$\Sigma_j v_j P_j^2 \frac{d}{d\lambda} J_{37} = \frac{1}{2\alpha}\left(1+\frac{6}{\alpha}+\frac{15}{\alpha^2}+\frac{15}{\alpha^3}\right)\left[-2\alpha\Sigma_j v_j P_j^2 \frac{d}{d\lambda} J_{17}/\left(1+\frac{1}{\alpha}\right)\right]$$
$$= -\sum_{n=4}^{\infty} \frac{(n-9)(n-7)(n-5)(n-3)(n-2)(n-1)n}{(n+11)(n+9)(n+7)(n+5)!}(-2\alpha)^n,$$

$$\Sigma_j v_j P_j^2 \frac{d}{d\lambda} J_{57} = -\frac{1}{2\alpha}\left(1+\frac{15}{\alpha}+\frac{105}{\alpha^2}+\frac{420}{\alpha^3}+\frac{945}{\alpha^4}+\frac{945}{\alpha^5}\right)\left[-2\alpha\Sigma_j v_j P_j^2 \frac{d}{d\lambda} J_{17}/\left(1+\frac{1}{\alpha}\right)\right]$$
$$= \sum_{n=2}^{\infty} \frac{(n-11)(n-9)(n-7)(n-5)(n-3)(n-1)n}{(n+13)(n+11)(n+9)(n+7)(n+5)(n+3)!}(-2\alpha)^n,$$

$$\Sigma_j v_j P_j^2 \frac{d}{d\lambda} J_{77} = -\frac{1}{15} + \frac{1}{2\alpha}\left(1+\frac{28}{\alpha}+\frac{378}{\alpha^2}+\frac{3150}{\alpha^3}+\frac{17325}{\alpha^4}+\frac{62370}{\alpha^5}+\frac{135135}{\alpha^6}+\frac{135135}{\alpha^7}\right)$$
$$\times\left[-2\alpha\Sigma_j v_j P_j^2 \frac{d}{d\lambda} J_{17}/\left(1+\frac{1}{\alpha}\right)\right] = -\sum_{n=2}^{\infty} \frac{(n-13)(n-11)(n-9)(n-7)(n-5)(n-3)(n-1)(n+2)}{(n+15)(n+13)(n+11)(n+9)(n+7)(n+5)(n+3)!}(-2\alpha)^n.$$

4. $G_n(\alpha_g, 2\xi-1, s)$ [a)]

$$4\alpha G_0 = \Big[1-e^{-2\alpha(1-\xi)}+2\alpha(1-\xi)\int_1^\infty \frac{dz}{z}\, e^{-2\alpha(1-\xi)z}\Big]+\Big[\xi\to 1-\xi\Big]$$

$$= 2\alpha\Big[-\ln(2\alpha(1-\xi))+1-\gamma+\xi\ln\Big(\frac{1-\xi}{\xi}\Big)\Big]+\sum_{n=1}^{\infty}\frac{1}{n(n+1)!}\Big[(2(1-\xi))^{n+1}+(2\xi)^{n+1}\Big](-\alpha)^{n+1},$$

$$4\alpha i G_1 = \Big[2\xi-1-\Big(\xi+\frac{1}{2\alpha}\Big)e^{-2\alpha(1-\xi)}+2\alpha\xi(1-\xi)\int_{2(1-\xi)}^{\infty}\frac{dz}{z}\, e^{-\alpha z}\Big]-\Big[\xi\to 1-\xi\Big]$$

$$= (2\xi-1)\alpha-2\alpha\xi(1-\xi)\ln\Big(\frac{1-\xi}{\xi}\Big)+\sum_{n=1}^{\infty}\frac{1}{n(n+2)!}\Big[(n+2\xi)(2(1-\xi))^{n+1}-(n+2(1-\xi))(2\xi)^{n+1}\Big](-\alpha)^{n+1},$$

$$4\alpha G_2 = \Big[6\xi(1-\xi)-1-\frac{1}{\alpha^2}+\Big[\xi(2\xi-1)+(1+2\xi)\frac{1}{2\alpha}+\frac{1}{\alpha^2}\Big]e^{-2\alpha(1-\xi)}-2\alpha\xi(1-\xi)(2\xi-1)\int_{2(1-\xi)}^{\infty}\frac{dz}{z}\, e^{-\alpha z}\Big]$$

$$+\Big[\xi\to 1-\xi\Big] = 2\alpha\Big[2\xi(1-\xi)-\frac{1}{6}+\xi(1-\xi)(2\xi-1)\ln\Big(\frac{1-\xi}{\xi}\Big)\Big]-\sum_{n=1}^{\infty}\frac{1}{n(n+3)!}\Big\{\Big[n^2+n(6\xi-1)+6\xi(2\xi-1)\Big]$$

$$\times(2(1-\xi))^{n+1}+\Big[\xi\to 1-\xi\Big](2\xi)^{n+1}\Big\}(-\alpha)^{n+1},$$

$$4\alpha i G_3 = \Big[(1-2\xi)\Big[1-10\xi(1-\xi)+\frac{5}{\alpha^2}\Big]+\Big[\xi(1-5\xi(1-\xi))+(1+5\xi^2)\frac{1}{2\alpha}+(1+\xi)\frac{5}{2\alpha^2}+\frac{15}{4\alpha^3}\Big]e^{-2\alpha(1-\xi)}$$

$$-2\alpha\xi(1-\xi)\Big[1-5\xi(1-\xi)\Big]\int_{2(1-\xi)}^{\infty}\frac{dz}{z}\, e^{-\alpha z}\Big]-\Big[\xi\to 1-\xi\Big] = 2\alpha\Big[\frac{1}{12}(1-2\xi)\Big[1-30\xi(1-\xi)\Big]$$

$$+\xi(1-\xi)\Big[1-5\xi(1-\xi)\Big]\ln\Big(\frac{1-\xi}{\xi}\Big)\Big]-\sum_{n=1}^{\infty}\frac{1}{n(n+4)!}\Big\{\Big[n^3+3n^2(4\xi-1)+2n(30\xi^2-18\xi+1)+24\xi(1-5\xi(1-\xi))\Big]$$

$$\times(2(1-\xi))^{n+1}-\Big[\xi\to 1-\xi\Big](2\xi)^{n+1}\Big\}(-\alpha)^{n+1},$$

$$4\alpha G_4 = \Big[1-20\xi(1-\xi)\Big[1-\frac{7}{2}\xi(1-\xi)+\frac{7}{2\alpha^2}\Big]+\frac{15}{\alpha^2}+\frac{21}{\alpha^4}-\Big[(2\xi-1)(1-7\xi(1-\xi))\xi+(1+2\xi-7\xi^2+14\xi^3)\frac{1}{2\alpha}$$

$$+(9+7\xi+14\xi^2)\frac{1}{2\alpha^2}+(3+2\xi)\frac{21}{4\alpha^3}+\frac{21}{\alpha^4}\Big]e^{-2\alpha(1-\xi)}+2\alpha\xi(1-\xi)(2\xi-1)\Big[1-7\xi(1-\xi)\Big]\int_{2(1-\xi)}^{\infty}\frac{dz}{z}\, e^{-\alpha z}\Big]$$

$$+\Big[\xi\to 1-\xi\Big] = 2\alpha\Big[\frac{1}{20}-\frac{1}{6}\xi(1-\xi)\Big[19-84\xi(1-\xi)\Big]+\xi(1-\xi)(1-2\xi)\Big[1-7\xi(1-\xi)\Big]\ln\Big(\frac{1-\xi}{\xi}\Big)\Big]$$

$$+\sum_{n=1}^{\infty}\frac{1}{n(n+5)!}\Big\{\Big[n^4+2n^3(10\xi-3)+n^2(180\xi^2-120\xi+11)+2n(420\xi^3-450\xi^2+110\xi-3)$$

$$+120\xi(2\xi-1)(1-7\xi(1-\xi))\Big](2(1-\xi))^{n+1}+\Big[\xi\to 1-\xi\Big](2\xi)^{n+1}\Big\}(-\alpha)^{n+1},$$

$$4\alpha i G_5 = \Big[(2\xi-1)\Big[1-28\xi(1-\xi)\Big(1-\frac{9}{2}\xi(1-\xi)\Big)+(1-6\xi(1-\xi))\frac{35}{\alpha^2}+\frac{189}{\alpha^4}\Big]-\Big[\xi\Big[1-14\xi(1-\xi)(1-3\xi(1-\xi))\Big]$$

$$+(1+14\xi^2-42\xi^3+42\xi^4)\frac{1}{2\alpha}+(1+\xi+3\xi^3)\frac{7}{\alpha^2}+(4+3\xi+3\xi^2)\frac{21}{2\alpha^3}+(2+\xi)\frac{63}{\alpha^4}+\frac{315}{2\alpha^5}\Big]e^{-2\alpha(1-\xi)}$$

$$+2\alpha\xi(1-\xi)\Big[1-14\xi(1-\xi)(1-3\xi(1-\xi))\Big]\int_{2(1-\xi)}^{\infty}\frac{dz}{z}\, e^{-\alpha z}\Big]-\Big[\xi\to 1-\xi\Big] = \frac{\alpha}{15}(2\xi-1)\Big[1-105\xi(1-\xi)(1-6\xi(1-\xi))\Big]$$

$$-2\alpha\xi(1-\xi)\Big[1-14\xi(1-\xi)(1-3\xi(1-\xi))\Big]\ln\Big(\frac{1-\xi}{\xi}\Big)+\sum_{n=1}^{\infty}\frac{1}{n(n+6)!}\Big\{\Big[n^5+10n^4(3\xi-1)+5n^3(84\xi^2-60\xi+7)$$

$$+10n^2(336\xi^3-378\xi^2+105\xi-5)+12n(1260\xi^4-1960\xi^3+910\xi^2-125\xi+2)$$

$$+720\xi(1-14\xi(1-\xi)(1-3\xi(1-\xi)))\Big](2(1-\xi))^{n+1}-\Big[\xi\to 1-\xi\Big](2\xi)^{n+1}\Big\}(-\alpha)^{n+1},$$

$$4\alpha G_6=\Big[42\xi(1-\xi)[1-\xi(1-\xi)(9-22\xi(1-\xi))]-1-[1-3\xi(1-\xi)(4-11\xi(1-\xi))]\frac{70}{\alpha^2}-[5-22\xi(1-\xi)]\frac{189}{\alpha^4}-\frac{1485}{\alpha^6}$$

$$+\big[\xi(2\xi-1)[1-\xi(1-\xi)(18-66\xi(1-\xi))]\big]+(1+2\xi-18\xi^2+102\xi^3-198\xi^4+132\xi^5)\frac{1}{2\alpha}$$

$$+(10+9\xi+18\xi^2-33\xi^3+66\xi^4)\frac{1}{\alpha^2}+(20+17\xi+11\xi^2+22\xi^3)\frac{9}{2\alpha^3}+(50+33\xi+22\xi^2)\frac{9}{\alpha^4}+(5+2\xi)\frac{495}{2\alpha^5}$$

$$+\frac{1485}{\alpha^6}\Big]e^{-2\alpha(1-\xi)}-2\alpha\xi(1-\xi)(2\xi-1)[1-\xi(1-\xi)(18-66\xi(1-\xi))]\int_{2(1-\xi)}^{\infty}\frac{dz}{z}e^{-\alpha z}\Big]+\big[\xi\to 1-\xi\big]$$

$$=\frac{\alpha}{105}\Big\{\xi(1-\xi)[819-210\xi(1-\xi)(47-132\xi(1-\xi))]-5\Big\}+2\alpha\xi(1-\xi)(2\xi-1)[1-\xi(1-\xi)(18-66\xi(1-\xi))]\ln\left(\frac{1-\xi}{\xi}\right)$$

$$-\sum_{n=1}^{\infty}\frac{1}{n(n+7)!}\Big\{\big[n^6+3n^5(14\xi-5)+5n^4(168\xi^2-126\xi+17)+15n^3(672\xi^3-784\xi^2+238\xi-15)$$

$$+2n^2(37800\xi^4-60480\xi^3+29820\xi^2-4725\xi+137)+12n(27720\xi^5-56700\xi^4+39480\xi^3-10780\xi^2$$

$$+959\xi-10)+5040\xi(2\xi-1)[1-\xi(1-\xi)(18-66\xi(1-\xi))]\big](2(1-\xi))^{n+1}+\big[\xi\to 1-\xi\big](2\xi)^{n+1}\Big\}(-\alpha)^{n+1},$$

$$4\alpha i G_7=\Big[(1-2\xi)\{1-6\xi(1-\xi)[9-11\xi(1-\xi)(9-26\xi(1-\xi))]+[3-11\xi(1-\xi)(4-13\xi(1-\xi))]\frac{42}{\alpha^2}+[5-26\xi(1-\xi)]\frac{693}{\alpha^4}$$

$$+\frac{19305}{\alpha^6}\}+\big[\xi\{1-3\xi(1-\xi)[9-11\xi(1-\xi)(6-13\xi(1-\xi))]\}\big]+(1+27\xi^2-198\xi^3+627\xi^4-858\xi^5+429\xi^6)$$

$$\times\frac{1}{2\alpha}+(9+9\xi+66\xi^3-143\xi^4+143\xi^5)\frac{3}{2\alpha^2}+(75+66\xi+66\xi^2+143\xi^4)\frac{9}{4\alpha^3}+(25+19\xi+13\xi^2+13\xi^3)\frac{99}{2\alpha^4}$$

$$+(45+26\xi+13\xi^2)\frac{495}{4\alpha^5}+(3+\xi)\frac{19305}{4\alpha^6}+\frac{135135}{8\alpha^7}\Big]e^{-2\alpha(1-\xi)}-2\alpha\xi(1-\xi)\{1-3\xi(1-\xi)[9-11\xi(1-\xi)$$

$$\times(6-13\xi(1-\xi))]\}\int_{2(1-\xi)}^{\infty}\frac{dz}{z}e^{-\alpha z}\Big]-\big[\xi\to 1-\xi\big]=\frac{\alpha}{140}(1-2\xi)\Big\{5-14\xi(1-\xi)[83-55\xi(1-\xi)$$

$$\times(23-78\xi(1-\xi))]\Big\}+2\alpha\xi(1-\xi)\Big\{1-3\xi(1-\xi)[9-11\xi(1-\xi)(6-13\xi(1-\xi))]\Big\}\ln\left(\frac{1-\xi}{\xi}\right)-\sum_{n=1}^{\infty}\frac{1}{n(n+8)!}$$

$$\times\Big\{\big[n^7+7n^6(8\xi-3)+7n^5(216\xi^2-168\xi+25)+35n^4(720\xi^3-864\xi^2+280\xi-21)+56n^3$$

$$\times(4950\xi^4-8100\xi^3+4158\xi^2-735\xi+29)+28n^2(71280\xi^5-148500\xi^4+107100\xi^3-31320\xi^2+3248\xi-63)$$

$$+144n(60060\xi^6-152460\xi^5+142450\xi^4-59850\xi^3+10962\xi^2-686\xi+5)+40320\xi\{1-3\xi(1-\xi)$$

$$\times(9-11\xi(1-\xi)(6-13\xi(1-\xi)))\}\big](2(1-\xi))^{n+1}-\big[\xi\to 1-\xi\big](2\xi)^{n+1}\Big\}(-\alpha)^{n+1}.$$

5. $G_n(\alpha_g, 2\xi-1, iy+z_1\nu_1-z_g\nu_g)\equiv\frac{1}{\eta}(G_{n1}+iG_{n2})$ a) b)

$$2G_{01}=\Big\{\frac{1}{2}\sin[2\eta(1-\xi)]+\eta(1-\xi)\int_1^{\infty}\frac{dz}{z}\cos[2\eta(1-\xi)z]\Big\}+\Big\{\xi\to 1-\xi\Big\}$$

$$=\eta\{-\ln[2\eta(1-\xi)]+1-\gamma\}+\eta\xi\ln\left(\frac{1-\xi}{\xi}\right)-\sum_{n=1}^{\infty}(-1)^n\frac{1}{4n(2n+1)!}[(2(1-\xi))^{2n+1}+(2\xi)^{2n+1}]\eta^{2n+1},$$

$$2G_{02}=-1+\Big\{\frac{1}{2}\cos[2\eta(1-\xi)]-\eta(1-\xi)\int_1^{\infty}\frac{dz}{z}\sin[2\eta(1-\xi)z]\Big\}+\Big\{\xi\to 1-\xi\Big\}$$

$$=-\frac{\pi}{2}\eta-\sum_{n=1}^{\infty}(-1)^n\frac{1}{2(2n-1)(2n)!}[(2(1-\xi))^{2n}+(2\xi)^{2n}]\eta^{2n},$$

$$2G_{11}=1-2\xi+\Big\{\frac{1}{2}\xi\cos[2\eta(1-\xi)]-\frac{1}{4\eta}\sin[2\eta(1-\xi)]\Big\}-\Big\{\xi\to 1-\xi\Big\}-\eta\xi(1-\xi)\int_{2(1-\xi)}^{2\xi}\frac{dz}{z}\sin(\eta z)$$

$$=-\sum_{n=1}^{\infty}(-1)^n\frac{1}{2(2n-1)(2n+1)!}\big[(2n-1+2\xi)(2(1-\xi))^{2n}-(\xi\to 1-\xi)(2\xi)^{2n}\big]\eta^{2n},$$

$$2G_{12}=-\left\{\frac{1}{2}\xi\sin[2\eta(1-\xi)]+\frac{1}{4\eta}\cos[2\eta(1-\xi)]\right\}+\left\{\xi\to 1-\xi\right\}-\eta\xi(1-\xi)\int_{2(1-\xi)}^{2\xi}\frac{dz}{z}\cos(\eta z)$$
$$=\frac{1}{2}(1-2\xi)\eta+\eta\xi(1-\xi)\ln\left(\frac{1-\xi}{\xi}\right)+\sum_{n=1}^{\infty}(-1)^n\frac{1}{4n(2n+2)!}\left[(2n+2\xi)(2(1-\xi))^{2n+1}-(\xi\to 1-\xi)(2\xi)^{2n+1}\right]$$
$$\times\eta^{2n+1},$$

$$2G_{21}=\left\{\frac{1}{2}\left[\xi(1-2\xi)+\frac{1}{\eta^2}\right]\sin[2\eta(1-\xi)]-\frac{1}{4\eta}(1+2\xi)\cos[2\eta(1-\xi)]\right\}+\left\{\xi\to 1-\xi\right\}+\eta\xi(1-\xi)(1-2\xi)$$
$$\times\int_{2(1-\xi)}^{2\xi}\frac{dz}{z}\cos(\eta z)=\left[2\xi(1-\xi)-\frac{1}{6}\right]\eta+\eta\xi(1-\xi)(2\xi-1)\ln\left(\frac{1-\xi}{\xi}\right)+\sum_{n=1}^{\infty}(-1)^n\frac{1}{4n(2n+3)!}\left\{n\to 2n\right\}\eta^{2n+1},$$

$$2G_{22}=1-6\xi(1-\xi)-\frac{1}{\eta^2}+\left\{\frac{1}{2}\left[\xi(1-2\xi)+\frac{1}{\eta^2}\right]\cos[2\eta(1-\xi)]+\frac{1}{4\eta}(1+2\xi)\sin[2\eta(1-\xi)]\right\}+\left\{\xi\to 1-\xi\right\}$$
$$+\eta\xi(1-\xi)(2\xi-1)\int_{2(1-\xi)}^{2\xi}\frac{dz}{z}\sin(\eta z)=\sum_{n=1}^{\infty}(-1)^n\frac{1}{2(2n-1)(2n+2)!}\left\{n\to 2n-1\right\}\eta^{2n},$$

$$2G_{31}=(2\xi-1)\left[1-10\xi(1-\xi)-\frac{5}{\eta^2}\right]-\left\{\frac{1}{2}\left[\xi(1-5\xi(1-\xi))-\frac{5}{2\eta^2}(1+\xi)\right]\cos[2\eta(1-\xi)]-\frac{1}{8\eta}\left(2+10\xi^2-\frac{15}{\eta^2}\right)\right.$$
$$\left.\times\sin[2\eta(1-\xi)]\right\}+\left\{\xi\to 1-\xi\right\}+\eta\xi(1-\xi)[1-5\xi(1-\xi)]\int_{2(1-\xi)}^{2\xi}\frac{dz}{z}\sin(\eta z)$$
$$=\sum_{n=1}^{\infty}(-1)^n\frac{1}{2(2n-1)(2n+3)!}\left\{n\to 2n-1\right\}\eta^{2n},$$

$$2G_{32}=\left\{\frac{1}{2}\left[\xi(1-5\xi(1-\xi))-\frac{5}{2\eta^2}(1+\xi)\right]\sin[2\eta(1-\xi)]+\frac{1}{8\eta}\left(2+10\xi^2-\frac{15}{\eta^2}\right)\cos[2\eta(1-\xi)]\right\}-\left\{\xi\to 1-\xi\right\}$$
$$+\eta\xi(1-\xi)[1-5\xi(1-\xi)]\int_{2(1-\xi)}^{2\xi}\frac{dz}{z}\cos(\eta z)=\frac{1}{12}(2\xi-1)[1-30\xi(1-\xi)]\eta-\eta\xi(1-\xi)[1-5\xi(1-\xi)]\ln\left(\frac{1-\xi}{\xi}\right)$$
$$-\sum_{n=1}^{\infty}(-1)^n\frac{1}{4n(2n+4)!}\left\{n\to 2n\right\}\eta^{2n+1},$$

$$2G_{41}=\left\{\frac{1}{2}\left[\xi(2\xi-1)(1-7\xi(1-\xi))-\frac{1}{2\eta^2}(9+7\xi+14\xi^2)+\frac{21}{\eta^4}\right]\sin[2\eta(1-\xi)]+\frac{1}{4\eta}\left[1+2\xi-7\xi^2+14\xi^3\right.\right.$$
$$\left.\left.-\frac{21}{2\eta^2}(3+2\xi)\right]\cos[2\eta(1-\xi)]\right\}+\left\{\xi\to 1-\xi\right\}+\eta\xi(1-\xi)(2\xi-1)[1-7\xi(1-\xi)]\int_{2(1-\xi)}^{2\xi}\frac{dz}{z}\cos(\eta z)$$
$$=\left[\frac{1}{20}-\frac{1}{6}\xi(1-\xi)(19-84\xi(1-\xi))\right]\eta+\eta\xi(1-\xi)(1-2\xi)[1-7\xi(1-\xi)]\ln\left(\frac{1-\xi}{\xi}\right)$$
$$-\sum_{n=1}^{\infty}(-1)^n\frac{1}{4n(2n+5)!}\left\{n\to 2n\right\}\eta^{2n+1},$$

$$2G_{42}=-1+20\xi(1-\xi)\left[1-\frac{7}{2}\xi(1-\xi)-\frac{7}{2\eta^2}\right]+\frac{15}{\eta^2}-\frac{21}{\eta^4}+\left\{\frac{1}{2}\left[\xi(2\xi-1)(1-7\xi(1-\xi))-\frac{1}{2\eta^2}(9+7\xi+14\xi^2)+\frac{21}{\eta^4}\right]\right.$$
$$\times\cos[2\eta(1-\xi)]-\frac{1}{4\eta}\left[1+2\xi-7\xi^2+14\xi^3-\frac{21}{2\eta^2}(3+2\xi)\right]\sin[2\eta(1-\xi)]\Big\}+\left\{\xi\to 1-\xi\right\}$$
$$+\eta\xi(1-\xi)(1-2\xi)[1-7\xi(1-\xi)]\int_{2(1-\xi)}^{2\xi}\frac{dz}{z}\sin(\eta z)=-\sum_{n=1}^{\infty}(-1)^n\frac{1}{2(2n-1)(2n+4)!}\left\{n\to 2n\right\}\eta^{2n},$$

$$2G_{51}=(1-2\xi)\left\{1-28\xi(1-\xi)\left[1-\frac{9}{2}\xi(1-\xi)\right]-\frac{35}{\eta^2}[1-6\xi(1-\xi)]+\frac{189}{\eta^4}\right\}+\left\{\frac{1}{2}\left[\xi(1-14\xi(1-\xi)(1-3\xi(1-\xi)))\right.\right.$$
$$-\frac{7}{\eta^2}(1+\xi+3\xi^3)+\frac{63}{\eta^4}(2+\xi)\Big]\cos[2\eta(1-\xi)]-\frac{1}{4\eta}\left[1+14\xi^2-42\xi^3+42\xi^4-\frac{21}{\eta^2}(4+3\xi+3\xi^2)+\frac{315}{\eta^4}\right]$$
$$\left.\sin[2\eta(1-\xi)]\right\}-\left\{\xi\to 1-\xi\right\}-\eta\xi(1-\xi)\left\{1-14\xi(1-\xi)[1-3\xi(1-\xi)]\right\}\int_{2(1-\xi)}^{2\xi}\frac{dz}{z}\sin(\eta z)$$
$$=-\sum_{n=1}^{\infty}(-1)^n\frac{1}{2(2n-1)(2n+5)!}\left\{n\to 2n-1\right\}\eta^{2n},$$

$$\begin{aligned}
2G_{52}=&-\Big\{\tfrac{1}{2}[\xi(1-14\xi(1-\xi)(1-3\xi(1-\xi)))-\tfrac{7}{\eta^2}(1+\xi+3\xi^3)+\tfrac{63}{\eta^4}(2+\xi)]\sin[2\eta(1-\xi)]+\tfrac{1}{4\eta}[1+14\xi^2-42\xi^3\\
&+42\xi^4-\tfrac{21}{\eta^2}(4+3\xi+3\xi^2)+\tfrac{315}{\eta^4}]\cos[2\eta(1-\xi)]\Big\}-\Big\{\xi\to1-\xi\Big\}-\eta\xi(1-\xi)\{1-14\xi(1-\xi)[1-3\xi(1-\xi)]\}\\
&\times\int_{2(1-\xi)}^{2\xi}\frac{dz}{z}\cos(\eta z)=\frac{1}{30}(1-2\xi)\{1-105\xi(1-\xi)[1-6\xi(1-\xi)]\}\eta+\eta\xi(1-\xi)\{1-14\xi(1-\xi)[1-3\xi(1-\xi)]\}\\
&\times\ln\left(\frac{1-\xi}{\xi}\right)+\sum_{n=1}^{\infty}(-1)^n\frac{1}{4n(2n+6)!}\{n\to2n\}\eta^{2n+1},
\end{aligned}$$

$$\begin{aligned}
2G_{61}=&\Big\{\tfrac{1}{2}[\xi(1-2\xi)(1-\xi(1-\xi)(18-66\xi(1-\xi)))+\tfrac{1}{\eta^2}(10+9\xi+18\xi^2-33\xi^3+66\xi^4)-\tfrac{9}{\eta^4}(50+33\xi+22\xi^2)+\tfrac{1485}{\eta^6}]\\
&\times\sin[2\eta(1-\xi)]-\tfrac{1}{4\eta}[1+2\xi-18\xi^2+102\xi^3-198\xi^4+132\xi^5-\tfrac{9}{\eta^2}(20+17\xi+11\xi^2+22\xi^3)+\tfrac{495}{\eta^4}(5+2\xi)]\\
&\times\cos[2\eta(1-\xi)]\Big\}+\Big\{\xi\to1-\xi\Big\}+\eta\xi(1-\xi)(1-2\xi)\{1-\xi(1-\xi)[18-66\xi(1-\xi)]\}\int_{2(1-\xi)}^{2\xi}\frac{dz}{z}\cos(\eta z)\\
=&\Big\{\tfrac{1}{10}\xi(1-\xi)[39-10\xi(1-\xi)(47-132\xi(1-\xi))]-\tfrac{1}{42}\Big\}\eta-\eta\xi(1-\xi)(1-2\xi)\{1-\xi(1-\xi)[18-66\xi(1-\xi)]\}\ln\left(\frac{1-\xi}{\xi}\right)\\
&+\sum_{n=1}^{\infty}(-1)^n\frac{1}{4n(2n+7)!}\{n\to2n\}\eta^{2n+1},
\end{aligned}$$

$$\begin{aligned}
2G_{62}=&1-42\xi(1-\xi)\{1-\xi(1-\xi)[9-22\xi(1-\xi)]\}-\tfrac{70}{\eta^2}\{1-\xi(1-\xi)[12-33\xi(1-\xi)]\}+\tfrac{189}{\eta^4}[5-22\xi(1-\xi)]-\tfrac{1485}{\eta^6}\\
&+\Big\{\tfrac{1}{2}[\xi(1-2\xi)[1-\xi(1-\xi)(18-66\xi(1-\xi))]+\tfrac{1}{\eta^2}(10+9\xi+18\xi^2-33\xi^3+66\xi^4)-\tfrac{9}{\eta^4}(50+33\xi+22\xi^2)\\
&+\tfrac{1485}{\eta^6}]\cos[2\eta(1-\xi)]+\tfrac{1}{4\eta}[1+2\xi-18\xi^2+102\xi^3-198\xi^4+132\xi^5-\tfrac{9}{\eta^2}(20+17\xi+11\xi^2+22\xi^3)+\tfrac{495}{\eta^4}(5+2\xi)]\\
&\times\sin[2\eta(1-\xi)]\Big\}+\Big\{\xi\to1-\xi\Big\}+\eta\xi(1-\xi)(2\xi-1)\{1-\xi(1-\xi)[18-66\xi(1-\xi)]\}\int_{2(1-\xi)}^{2\xi}\frac{dz}{z}\sin(\eta z)\\
=&\sum_{n=1}^{\infty}(-1)^n\frac{1}{2(2n-1)(2n+6)!}\{n\to2n-1\}\eta^{2n},
\end{aligned}$$

$$\begin{aligned}
2G_{71}=&(2\xi-1)\{1-6\xi(1-\xi)[9-11\xi(1-\xi)(9-26\xi(1-\xi))]-\tfrac{42}{\eta^2}[3-11\xi(1-\xi)(4-13\xi(1-\xi))]+\tfrac{693}{\eta^4}[5-26\xi(1-\xi)]\\
&-\tfrac{19305}{\eta^6}\}-\Big\{\tfrac{1}{2}[\xi[1-3\xi(1-\xi)(9-11\xi(1-\xi)(6-13\xi(1-\xi)))]-\tfrac{3}{2\eta^2}(9+9\xi+66\xi^3-143\xi^4+143\xi^5)\\
&+\tfrac{99}{2\eta^4}(25+19\xi+13\xi^2+13\xi^3)-\tfrac{19305}{4\eta^6}(3+\xi)]\cos[2\eta(1-\xi)]-\tfrac{1}{4\eta}[1+27\xi^2-198\xi^3+627\xi^4-858\xi^5\\
&+429\xi^6-\tfrac{9}{2\eta^2}(75+66\xi+66\xi^2+143\xi^4)+\tfrac{495}{2\eta^4}(45+26\xi+13\xi^2)-\tfrac{135135}{4\eta^6}]\sin[2\eta(1-\xi)]\Big\}+\Big\{\xi\to1-\xi\Big\}\\
&+\eta\xi(1-\xi)\{1-3\xi(1-\xi)[9-11\xi(1-\xi)(6-13\xi(1-\xi))]\}\int_{2(1-\xi)}^{2\xi}\frac{dz}{z}\sin(\eta z)\\
=&\sum_{n=1}^{\infty}(-1)^n\frac{1}{2(2n-1)(2n+7)!}\{n\to2n-1\}\eta^{2n},
\end{aligned}$$

$$\begin{aligned}
2G_{72}=&\Big\{\tfrac{1}{2}[\xi[1-3\xi(1-\xi)(9-11\xi(1-\xi)(6-13\xi(1-\xi)))]-\tfrac{3}{2\eta^2}(9+9\xi+66\xi^3-143\xi^4+143\xi^5)+\tfrac{99}{2\eta^4}(25+19\xi\\
&+13\xi^2+13\xi^3)-\tfrac{19305}{4\eta^6}(3+\xi)]\sin[2\eta(1-\xi)]+\tfrac{1}{4\eta}[1+27\xi^2-198\xi^3+627\xi^4-858\xi^5+429\xi^6\\
&-\tfrac{9}{2\eta^2}(75+66\xi+66\xi^2+143\xi^4)+\tfrac{495}{2\eta^4}(45+26\xi+13\xi^2)-\tfrac{135135}{4\eta^6}]\cos[2\eta(1-\xi)]\Big\}-\Big\{\xi\to1-\xi\Big\}\\
&+\eta\xi(1-\xi)\{1-3\xi(1-\xi)[9-11\xi(1-\xi)(6-13\xi(1-\xi))]\}\int_{2(1-\xi)}^{2\xi}\frac{dz}{z}\cos(\eta z)=(2\xi-1)\Big\{\frac{1}{56}-\frac{1}{40}\xi(1-\xi)\\
&\times[166-110\xi(1-\xi)(23-78\xi(1-\xi))]\Big\}\eta-\eta\xi(1-\xi)\{1-3\xi(1-\xi)[9-11\xi(1-\xi)(6-13\xi(1-\xi))]\}\ln\left(\frac{1-\xi}{\xi}\right)\\
&-\sum_{n=1}^{\infty}(-1)^n\frac{1}{4n(2n+8)!}\{n\to2n\}\eta^{2n+1}.
\end{aligned}$$

6. $F_n(\alpha_j,\xi,\mu,s)$, $\mu>0$ [c)]

$$4\alpha F_0 = 1-e^{-2\alpha\xi/\mu} = -\sum_{n=1}^{\infty}\frac{1}{n!}\left(-2\alpha\frac{\xi}{\mu}\right)^n,$$

$$4\alpha i F_1 = 2\xi-1-\frac{\mu}{\alpha}+\left(1+\frac{\mu}{\alpha}\right)e^{-2\alpha\xi/\mu} = \sum_{n=1}^{\infty}\frac{1}{(n+1)!}(n+1-2\xi)\left(-2\alpha\frac{\xi}{\mu}\right)^n,$$

$$4\alpha F_2 = 6\xi(1-\xi)-1-3(1-2\xi)\frac{\mu}{\alpha}-3\left(\frac{\mu}{\alpha}\right)^2+\left[1+3\frac{\mu}{\alpha}+3\left(\frac{\mu}{\alpha}\right)^2\right]e^{-2\alpha\xi/\mu}$$
$$= \sum_{n=1}^{\infty}\frac{1}{(n+2)!}\left\{n^2+3n(1-2\xi)+2[1-6\xi(1-\xi)]\right\}\left(-2\alpha\frac{\xi}{\mu}\right)^n,$$

$$4\alpha i F_3 = (1-2\xi)[1-10\xi(1-\xi)]+6[1-5\xi(1-\xi)]\frac{\mu}{\alpha}+15(1-2\xi)\left(\frac{\mu}{\alpha}\right)^2+15\left(\frac{\mu}{\alpha}\right)^3-\left[1+6\frac{\mu}{\alpha}+15\left(\frac{\mu}{\alpha}\right)^2+15\left(\frac{\mu}{\alpha}\right)^3\right]e^{-2\alpha\xi/\mu}$$
$$= -\sum_{n=1}^{\infty}\frac{1}{(n+3)!}\left\{n^3+6n^2(1-2\xi)+n[11-60\xi(1-\xi)]+6(1-2\xi)[1-10\xi(1-\xi)]\right\}\left(-2\alpha\frac{\xi}{\mu}\right)^n,$$

$$4\alpha F_4 = 1-20\xi(1-\xi)[1-\tfrac{7}{2}\xi(1-\xi)]+10(1-2\xi)[1-7\xi(1-\xi)]\frac{\mu}{\alpha}+45[1-\tfrac{14}{3}\xi(1-\xi)]\left(\frac{\mu}{\alpha}\right)^2+105(1-2\xi)\left(\frac{\mu}{\alpha}\right)^3+105\left(\frac{\mu}{\alpha}\right)^4$$
$$-\left[1+10\frac{\mu}{\alpha}+45\left(\frac{\mu}{\alpha}\right)^2+105\left(\frac{\mu}{\alpha}\right)^3+105\left(\frac{\mu}{\alpha}\right)^4\right]e^{-2\alpha\xi/\mu} = -\sum_{n=1}^{\infty}\frac{1}{(n+4)!}\left\{n^4+10n^3(1-2\xi)+5n^2[7-36\xi(1-\xi)]\right.$$
$$\left.+10n(1-2\xi)[5-42\xi(1-\xi)]+24-480\xi(1-\xi)[1-\tfrac{7}{2}\xi(1-\xi)]\right\}\left(-2\alpha\frac{\xi}{\mu}\right)^n,$$

$$4\alpha i F_5 = (2\xi-1)\left\{1-28\xi(1-\xi)[1-\tfrac{9}{2}\xi(1-\xi)]\right\}-15\left\{1-14\xi(1-\xi)[1-3\xi(1-\xi)]\right\}\frac{\mu}{\alpha}-105(1-2\xi)[1-6\xi(1-\xi)]\left(\frac{\mu}{\alpha}\right)^2$$
$$-420[1-\tfrac{9}{2}\xi(1-\xi)]\left(\frac{\mu}{\alpha}\right)^3-945(1-2\xi)\left(\frac{\mu}{\alpha}\right)^4-945\left(\frac{\mu}{\alpha}\right)^5+\left[1+15\frac{\mu}{\alpha}+105\left(\frac{\mu}{\alpha}\right)^2+420\left(\frac{\mu}{\alpha}\right)^3+945\left(\frac{\mu}{\alpha}\right)^4+945\left(\frac{\mu}{\alpha}\right)^5\right]$$
$$\times e^{-2\alpha\xi/\mu} = \sum_{n=1}^{\infty}\frac{1}{(n+5)!}\left\{n^5+15n^4(1-2\xi)+5n^3[17-84\xi(1-\xi)]+15n^2(1-2\xi)[15-112\xi(1-\xi)]\right.$$
$$\left.+2n[137-\xi(1-\xi)(2310-7560\xi(1-\xi))]+120(1-2\xi)[1-\xi(1-\xi)(28-126\xi(1-\xi))]\right\}\left(-2\alpha\frac{\xi}{\mu}\right)^n,$$

$$4\alpha F_6 = 42\xi(1-\xi)\left\{1-9\xi(1-\xi)[1-\tfrac{22}{9}\xi(1-\xi)]\right\}-1-21(1-2\xi)\left\{1-18\xi(1-\xi)[1-\tfrac{11}{3}\xi(1-\xi)]\right\}\frac{\mu}{\alpha}-210\left\{1-12\xi(1-\xi)\right.$$
$$\left.\times[1-\tfrac{11}{4}\xi(1-\xi)]\right\}\left(\frac{\mu}{\alpha}\right)^2-1260(1-2\xi)[1-\tfrac{11}{2}\xi(1-\xi)]\left(\frac{\mu}{\alpha}\right)^3-4725[1-\tfrac{22}{5}\xi(1-\xi)]\left(\frac{\mu}{\alpha}\right)^4-10395(1-2\xi)\left(\frac{\mu}{\alpha}\right)^5$$
$$-10395\left(\frac{\mu}{\alpha}\right)^6+\left[1+21\frac{\mu}{\alpha}+210\left(\frac{\mu}{\alpha}\right)^2+1260\left(\frac{\mu}{\alpha}\right)^3+4725\left(\frac{\mu}{\alpha}\right)^4+10395\left(\frac{\mu}{\alpha}\right)^5+10395\left(\frac{\mu}{\alpha}\right)^6\right]e^{-2\alpha\xi/\mu}$$
$$= \sum_{n=1}^{\infty}\frac{1}{(n+6)!}\left\{n^6+21n^5(1-2\xi)+35n^4[5-24\xi(1-\xi)]+105n^3(1-2\xi)[7-48\xi(1-\xi)]+56n^2[29-\xi(1-\xi)\right.$$
$$\times(435-1350\xi(1-\xi))]+252n(1-2\xi)[7-\xi(1-\xi)(160-660\xi(1-\xi))]+720$$
$$\left.-30240\xi(1-\xi)[1-\xi(1-\xi)(9-22\xi(1-\xi))]\right\}\left(-2\alpha\frac{\xi}{\mu}\right)^n,$$

$$4\alpha i F_7 = (1-2\xi)\left\{1-54\xi(1-\xi)[1-11\xi(1-\xi)(1-\tfrac{26}{9}\xi(1-\xi))\right\}+28\left\{1-27\xi(1-\xi)[1-\tfrac{22}{3}\xi(1-\xi)(1-\tfrac{13}{6}\xi(1-\xi))]\right\}\frac{\mu}{\alpha}$$
$$+378(1-2\xi)\left\{1-\tfrac{44}{3}\xi(1-\xi)[1-\tfrac{13}{4}\xi(1-\xi)]\right\}\left(\frac{\mu}{\alpha}\right)^2+3150\left\{1-11\xi(1-\xi)[1-\tfrac{13}{5}\xi(1-\xi)]\right\}\left(\frac{\mu}{\alpha}\right)^3$$
$$+17325(1-2\xi)[1-\tfrac{26}{5}\xi(1-\xi)]\left(\frac{\mu}{\alpha}\right)^4+62370[1-\tfrac{13}{3}\xi(1-\xi)]\left(\frac{\mu}{\alpha}\right)^5+135135(1-2\xi)\left(\frac{\mu}{\alpha}\right)^6+135135\left(\frac{\mu}{\alpha}\right)^7$$
$$-\left[1+28\frac{\mu}{\alpha}+378\left(\frac{\mu}{\alpha}\right)^2+3150\left(\frac{\mu}{\alpha}\right)^3+17325\left(\frac{\mu}{\alpha}\right)^4+62370\left(\frac{\mu}{\alpha}\right)^5+135135\left(\frac{\mu}{\alpha}\right)^6+135135\left(\frac{\mu}{\alpha}\right)^7\right]e^{-2\alpha\xi/\mu}$$
$$= -\sum_{n=1}^{\infty}\frac{1}{(n+7)!}\left\{n^7+28n^6(1-2\xi)+14n^5[23-108\xi(1-\xi)]+280n^4(1-2\xi)[7-45\xi(1-\xi)]+7n^3[967\right.$$
$$-\xi(1-\xi)(13320-39600\xi(1-\xi))]+28n^2(1-2\xi)[469-\xi(1-\xi)(9270-35640\xi(1-\xi))]+36n[363$$
$$\left.-\xi(1-\xi)(12488-\xi(1-\xi)(103180-240240\xi(1-\xi)))]+5040(1-2\xi)[1-\xi(1-\xi)(54-\xi(1-\xi)(594-1716\xi(1-\xi)))]\right\}$$
$$\times\left(-2\alpha\frac{\xi}{\mu}\right)^n.$$

7. $F_n(\alpha_q,\xi,\mu,iy+\Sigma_1 v_1-\Sigma_q v_q)\equiv\frac{1}{\eta}(F_{n1}+iF_{n2}),\ \mu>0$ [b)c)]

$$4F_{01}=\sin\left(2\eta\frac{\xi}{\mu}\right)=-\sum_{n=1}^{\infty}(-1)^n\frac{1}{(2n-1)!}\left(2\eta\frac{\xi}{\mu}\right)^{2n-1},$$

$$4F_{02}=\cos\left(2\eta\frac{\xi}{\mu}\right)-1=\sum_{n=1}^{\infty}(-1)^n\frac{1}{(2n)!}\left(2\eta\frac{\xi}{\mu}\right)^{2n},$$

$$4F_{11}=1-2\xi-\cos\left(2\eta\frac{\xi}{\mu}\right)+\frac{\mu}{\eta}\sin\left(2\eta\frac{\xi}{\mu}\right)=-\sum_{n=1}^{\infty}(-1)^n\frac{1}{(2n+1)!}(2n+1-2\xi)\left(2\eta\frac{\xi}{\mu}\right)^{2n},$$

$$4F_{12}=-\frac{\mu}{\eta}+\sin\left(2\eta\frac{\xi}{\mu}\right)+\frac{\mu}{\eta}\cos\left(2\eta\frac{\xi}{\mu}\right)=-\sum_{n=1}^{\infty}(-1)^n\frac{1}{(2n)!}(2n-2\xi)\left(2\eta\frac{\xi}{\mu}\right)^{2n-1},$$

$$4F_{21}=3(1-2\xi)\frac{\mu}{\eta}-\left[1-3\left(\frac{\mu}{\eta}\right)^2\right]\sin\left(2\eta\frac{\xi}{\mu}\right)-3\frac{\mu}{\eta}\cos\left(2\eta\frac{\xi}{\mu}\right)=\sum_{n=1}^{\infty}(-1)^n\frac{1}{(2n+1)!}\{n\to 2n-1\}\left(2\eta\frac{\xi}{\mu}\right)^{2n-1},$$

$$4F_{22}=1-6\xi(1-\xi)-3\left(\frac{\mu}{\eta}\right)^2-\left[1-3\left(\frac{\mu}{\eta}\right)^2\right]\cos\left(2\eta\frac{\xi}{\mu}\right)+3\frac{\mu}{\eta}\sin\left(2\eta\frac{\xi}{\mu}\right)=-\sum_{n=1}^{\infty}(-1)^n\frac{1}{(2n+2)!}\{n\to 2n\}\left(2\eta\frac{\xi}{\mu}\right)^{2n},$$

$$4F_{31}=(2\xi-1)[1-10\xi(1-\xi)]+15(1-2\xi)\left(\frac{\mu}{\eta}\right)^2+\left[1-15\left(\frac{\mu}{\eta}\right)^2\right]\cos\left(2\eta\frac{\xi}{\mu}\right)-3\frac{\mu}{\eta}\left[2-5\left(\frac{\mu}{\eta}\right)^2\right]\sin\left(2\eta\frac{\xi}{\mu}\right)$$
$$=\sum_{n=1}^{\infty}(-1)^n\frac{1}{(2n+3)!}\{n\to 2n\}\left(2\eta\frac{\xi}{\mu}\right)^{2n},$$

$$4F_{32}=3\frac{\mu}{\eta}\left\{2[1-5\xi(1-\xi)]-5\left(\frac{\mu}{\eta}\right)^2\right\}-\left[1-15\left(\frac{\mu}{\eta}\right)^2\right]\sin\left(2\eta\frac{\xi}{\mu}\right)-3\frac{\mu}{\eta}\left[2-5\left(\frac{\mu}{\eta}\right)^2\right]\cos\left(2\eta\frac{\xi}{\mu}\right)$$
$$=\sum_{n=1}^{\infty}(-1)^n\frac{1}{(2n+2)!}\{n\to 2n-1\}\left(2\eta\frac{\xi}{\mu}\right)^{2n-1},$$

$$4F_{41}=5(2\xi-1)\frac{\mu}{\eta}\left\{2[1-7\xi(1-\xi)]-21\left(\frac{\mu}{\eta}\right)^2\right\}+\left[1-45\left(\frac{\mu}{\eta}\right)^2+105\left(\frac{\mu}{\eta}\right)^4\right]\sin\left(2\eta\frac{\xi}{\mu}\right)$$
$$+5\frac{\mu}{\eta}\left[2-21\left(\frac{\mu}{\eta}\right)^2\right]\cos\left(2\eta\frac{\xi}{\mu}\right)=-\sum_{n=1}^{\infty}(-1)^n\frac{1}{(2n+3)!}\{n\to 2n-1\}\left(2\eta\frac{\xi}{\mu}\right)^{2n-1},$$

$$4F_{42}=20\xi(1-\xi)\left[1-\frac{7}{2}\xi(1-\xi)\right]-1+45\left[1-\frac{14}{3}\xi(1-\xi)\right]\left(\frac{\mu}{\eta}\right)^2-105\left(\frac{\mu}{\eta}\right)^4+\left[1-45\left(\frac{\mu}{\eta}\right)^2+105\left(\frac{\mu}{\eta}\right)^4\right]\cos\left(2\eta\frac{\xi}{\mu}\right)$$
$$-5\frac{\mu}{\eta}\left[2-21\left(\frac{\mu}{\eta}\right)^2\right]\sin\left(2\eta\frac{\xi}{\mu}\right)=\sum_{n=1}^{\infty}(-1)^n\frac{1}{(2n+4)!}\{n\to 2n\}\left(2\eta\frac{\xi}{\mu}\right)^{2n},$$

$$4F_{51}=(1-2\xi)\left\{1-28\xi(1-\xi)\left[1-\frac{9}{2}\xi(1-\xi)\right]-105[1-6\xi(1-\xi)]\left(\frac{\mu}{\eta}\right)^2+945\left(\frac{\mu}{\eta}\right)^4\right\}-\left[1-105\left(\frac{\mu}{\eta}\right)^2+945\left(\frac{\mu}{\eta}\right)^4\right]$$
$$\times\cos\left(2\eta\frac{\xi}{\mu}\right)+15\frac{\mu}{\eta}\left[1-28\left(\frac{\mu}{\eta}\right)^2+63\left(\frac{\mu}{\eta}\right)^4\right]\sin\left(2\eta\frac{\xi}{\mu}\right)=-\sum_{n=1}^{\infty}(-1)^n\frac{1}{(2n+5)!}\{n\to 2n\}\left(2\eta\frac{\xi}{\mu}\right)^{2n},$$

$$4F_{52}=-15\frac{\mu}{\eta}\left\{1-14\xi(1-\xi)[1-3\xi(1-\xi)]-28\left[1-\frac{9}{2}\xi(1-\xi)\right]\left(\frac{\mu}{\eta}\right)^2+63\left(\frac{\mu}{\eta}\right)^4\right\}+\left[1-105\left(\frac{\mu}{\eta}\right)^2+945\left(\frac{\mu}{\eta}\right)^4\right]\sin\left(2\eta\frac{\xi}{\mu}\right)$$
$$+15\frac{\mu}{\eta}\left[1-28\left(\frac{\mu}{\eta}\right)^2+63\left(\frac{\mu}{\eta}\right)^4\right]\cos\left(2\eta\frac{\xi}{\mu}\right)=-\sum_{n=1}^{\infty}(-1)^n\frac{1}{(2n+4)!}\{n\to 2n-1\}\left(2\eta\frac{\xi}{\mu}\right)^{2n-1},$$

$$4F_{61}=21(1-2\xi)\frac{\mu}{\eta}\left\{1-18\xi(1-\xi)\left[1-\frac{11}{3}\xi(1-\xi)\right]-60\left[1-\frac{11}{2}\xi(1-\xi)\right]\left(\frac{\mu}{\eta}\right)^2+495\left(\frac{\mu}{\eta}\right)^4\right\}-\left[1-210\left(\frac{\mu}{\eta}\right)^2\right.$$
$$\left.+4725\left(\frac{\mu}{\eta}\right)^4-10395\left(\frac{\mu}{\eta}\right)^6\right]\sin\left(2\eta\frac{\xi}{\mu}\right)-21\frac{\mu}{\eta}\left[1-60\left(\frac{\mu}{\eta}\right)^2+495\left(\frac{\mu}{\eta}\right)^4\right]\cos\left(2\eta\frac{\xi}{\mu}\right)$$
$$=\sum_{n=1}^{\infty}(-1)^n\frac{1}{(2n+5)!}\{n\to 2n-1\}\left(2\eta\frac{\xi}{\mu}\right)^{2n-1},$$

$$4F_{62}=1-42\xi(1-\xi)\left\{1-9\xi(1-\xi)\left[1-\frac{22}{9}\xi(1-\xi)\right]\right\}-210\left\{1-12\xi(1-\xi)\left[1-\frac{11}{4}\xi(1-\xi)\right]\right\}\left(\frac{\mu}{\eta}\right)^2+4725\left[1-\frac{22}{5}\xi(1-\xi)\right]\left(\frac{\mu}{\eta}\right)^4$$
$$-10395\left(\frac{\mu}{\eta}\right)^6-\left[1-210\left(\frac{\mu}{\eta}\right)^2+4725\left(\frac{\mu}{\eta}\right)^4-10395\left(\frac{\mu}{\eta}\right)^6\right]\cos\left(2\eta\frac{\xi}{\mu}\right)+21\frac{\mu}{\eta}\left[1-60\left(\frac{\mu}{\eta}\right)^2+495\left(\frac{\mu}{\eta}\right)^4\right]\sin\left(2\eta\frac{\xi}{\mu}\right)$$
$$=-\sum_{n=1}^{\infty}(-1)^n\frac{1}{(2n+6)!}\{n\to 2n\}\left(2\eta\frac{\xi}{\mu}\right)^{2n},$$

$$4F_{71}=(2\xi-1)\Big\{1-54\xi(1-\xi)[1-11\xi(1-\xi)(1-\tfrac{26}{9}\xi(1-\xi))]-378[1-\tfrac{44}{3}\xi(1-\xi)(1-\tfrac{13}{4}\xi(1-\xi))](\tfrac{\mu}{\eta})^2$$
$$+17325[1-\tfrac{26}{5}\xi(1-\xi)](\tfrac{\mu}{\eta})^4-135135(\tfrac{\mu}{\eta})^6\Big\}+[1-378(\tfrac{\mu}{\eta})^2+17325(\tfrac{\mu}{\eta})^4-135135(\tfrac{\mu}{\eta})^6]\cos(2\eta\tfrac{\xi}{\mu})$$
$$-7\tfrac{\mu}{\eta}[4-450(\tfrac{\mu}{\eta})^2+8910(\tfrac{\mu}{\eta})^4-19305(\tfrac{\mu}{\eta})^6]\sin(2\eta\tfrac{\xi}{\mu})=\sum_{n=1}^{\infty}(-1)^n\frac{1}{(2n+7)!}\{n\to 2n\}(2\eta\tfrac{\xi}{\mu})^{2n},$$
$$4F_{72}=7\tfrac{\mu}{\eta}\Big\{4[1-27\xi(1-\xi)[1-\tfrac{22}{3}\xi(1-\xi)(1-\tfrac{13}{6}\xi(1-\xi))]]-450[1-11\xi(1-\xi)(1-\tfrac{13}{5}\xi(1-\xi))](\tfrac{\mu}{\eta})^2$$
$$+8910[1-\tfrac{13}{3}\xi(1-\xi)](\tfrac{\mu}{\eta})^4-19305(\tfrac{\mu}{\eta})^6-[1-378(\tfrac{\mu}{\eta})^2+17325(\tfrac{\mu}{\eta})^4-135135(\tfrac{\mu}{\eta})^6]\sin(2\eta\tfrac{\xi}{\mu})$$
$$-7\tfrac{\mu}{\eta}[4-450(\tfrac{\mu}{\eta})^2+8910(\tfrac{\mu}{\eta})^4-19305(\tfrac{\mu}{\eta})^6]\cos(2\eta\tfrac{\xi}{\mu})=\sum_{n=1}^{\infty}(-1)^n\frac{1}{(2n+6)!}\{n\to 2n-1\}(2\eta\tfrac{\xi}{\mu})^{2n-1}.$$

8. $A_n(\alpha_q, s)$

$$\alpha A_1=1-e^{-\alpha}=-\sum_{n=1}^{\infty}\frac{1}{n!}(-\alpha)^n,$$
$$\alpha A_3=\frac{3}{2}-\frac{5}{\alpha^2}+\Big(1+\frac{5}{\alpha}+\frac{5}{\alpha^2}\Big)e^{-\alpha}=\sum_{n=1}^{\infty}\frac{1}{(n+2)!}(n-3)(n+1)(-\alpha)^n,$$
$$\alpha A_5=\frac{15}{8}-\frac{35}{2\alpha^2}+\frac{378}{\alpha^4}-\Big(\frac{71}{8}+\frac{91}{2\alpha}+\frac{343}{2\alpha^2}+\frac{378}{\alpha^3}+\frac{378}{\alpha^4}\Big)e^{-\alpha}=-\sum_{n=1}^{\infty}\frac{1}{8(n+4)!}(n+1)(n+3)(71n^2+62n+120)(-\alpha)^n,$$
$$\alpha A_7=\frac{35}{16}-\frac{315}{8\alpha^2}+\frac{2079}{\alpha^4}-\frac{115830}{\alpha^6}+\Big(\frac{367}{4}+\frac{5265}{8\alpha}+\frac{30609}{8\alpha^2}+\frac{17226}{\alpha^3}+\frac{55836}{\alpha^4}+\frac{115830}{\alpha^5}+\frac{115830}{\alpha^6}\Big)e^{-\alpha}$$
$$=\sum_{n=1}^{\infty}\frac{1}{8(n+6)!}(n+3)(n+5)[(n+4)(n+6)(734n^2-3063n+21547)-4752(29n+109)](-\alpha)^n.$$

9. $C_n^a(\alpha_q, s)$

$$2\alpha C_0^a=1-e^{-2\alpha}+2\alpha\int_1^{\infty}\frac{dz}{z}e^{-2\alpha z}=2\alpha[1-\gamma-\ln(2\alpha)]+\sum_{n=1}^{\infty}\frac{1}{n(n+1)!}(-2\alpha)^{n+1},$$
$$2\alpha i C_1^a=1-\frac{1}{2\alpha}+\frac{1}{2\alpha}e^{-2\alpha}=-\sum_{n=0}^{\infty}\frac{1}{(n+2)!}(-2\alpha)^{n+1},$$
$$2\alpha C_2^a=-1+\frac{3}{2\alpha}-\frac{1}{\alpha^2}+\frac{1}{\alpha}\Big(\frac{1}{2}+\frac{1}{\alpha}\Big)e^{-2\alpha}=-\sum_{n=0}^{\infty}\frac{1}{(n+3)!}(n-1)(-2\alpha)^{n+1},$$
$$2\alpha i C_3^a=-1+\frac{3}{\alpha}-\frac{5}{\alpha^2}+\frac{15}{4\alpha^3}-\frac{1}{2\alpha}\Big(1+\frac{5}{\alpha}+\frac{15}{2\alpha^2}\Big)e^{-2\alpha}=\sum_{n=0}^{\infty}\frac{1}{(n+4)!}(n-1)(n-2)(-2\alpha)^{n+1},$$
$$2\alpha C_4^a=1-\frac{5}{\alpha}+\frac{15}{\alpha^2}-\frac{105}{4\alpha^3}+\frac{21}{\alpha^4}-\frac{1}{\alpha}\Big(\frac{1}{2}+\frac{9}{2\alpha}+\frac{63}{4\alpha^2}+\frac{21}{\alpha^3}\Big)e^{-2\alpha}=\sum_{n=0}^{\infty}\frac{1}{(n+5)!}(n-1)(n-2)(n-3)(-2\alpha)^{n+1},$$
$$2\alpha i C_5^a=1-\frac{15}{2\alpha}+\frac{35}{\alpha^2}-\frac{105}{\alpha^3}+\frac{189}{\alpha^4}-\frac{315}{2\alpha^5}+\frac{1}{\alpha}\Big(\frac{1}{2}+\frac{7}{\alpha}+\frac{42}{\alpha^2}+\frac{126}{\alpha^3}+\frac{315}{2\alpha^4}\Big)e^{-2\alpha}$$
$$=-\sum_{n=0}^{\infty}\frac{1}{(n+6)!}(n-1)(n-2)(n-3)(n-4)(-2\alpha)^{n+1},$$
$$2\alpha C_6^a=-1+\frac{21}{2\alpha}-\frac{70}{\alpha^2}+\frac{315}{\alpha^3}-\frac{945}{\alpha^4}+\frac{3465}{2\alpha^5}-\frac{1485}{\alpha^6}+\frac{1}{\alpha}\Big(\frac{1}{2}+\frac{10}{\alpha}+\frac{90}{\alpha^2}+\frac{450}{\alpha^3}+\frac{2475}{2\alpha^4}+\frac{1485}{\alpha^5}\Big)e^{-2\alpha}$$
$$=-\sum_{n=0}^{\infty}\frac{1}{(n+7)!}(n-1)(n-2)(n-3)(n-4)(n-5)(-2\alpha)^{2n+1},$$
$$2\alpha i C_7^a=-1+\frac{14}{\alpha}-\frac{126}{\alpha^2}+\frac{1575}{2\alpha^3}-\frac{3465}{\alpha^4}+\frac{10395}{\alpha^5}-\frac{19305}{\alpha^6}+\frac{135135}{8\alpha^7}-\frac{1}{2\alpha}\Big(1+\frac{27}{\alpha}+\frac{675}{2\alpha^2}+\frac{2475}{\alpha^3}+\frac{22275}{2\alpha^4}$$
$$+\frac{57915}{2\alpha^5}+\frac{135135}{4\alpha^6}\Big)e^{-2\alpha}=\sum_{n=0}^{\infty}\frac{1}{(n+8)!}(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)(-2\alpha)^{n+1}.$$

10. $C_n^b(\alpha_q, s)$

$$2\alpha C_0^b = 1 - e^{-2\alpha} + 2\alpha \int_1^\infty \frac{dz}{z^2} e^{-2\alpha z} = 4\alpha + 4\alpha^2[\gamma + \ln(2\alpha) - \tfrac{3}{2}] + 2\sum_{n=2}^\infty \frac{1}{(n-1)(n+1)!}(-2\alpha)^{n+1},$$

$$2\alpha i C_1^b = 1 - \frac{2}{3\alpha} + \left(\frac{1}{3} + \frac{2}{3\alpha}\right)e^{-2\alpha} - \frac{2\alpha}{3}\int_1^\infty \frac{dz}{z^2} e^{-2\alpha z} = \frac{4\alpha^2}{3}[\tfrac{5}{6} - \ln(2\alpha) - \gamma] - 2\sum_{n=2}^\infty \frac{1}{(n-1)(n+2)!} n(-2\alpha)^{n+1},$$

$$2\alpha C_2^b = -1 + \frac{2}{\alpha} - \frac{3}{2\alpha^2} + \frac{1}{\alpha}\left(1 + \frac{3}{2\alpha}\right)e^{-2\alpha} = -2\sum_{n=1}^\infty \frac{1}{(n+3)!} n(-2\alpha)^{n+1},$$

$$2\alpha i C_3^b = -1 + \frac{4}{\alpha} - \frac{15}{2\alpha^2} + \frac{6}{\alpha^3} - \frac{1}{\alpha}\left(1 + \frac{9}{2\alpha} + \frac{6}{\alpha^2}\right)e^{-2\alpha} = 2\sum_{n=1}^\infty \frac{1}{(n+4)!} n(n-2)(-2\alpha)^{n+1},$$

$$2\alpha C_4^b = 1 - \frac{20}{3\alpha} + \frac{45}{2\alpha^2} - \frac{42}{\alpha^3} + \frac{35}{\alpha^4} - \frac{1}{\alpha}\left(1 + \frac{17}{2\alpha} + \frac{28}{\alpha^2} + \frac{35}{\alpha^3}\right)e^{-2\alpha} = 2\sum_{n=1}^\infty \frac{1}{(n+5)!} n(n-2)(n-3)(-2\alpha)^{n+1},$$

$$2\alpha i C_5^b = 1 - \frac{10}{\alpha} + \frac{105}{2\alpha^2} - \frac{168}{\alpha^3} + \frac{315}{\alpha^4} - \frac{270}{\alpha^5} + \frac{1}{\alpha}\left(1 + \frac{27}{2\alpha} + \frac{78}{\alpha^2} + \frac{225}{\alpha^3} + \frac{270}{\alpha^4}\right)e^{-2\alpha}$$
$$= -2\sum_{n=1}^\infty \frac{1}{(n+6)!} n(n-2)(n-3)(n-4)(-2\alpha)^{n+1},$$

$$2\alpha C_6^b = -1 + \frac{14}{\alpha} - \frac{105}{\alpha^2} + \frac{504}{\alpha^3} - \frac{1575}{\alpha^4} + \frac{2970}{\alpha^5} - \frac{10395}{4\alpha^6} + \frac{1}{\alpha}\left(1 + \frac{39}{2\alpha} + \frac{171}{\alpha^2} + \frac{1665}{2\alpha^3} + \frac{4455}{2\alpha^4} + \frac{10395}{4\alpha^5}\right)e^{-2\alpha}$$
$$= -2\sum_{n=1}^\infty \frac{1}{(n+7)!} n(n-2)(n-3)(n-4)(n-5)(-2\alpha)^{n+1},$$

$$2\alpha i C_7^b = -1 + \frac{56}{3\alpha} - \frac{189}{\alpha^2} + \frac{1260}{\alpha^3} - \frac{5775}{\alpha^4} + \frac{17820}{\alpha^5} - \frac{135135}{4\alpha^6} + \frac{30030}{\alpha^7} - \frac{1}{\alpha}\left(1 + \frac{53}{2\alpha} + \frac{325}{\alpha^2} + \frac{4675}{2\alpha^3} + \frac{20625}{2\alpha^4}\right.$$
$$\left. + \frac{105105}{4\alpha^5} + \frac{30030}{\alpha^6}\right)e^{-2\alpha} = 2\sum_{n=1}^\infty \frac{1}{(n+8)!} n(n-2)(n-3)(n-4)(n-5)(n-6)(-2\alpha)^{n+1}.$$

11. $C_n^b(\alpha_q, iy + z_1 v_1 - z_q v_q) \equiv \frac{1}{\eta}(C_{n1}^b + i C_{n2}^b)$

$$2C_{01}^b = 2\eta\cos(2\eta) + \sin(2\eta) - (2\eta)^2 \int_1^\infty \frac{dz}{z}\sin(2\eta z) = 4\eta - 2\pi\eta^2 - 2\sum_{n=1}^\infty (-1)^n \frac{1}{(2n-1)(2n+1)!}(2\eta)^{2n+1},$$

$$2C_{02}^b = -2\eta\sin(2\eta) + \cos(2\eta) - 1 - (2\eta)^2 \int_1^\infty \frac{dz}{z}\cos(2\eta z) = (2\eta)^2[\gamma + \ln(2\eta) - \tfrac{3}{2}] - \sum_{n=2}^\infty (-1)^n \frac{1}{(n-1)(2n)!}(2\eta)^{2n},$$

$$2C_{11}^b = \frac{2\eta}{3}\sin(2\eta) - 1 - \frac{1}{3}\cos(2\eta) + \frac{2}{3\eta}\sin(2\eta) + \frac{1}{3}(2\eta)^2 \int_1^\infty \frac{dz}{z}\cos(2\eta z)$$
$$= \frac{1}{3}(2\eta)^2[\tfrac{5}{6} - \ln(2\eta) - \gamma] + \sum_{n=2}^\infty (-1)^n \frac{1}{(n-1)(2n+1)!}(2n-1)(2\eta)^{2n},$$

$$2C_{12}^b = \frac{2\eta}{3}\cos(2\eta) + \frac{1}{3}\sin(2\eta) - \frac{2}{3\eta}[1 - \cos(2\eta)] - \frac{1}{3}(2\eta)^2 \int_1^\infty \frac{dz}{z}\sin(2\eta z)$$
$$= -\frac{2}{3}\pi\eta^2 - 4\sum_{n=1}^\infty (-1)^n \frac{1}{(2n-1)(2n+2)!} n(2\eta)^{2n+1},$$

$$2C_{21}^b = -\frac{2}{\eta}[1 + \tfrac{1}{2}\cos(2\eta)] + \frac{3}{2\eta^2}\sin(2\eta) = 4\sum_{n=1}^\infty (-1)^n \frac{1}{(2n+3)!} n(2\eta)^{2n+1},$$

$$2C_{22}^b = 1 + \frac{1}{\eta}\sin(2\eta) - \frac{3}{2\eta^2}[1 - \cos(2\eta)] = 2\sum_{n=1}^\infty (-1)^n \frac{1}{(2n+2)!}(2n-1)(2\eta)^{2n},$$

$$2C_{31}^b = 1 - \frac{1}{\eta}\sin(2\eta) - \frac{3}{2\eta^2}[5 + 3\cos(2\eta)] + \frac{6}{\eta^3}\sin(2\eta) = -2\sum_{n=1}^\infty (-1)^n \frac{1}{(2n+3)!}(2n-1)(2n-3)(2\eta)^{2n},$$

$$2C_{32}^b = \frac{1}{\eta}[4 - \cos(2\eta)] + \frac{9}{2\eta^2}\sin(2\eta) - \frac{6}{\eta^3}[1 - \cos(2\eta)] = 8\sum_{n=2}^\infty (-1)^n \frac{1}{(2n+4)!} n(n-1)(2\eta)^{2n+1},$$

$$2C_{41}^{b}=\frac{1}{\eta}\left[\frac{20}{3}+\cos(2\eta)\right]-\frac{17}{2\eta^2}\sin(2\eta)-\frac{14}{\eta^3}[3+2\cos(2\eta)]+\frac{35}{\eta^4}\sin(2\eta)$$
$$=-8\sum_{n=2}^{\infty}(-1)^n\frac{1}{(2n+5)!}n(n-1)(2n-3)(2\eta)^{2n+1},$$

$$2C_{42}^{b}=-1-\frac{1}{\eta}\sin(2\eta)+\frac{1}{2\eta^2}[45-17\cos(2\eta)]+\frac{28}{\eta^3}\sin(2\eta)-\frac{35}{\eta^4}[1-\cos(2\eta)]$$
$$=-4\sum_{n=1}^{\infty}(-1)^n\frac{1}{(2n+4)!}(2n-1)(2n-3)(n-2)(2\eta)^{2n},$$

$$2C_{51}^{b}=-1+\frac{1}{\eta}\sin(2\eta)+\frac{3}{2\eta^2}[35+9\cos(2\eta)]-\frac{78}{\eta^3}\sin(2\eta)-\frac{45}{\eta^4}[7+5\cos(2\eta)]+\frac{270}{\eta^5}\sin(2\eta)$$
$$=4\sum_{n=1}^{\infty}(-1)^n\frac{1}{(2n+5)!}(2n-1)(2n-3)(n-2)(2n-5)(2\eta)^{2n},$$

$$2C_{52}^{b}=\frac{1}{\eta}[\cos(2\eta)-10]-\frac{27}{2\eta^2}\sin(2\eta)+\frac{6}{\eta^3}[28-13\cos(2\eta)]+\frac{225}{\eta^4}\sin(2\eta)-\frac{270}{\eta^5}[1-\cos(2\eta)]$$
$$=-16\sum_{n=3}^{\infty}(-1)^n\frac{1}{(2n+6)!}n(n-1)(2n-3)(n-2)(2\eta)^{2n+1},$$

$$2C_{61}^{b}=-\frac{1}{\eta}[14+\cos(2\eta)]+\frac{39}{2\eta^2}\sin(2\eta)+\frac{9}{\eta^3}[56+19\cos(2\eta)]-\frac{1665}{2\eta^4}\sin(2\eta)-\frac{1485}{2\eta^5}[4+3\cos(2\eta)]$$
$$+\frac{10395}{4\eta^6}\sin(2\eta)=16\sum_{n=3}^{\infty}(-1)^n\frac{1}{(2n+7)!}n(n-1)(2n-3)(n-2)(2n-5)(2\eta)^{2n+1},$$

$$2C_{62}^{b}=1+\frac{1}{\eta}\sin(2\eta)-\frac{3}{2\eta^2}[70-13\cos(2\eta)]-\frac{171}{\eta^3}\sin(2\eta)+\frac{45}{2\eta^4}[70-37\cos(2\eta)]+\frac{4455}{2\eta^5}\sin(2\eta)$$
$$-\frac{10395}{4\eta^6}[1-\cos(2\eta)]=8\sum_{n=1}^{\infty}(-1)^n\frac{1}{(2n+6)!}(2n-1)(2n-3)(n-2)(2n-5)(n-3)(2\eta)^{2n},$$

$$2C_{71}^{b}=1-\frac{1}{\eta}\sin(2\eta)-\frac{1}{2\eta^2}[378+53\cos(2\eta)]+\frac{325}{\eta^3}\sin(2\eta)+\frac{275}{2\eta^4}[42+17\cos(2\eta)]-\frac{20625}{2\eta^5}\sin(2\eta)$$
$$-\frac{1155}{4\eta^6}[117+91\cos(2\eta)]+\frac{30030}{\eta^7}\sin(2\eta)=-8\sum_{n=1}^{\infty}(-1)^n\frac{1}{(2n+7)!}(2n-1)(2n-3)(n-2)(2n-5)(n-3)(2n-7)(2\eta)^{2n},$$

$$2C_{72}^{b}=\frac{1}{\eta}\left[\frac{56}{3}-\cos(2\eta)\right]+\frac{53}{2\eta^2}\sin(2\eta)-\frac{5}{\eta^3}[252-65\cos(2\eta)]-\frac{1675}{2\eta^4}\sin(2\eta)+\frac{165}{2\eta^5}[216-125\cos(2\eta)]$$
$$+\frac{105105}{4\eta^6}\sin(2\eta)-\frac{30030}{\eta^7}[1-\cos(2\eta)]=32\sum_{n=4}^{\infty}(-1)^n\frac{1}{(2n+8)!}n(n-1)(2n-3)(n-2)(2n-5)(n-3)(2\eta)^{2n+1}.$$

12. $C_n^c(\alpha_1, s)$

$$2\alpha C_0^c=1-e^{-2\alpha/\mu_1}=-\sum_{n=1}^{\infty}\frac{1}{n!}\left(-\frac{2\alpha}{\mu_1}\right)^n,$$

$$2\alpha i C_1^c=1-\frac{\mu_1}{\alpha}+\left(1+\frac{\mu_1}{\alpha}\right)e^{-2\alpha/\mu_1}=\sum_{n=2}^{\infty}\frac{1}{(n+1)!}(n-1)\left(-\frac{2\alpha}{\mu_1}\right)^n,$$

$$2\alpha C_2^c=-1+3\frac{\mu_1}{\alpha}-3\left(\frac{\mu_1}{\alpha}\right)^2+\left[1+3\frac{\mu_1}{\alpha}+3\left(\frac{\mu_1}{\alpha}\right)^2\right]e^{-2\alpha/\mu_1}=\sum_{n=3}^{\infty}\frac{1}{(n+2)!}(n-1)(n-2)\left(-\frac{2\alpha}{\mu_1}\right)^n,$$

$$2\alpha i C_3^c=-1+6\frac{\mu_1}{\alpha}-15\left(\frac{\mu_1}{\alpha}\right)^2+15\left(\frac{\mu_1}{\alpha}\right)^3-\left[1+6\frac{\mu_1}{\alpha}+15\left(\frac{\mu_1}{\alpha}\right)^2+15\left(\frac{\mu_1}{\alpha}\right)^3\right]e^{-2\alpha/\mu_1}=-\sum_{n=4}^{\infty}\frac{1}{(n+3)!}(n-1)(n-2)(n-3)\left(-\frac{2\alpha}{\mu_1}\right)^n,$$

$$2\alpha C_4^c=1-10\frac{\mu_1}{\alpha}+45\left(\frac{\mu_1}{\alpha}\right)^2-105\left(\frac{\mu_1}{\alpha}\right)^3+105\left(\frac{\mu_1}{\alpha}\right)^4-\left[1+10\frac{\mu_1}{\alpha}+45\left(\frac{\mu_1}{\alpha}\right)^2+105\left(\frac{\mu_1}{\alpha}\right)^3+105\left(\frac{\mu_1}{\alpha}\right)^4\right]e^{-2\alpha/\mu_1}$$
$$=-\sum_{n=5}^{\infty}\frac{1}{(n+4)!}(n-1)(n-2)(n-3)(n-4)\left(-\frac{2\alpha}{\mu_1}\right)^n,$$

$$2\alpha i C_5^c = 1-15\frac{\mu_1}{\alpha}+105\left(\frac{\mu_1}{\alpha}\right)^2-420\left(\frac{\mu_1}{\alpha}\right)^3+945\left(\frac{\mu_1}{\alpha}\right)^4-945\left(\frac{\mu_1}{\alpha}\right)^5+\left[1+15\frac{\mu_1}{\alpha}+105\left(\frac{\mu_1}{\alpha}\right)^2+420\left(\frac{\mu_1}{\alpha}\right)^3+945\left(\frac{\mu_1}{\alpha}\right)^4+945\left(\frac{\mu_1}{\alpha}\right)^5\right]$$
$$\times e^{-2\alpha/\mu_1} = \sum_{n=6}^{\infty}\frac{1}{(n+5)!}(n-1)(n-2)(n-3)(n-4)(n-5)\left(-\frac{2\alpha}{\mu_1}\right)^n,$$

$$2\alpha C_6^c = -1+21\frac{\mu_1}{\alpha}-210\left(\frac{\mu_1}{\alpha}\right)^2+1260\left(\frac{\mu_1}{\alpha}\right)^3-4725\left(\frac{\mu_1}{\alpha}\right)^4+10395\left(\frac{\mu_1}{\alpha}\right)^5-10395\left(\frac{\mu_1}{\alpha}\right)^6+\left[1+21\frac{\mu_1}{\alpha}+210\left(\frac{\mu_1}{\alpha}\right)^2+1260\left(\frac{\mu_1}{\alpha}\right)^3\right.$$
$$\left.+4725\left(\frac{\mu_1}{\alpha}\right)^4+10395\left(\frac{\mu_1}{\alpha}\right)^5+10395\left(\frac{\mu_1}{\alpha}\right)^6\right]e^{-2\alpha/\mu_1} = \sum_{n=7}^{\infty}\frac{1}{(n+6)!}(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)\left(-\frac{2\alpha}{\mu_1}\right)^n,$$

$$2\alpha i C_7^c = -1+28\frac{\mu_1}{\alpha}-378\left(\frac{\mu_1}{\alpha}\right)^2+3150\left(\frac{\mu_1}{\alpha}\right)^3-17325\left(\frac{\mu_1}{\alpha}\right)^4+62370\left(\frac{\mu_1}{\alpha}\right)^5-135135\left(\frac{\mu_1}{\alpha}\right)^6+135135\left(\frac{\mu_1}{\alpha}\right)^7$$
$$-\left[1+28\frac{\mu_1}{\alpha}+378\left(\frac{\mu_1}{\alpha}\right)^2+3150\left(\frac{\mu_1}{\alpha}\right)^3+17325\left(\frac{\mu_1}{\alpha}\right)^4+62370\left(\frac{\mu_1}{\alpha}\right)^5+135135\left(\frac{\mu_1}{\alpha}\right)^6+135135\left(\frac{\mu_1}{\alpha}\right)^7\right]e^{-2\alpha/\mu_1}$$
$$= -\sum_{n=8}^{\infty}\frac{1}{(n+7)!}(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)(n-7)\left(-\frac{2\alpha}{\mu_1}\right)^n.$$

13. $C_n^c(\alpha_\eta, i\eta+\tau_1\nu_1-\tau_\eta\nu_\eta) \equiv \frac{1}{\eta}\left(C_{n1}^c+iC_{n2}^c\right)$

$$2C_{01}^c = \sin\left(\frac{2\eta}{\mu_1}\right) = \sum_{n=0}^{\infty}(-1)^n\frac{1}{(2n+1)!}\left(\frac{2\eta}{\mu_1}\right)^{2n+1},$$

$$2C_{02}^c = \cos\left(\frac{2\eta}{\mu_1}\right)-1 = \sum_{n=1}^{\infty}(-1)^n\frac{1}{(2n)!}\left(\frac{2\eta}{\mu_1}\right)^{2n},$$

$$2C_{11}^c = 1-\cos\left(\frac{2\eta}{\mu_1}\right)+\frac{\mu_1}{\eta}\sin\left(\frac{2\eta}{\mu_1}\right) = -\sum_{n=1}^{\infty}(-1)^n\frac{1}{(2n+1)!}(2n-1)\left(\frac{2\eta}{\mu_1}\right)^{2n},$$

$$2C_{12}^c = \sin\left(\frac{2\eta}{\mu_1}\right)-\frac{\mu_1}{\eta}\left[1-\cos\left(\frac{2\eta}{\mu_1}\right)\right] = 2\sum_{n=1}^{\infty}(-1)^n\frac{1}{(2n+2)!}\,n\left(\frac{2\eta}{\mu_1}\right)^{2n+1},$$

$$2C_{21}^c = -\sin\left(\frac{2\eta}{\mu_1}\right)-3\frac{\mu_1}{\eta}\left[1+\cos\left(\frac{2\eta}{\mu_1}\right)\right]+3\left(\frac{\mu_1}{\eta}\right)^2\sin\left(\frac{2\eta}{\mu_1}\right) = -2\sum_{n=1}^{\infty}(-1)^n\frac{1}{(2n+3)!}\,n(2n-1)\left(\frac{2\eta}{\mu_1}\right)^{2n+1},$$

$$2C_{22}^c = 1-\cos\left(\frac{2\eta}{\mu_1}\right)+3\frac{\mu_1}{\eta}\sin\left(\frac{2\eta}{\mu_1}\right)-3\left(\frac{\mu_1}{\eta}\right)^2\left[1-\cos\left(\frac{2\eta}{\mu_1}\right)\right] = -2\sum_{n=2}^{\infty}(-1)^n\frac{1}{(2n+2)!}(2n-1)(n-1)\left(\frac{2\eta}{\mu_1}\right)^{2n},$$

$$2C_{31}^c = 1+\cos\left(\frac{2\eta}{\mu_1}\right)-6\frac{\mu_1}{\eta}\sin\left(\frac{2\eta}{\mu_1}\right)-15\left(\frac{\mu_1}{\eta}\right)^2\left[1+\cos\left(\frac{2\eta}{\mu_1}\right)\right]+15\left(\frac{\mu_1}{\eta}\right)^3\sin\left(\frac{2\eta}{\mu_1}\right)$$
$$= 2\sum_{n=2}^{\infty}(-1)^n\frac{1}{(2n+3)!}(2n-1)(n-1)(2n-3)\left(\frac{2\eta}{\mu_1}\right)^{2n},$$

$$2C_{32}^c = -\sin\left(\frac{2\eta}{\mu_1}\right)+6\frac{\mu_1}{\eta}\left[1-\cos\left(\frac{2\eta}{\mu_1}\right)\right]+15\left(\frac{\mu_1}{\eta}\right)^2\sin\left(\frac{2\eta}{\mu_1}\right)-15\left(\frac{\mu_1}{\eta}\right)^3\sin\left(\frac{2\eta}{\mu_1}\right)$$
$$= -4\sum_{n=2}^{\infty}(-1)^n\frac{1}{(2n+4)!}\,n(2n-1)(n-1)\left(\frac{2\eta}{\mu_1}\right)^{2n+1},$$

$$2C_{41}^c = \sin\left(\frac{2\eta}{\mu_1}\right)+10\frac{\mu_1}{\eta}\left[1+\cos\left(\frac{2\eta}{\mu_1}\right)\right]-45\left(\frac{\mu_1}{\eta}\right)^2\sin\left(\frac{2\eta}{\mu_1}\right)-105\left(\frac{\mu_1}{\eta}\right)^3\left[1+\cos\left(\frac{2\eta}{\mu_1}\right)\right]+105\left(\frac{\mu_1}{\eta}\right)^4\sin\left(\frac{2\eta}{\mu_1}\right)$$
$$= 4\sum_{n=2}^{\infty}(-1)^n\frac{1}{(2n+5)!}\,n(2n-1)(n-1)(2n-3)\left(\frac{2\eta}{\mu_1}\right)^{2n+1},$$

$$2C_{42}^c = \cos\left(\frac{2\eta}{\mu_1}\right)-1-10\frac{\mu_1}{\eta}\sin\left(\frac{2\eta}{\mu_1}\right)+45\left(\frac{\mu_1}{\eta}\right)^2\left[1-\cos\left(\frac{2\eta}{\mu_1}\right)\right]+105\left(\frac{\mu_1}{\eta}\right)^3\sin\left(\frac{2\eta}{\mu_1}\right)-105\left(\frac{\mu_1}{\eta}\right)^4\left[1-\cos\left(\frac{2\eta}{\mu_1}\right)\right]$$
$$= 4\sum_{n=3}^{\infty}(-1)^n\frac{1}{(2n+4)!}(2n-1)(n-1)(2n-3)(n-2)\left(\frac{2\eta}{\mu_1}\right)^{2n},$$

$$2C_{51}^c = -1-\cos\left(\frac{2\eta}{\mu_1}\right)+15\frac{\mu_1}{\eta}\sin\left(\frac{2\eta}{\mu_1}\right)+105\left(\frac{\mu_1}{\eta}\right)^2\left[1+\cos\left(\frac{2\eta}{\mu_1}\right)\right]-420\left(\frac{\mu_1}{\eta}\right)^3\sin\left(\frac{2\eta}{\mu_1}\right)-945\left(\frac{\mu_1}{\eta}\right)^4\left[1+\cos\left(\frac{2\eta}{\mu_1}\right)\right]$$
$$+945\left(\frac{\mu_1}{\eta}\right)^5\sin\left(\frac{2\eta}{\mu_1}\right) = -4\sum_{n=3}^{\infty}(-1)^n\frac{1}{(2n+5)!}(2n-1)(n-1)(2n-3)(n-2)(2n-5)\left(\frac{2\eta}{\mu_1}\right)^{2n},$$

$$2C'^{\,c}_{52}=\sin\left(\frac{2\eta}{\mu_1}\right)-15\frac{\mu_1}{\eta}\left[1-\cos\left(\frac{2\eta}{\mu_1}\right)\right]-105\left(\frac{\mu_1}{\eta}\right)^2\sin\left(\frac{2\eta}{\mu_1}\right)+420\left(\frac{\mu_1}{\eta}\right)^3\left[1-\cos\left(\frac{2\eta}{\mu_1}\right)\right]+945\left(\frac{\mu_1}{\eta}\right)^4\sin\left(\frac{2\eta}{\mu_1}\right)$$
$$-945\left(\frac{\mu_1}{\eta}\right)^5\left[1-\cos\left(\frac{2\eta}{\mu_1}\right)\right]=8\sum_{n=3}^{\infty}(-1)^n\frac{1}{(2n+6)!}\,n(2n-1)(n-1)(2n-3)(n-2)\left(\frac{2\eta}{\mu_1}\right)^{2n+1},$$

$$2C'^{\,c}_{61}=-\sin\left(\frac{2\eta}{\mu_1}\right)-21\frac{\mu_1}{\eta}\left[1+\cos\left(\frac{2\eta}{\mu_1}\right)\right]+210\left(\frac{\mu_1}{\eta}\right)^2\sin\left(\frac{2\eta}{\mu_1}\right)+1260\left(\frac{\mu_1}{\eta}\right)^3\left[1+\cos\left(\frac{2\eta}{\mu_1}\right)\right]-4725\left(\frac{\mu_1}{\eta}\right)^4\sin\left(\frac{2\eta}{\mu_1}\right)$$
$$-10395\left(\frac{\mu_1}{\eta}\right)^5\left[1+\cos\left(\frac{2\eta}{\mu_1}\right)\right]+10395\left(\frac{\mu_1}{\eta}\right)^6\sin\left(\frac{2\eta}{\mu_1}\right)=-8\sum_{n=3}^{\infty}(-1)^n\frac{1}{(2n+7)!}\,n(2n-1)(n-1)(2n-3)(n-2)(2n-5)\left(\frac{2\eta}{\mu_1}\right)^{2n+1},$$

$$2C'^{\,c}_{62}=1-\cos\left(\frac{2\eta}{\mu_1}\right)+21\frac{\mu_1}{\eta}\sin\left(\frac{2\eta}{\mu_1}\right)-210\left(\frac{\mu_1}{\eta}\right)^2\left[1-\cos\left(\frac{2\eta}{\mu_1}\right)\right]-1260\left(\frac{\mu_1}{\eta}\right)^3\sin\left(\frac{2\eta}{\mu_1}\right)+4725\left(\frac{\mu_1}{\eta}\right)^4\left[1-\cos\left(\frac{2\eta}{\mu_1}\right)\right]$$
$$+10395\left(\frac{\mu_1}{\eta}\right)^5\sin\left(\frac{2\eta}{\mu_1}\right)-10395\left(\frac{\mu_1}{\eta}\right)^6\left[1-\cos\left(\frac{2\eta}{\mu_1}\right)\right]=-8\sum_{n=4}^{\infty}(-1)^n\frac{1}{(2n+6)!}\,(2n-1)(n-1)(2n-3)(n-2)(2n-5)(n-3)\left(\frac{2\eta}{\mu_1}\right)^{2n},$$

$$2C'^{\,c}_{71}=1+\cos\left(\frac{2\eta}{\mu_1}\right)-28\frac{\mu_1}{\eta}\sin\left(\frac{2\eta}{\mu_1}\right)-378\left(\frac{\mu_1}{\eta}\right)^2\left[1+\cos\left(\frac{2\eta}{\mu_1}\right)\right]+3150\left(\frac{\mu_1}{\eta}\right)^3\sin\left(\frac{2\eta}{\mu_1}\right)+17325\left(\frac{\mu_1}{\eta}\right)^4\left[1+\cos\left(\frac{2\eta}{\mu_1}\right)\right]$$
$$-62370\left(\frac{\mu_1}{\eta}\right)^5\sin\left(\frac{2\eta}{\mu_1}\right)-135135\left(\frac{\mu_1}{\eta}\right)^6\left[1+\cos\left(\frac{2\eta}{\mu_1}\right)\right]+135135\left(\frac{\mu_1}{\eta}\right)^7\sin\left(\frac{2\eta}{\mu_1}\right)$$
$$=8\sum_{n=4}^{\infty}(-1)^n\frac{1}{(2n+7)!}\,(2n-1)(n-1)(2n-3)(n-2)(2n-5)(n-3)(2n-7)\left(\frac{2\eta}{\mu_1}\right)^{2n},$$

$$2C'^{\,c}_{72}=-\sin\left(\frac{2\eta}{\mu_1}\right)+28\frac{\mu_1}{\eta}\left[1-\cos\left(\frac{2\eta}{\mu_1}\right)\right]+378\left(\frac{\mu_1}{\eta}\right)^2\sin\left(\frac{2\eta}{\mu_1}\right)-3150\left(\frac{\mu_1}{\eta}\right)^3\left[1-\cos\left(\frac{2\eta}{\mu_1}\right)\right]-17325\left(\frac{\mu_1}{\eta}\right)^4\sin\left(\frac{2\eta}{\mu_1}\right)$$
$$+62370\left(\frac{\mu_1}{\eta}\right)^5\left[1-\cos\left(\frac{2\eta}{\mu_1}\right)\right]+135135\left(\frac{\mu_1}{\eta}\right)^6\sin\left(\frac{2\eta}{\mu_1}\right)-135135\left(\frac{\mu_1}{\eta}\right)^7\left[1-\cos\left(\frac{2\eta}{\mu_1}\right)\right]$$
$$=-16\sum_{n=4}^{\infty}(-1)^n\frac{1}{(2n+8)!}\,n(2n-1)(n-1)(2n-3)(n-2)(2n-5)(n-3)\left(\frac{2\eta}{\mu_1}\right)^{2n+1}.$$

---

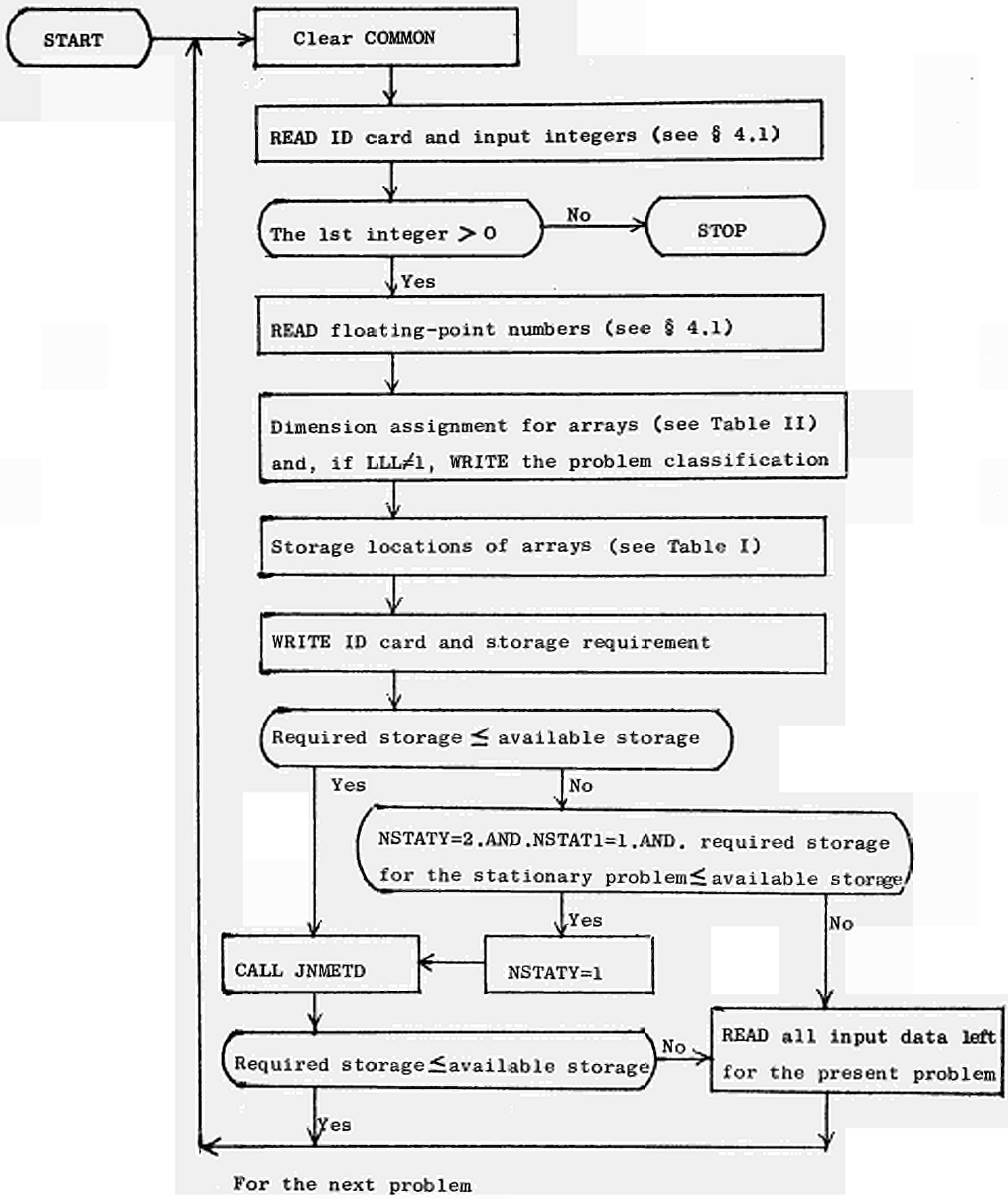
a) $[\,\xi \rightarrow 1-\xi\,]$ indicates that the expression is the same as shown just before except for replacing ξ by $1-\xi$.

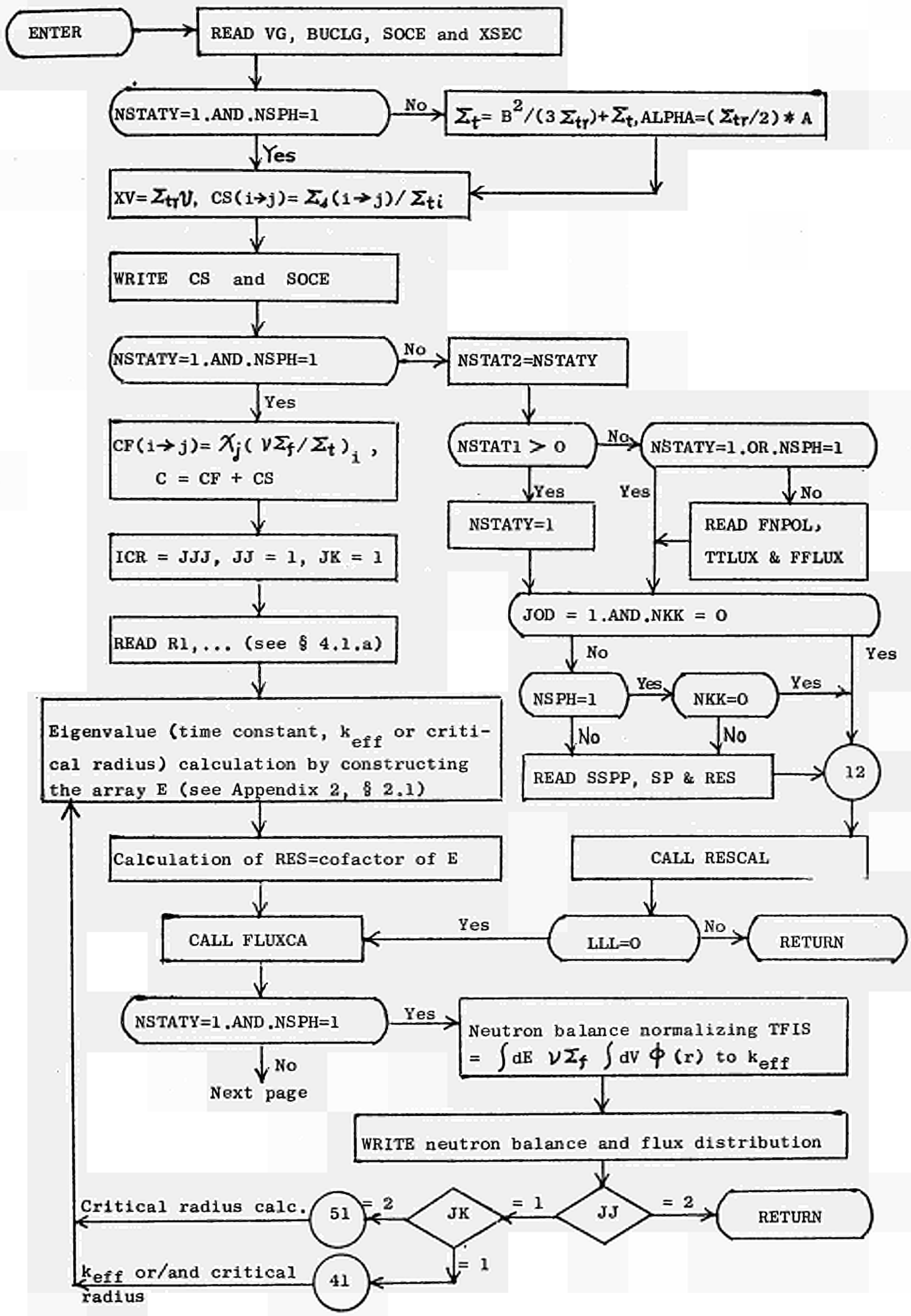
b) $\{n \rightarrow 2n-1\}$ or $\{n \rightarrow 2n\}$ in the series expansion of G_{m1} or G_{m2} (F_{m1} or F_{m2}) shows that the expression is the same as shown in braces of the series expansion of G_m (see Section 4) [F_m (see Section 6)] except for replacing n by 2n-1 or 2n.

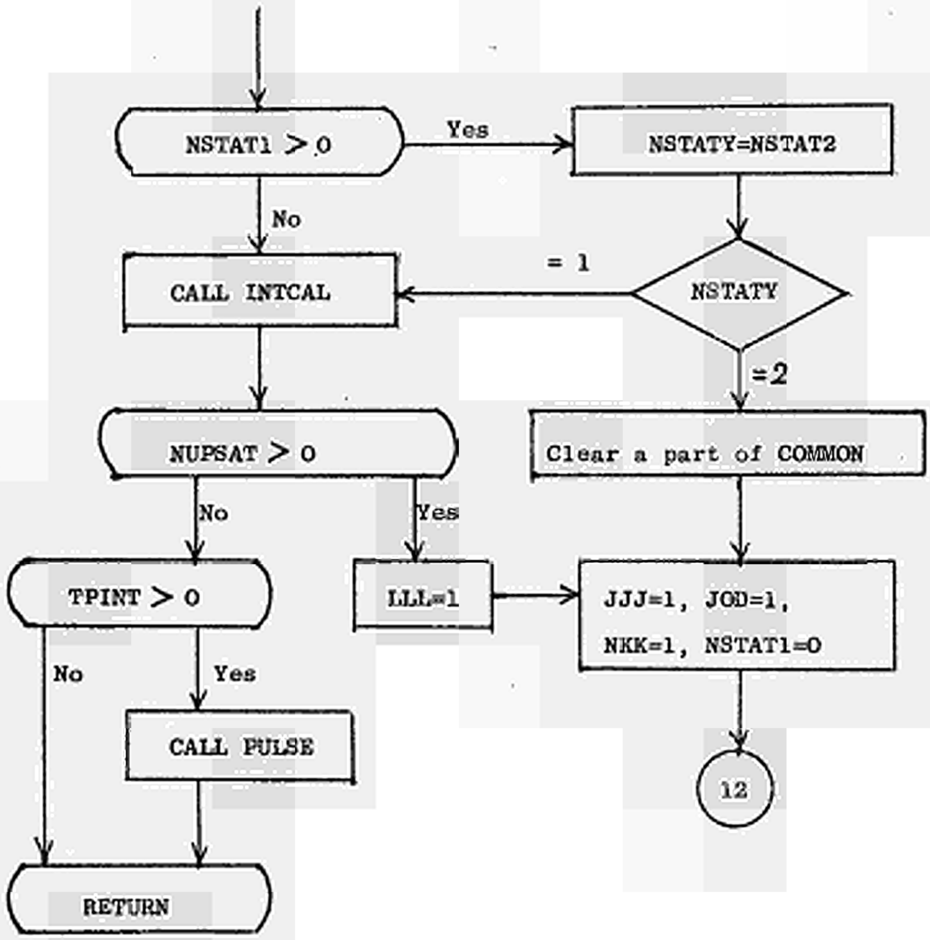
c) For $\mu<0$, $F_n(\alpha_q,\xi,\mu,s)=(-1)^n F_n(\alpha_q,1-\xi,-\mu,s)$.

Appendix 2. Flow Diagrammes of Computer Programmes

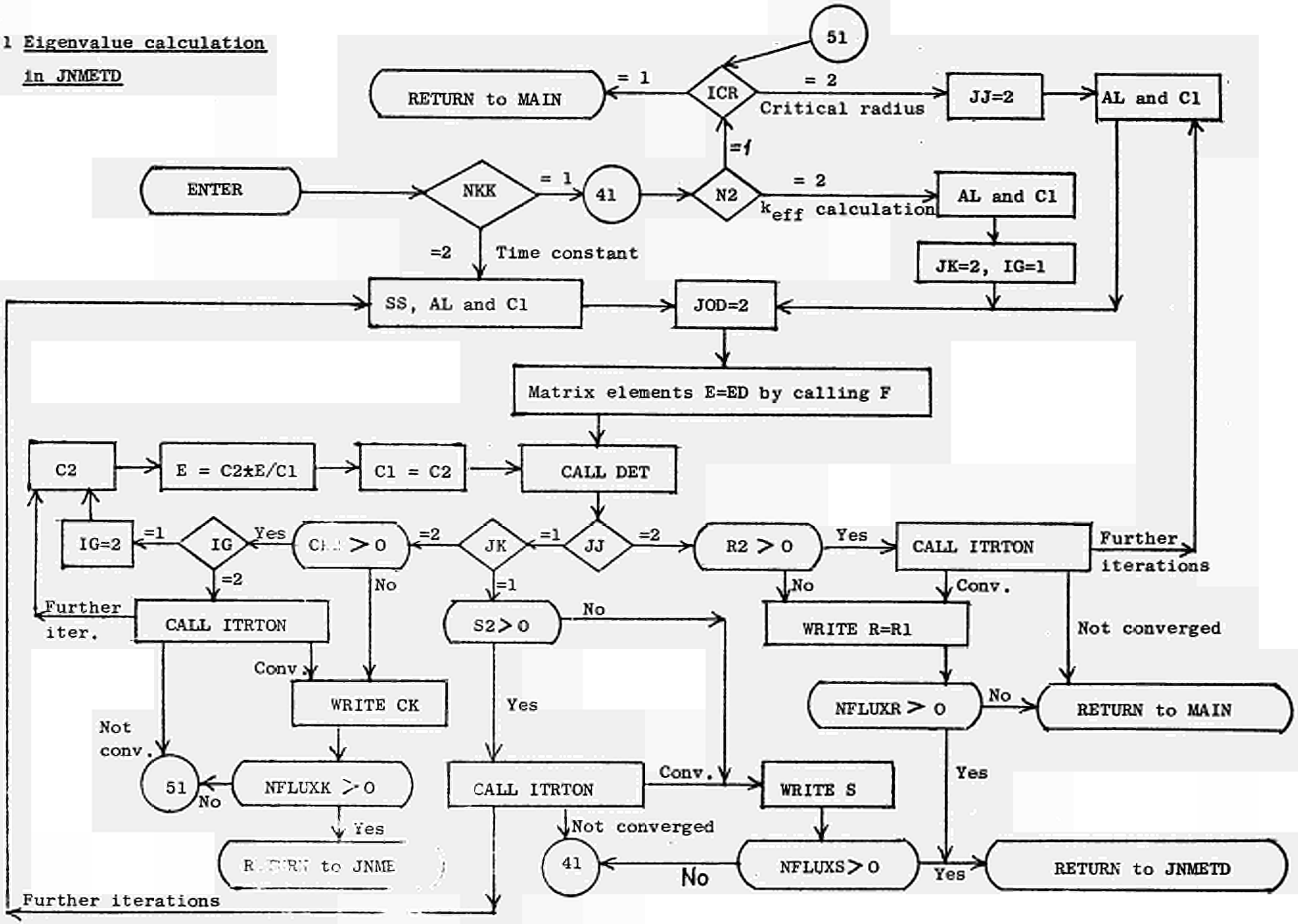
1. MAIN



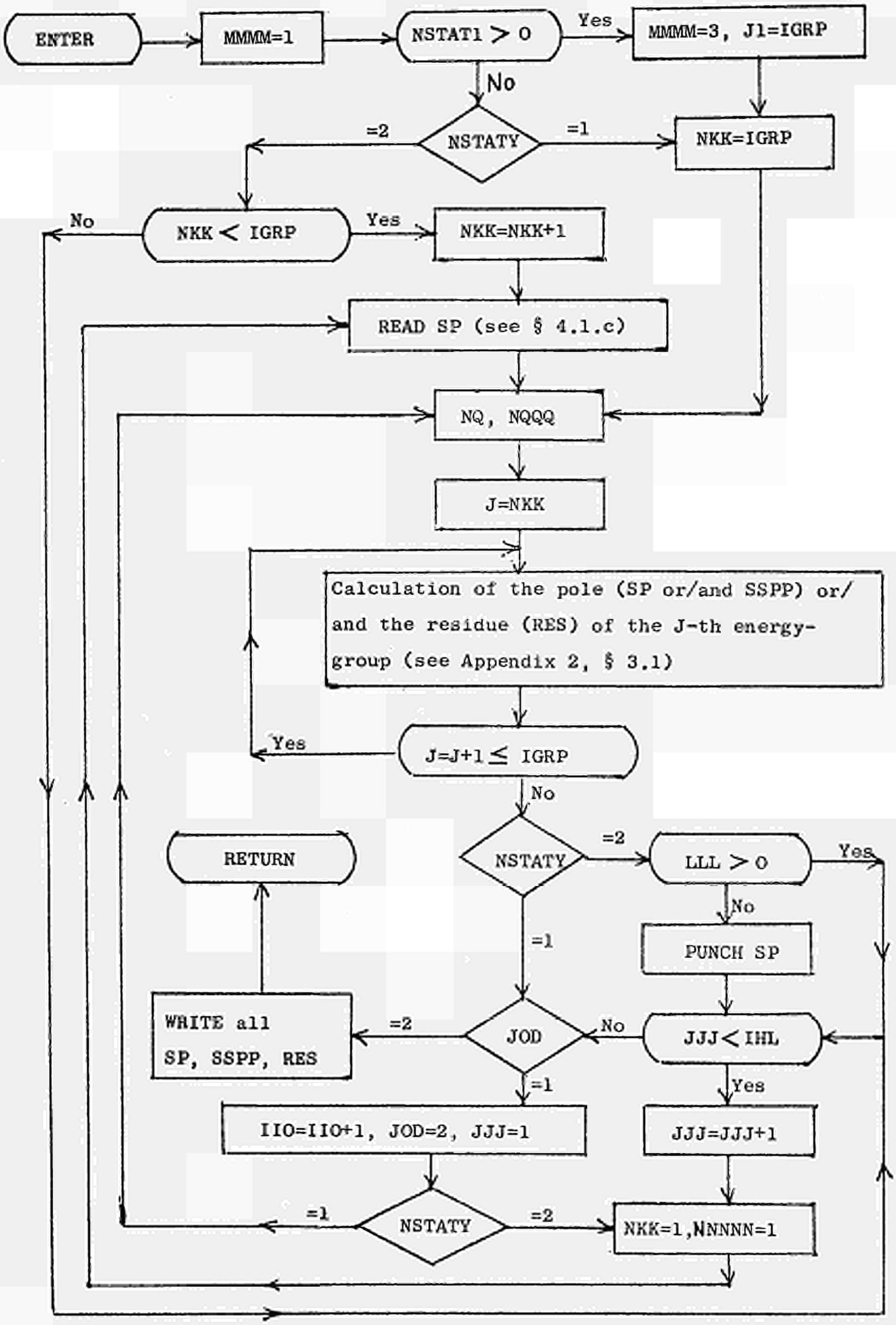

ENTER
READ VG, BUCLG, SOCE and XSEC
NSTATY=1.AND.NSPH=1
No
$\Sigma_t = B^2/(3\Sigma_{tr}) + \Sigma_t$, ALPHA=$(\Sigma_{tr}/2)$ * A
Yes
XV=$\Sigma_{tr}V$, CS(i→j)= Σ_s(i→j)/Σ_{ti}
WRITE CS and SOCE
NSTATY=1.AND.NSPH=1
No
NSTAT2=NSTATY
Yes
CF(i→j)= $\chi_j(\nu\Sigma_f/\Sigma_t)_i$,
C = CF + CS
NSTAT1 > 0
No
NSTATY=1.OR.NSPH=1
Yes
Yes
No
NSTATY=1
READ FNPOL, TTLUX & FFLUX
ICR = JJJ, JJ = 1, JK = 1
JOD = 1.AND.NKK = 0
READ R1,... (see § 4.1.a)
No
Yes
NSPH=1
Yes
NKK=0
Yes
No
No
Eigenvalue (time constant, k_{eff} or critical radius) calculation by constructing the array E (see Appendix 2, § 2.1)
READ SSPP, SP & RES
12
Calculation of RES=cofactor of E
CALL RESCAL
CALL FLUXCA
Yes
LLL=0
No
RETURN
NSTATY=1.AND.NSPH=1
Yes
Neutron balance normalizing TFIS = $\int dE\, \nu\Sigma_f \int dV\, \phi(r)$ to k_{eff}
No
Next page
WRITE neutron balance and flux distribution
Critical radius calc.
51
= 2
JK
= 1
JJ
= 2
RETURN
= 1
k_{eff} or/and critical radius
41


NSTAT1 > 0
Yes
NSTATY=NSTAT2
No
= 1
CALL INTCAL
NSTATY
=2
NUPSAT > 0
Clear a part of COMMON
No
Yes
TPINT > 0
LLL=1
JJJ=1, JOD=1,
NKK=1, NSTAT1=0
No
Yes
CALL PULSE
12
RETURN

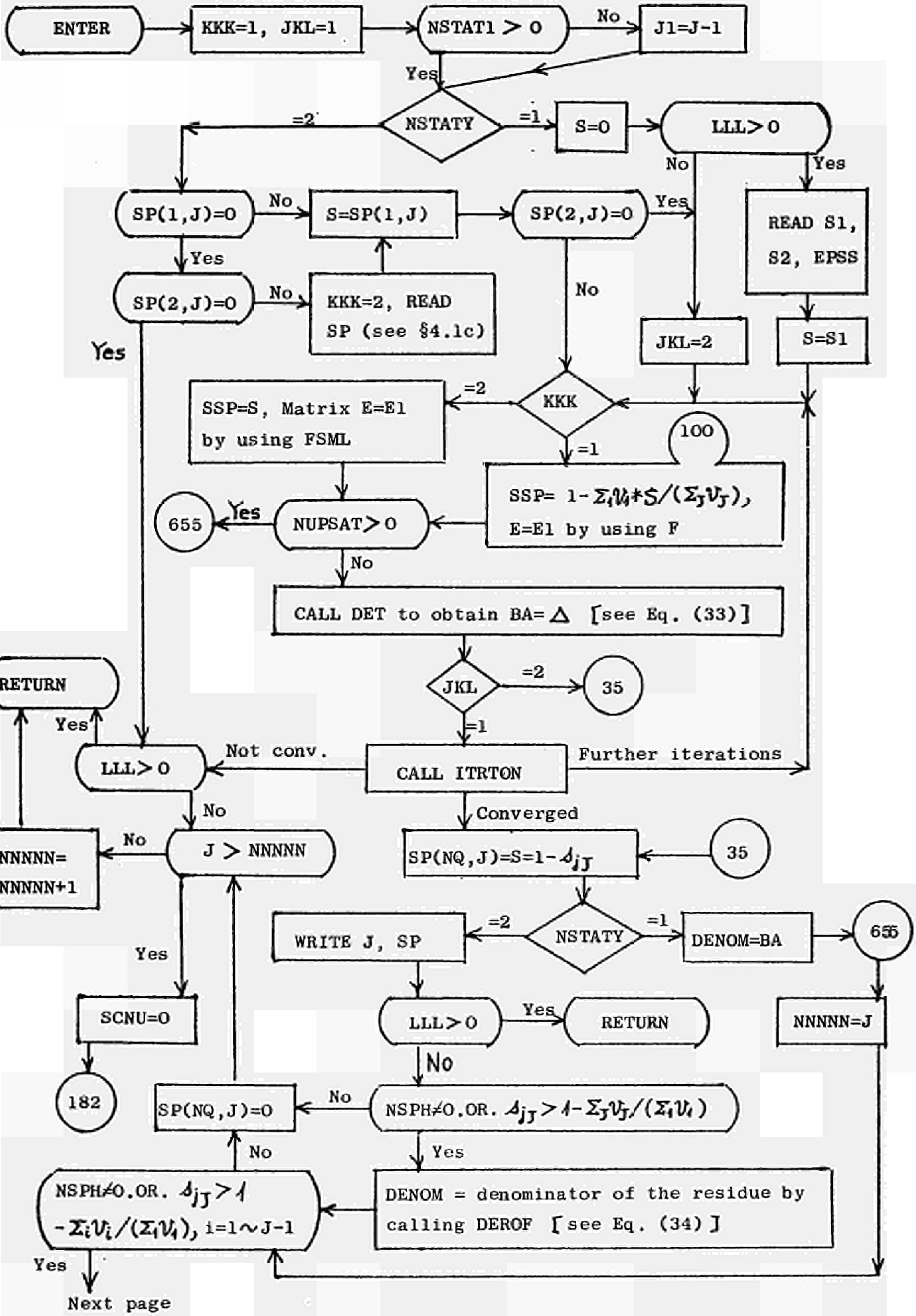
2.1 Eigenvalue calculation in JNMETD

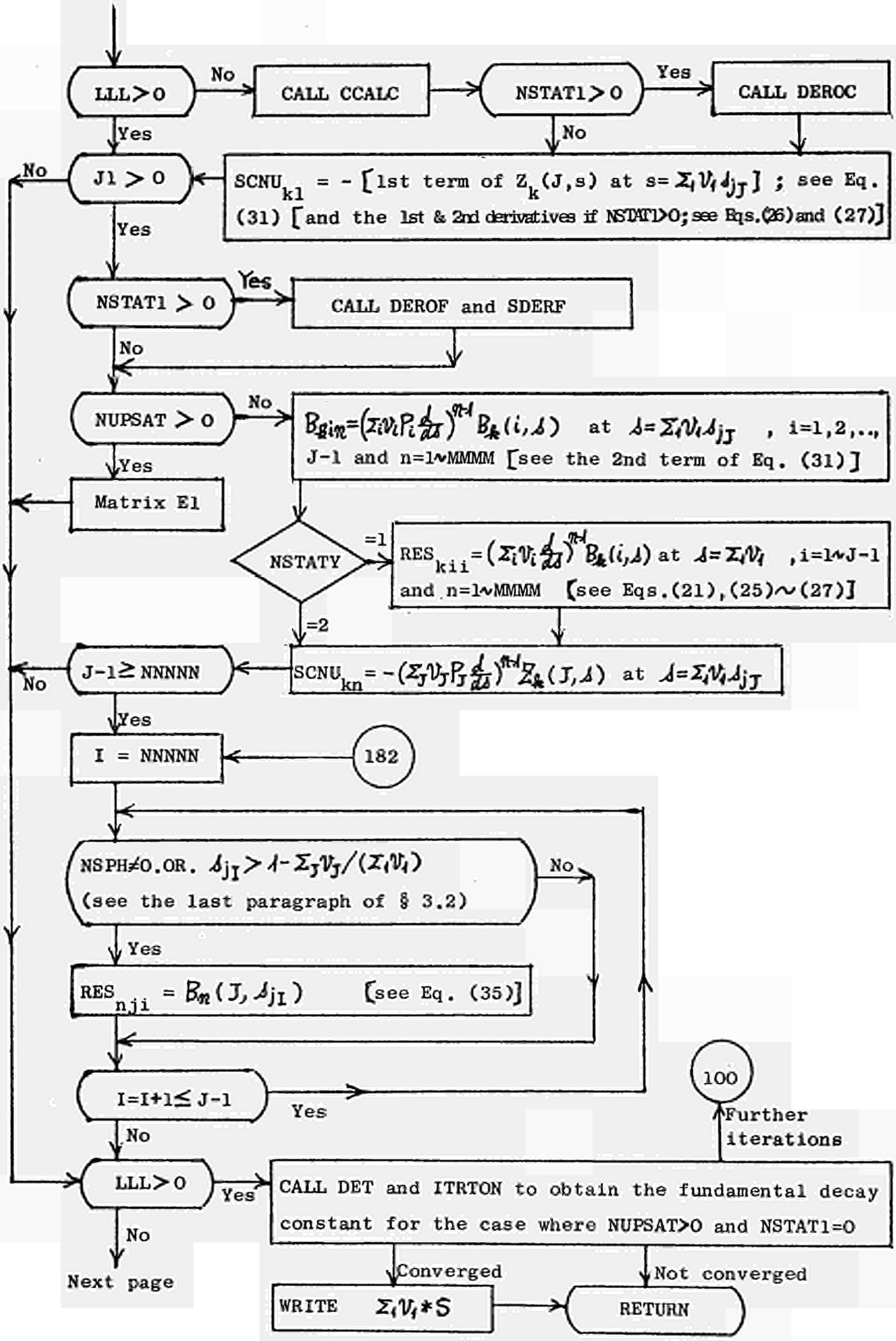


3. RESCAL

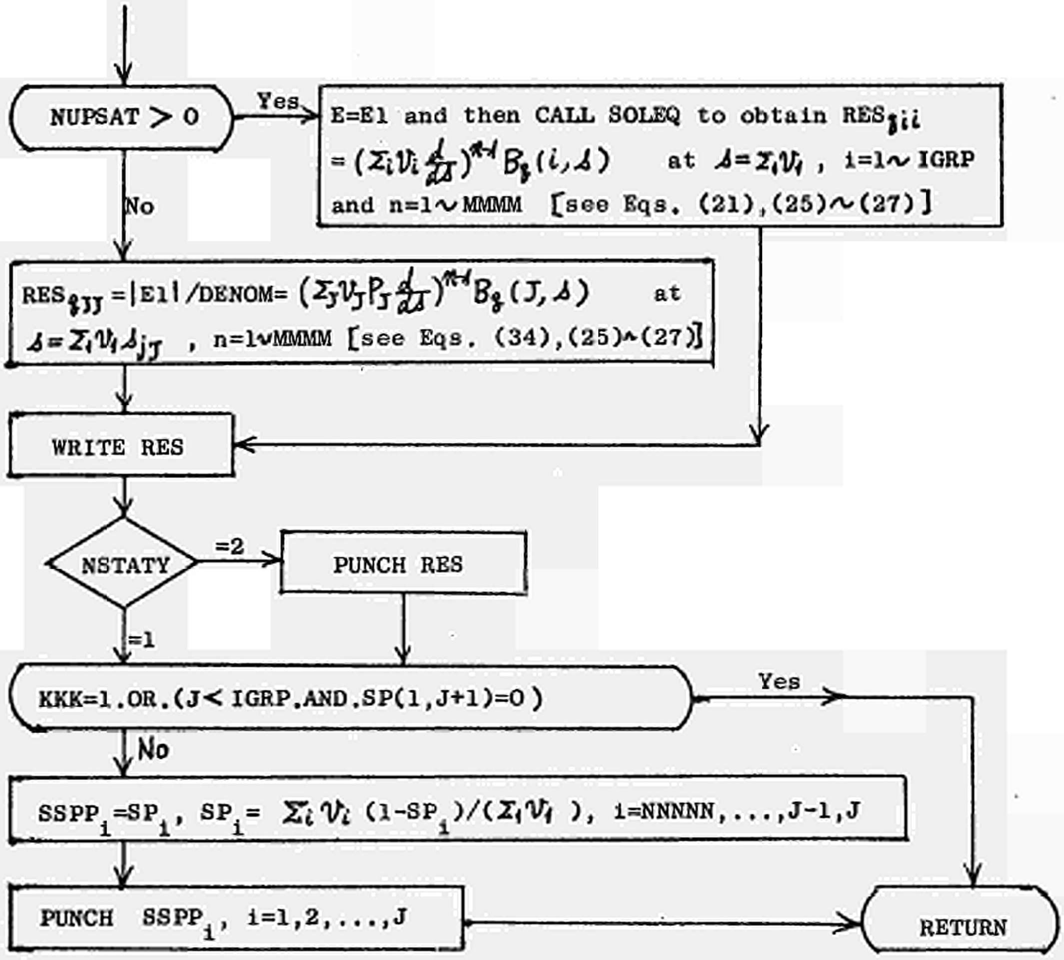

ENTER
MMMM=1
NSTAT1 > 0
Yes
MMMM=3, J1=IGRP
No
NSTATY
=2
=1
NKK=IGRP
No
NKK < IGRP
Yes
NKK=NKK+1
READ SP (see § 4.1.c)
NQ, NQQQ
J=NKK
Calculation of the pole (SP or/and SSPP) or/
and the residue (RES) of the J-th energy-
group (see Appendix 2, § 3.1)
Yes
J=J+1 ≤ IGRP
No
RETURN
NSTATY
=2
LLL > 0
Yes
No
PUNCH SP
=1
WRITE all
SP, SSPP, RES
=2
JOD
No
JJJ < IHL
=1
Yes
IIO=IIO+1, JOD=2, JJJ=1
JJJ=JJJ+1
=1
NSTATY
=2
NKK=1,NNNNN=1

3.1 Calculation of the pole (SP or/and SSPP) or/and the residue (RES) of the J-th group in RESCAL



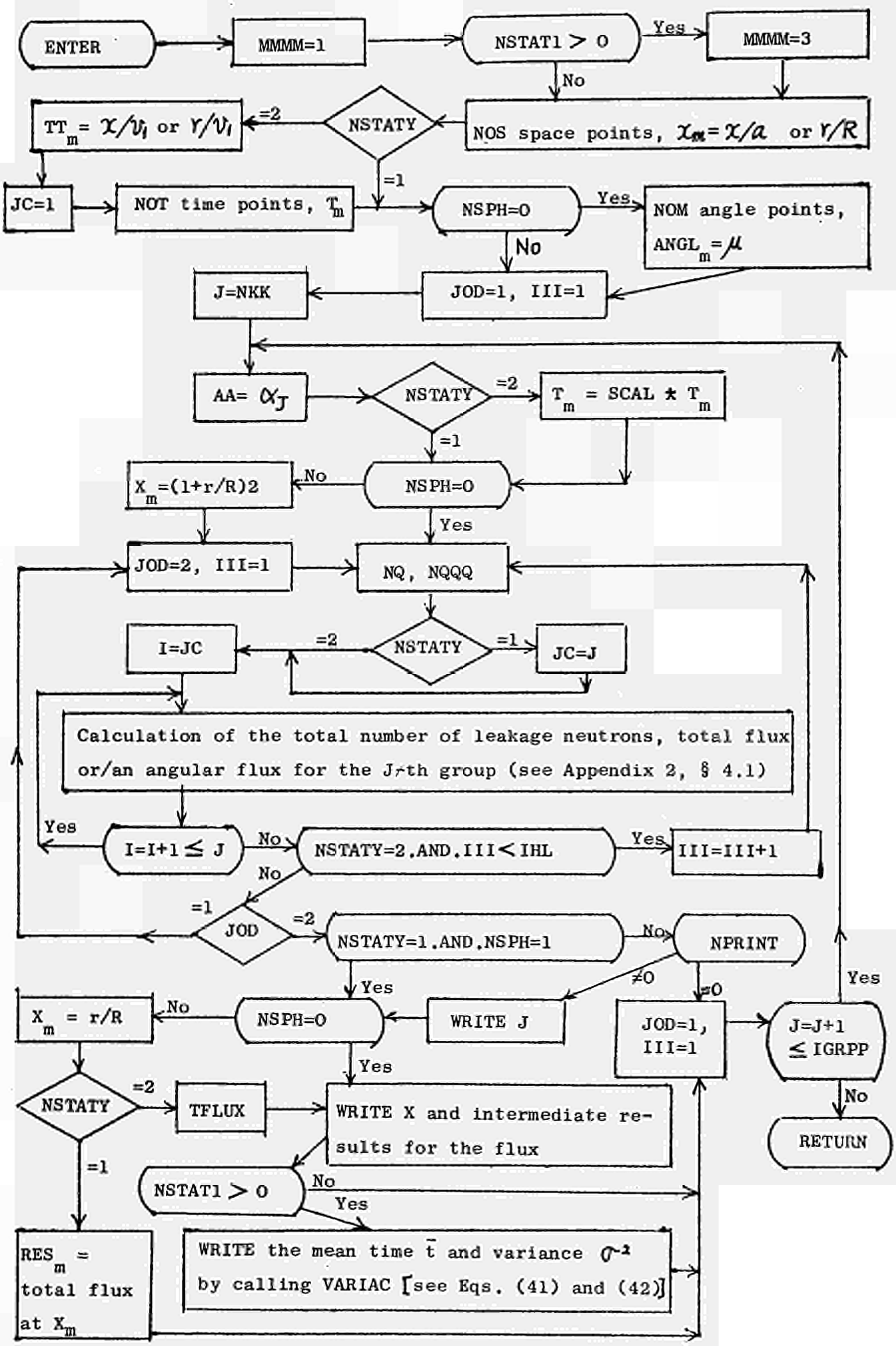


LLL>0
No
CALL CCALC
NSTAT1>0
Yes
CALL DEROC
No
Yes
No
J1 > 0
SCNU$_{k1}$ = - [1st term of Z_k(J,s) at s= $\Sigma_i \nu_i s_{jJ}$] ; see Eq. (31) [and the 1st & 2nd derivatives if NSTAT1>0; see Eqs.(26) and (27)]
Yes
NSTAT1 > 0
Yes
CALL DEROF and SDERF
No
NUPSAT > 0
No
$B_{gin}=(\Sigma_i\nu_i P_i \frac{d}{ds})^{n-1} B_k(i,s)$ at $s=\Sigma_i \nu_i s_{jJ}$, i=1,2,.., J-1 and n=1~MMMM [see the 2nd term of Eq. (31)]
Yes
Matrix E1
NSTATY
=1
RES$_{kii}$ = $(\Sigma_i \nu_i \frac{d}{ds})^{n-1} B_k(i,s)$ at $s=\Sigma_i \nu_i$, i=1~J-1 and n=1~MMMM [see Eqs.(21),(25)~(27)]
=2
J-1≥ NNNNN
No
SCNU$_{kn}$ = $-(\Sigma_J \nu_J P_J \frac{d}{ds})^{n-1} Z_k(J,s)$ at $s=\Sigma_i \nu_i s_{jJ}$
Yes
I = NNNNN
182
NSPH≠0.OR. $s_{jI} > 1 - \Sigma_J \nu_J / (\Sigma_i \nu_i)$ (see the last paragraph of § 3.2)
No
Yes
RES$_{nji}$ = $B_n(J, s_{jI})$ [see Eq. (35)]
I=I+1≤ J-1
Yes
100
Further iterations
No
LLL> 0
Yes
CALL DET and ITRTON to obtain the fundamental decay constant for the case where NUPSAT>0 and NSTAT1=0
No
Next page
Converged
Not converged
WRITE $\Sigma_i \nu_i * S$
RETURN

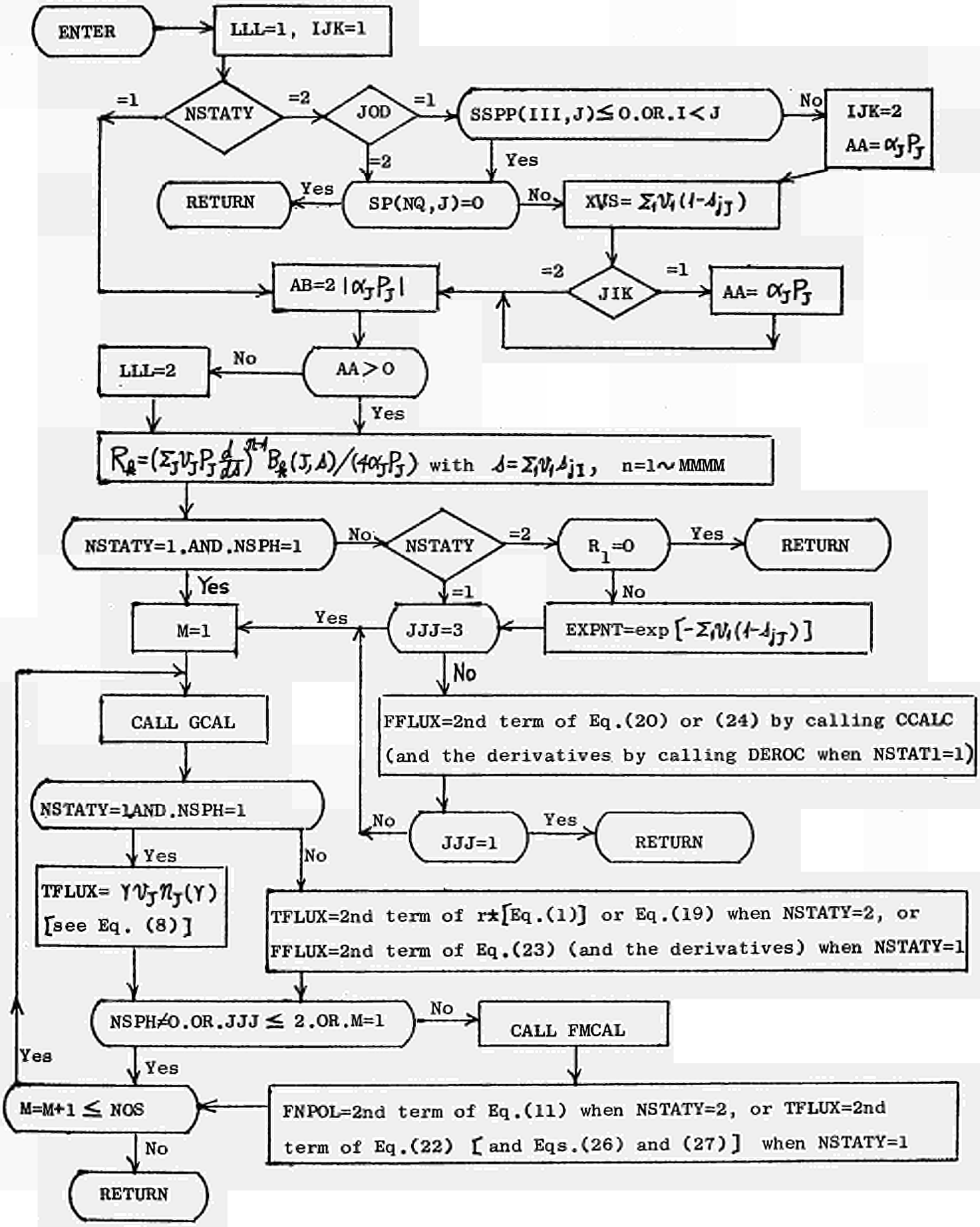


NUPSAT > 0
Yes
No
E=E1 and then CALL SOLEQ to obtain RES
at
i=1~IGRP
and n=1~MMMM [see Eqs. (21),(25)~(27)]
RES
=|E1|/DENOM=
at
, n=1~MMMM [see Eqs. (34),(25)~(27)]
WRITE RES
NSTATY
=2
PUNCH RES
=1
KKK=1.OR.(J< IGRP.AND.SP(1,J+1)=0)
Yes
No
i=NNNNN,...,J-1,J
PUNCH SSPP
i=1,2,...,J
RETURN

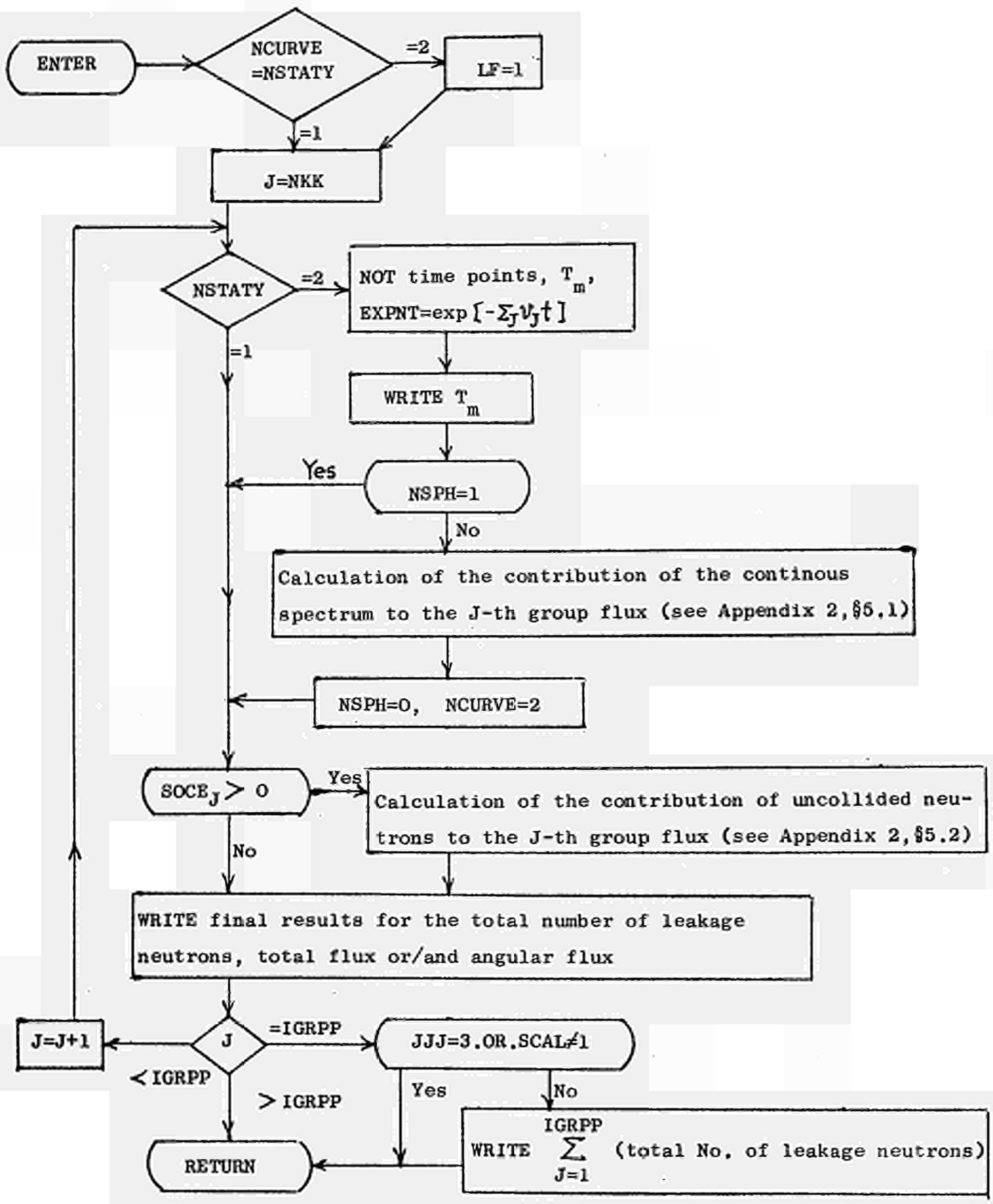
4. FLUXCA


ENTER
MMMM=1
NSTAT1 > 0
Yes
MMMM=3
No
NOS space points, Xm = X/a or r/R
NSTATY
=2
TTm = X/v1 or r/v1
=1
JC=1
NOT time points, Tm
NSPH=0
Yes
NOM angle points, ANGLm = μ
No
JOD=1, III=1
J=NKK
AA= αJ
NSTATY
=2
Tm = SCAL * Tm
=1
NSPH=0
No
Xm =(1+r/R)2
Yes
JOD=2, III=1
NQ, NQQQ
NSTATY
=2
I=JC
=1
JC=J
Calculation of the total number of leakage neutrons, total flux or/an angular flux for the J-th group (see Appendix 2, § 4.1)
Yes
I=I+1 ≤ J
No
NSTATY=2.AND.III < IHL
Yes
III=III+1
No
=1
JOD
=2
NSTATY=1.AND.NSPH=1
No
NPRINT
≠0
=0
Yes
WRITE J
NSPH=0
No
Xm = r/R
Yes
JOD=1, III=1
J=J+1 ≤ IGRPP
Yes
No
RETURN
NSTATY
=2
TFLUX
WRITE X and intermediate results for the flux
=1
NSTAT1 > 0
No
Yes
WRITE the mean time t̄ and variance σ²
by calling VARIAC [see Eqs. (41) and (42)]
RESm = total flux at Xm

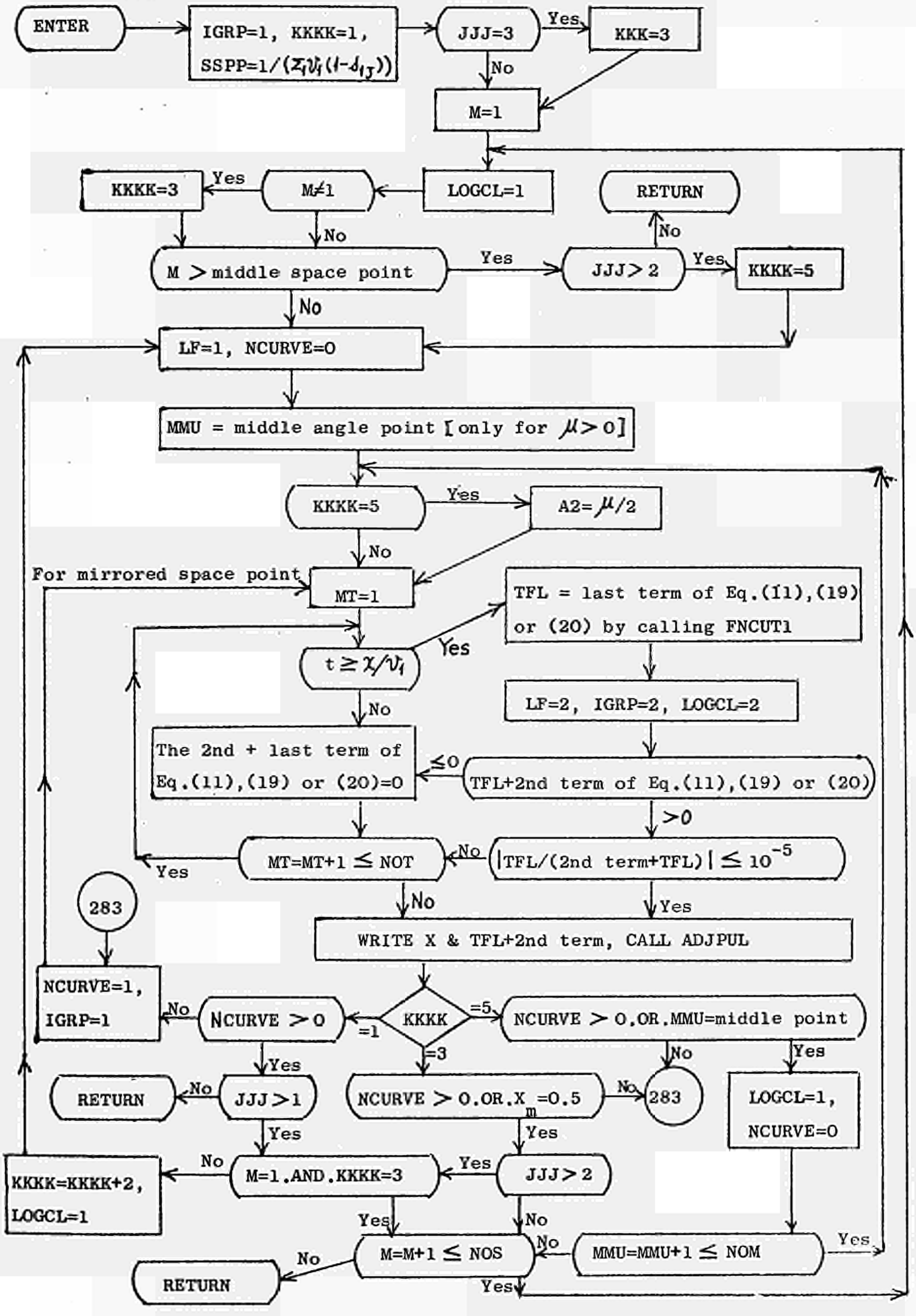
4.1 Calculation of the total number of leakage neutrons, total flux or/and angular flux for the J-th group in FLUXCA



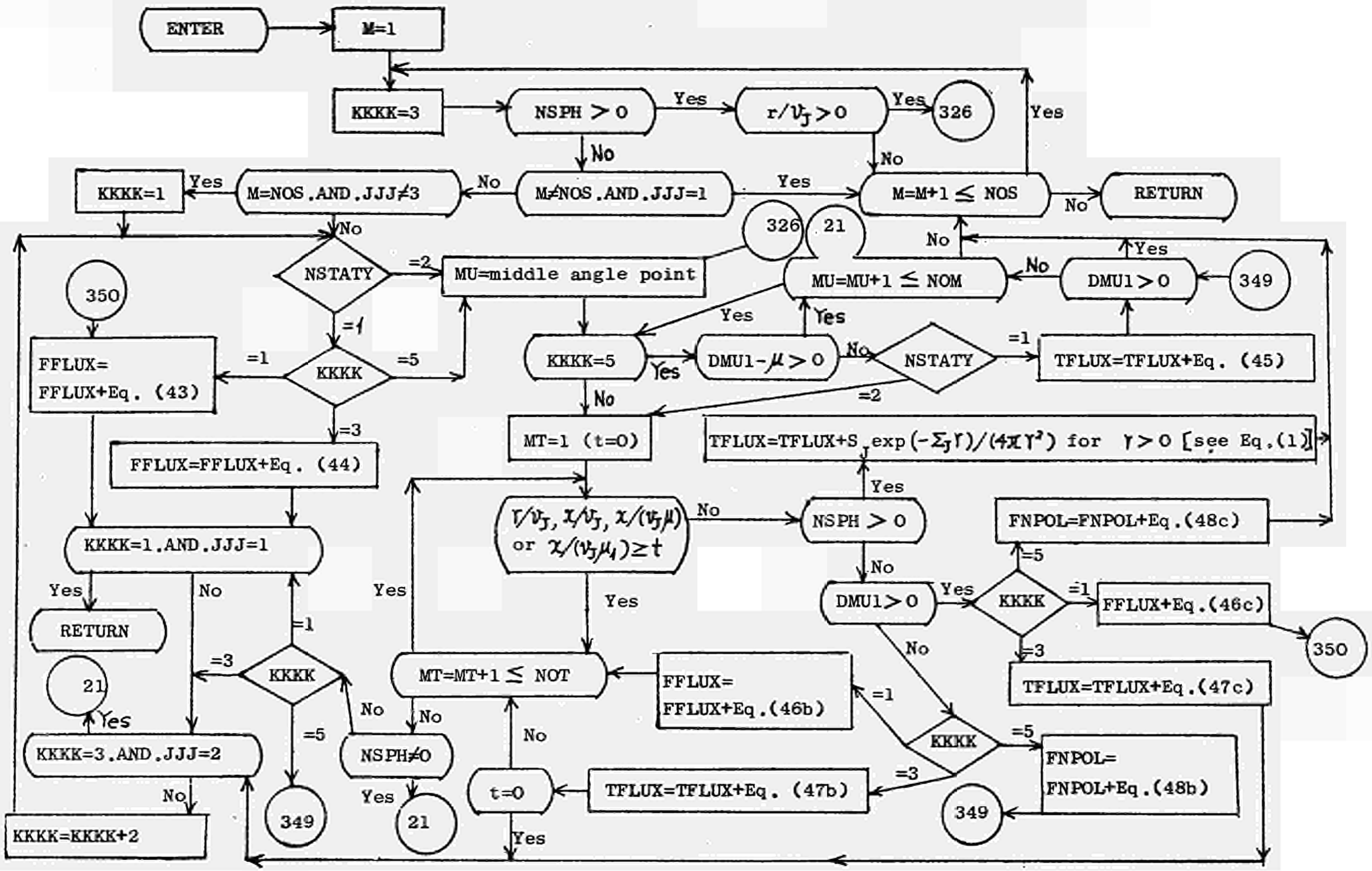
5. INTCAL


ENTER
NCURVE =NSTATY
=2
LF=1
=1
J=NKK
NSTATY
=2
NOT time points, T_m, EXPNT=exp$[-\Sigma_J v_J t]$
=1
WRITE T_m
Yes
NSPH=1
No
Calculation of the contribution of the continous spectrum to the J-th group flux (see Appendix 2,§5.1)
NSPH=0, NCURVE=2
$SOCE_J > 0$
Yes
Calculation of the contribution of uncollided neutrons to the J-th group flux (see Appendix 2,§5.2)
No
WRITE final results for the total number of leakage neutrons, total flux or/and angular flux
J=J+1
J
=IGRPP
JJJ=3.OR.SCAL≠1
< IGRPP
> IGRPP
Yes
No
RETURN
WRITE $\sum_{J=1}^{IGRPP}$ (total No. of leakage neutrons)

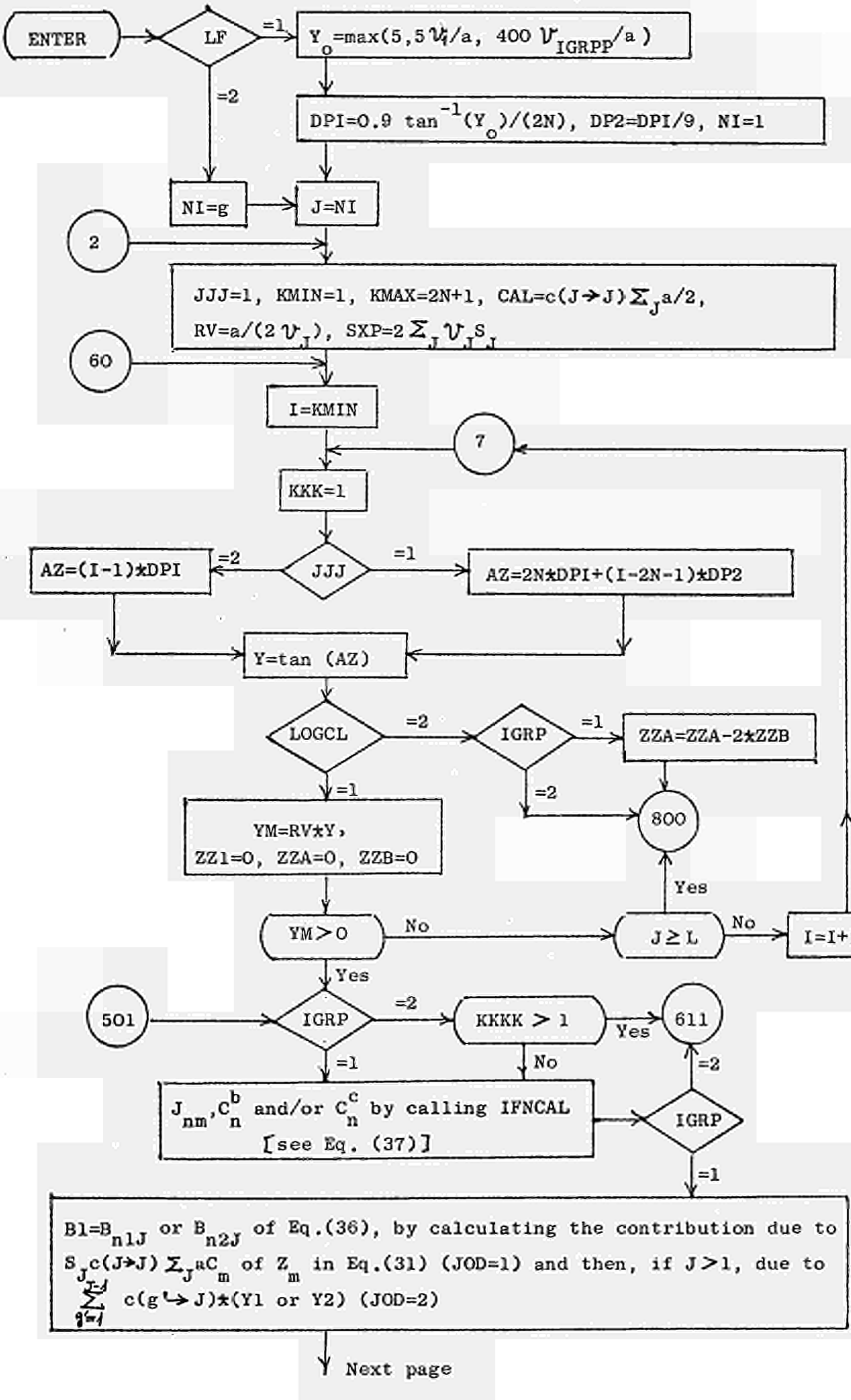
5.1 Calculation of the contribution of the continuous spectrum to the J-th group flux in INTCAL

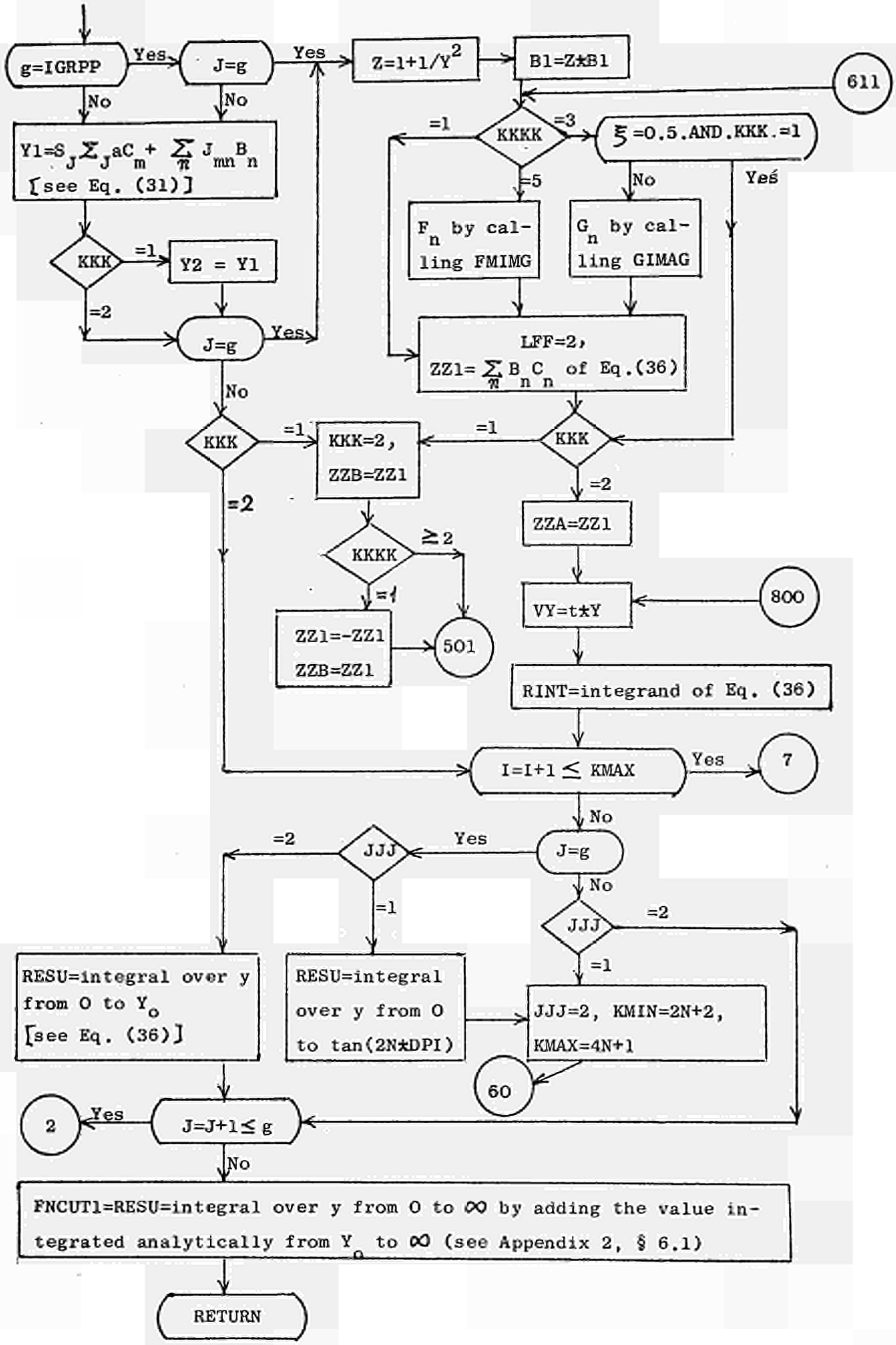


5.2 Calculation of the contribution of uncollided neutrons to the J-th group flux in INTCAL

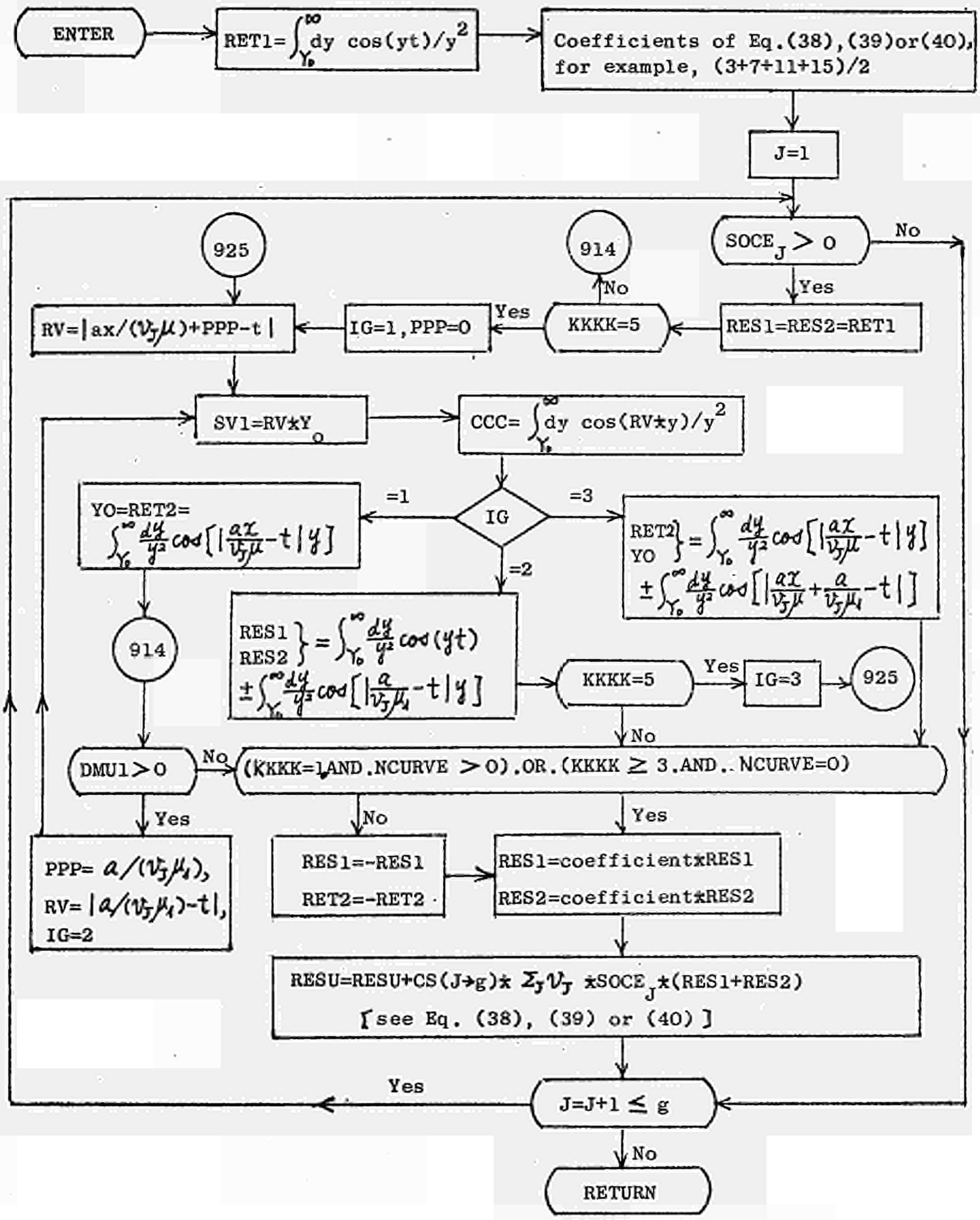


6. FNCUT1 (g,t,ξ,....)

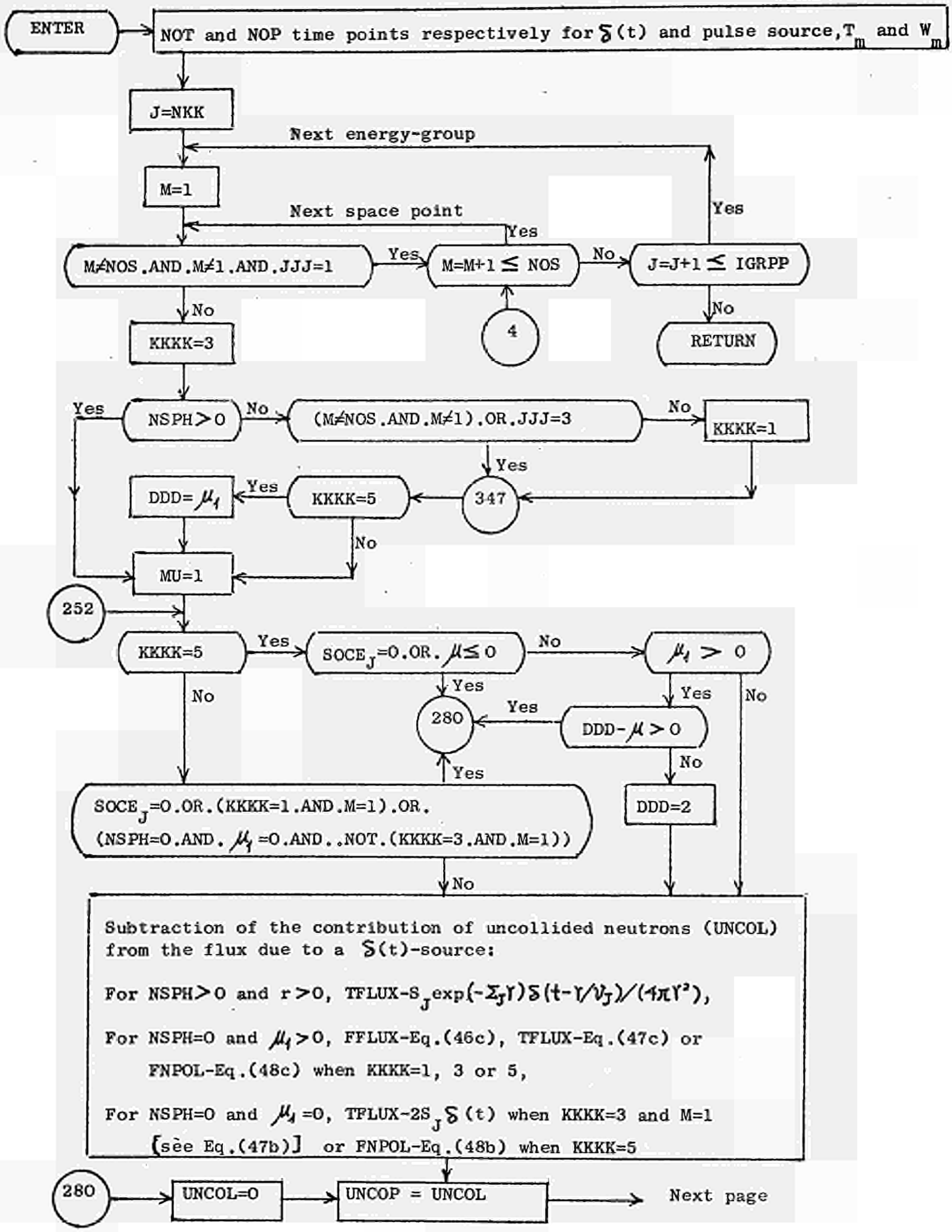

ENTER
LF
=1
=2
Y_o=max(5,5ν_t/a, 400 ν_{IGRPP}/a)
DPI=0.9 $\tan^{-1}(Y_o)$/(2N), DP2=DPI/9, NI=1
NI=g
J=NI
2
JJJ=1, KMIN=1, KMAX=2N+1, CAL=c(J→J) Σ_Ja/2,
RV=a/(2 ν_J), SXP=2 $\Sigma_J \nu_J S_J$
60
I=KMIN
7
KKK=1
JJJ
AZ=(I-1)*DPI
AZ=2N*DPI+(I-2N-1)*DP2
Y=tan (AZ)
LOGCL
IGRP
ZZA=ZZA-2*ZZB
800
YM=RV*Y,
ZZ1=0, ZZA=0, ZZB=0
YM>0
No
Yes
J≥L
I=I+1
501
IGRP
KKKK > 1
611
J_{nm},C_n^b and/or C_n^c by calling IFNCAL
[see Eq. (37)]
IGRP
B1=B_{n1J} or B_{n2J} of Eq.(36), by calculating the contribution due to
S_Jc(J→J) Σ_JaC_m of Z_m in Eq.(31) (JOD=1) and then, if J>1, due to
$\sum_{g'=1}^{J-1}$ c(g'→J)*(Y1 or Y2) (JOD=2)
Next page

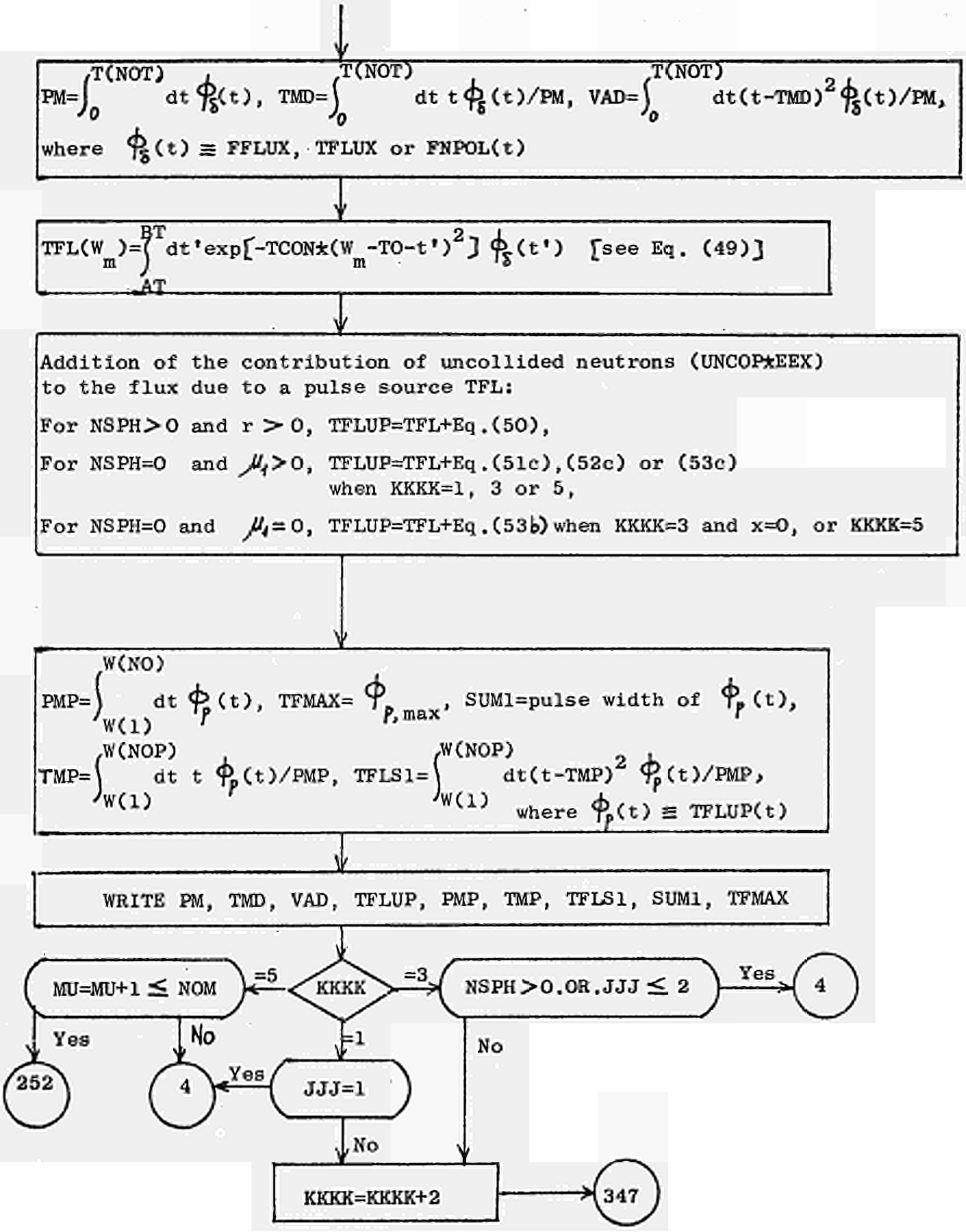

g=IGRPP
Yes
No
J=g
Yes
No
Z=1+1/Y^2
B1=Z*B1
611
Y1=S_J Σ_J aC_m + Σ_π J_mn B_n
[see Eq. (31)]
=1
KKKK
=3
ξ=0.5.AND.KKK.=1
=5
No
Yes
F_n by calling FMIMG
G_n by calling GIMAG
KKK
=1
Y2 = Y1
=2
J=g
Yes
LFF=2,
ZZ1= Σ_π B_n C_n of Eq.(36)
No
KKK
=1
KKK=2,
ZZB=ZZ1
=1
KKK
=2
=2
ZZA=ZZ1
KKKK
≥2
=1
VY=t*Y
800
ZZ1=-ZZ1
ZZB=ZZ1
501
RINT=integrand of Eq. (36)
I=I+1 ≤ KMAX
Yes
7
No
=2
JJJ
Yes
J=g
No
=1
JJJ
=2
=1
RESU=integral over y from 0 to Y_o
[see Eq. (36)]
RESU=integral over y from 0 to tan(2N*DPI)
JJJ=2, KMIN=2N+2,
KMAX=4N+1
60
2
Yes
J=J+1≤ g
No
FNCUT1=RESU=integral over y from 0 to ∞ by adding the value integrated analytically from Y_o to ∞ (see Appendix 2, § 6.1)
RETURN

6.1 Analytical intgration over y from Y_0 to ∞ in FNCUT1 (see the second half of §3.3)



7. PULSE


ENTER
NOT and NOP time points respectively for δ(t) and pulse source, T_m and W_m
J=NKK
Next energy-group
M=1
Next space point
Yes
M≠NOS.AND.M≠1.AND.JJJ=1
Yes
M=M+1 ≤ NOS
No
J=J+1 ≤ IGRPP
Yes
No
4
No
RETURN
KKKK=3
Yes
NSPH > 0
No
(M≠NOS.AND.M≠1).OR.JJJ=3
No
KKKK=1
Yes
DDD=μ1
Yes
KKKK=5
347
No
MU=1
252
KKKK=5
Yes
SOCE_J=0.OR. μ≤0
No
μ1 > 0
Yes
Yes
No
No
280
Yes
DDD-μ > 0
No
Yes
SOCE_J=0.OR.(KKKK=1.AND.M=1).OR.
(NSPH=0.AND. μ1 =0.AND..NOT.(KKKK=3.AND.M=1))
DDD=2
No
Subtraction of the contribution of uncollided neutrons (UNCOL) from the flux due to a δ(t)-source:
For NSPH>0 and r>0, TFLUX-S_J exp(-Σ_J r) δ(t-r/v_J)/(4πr²),
For NSPH=0 and μ1>0, FFLUX-Eq.(46c), TFLUX-Eq.(47c) or FNPOL-Eq.(48c) when KKKK=1, 3 or 5,
For NSPH=0 and μ1=0, TFLUX-2S_J δ(t) when KKKK=3 and M=1 [see Eq.(47b)] or FNPOL-Eq.(48b) when KKKK=5
280
UNCOL=0
UNCOP = UNCOL
Next page


$PM=\int_0^{T(NOT)} dt\, \phi_S(t)$, $TMD=\int_0^{T(NOT)} dt\, t\, \phi_S(t)/PM$, $VAD=\int_0^{T(NOT)} dt (t-TMD)^2 \phi_S(t)/PM$,
where $\phi_S(t) \equiv$ FFLUX, TFLUX or FNPOL(t)
$TFL(W_m)=\int_{AT}^{BT} dt' \exp[-TCON*(W_m-TO-t')^2]\, \phi_S(t')$ [see Eq. (49)]
Addition of the contribution of uncollided neutrons (UNCOP*EEX) to the flux due to a pulse source TFL:
For NSPH$>$0 and r $>$ 0, TFLUP=TFL+Eq.(50),
For NSPH=0 and $\mu_1>0$, TFLUP=TFL+Eq.(51c),(52c) or (53c) when KKKK=1, 3 or 5,
For NSPH=0 and $\mu_1=0$, TFLUP=TFL+Eq.(53b) when KKKK=3 and x=0, or KKKK=5
$PMP=\int_{W(1)}^{W(NO)} dt\, \phi_p(t)$, $TFMAX=\phi_{p,max}$, SUM1=pulse width of $\phi_p(t)$,
$TMP=\int_{W(1)}^{W(NOP)} dt\, t\, \phi_p(t)/PMP$, $TFLS1=\int_{W(1)}^{W(NOP)} dt (t-TMP)^2\, \phi_p(t)/PMP$,
where $\phi_p(t) \equiv$ TFLUP(t)
WRITE PM, TMD, VAD, TFLUP, PMP, TMP, TFLS1, SUM1, TFMAX
MU=MU+1 $\leq$ NOM
=5
KKKK
=3
NSPH$>$0.OR.JJJ $\leq$ 2
Yes
4
Yes
No
No
=1
252
4
Yes
JJJ=1
No
KKKK=KKKK+2
347

1. STATIONARY PROBLEM FOR A SPHERE TO EVALUATE TIME-CONSTANT, K_{eff} AND CRITICAL RADIUS, AND THE FLUX DISTRIBUTION BY THE USE OF A 1 ENERGY-GROUP MODEL AND THE j_7 APPROXIMATION

	1 I 20 I 40 I 60 I 80
ID	TEST CASE 1 1-GROUP,NOS=21
	7 1 1 1 3 4 4 1 2 2 2 1 3 1 21 1 1 1 9
	2.2 +0 .00001 +0
ν	1.
χ	1.
$\nu\Sigma_f$	1.3
XSEC	.5 1. 1. .5
	1.1 1.19 -.1008 -.101 .9437 .9435 .00001 .00001
	1 I 20 I 40 I 60 I 80

2. STATIONARY PROBLEM FOR A SLAB (WITH UP-SCATTERING OF NEUTRONS) TO OBTAIN 7TH GROUP ANGULAR FLUX, TOTAL FLUX AND TOTAL NUMBER OF LEAKAGE NEUTRONS DUE TO A POINT ISOTROPIC BOUNDARY SOURCE BY THE USE OF A 7-GROUP MODEL AND THE j_7 APPROXIMATION (ALSO THE 1ST AND 2ND TIME MOMENTS OF THE FLUXES DUE TO $S(t)$-SOURCE AND THE FUNDAMENTAL DECAY CONSTANT ARE CALCULATED)

```
                          20                      40                      60                      80
1         I         I         I         I         I         I         I         I
```

ID	TEST CASE 2 7-GROUP, NOS=2, NOM=3, JJJJ=4 FOR 7TH GROUP
	7 1 1 1 7 3 4 9 1 1 7 4 1 3 2 1 15 1 1
	9. +0
v_g	285. 171.2 82.24 18.45 2.118 .2402 .024484
$(B_y^2+B_z^2)_g$	.009 .012 .016 .018 .019 .019 .021
S_g	.10757 .36278 .50403 .02559 .00003
XSEC	.00133721.08457866.08457866-.0305552
	.1277501 .1277501 -.0377562.08437803
	.2766694 .2766694 .06970756.14904233.02674421
	.51975166.51975166.21350756.20038489.0161576 .00267442
	.00026137.70096002.70096002.48070889.30493378.00657692.00030642
	.00204551.74581862.74581862.48544689.21846222.00131032
	.01947 2.11599 2.11599 2.09652 .25832622.00152754
	.00004 .000038 .0000001

```
1         I         I         I         I         I         I         I         I
                          20                      40                      60                      80
```

3. TIME-DEPENDENT PROBLEM FOR A SPHERE FOR EVALUATING THE TIME-DEPENDENT TOTAL FLUX DUE TO THE INCIDENCE OF A $\delta(t)$-SOURCE AT THE CENTRE, BY THE USE OF A 1-GROUP MODEL AND THE J_5 APPROXIMATION (USING PREVIOUSLY OBTAINED PUNCHED CARDS FOR THE POLES AND RESIDUES)

```
     1       I      20       I      40       I      60       I      80
ID   TEST CASE 3              1-GROUP,NOS=6,NOT=40
      5  1  1  2  1  3  4  4  1  2  3  1     1    3 40    6  1     20
   { 1.4       +0          1.       -7.07      +01.      +01.      +0
   {
ν     1.

S     1.
XSEC          1.       1.       1.
1-λ1  0.24421241D 01
    { 0.18110943E 01
    { 0.40183169E 00
    {-0.10175411E-01
1-λ2  0.79124205D 01
    {-0.77125874E 01
    {-0.27038589E 02
    { 0.42687244E 01
1-λ3  0.13313983D 02
    { 0.14525379E 04
    { 0.51024375E 04
    { 0.70917461E 04
     1       I      20       I      40       I      60       I      80
```

4. TIME-DEPENDENT PROBLEM FOR A SLAB TO OBTAIN THE TIME-DEPENDENT ANGULAR FLUX, TOTAL FLUX AND THE TOTAL NUMBER OF LEAKAGE NEUTRONS FROM THE SLAB WITH A MONODIRECTIONAL BOUNDARY SOURCE (μ_1=1) OF THE TIME BEHAVIOUR DESCRIBED BY A RECTANGULAR PULSE, BY THE USE OF A 1-GROUP MODEL AND THE j_5 APPROXIMATION (ALSO THE FLUXES DUE TO THE S(t)-SOURCE AND THE FIRST 3 TIME MOMENTS ARE CALCULATED AND COMPARED WITH THE STATIONARY VALUES)

```
                       20                  40                  60                  80
      1       I        I        I          I        I          I        I          I
ID    TEST CASE 4                 1-GROUP,NOS=3,NOM=3,NOT=32,JJJJ=4,N=70,NOP=40
        5  1  1  2  1  3  4  4    1  1   70     1     4 32  3  3  1    40 14  1
     { 10.        +01.        +01.       -72.        +01.       +01.       +0
     {                                    1.         +0          2.       +0
v       1.
(B1²+B2²)
S       1.
XSEC  .1        1.        1.        .9
     { .12744   .1274
     { .36297   .363
1-Δ1 {
     { .21181   .2118
     { .59495   .595
      1       I        I        I          I        I          I        I          I
                       20                  40                  60                  80
```

Table I Locations of the first elements of Real*8 (or•Real*4) arrays stored in the floating COMMON and their dimensions

Location	Array name (dimension)					
IA(71)	ALPHA or AL(IGRP)					
IA(72)	XV(IGRP)					
IA(73)	SP(IA(1),IA(2))					
IA(74)	A(IA(3))					
IA(75)	DELTA or AL1(IA(4))	IA(61)	• FNPOL(IA(6))			
IA(76)	E(IA(5),IA(69))	IA(34)	• TFLUX(IA(7))	when NSTATY=1		
IA(77)	ED or E1(IA(5),IA(69))	IA(36)	• FFLUX(IA(8))			
IA(78)	SS(IGRP)					
IA(61)	• FNPOL(IA(6))					
IA(34)	• TFLUX or TTLUX(IA(7))	when NSTATY=2				
IA(36)	• FFLUX(IA(8))					
IA(79)	C1(IA(9),IA(9))	IA(79)	X(IA(16))	IA(79)	B(IA(11),IA(12),IA(15))	
IA(80)	C2(IA(10),IA(10))	IA(82)	RC(IA(17))	IA(84)	SCNU(IA(13),IA(15))	
		IA(83)	ABB or DELTA(IA(18))			
		IA(85)	SO, AA or DELTA(IA(14))			
		IA(86)	SN(IA(17))			
IA(106)	AL2(IA(20))					
IA(145)	B(IA(20))					
IA(87)	VG(IA(19))	IA(66)	•ZZA(IA(25),IA(26))	IA(66)	• W(IA(68))	
IA(88)	E2(IA(11),IA(11))	IA(91)	•ZZB(IA(25),IA(26))	IA(139)	• TFLUP(IA(68))	
IA(89)	CNU(IA(21),IA(22),IA(15))	IA(92)	•ZZ1(IA(27))			
IA(90)	R(IA(23),IA(12))	IA(93)	•B1(IA(40),IA(25), IA(26))			
IA(95)	SSS or AIN(8)	IA(95)	F1(IA(30))			
IA(96)	FG(IA(29))	IA(101)	F2(IA(31))			
IA(97)	AP(10)	IA(102)	F3(IA(32))			
IA(98)	SG(10)	IA(103)	C(IA(33))			
		IA(104)	F8 or F4 (IA(30))			
		IA(105)	F9 or F6 (IA(35))			
IA(99)	EXPN or VG(10)	IA(99)	A1(IA(33))			
		IA(107)	R(IA(37))			

to be continued

Table I (continued)

Location	Array name (dimension)				
IA(108)	X5, XY or AY (IA(39))				
IA(109)	RC(IA(39))				
IA(110)	F7(IA(40))	IA(110)	ABB(IA(42))	IA(110)	SO(IA(45))
IA(111)	F5(IA(41))	IA(112)	SO(IA(43))		
		IA(113)	SN(IA(44))		
IA(114)	FN1(IA(40))	IA(114)	RS1(IA(38),IA(49),IA(94))		
IA(115)	FN2(IA(47))	IA(144)	AC or AA(IA(28))		
IA(116)	FN3(IA(40))				
IA(117)	FN4(IA(27))				
IA(118)	X11(IA(50))				
IA(119)	F4(IA(51))				
IA(120)	F6(IA(40))				
IA(121)	FN5(IA(53))				
IA(138)	YD1(IA(31))				
IA(122)	• CS(IGRP,IGRP)				
IA(123)	• SOCE(IGRP)				
IA(124)	• RES(IA(54),IA(55),IA(55))	IA(124)	• Y1(IA(50),IA(59),IA(60))		
IA(125)	• SSPP(IA(56),IA(57))				
IA(126)	• AL3(IA(20))	IA(126)	• ANGL(IA(62))		
IA(127)	• AL4(IA(20))	IA(129)	• EXPNT(IA(63))		
IA(128)	• EX(IA(20))	IA(130)	• T(IA(64))		
		IA(131)	• TT(IA(65))		
		IA(132)	• R(IA(17))		
IA(133)	• CF(IA(9),IA(9)	IA(133)	• Y2(IA(50),IA(59),IA(60))		
IA(134)	• C(IA(9),IA(9))				
IA(135)	• BUCLG(IGRP)				
IA(136)	• VG(IGRP)				
IA(137)	• XSEC(IHL,IGRP)				

Table II Computed integers for specifying the array dimensions [JHL ≡ (IIO+1)/2, NNNN ≡ NNN+1, IGRPP≡ JNKK+NNN and NOSMN≡ NOS*NOM*
*NNNN]

NSTATY	1					2		
NSPH	1	0				1	0	
NSTAT1	0	0		1		0	0	1
NUPSAT	0	0	1	0	1	0	0	0
IA(1)	2	JHL+3		Max(JHL+3,6)		IIO+3		
IA(2)	2	Max(IGRP,2)		Max(IGRP,2)		Max(IGRP,2)		
IA(3)	0	0		11		11		
IA(4)	Max(IGRP,10)							
IA(5)	JHL*IGRP	JHL	JHL*IGRP	JHL	JHL*IGRP	JHL		
IA(6)	0					0	• NOSMN*NOT (for JJJJ=3 or 4)	• NOSMN*(NOT+1) (for JJJJ=3 or 4)
IA(7)	• NOS*NNNN	NOSMN (for JJJJ=3or4)		3*NOSMN (for JJJJ=3 or 4)		• NOS*NNNN*NOT (for JJJJ=2,3 or 4)		• Max(NOS*NNNN*(NOT+1), 3*NOSMN) (for JJJJ=3 or 4) • NOS*NNNN*(NOT+1) (JJJJ=2)
IA(8)	0	2*NNNN (for JJJJ=1), NOS*NNNN (JJJJ=3), (NOS+2)*NNNN(JJJJ=2 or 4)		6*NNNN (for JJJJ=1) 3*NOS*NNNN (JJJJ=3) 3*(NOS+2)*NNNN(JJJJ=2 or 4)		0	• 2*NOT*(NNN+2) (JJJJ=1,2or 4)	• Max(2*NOT*(NNN+2)+2*NNNN, 6*NNNN) (for JJJJ=1) • 3*NOS*NNNN (for JJJJ=3), • Max(2*NOT*(NNN+2)+2*NNNN, 3*(NOS+2)*NNNN) (for JJJJ=2 or 4)

(To be continued)

Table II (Continued)

	NSTATY=1		NSTATY=2	
	NSPH=1	NSPH=0	NSPH=1	NSPH=0
IA(9)	IGRP	0	0	
IA(10)	IGRP (for N=2)	0	0	
IA(11)	0	(• JHL (IGRP>1 and NUPSAT=0)	• JHL (for IGRP>1)	
IA(12)	0	• IGRP-1	• IGRP-1	
IA(13)	0	JHL	JHL	
IA(14)	• 12	• 12	• 12	
IA(15)	1	1 (for NSTAT1=0) 3 (for NSTAT1=1)	1	1 (for NSTAT1=0) 3 (for NSTAT1=1)
IA(16)	• NOS			
IA(17)	• 4			
IA(18)	11			
IA(19)	0	4 (for NSTAT1=0) 10 (for NSTAT1=1)	• 10	
IA(20)	• IGRP	0	0	
IA(21)	0	• JHL	• JHL	
IA(22)	0	• IGRP	• IGRP	
IA(23)	0	• 10 (for IGRP>1)	• 10 (for IGRP>1)	
IA(24)⊗	0	0	0	• 4
IA(25)	0	0	0	• 2
IA(26)	0	0	0	• 4*N+1
IA(27)	0	0	0	• 6
IA(28)	0	20 (for NSTAT1≠1)	0	20 (for NSTAT1=1)
IA(29)	0	10	0	10
IA(30)	0			• 20
IA(31)	0			• 10
IA(32)	0			• 12
IA(33)	0			• JHL
IA(35)	0			• 8
IA(37)	0			• IIO-1
IA(38)	0	10 (for NSTAT1=1)	0	10 (for NSTAT1≠1)

(To be continued)

Table II (Continued)

<table>
<tr><th rowspan="2"></th><th colspan="2">NSTATY=1</th><th colspan="2">NSTATY=2</th></tr>
<tr><th>NSPH=1</th><th>NSPH=0</th><th>NSPH=1</th><th>NSPH=0</th></tr>
<tr><td>IA(39)</td><td colspan="3">0</td><td>• 8 (for JJJJ=2,3 or 4)</td></tr>
<tr><td>IA(40)</td><td colspan="3">0</td><td>• IIO+1</td></tr>
<tr><td>IA(41)</td><td colspan="3">0</td><td>• 16 (for IIO ≥ 5)</td></tr>
<tr><td>IA(42)</td><td colspan="3">0</td><td>• 3</td></tr>
<tr><td>IA(43)</td><td colspan="3">0</td><td>• 4 (JJJJ=2,3 or 4)</td></tr>
<tr><td>IA(44)</td><td colspan="3">0</td><td>• 2</td></tr>
<tr><td>IA(45)</td><td colspan="3">0</td><td>• 7 (for JJJJ= 3 or 4)</td></tr>
<tr><td>IA(47)</td><td colspan="3">0</td><td>• 6 (for IIO ≥ 5)</td></tr>
<tr><td>IA(49)</td><td>0</td><td>IGRP (for NSTAT1=1)</td><td>0</td><td>IGRP (for NSTAT=1)</td></tr>
<tr><td>IA(50)</td><td colspan="3">0</td><td>• IIO+1 (for IGRPP > 1)</td></tr>
<tr><td>IA(51)</td><td colspan="3">0</td><td>• IIO+1 (for IIO ≥ 5)</td></tr>
<tr><td>IA(53)</td><td colspan="3">0</td><td>• 8 (for IIO=7)</td></tr>
<tr><td>IA(54)</td><td>$\mathrm{Max}\begin{pmatrix}(JHL+1)*JHL, \\ NOS\end{pmatrix}$</td><td>(JHL+1)*JHL(NSTAT1=0),
Max(2*(JHL)^2,6*JHL)
(for NSTAT1=1)</td><td>2*(JHL)^2</td><td>• 2*(JHL)^2 (NSTAT1=0),
• Max(2*(JHL)^2,6*JHL)
(for NSTAT1=1)</td></tr>
<tr><td>IA(55)</td><td colspan="4">• IGRP</td></tr>
<tr><td>IA(56)</td><td>0</td><td>1</td><td>0</td><td>• JHL</td></tr>
<tr><td>IA(57)</td><td>0</td><td>• IGRP</td><td>0</td><td>• IGRP</td></tr>
<tr><td>IA(59)</td><td>0</td><td>0</td><td>0</td><td>• IGRPP-1</td></tr>
<tr><td>IA(60)</td><td>0</td><td>0</td><td>0</td><td>• 4*N+1 (if IGRPP > 1)</td></tr>
<tr><td>IA(62)</td><td>0</td><td>• NOM</td><td>0</td><td>• NOM</td></tr>
<tr><td>IA(63)</td><td colspan="2">1</td><td colspan="2">• NOT</td></tr>
<tr><td>IA(64)</td><td colspan="2">0</td><td colspan="2">• NOT</td></tr>
<tr><td>IA(65)</td><td colspan="2">0</td><td colspan="2">• NOS</td></tr>
<tr><td>IA(67)▲</td><td>0</td><td>NOS*NNNN (JJJJ=2 or 4 and NSTAT1=0),
3*NOS*NNNN(JJJJ=2 or 4 and NSTAT1=1)</td><td>0</td><td>3*NOS*NNNN(JJJJ=2 or 4 and NSTAT1=1)</td></tr>
<tr><td>IA(68)</td><td>0</td><td>0</td><td colspan="2">NFLUXK (if TPINT > 0)</td></tr>
<tr><td>IA(69)</td><td>JHL*IGRP</td><td>JHL (for NUPSAT=0),
JHL*IGRP+2
(for NUPSAT=1)</td><td colspan="2">JHL</td></tr>
<tr><td>IA(94)</td><td>0</td><td>2 (for NSTAT1=1)</td><td>0</td><td>2 (for NSTAT1=1)</td></tr>
</table>

• Only if LLL=0 (or NFLUXR, NFLUXS or NFLUXK > 0 for NSTATY=1 and NSPH=1)

⊛ Dimension for the integer array II at the location IA(140)

▲ Dimension for the stationary total flux which is included in IA(8)

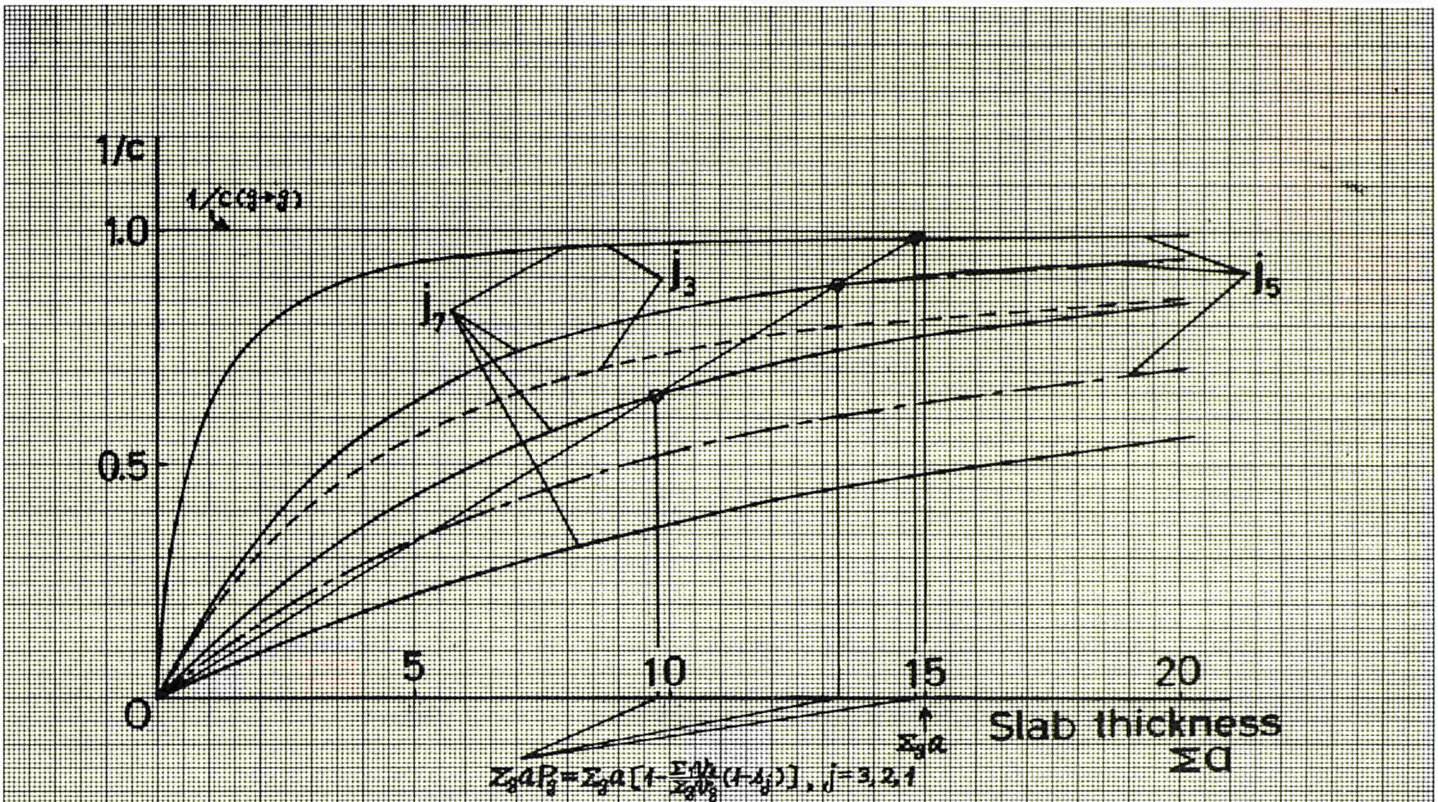


Fig. 1 Numerical values of 1/c as a function of slab thickness Σa in the j_3, j_5 and j_7 approximation, and an illustration how to find poles $\Sigma_i v_i S_j$ in the j_7 appr. for the g-th group [$c(g \to g)=1$ & $\Sigma_g a=15$]

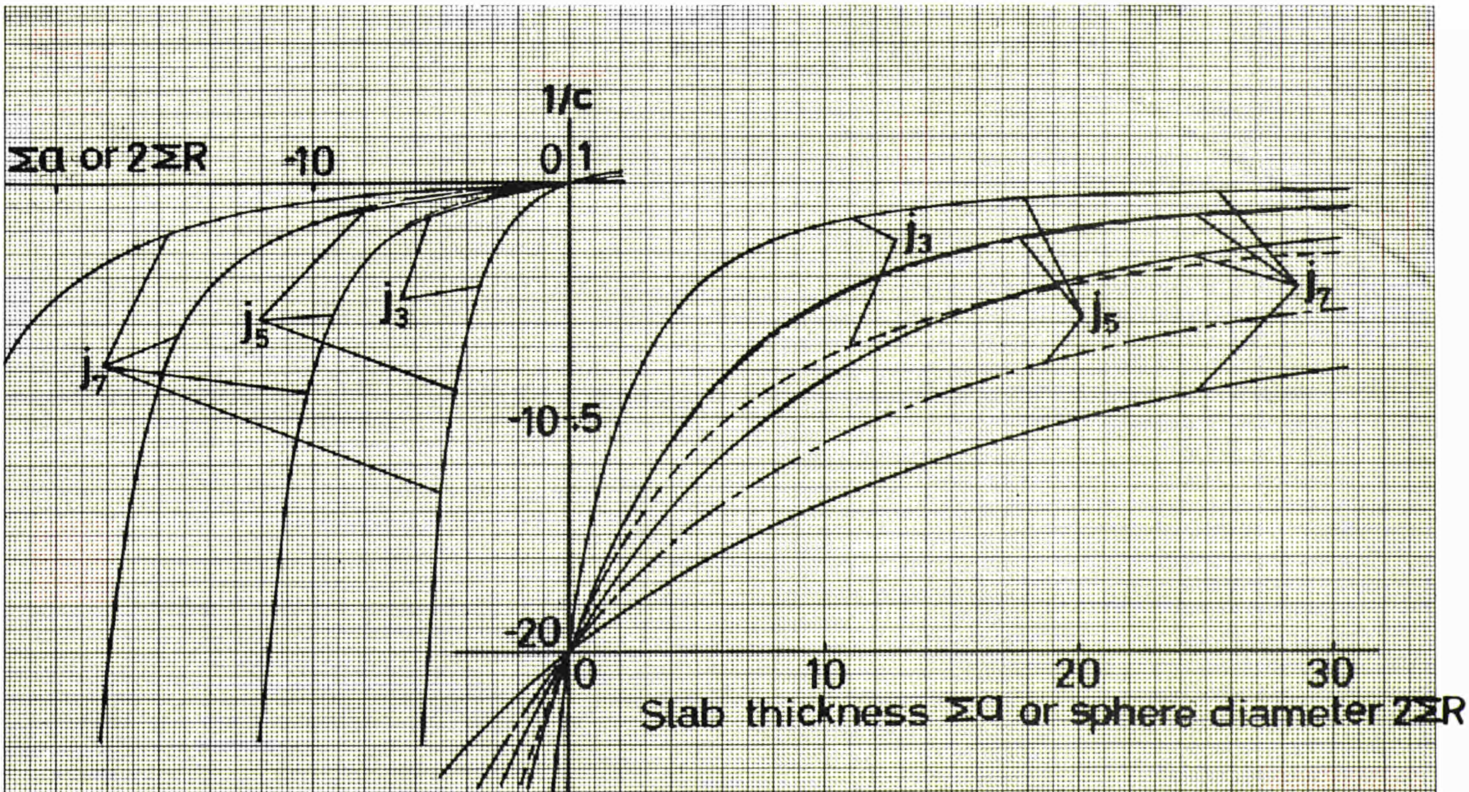


Fig. 2 Values of 1/c as a function of slab thickness Σa or sphere diameter $2\Sigma R$ in the j_3, j_5 and j_7 approximation

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Alfred Nobel

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