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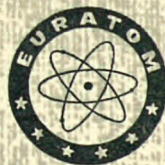
COMMISSION OF THE EUROPEAN COMMUNITIES

A THERMO-HYDRODYNAMIC MODEL FOR THE  
THEORETICAL STUDY OF THE BOILING AND  
EJECTION OF THE COOLANT FROM A REACTOR  
CHANNEL DUE TO A DIRECT CONTACT  
WITH HOT MOLTEN FUEL

by

J. RANGLES

1971



Joint Nuclear Research Centre  
Ispra Establishment - Italy

Reactor Physics Department  
Reactor Theory and Analysis



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## ABSTRACT

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A simple numerical method of solving the complete set of equations jointly is proposed and has been embodied in a new computer programme TOMOF (thermohydrodynamics of a mixture of two fluids). This programme has been tested with a wide range of data and has proved that the chosen numerical method is effective.

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## KEYWORDS

THERMODYNAMICS  
FLOW MODELS  
BOILING  
COOLANT LOOPS  
FUELS  
PRESSURE

VAPORS  
PROGRAMMING  
URANIUM DIOXIDE  
NATRIUM  
PEAKS

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Introduction \*)

Ever since the famous SPERT experiment in which the fuel in a water reactor assembly was allowed to melt, mix with the coolant and cause a destructive vapour explosion, the problem of direct contact fuel/coolant accidents has been "in the air".

Such accidents are of particular current interest in the LMFBR field, since even the experts are not yet entirely happy about the idiosyncracies of liquid sodium cooling and, in the absence of more detailed knowledge, it is reasonable to assume pessimistically, that the direct contact accident is a concrete, if small, possibility. This assumption is encouraged also by the well known positive coolant void coefficient of reactivity in the central region of the LMFBR which may be capable of amplifying a direct contact accident into a dangerous nuclear excursion.

Many experiments are underway around the world, Ispra included, to directly observe the effects of introducing molten  $UO_2$  into a channel filled with liquid sodium. By this means the possibility is afforded both of observing the hazards produced (pressure pulses, voiding, etc.) with the materials and dimensions existing in the real system, and of providing the data needed for a theoretical understanding of the phenomenon. The purpose of the present paper is directed entirely towards the latter aspect: the development of an appropriate thermo-hydrodynamic model.

In formulating this model, we do not restrict ourselves to particular "fuels" or particular "coolants", but consider only the essential features of a "direct contact" situation. These features are summarized by the following chronological series of events.

1. A dense liquid (fuel) at high temperature  $T_{fo}$  is dispersed in globules into a certain region (interacting zone) of a channel filled with a light liquid (coolant) whose normal boiling temperature  $T_b$  is less than  $T_{fo}$ .
2. The heat transferred from the fuel causes the coolant in the interacting zone to boil.
3. The vapour pressure so generated leads to the ejection of the unboiled coolant in the remainder of the channel.

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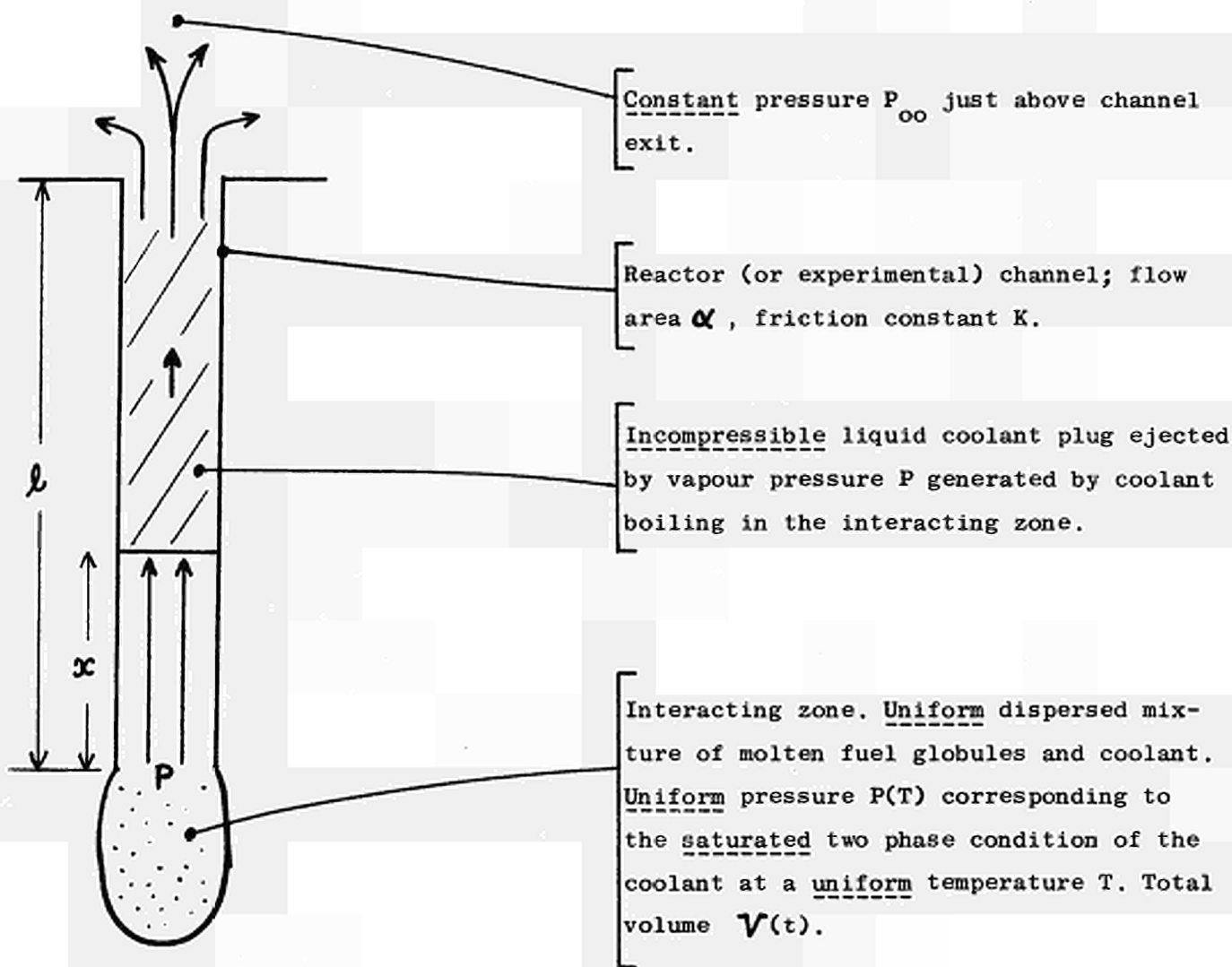
\*) Manuscript received on 21 October 1970

The accompanying diagram will serve to clarify these events and highlight some of the assumptions made to facilitate their mathematical description. These assumptions are indicated by the broken underlining of certain key words. Also provided below is a list of the more important physical symbols used in the theory and an indication of some further assumptions made.

The work presented in this article draws on many references, but the report of DUFFEY (1) is by far the most important.



Configuration employed as the basis for the mathematical description of the coolant boiling and ejection process; assumptions indicated by broken underlining:



Glossary of the more important physical quantities and the assumptions made about them:

Interacting Zone

1) Molten Fuel Globules

Total mass	$m_f$	
Initial temp.	$T_{fo}$	(assumed uniform)
<u>Mean</u> temp. at time t	$T_f(t)$	
<u>Mean</u> linear size of globules	$d_f$	
Specific heat (const.press.)	$C_p^f$	(assumed constant)
Density	$\rho_f$	(assumed constant)
Thermal conductivity	$k$	(assumed constant)
Thermal diffusivity	$\kappa$	(assumed constant)
Boundary layer resistance	$h$	(assumed constant)

2) Coolant in the Liquid Phase

Total mass	$m_l(t)$	
Initial temp.	$T_o$	(assumed uniform)
Temp. at time t	$T(t)$	(assumed uniform)
Specific heat (const.press.)	$C_p^l$	(assumed constant)
Density	$\rho_l$	(assumed constant)

3) Coolant in the Vapour Phase

Total mass	$m_v(t)$	(assumed a perfect gas)
Temp. at time t	$T(t)$	(assumed uniform)
Specific heat (const.press.)	$C_p^v$	(assumed constant)
Latent heat of evaporation	$L$	(assumed constant)
Density	$\rho_v$	
Pressure	$P(T)$	(assumed uniform)

Channel

Flow area	$\alpha$	(assumed constant)
Friction press. drop coef.	$K$	(assumed constant)



1. Mass Conservation and Kinematic Relations

We imagine that at  $t = 0$ , the interacting zone contains only fuel and coolant in the liquid state, the respective temperatures being  $T_{fo}$  and  $T_o$ . The initial volume of the interacting zone is thus

$$V_o = \frac{m_f}{\rho_f} + \frac{m}{\rho_l} \quad (1.1)$$

where  $m$  is the total mass of coolant mixed with the fuel and the dependence of the densities on temperature and pressure will be ignored.

With time, however, heat flows from the fuel to the coolant and eventually (after some time  $t_o$ ) the coolant begins to boil. If  $m_v$  is the mass of vapour and  $m_l$  the mass of liquid coolant, then, by the principle of mass conservation we must have

$$m_l + m_v = m \quad (1.2)$$

Because of this vapour formation, the volume of the interacting region increases from  $V_o$  to

$$\begin{aligned} V &= \frac{m_f}{\rho_f} + \frac{m_l}{\rho_l} + \frac{m_v}{\rho_v} \\ &= \frac{m_f}{\rho_f} + \frac{m}{\rho_l} + \frac{m_v}{\rho_v} \left(1 - \frac{\rho_v}{\rho_l}\right) \end{aligned}$$

Since we shall be concerned with pressures and temperatures fairly well below the critical point, we shall have  $\rho_v \ll \rho_l$  and

$$V = V_o + \frac{m_v}{\rho_v} \quad (1.3)$$

On the other hand, the expansion of the interacting region implies the displacement of the coolant plug through a distance  $x$  given by

$$\alpha x = V - V_0$$

where  $\alpha$  is the flow area of the plug. Thus, from equation (1.3) we get

$$m_v = \alpha \rho_v x \tag{1.4}$$

## 2. Pressure in the Interacting Zone

The vapour pressure  $P$  generated by the coolant must satisfy (assuming saturated conditions) the Clapeyron equation:

$$\frac{dP}{dT} = \frac{L}{T(v_v - v_l)} \tag{2.1}$$

where  $L$ ,  $v_v$  ( $=1/\rho_v$ ) and  $v_l$  ( $=1/\rho_l$ ) are the latent heat and specific volumes of the vapour and liquid phases of the coolant respectively. We shall be interested only in pressures and temperatures low enough (viz.  $P \lesssim 100$  atm;  $T \lesssim 2100$  °K) to allow the assumptions

$$L = \text{constant} \tag{2.2}$$

and

$$v_v = \frac{RT}{P} \gg v_l \tag{2.3}$$

the latter equation being that of an ideal gas free of chemical reactions. With these approximations, (2.1) becomes



$$\frac{dP}{dT} = \frac{LP}{RT^2} \quad (2.4)$$

which integrates immediately to give the saturated (P,T) law:

$$\begin{aligned}
 &P = A e^{-B/T} \\
 \text{with} &A = P_a e^{L/RT_b} \\
 \text{and} &B = \frac{L}{R}
 \end{aligned}
 \quad (2.5)$$

where  $P_a$  ( $= 1.0133 \times 10^6 \text{ dyn/cm}^2$ ) is atmospheric pressure,  $T_b$  the normal boiling temperature of the coolant and the gas constant  $R$  for the coolant is given by

$$R = \frac{8.3147 \times 10^7}{M} \text{ erg/gm}^\circ\text{K} \quad (2.6)$$

$M$  being the molecular mass. For sodium,  $M = 23$  and  $R = 3.6151 \times 10^6 \text{ erg/gm}^\circ\text{K}$ . The above theory ignores all chemical effects such as dissociation or association and thus, for example, fails to account for the dimerization which occurs in saturated sodium vapour.

### 3. Heat Transfer and Coolant Boiling in the Interacting Zone

As described in section 1, the flow of heat from the mass  $m_f$  of fuel into the mass  $m$  of coolant causes the latter to boil and the plug of coolant above the interacting zone to be ejected. The heat exchanges within the mixture during this process are as follows:

$dQ_f$	$= m_f(dE_f + Pdv_f)$	to the fuel
$dQ_l$	$= m_l(dE_l + Pdv_l)$	to the liquid phase of the coolant
$dQ_v$	$= m_v(dE_v + Pdv_v)$	" " vapour " " " "
$dQ_{l \rightarrow v}$	$= Ldm_v$	" " production of new vapour

Since the process will be somewhat fast, we can ignore the heat transfer to the mixture as a whole and write

$$dQ_f + dQ_l + dQ_v + dQ_{l \rightarrow v} = 0. \quad (3.1)$$

In the above expressions, the E's denote specific internal energies. Ignoring, as before, the variations in the densities of the liquids, equation (3.1) gives

$$m_f dE_f + m_l dE_l + m_v (dE_v + P dv_v) + L dm_v = 0. \quad (3.2)$$

Now, the general thermodynamic relation for changes of specific internal energy as a function of P and T is

$$dE = \left[ C_p - P \left( \frac{\partial v}{\partial T} \right)_P \right] dT - \left[ T \left( \frac{\partial v}{\partial T} \right)_P + P \left( \frac{\partial v}{\partial P} \right)_T \right] dP \quad (3.3)$$

so that for the liquid phases, we get (again ignoring the volume changes):

$$dE_f = C_p^f dT_f$$

and

$$dE_l = C_p^l dT$$

For the vapour phase, we see from (2.3) that

and

$$\left. \begin{aligned} \left( \frac{\partial v_v}{\partial T} \right)_P &= \frac{R}{P} \\ \left( \frac{\partial v_v}{\partial P} \right)_T &= -\frac{RT}{P^2} \end{aligned} \right\} \quad (3.4)$$



from which it follows that the second term of (3.3) vanishes identically and

$$dE_v = (C_p^v - R) dT$$

Furthermore, formulae (3.4) indicate that

$$P dv_v = R dT - \frac{RT}{P} dP$$

and if  $dP$  is eliminated by using (2.4), this gives

$$P dv_v = \left( R - \frac{L}{T} \right) dT$$

Assembling these expressions into equation (3.2) and eliminating  $m_e$  by means of (1.2), we get finally

$$m_f C_p^f dT_f + \left\{ m C_p^l + m_v \left( C_p^v - C_p^l - \frac{L}{T} \right) \right\} dT + L dm_v = 0. \quad (3.5)$$

This equation represents an overall condition on the internal heat exchanges within the mixture, namely, that there is no heat transfer to/from the environment. It does not describe the heat exchange process itself which requires a quite separate analysis based on the heat diffusion equation for the fuel globules and the surrounding coolant and the conditions which exist at the boundary between them. Such an analysis is given in the appendix where it is assumed that the fuel globules are spherical, with a diameter equal to the mean size  $d_f$  which occurs in practice. In addition, because of the turbulence induced by vapour formation in the coolant and the tendency of vapour blanketing to occur on hot surfaces (combined perhaps with a high coolant thermal conductivity) it is assumed that the only significant spatial temperature variation in the coolant occurs in a purely resistive skin layer next to the globules. On the basis of this picture, it is shown in the appendix that the mean fuel temperature  $T_f$  and (uniform) coolant temperature  $T$  are approximately related by the equation

$$\tau \dot{T}_f + T_f = T \quad (3.6)$$

where the time constant  $\tau$  embodies the hold-up of heat in the fuel due to its heat capacity and thermal conductivity and the thermal resistance at the fuel/coolant interface. Denoting the latter by  $h$ , and the mean volume/surface ratio of the globules by  $\sigma$  ( $=d_f/6$ ), it is shown that  $\tau$  is given by

$$\tau = \sigma \rho_f C_p^f \left( h + \frac{3}{5} \frac{\sigma}{k} \right) \quad (3.7)$$

the condition for the validity of (3.6) and (3.7) being

$$V = \frac{hk}{3\sigma} \gtrsim 1 \quad (3.8)$$

which implies that the effect of the fuel thermal conductivity (second term in (3.7)) must never exceed 20% of the effect of the boundary thermal resistance (first term). To compute orders of magnitude we can assume  $k \sim 0.1$  watt/cm<sup>o</sup>K,  $h \sim 10^o$ K/watt cm<sup>-2</sup>,  $\sigma \sim 0.1$  cm ( $d_f \sim 0.6$  cm),  $\rho_f \sim 10$  gm/cm<sup>3</sup> and  $C_p^f \sim 0.3$  J/gm<sup>o</sup>K. Then

$$V \sim 3$$

in support of the validity of the approximations (3.6) and (3.7), and

$$\tau \sim 3 \text{ sec}$$

#### 4. Equation of Motion of the Coolant Plug

The assumptions made in order to simplify the derivation of an equation of motion for the coolant plug are: (a) the coolant plug is incompressible and (b) the pressure  $P_{oo}$  a little beyond the channel exit is constant. Assumption (a) is valid only so long as the speed of the coolant plug is

well below the speed of sound in the liquid phase of the coolant. Since there is a pressure loss of about  $\rho_l \dot{x}^2$  as the coolant bursts from the channel, it follows from assumption (b) that the pressure at the exit is  $P_{oo} + \rho_l \dot{x}^2$ . Thus we can write the following statements:

$$\begin{aligned} \text{mass of plug} &= \rho_l \alpha (l-x) ; \\ \text{force on it due to pressure} &= (P - P_{oo} - \rho_l \dot{x}^2) \alpha ; \\ \text{" " " " " friction} &= -\rho_l \alpha (l-x) K \dot{x}^2 ; \\ \text{" " " " " gravity} &= -\rho_l \alpha (l-x) g ; \end{aligned}$$

where, for example

$$g = \begin{cases} + 980.66 \text{ cm/sec}^2 & \text{if the coolant is ejected upwards} \\ 0 & \text{" " " " " " " horizontally} \\ - 980.66 & \text{" " " " " " " downwards} \end{cases} \quad (4.1)$$

and K is a constant giving the pressure gradient due to frictional drag:

$$\left( \frac{\partial P}{\partial x} \right)_{\text{Friction}} = -\rho_l K \dot{x}^2 \quad (4.2)$$

Combining the above forces into Newton's second law of motion, we get

$$\ddot{x} = \frac{P - P_{oo} - \rho_l \dot{x}^2}{\rho_l (l-x)} - K \dot{x}^2 - g \quad (4.3)$$



### 5. Reduction of the Equations

With the derivation of the above equation of motion, the problem is completely defined. We have the five unknowns  $m_v$ ,  $P$ ,  $T_f$ ,  $T$  and  $x$  and five equations (1.4), (2.5), (3.5), (3.6) and (4.3). Because of the simplicity of the first two of these, however, it is easy to reduce the problem essentially to a set of three equations in the variables  $T_f$ ,  $T$  and  $x$ .

Eliminating  $\rho_v$  from (1.4) by means of equation (2.3), we have

$$m_v = \frac{\alpha P x}{RT} \quad (5.1)$$

of which the differential is

$$dm_v = \frac{\alpha}{R} \left\{ \frac{x}{T} dP - \frac{Px}{T^2} dT + \frac{P}{T} dx \right\}.$$

From (2.4) or (2.5), we have

$$dP = \frac{BP}{T^2} dT$$

which reduces the above expression to a function of  $T$  and  $x$ :

$$dm_v = \frac{\alpha P}{RT} \left\{ \frac{B-T}{T^2} x dT + dx \right\} \quad (5.2)$$

Substituting expressions (5.1) and (5.2) into equation (3.5), using  $B = L/R$  from (2.5) and rearranging slightly, we then obtain

$$dT_f + \left\{ \beta \frac{C_P^l}{C_P^g} + x F(T) \right\} dT + G(T) dx = 0 \quad (5.3)$$

where

$$\beta = \frac{m}{m_f} \quad (5.4)$$

is the mass of coolant per unit mass of fuel in the interacting zone,  
and

$$F(T) = \frac{\alpha}{m_f C_P^f R} \frac{P(T)}{T} \left( C_P^v - C_P^l - \frac{2L}{T} + \frac{L^2}{RT^2} \right) \quad (5.5)$$

and

$$G(T) = \frac{\alpha L}{m_f C_P^f R} \frac{P(T)}{T} \quad (5.6)$$

are functions only of  $T$ ,  $P(T)$  being given by (2.5). Equations (3.6), (4.3) and (5.3) now form the above mentioned trio for the evaluation of  $T_f$ ,  $T$  and  $x$ .

#### 6. Solution of the Equations before the Onset of Coolant Evaporation

During a certain initial time interval  $0 \leq t \leq t_0$  the heat flowing into the coolant merely causes its temperature to rise, the latter being too low for boiling to occur. During this interval, the vapour mass  $m_v$  is zero and hence by (1.4) or (5.1)

$$x = 0 \quad \text{for} \quad 0 \leq t \leq t_0 \quad (6.1)$$

With this condition, equation (4.3) is irrelevant, (3.6) of course remains unchanged:

$$\tau \dot{T}_f + T_f = T \quad (6.2)$$

and equation (5.3) reduces to

$$dT_f + \beta \frac{C_P^l}{C_P^f} dT = 0 \quad (6.3)$$

The solutions to equations (6.2) and (6.3), with the boundary conditions

$$\left. \begin{array}{l} T = T_o \\ T_f = T_{fo} \end{array} \right\} \text{ at } t = 0, \quad (6.4)$$

are

$$T = T_m - (T_m - T_o) e^{-t/\tau_1} \quad (6.5)$$

and

$$T_f = T_m + (T_{fo} - T_m) e^{-t/\tau_1} \quad (6.6)$$

where

$$T_m = \frac{T_{fo} + \beta \frac{C_p^l}{C_p^f} T_o}{1 + \beta \frac{C_p^l}{C_p^f}} \quad (6.7)$$

is the final (asymptotic) temperature which both the fuel and coolant would reach in the absence of boiling and

$$\tau_1 = \frac{\beta \frac{C_p^l}{C_p^f} \tau}{1 + \beta \frac{C_p^l}{C_p^f}} \quad (6.8)$$

is the hold-up time constant of the mixture.

The above solutions are only valid as long as the vapour pressure  $P(T)$  corresponding to the temperature  $T$  is less than the pressure  $P_{oo} + \rho_l g$  in the interacting zone. By the definition of  $t_o$ , it follows that when



$t = t_0$ , these two pressures are equal:

$$P = P_{00} + \rho_l l g \quad \text{at } t = t_0 \quad (6.9)$$

at which time the temperature is equal to the corresponding boiling point  $T_{b1}$ :

$$T = T_{b1} \quad (6.10)$$

Combining (6.9) and (6.10) with (2.5), we get

$$A e^{-B/T_{b1}} = P_{00} + \rho_l l g$$

i.e. inverting and substituting for A and B from (2.5):

$$T_{b1} = \frac{T_b}{1 + \frac{RT_b}{L} \ln\left(\frac{P_a}{P_{00} + \rho_l l g}\right)} \quad (6.11)$$

The time  $t_0$  at which evaporation begins follows from (6.10) and (6.5):

$$t_0 = \tau_1 \ln\left(\frac{T_m - T_0}{T_m - T_{b1}}\right) \quad (6.12)$$

7. Solution of the Equations after the Onset of Coolant Evaporation

When  $t > t_o$ , all three of the equations (3.6), (4.3) and (5.3) come into play simultaneously and means must be sought for obtaining their joint solution. For convenience, they are repeated here:

$$\tau \dot{T}_f + T_f = T \quad (7.1)$$

$$\ddot{x} = \frac{P(T) - P_{oo} - \rho_l \dot{x}^2}{\rho_l (l - x)} - K \dot{x}^2 - g \quad (7.2)$$

and

$$dT_f + \{a + x F(T)\} dT + G(T) dx = 0 \quad (7.3)$$

where

$$a = \beta \frac{C_p^l}{C_p^f} \quad (7.4)$$

$$P(T) = A e^{-B/T} \quad (7.5)$$

$$G(T) = \frac{\alpha L}{m_f C_p^f R} \frac{P(T)}{T} \quad (7.6)$$

$$F(T) = \frac{G(T)}{L} \left( C_p^v - C_p^l - \frac{2L}{T} + \frac{L^2}{RT^2} \right) \quad (7.7)$$

The boundary conditions at  $t = t_0$  are:

$$\begin{array}{ll}
 T = T_{b1} & \text{(a)} \\
 T_f = T_m + (T_{f0} - T_m) e^{-t_0/\tau_1} & \text{(b)} \\
 x = 0 & \text{(c)} \\
 \dot{x} = 0 & \text{(d)} \\
 \ddot{x} = 0 & \text{(e)}
 \end{array} \quad \left. \vphantom{\begin{array}{l} (a) \\ (b) \\ (c) \\ (d) \\ (e) \end{array}} \right\} \quad (7.8)$$

the latter condition being a consequence of the instantaneous static equilibrium embodied in (6.9). Conditions (c), (d) and (e) make possible an explicit solution of equation (7.2) for times very near to  $t_0$ . For these times it is accurate to put  $\ddot{x} = 0$  in (7.2) and, recalling (6.9), employ the expansion

$$P = P_{00} + \rho_l l g + \gamma(t - t_0)$$

where

$$\gamma = \left( \frac{dP}{dt} \right)_{t=t_0}$$

With these substitutions, (7.2) becomes

$$\ddot{x} = \frac{\gamma}{\rho_l l} (t - t_0)$$

of which the integral (using boundary conditions (c) and (d)) is

$$x = \frac{\gamma}{6 \rho_l l} (t - t_0)^3$$

i.e. for  $t$  very near to  $t_0$ :

$$x = \frac{1}{6\rho_l l} \left( \frac{dP}{dt} \right)_{t=t_0} (t-t_0)^3 \quad (7.9)$$

Because of the complicating non-linearities in equations (7.2) and (7.3), the evolution of  $T$ ,  $T_f$  and  $x$  with time can only be obtained approximately from a finite difference representation. The system being used at the moment, the simplest possible, is described below.

### 8. Finite Difference Representation

Introducing an equally spaced chain of time points  $t_n$ :

$$t_n = n \Delta, \quad n = 0, 1, 2, \dots \quad (8.1)$$

with the origin  $n = 0$  coincident with the boiling onset time  $t_0$  and writing

$$\left. \begin{aligned} T(t_n) &= T_n, & \Delta T_n &= T_{n+1} - T_n; \\ T_f(t_n) &= T_n^f, & \Delta T_n^f &= T_{n+1}^f - T_n^f; \\ x(t_n) &= x_n, & \Delta x_n &= x_{n+1} - x_n; \end{aligned} \right\} \quad (8.2)$$

for the unknown variables and

$$\left. \begin{aligned} P(T_n) &= P_n \\ G(T_n) &= G_n \\ F(T_n) &= F_n \end{aligned} \right\} \quad (8.3)$$

for the functions of temperature, we can deduce formulae correct to first order in  $\Delta$  for equations (7.1)-(7.3):

$$\Delta T_n^f = \frac{\Delta}{\tau} (T_n - T_n^f) \quad (8.4)$$



$$\Delta x_n = \Delta x_{n-1} + \Delta^2 \left\{ \frac{P_n - P_{o0}}{\rho_l (l - x_n)} - g \right\} - \left\{ K + \frac{1}{l - x_n} \right\} \Delta x_{n-1}^2 \quad (8.5)$$

$$\Delta T_n^f + \{ a + x_n F_n \} \Delta T_n + G_n \Delta x_n = 0 \quad (8.6)$$

The above equations are valid for  $n \geq 1$ . The temperatures at  $n = 1$  can be obtained by extrapolating the solutions (6.5) and (6.6) to  $t = t_0 + \Delta$  :

$$T_1 = T_m - (T_m - T_0) e^{-(t_0 + \Delta)/\tau_1} \quad (8.7)$$

$$T_1^f = T_m + (T_{f0} - T_m) e^{-(t_0 + \Delta)/\tau_1} \quad (8.8)$$

The displacement  $x_1$  at  $n = 1$  can be obtained from (7.9):

$$x_1 = \Delta x_0 = \frac{P_1 - P_0}{6 \rho_l l} \Delta^2 \quad (8.9)$$

where  $P_1 = P(T_1)$  and  $P_0 = P(T_{b1})$ . With the values of all three variables at the first time point thus given by equations (8.7)-(8.9), a complete step-by-step solution of equations (8.4)-(8.6) is a straightforward matter and a computer programme has been written to perform all the necessary arithmetic. This programme has been named TOMOF (Thermohydrodynamics of a Mixture of two Fluids).

9. Preliminary Results of the TOMOF Programme for the Ispra UO<sub>2</sub>/Na Direct Contact Rig

The experimental rig designed at Ispra by KOTTOWSKI et al (2) for the observation of the boiling and ejection of a sodium coolant due to a direct contact with molten UO<sub>2</sub> will have the following essential features.

Channel length	$l = 100 \text{ cm}$
" flow area (annular)	$\alpha = 2.356 \text{ cm}^2$
" friction coefficient	$K = 0.0125 \text{ cm}^{-1}$
Ambient pressure	$P_{oo} = 1 \text{ atm}$

In the early experiments, only 2.4 gm of molten UO<sub>2</sub> (i.e. at about 3070°K) will be used, though larger quantities will be employed later. This UO<sub>2</sub> will be brought into contact with sodium at a variety of temperatures, typically 970°K, by allowing the sodium to fall from the channel into the interacting chamber via a punctured diaphragm. Thus, we shall assume in the calculations the following parameters for the UO<sub>2</sub>/Na mixture:

Total mass of UO <sub>2</sub>	$m_f = 2.4 \text{ gm}$
Initial temperature of Na	$T_o = 970^\circ\text{K}$
" " " UO <sub>2</sub>	$T_{fo} = 3070^\circ\text{K}$

Two further parameters of the mixture are the mean UO<sub>2</sub> globule size  $d_f$  and the mass  $B$  of sodium per unit mass of UO<sub>2</sub> which engages in the heat transfer and boiling process. The values of both of these parameters are highly uncertain, but it seems likely that they will have the orders of magnitude:  $d_f \sim 0.1 \text{ cm}$  and  $B \sim 0.1$ , the latter signifying that the volume of interacting Na will be at least as great as that of the UO<sub>2</sub>. To ensure adequate coverage of the above magnitudes, we shall perform calculations for

$$d_f = 0.01, 0.015, 0.02, 0.04, 0.06, 0.1, 0.2, 0.4 \text{ cm}$$

and

$$B = 0.1, 0.15, 0.2, 0.25, 0.3.$$

For the remaining material constants, we shall assume:

UO <sub>2</sub> density	$\rho_f = 9.7 \text{ gm/cm}^3$
" specific heat (cons. press.)	$C_P^f = 0.35 \text{ J/gm } ^\circ\text{K}$
" thermal conductivity	$k = 0.1 \text{ W/cm } ^\circ\text{K}$
Boundary layer (UO <sub>2</sub> /Na) resistance	$h = 1.7 \text{ } ^\circ\text{K/W cm}^{-2}$
Na molecular mass	$M = 23$
" density (liquid)	$\rho_l = 0.8 \text{ gm/cm}^3$
" sp. ht.. (cons. press. liq. ph.)	$C_P^l = 1.3 \text{ J/gm } ^\circ\text{K}$
" " " ( " " vap. " )	$C_P^v = 2.7 \text{ J/gm } ^\circ\text{K}$
" latent heat vaporization	$L = 3900 \text{ J/gm}$
" normal boiling temp.	$T_b = 1155 \text{ } ^\circ\text{K}$

Typical results obtained by using the above data in the TOMOF programme are plotted in Figure 1 which shows the sodium vapour pressure in the interacting zone as a function of time during the boiling and ejection process for  $\beta = 0.1$  and for all of the above values of  $d_f$ . It will be noted that in every case, the pressure rise  $\Delta P(t)$  above the initial value of 1.08 atm (eq. (6.9)) has the form of a pulse whose maximum value  $\Delta P_{\max}$  occurs relatively early in the excursion. For the smaller values of  $d_f$ , the pulses are sharply peaked, the half-width  $t_{1/2}$  being small and  $\Delta P_{\max}$  large. As  $d_f$  increases, however, the pulses become much smaller and broader, reflecting the decreased rate of heat transfer for the larger fuel globules.

This effect is further illustrated in Tables 1 and 2 which present the maximum pressure rise  $\Delta P_{\max}$  and pulse half-width  $t_{1/2}$  respectively for all the assumed values of  $d_f$  and  $\beta$ . The first column of these tables corresponds to the above discussed results for  $\beta = 0.1$  plotted in Figure 1 and the other four columns correspond to the other assumed values of  $\beta$ . The sensitive dependence of  $\Delta P_{\max}$  and  $t_{1/2}$  on both  $d_f$  and  $\beta$  is clearly exhibited. Because of this sensitivity, it should be easy to obtain from the measured values of  $\Delta P_{\max}$  and  $t_{1/2}$  the values of  $d_f$  and  $\beta$  which give agreement between the theory and experiment. The value of  $d_f$  thus revealed can then be checked for consistency against the fuel globule dimensions observed when the experiment is dismantled. The value of  $\beta$ , though probably not verifiable, is of considerable interest.

The calculated values of the ejection speed  $V_{ej}$  of the sodium plug at the instant of removal from the channel and the time  $t_{ej}$  between this moment and the moment when boiling began are given as functions of  $d_f$  and  $\beta$  in Tables 3 and 4 respectively.

It is interesting to note from Table 3 that  $V_{ej}$ , while being sensitive to variations in  $d_f$ , is practically independent of  $\beta$ . Hence, from an experimental value of  $V_{ej}$  it should be simple to infer the value of  $d_f$  required by the theory to give the same value of  $V_{ej}$ . This value of  $d_f$  can then be checked against the observed globule size and that determined from the  $\Delta P_{max}$  and  $t_{1/2}$  measurements as above.

The behaviour of  $t_{ej}$  with respect to  $d_f$  and  $\beta$  as displayed in Table 4 is similar to that of  $V_{ej}$ , i.e. strongly dependent on  $d_f$  but rather insensitive to  $\beta$ . Thus, from an experimental value of  $t_{ej}$ , we may again infer which value of  $d_f$  should go in the theory (to give the same  $t_{ej}$ ) and thereby have the third estimate of  $d_f$  to compare with the observed value.

If these three values are all in good agreement with experiment, the theory will have attained a high degree of plausibility.

The existence in the theory of the two somewhat ambiguous parameters  $d_f$  and  $\beta$  has stimulated a search for derived mathematical entities which depend only slightly on these parameters. This search has led to the formulation of the following two definitions of the pressure impulse.

- (a) The integral of the pressure rise with respect to time between  $t = 0$  (boiling onset) and  $t = t_{ej}$  (complete ejection of the sodium plug):

$$I_1 = \int_0^{t_{ej}} \Delta P(t) dt \quad (9.1)$$



(b) The product of the absolute maximum pressure (not rise) and the pulse half-width:

$$I_2 = P_{\max} T_{1/2} \quad (9.2)$$

$$\text{where } P_{\max} = P_0 + \Delta P_{\max}.$$

The values of  $I_1$  and  $I_2$  are displayed as functions of  $d_f$  and  $\beta$  in Tables 5 and 6 respectively.

From Table 5 we see that  $I_1$  is insensitive to  $\beta$  but varies with  $d_f$  by at least the same amount as  $V_{ej}$ .  $I_1$  therefore fails to provide the required constant.

On the other hand, Table 6 shows that the quantity  $I_2$  is substantially the same over a very wide range of both  $d_f$  and  $\beta$ . In fact, for the conditions of interest in this section, it appears true to write

$$P_{\max} T_{1/2} \approx 300 \text{ atm msec} \quad (9.3)$$

over the whole expected range of  $d_f$  and  $\beta$ . It will be useful to see if this rough prediction is fulfilled by the experiments.

10. Appendix - Heat Transfer between the Fuel Globules and Coolant

To derive a simple equation connecting the mean temperature  $T_f$  of the fuel and the temperature  $T$  of the coolant in the interacting zone, we assume that the fuel is quickly broken into globules of mean linear size  $d_f$ . It is assumed that these globules remain close together so that the coolant experiences the arrival of heat from all directions. The fuel temperatures of interest in this problem are far above the normal boiling point  $T_b$  of the coolant and heat transfer will therefore be accompanied by vigorous bubble formation and turbulence near the surface of the globules. Such turbulence (combined with a high thermal conductivity for some coolants) will tend to suppress spatial variations of temperature in all regions of the coolant except a thin layer next to the fuel. Thus, it appears reasonable to assume that the coolant has a uniform temperature  $T$  except inside a purely resistive skin layer (mainly vapour) on the surface of the globules.

Representing a typical fuel globule by a sphere of radius  $r_0 = d_f/2$ , denoting the above thermal resistance at the boundary with the coolant by  $h$ , letting  $T'(r,t)$ ,  $k$  and  $\kappa$  be its internal temperature distribution, thermal conductivity and thermal diffusivity respectively and letting  $T(t)$  denote the bulk temperature of the coolant, then we can write

$$\frac{\partial T'}{\partial t} = \kappa \frac{\partial}{\partial r} \left( r^2 \frac{\partial T'}{\partial r} \right) \quad (10.1)$$

inside the fuel and

$$T' - T = -hk \frac{\partial T'}{\partial r} \quad \text{at } r = r_0 \quad (10.2)$$

at the boundary with the coolant. At the centre of the fuel globule, the temperature gradient is obviously zero:

$$\frac{\partial T'}{\partial r} = 0 \quad \text{at } r = 0 \quad (10.3)$$

The mean fuel temperature is given by

$$T_f(t) = \frac{3}{r_0^3} \int_0^{r_0} T'(r, t) r^2 dr \quad (10.4)$$

Quite independently of the above equations, one would expect, intuitively, that an equation of the form

$$\tau \dot{T}_f = T - T_f \quad (10.5)$$

should describe approximately the behaviour of  $T_f$ . The reason for this expectation is that  $T_f$  will decrease if  $T_f > T$  (and vice versa) and the rate of change must be governed by the difference  $T - T_f$  and by some time constant  $\tau$  embodying the effects of the heat capacity and thermal conductivity of the fuel and the resistance  $h$  at its surface. The attack on equations (10.1) - (10.4) will be performed with a view to establishing an expression for  $\tau$  and determining the domain of validity of (10.5). This will be accomplished by considering two widely differing cases.

(a) Oscillating Coolant Temperature

For this case, we assume that a harmonic fluctuation

$$T = T_o(\omega) e^{i\omega t} \quad (10.6)$$

is imposed on the coolant temperature and we ask what the response of  $T'(r, t)$  and  $T_f(t)$  will be. First, it is clear that (after the decay of transients) the time dependence of  $T'$  and  $T_f$  must also be oscillatory and therefore that

$$T'(r, t) = T_o'(\omega, r) e^{i\omega t} \quad (10.7)$$

and

$$T_f(t) = T_{of}(\omega) e^{i\omega t} \quad (10.8)$$

Secondly, substituting (10.6) and (10.8) into (10.5), it would appear that we must have, approximately,

$$T_{of}(\omega) = \frac{T_o(\omega)}{1 + i\omega\tau} \quad (10.9)$$

Passing on to the detailed analysis of the heat flow within and out of the fuel, we substitute (10.7) into equation (10.1) to get

$$\frac{\partial^2 T_0'}{\partial r^2} + \frac{2}{r} \frac{\partial T_0'}{\partial r} - \frac{i\omega}{\kappa} T_0' = 0$$

of which the solution is

$$T_0' = A(\omega) \frac{\sin(zr/r_0)}{r/r_0} \quad (10.10)$$

where

$$z = \sqrt{-\frac{i\omega r_0^2}{\kappa}} \quad (10.11)$$

is a dimensionless complex constant. A second term in  $\cos zr/r_0$  is excluded from (10.10) by condition (10.3). Substituting (10.6) and (10.7) with (10.10) into (10.2), we obtain

$$A(\omega) \left\{ \sin z + \frac{hk}{r_0} (z \cos z - \sin z) \right\} = T_0(\omega)$$

and substitution of (10.7) into (10.4), with application of (10.10), gives

$$T_{of}(\omega) = \frac{3A(\omega)}{z^2} (\sin z - z \cos z)$$

Elimination of  $A(\omega)$  from these two equations leads to

$$T_{of}(\omega) = \frac{T_0(\omega)}{\frac{1}{3} \left\{ \frac{z^2}{1 - z \cot z} - \frac{hk}{r_0} z^2 \right\}} \quad (10.12)$$

The function  $z \cot z$  can be expanded into a power series in  $z$  which for  $|z| \ll 1$  is rapidly convergent:

$$z \cot z = 1 - \frac{z^2}{3} - \frac{z^4}{45} \dots$$

so that to order  $z^2$

$$\frac{z^2}{1 - z \cot z} = 3 \left( 1 - \frac{z^2}{15} \right)$$

Substituting this into (10.12) and using (10.11), we then see that for  $\omega r_0^2 / \kappa \ll 1$

$$T_{of}(\omega) = \frac{T_o(\omega)}{1 + \frac{i\omega r_0^2}{15\kappa} \left( 1 + \frac{5hk}{r_0} \right)} \quad (10.13)$$

and comparison with equation (10.9) immediately provides the formula

$$\tau = \frac{r_0^2}{15\kappa} \left( 1 + \frac{5kh}{r_0} \right) \quad (10.14)$$

for the time constant in equation (10.5).

Substituting

$$\kappa = \frac{k}{\rho_f C_P^f}$$

for the diffusivity of the fuel, we get

$$\tau = \frac{1}{3} \rho_f C_P^f r_0 \left( h + \frac{r_0}{5k} \right)$$

or, in terms of the volume/surface ratio of the globule,  $\sigma = r_0/3$ :

$$\tau = \sigma \rho_f C_p^f \left( h + \frac{3}{5} \frac{\sigma}{k} \right) \quad (10.15)$$

(b) Step Drop in Coolant Temperature

For this case, it is assumed that for  $t < t_0$  the fuel and coolant temperatures are both equal to  $T_0$  and that, at  $t = 0$ , the coolant temperature suddenly drops to zero and remains there. With such a disturbance, the fuel temperature  $T'(r,t)$  begins its evolution from a uniform distribution:

$$T'(r, 0) = T_0 \quad (10.16)$$

According to the approximate equation (10.5), this evolution is given, in terms of the mean fuel temperature  $T_f$ , by

$$T_f = T_0 e^{-t/\tau} \quad (10.17)$$

On the other hand, the detailed behaviour of the fuel temperature can be obtained by putting  $T = 0$  in (10.2):

$$T' + hk \frac{\partial T'}{\partial r} = 0 \quad \text{at } r = r_0 \quad (10.18)$$

and then solving the complete diffusion equation (10.1) with (10.3) and (10.18) as the boundary conditions. Such a procedure leads straightforwardly to the result

$$T'(r, t) = \sum_{\lambda} A(\lambda) e^{-\lambda^2 \kappa t / r_0^2} \frac{\sin \lambda r / r_0}{r / r_0} \quad (10.19)$$

where the eigenvalues  $\lambda$  are determined by substituting (10.19) into (10.18):

$$\left( 1 - \frac{hk}{r_0} \right) \sin \lambda + \frac{hk}{r_0} \lambda \cos \lambda = 0 \quad (10.20)$$



and the coefficients  $A(\lambda)$  are obtained from the initial condition (10.16).

Substitution of (10.19) into (10.16) leads to

$$\sum_{\lambda} A(\lambda) \frac{\sin \lambda r/r_0}{r/r_0} = T_0$$

and by using the orthogonality of the functions

$$F_{\lambda}(x) = \frac{\sin \lambda x}{x}$$

i.e.

$$\int_0^1 F_{\lambda}(x) F_{\mu}(x) x^2 dx = \delta_{\lambda\mu} \int_0^{\lambda} \sin^2 \xi d\xi$$

we get

$$A(\lambda) = T_0 \frac{\int_0^{\lambda} \xi \sin \xi d\xi}{\lambda \int_0^{\lambda} \sin^2 \xi d\xi} \quad (10.21)$$

The mean fuel temperature is obtained by integrating (10.19) in the way prescribed by (10.4):

$$T_f(t) = \sum_{\lambda} A(\lambda) e^{-\lambda^2 \kappa t / r_0^2} \left( \frac{3}{r_0^3} \int_0^{r_0} \frac{\sin \lambda r / r_0}{r / r_0} r^2 dr \right)$$

Substituting for  $A(\lambda)$  from (10.21), this gives

$$T_f(t) = T_0 \sum_{\lambda} \frac{3 \left( \int_0^{\lambda} \xi \sin \xi d\xi \right)^2}{\lambda^3 \int_0^{\lambda} \sin^2 \xi d\xi} e^{-\lambda^2 \kappa t / r_0^2}$$

and using the formulae

$$\int_0^\lambda \xi \sin \xi d\xi = \sin \lambda - \lambda \cos \lambda$$

$$\int_0^\lambda \sin^2 \xi d\xi = \frac{1}{2} (\lambda - \sin \lambda \cos \lambda)$$

we get finally

$$T_f(t) = T_0 \sum_{\lambda} \frac{6(\sin \lambda - \lambda \cos \lambda)^2}{\lambda^3 (\lambda - \sin \lambda \cos \lambda)} e^{-\lambda^2 \kappa t / r_0^2} \quad (10.22)$$

Equation (10.22) offers an immediate test of the validity of the intuitive approximation (10.5) and (10.17). If conditions are such that the first term of (10.22) - that with the lowest eigenvalue  $\lambda_1$  - is much larger than all the other terms combined, then (10.17) is valid and the hold-up time constant  $\tau$  is given by

$$\tau = \frac{r_0^2}{\lambda_1^2 \kappa} \quad (10.23)$$

Let us consider the behaviour of the series (10.22) as a function of the parameter

$$p = \frac{h \kappa}{r_0}$$

which alone determines the eigenvalues in equation (10.20).

Case 1:  $p = \frac{h \kappa}{r_0} \ll 1$

In this case, examination of (10.20) indicates that the lowest eigenvalue  $\lambda_1$  must be very near to  $\pi$ . Let us write

$$\lambda_1 = \pi(1 - \varepsilon)$$

where  $\varepsilon \ll 1$ . Then

$$\sin \lambda_1 = \sin \varepsilon \pi = \varepsilon \pi + O(\varepsilon^3); \quad \cos \lambda_1 = -1 + O(\varepsilon^2)$$

and therefore, by (10.20)

$$\varepsilon = p + O(p^3)$$

Hence, for  $\frac{hk}{r_0} \ll 1$

$$\lambda_1 = \pi \left(1 - \frac{hk}{r_0}\right)$$

and

$$T_f(t) = \frac{6}{\pi^2} T_0 e^{-t/\tau} + \text{faster decay terms}$$

where

$$\tau = \frac{r_0^2}{\pi^2 \kappa (1 - hk/r_0)} \quad (10.24)$$

These results show that the first, slowest decaying term of (10.22) accounts initially for only about 61% of the fuel temperature excess and, therefore, that the "intuitive" formulae (10.5) and (10.17) are rather inaccurate in this case.

Case 2:

$$p = \frac{hk}{r_0} = 1$$

In this case, the smallest solution of (10.20) is

$$\lambda_1 = \frac{\pi}{2}$$

and therefore

$$T_f(t) = \frac{96}{\pi^4} T_0 e^{-t/\tau} + \text{faster decaying terms}$$

where

$$\tau = \frac{4 r_0^2}{\pi^2 \kappa}$$

Thus, we see that for  $\frac{hk}{r} = 1$ , the first term of (10.22) is completely dominant, accounting for 98.6% of the fuel temperature excess even at  $t = 0$ . The intuitive approximations (10.5) and (10.17) are therefore very accurate in this case.

Case 3:

$$p = \frac{hk}{r_0} \gg 1$$

In this case, (10.20) can be written

$$\lambda \cos \lambda - \left(1 - \frac{1}{p}\right) \sin \lambda = 0$$

from which it is obvious that the smallest eigenvalue is very small:  $\lambda_1 \ll 1$ .

Because of this, the functions can be expanded in power series:  $\sin \lambda_1 =$

$\lambda_1 - \lambda_1^3/6$ ;  $\cos \lambda_1 = 1 - \lambda_1^2/2$ , and therefore

$$\lambda_1 - \frac{\lambda_1^3}{2} - \left(1 - \frac{1}{p}\right) \left(\lambda_1 - \frac{\lambda_1^3}{6}\right) = 0$$

i.e.  $\lambda_1 = \sqrt{\frac{3}{p}}$

$$\lambda_1 = \sqrt{\frac{3}{p}}$$

In addition

$$\sin \lambda_1 - \lambda_1 \cos \lambda_1 = \frac{\lambda_1^3}{3}$$

$$\lambda_1 - \sin \lambda_1 \cos \lambda_1 = \frac{2}{3} \lambda_1^3$$

Thus, (9.22) becomes

$$T_f(t) = T_0 e^{-t/\tau}$$

where

$$\tau = \frac{p r_0^2}{3 \kappa} = \frac{h k r_0}{3 \kappa} \quad (10.26)$$

The faster decaying terms are completely negligible here and the intuitive description given by (10.5) and (10.17) is therefore exact in this case.

Thus, we have established that the simple equation

$$\tau \dot{T}_f + T_f = T$$

gives a very good description of the mean fuel temperature for a step change in the coolant temperature  $T$  provided that

$$\frac{h k}{r_0} \gg 1 \quad (10.27)$$

Now let us compare the formulae (10.24) - (10.26) for the time constant  $\tau$  with that derived in the analysis of harmonically varying temperatures (equation (10.14)). In the following Table the values of  $\tau$  calculated by the two methods are shown explicitly:

$\frac{hk}{r_o}$	$\tau$	
	harmonic	step
small	$\frac{1}{15} \frac{r_o^2}{\mathcal{K}}$	$\frac{1}{9.9} \frac{r_o^2}{\mathcal{K}}$
1	$0.4 \frac{r_o^2}{\mathcal{K}}$	$0.405 \frac{r_o^2}{\mathcal{K}}$
large	$\frac{hk r_o}{3 \mathcal{K}}$	$\frac{hk r_o}{3 \mathcal{K}}$

Thus, we see very clearly that if the condition (10.27) for the validity of the intuitive description is satisfied, the time constant  $\tau$  is independent of the manner of exciting temperature variations. We therefore conclude that the intuitive equation (10.5) connecting the mean fuel temperature and coolant temperature is quite general provided equations (10.14) (or (10.15)) and (10.27) are observed.

#### References

1. Duffey, R.B., "Channel Voiding due to Fuel-Coolant Interactions in Sodium-Cooled Fast Reactors: A Theoretical Model", CEGB report RD/B/N1609.
2. Kottowski, H. et al. Private Communication.



TABLE 1

Maximum Pressure Rise  $\Delta P_{\max}$  (atm) during Ejection as a Function of  $d_f$  (cm) and  $\beta$ .

$d_f \backslash \beta$	0.1	0.15	0.2	0.25	0.3
0.01	44.9	34.7	28.2	23.5	19.9
0.015	34.5	27.0	22.1	18.6	16.0
0.02	28.3	22.3	18.4	15.6	13.5
0.04	17.0	13.6	11.4	9.81	8.60
0.06	12.4	9.98	8.42	7.31	6.46
0.1	8.19	6.63	5.64	4.93	4.39
0.2	4.48	3.65	3.13	2.76	2.47
0.4	2.31	1.89	1.63	1.44	1.29

TABLE 2

Pressure Pulse Half-Width  $t_{1/2}$  (msec) as a Function of  $d_f$  (cm) and  $\beta$

$d_f \backslash \beta$	0.1	0.15	0.2	0.25	0.3
0.01	6.75	9.44	12.1	14.8	17.6
0.015	8.52	12.0	15.4	18.8	22.3
0.02	10.1	14.3	18.4	22.5	26.6
0.04	15.6	22.3	28.9	35.4	41.7
0.06	20.1	29.0	37.8	46.2	54.2
0.1	27.6	40.1	52.3	63.7	74.0
0.2	40.6	58.9	76.3	91.8	104.9
0.4	54.4	76.4	97.1	115.8	132.2

TABLE 3

Ejection speed  $V_{ej}$  (cmsec) of Sodium Plug from the Channel as a Function of  $d_f$  (cm) and  $\beta$ .

$d_f \backslash \beta$	0.1	0.15	0.2	0.25	0.3
0.01	cenc	2457	2384	2312	2241
0.015	cenc	2413	2346	2279	2212
0.02	cenc	2357	2296	2235	2174
0.04	2178	2136	2094	2050	2005
0.06	1988	1959	1926	1892	1856
0.1	1719	1701	1680	1657	1631
0.2	1326	1320	1310	1297	1282
0.4	920	920	920	915	909

cenc = complete ejection not computed

TABLE 4

Ejection time  $t_{ej}$  (m sec) of Sodium Plug from the Channel as a Function of  $d_f$  (cm) and  $\beta$ .

$d_f \backslash \beta$	0.1	0.15	0.2	0.25	0.3
0.01	cenc	43.0	45.0	48.1	50.8
0.015	cenc	46.4	49.1	51.8	54.6
0.02	cenc	49.5	52.3	55.1	58.0
0.04	56.3	59.6	62.8	66.1	69.4
0.06	63.9	67.7	71.3	75.0	78.6
0.1	76.4	80.9	85.2	89.5	93.7
0.2	100.9	106.6	112.2	117.6	123.0
0.4	141.4	148.6	155.6	162.5	169.3

cenc = complete ejection not computed

TABLE 5

$I_1$ : Integral of Pressure Rise over Time up to the Moment of Ejection, i.e. Ejection Impulse (atm msec) as a Function of  $d_f$  (cm) and  $\beta$

$d_f \backslash \beta$	0.1	0.15	0.2	0.25	0.3
0.01	cenc	530	507	484	462
0.015	cenc	496	476	457	437
0.02	cenc	469	451	434	416
0.04	410	397	384	371	358
0.06	364	353	342	331	320
0.1	307	298	289	280	272
0.2	233	227	220	214	207
0.4	163	160	156	151	147

cenc = complete ejection not computed

TABLE 6

$I_2$ : (maximum pressure, atm) x (pressure pulse half-width, m sec) as a Function of  $d_f$  (cm) and  $\beta$

$d_f \backslash \beta$	0.1	0.15	0.2	0.25	0.3
0.01	310	338	355	364	369
0.015	303	336	358	371	380
0.02	297	334	359	376	387
0.04	282	327	361	386	403
0.06	271	321	359	388	408
0.1	255	309	351	383	404
0.2	226	279	321	352	372
0.4	185	227	262	291	313

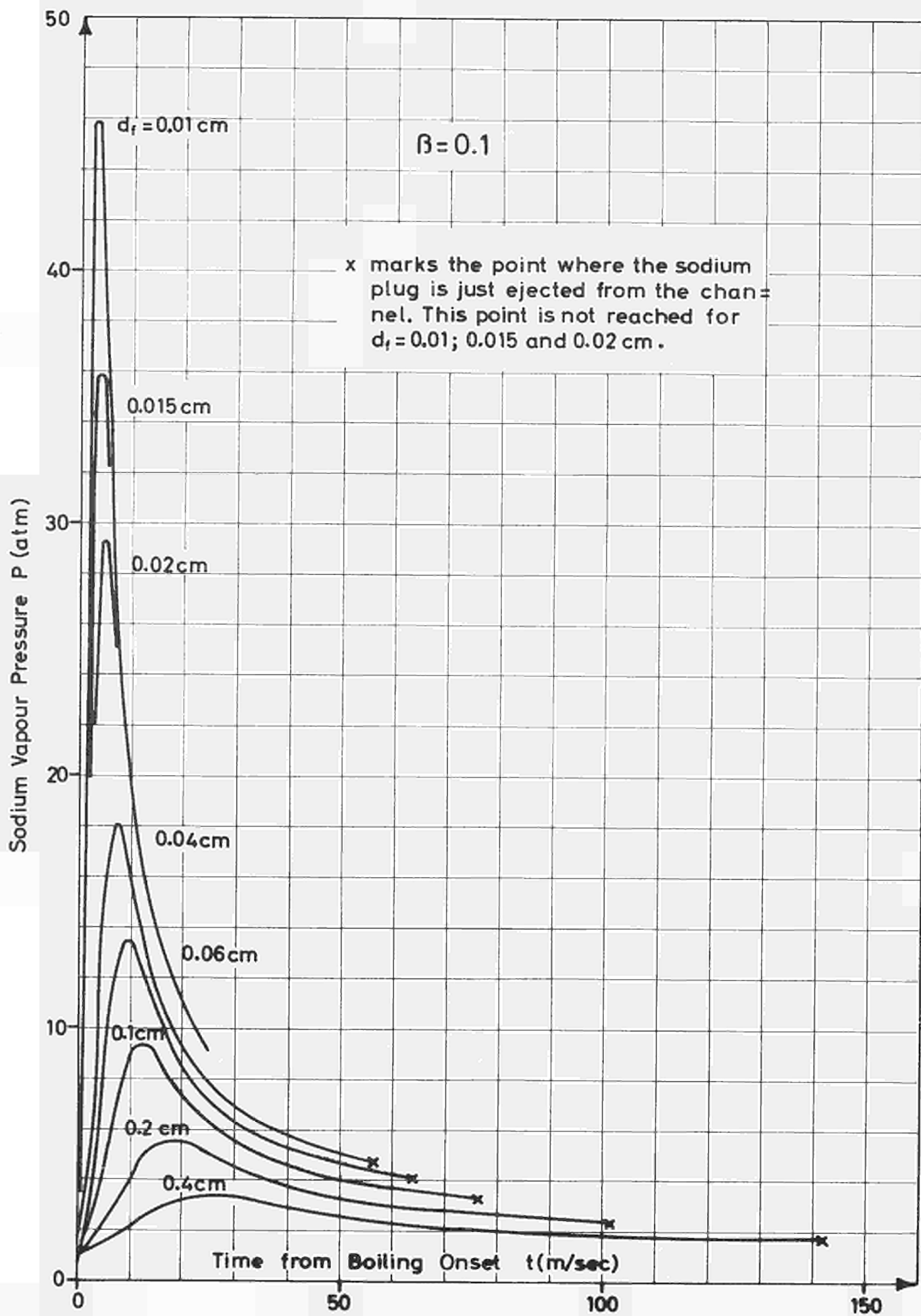


FIGURE 1. Example of the Pressure Pulses expected for the Ispra  $UO_2/N_2$  Direct Contact Rig



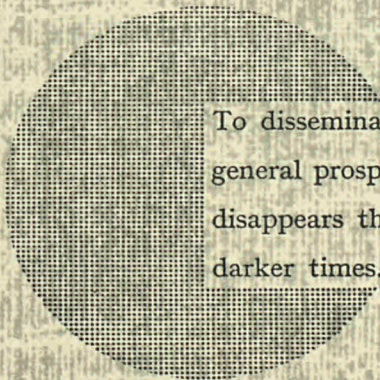
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