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THE PART OF THERMAL NON-EQUILIBRIUM IN BOILING CO-CURRENT FLOW

by

H. WUNDT

1968

Joint Nuclear Research Center
Ispra Establishment - Italy

Reactor Physics Department
Research Reactors

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SUMMARY

Various hypotheses on the part of thermal non-equilibrium in boiling processes can be found in the literature. In this paper it is attempted not to make assumptions ad hoc (e.g. in order to establish quickly a computer programme), but to consider carefully the pertinent elementary mechanisms. This leads to a number of up to now qualitative relationships. Emphasis is laid on the radial temperature and vapor distributions in co-current gas-liquid flow through a heated vertical tube. The structure of these relationships may be helpful in establishing correlations for nucleate boiling heat transfer, because the manner how various parameters (e.g. surface tension and surface roughness) influence the mechanism may better be overlooked.

KEYWORDS

BOILING
FLUID FLOW
HEAT TRANSFER
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TEMPERATURE
BUBBLES
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THERMODYNAMICS

Contents

THE PART OF THERMAL NON-EQUILIBRIUM IN BOILING CO-CURRENT FLOW

1. Bubble motion without boiling (*)

The mechanism of nucleate boiling has been investigated by various authors (see e.g. [1]) in a qualitative and experimental manner. In this paper, the pecularities of nucleate boiling in forced co-current two-phase flow shall be discussed.

In order to understand the full mechanism in all its details, a rigoròus solution of the hydrodynamic equations of motion would be necessary, together with energetic considerations from thermodynamics, as soon as the temperature does not remain constant.

Already for a single phase turbulent flow a rigorous solution is not possible, as well known, for one can at best describe the main stream behavior, whereas the turbulent random oscillations are eliminated by means of an appropriate time averaging procedure. More precisely, the turbulent flow is constitutionally never stationary; steady flow patterns are only pretended.

Of course, in most cases the fine structure is not interesting so that satisfactory engineering formulas, for the pressure drop e.g., may be well established. These formulas are of the "correlation type" because they correlate the searched effect to some input data by means of the "dimensionless parameters" of the problem with adaptable coefficients.

The pith of this consideration is that one is obliged to fall back to (approximative) dimension-analytical descriptions, as it was also done by all other authors. This is the more the case for boiling flows which we are going to consider.

Boiling flow, too, is never stationary, but in a much more extensive sense than is ordinary turbulent flow. Instead of eddies, the compactly moving volumes are now vapor bubbles. We will roughly estimate their rise velocity due to buoyancy.

(*) **Manuscript received on August 8, 1968.**

We assume a vertical pipe because otherwise the vapor would accumulate along the top generating line, thus providing a priori some phase separation.

We apply STOKES' viscosity law for falling balls to spheres with less density than the surrounding liquid (with the liquid coordinates as reference system):

$$v_v = \frac{2}{9}\frac{r^2 g}{\mu_l}(\rho_l - \rho_v) \tag{1.1}$$

Here

v_v = (upwards-) velocity of spherical vapor bubble, asymptotic for large times

r = bubble radius

g = acceleration of gravity

μ_l = dynamic viscosity of liquid

ρ_l = density of liquid

ρ_v = density of vapor.

This law which is not quite easy to derive is not entirely rigorous because of a certain linearization applied in the equations of motion, and holds only far from other liquid boundaries. Nevertheless, it gives a good idea for spheres which are not too large; the numerical coefficient 2/9 may be fitted, if necessary. The structure of this law is appropriate to be incorporated into dimensional-analytical correlations.

Let v_l be the liquid velocity in a (wide) vertical pipe, then the (axial)"slip ratio" $S = v_v/v_l$ (now with respect to an "inertial" system) is found to be

$$S = 1 + \frac{2}{9}\frac{r^2 g}{\mu_l v_l}(\rho_l - \rho_v). \tag{1.2}$$

The essential point is that the rise velocity depends strongly on the bubble radius (factor r^2). Thus, larger bubbles move faster than smaller ones and catch them up. On account of their surface tension they add together, becoming so still larger and moving still faster. One large bubble therefore clears away the whole downstream space from all smaller bubbles until it reaches a free surface. A pulsating shock flow results, the periods of which are in general too slow for being averaged out reasonably. The deviations from the main flow represent no longer a "microstructure" as for ordinary turbulence but prove to be an essential aspect of the flow pattern.

2. Void increase in flow direction

The picture, as developed in the previous chapter, assures no influence of the pipe walls, i.e. no radial dependence of the void distribution. This is not realistic for two reasons. First, large bubbles which only streak other bubbles laterally will draw them along their own path, thus provoking a radial velocity component. Secondly, the liquid viscosity is generally larger than that of the vapor so that the liquid, due to shear stresses, is braked in the wall boundary layer. The well known consequence is that the vapor bubbles tend to the pipe axis for better movement. The overall motion is however not a straight one because of random lateral momenta occurring when the central bubble joins other ones coming from the side.

Let us now calculate the vapor contents increase along the flow direction z if a heat source Q(z) per surface unit is assumed. Provisionally, we disregard any radial dependence of the flow so that - for a circular cylinder with radius R, e.g. - also a uniform volumetric heat production $q(z) = \frac{2}{R} Q(z)$ may be assumed.

The evaporation heat h_{lv} depends on the pressure p only; if the pressure drop is small against the static pressure, the z-variation of h_{lv} is inconsiderable. The vapor (mass-) production per unit volume is then

$$\sigma_v(z) = \frac{q(z)}{h_{lv}(p)} , \qquad (2.1)$$

provided that liquid and vapor are in thermodynamic equilibrium, i.e. that no boiling delay occurs. This point will be considered in more detail below.

The mass fraction x (= local ratio steam mass by total mass; not steam flow rate by total flow rate which is different from the former one for slip ratios $\neq 1$) increases according to

$$x(z) = \frac{1}{v_o \rho_l} \int_o^z \sigma_v(\zeta) d\zeta . \qquad (2.2)$$

$z = 0$ is the point where boiling just begins ($x = 0$). The liquid velocity at this point is v_o so that $v_o \rho_l$ is the mass flow rate per unit of cross sectional area which is z-independent if there is no mass storage along the pipe. For transient processes the storage must indeed be considered.

Sometimes (for calculating reactivity effects in nuclear reactors, e.g.) the knowledge of the vapor volume fraction is preferred. The relationship between α and x is [2]

$$\alpha = \frac{\xi x}{\xi x + (1-x)} \qquad \text{with } \xi = \frac{\rho_l}{\rho_v} , \qquad (2.3)$$

independent of any phase slip.

As outlined at the beginning of this chapter, the vapor, by means of radial velocity components, tends to concentrate in the pipe axis. Thus, the fractions x or α, as calculated from (2.2) or (2.3), are mostly higher in the axis and lower at the wall, but are always valid in the radial average. Experience shows that, from a certain high steam contents downstream, also a spray flow pattern may occur where the wall is practically unwetted. The liquid part moves near the axis in form of droplets. This may be explained by considering that the liquid has lost any contact with the wall so that its proper viscosity becomes irrelevant. The droplets are transported like solid bodies. The minimum surface principle for the surface tension is however violated in this flow pattern. For all wetting liquids the droplets would remain at the wall, once arrived there. In our case, they are sufficiently small in order to be evaporated immediately. Spray flow is, in general, not possible without wall heating.

3. Heat transfer coefficients

The local heat transfer coefficients $h(z)$ occurring along z depend on several quantities among which the local void fraction α_o (at the wall) and an axial velocity "near" the wall (at the inner side of the laminar boundary sublayer, say) are probably the most important ones. The temperature differences to which these heat transfer coefficients refer must first be clearly defined. The liquid reference temperature may not be taken too far from the wall,for the properly pertinent quantity is the temperature gradient at the wall which may not be falsified by linearization of a major part of the more or less bended (radial) temperature profile. A reasonable answer to this question is given only at the end of this report.

In the author's opinion, and in equivalence to OHM's electrical resistence law, the temperature difference - corresponding to the voltage difference - should appear to the first power only, and all other influences debited only to the heat transfer coefficient - corresponding to the reciprocal electrical resistance -, thus

$$Q = h(\dots)\cdot \Delta T \,. \qquad (3.1)$$

The known JENS-LOTTES [3] correlation $Q = A(K\cdot \Delta T)^4$ easily prejudices the physical comprehension and is, besides, not independent on the units' system applied.

In (3.1) it is tacitly assumed that h itself does not depend directly on ΔT, but only on other quantities which, however, in turn may be related to temperatures (via the state equation, e.g.).

The only sure statements are that the heat transfer is better from the wall to a liquid than to a gas, and that "stirring" of the boundary layer considerably improves the heat transfer. Combination of the two effects leads to the following qualitative behaviour for $h(\alpha_o)$:

For $\alpha_o = 0$, the heat transfer coefficient h is on a "mean" level, namely that one for a liquid*). As soon as a few bubbles appear for small α_o, the wetted surface fraction is somewhat decreased, but the boundary layer agitation is such that the total effect is a gradual improvement of h. This behaviour continues up to a certain α_o where the agitation is no longer strengthened by still more bubbles, because those ones immediately fuse due to the surface tension, but the diminution of the wetted surface becomes preponderant so that h goes through a maximum. For still higher α_o, h decreases continuously until reaching the low value for an entirely dry surface, $\alpha_o = 1$.

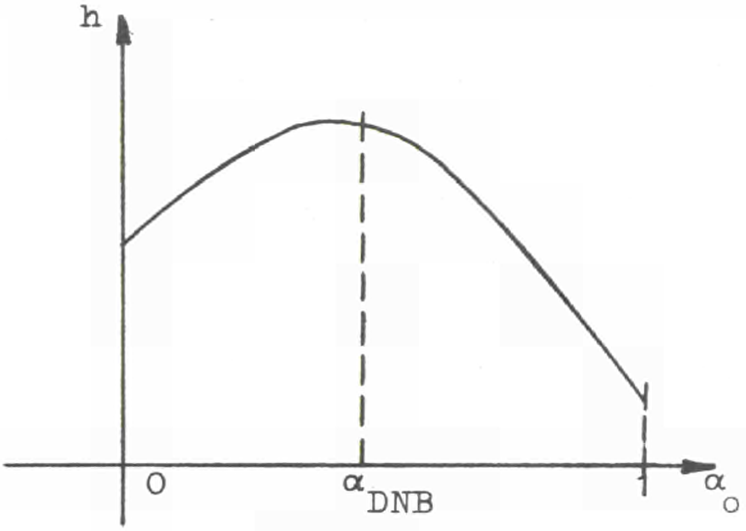

Figure 1
Qualitative behaviour of $h(\alpha_o)$

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*) Of course, α_o is to be understood as a time averaged quantity

We now must consider that α_o, the wall value, is not the α calculated in chapter 2, formulas (2.2) and (2.3), but in general lower, as mentioned there. The ratio between α_o and α is by no means a constant along z; α_o, the near-wall-void-fraction, exhibits rather a sudden upward jump at a certain $z = z_{DNB}$ though the mean α increases only smoothly with z. Physically, the cohesion of the liquid film at the wall bursts for a certain $\alpha_o = \alpha_{DNB}$, obviously a surface tension effect. As a consequence, for a given heat production Q, the temperature difference ΔT must go up correspondingly when h suddenly breaks down towards the right-hand part of fig. 1: the well-known phenomenon of "departure from nucleate boiling" of "burnout".

For completeness, we may remark that, for non-autonomous heating, i.e. for heat exchangers, the effect of a breakdown of h lets ΔT unchanged but diminishes simply Q correspondingly, according to (3.1). In this case, the real burnout effect, above all a wall material melting, cannot occur, though the conditions on the liquid side are the same as above.

The influence of the axial velocity on the heat transfer coefficient seems to be relatively simple. The stirring effect of the boundary layer is improved by higher mass flow rates per cross section unit on account of better dispatch of the bubbles generated. h may thus be assumed to be a monotonically increasing function of v_o. A better comprehension may however be obtained from the reasonings of chapter 4.

4. Radial temperature and vapor distributions

The problem obviously condenses to the task of finding a relationship between α_o and α, or, in other words, the radial void distribution. As we shall see, this is in turn tightly coupled with the radial temperature profile of the liquid.

(2.3) gave namely α versus x, (2.2) x versus σ_v, (2.1) σ_v versus $q = \frac{2}{R} Q$, (3.1) Q versus h, but fig. 1 only h versus α_o. The loop is not yet closed.

First, it must be repeated that $\alpha(r)$ is not a time-independent profile for each z on account of the essential non-stationary character of the flow. We therefore content ourselves to search for sufficiently long time averages of these profiles. They prove to be the characteristics of the various boiling "flow patterns".

We begin with the fact that "subcooled boiling" exists; this is boiling with subsequent recondensation of the bubbles in the central stream, without net vapor production. If the liquid entrance temperature is below the wall temperature and sufficiently below saturation, this phenomenon simply occurs because of the negatively bended temperature profile.

We remark that unequal radial temperatures are necessary to explain the subcooled boiling. But also for ordinary nucleate boiling without complete recondensation this temperature unevenness proves to be important, either across a regular profile or locally between liquid flow and transported vapor bubbles. These two possibilities, however, cannot be separated one from the other, for the bubbles are transported radially through the profile an reinfluence it. The origin of the profile is indeed nearly exclusively due to convective heat transport, either by liquid or by vapor.

It is convenient to separate the bubble motion into two parts, the first one as long as the bubble adheres still to the heating surface, the second one as long as it moves through the liquid.

The heat transfer being by far better to the liquid than to the vapor, it is obvious and confirmed by measurements that a great deal of the heat makes the detour through the liquid before it arrives at the vapor bubble. Indeed, the bubble growth results from a liquid mass sink so that the liquid, on account of the second law of thermodynamics, must be hotter than the vapor in the bubble, in the vicinity of the wall, i.e. in its first motion part.

One has found that the vapor has practically always saturation and the liquid is superheated, for water by some tenths of a °C in practice. This is the result of nucleate boiling observations when the liquid is heated from a horizontal plate, with free surface. Here, one has no perceptible temperature gradient through bulk liquid.

In this case, the bubbles grow further when rising through the superheated liquid by supplementary evaporation.

Consider now our case of a heated vertical tube. From the entrance at the bottom, the liquid temperature profile is first curved negatively. Though the liquid temperature is a bit above saturation near the wall, it is generally well below saturation in the axis. In this entrance region, recondensation instead of further evaporation occurs in the bulk stream.

Downstream, the liquid temperature profile is flattened gradually but remains always negatively curved. The question is if the consecutive profiles intersect the saturation temperature level or not. For not too narrow tubes, the intersection may persist so that an outer region of evaporation and an inner region of condensation exist.

If the pressure drop is negligible, we have constant saturation temperature along the flow. Strictly speaking, the saturation temperature falls downstream with decreasing pressure. Already a few tenths of a degree are relevant because the profile elevation near the wall above saturation temperature is in this order of magnitude, as we saw.

Whereas the temperature gradient in the outer region is accentuated by the heat withdrawal by evaporation, the profile is strongly flattened by the recondensation in the central zone. However, differently from common heat transport problems, a point will be reached for the liquid enthalpy balance where the heat supply can no more be received. The rejected part flows downwards in form of bubbles.

Normally, this point is considered to be the transition from subcooled to ordinary nucleate boiling. If the process were a stationary one, the liquid temperature profile minimum would just have reached the saturation level. Since this is not the case and since condensation consumes times, a bubble can well first be carried away axially and then, nevertheless, condensate afterwards if, for instance, the heating is finished from this z-position downstream.

Due to the non-stationarity, a central temperature below saturation is compatible with net vapor release from a cross-sectional layer. The transition from subcooled to nucleate boiling is not sharp, in particular for high axial velocities.

Thus, an as high as possible supply of (relatively) subcooled water improves the overall heat transfer by expanding the recondensation zones downstream.

In the author's opinion, this is indeed the true point where the velocity comes into play. We prefer to no more attribute the velocity influence to the heat transfer coefficient h, i.e. to a wall-located process, but to a bulk-stream process. Strictly spoken, the axial velocity is, on account of the adhesion conditions, zero at the wall so that the former considerations were not consistent. *)

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*) It may not be suppressed that heat transfer coefficients for ordinary single phase flow are said to depend on a REYNOLDS' number built with the bulk velocity. In these cases, however, the relationship between the central velocity and the gradient at the wall is more or less known (laminar or 1/7-profiles, e.g.).

From the foregoing it seems that one is already rather near the possibility to formulate the bulk stream processes mathematically. However, two important problems are unsolved. The first one is, how large is the heat transfer coefficient from a slightly superheated liquid to a saturated bubble of its own vapor, including evaporation, and how large is this coefficient from saturated vapor to its slightly subcooled liquid, including condensation? The experimental data which were available to the author, were not sufficient. The values are certainly extremely high. It is understood that the heat transfer coefficients in question shall refer to the local superheating or subcooling temperature difference.

The second problem is to know the velocity evolution because only this one tells how far a bubble may be transported into regions of different temperature. A solution is not possible without describing the pressure field simultaneously. Obviously, the pressure drop problem is insolubly coupled with the heat transfer problem. The two processes may never be described by entirely independent correlations.

On account of this question, we are also not able up to now to overlook the initially stressed relationship between α and α_o. We will but discuss in the following some features which may be useful to advance towards our goal.

5. The overpressure of bubbles and droplets

Thermodynamic theory shows that the vapor pressure of small spherical liquid drops must be higher than that of the surroundings by an amount of

$$\Delta p = \frac{2\sigma}{r} \qquad (5.1)$$

where σ is the surface tension and r the bubble radius.

Here the state variable "chemical potential" *) is not a pure intensive quantity as are the temperature T and the pressure p. It depends rather on the geometrical extension of one phase. As a consequence, the vapor pressure p_r of each curved surface with curvature radius r (in the case of a doubly curved surface the pertinent length is the sum of the main curvatures $\frac{1}{r_1} + \frac{1}{r_2}$) is higher than that of a plane surface (p_∞).

This reasoning holds equivalently for the existence of vapor bubbles in a liquid: the pressure on the concave side must be higher in order to prevent the collapse. The pertaining saturation temperature is also higher.

The larger is the liquid temperature excess above saturation, the smaller is the bubble radius. In equilibrium conditions, bubbles are therefore smallest near the heating surface.

Another equivalence is the following: just like a droplet which cannot be created - far from solid surfaces - by condensation, starting from radius zero (see 5.1), a bubble cannot be generated by evaporation because the necessary vapor overpressure would be infinite at the beginning. This is indeed the statement of classical equilibrium thermodynamics. As a matter of fact, only the random oscillatory movements of statistical mechanics open the comprehension why these processes can nevertheless initiate spontaneously. We will consider the necessary conditions for them in chapter 7.

Obviously, certain overpressures or, by means of the state equation, also overtemperatures are physically not unstable; these "oversaturations" must not lead to direct phase changes.

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*) The expression "chemical potential" is perhaps improper because this function is relevant also for processes without chemical reactions, namely for one-component systems with more than one phase; see pertinent textbooks.

On the other hand, overtemperatures as existing near the wall facilitate spontaneous nucleation. On reasons which become understandable in chapter 7, the curvature of cavities has a similar effect. The rougher the heating surface, the lower is normally the liquid overtemperature with respect to saturation; it is highest for carefully polished surfaces.

Another strange result is that a thermodynamically stable two-phase mixture has no uniform pressure over a sufficiently extended volume (see also chapter 6).

6. The locus of metastable states in diagrams

Before calculating the probability of forming a nucleus for bubbles or droplets we will show the location of such oversaturated states in an ordinary (p,v)- (pressure, specific volume) - diagram.

Of course, for a perfect gas, phase changes do not exist. We must at least consider the next refinement, the VAN DER WAALS gas, which gives good qualitative (not quantitative) comprehension. If we make all three variables p, v, T dimensionless by dividing them by their "critical" values (pertaining to the critical point), the new reduced variables, for simplicity anew named p, v, T, obey the state equation (the so-called equation of corresponding states, valid for all one-component systems)

$$p = \frac{8T}{3v-1} - \frac{3}{v^2} . \qquad (6.1)$$

T is the family parameter so that the isothermal lines are directly visible.

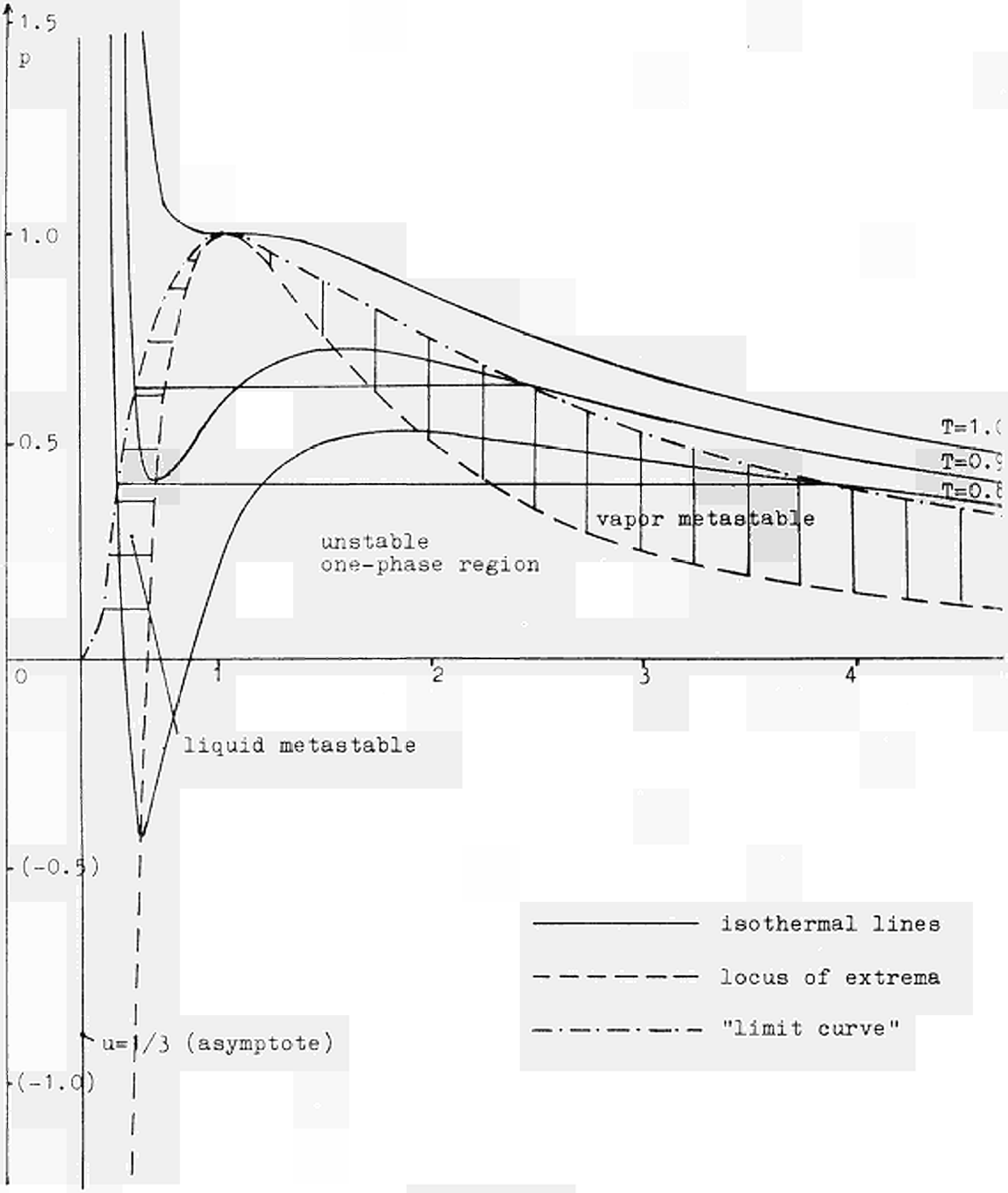

Figure 2

Regions of metastable states
in (p,v)-diagram

These isothermes exhibit the famous minima and maxima, as long as $T < 1$; the curve $T = 1$ has a horizontal point of inflexion. In the equilibrium thermodynamics, the left and right hand branches of each curve are connected by a straight horizontal line (constant pressure) which represents the states of two-phase mixtures. The left-hand branch means pure liquid, the right-hand branch means pure vapor. The p-level of these horizontal lines, indicating the saturation pressure to each temperature T, is found by equalizing the two areas between the complete isothermal line and the horizontal line. This condition is a consequence of CLAUSIUS-CLAPEYRON's equation.

The locus of the branchings of the horizontal saturation pressure lines from the original isothermal lines is called the "limit curve". Its explicit formulation cannot be given because of the third order degree in v of the above equation (6.1) (casus irreducibilis).

Another curve, the locus of all extrema of (6.1), can also be constructed. Its equation is

$$p_{ext} = \frac{3}{v_{ext}^{2}} - \frac{2}{v_{ext}^{3}} . \qquad (6.2)$$

Note that these curves may also be drawn for regions which are physically meaningless, e.g. for $v < 0$ or $p < 0$.

We now consider an isothermal state change when coming from the right-hand side. When reaching the saturation level, it is generally argued that one must now follow the straight line. All available state diagrams, also translated into any other pairs of state variables, show exclusively these two-phase states which are "stable in the large". They are characterized by the fact that the entropy s there has an absolute maximum for any given pair of other state variables.

Now, instead of entering the two-phase region when reaching the limit curve, we further follow the isothermal line in the (p,v)-diagram, according to (6.1) which leads at first to a higher pressure for the same specific volume, without partial condensation, of course. We find that these oversaturated vapor states are also stable but the corresponding entropy (for the same T,v-pair) is lower than for the two-phase state. The entropy has indeed a maximum also for these states but the maximum is a relative one. We call such states "metastable".

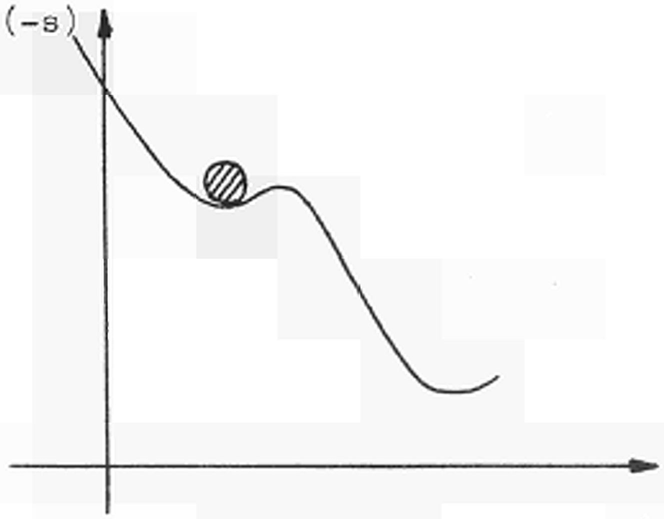

Figure 3
Metastable state (mechanical analogue)

Molecular or other perturbations which are sufficiently large to lower temporarily the entropy such that the dam surrounding the relative maximum is overpassed, let the vapor state drop down to the absolute entropy maximum, i.e. to the corresponding two-phase state (spontaneous condensation, sometimes in an explosive manner). The necessary perturbation amount will be roughly estimated in the next chapter. Note that the previously uniform pressure splits up into the two phase pressures, necessary because of boundary curvatures. Therefore, the horizontal two phase isothermal lines are in reality somewhat extended stripes according to the bubble or drop radii.

When further following the above isotherms, we pass through its maximum. The points on the other side must be unstable, because a volume diminution would lead to a pressure diminution, thus to an irresistible collapse.

After the minimum of the isotherms, anew a region of "metastable states" occurs (physically possible as far as we are not in the negative p-region). Here, the liquid is superheated and may evaporate spontaneously after a sufficiently large entropy perturbation. Finally, the limit curve is reached where the isothermal line continues with its normal stable liquid branch.

As a whole, we find that just the region between the limit curve and the curve joining the extrema of the isothermal lines is the region of metastable states.

7. The nucleation probability

For convenience, we speak about the nucleation of liquid droplets from a vapor; the bubble birth problem is similar.

As shown in chapter 5, the vapor pressure of a droplet is the larger, the smaller is the droplet. For a given oversaturation only such droplets tend to grow, the radius r of which is larger than a certain value r_N, whereas droplets with smaller r tend to re-evaporate. N symbolizes the number of molecules in the critical droplet.

Statistical mechanics theory gives an expression for the vapor pressure p_N of a droplet of N molecules:

$$x = \ln \frac{p_N}{p_\infty} = \frac{2\sigma V}{kTr_N} , \qquad (7.1)$$

where V is the volume of one molecule and k the BOLTZMANN constant. p_∞ and σ are defined in chapter 5. x may be our measure of "oversaturation".

We stated that a certain entropy diminution Δs is necessary for nucleation. Therefore we assume the nucleation frequency F to be proportional to

$$F = e^{-\frac{\Delta s}{k}} . \tag{7.2}$$

In order to calculate Δs to build a droplet of radius r_N, we generate it reversibly from its vapor:

a) removal of N molecules from the vapor volume,
b) expansion from p to p_∞,
c) condensation of the N molecules on a plane surface,
d) bending of the surface to the droplet surface S_N.

The works for a) and c) outweigh one another. The work for b) is $- NkT \ln \frac{p}{p_\infty}$ according to elementary theory and the contribution of d) is the surface work $+ \sigma S_N$.

Considering that

$$\frac{V}{r_N} = \frac{4\pi r_N^3}{3Nr_N} = \frac{S_N}{3N} , \tag{7.3}$$

we find with (7.1) that the work b) is equal to $- \frac{2}{3} \sigma S_N$. The whole work is

$$W = + \frac{\sigma S_N}{3} . \tag{7.4}$$

An equivalent heat amount Q must be removed from the system in order to ensure unchanged energy.

Hence, the corresponding entropy change is

$$\Delta s = \frac{Q}{T} = \frac{1}{3} \frac{\sigma S_N}{T} , \tag{7.5}$$

and the nucleation frequency is

$$F = C \cdot e^{-\frac{1}{3}\frac{\sigma S_N}{kT}} \quad . \tag{7.6}$$

Now, if we solve (7.1) with respect to r_N, the exponent of (7.6) proves to be

$$-\frac{1}{3}\frac{\sigma S_N}{kT} = -\frac{16\pi}{3}\frac{\sigma^3 V^2}{(kT)^3}\cdot\frac{1}{x^2} \quad . \tag{7.7}$$

Note the strange dimensionless parameter combinations occurring here, which are probably important for subsequent correlations.

For water of room temperature one gets approximatively

$$F = C \cdot e^{-\frac{116}{x^2}} \quad . \tag{7.8}$$

The logarithmic pressure ratio x (see 7.1) has an overwhelming effect on the nucleation frequency (exponent x^{-2}!).

If we expect a nucleation frequency in the order of 1 droplet per cm^3 and sec, F = 1, we get

$$x = \sqrt{\frac{116}{\ln C}} \quad . \tag{7.9}$$

If each molecular collision per cm^3 and sec leads to a nucleation, an admittedly crude hypothesis, the value of C would be roughly e^{58} for water with saturation pressure of 10^{-2} atmospheres at room temperature.

In this case, one gets $x = \sqrt{2}$, or $\frac{p_\infty}{p} = 4{,}12$.

The dependence of F upon x is so strong that already x = 1,5 would lead to $F = 10^3\ cm^{-3}\ sec^{-1}$. This means that the overpressure for spontanoues nucleation is as high as four times the saturation pressure. The corresponding water superheat would be in the order of 20°C, a value which is considerably higher than observed values. Nevertheless, for very accurately smoothed heating surfaces which must evidently be supposed here to eliminate cavity effects, the result is not quite unreasonable.

We will not stress so much the quantitative indications given here but the general structure of relationships coming into play.

8. General conclusions

We have seen that the vapor can be considered to be at saturation temperature in a boiling co-current flow. However, the liquid is superheated near the wall; its temperature falls towards the tube axis, intersecting or not the saturation temperature level, according to circumstances and to the upstream history.

The liquid superheat amount as well as the nucleation frequency depend on the surface roughness. The temperature difference ΔT occurring in chapter 3 to define the heat transfer coefficient is obviously the difference between wall temperature and that of the superheated liquid. For rough surfaces the liquid temperature profile is sufficiently flat so that no major problem occurs in defining a liquid temperature, but for smooth surfaces one could have troubles because of pronounced non - equilibrium states with latent vapor retention.

A number of phenomena has been qualitatively formulated which are hoped to provide better insight for subsequent dimension-analytical correlations. The title of this report already indicates the deliberate restriction to such a descriptive treatment. This way has been followed because a rigorous treatment is not possible, as said at the beginning, and because otherwise the theory should be underset by numerical values obtainable from measurements only. In order to perform the corresponding experiments suitably, the skeleton of the theory must already be available.

The author thinks it proper not to wait until a complete theory has outgrown from the above considerations but to communicate them at an early stage.

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>
> Alfred Nobel

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