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THE BALANCED OSCILLATOR TESTS IN SEFOR*

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Abstract

In SEFOR, Balanced Oscillator Tests will be performed to measure reactivity coefficients. Two types of Balanced Oscillator Tests are planned for SEFOR.

1st Balanced Oscillator Test (1st B.O.T.) in which, while the coolant temperature is kept constant, the transfer functions between reactivity and power and between power and coolant flow are measured.

2nd Balanced Oscillator Test (2nd B.O.T.) in which, while the reactor power is kept constant, the transfer function between reactivity and coolant temperature is measured.

Particular effort has been devoted to studying possible ways of obtaining balancing conditions in the reactor plant. Two different types of control are being considered.

1. Open Loop Control in which predetermined input signals will be given to the reactor plant in order to hold constant the coolant temperature (for the 1st B.O.T.) and the power (for the 2nd B.O.T.).

2. Closed Loop Control in which the balance condition (constancy of the coolant temperature for the 1st B.O.T. and power for the 2nd B.O.T.) is reached with the aid of closed loops control.

Both control methods are described in this paper and their performance characteristics are also analysed.

1. INTRODUCTION

In SEFOR (Southwest Experimental Fast Oxide Reactor) "Balanced Oscillator Tests" will be performed to measure reactivity coefficients and thermal parameters of fuel rods. Two types of "Balanced Oscillator Tests" are planned.

1st Balanced Oscillator Test (1st B.O.T.) in which, while the coolant temperature, Θ, is kept constant, the transfer functions between reactivity and power and between power and primary coolant flow are measured.

2nd Balanced Oscillator Test (2nd B.O.T.) in which, while the reactor power is kept constant, the transfer function between reactivity and coolant temperature is measured.

The description of these "Balanced Oscillator Tests" and the analysis of the obtainable results have already been the subject of previous publications (Ref. 1; 2; 3; and 4) and partially of another paper (Ref. 5) already presented to this conference. Here we want to examine these tests from the point of view of the possible ways of performing them in SEFOR. Fig. 1-1 shows the cooling system of the reactor plant. The heat produced in the core is removed by the primary coolant, Sodium, and is then transferred to the secondary Sodium circuit. From here the heat is rejected to the atmosphere by open circuit forced-air cooling. Fig. 1-2 shows a schematic block diagram of reactor transfer functions.

Two different types of balancing may be considered (fig. 1-2).

1. To choose the input oscillating signals of reactivity "Δk", primary coolant flow "Δw_p" and secondary coolant flow "Δw_s" in such a way that inlet and outlet coolant reactor temperatures remain constant ($\Delta\Theta_i = \Delta\Theta_{out} = 0$). This test is called 1st B.O.T. and the condition $\Delta\Theta_i = \Delta\Theta_{out} = 0$ is called "balance condition for the 1st B.O.T."
2. To choose the input oscillating signal Δk, Δw_p and Δw_s in such a way that $\Delta P=0$ or at least that $P(t)$ does not contain oscillations at the fundamental frequency. This test is called 2nd B.O.T. and the condition $P=0$ /or $P(t)$ not containing the fundamental harmonic/ is called "balance condition for the B.O.T."

2. METHODS OF PERFORMING THE BALANCED OSCILLATOR TESTS

Two different methods have been thought to be used to perform Balanced Oscillator Tests in SEFOR: They are

1. Open Loop Control System
2. Closed Loop Control System

With the "Open Loop Control System" (fig. 1.2) the operator sets predetermined input control values which produce balanced conditions in the plant. These required input signals are found by an analytical procedure carried out before the test is performed. This analytical procedure is described in paragraph 2.1. In the "Closed Loop System", instead, the balanced condition is reached with the aid of controllers and feedback loops.

At the present time it is intended to provide SEFOR at least with the open loop control system. The "Closed Loop Control System" is being studied at Karlsruhe. The type of "Closed Loop Control System" shown in this paper can be basically applied to any reactor, but the numerical evaluations refer to SEFOR. Open and Closed Loop Control Systems are described respectively in sections 2.1 and 2.2.

2.1 Open Loop Control System

2.1.1 Test Variables

In the Balanced Oscillator Tests (B.O.T.), there are three dependent variables which must be either oscillated or held constant, depending on the desired results. These variables are:

1. Reactor Power, P
2. Reactor Coolant temperature rise, $\Theta_{out} - \Theta_i$
3. Reactor coolant inlet temperature, Θ_i

Control of these variables is accomplished by oscillation of three independent variables:

1. Reactivity, Δk
2. Primary coolant flow, Δw_p
3. Secondary coolant flow, Δw_s

2.1.2 Effects of Oscillation

The ability to hold one or two variables constant while the others are oscillated is subject to the effects of oscillation on the dependent variables. Studies of these effects have been made on the analog model of the SEFOR reactor and show the following results:

1. When an independent variable is oscillated sinusoidally about its average value, the resulting oscillations of the dependent variables are also sinusoidal at the fundamental frequency, but with different phase and amplitude for each variable.
2. Oscillations at harmonic frequencies may also be present, but their amplitudes are small enough to be disregarded.
   The one exception to this is the coolant temperature rise oscillation caused by primary coolant flow oscillation. This causes a problem in the second B.O.T., which will be discussed later.
3. The amplitude of oscillation for a dependent variable is proportional to the amplitude of oscillation for the independent variable for the amplitudes of interest in the B.O.T.
4. The phase relationship between the two variables does not change with amplitude for a given set of test conditions.
5. The effects of oscillating more than one independent variable can be found by vector addition of the individual effects on a given dependent variable, using vector magnitude to represent amplitude and vector direction to represent phase angle.

These results then lead to the conclusion that a balanced oscillator test can be set up for the SEFOR reactor using an open loop control system, and this was verified on the analog model.

2.1.3 Method of Control

In the open loop control system for the SEFOR plant, each of the three independent variables is controlled from the rod oscillator device, thus maintaining identical frequencies for each variable. This device converts the rotating motion of a constant speed drive to vertical reciprocating motion through a Scotch-yoke mechanism. The speed and amplitude of this device are adjustable over a wide range, but will remain fixed during a test. The reciprocating motion is applied directly to a poison rod in the center of the core, thus achieving the desired sinusoidal oscillation of reactivity.

The motion of the rod oscillator is also used to generate control signals for the primary and secondary coolant pumps. The phase and amplitude of these signals can be independently adjusted so as to achieve the desired balanced conditions. Thus, all three independent variables will be oscillated at identical frequencies with independently adjustable phase and amplitude.

2.1.4 First Balanced Oscillator Test

Simulated tests of the open loop system on an analog computer have shown that, for the first balanced oscillator test, coolant temperatures can be balanced with an accuracy of ± 1 1/2 ^{o}F at 20 MW when the proper phase and amplitude are set for each independent variable. The method used to determine these phase and amplitude values is to first measure the effects of oscillating one independent variable at a time. Typical values for these effects are shown by the curves in Appendix I. Determination of these effects can be done experimentally in a minimum amount of time, since analog results have shown that less than 3 cycles are required to achieve stable conditions after starting or changing an oscillation. Once the individual effects are known, vector diagrams can be drawn as shown in Figures 2.1-1; 2.1-2 and 2.1-3. In Figure 2.1-1 the vector sums for inlet temperature and temperature rise have been set to zero by varying the primary and secondary coolant flow vectors. This can be done quite easily by using primary coolant flow to balance temperature rise and secondary coolant flow to balance inlet temperature. A computer code has been written to solve the required vector equations. A similar procedure could be used experimentally. However, the analytical procedure is obviously less costly, and analog results have indicated that analysis will yield balanced conditions.

2.1.5 Second Balanced Oscillator Test

The independent vectors for the second B.O.T., in which power is held constant while reactivity is oscillated, can be found by an analytical procedure similar to that used in the first B.O.T., using the same data for effects of individual oscillators. Since only one parameter, reactor power, is required to be balanced, it is only necessary to oscillate one additional parameter, primary coolant flow. However, oscillation of the secondary coolant flow may also be used to reduce the control sensitivity required or to reduce the flow amplitude required for a given reactivity amplitude.

Figure 2.1-2 shows a typical vector diagram for the second B.O.T. in which the secondary coolant flow is oscillated so as to hold the reactor inlet temperature constant. However, this requires a rather large amplitude for the secondary flow oscillation which is undesirable. Figure 2.1-3 shows the vector diagram for the same experiment, but with the inlet temperature oscillating. In this case the secondary coolant flow vector was arbitrarily chosen to illustrate one of the many possible choices. The final choice for this vector will be based primarily on the ease of reaching a balanced condition if the initial vector choice is slightly in error.

2.1.6 Control Sensitivity Requirements

In order to reduce the reactivity effect due to the reactor power within the limits of ± 0,15 % it is necessary to balance the reactor power within ± 0,02 MW in the second B.O.T.. Comparison of this requirement to the requirement of balancing temperatures to approximately 1 % of the temperature rise (1 1/2 ^{o}F) in the first B.O.T. indicates that the second B.O.T. will be more difficult to balance than the first. An estimate of the control sensitivity required to balance a parameter can be obtained by calculating the effect of small changes in the phase or amplitude of an independent variable. As shown by the vector diagrams in Figure 2.1-4a and 2.1-4b, these effects can be calculated for a balanced variable from the equations:

$$\Delta E = A_o \cdot C \cdot (\Delta A / A_o) \text{ for amplitude changes, and}$$

$$\Delta E = A_o \cdot C \cdot \sin \Delta\Theta \text{ for phase angle changes less than 10 degrees}$$

where:

ΔE = change in amplitude of balanced variable.

ΔA = change in amplitude of independent variable.

A_o = initial amplitude of independent variable.

C = ratio of dependent variable amplitude to independent variable amplitude

$\Delta\Theta$ = change in phase angle of the independent variable.

Comparison of these effects to the allowable limits of unbalance will then give an indication of the control sensitivity required to obtain a balanced condition. Table 1 shows these values for the vector diagrams shown in Figures 2.1-1; 2.1-2 and 2.1-3. Changes of 10 % in amplitude or 5.7 degrees in phase angle, were chosen for the table values because the resulting changes in amplitude of the dependent variables are equal.

2.1.7 Harmonics in the Second B.O.T.

It has been observed that in the second B.O.T., sinusoidal oscillation of the primary coolant flow introduces a harmonic of significant amplitude into the reactivity feedback. Since the reactivity feedback is balanced against the fundamental sine wave of the reactivity input, this results in a power oscillation at twice the fundamental frequency with an amplitude approximately equal to or slightly greater than the allowable limits for amplitude of the fundamental. (Ref. 2).

The source of the harmonic can be found in the equation relating coolant flow and temperature rise at constant power:

$$\Theta_{out}-\Theta_i = P_o/w_p \qquad (1)$$

For sinusoidal flow oscillations,

$$w_p = w_{po} + A \sin \omega t; \qquad (2)$$

$$\Theta_{out}-\Theta_i = (\Theta_{out}-\Theta_i)_o+f(t); \qquad (3)$$

$$P_o = w_o \cdot (\Theta_{out}-\Theta_i)_o ; \qquad (4)$$

Solving these equations for the temperature oscillation, f(t), yields

$$f(t) = \frac{-A \sin \omega t}{w_o+A \sin \omega t} (\Theta_{out}-\Theta_i)_o \qquad (5)$$

This function contains many harmonics of generally small amplitudes, but the first harmonic (cos 2 ωt) has a significant amplitude. Mathematically, this problem can be eliminated by using a function generator to correct the coolant flow oscillation so that it is

$$w = w_o + \frac{B \sin \omega t}{1-B \sin \omega t} w_o$$

This function was found by arbitrarily choosing a temperature rise function of

$$(\Theta_{out}-\Theta_i) = (\Theta_{out}-\Theta_i)_o\,(1-B \sin \omega t)$$

Tests on the analog model have indicated that correction of the primary flow oscillation in the above manner does reduce the amplitude of the first harmonic. However, an increase in the amplitude of the second harmonic was also noted, which may be due to transient effects in the IHX (Ref.2). Thus it appears that final elimination of the harmonics in the power oscillation will have to be done by analytical methods in the data-reduction process.

2.1.8 Time Required to Achieve a Balanced Condition

The time required to obtain a balanced condition in either B.O.T. will depend on the oscillator frequency, since two to three cycles are required to achieve stable conditions after the oscillator is started or a change is made. Setting of the desired phase and amplitude should be accomplished in a minimum amount of time through proper calibration and test procedures. The oscillator settings determined by vector diagrams for the first B.O.T. should produce the required balanced conditions with no further adjustments required. However, the second B.O.T. requires more accurate control settings, and is expected to require an average of 10 cycles per test to obtain the final balanced condition.

2.1.9 Conclusions for the open loop control system

1. Analog studies indicate that it will be possible to perform balanced oscillator tests on SEFOR through controlled oscillations of reactivity, primary coolant flow, and secondary coolant flow.
2. An open loop control system in which the phase and amplitude of the independent variable oscillations are manually adjusted will provide adequate control to obtain balanced conditions within reasonable limits.
   a) Balanced parameter limits of ± 1 1/2 ^{o}F in the first B.O.T. appear to be reasonable.
   b) The balanced parameter limit of ± 0,1 % of reactor power in the second B.O.T. are more difficult to achieve, but there is a hope of approaching this limit because the assumed error for phase and amplitude in table 1 are pessimistic.

3. A vector analysis method of combining the effects of oscillating each independent variable can be used to determine the oscillator phase and amplitude requirements for each balanced oscillator test.

4. It is expected that 10 cycles or less will be required to achieve balanced conditions after the pre-calculated amplitudes and phase angles are set.

5. The open loop control system provides a simple and effective mean of running a balanced oscillator test and eliminates control problems normally associated with large phase lags and parameter inter-actions.

Table 1

OSCILLATION AMPLITUDE OF A BALANCED VARIABLE
CAUSED BY CHANGING AN INDEPENDENT VARIABLE
BY 10 % OF ITS AMPLITUDE OR
BY 5.7 DEGREES IN PHASE ANGLE

B.O.T.	Independent Variable	Initial Amplitude	BALANCED VARIABLE		
			Temperature Rise	Inlet Temperature	Power
I	W_P	5.8 %	0.9 °F	0.5°F	--
	W_S	9.5 %	0.1 °F	0.4°F	--
	ΔK/K	10 ¢	0.9 °F	0.2°F	--
II	W_P	8.5 %	--	0.7°F	.22 MW
	W_S	*19.1 %	--	0.7°F	.11 MW
	ΔK/K	3 ¢	--	0.1°F	.14 MW
II	W_P	4.1 %	--	(Oscillating	.10 MW
	W_S	6.0 %	--	inlet	.03 MW
	ΔK/K	3 ¢	--	Temperature)	.13 MW

Notes: (1) Average Reactor Power = 20 MW

(2) Average Coolant Flow Rate = 80 % of Design Value (4310 GPM)

(3) Maximum Allowable Amplitude of:
Balanced Temperature = 1 1/2 °F;
Balanced Power = 0.02 MW.

* Maximum Allowable Flow Amplitude is 20 %.

2.2 Closed Loop Control System

2.2.1 Generals

Fig. 2.2-1 shows a schematic block diagram of the connections of the Closed Loop Control System to the plant in the case of the 1st B.O.T.

The reactor is fed with a sinusoidal reactivity signal at frequency "f_o"

$$\Delta K = \Delta K_m \sin 2\pi f_o t \qquad (1)$$

which is produced by the "Frequency and Sinus Function Generator". The input signal to the "Controller Nr. 1" is the difference between the signal of outlet and inlet reactor coolant temperatures, $(\Theta_{out}-\Theta_i)$. The output signal acts on the "Primary Pump", which will tend to change the primary Sodium flow in such a way that

$$\Theta_{out}-\Theta_i = \text{const.} \qquad (2)$$

The input signal to the "Controller No. 2" is the outlet primary heat exchanger Sodium temperature, T_{out}. Its output signal acts on the "Secondary Pump" which will tend to change the secondary coolant flow in such a way that

$$T_{out} = \text{const.} \qquad (3)$$

The "Transfer Function Analyser" (T.F.A.) measures the transfer functions respectively between power (P) and reactivity (ΔK) and between primary coolant flow (w_p) and power (P).

Fig. 2.2-2A shows a schematic block diagram of the connections of the Closed Loop Control System to the plant in the case of the 2nd B.O.T.

The plant is fed with a sinusoidal signal at frequency f_o on the primary pump

$$\Delta w_p = \Delta w_{pm} \sin (2\pi f_o t) \qquad (4)$$

The secondary pump can either be controlled to keep "T_{out}" constant (as in the 1st B.O.T.) or be fed with a sinusoidal signal:

$$\Delta w_s = \Delta w_{sm} \sin (2\pi f_o t + \alpha) \qquad (5)$$

with Δw_{pm} and α chosen in such a way they produce the maximum possible change of the reactor average coolant temperature, $\Delta\bar{\Theta}$, compatible with the safety and the limitations of the plant. Both possible control schemes of the secondary pump have not been shown in Fig. 2.2-4A. The input signal to the "Controller" is the power P, which is measured by a flux detector. Its output signal acts on a control rod to produce a change of reactivity, Δk, which will tend to keep the power constant:

$$P = \text{const.} \qquad (6)$$

The T.F.A. measures the transfer function between the reactivity (Δk) and the average coolant temperature ($\bar{\Theta}$) so defined:

$$\bar{\Theta} = \frac{\Theta_i + \Theta_{out}}{2} \qquad (7)$$

An alternative to the scheme of Fig. 2.2-4A is that of Fig. 2.2-2B in which the power P is kept constant by acting on the primary coolant flow.

The two control loops of fig. 2.2-1 and those of fig. 2.2-2A and 2.2-2B can be schematically described by the diagram shown in fig. 2.2-3. In fig. 2.2-3, "U" is the controlled variable which is intended to be kept constant. When an input signal "I" is introduced in the plant, this will produce a change "U_1" of U through the transfer function $P_1(s)$. The controlled variable "U" is measured by the Feedback circuit which has the transfer function "H(s)". The output signal "Y" from this circuit is compared to the reference "R" and the difference "ε" feeds the "Controller". The output signal "γ" from the Controller acts on the plant and produces a change "U_2" of "U" which tends to compensate for the previous change "U_1" due to the input signal "I". The plant transfer function between U_2 and γ is indicated by $P_2(s)$.

The Controller consists of two parts which we call "Regulator" and High Gain Unit (H.G.U.), having, respectively, transfer functions G(s) and M(s).

The function of the "Regulator" is to amplify the input signal "ε" and to correct it (by means of a corrective network) to get phase advance of "ε" and stability.

The function of the "High Gain Unit" is to suppress the oscillations of the controlled variable "U" at the frequency "f_o" at which the B.O.T. is performed. This means that any disturbance "U_1" of U at the frequency "f_o" is compensated by an oscillation "U_2" having the same amplitude as U_1. The way in which the H.G.U. operates will be described in the following paragraph.

The H.G.U. can be connected or disconnected from the loop operating the switch "S_w" (fig. 2.2-3) without affecting the stability of the system.

2.2.2 Basic analytical considerations

Looking at fig. 2.2.3, we can write the following equations in the Laplace domain:

$$U = U_1 + U_2 \quad (1)$$

$$U_2 = \gamma \, P_2(s) \quad (2)$$

$$\gamma = \eta + \lambda \quad (3)$$

$$\lambda = \gamma \, M(s) \quad (4)$$

$$\eta = \varepsilon \, G(s) \quad (5)$$

$$\varepsilon = R - Y \quad (6)$$

$$Y = U \, H(s) \quad (7)$$

From eqs. 2 to 7 we get (for R a const):

$$\frac{U_2}{U} = - K(s) \quad (8)$$

where

$$K(s) = W(s) \frac{1}{1-M(s)} \quad (9)$$

and

$$W(s) = P_2(s) \cdot H(s) \cdot G(s) \quad (10)$$

From eqs. 1 and 8, we get

$$\frac{U}{U_1} = \frac{1}{1+K(s)} \quad (11)$$

The transfer function of eq. 11 is called "closed loop transfer function", while that of eq. 8 is called "open loop transfer function". The reason of the second denomination is due to the fact that eq. 8 would represent the transfer function of the loop supposed to be ideally cut at the point where U_1 and U_2 are added.

From eqs. 8 and 11 it is clear that the properties of the "closed loop transfer function $\frac{U}{U_1}$ can be derived by analysing the open loop transfer function "K(s)". The second control loop of the 1st B.O.T. is the worst from the point of view of stability because of the time constants involved. We shall therefore discuss this loop here.

In this case in fig. 2.2-3, "U" would represent the primary heat exchanger outlet coolant temperature "T_{out}" and "γ" the signal to the secondary pump.

"$P_2(s)$" includes the transfer function of the secondary pump $(\frac{1}{1+2s})$, that between primary heat exchanger outlet coolant temperature and secondary coolant flow and the 1.5 sec. time constant of the lower mixing of the heat exchanger.

The feedback transfer function H(s) includes the time constant of the thermocouple (supposed to be 1 sec) and the 1 sec time lag between the place at which the thermocouple is mounted and the outlet of the primary heat exchanger. This situation would correspond to a coolant flow equal to 80 % of its rated value.

Curve No. 1 of Fig. 2.2-4 shows the polar diagram of the frequency response of the function

$$W(j\,2\pi f) = P_2(j\,2\pi f)\,H(j\,2\pi f)\,G(j\,2\pi f) \qquad (12)$$

where

$$G(s) = G_o \frac{(1+4s)}{(1+0.2s)} \qquad (13)$$

with G_o chosen in such a way that

$$W(o) = 1 \qquad (14)$$

Let us suppose for the moment that the H.G.U. is a "Low Pass Filter" (L.P.F.). If the input signal "γ" is at low frequency, it will pass through the filter and the output "λ" will be added to the signal "η" from the "Regulator". If instead "γ" is at higher frequency, it will be attenuated and shifted by the filter and therefore the output signal "λ" will not have practically any regulating action.

This means that the controller would be able to give a precise control at low frequencies, while at higher frequencies instead would become less accurate.

This loss of accuracy is due to two causes :

a) the higher the frequency, the more delayed in phase is the signal "λ" in respect to "η"

b) the higher the frequency, the bigger is the attenuation between "λ" and "γ".

The first cause can be eliminated by making a phase correction with a device which changes the phase without changing the amplitude, for example a pure time delay.

If we incorporate in our H.G.U. a memory in cascade to the "Low Pass Filter", the frequency response of the transfer function M(s) would be of the type

$$M(j\ 2\pi\ f) = A \frac{\exp(-j\ \Psi\ f/f_o)}{(1+j\ f/f_m)^n} \qquad (15)$$

where "f_m" is the cut-off frequency of the filter.
The angle "Ψ" must be chosen in such a way that, at the frequency "f_o" of the experiment, the sum of "Ψ" and of the phase shift "ϕ_o" of the filter gives 2π, that is

$$\Psi + \phi_o = 2\pi \qquad (16)$$

The curve of Fig. 2.2-5 shows the polar diagram of the frequency response of

$$\frac{1}{1-M(j\ f/f_o)} \qquad (17)$$

as function of $\frac{f}{f_o}$ where M(s) is given by eq. 15 with

$$n=4\ , \quad \frac{f_o}{f_m} = 0.2 \quad \text{and } A=1 \qquad (18)$$

This curve shows a high gain at the frequency "f_o". At the higher harmonics $2f_o$, $3f_o$ etc., the gain presents also a maximum value which is becoming smaller as the order of the harmonic increases.

If we now introduce this modified H.G.U. in our control loop, the frequency response of the open loop transfer function K(j 2π f) becomes the curve No. 2 of fig. 2.2-4 if the system is set at the frequency

$$f_o = 0.02 \text{ cps.} \qquad (19)$$

The gain at frequency "ϕ_o" is about 12.5 which means that the amplitude of the oscillations of the controlled variable "U" at this frequency is reduced to $\frac{1}{12.5}$ = 8 % of that of the disturbance "U_1". The curve No. 2 of fig. 2.2-4 shows (according to the Nyquist criterion) that the closed loop will be stable because the open loop transfer function does not encircle the point "-1". This conclusion can be easily drawn if one thinks that stability means that the characteristic transcendental equation of the closed loop

$$W(s)\ \frac{1}{1-M(s)} + 1 = 0 \qquad (20)$$

must not have any root with real part positive. If one puts in eq. 20

$$S = \alpha + j\ 2\pi\ f \qquad (21)$$

for $\alpha > o$ one realizes that for the same value of "f" the function W(s) becomes smaller in modulus and phase shift. This means that for a given frequency the corresponding point on the curve (curve No. 1 of fig. 2.2-4) tends to move from the left to the right (as indicated by the small arrow), while the curve tend to squeeze itself towards the origin. At the same time the function $1/[1-M(s)]$ of fig. 2.2-5 tends for $\alpha > o$ to squeeze itself towards the point 1 . Because of all these effects, the envelope of the lobes of curve No. 2 in fig. 2.2-4 for $\alpha > o$ tends to squeeze itself towards the origin. This behaviour of the function $W(s)/[1-M(s)]$ seen on the Nyquist diagram (curve No. 2 of fig. 2.2-4) ensures us that the characteristic equation 20 is always for $\alpha > o$ different from -1

$$W(s)\ \frac{1}{1-M(s)} \neq -1 \qquad (22)$$

which means that the system is stable. In order to improve the accuracy of the system, one can use a L.P.F. with a damping factor ζ different from 1. The frequency response of the transfer function "M(s)" of the H.G.U. would be of the type

$$M(j\ 2\pi\ f) = A\ \frac{\exp(-j\ \psi\ f/f_o)}{\left[1+2j\ \zeta\ f/f_m-(f/f_m)^2\right]^n} \qquad (23)$$

The damping faktor ζ must be chosen in such a way that the modulus of $M(j\ 2\pi\ f)$ has its maximum value at $f = f_o$. Then "A" is chosen so that the modulus "C" of $M(j\ 2\pi f)$ at $f=f_o$ is as close as possible to 1.

The curve of fig. 2.2-6 shows the polar diagram of the equation 23 as function of f/f_o with

$$n=1; \quad \frac{f_o}{f_m} = 0.5\ ; \quad \zeta = 0.6\ ; \quad C = 0.98. \qquad (24)$$

The gain "A" of H.G.U. at frequency "f_o" would be

$$A = C\left[1-2(1-2\zeta^2)\left(\frac{f_o}{f_m}\right)^2 + \left(\frac{f_o}{f_m}\right)^4\right]^{n/2} = 0.94 \qquad (25)$$

Curve No. 2 of fig. 2.2-7 is the polar diagram of the open loop frequency response $K(j\ 2\pi\ f)$ at $f_o = 0.02$ cps. in the case in which $M(j\ 2\pi\ f)$ is defined by eqs. 23

and 24. The system also in this case is stable and the gain at $f=f_o$ has become about 50, which means a precision of about 2 %.

Preliminary work has been carried out on the analog computer just to see if this type of control system would have major troubles or not. The model used was an old model of the SEFOR plant and the control system was not optimized.

The 1st B.O.T. was carried out at a frequency of 0.0025 c.p.s. (fig. 2.2-8). The amplitude of the input reactivity signal was 10 ¢. The trace of the average coolant temperature Θ is shown: its behaviour is in agreement with the results of the simplified analysis developed in Appendix 2. The control scheme adopted in fig. 2.2-8 is slightly different from the control scheme of fig. 2.2-1. In fig. 2.2-8 the input signal to the first controller is the reactor outlet coolant temperature and not the difference between reactor outlet and inlet coolant temperature.

The 2nd B.O.T. was also carried out on the analog computer at a frequency of 0.0025 c.p.s. (fig. 2.2-9). Primary (w_p) and secondary (w_s) coolant flows were oscillated in phase and with an amplitude equal to 10 % of their rated values. Fig. 2.2-10 shows the trace of the power "P": its behaviour also in this case is in agreement with the results of the simplified analysis developed in Appendix 2. In both the Θ and P traces of figs. 2.2-8 one can observe a change of the average value at the time at which the switches "B" or "C" are operated. This change of the average values were due to the fact that the control loops and the memories were not exactly set at the same d.c.level as that of the circuit simulating the plant.

2.2.3 Final Comments on the Closed Loop Control System

The type of Closed Loop Control System which has been described in the preceeding paragraphs has the following characteristics.

1. It allows to reach a very high precision at the frequency "f_o" of the experiment. This is obtained by setting the gain of the H.G.U. at f_o as near as possible to 1 and the phase delay "ψ" of the memory in such a way that:

$$\psi + \phi_o = 2\pi \qquad (1)$$

   where ϕ_o is the phase shift of the L.P.F. at the frequency "f_o". The precision is limited by the practical limitations of carrying out these two settings. "C" can be set within ± 1 %. If we choose for "C" the value 0,98, the open loop gain will be 50 and therefore the error "E" is 2 %, i.e., the oscillation of the controlled variable at frequency "f_o" will be reduced to 2 % of the disturbance. This is valid if the phase in supposed to be set perfectly. Fig. 2.2-10 shows the error "E" as function of the difference "$\delta\psi$" between the set value of ψ and its corrected value (see Appendix 3). It appears that "E" is sensitive to this phase setting error, $\delta\psi$. The memory will have a paper tape which moves under the writing and the reading heads. The tape will have a line of holes and the distance between hole and hole will correspond to an angle of 1 degree. In this way 360^o will be given by 360 holes. The value of ψ is set by choosing the right length of the tape between the two heads, that is, by counting the right number of holes. The system is by itself capable to have a sensitivity of ± 0.5 degree. The precision in setting "ψ" will therefore depend upon the way in which the calibration of memory plus filter is carried out. The reproducibility of the setting of ψ is perfect.

2. The open loop gain drops as the frequency moves a little from the selected frequency "f_o". Synchronization between the tape speed and the frequency "f_o" is therefore required. The movement of the tape must be derived by the "Frequency and Sine Function Generator", shown in the schemes of figs. 2.2-1; 2.2-2A and 2.2-2B.

3. The system is capable to provide a high open loop gain at very low frequencies (figs. 2.2-4 and 2.2-6) which means that it can cope with the drift of the plant.

   The time needed by the system to reach the balance condition, that is, to compensate for the disturbances U_1 (2.2-3), can be evaluated as follows. We can say that an any new cycle the output U will be reduced approximately by a factor (Appendix 2)

$$|1 + W(j\ 2\pi\ f_o)| \qquad (2)$$

   In the case of the 2nd loop of the 1st B.O.T., we have (fig. 2.2-6)

$$|1 + W(j\ 2\pi\ 0.02)| = 1.85 \qquad (3)$$

   which means that the amplitude of the temperature oscillation will be reduced in 6 cycles to about 2,5 % of its initial value.

Looking at fig. 2.2-6, we see that the modulus of the functions $K(j\ 2\pi\ f)$ and $W(j\ 2\pi\ f)$ can be increased by a factor of 1.5 without having problems of stability. If this is done, the reduction factor becomes 2.4 instead of 1.5 (as given by eq. 3) and the same reduction of amplitude to 2,5 % will be reached in only 4 cycles.

4. It is very interesting to notice that, after few cycles, the memory has already instored the right corrective signal so that the control loop can even be open while the tape continues to feed the plant with the right corrective signal. In this case it is more convenient to have a second reading head (at 360^{o} from the writing head) which gives the signal to the writing head in such a way that the signal remains in the tape always unchanged. The first reading head will continue to feed the plant. This feature seems valuable if one plans to repeat a B.O.T. In this case the right corrective signal already exists instored in the tape which can feed the plant directly. An additional control loop able only to cope with the drift of the plant could be added to the system. For a better precision it would be convenient to have this second loop working in parallel to the tape, even when the tape is recording the corrective signal.

5. For safety purposes the control system must have the following two features:

   a) The controller must be designed in such a way that the output signal cannot in any case exceed some extreme values fixed by Safety Considerations.

   b) A device must be incorporated which switches the controlling variable (coolant flow or reactivity) to a fixed constant value before the signal reaches one of the above mentioned extreme values. This device also gives an alarm signal.

6. Other types of closed loop control systems are under investigation at Karlsruhe. We mention here particularly systems having a Band Pass Filter either directly on the main control line or as a positive feedback on it.

7. The closed loop control system has the following properties.

   a) It allows to obtain the balance condition with a very good precision.

   b) The closed loop control system releases the operator of the tediousness of carrying out the experiments and of making all the analysis which precedes them as shown in para 2.1. Reactor operating time will be reduced in the second B.O.T. where the balanced power requires greater precision. The balance condition is obtained in a lower number of cycles than that of the open loop. The time saved is not only that needed to reach the balance condition, but also that needed to evaluate the individual transfer function whose knowledge is necessary to carry out the vector analysis shown in section 2.1.

3. REFERENCES

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Appendix I

OSCILLATOR EFFECTS

The curves in this appendix show the effect on each of three dependent variables produced by individual oscillation of each of three independent variables. These data are used to construct vector diagrams and thus determine control settings required for balanced conditions.

It should be noted that phase angle is defined as the lag angle (time) between maximum values of the dependent and independent variables. This definition of phase angle was chosen to avoid confusion in the use of the data for vector diagrams.

The primary cold leg temperature refers to the temperature at the IHX outlet as measured with a thermocouple having a 5-sec time constant. The core temperature rise refers to the actual value of core outlet temperature minus core inlet temperature. Reactor power refers to neutron flux.

Appendix II

EVALUATION OF THE BEHAVIOUR OF THE CONTROLLED VARIABLE IN THE TIME DOMAIN

Let us consider the closed loop control system of fig. 2.2-3. We write eq. 11 of sect. 2.2.2

$$\frac{U}{U_1} = \frac{1}{1+K(s)} \tag{1}$$

where

$$K(s) = W(s)\,\frac{1}{1-M(s)} \tag{2}$$

and

$$W(s) = P_2(s)\ H(s)\ G(s) \tag{3}$$

We suppose that $U_1(t)$ is a sinusoidal function in the time domain. We have

$$U_1(t) = U_o \sin 2\pi\, f_o\, t \tag{4}$$

In the Laplace domain we have

$$U_1 = U_o\,\frac{2\, f_o\, \pi}{s^2+(2\pi\, f_o)^2} \tag{5}$$

Eq. 1 becomes

$$U = U_o\,\frac{2\pi\, f_o}{s^2+(2\pi\, f_o)^2}\ \frac{1}{1+W(s)\,\frac{1}{1-M(s)}} \tag{6}$$

From eq. 6 in the time domain we get

$$U(t) = U_o L^{-1}\left\{\frac{2\pi\, f_o}{s^2+(2\pi\, f_o)^2}\ \frac{1}{1+W(s)\,\frac{1}{1-M(s)}}\right\}\ , \tag{7}$$

where the symbol L^{-1} indicates antitransformation.

We shall solve eq. 7 in the case

$$M(s) = AF(s)\ \ \exp.(-\,\psi\, s/2\pi\, f_o) \tag{8}$$

where F(s) is the transfer function of the L.P.F.

Taking into account eq. 8, eq. 6 becomes

$$U = U_o \frac{2\pi f_o}{s^2+(2\pi f_o)^2} \frac{1}{1+W(s)} \frac{1-AF(s)\exp(-\psi s/2\pi f_o)}{1-[AF(s)\exp(-\psi s/2\pi f_o)]/(1+W(s))} \qquad (9)$$

Eq. 9 can be written as follows

$$U = U_o \frac{2\pi f_o}{s^2+(2\pi f_o)^2} [1-AF(s)\exp(-\psi s/2\pi f_o)] \sum_{n=1}^{n=\infty} \left\{ (-1)^n \frac{[AF(s)]^{n-1}}{[1+W(s)]^n} \exp\left[-(n-1) \frac{\psi s}{2\pi f_o}\right]\right\} \qquad (10)$$

$$U = U_o \frac{2\pi f_o}{s^2+(2\pi f_o)^2} \sum_{n=1}^{n=\infty} \left\{(-1)^{n-1} \frac{[AF(s)]^{n-1}}{[1+W(s)]^n} \left[\exp\left[-(n-1)\frac{\psi s}{2\pi f_o}\right] - AF(s)\exp(-n \frac{\psi s}{2\pi f_o})\right]\right\} \qquad (11)$$

We shall antitransform eq. 11 in the particular case

$$A.F(s) = 1 \qquad (12)$$

$$W(s) = W_o = \text{const.} \qquad (13)$$

$$\psi = 2\pi \qquad (14)$$

Taking into account eqs. 12; 13 and 14, eq. 11 becomes

$$U = U_o \frac{2\pi f_o}{s^2+(2\pi f_o)^2} \sum_{n=1}^{n=\infty} \left\{(-1)^{n-1} \frac{1}{(1+W_o)^n} \left[\exp[-(n-1) \frac{s}{f_o}] - \exp(-n \frac{s}{f_o})\right]\right\} \qquad (15)$$

The antitransform of eq. 15 is:

$$U(t) = U_o \sin(2\pi f_o t) \sum_{n=1}^{n=\infty} \left\{\frac{1}{(1+W_o)^n} [1(t-\frac{n-1}{f_o}) - 1(t-\frac{n}{f_o})]\right\} \qquad (16)$$

where

n = "nth" oscillation

and

1(t) indicates step function.

Eq. 16 is shown in fig. 2.2-11. The controlled variable U(t) oscillates with an amplitude which is decreasing with the time and taking the following values:

1st oscillation: $\frac{U_o}{1+W_o}$

2nd oscillation: $\dfrac{U_o}{(1+W_o)^2}$

"n"th oscillation: $\dfrac{U_o}{(1+W_o)^n}$

At any cycle, therefore, the oscillation decreases by a factor $(1+W_o)$. In the real case this factor will be approximately

$$|1+W(j\ 2\pi\ f_o)|$$

if the phase shift of $W(j\ 2\pi\ f_o)$ is not too large.

Appendix 3

DEPENDENCE OF THE PRECISION UPON THE ERROR "$\delta\psi$" IN SETTING THE PHASE

The error "E" of the controlled variable "U" is given by eq. 11 of **sect.** 2.2.-2

$$E = \left|\frac{U}{U_1}\right| = \left|\frac{1}{1+K(j\ 2\pi\ f_o)}\right| \approx \left|\frac{1}{K(j\ 2\pi\ f_o)}\right| = \left|\frac{1}{W(j\ 2\pi\ f_o)}\right| \cdot \left|1-C\ e^{j\delta\psi}\right| \qquad (1)$$

where C is defined by eq. 25 of para. 2.2.-2.

Eq. 1 can be written as follows :

$$E = \left|\frac{1}{W(j\ 2\pi\ f_o)}\right| \cdot \left|1-C\ \cos\ \delta\psi + j\ C\ \text{sind}\ \delta\ \psi\right| = \left|\frac{1}{W(j\ 2\pi\ f_o)}\right| \sqrt{(1-C\ \cos\ \delta\psi)^2+(C\sin\delta\psi)^2} \qquad (2)$$

Since

$$\cos\ \delta\ \psi \simeq 1 \qquad (3)$$

and

$$\sin\ \delta\ \psi \simeq \delta\ \psi \qquad (4)$$

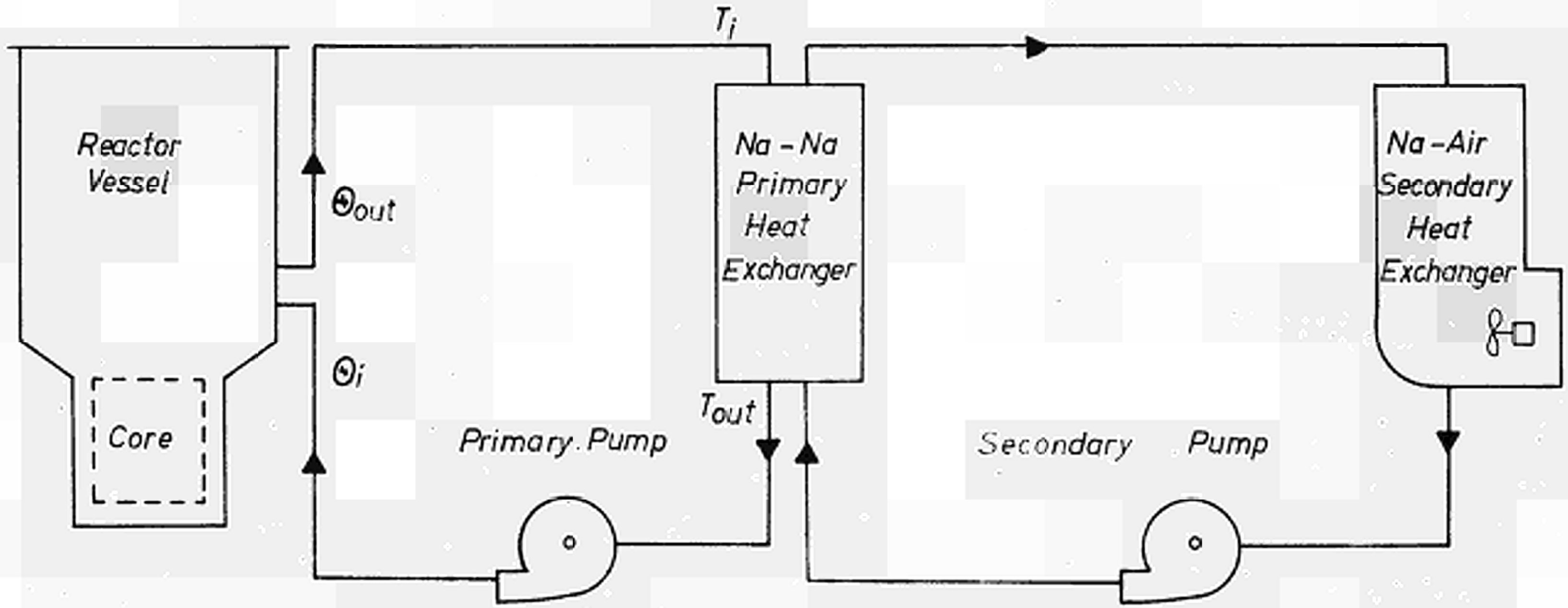
eq. 2 becomes

$$E \simeq \frac{1-C}{|W\ (j\ 2\pi\ f_o)|} \sqrt{1+\left(\frac{C}{1-C}\ \delta\psi\right)^2} \qquad (5)$$

Fig. 2.2.-10 shows the ratio

$$\frac{E}{E_{id}} = \frac{E|W(j\ 2\pi\ f_o)|}{1-C} = \sqrt{1+\left(\frac{C}{1-C}\ \delta\psi\right)^2} \qquad (6)$$

as function of "$\delta\psi$" for different values of "C".



Θ_i = reactor inlet temperature

Θ_{out} = reactor outlet temperature

T_i = Prim. Heat Exch. inlet temperature

T_{out} = Prim. Heat Exch. outlet temperature

Fig. 1-1 SCHEMATIC REACTOR FLOW DIAGRAM

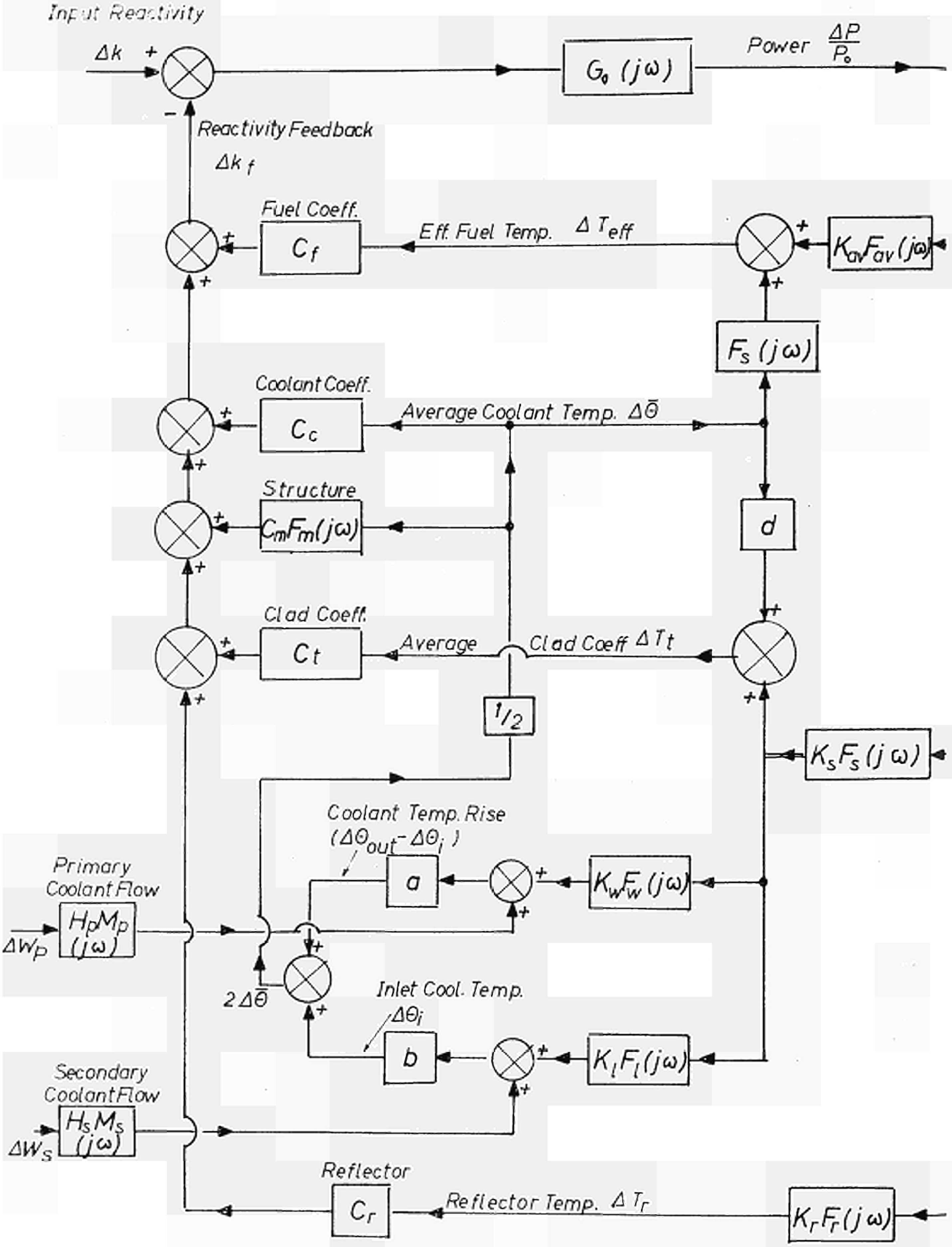
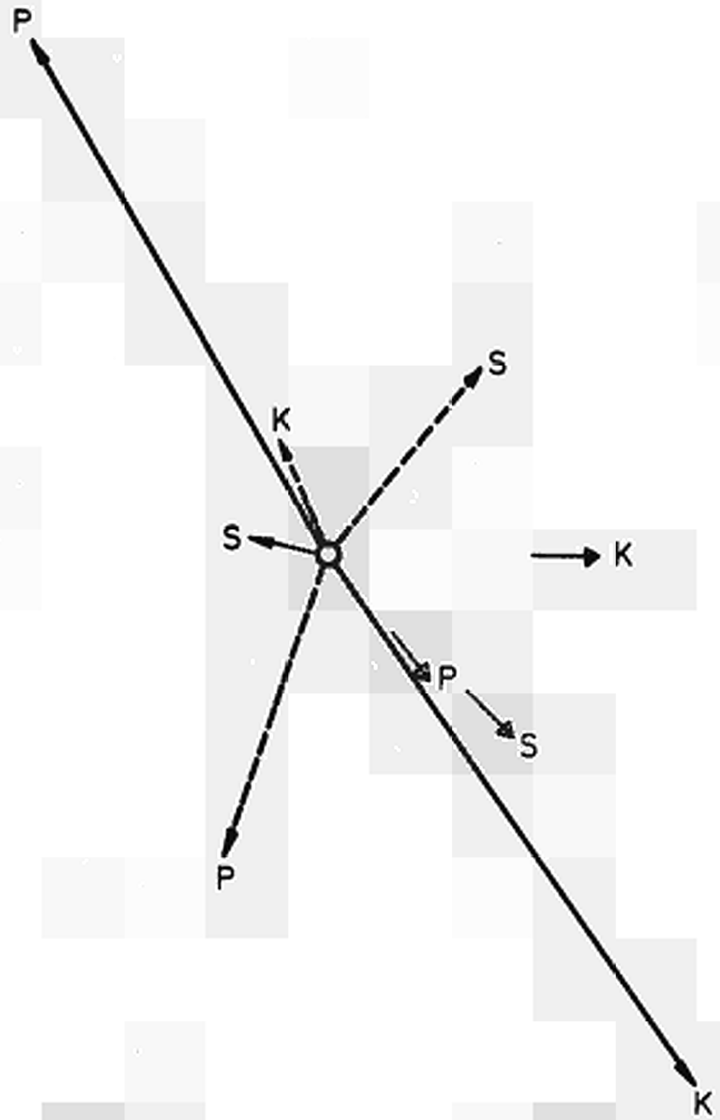


Fig. 1 - 2 Schematic Block Diagram of Reactor Transfer Functions

SEFOR

VECTOR DIAGRAMS FOR

THE FIRST BALANCED OSCILLATOR TEST



LEGEND		NET AMPLITUDE	PHASE ANGLE
———	COOLANT TEMPERATURE RISE	0	
- - - -	PRIMARY COLD LEG TEMPERATURE	0	
P	PRIMARY COOLANT FLOW	5.8%	- 52°
S	SECONDARY COOLANT FLOW	9.5%	- 45°
K	REACTIVITY	10 ¢	0

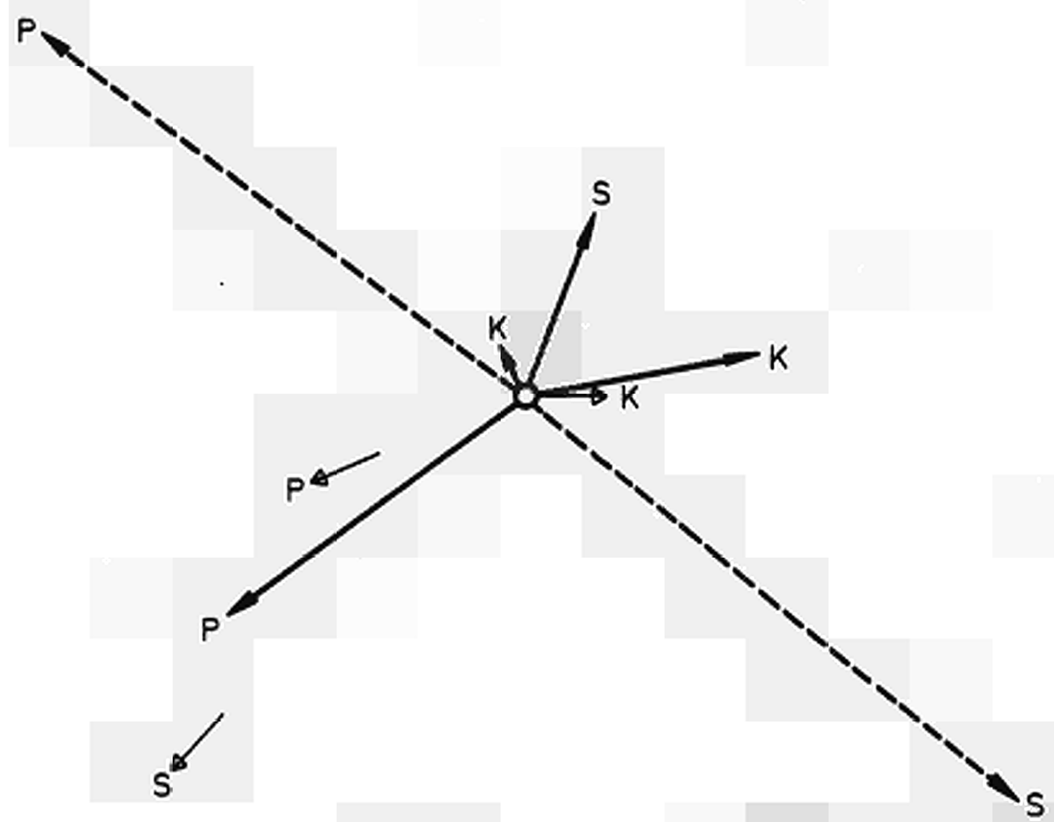
FIG. 2.1-1

SEFOR

VECTOR DIAGRAMS FOR

THE SECOND BALANCED OSCILLATOR TEST

WITH INLET TEMPERATURE BALANCED



LEGEND		NET AMPLITUDE	PHASE ANGLE
———	REACTOR POWER	0	
------	COOLANT INLET TEMPERATURE	0	
P	PRIMARY COOLANT FLOW	8.5%	- 159°
S	SECONDARY COOLANT FLOW	19.1%	- 34°
K	REACTIVITY	3¢	0

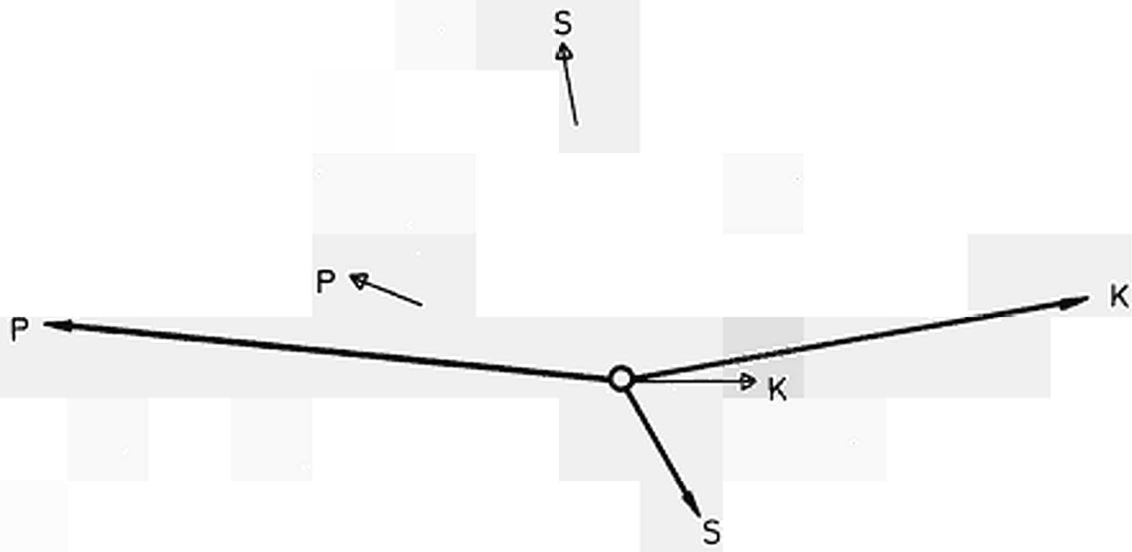
FIG. 2.1-2

SEFOR

VECTOR DIAGRAMS FOR

THE SECOND BALANCED OSCILLATOR TEST

WITH PRIMARY COLD LEG TEMPERATURE OSCILLATING

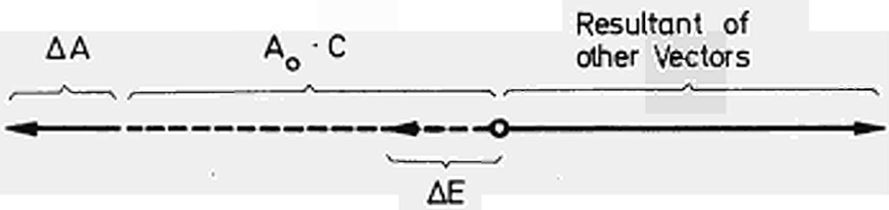


	LEGEND	NET AMPLITUDE	PHASE ANGLE
	REACTOR POWER	0	
P	PRIMARY COOLANT FLOW	8.6 %	160°
S	SECONDARY COOLANT FLOW	10 %	100°
K	REACTIVITY	4¢	0°

FIG. 2.1-3

EFFECT OF CHANGE IN AMPLITUDE AT BALANCED CONDITION

$$\Delta E = \frac{\Delta A}{A_o} \cdot A_o \cdot C$$



EFFECT OF CHANGE IN PHASE ANGLE AT BALANCED CONDITION

$$\Delta E = A_o \cdot C \cdot \frac{\sin \Delta\Theta}{\sin \beta} \approx A_o \cdot C \cdot \sin \Delta\Theta$$

for $\Delta\Theta < 10°$

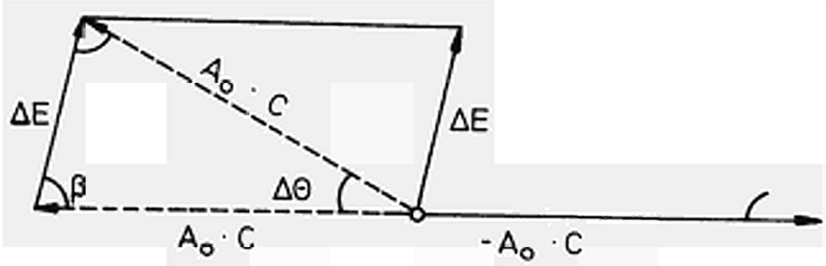


FIG. 2.1-4b

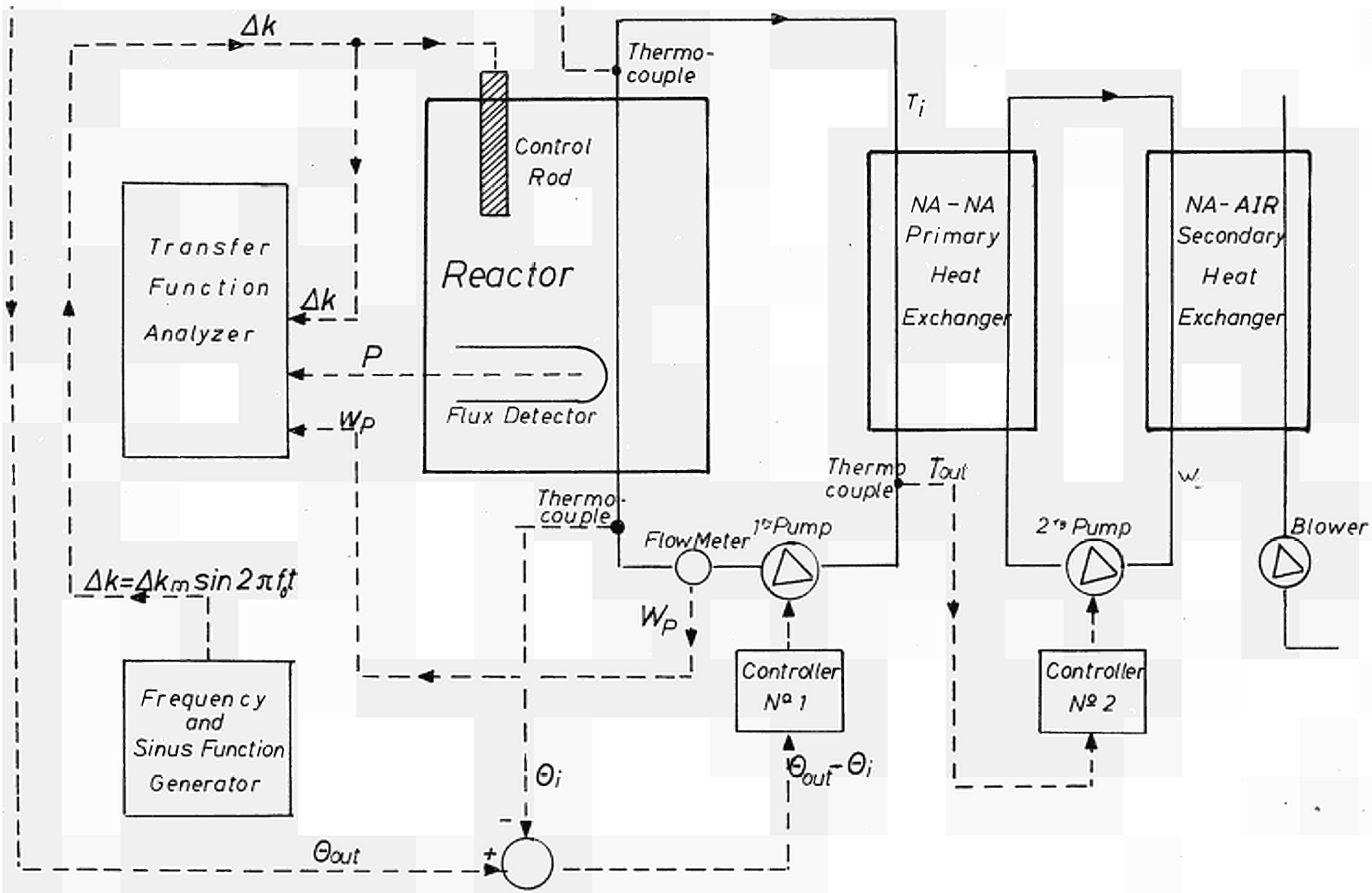


Fig. 2.2-1 1st Balanced Oscillator Experiment – Schematic block diagram of the connections of the Closed Loop Control System to the Plant

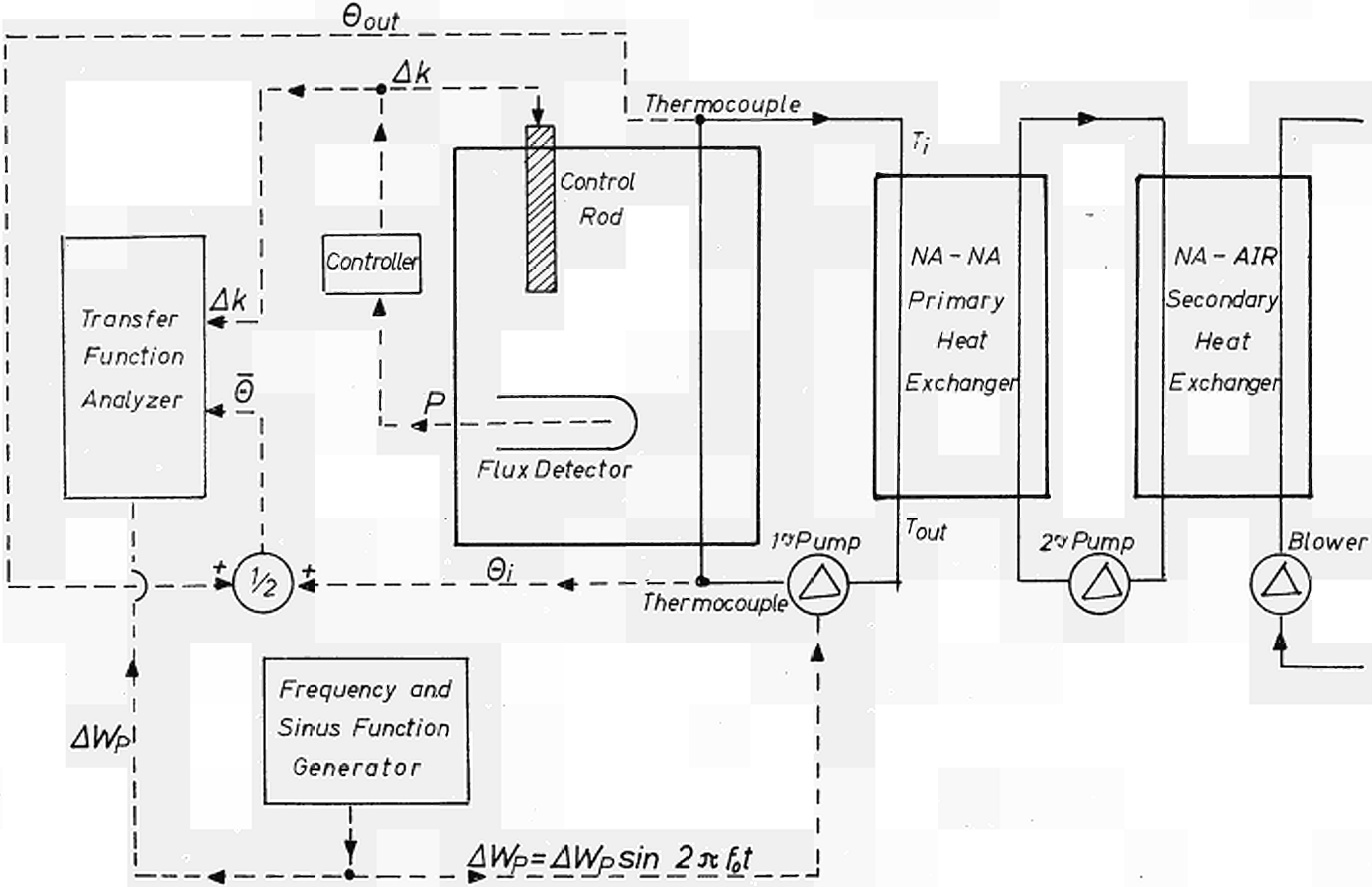


Fig. 2.2-2A 2nd Balanced Oscillator Experiment –Schematic block diagram of the connections of the Closed Loop Control System to the Plant –

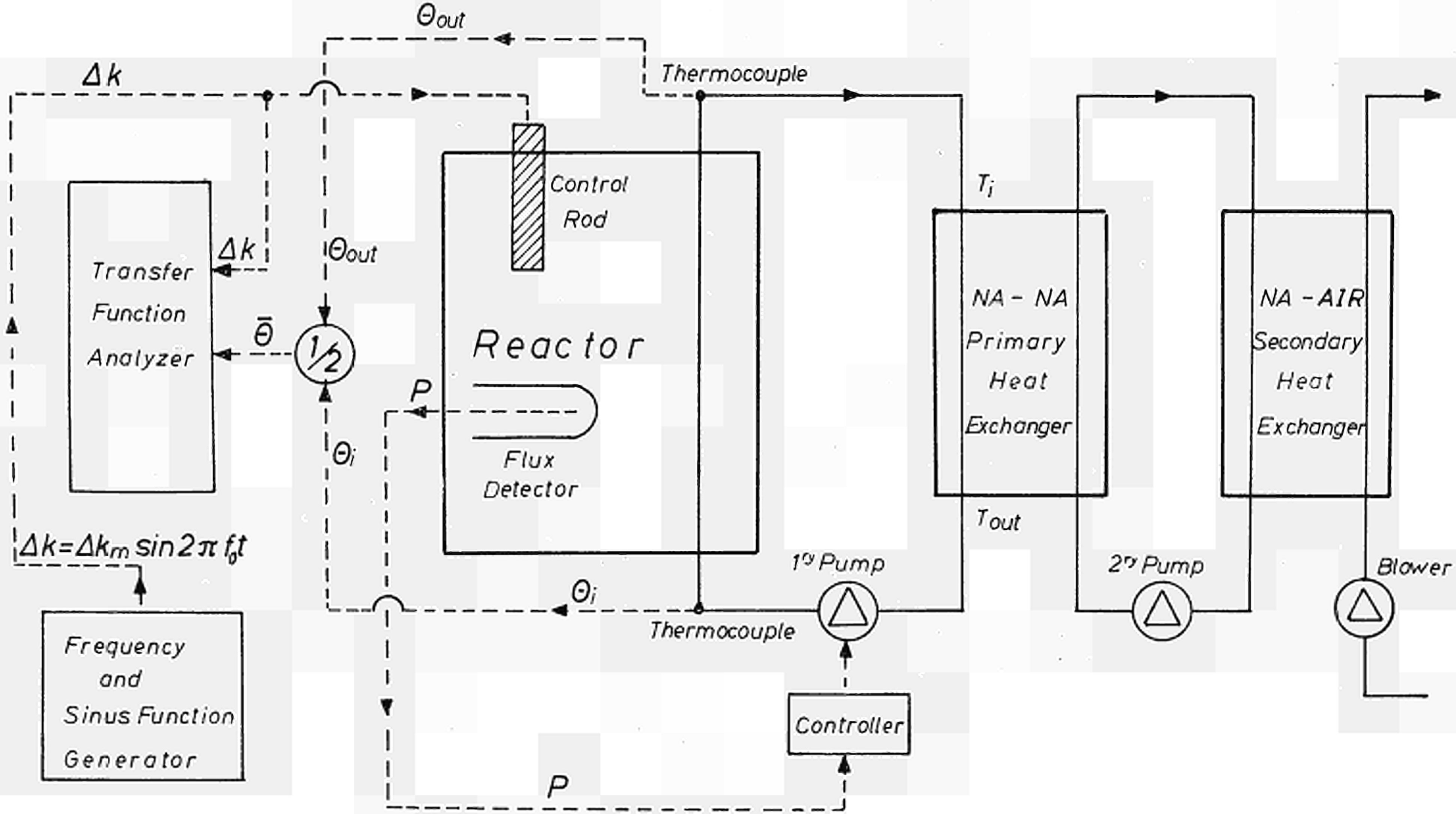


Fig. 2.2-2B 2nd Balanced Oscillator Experiment - Schematic block diagram of the connections of the Closed Loop Control System to the Plant - 2nd Philosophy Design

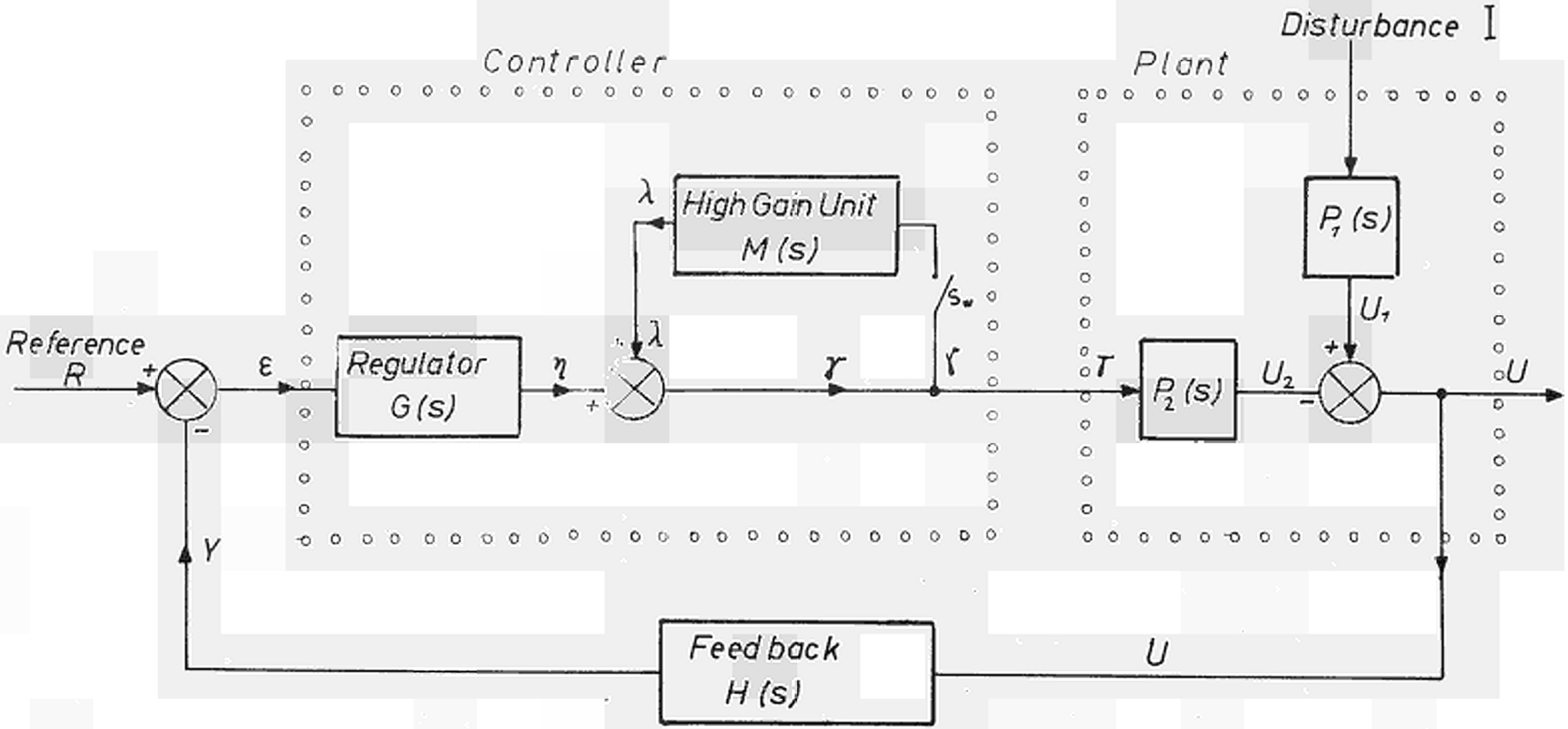
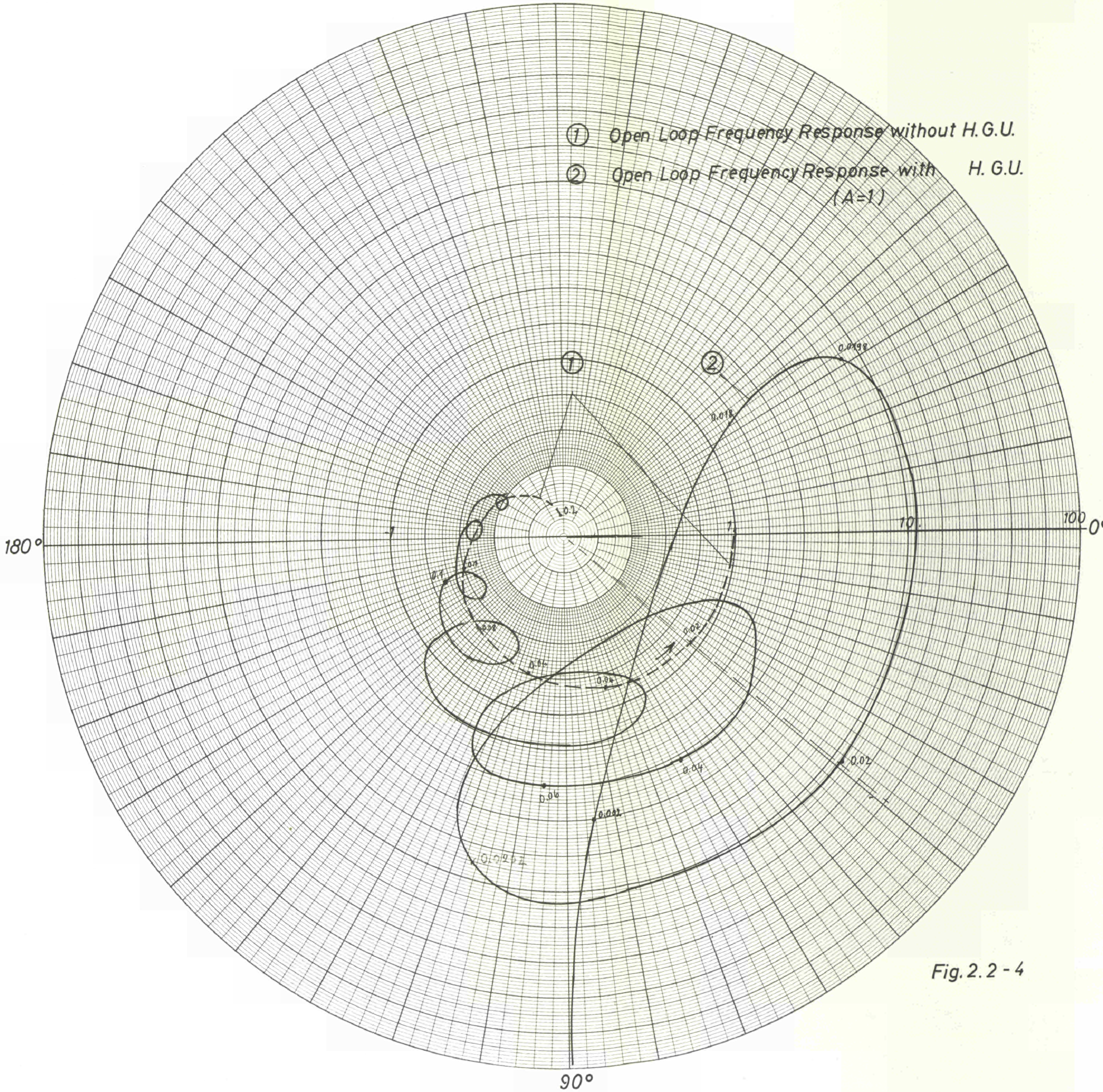


Fig. 2. 2-3 Schematic Block Diagram of the Closed Control Loop



① Open Loop Frequency Response without H.G.U.
② Open Loop Frequency Response with H.G.U.
(A=1)
180°
0°
90°
1
10
100

Fig. 2.2 - 4

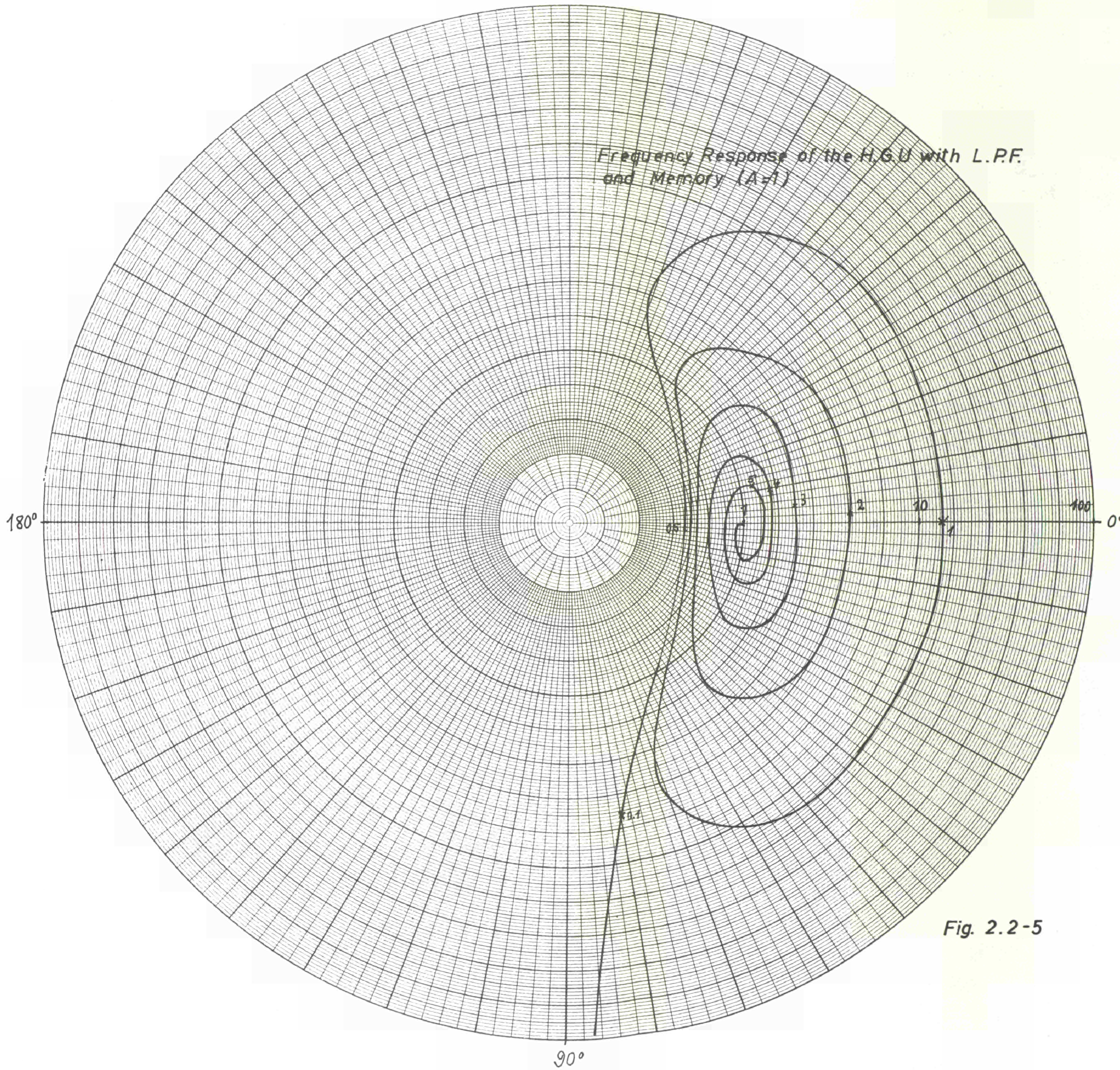

Frequency Response of the H.G.U with L.P.F.
and Memory (A=1)
180°
0°
90°
05
7
5
4
3
2
10
1
100
0.1

Fig. 2.2-5

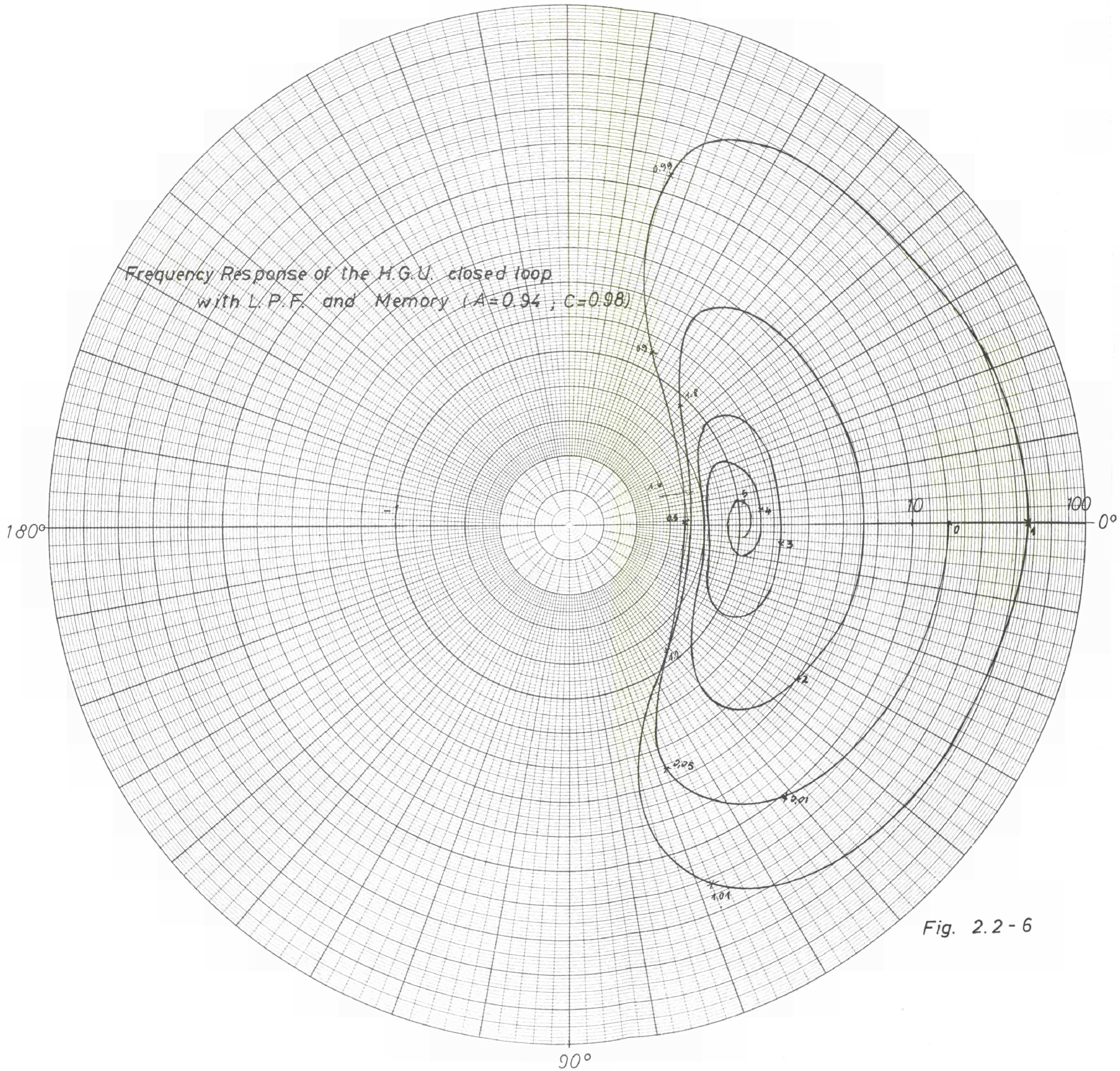
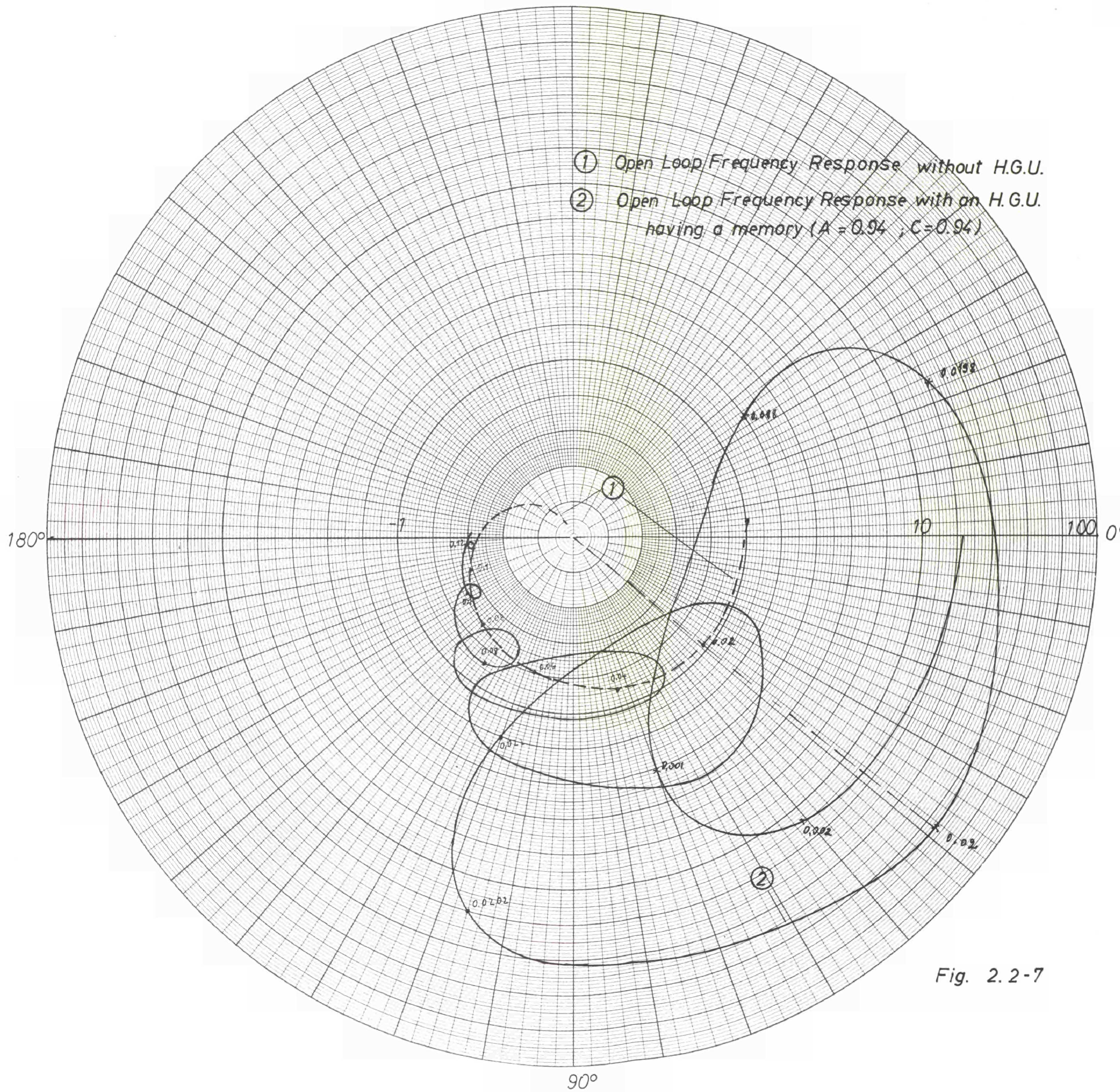

Frequency Response of the H.G.U. closed loop
with L.P.F. and Memory (A=0.94 ; C=0.98)
180°
0°
90°
10
100

Fig. 2.2 - 6



① Open Loop Frequency Response without H.G.U.
② Open Loop Frequency Response with an H.G.U. having a memory (A = 0.94 ; C = 0.94)
180°
0°
90°
-1
10
100

Fig. 2.2-7

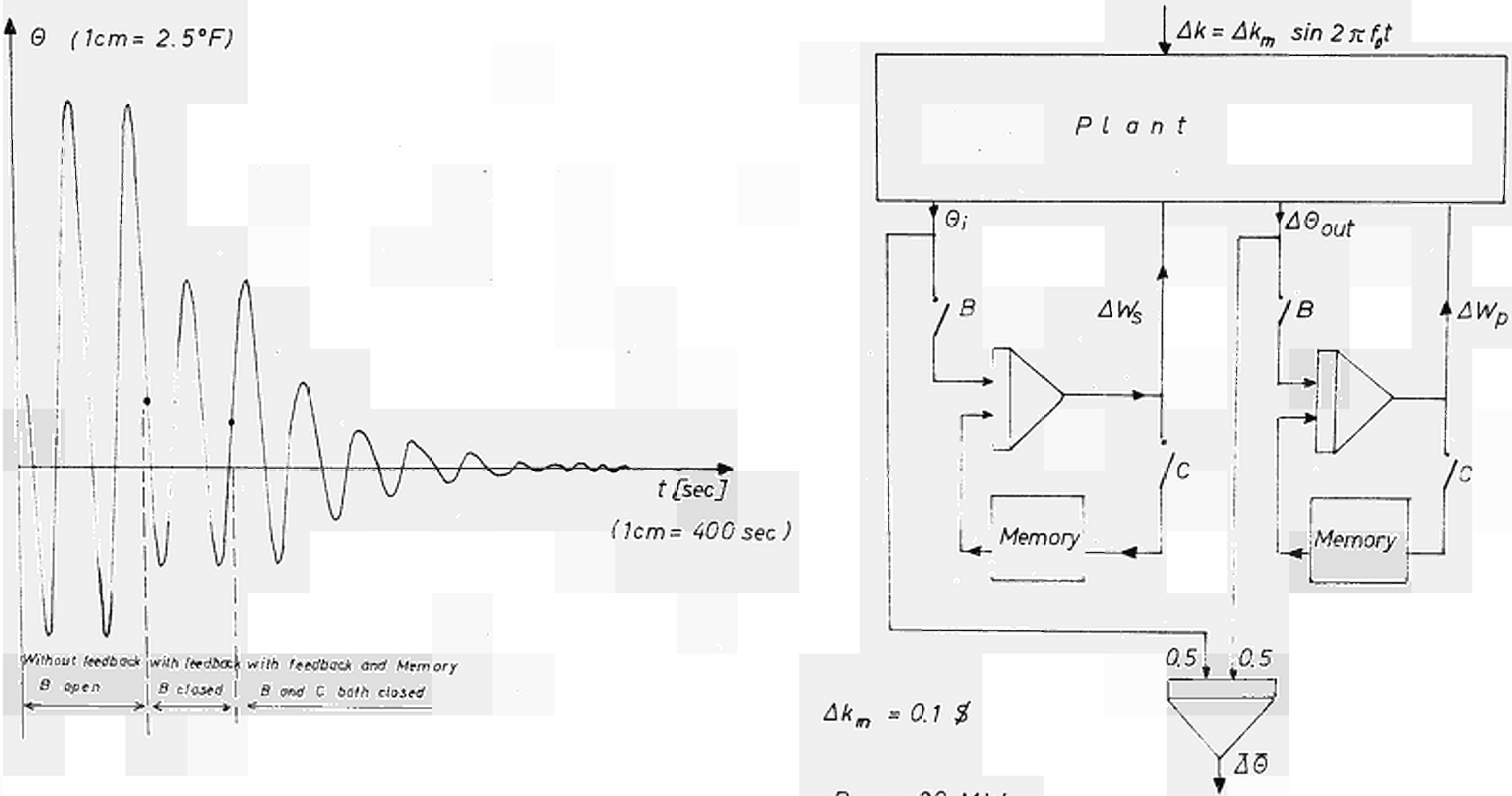

Θ (1cm = 2.5°F)
t [sec]
(1cm = 400 sec)
Without feedback
B open
with feedback
B closed
with feedback and Memory
B and C both closed
Δk = Δk_m sin 2π f_0 t
Plant
Θ_i
ΔW_S
ΔΘ_out
ΔW_p
B
C
Memory
0.5
ΔΘ
Δk_m = 0.1 $
P_o = 20 MW

Fig. 2.2-8

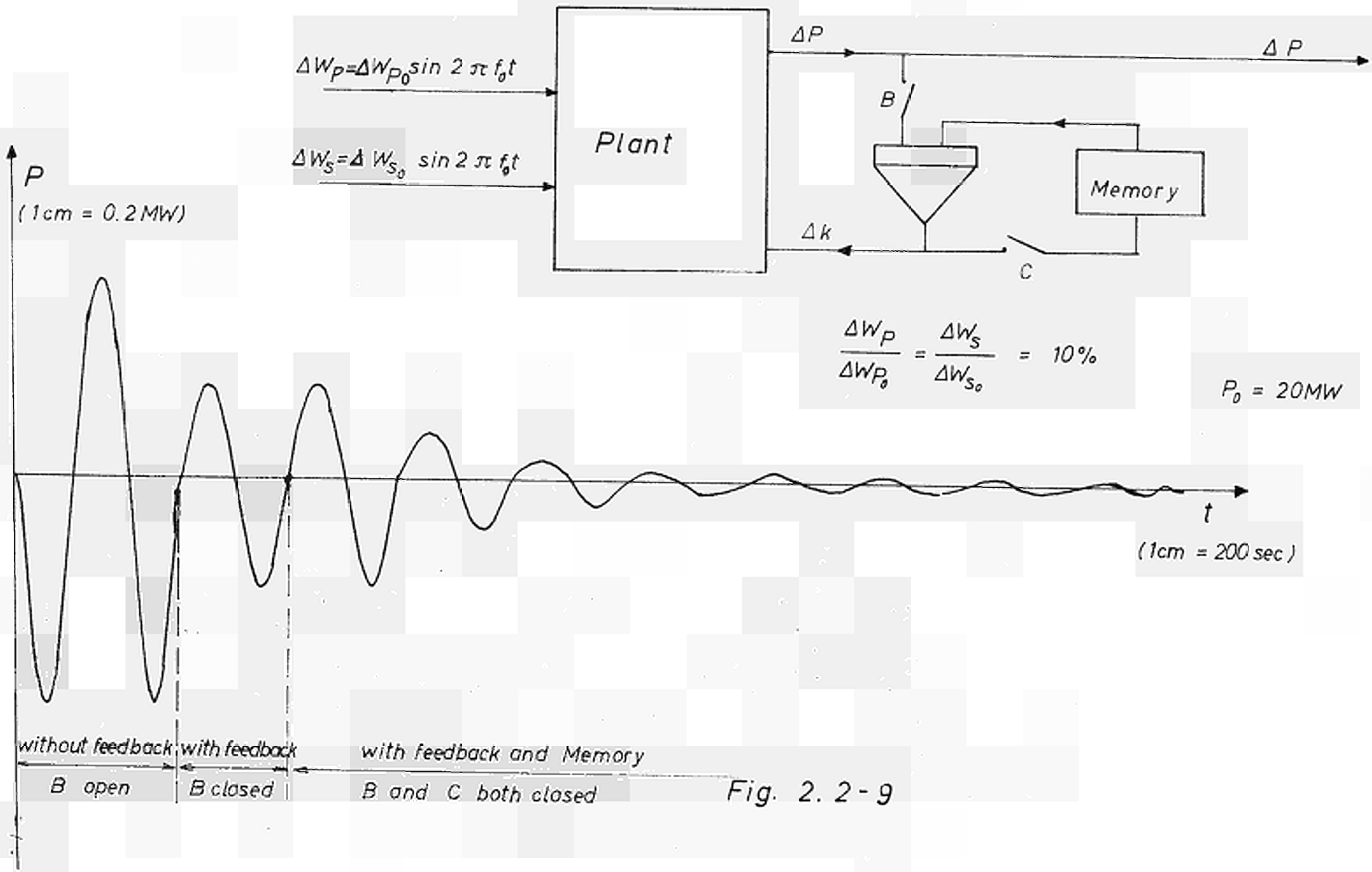

ΔW_P=ΔW_P0 sin 2π f0t
ΔW_S=Δ W_S0 sin 2π f0t
Plant
ΔP
Δ P
B
Memory
Δk
C
ΔW_P / ΔW_P0 = ΔW_S / ΔW_S0 = 10%
P0 = 20MW
P
(1cm = 0.2MW)
t
(1cm = 200 sec)
without feedback
B open
with feedback
B closed
with feedback and Memory
B and C both closed

Fig. 2. 2-9

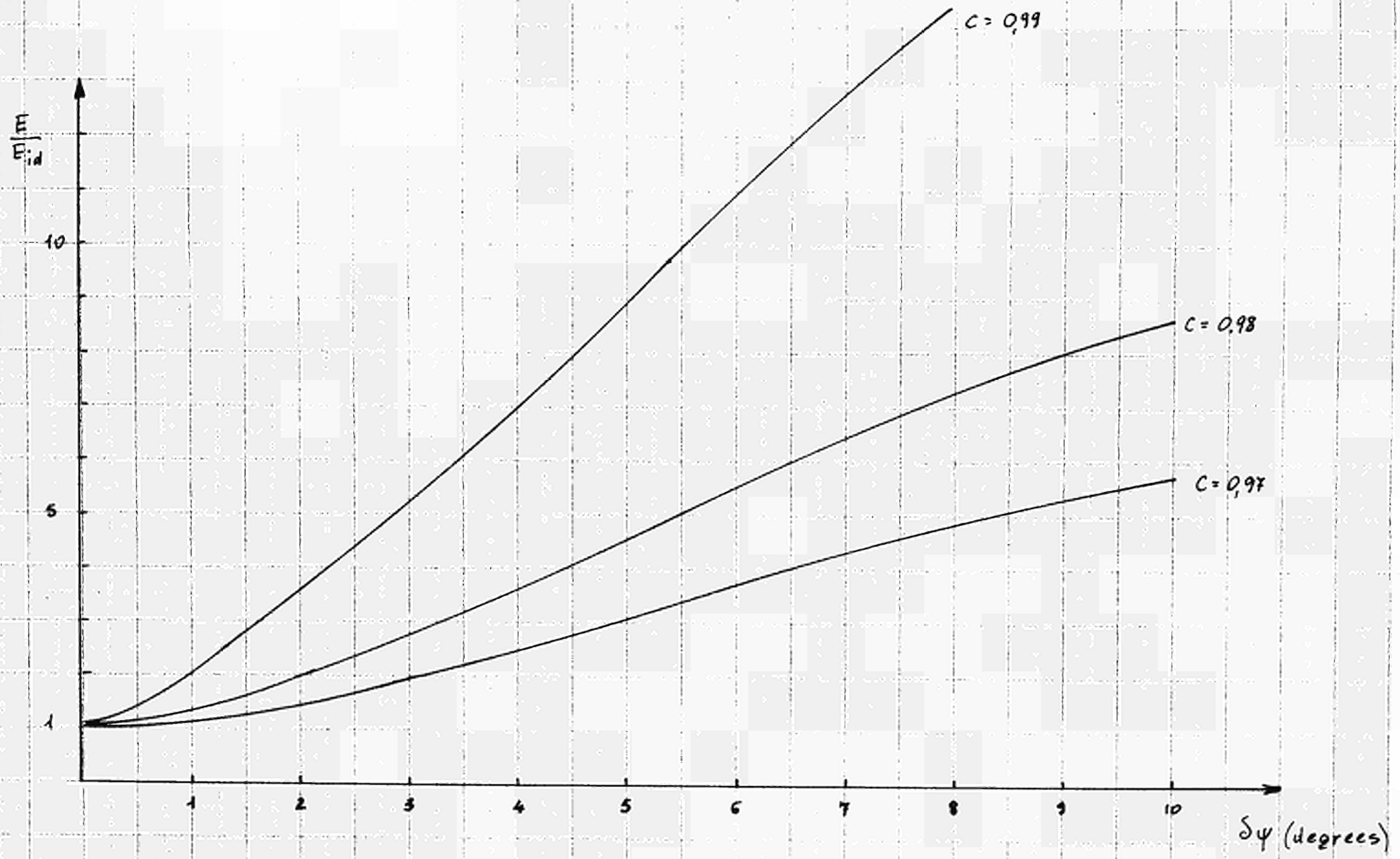


Fig. 2.2-10 Error "E/E_{id}" as function of the error $\delta\psi$ of phase setting

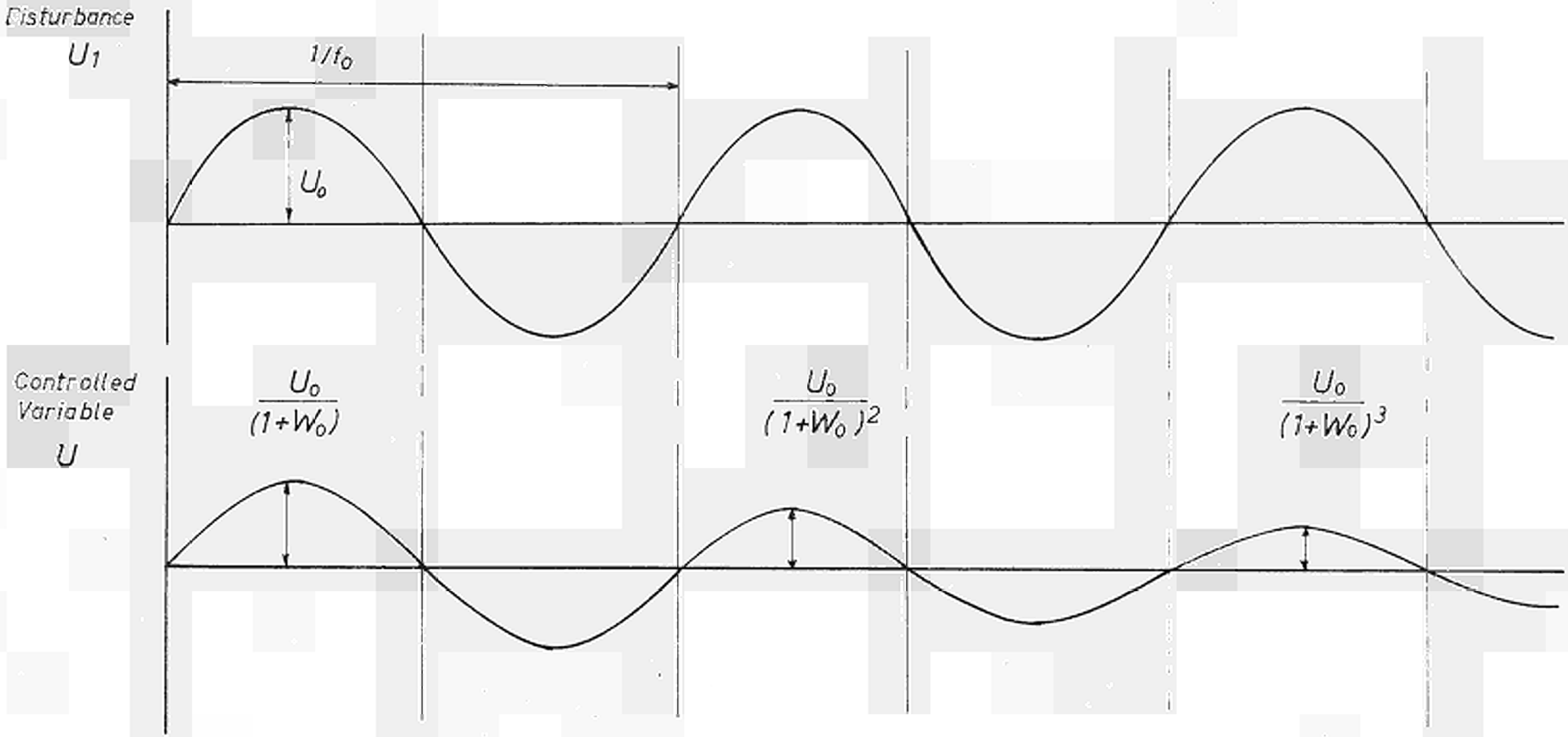
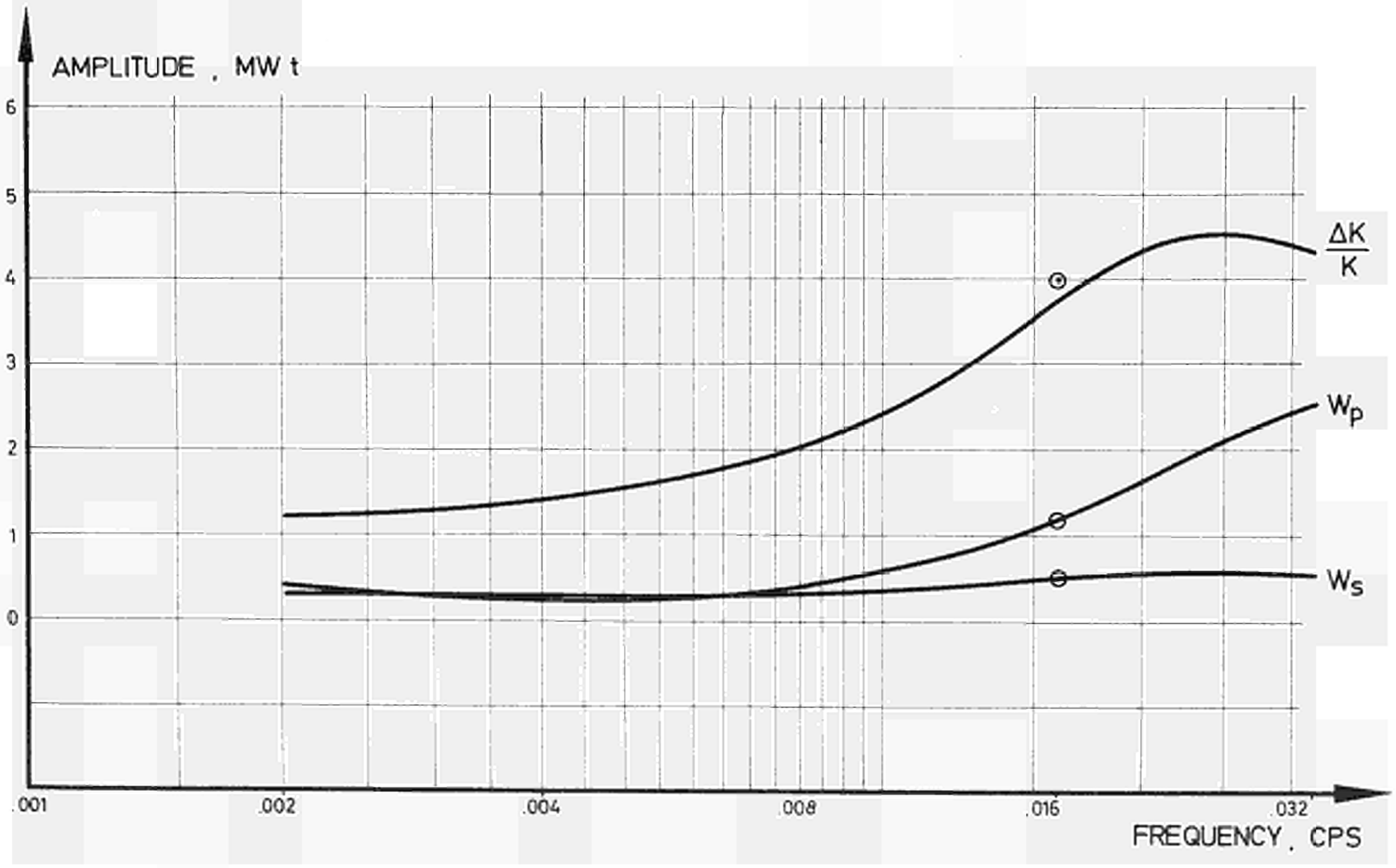


Fig. 2.2 - 11 Behaviour of the Controlled Variable U in the ideal case of Appendix 4

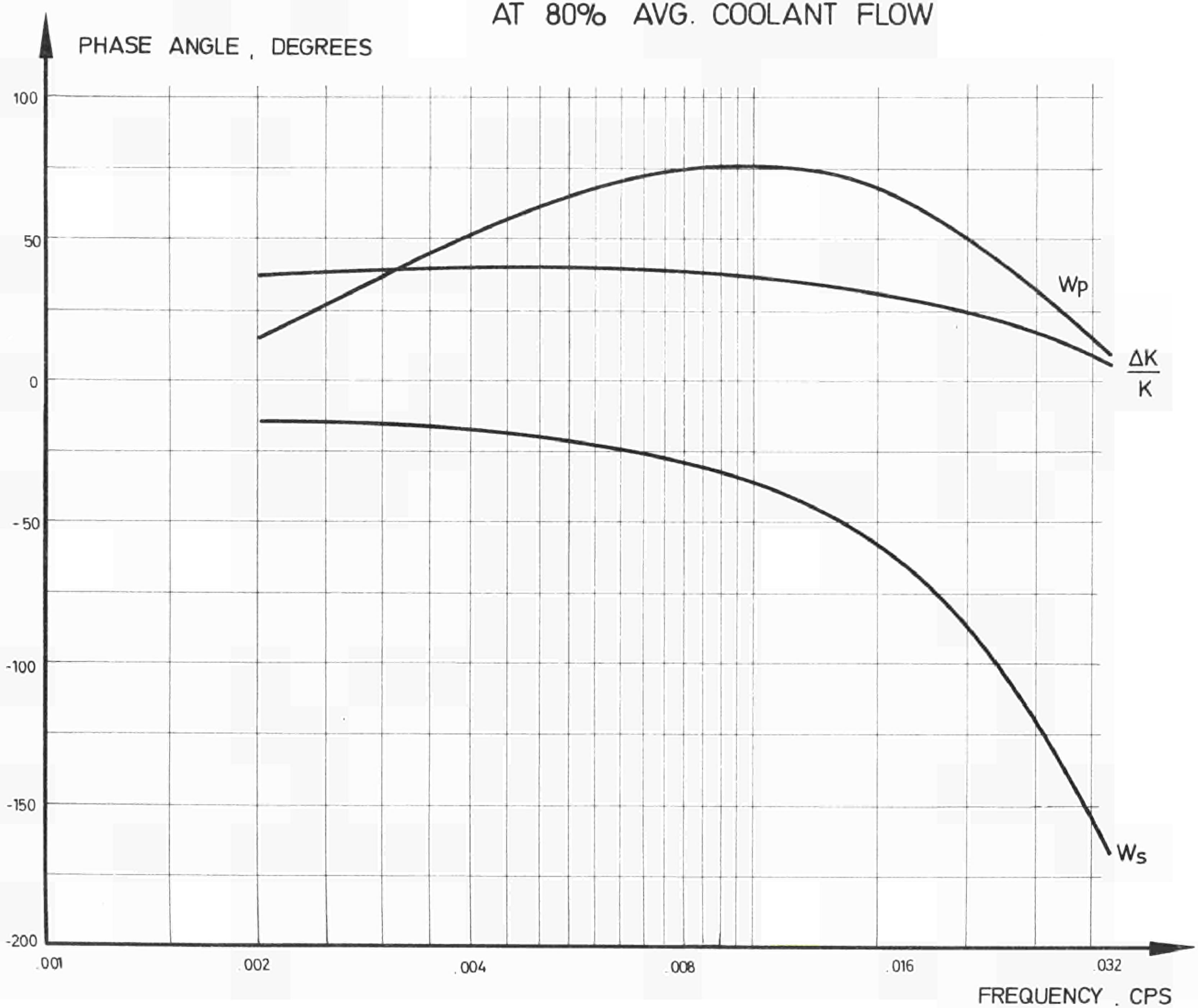
Appendix 1 - Fig. 1

OSCILLATOR EFFECTS ON NEUTRON FLUX
AT 80% AVG. COOLANT FLOW
FOR ± 10 ¢ AND ± 10 % OSCILLATOR AMP.

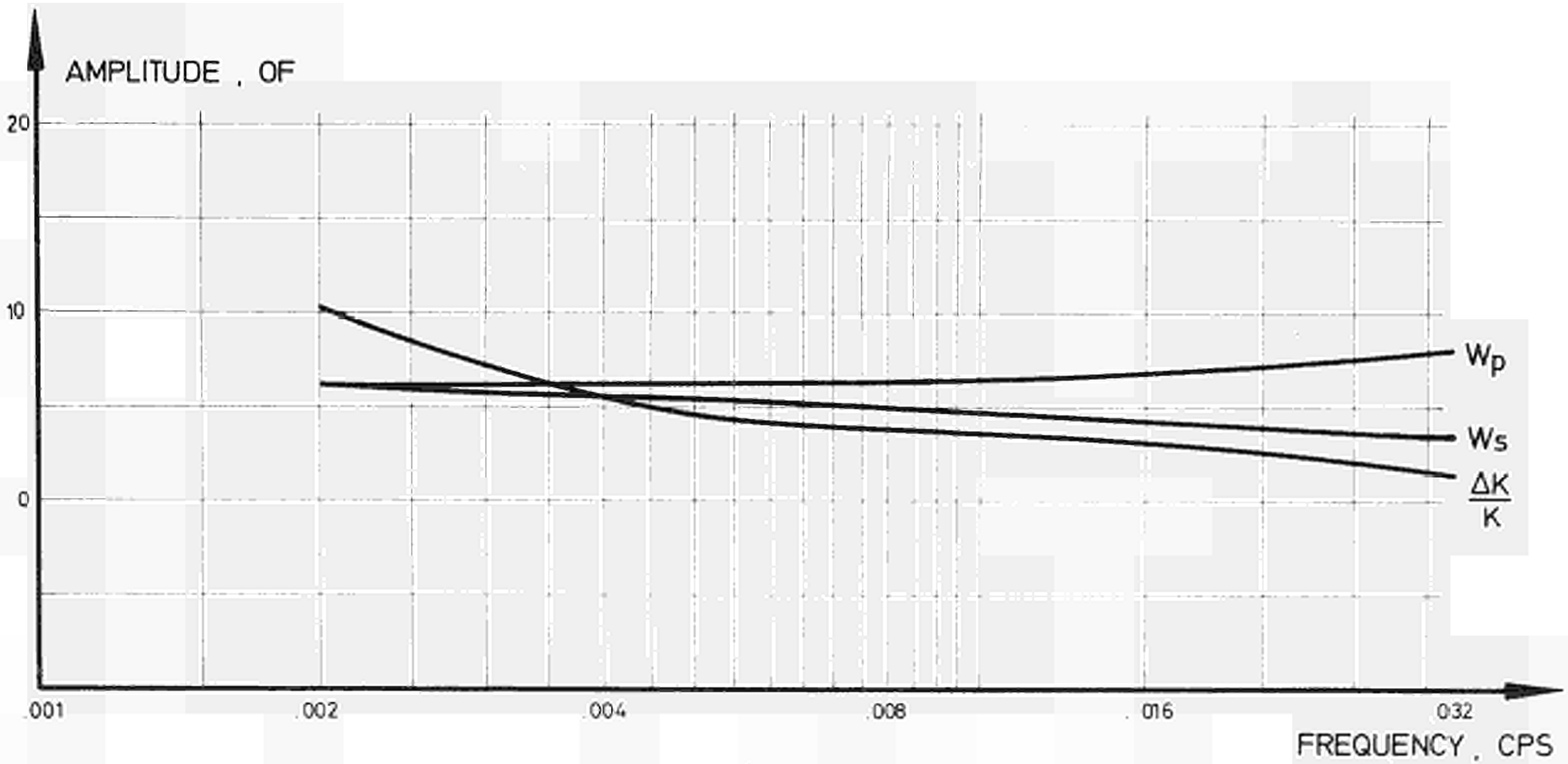


Appendix 1 - Fig. 2

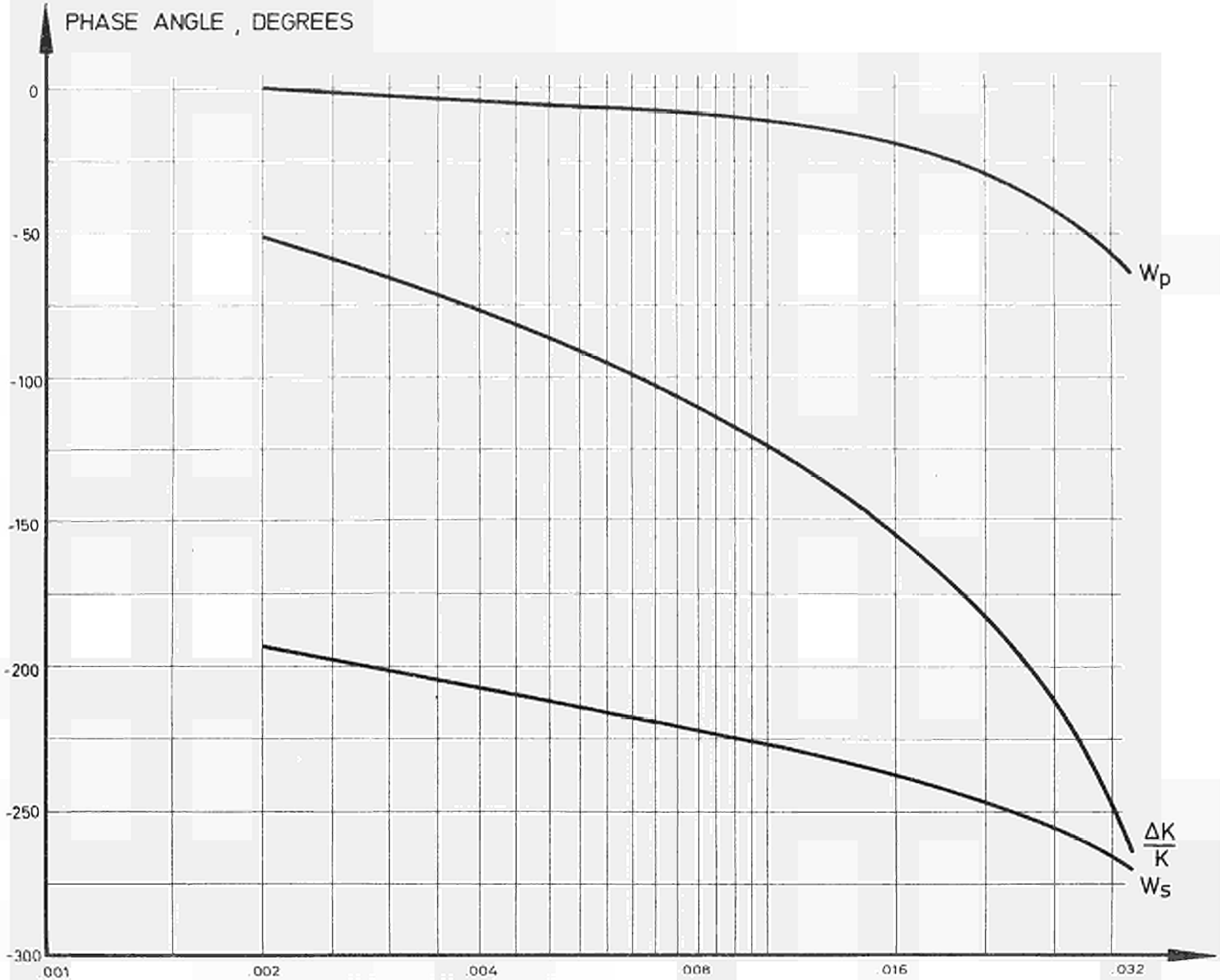
OSCILLATOR EFFECTS ON NEUTRON FLUX AT 80% AVG. COOLANT FLOW



Appendix 1 - Fig. 3 OSCILLATOR EFFECTS ON PRIMARY COLD LEG TEMPERATURE AT 80 % AVG. COOLANT FLOW FOR ± 10 ¢ AND ± 10 % OSCILLATOR AMP.

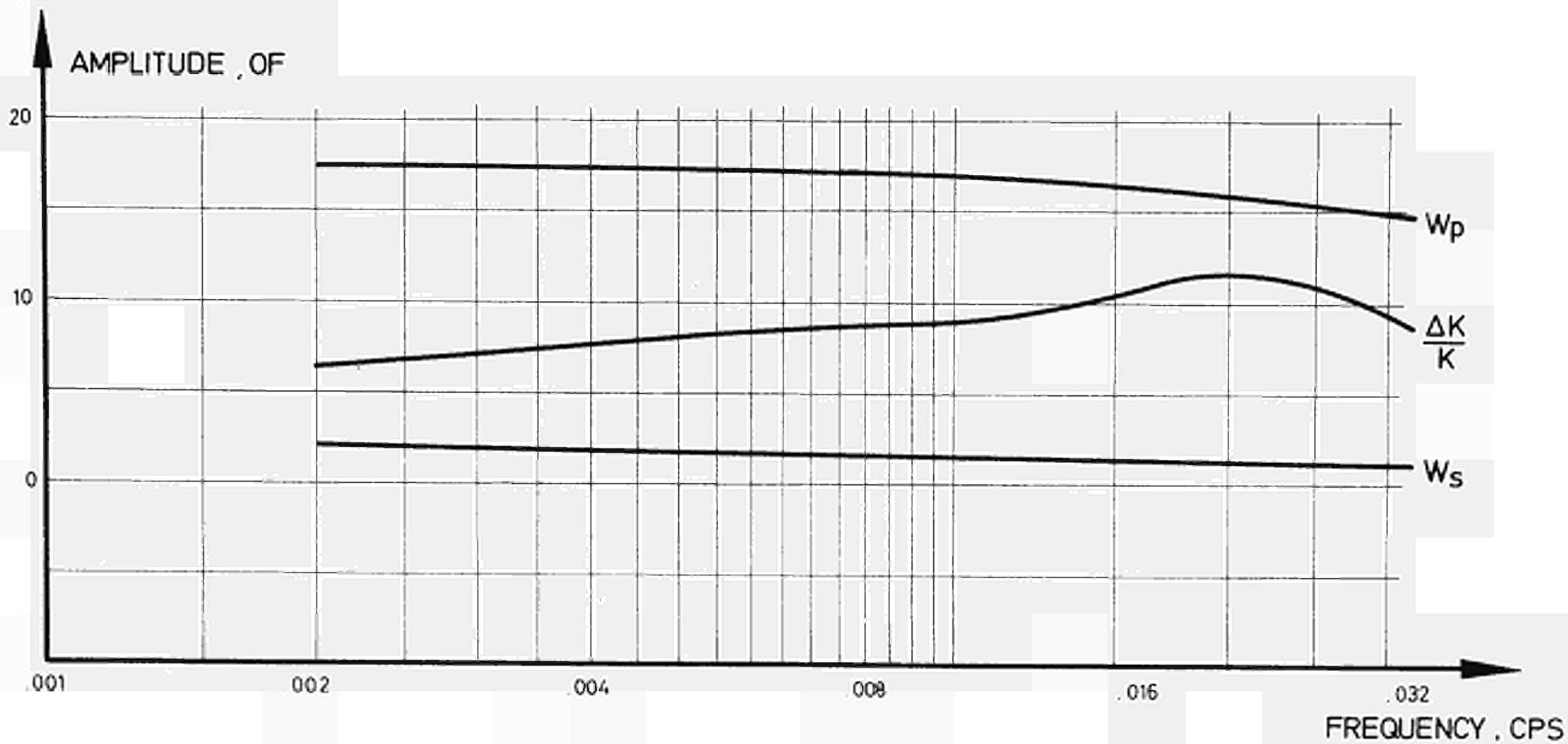


Appendix 1 - Fig. 4 OSCILLATOR EFFECTS ON PRIMARY COLD LEG TEMPERATURE AT 80% AVG. COOLANT FLOW



OSCILLATOR EFFECTS ON CORE COOLANT ΔT
AT 80% AVG. COOLANT FLOW
FOR ± 10¢ AND ± 10 % OSCILLATOR AMP.

Appendix 1 - Fig. 5



Appendix 1 - Fig. 6 OSCILLATOR EFFECTS ON CORE COOLANT ΔT AT 80% AVG. COOLANT FLOW