# **EUR 3908** e

## EUROPEAN ATOMIC ENERGY COMMUNITY - EURATOM

# A SIMPLIFIED METHOD FOR KINEMATIC CALCULATIONS OF NUCLEAR REACTIONS

by

J. WINTER and H. SCHMID

1968



Joint Nuclear Research Center Geel Establishment - Belgium

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European Atomic Energy Community — EURATOM Joint Nuclear Research Center — Geel Establishment (Belgium) Central Bureau for Nuclear Measurements — CBNM Brussels, July 1968 - 16 Pages - 2 Figures - FB 40

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reactions may be calculated by means of only one equation. To facilitate handling, this equation has been tabulated. In the single valued region  $(0 \le \eta \le 1)$  the incremental step of the parameter  $\eta$  is 0.01 and the angle  $\Theta$  increases in steps of 5 deg. from 0 to 180 deg. In the double valued region  $(\eta > 1)$  tabulation of  $\eta$  is given only up to  $\eta = 2$ , the incremental steps being 0.02 and 5 deg.

This report consists essentially of a reprint of a publication in "Nuclear Instruments and Methods" 59 (1968) 167-169. For the practical use, however, it is supplemented by the above mentioned tables.

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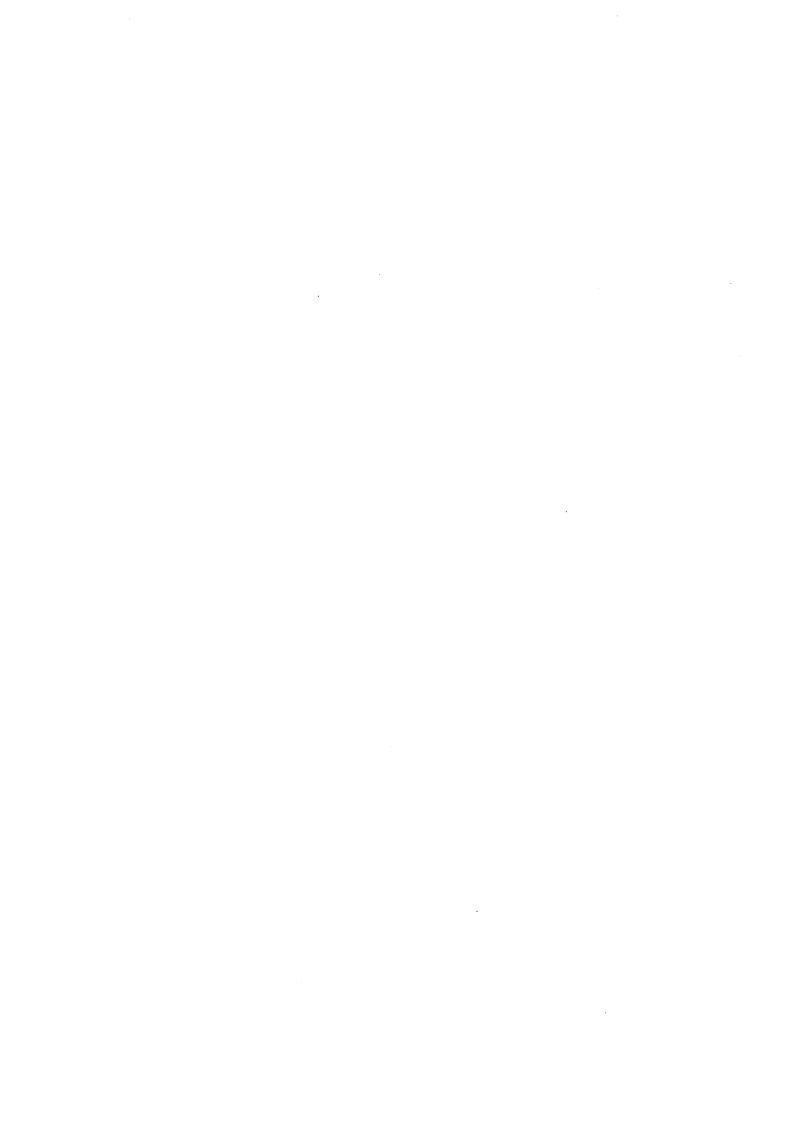
### SUMMARY

A simplified method of presentation of kinematic relationships in nuclear reactions is given. It allows to obtain in a convenient way the energy of the reaction products as a function of the angle of emission in the laboratory frame for non-relativistic reactions. It has been shown that after the introduction of normalized quantities the kinematics of all non-relativistic reactions may be calculated by means of only one equation. To facilitate handling, this equation has been tabulated. In the single valued region  $(0 \le \eta \le 1)$  the incremental step of the parameter  $\eta$  is 0.01 and the angle  $\theta$  increases in steps of 5 deg. from 0 to 180 deg. In the double valued region  $(\eta > 1)$  tabulation of  $\eta$  is given only up to  $\eta = 2$ , the incremental steps being 0.02 and 5 deg.

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### **KEYWORDS**

NUCLEAR REACTIONS
ANGULAR DISTRIBUTION
DIAGRAMS
EQUATIONS
TABLES
ENERGY
PARTICLES



# A simplified method for kinematic calculations of nuclear reactions \*

The formulas describing the non-relativistic kinematics of nuclear reactions are rather combersone. To facilitate handling they have been represented for some reactions in form of tables and diagrams 1-5. The number of reactions and quantities of interest is numerous and therefore also the number of diagrams. A very general representation was proposed by Dickens 5 but even there for each type of reaction, e.g. (p,p), (p,d) ... another diagram must be used.

In this paper a general formula for all non-relativistic reactions is given in terms of normalized quantities. The formula may be represented by one nomogram or one table. The normalized quantities themselves are in simple relation to the physical quantities and can be calculated easily.

We consider the following case: A projectile particle with mass  $m_1$  and kinetic energy  $E_1$  collides with a target particle  $m_2$  being at rest  $(E_2 = 0)$  in the lab. (laboratory) frame. Two particles of masses  $m_3$  and  $m_4$  are produced. The reaction shortly written 1(2,3)4 will be calculated under the following assumptions: conservation of: energy, linear momentum and mass. From these follows for the absolute velocity  $|v_3|$  and energy  $E_3$  of particle 3 in the CM (center of mass) frame and for the velocity of the center of mass  $|v_{\rm CM}|$ 

$$\begin{aligned} |v'_{3}| &= \left[ \left\{ 2 \, m_{2} m_{4} / (m_{3} \, M^{2}) \right\} (E_{1} - E_{\text{th}})^{1/2}; \\ E'_{3} &= \frac{1}{2} \, m_{3} \, |v'_{3}|^{2} = (m_{2} \, m_{4} / M^{2}) (E_{1} - E_{\text{th}}); \\ |v_{\text{CM}}| &= m_{1} \, |v_{1}| / M \end{aligned}$$
(1a, b)

With

$$m_1 + m_2 = m_3 + m_4 = M^{**};$$
 $E_{\text{th}} = -QM/m_2 = |Q|M/m_2 \text{ for } Q < 0$ 
 $Q = E_3 + E_4 - E_1^{**}.$  (3a, b)

The dynamical variables of a particle expressed in the CM frame will be distinguished by attaching primes to them. The following calculation is valid for particle 3. Interchanging the indices 3 and 4 yields the equations for particle 4. One observes that  $|v_3|$  and  $|v_{\rm CM}|$  are constant if  $E_1$  is kept constant. As shown in fig. 1 the transition from the CM frame to lab. frame is simply the vector addition of  $v_{\rm CM}$  to  $v'_{\rm 3}$ . The shape of the resulting vector diagram depends only on the ratio  $|v_{\rm cm}|/|v_3|$ . It is therefore convenient to introduce the kinematic parameter

$$\eta_3 = |v_{\text{CM}}|/|v_3|; \qquad \eta_3^2 = \{ (m_1 m_3)/(m_2 m_4) \} E_1/(E_1 - E_{\text{th}})$$
(4a, b)

<sup>\*</sup> Text also published in Nuclear Instruments and Methods 59 (1968) 167-169 by North-Holland

Publishing Co.

\*\*\* In a non-relativistic calculation the conservation of energy and mass must be postulated separately.

Q must be introduced by means of the relativistic eq.  $Q = \Delta mc^2 = (m_1 + m_2 - m_3 - m_4)c^2$ . The necessary condition for non-relativistic calculation is  $\Delta m \ll M$ .

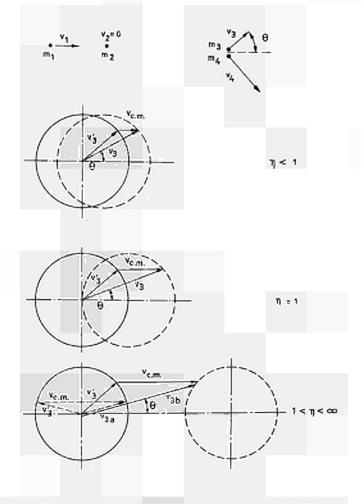


Fig. 1. Transition from the cm frame to the lab. frame for different values of the parameter η.

The six quantities  $m_1$ ,  $m_2$ ,  $m_3$ ,  $m_4$ ,  $E_1$  and  $E_{th}$  are in this way combined in one single parameter. From fig. 1 one obtains.

$$|v_3|^2 \sin \theta = |v'_3|^2 - [|v_3| \cos \theta - |v_{CM}|]^2.$$
 (5)

With eq. (3a) and a second normalized quantity x

$$x = |v_3|/|v_3| = (E_3/E_3)^{1/2};$$
 (6)

we obtain from eq. (6) finally the desired general relation

$$x = \pm \; (1 - \eta^2 \sin^2 \theta)^{1/2} + \eta \; \cos \theta,$$

or

$$\cos \theta = (2x\eta)^{-1}(x^2 + \eta^2 - 1).$$
 (7)

This equation in connection with the definitions of x and  $\eta$  is equivalent to the general formula  $^{c}$ 

$$Q = E_3 \left\{ 1 + (m_3/m_4) \right\} - E_1 \left\{ 1 - (m_1/m_4) \right\} - \left\{ (2\cos\theta)/m_4 \right\} (m_1 m_3 E_1 E_3)^{1/2} \quad (8)$$

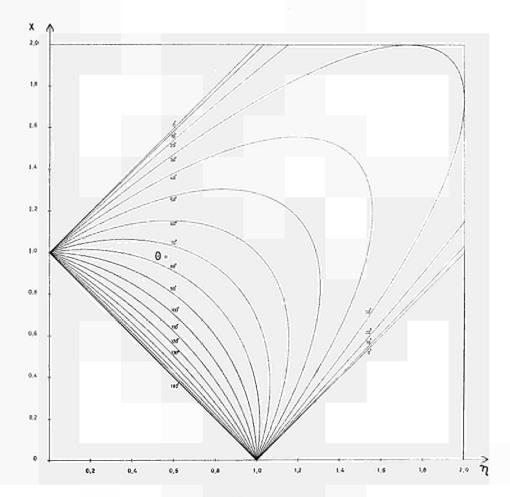


Fig. 2. Nomogram of eq. (7).

Eq. (7) allows to calculate the energy of particle 3 via the quantity x as a function of the angle  $\theta$  of emission in the lab. frame.

In order to explain the meaning of  $\eta$  we consider e.g. the dependence of  $\eta$  on  $E_1$  for reactions with a negative  $Q(E_{th} > 0)$ .

- 1.  $\eta < 0$ . No reaction,  $E_1 < E_{\text{th}}$ .
- 2.  $\eta = \infty$ . Beginning of the reaction.  $E_1 = E_{10}$ .
- 3.  $1 < \eta < \infty$ . Region of double valued energy. At each angle  $\theta$  ( $\theta < \theta_{max}$ ) there are particles with two different energies  $E_{th} < E_1 < E_{bs}$ .
- η = 1. This condition gives that E<sub>1</sub> at which just no particle 3 can be found at angles θ greater than 90°, E<sub>1</sub> = E<sub>b3</sub>.
  - 5.  $(m_1m_3)/(m_2m_4) \le \eta \le 1$ . Region of single valued energy  $E_1 > E_{\rm b3}$ .

For reactions with a positive Q value  $\eta$  increases with increasing  $E_1$ . The other aspects being the same. The possible range for x is also given by  $\eta$ .

$$x_{\max} = 1 \pm \eta \text{ for } \eta < 1 \text{ and } x_{\max} = \eta \pm 1 \text{ for } \eta > 1$$
 (9)

For  $1 < \eta < \infty$  the maximum angle  $\theta_{max}$  at which particle 3 can be found may be calculated with

$$\sin \theta_{\text{max}} = 1/\eta_3; \quad x_{\text{max}} = (\eta_3^2 - 1)^{1/2}.$$
 (10)

Example: 'Li(p,n)'Be.

Incident proton energy 2200 keV; Threshold energy  $E_{th}=1880.4$  keV. The kinematic parameter  $\eta$  is calculated with eq. (4b),  $\eta=0.37$ . From this we conclude that  $E_3$  is a single valued function of  $\theta$ . With eq. (1b) one obtains for the neutron energy in the CM frame  $E'_3=244.4$  keV. According to eqs. (6) and (9) the neutron energy  $E_3$  in the lab. frame varies between  $(1-\eta)^2$   $E'_3=94.85$  keV and  $(1+\eta)^2$   $E'_3=463.4$  keV. The angle for one desired energy  $E_3$  inside this range may easily be found by means of fig. 2 or by evaluating eq. (7).

The variation of  $E_3$  with the variation of  $E_1$  may be calculated from:

$$\partial E_3/\partial E_1 = (m_1 m_2/M^2) \times [\{M/(m_1 m_3)\}(m_4 - m_1) + \frac{1}{2} (x/\eta)^2]/\{1 - \frac{1}{2} (\eta/x)^2\}$$
 (12) which follows from eq. (8) and from eqs. (3b), (6) and (7).

One may conclude that especially for the kind of questions mentioned above kinematic calculations can be simplified by introducing normalized quantities,

The stimulating discussions with K. H. Böckhoff and W. Stüber are gratefully acknowledged.

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0.73000 0.72006 0.76032 0.75142 0.29 0.80504 J. 73828 0.72963 0.71078 0.81566 0.79726 0.78040 0.76511 0.73934 0.72888 0.72004 0.71281 0.70720 0.70320 0.70080 0.70000 0.77213 0.76380 0.75540 0.75648 0.74779 0.73905 0.74247 0.73348 0.72445 0.73013 0.72088 0.71160 0.71945 0.70999 0.70051 0.71043 0.7007 0.6911 0.70306 0.69329 0.68351 0.69326 0.68332 0.67337 0.69734 0.69000 0.78941 0.69081 0.80083 0.78149 0.68083 0.32 0.68000 0.67759 0.67084 0.67000 0.70229 0.69294 0.68355 0.67414 0.66468 0.66342 0.68148 0.66085 0.78567 0.76542 0.74694 0.73025 0.71537 0.69100 0.67372 0.66771 0.66000 0.77796 0.77016 0.76227 0.75430 0.73841 0.72982 0.72116 0.71243 0.68146 0.67190 0.66231 0.65269 0.72140 0.71250 0.70354 0.70625 0.69708 0.68787 0.65087 0.75727 0.66392 0.65781 0.74904 0.74074 0.64792 0.66209 0.36 0.37 0.38 0.64000 0.65410 0.64351 0.64088 0.63356 0.64427 0.63089 0.63000 0.73236 0.69452 0.67861 0.63444 0.62810 0.62359 0.62090 0.64262 0.62000 0.62458 0.61472 0.60485 0.59496 0.70364 0.63286 0.74624 0.68545 0.66931 0.65519 0.64305 0.61818 0.61363 0.61091 0.61000 0.73808 0.72984 0.72150 0.71537 0.65996 0.65057 0.64113 0.64566 0.63610 0.62651 0.63339 0.62369 0.61397 0.67632 0.66714 0.65790 0.60826 0.59832 0.58839 0.60366 0.60091 0.40 6.60000 0.59000 0.41 0.68585 0.67686 0.66779 0.6132 0.42 0.69805 0.6034 0.58372 0.58093 0.57093 0.56094 0.55094 0.43 0.71308 0.68927 0.64860 0.63164 0.61687 0.60422 0.59364 0.58506 0.57844 0.57374 0.57000 0.59445 0.58465 0.57482 0.56496 0.63924 0.62983 0.62035 0.62211 0.60720 0.59750 0.58776 0.58371 0.57391 0.56402 0.56849 0.55853 0.54856 0.56376 0.44 0.70455 0.68041 0.65866 0.57515 0.56000 0.67147 0.56522 0.55000 0.68722 0.66245 0.60291 0.54379 0.54095 0.64017 0.54000 0.46 0.58776 0.57788 0.557831 0.554842 0.538449 0.52853 0.51852 0.50848 0.59323 0.58351 0.57374 0.56392 0.5541 0.5441 0.5342 0.5242 0.53380 0.52381 0.51382 0.50382 0.67841 0.65334 0.63083 0.61082 0.54534 0.53859 0.53095 0.53000 0.53538 0.52541 0.51542 0.50543 0.60123 0.59157 0.58186 0.57209 0.55508 0.54517 0.53523 0.52527 0.52861 0.51862 0.50863 0.64414 0.62141 0.61191 0.60235 0.48 0.52095 0.66951 0.52000 0.66050 0.51000 0.62549 0.50095 0.50 0.59279 0.58299 0.57319 0.56332 0.553332 0.54335 0.52306 0.54279 0.61603 0.49863 0.51 0.64218 0.55406 0.5142 0.49382 0.49095 0.49000 0.60648 0.59684 0.58712 0.57729 0.56738 0.54414 0.53418 0.52416 0.51410 0.51528 0.50525 0.49521 0.48513 0.56225 0.55235 0.54239 0.49542 0.48540 0.47536 0.48000 0.52 0.63286 0.5042 0.48862 0.48381 0.48095 0.47000 0.4942 0.47861 0.47381 0.47095 0.62344 0.46095 0.61391 0.54239 0.53237 0.52228 0.51213 0.50191 0.49163 0.48127 0.60428 0.49840 0.4741 0.46532 0.45856 0.45378 0.45094 0.45000 0.60428 0.594537 0.57470 0.554467 0.55544407 0.5533365 0.5512351 0.49095 0.479045 0.56 0.57 0.58 0.59 0.50398 0.48829 0.47503 0.46406 0.45526 0.44852 0.44377 0.44094 0.44600 0.55736 0.54725 0.53705 0.49382 0.48360 0.47333 0.47813 0.46793 0.45770 0.46489 0.45473 0.44454 0.44519 0.43511 0.42501 0.43375 0.42373 0.41370 0.43093 0.42093 0.41092 0.43000 0.45390 0.43848 0.4438 0.42843 0.41000 0.50244 0.40000 0.52674 0.46301 0.44743 0.43432 0.41490 0.40831 0.40367 0.40092 0.60 0.51633 0.61 0.47086 0.45264 0.43711 0.42408 0.4133 0.40478 0.39824 0.39364 0.39091 0.46037 0.48149 0.44221 0.42676 0.41636 0.40593 0.39465 0.38817 0.38361 0.38090 0.50581 0.41380 0.4031 0.62 0.63 0.37089 0.37000 0.48447 0.46019 0.43919 0.42119 0.39316 0.3826 0.37435 0.36799 0.36353 0.36088 0.36000 0.64 0.42119 0.41060 0.339996 0.38926 0.37850 0.36769 0.35682 0.34589 0.39545 0.38494 0.37438 0.36378 0.44941 0.43853 0.42757 0.42850 0.41773 0.40689 0.3724 0.3621 0.3518 0.35348 0.34344 0.33339 0.35087 0.65 0.47364 0.38280 0.36418 0.35790 0.35000 0.34000 0.37240 0.36198 0.35153 0.35399 0.34380 0.33359 0.34779 0.33768 0.32756 0.66 0.46259 0.34086 0.67 0.46821 0.44047 0.41652 0.39598 0.3415 0.32333 0.32083 0.32000 0.68 0.42918 0.41777 0.40624 0.39458 0.45683 0.40537 0.38499 0.35314 0.34105 0.3311 0.32337 0.31744 0.31328 0.31082 0.31000 0.69 0.33114 0.3208 0.3104 0.30004 0.28962 0.27917 0.26871 0.25822 0.24771 0.31744 0.30730 0.29716 0.28702 0.27686 0.26670 0.25653 0.24636 0.33053 0.31999 0.30941 0.29881 0.30322 0.29316 0.28309 0.27302 0.44530 0.39412 0.38278 0.37133 0.70 0.37393 0.36280 0.35158 0.34245 0.33172 0.32095 0.30080 0.29079 0.28077 0.30000 0.31313 0.31313 0.30289 0.29263 0.28235 0.27207 0.26177 0.25146 0.24114 0.23080 0.22045 0.34589 0.33491 0.32386 0.31275 0.30159 0.29036 0.27777 0.42179 0.28000 0.72 0.40981 0.39766 0.38535 0.37286 0.36020 0.34029 0.32891 0.31746 0.30593 0.29431 0.27000 0.26000 0.25000 0.38280 0.35979 0.31014 0.27075 0.31014 0.29928 0.27743 0.26644 0.25540 0.24432 0.23210 0.21078 0.19951 0.176840 0.298817 0.28817 0.26681 0.25608 0.24532 0.23452 0.22369 0.21284 0.34814 0.33639 0.32453 0.31256 0.37089 0.26073 0.25072 0.24070 0.26295 0.75 0.25288 0.34666 0.24000 0.76 0.23618 0.22599 0.21579 0.20559 0.23068 0.22065 0.21063 0.20061 0.23000 0.23272 0.36020 0.34733 0.32111 0.30769 0.29400 0.28025 0.25185 0.23731 0.20746 0.19212 0.16057 0.28260 0.288260 0.27081 0.25894 0.24697 0.23491 0.22276 0.21052 0.27707 0.26772 0.25630 0.24482 0.23327 0.22166 0.20997 0.19823 0.32187 0.30925 0.29649 0.28357 0.27049 0.23719 0.22664 0.21607 0.20547 0.22000 0.78 0.79 0.30048 0.28829 0.27598 0.22263 0.21255 0.20246 0.80 0.26355 0.19971 0.19538 0.19236 0.19059 0.19000 0.81 0.25100 0.23833 0.22552 0.21259 0.20194 0.19486 0.18932 0.18516 0.18227 0.18056 0.18000 0.17054 0.25724 0.24382 0.23023 0.21646 0.17217 0.17000 0.19102 0.18006 0.16907 0.18423 0.17892 0.17494 0.16470 0.15446 0.83 0.16000 0.84 0.16289 0.15219 0.14147 0.13072 0.15000 0.85 0.19818 0.18641 0.15807 0.15196 0.15049 0.19952 0.18632 0.17297 0.15948 0.18574 0.17320 0.16056 0.17452 0.16256 0.15052 0.14763 0.13717 0.12670 0.14185 0.13174 0.12162 0.14046 0.16540 0.15805 0.14422 0.14699 0.13590 0.12477 0.11361 0.13396 0.12370 0.87 0.20250 0.18834 0.15393 0.14241 0.13084 0.13000 0.12000 0.12040 0.14782 0.13842 0.11995 0.11622 0.11343 0.11037 0.11000 0.17398 0.10000 0.90 0.91 0.92 0.93 0.13497 0.12201 0.10894 0.11922 0.10755 0.09582 0.15942 0.14583 0.12624 0.10916 0.10572 0.10316 0.10139 0.10034 0.11398 0.09521 0.09126 0.09031 0.16057 0.14432 0.12773 0.14463 0.10241 0.09835 0.13204 0.09288 0.08028 0.08000 0.07000 0.06000 0.09575 0.08245 0.06903 0.05548 0.08404 0.07229 0.07100 0.11437 0.10396 0.08923 0.07991 0.07666 0.07415 0.06022 0.07220 0.06031 0.04836 0.11078 0.09888 0.08967 0.07674 0.06860 0.06578 0.06360 0.06198 0.06087 0.05487 0.95 0.96 0.97 0.09344 0.07570 0.05752 0.07520 0.05303 0.05073 0.05000 0.08313 0.05726 0.04588 0.05167 0.06416 0.04015 0.04000 0.03102 0.02069 0.01035 0.03299 0.02202 0.01102 0.03000 0.03011 0.02007 0.01004 0.05079 0.04572 0.04181 0.03876 0.03636 0.03447 0.03186 0.03045 0.98 0.99 1.00 0.02801 0.02593 0.02430 0.02126 0.02030 0.01015 0. 0.03887 0.03419 0.03068 0.02302 0.01000 0.01971 0.01726 0.01545 0.01153



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n	x /x	x /x	x <sub>1</sub> /x <sub>2</sub>	X / X	x /x 1 2	1 2	1 2	x /x	x /x 1 2	1 2	1 Z	1 2	x /x	1 2	1 2	1 2	1 2	1 2	1/1/2
1.00	2.00000	1.99239	1.96962	1.93185	1.87939	1.81262	1.73205	1.63830	1.53209	1.41421	1.28558	1.14715	1.00000	0.	0.	0.	0.34730	0.17431	0.
1.02	0. 2.02000 0.02000	0. 2.01216 0.02008	1.98869	1.94977	1.89566	1.82675	1.74352	1.64653	0.02629	1.41393	1.27972	1.13449	0.97872	0.04973	0.63400	0.43515			
1.04	2.04000	2.03193	2.00776	1.96766	1.91188	1.84079	1.75483	1.65452	1.54040	0.05775	0.06411	1.12020	0.95451 0.08549 0.92661	0.77356 0.10549 0.72561	0.56765 0.14375 0.45107				
1.06	2.06000	2.05169	2.02681	1.98551 0.06225 2.00334	1.92804 0.06411 1.94415	1.85473 0.06664 1.86857	1.76599 0.06999 1.77697	1.66225 0.07436 1.66971	1.54396 0.08005 1.54710	1.41150 0.08757 1.40928	1.26500 0.09771 1.25593	1.10403 0.11195 1.08565	0-13339	0.17034	0.27402				
1.08	2.08000 0.08000 2.10000	2.07145 0.08033 2.09121	2.04585 0.08134 2.06488	0.08306	0.08559	0.08905	0.09364	0.09966	0.10756	0.11807 1.40631	0.13249	0.15327 1.06461	0.18616	0.54307					
1.12	2.12000	0.10042	0.10170	0.10390	0.10713	0.11157	0.11746	0.12523	0.13550	0.14933	0.16860	0.19725	0.24586 0.80331 0.31669	0.38669					
	2.14000	0.12051	0.12208	0.12477 2.05664 0.14567	0.12873 1.99211 0.15039		0.14146 1.80891 0.16562	0.15109 1.69043 0.17723	0.16391 1.55376 0.19282	0.18139 1.39788 0.21432	0.20622 1.21998 0.24558	0.24456 1.01159 0.29617	0.72906						
1.16	0.14000 2.16000 0.16000	0.14061 2.15046 0.16071	0.14247 2.12188 0.16287	2.07434	2.00797	1.92291	1.81921	1.69675	1.55497	1.39226	1.20429 0.28697	0.97694							
1.18	2.18000	2.17021 0.18081	2.14086 0.18329	2.09201 0.18757	2.02378 0.19389	1.93622	1.82931	0.23045	1.55562	1.38557 0.28321 1.37768	1.18617 0.33081 1.16501	0.93311							
1.20	0.20000	2.18995	2.15982	2.10966 0.20857	2.03953 0.21574 2.05521	1.94943 0.22571 1.96253	1.83923 0.23923 1.84895	1.70842 0.25755 1.71374	1.55567 0.28284 1.55508	0.31938	0.37768	0.87199 0.50459 0.73538							
1.22	2.22000 0.22000 2.24000	2.20969 0.22103 2.22942	2.17877 0.22416 2.19771	2.12727 0.22959 2.14485	0.23764	0.24886	0.26415	1.71870	0.31407 1.55380	0.35690 1.35765	0.42843	0.66414							
1.26	0.24000	0.24114	0.24462 2.21663	0.25065	0.25961 2.08639	0.27213	0.28927	0.31279	0.34599	0.39598	0.48449								
1.28	0.26000	0.26125	2.23554	0.27174	0.28164 2.10189 0.30373	0.29552 2.00113 0.31902	0.31460 1.87689 0.34014	0.34098 1.72747 0.36956	0.37867 1.54891 0.41216	0.43686 1.33030 0.47989	0.54846 1.01910 0.62643								
1.30	0.28000 2.30000 0.30000	0.28137 2.28861 0.30149	0.28557 2.25444 0.30606	0.29286 2.19740 0.31401	2.11732 0.32588	2.01376	1.88577	1.73124	1.54516	1.31294	0.92655 0.74470								
1.32	2.32000	2.30834	2.27332	2.21485	2.13268 0.34811	2.02626	1.89442	1.73456	1.54041	1.29227									
1.34	2.34000 0.34000	2.32806	2.29219	2.23227	2.14798	2.03865 0.39026 2.05090	1.90284 0.41811 1.91101	1.73740 0.45793 1.73975	1.53454 0.51846 1.52740	1.26721 0.62784 1.23589									
1.36	2.36000 0.36000 2.38000	2.34778 G.36188 2.36749	2.31105 0.36763 2.32990	2.24966 0.37766 2.26702	2.16322 0.39275 2.17838	0.41426	0.44458	0.48835	0.55624	0.68744									
1.40	0.38000	0.38201	0.38817 2.34873	0.39894	0.41517 2.19348	0.43839 2.07502	0.47131	0.51931	0.59546	0.75717									
1.42	2.42000	0.40215	0.40873	0.42025	0.43766 2.20851 0.46022	0.46265 2.08687 0.48705	0.49829 1.93396 0.52555	0.55084 1.74339 0.58300	0.63637 1.49628 0.67928	0.84853									
1.44	0.42000 2.44000 0.44000	0.42228 2.42661 0.44243	0.42931 2.38636 0.44989	0.44160 2.31889 0.46298	2.22346	2.09858	1.94105	1.74332	1.48157										
1.46	2.46000	2.44632 0.46257	2.40515	2.33611	2.23835 0.50555	2.11015	1.94784	0.64940	0.77306										
	2.48000	2.46601	2.42393	2.35330	2.25317 0.52832 2.26791	2.12158	1.95433 0.60911	1.74091 0.68378 1.73841	1.44193 0.82556 1.41431										
1.50	2.50000 0.50000 2.52000		0.51173	0.52733	0.55117	0.58607	1.96628	1.73491	0.88382 1.37740										
1.54	0.52000	0.52303	0.53237	0.54884	0.57409 2.29717	0.61120 2.15494	0.66644	0.75531	0.95132										
1.56	0.54000 2.56000	0.54319 2.54478	2.49891	2.42171	0.59708	0.63649 2.16573	1.97678	0.79270 1.72439 0.83137	1.03789										
1.58	0.56000 2.58000 0.58000	2.56446		0.59198 2.43873 0.61360	0.62015 2.32613 0.64330	0.66195 2.17636 0.68757	0.75521	0.87152											
1.60		2.58414	2.53632 0.61506	2.45571 0.63525	2.34049 0.66653	2.18682 0.71336	1.98564	0.91342											
1.62	2.62000	2.60352 0.62385	2.55501 0.63577	2.47265	2.35477	2.19710	1.98939 0.81653 1.99265	1.69662 0.95743 1.68275											
1.64	0.64000	0.64403	0.65649	2.48956 0.67867 2.50644	2.36897 0.71322 2.38309	2.20719 0.76550 2.21710	0.84792	1.00407											
1.68	0.66000	0.66420	2.61098	0.70044 2.52327	0.73669 2.39712	0.79185	0.87984	1.05412 1.64350 1.10886											
	0.68000 2.70000	2.68249	0.69798	0.72224 2.54008 0.74407	0.76025 2.41107 0.78388	0.81839 2.23630 0.84515	0.91234 1.99903 0.94546	1.61441											
1.72	0.70000 2.72000 0.72000	2.70216	2.64822	2.55684 0.76595 2.57357	2.42493 0.80761	2.24559 0.87211	1.99986	1.57239											
1.74	2.74000	2.72181	2.66683 0.76031	0.78786	0.83142	2.25466 0.89930	1.99994	1.48818											
	2.76000 0.76000	2.74147	2.68542 0.78111	2.59025 0.80980	2.45239 0.85533 2.46598	2.26350 0.92671 2.27209	1.04923												
1.78	2.78000 0.78000 2.80000	0.78533	0.80193	0.83179	0.87933	0.95436	1.08556												
1.82	0.80000	0.80553 2.80041	0.82276	0.85382 2.64009	0.90342 2.49288	0.98227 2.28853	1.12296												
1.84	0.82000	0.82574 2.82006	2.75963	2.65663	0.92760 2.50618 0.95189	2.29634	1.98541												
1.86	0.84000 2.86000	2.83970	2.77815	2.67312	2.51939		1.97837												
1.88	0.86000 2.88000 0.88000	2.85933	2.79665	2.68958 0.94231	2.53249 1.00075	2.31109	1.96930 1.28695												
	2.90000	2.87896 0.90658	2.81514 0.92713	2.70599 0.96453	1.02534	1.12597	1.33320												
	2.92000	0.92680	0.94805	0.98679	2.55838 1.05004 2.57116	1.15566	1.38277												
1.94	0.94000	0.94702	0.96898 2.87052	2.75499	1.07485	1.18570	1.43698												
	0.96000	0.96724	0.98992	2.77124	2.59639	1.21613	1.85580												
2.00	0.98000	0.98747	2.90/3/	2.78745	2.60883	2.34701	1.73205					*							
	1.00000	1.00770	1.03186	1.0/625	1.14994	1.2/822	1.13205												

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Alfred Nobel

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