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EUROPEAN ATOMIC ENERGY COMMUNITY - EURATOM

STUDIES ON FILM THICKNESS AND VELOCITY DISTRIBUTION OF TWO-PHASE ANNULAR FLOW

by

L. BIASI*, G.C. CLERICI*, R. SALA** and A. TOZZI*

*ARS S.p.A. and «Istituto di Scienze Fisiche», University of Milan
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1968

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Other quantities which are to be determined are the film thickness and the liquid and gas film flowrate. The entrained liquid flowrate and the core gas flowrate can then be obtained from these latter by using some balance equations.

An equation relating the liquid film thickness to the physical and geometrical parameters of the system is obtained. By solving this equation the film thickness, the liquid and gas film flowrates are calculated. The results are compared with some experimental data at different conditions.

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SUMMARY

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KEYWORDS

FILMS
THICKNESS
VELOCITY
DISTRIBUTION
TWO-PHASE FLOW
FLUID FLOW
PRESSURE
DIFFERENTIAL EQUATIONS

CONTENTS

STUDIES ON FILM THICKNESS AND VELOCITY DISTRIBUTION OF TWO-PHASE ANNULAR FLOW (+)

INTRODUCTION

This report contains a theoretical study on the fluidodynamics of a two-phase annular dispersed flow in adiabatic conditions. Assuming as initial condition a situation of equilibrium, the main quantities necessary for a fully macroscopic description of the system are the total pressure drop, the gaseous and liquid phase distribution, the velocity profiles. If the flow field is subdivided into two region "film" and "core", according to a usual representation, the other quantities which are to be determined are the film thickness and the liquid and gas film flowrate. Then the entrained liquid flowrate and the core gas flowrate can be obtained from these latter by using some balance equations. The entrainment and diffusion liquid droplet velocities are also necessary but only for the description of non equilibrium conditions.

The knowledge of the equilibrium conditions here analyzed may be a useful step for this determination.

An analytical solution of two-phase annular flow with liquid entrainment has been previously presented by S. Levy.[1/] By assuming the knowledge of the total pressure drop for unit length, Levy builds up a quantity F depending on the film thickness only.

The relation between the function F and the film thickness is obtained by some plots correlating experimental measurements performed at CISE. Actually a fully analytical description of this type of flow though referred to time averaged quantities is exceedingly difficult. For this reason a similar approach was followed in the present work. The basic assumption is the knowledge of the total pressure drop together with the average void fraction α .

(+) Manuscript received on January 12, 1968.

They are two well-studied quantities and several empirical correlations give their value with good accuracy. Under these two assumptions, an equation relating the liquid film thickness to the physical and geometrical parameters of the system is obtained. By solving this equation the film thickness, the liquid and gas film flowrates are then calculated. The results are compared with some experimental data at different conditions.

1 - DESCRIPTION OF THE MODEL

The model here presented is based on a subdivision of the flow field into two regions:an annulus around the solid walls of the duct (film) and a central zone (core). Both phases are assumed to be present everywhere.The film is further subdivided into a laminar sublayer and a turbulent region. In single phase systems there exists also a buffer zone where the dynamic and eddy viscosity are comparable On the other hand in two-phase systems the mass transfer between film and core can reduce the extension of this transition region. Therefore it can be reasonable to neglect it when the film thickness is much smaller then the duct radius. The core is assumed to be completely turbulent. It is also stated that the motion conditions in the film are mainly affected by the physical properties of the liquid phase, whilst in the core by those of the gaseous phase. Thus, instead of establishing an actual separation of the phases, a separation is postulated in the motion conditions between the two regions. The velocity distribution in the turbulent region is obtained by using Von Karman assumption on the mixing length and taking into account the actual shear stress distribution. The integration constants are calculated by matching the velocities and their derivatives at the boundary between the laminar sublayer and the turbulent region, and at the film-core interface. Prandtl's universal velocity distribution was also tested for the core region. In both cases the value of the mixing length constant was modified as suggested by the experimental results obtained at CISE. A single expression of the local void fraction α as a function of the film thickness is introduced. A set of equations is then derived which leads to the determination of the film thickness and partial mass flowrates.

2 - DISTRIBUTION OF THE LIQUID AND GASEOUS PHASE.

The experimental results show that the void fraction α is a function of the distance from the duct wall. As a first approximation for the void fraction α a linear trend has been assumed starting from zero at the wall up to a value α^* at the film-core interface. In the core it is assumed to have a constant value equal to α^*. Denoting by R the duct radius, Δ the film thickness, b the core radius and y the spatial coordinate, it is

(1) $$\alpha = \begin{cases} \alpha^* \dfrac{R-y}{\Delta} & b \leq y \leq R \\ \alpha^* & 0 \leq y \leq b \end{cases}$$

as shown in fig. 1

If $\bar{\alpha}$ is the average value of α , the relation between α^* and $\bar{\alpha}$ is

(2) $$\pi R^2 \alpha = \pi \int_0^b \alpha^* y \, dy + \pi \int_b^R \alpha^* \frac{R-y}{\Delta} y \, dy$$

which gives

(3) $$\alpha^* = \frac{R^2 \bar{\alpha}}{b^2 + \Delta(b + \Delta/3)}$$

Thus, the film and core density ρ_f and ρ_c are given by

(4) $$\rho_f = \rho_L \left(1 - \alpha^* \frac{R-y}{\Delta}\right) + \rho_G \, \alpha^* \, \frac{R-y}{\Delta}$$

(5) $$\rho_c = \rho_L (1 - \alpha^*) + \rho_G \alpha^*$$

where ρ_L and ρ_G are the lîquid and gas density.

Trend of α as a function of the distance from the duct wall.

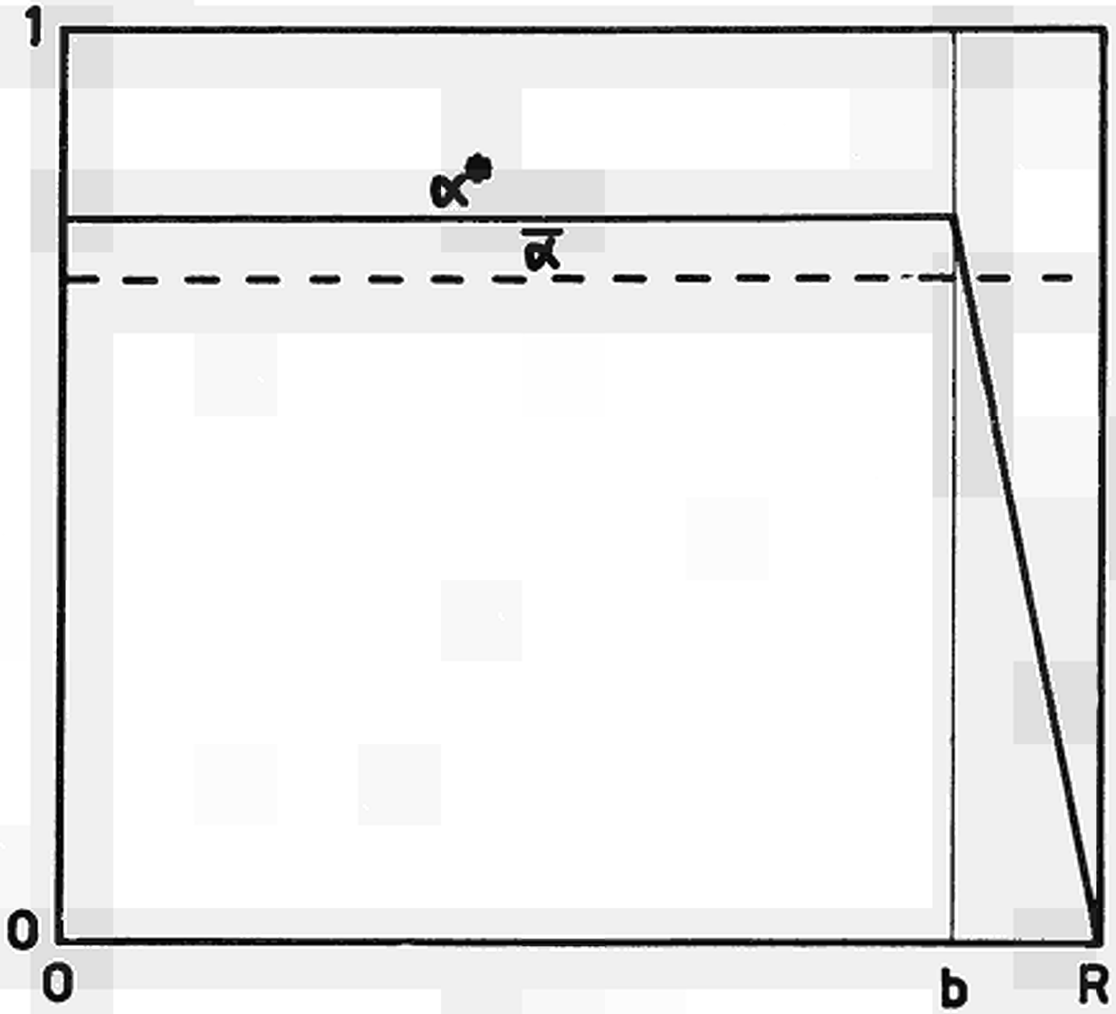

Fig. 1

3 - FILM VELOCITY DISTRIBUTION

Since the ratio between the film thickness and the duct radius is usually very small, the cylindrical geometry of fig. 2 is substituted with the plane geometry of fig. 3 in the calculation of the film velocity profile. The validity of this approximation is examined in the appendix. Assuming in the film

(6) $$\tau = K_1 y + K_2$$

the constant K_2 , usually equal to zero for single phase systems, is now determined by matching the shear stresses at the film-core boundary. τ_o and τ_c being the values of the shear stress at the wall and at y=b, they are given by

(7) $$\tau_o = \left\{ \frac{dp}{dx} - g\left[\rho_G \bar{\alpha} + (1-\bar{\alpha})\rho_L\right]\right\} \frac{R}{2}$$

(8) $$\tau_c = \left\{ \frac{dp}{dx} - g\rho_c \right\} \frac{b}{2}$$

Equation (6) can be rewritten as

(9) $$\tau = (\tau_o - \tau_c)\,\frac{y-b}{\Delta} + \tau_c$$

where $\frac{dp}{dx}$ is the total pressure drop per unit length and g is the gravitational constant. Following Von Karman, the shear stress τ is connected with the velocity profile by

(10) $$\tau = \rho \chi^2 \frac{(du/dy)^4}{(d^2u/dy^2)^2}$$

Circular geometry of the system.

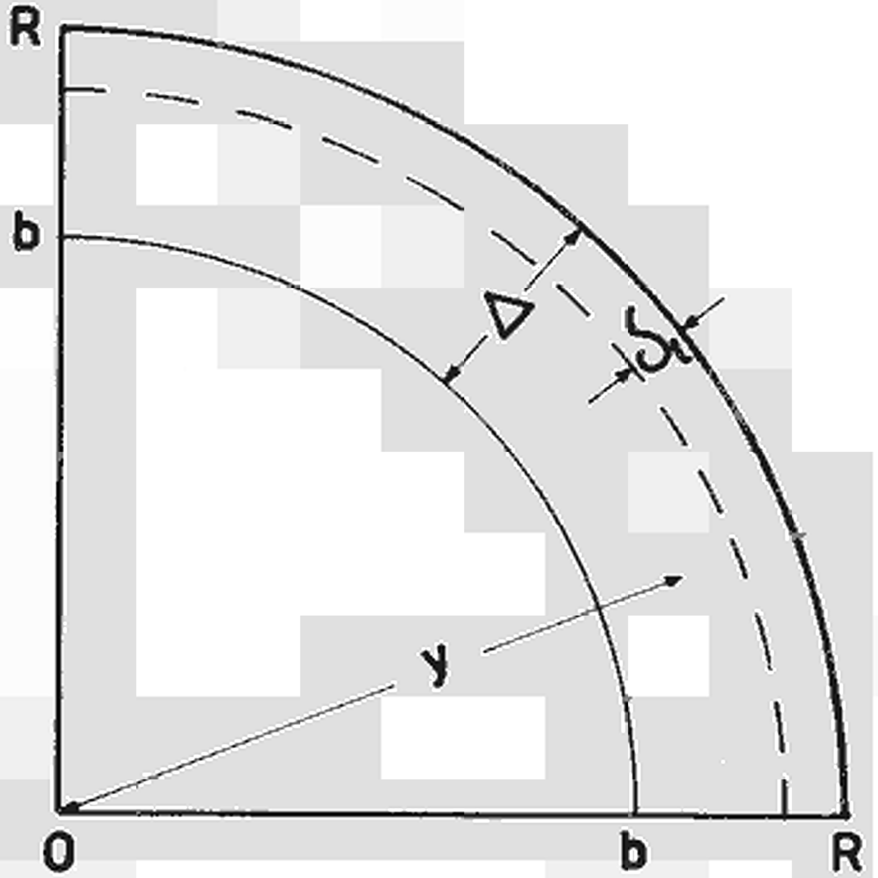

Fig. 2

Equivalent plane geometry.

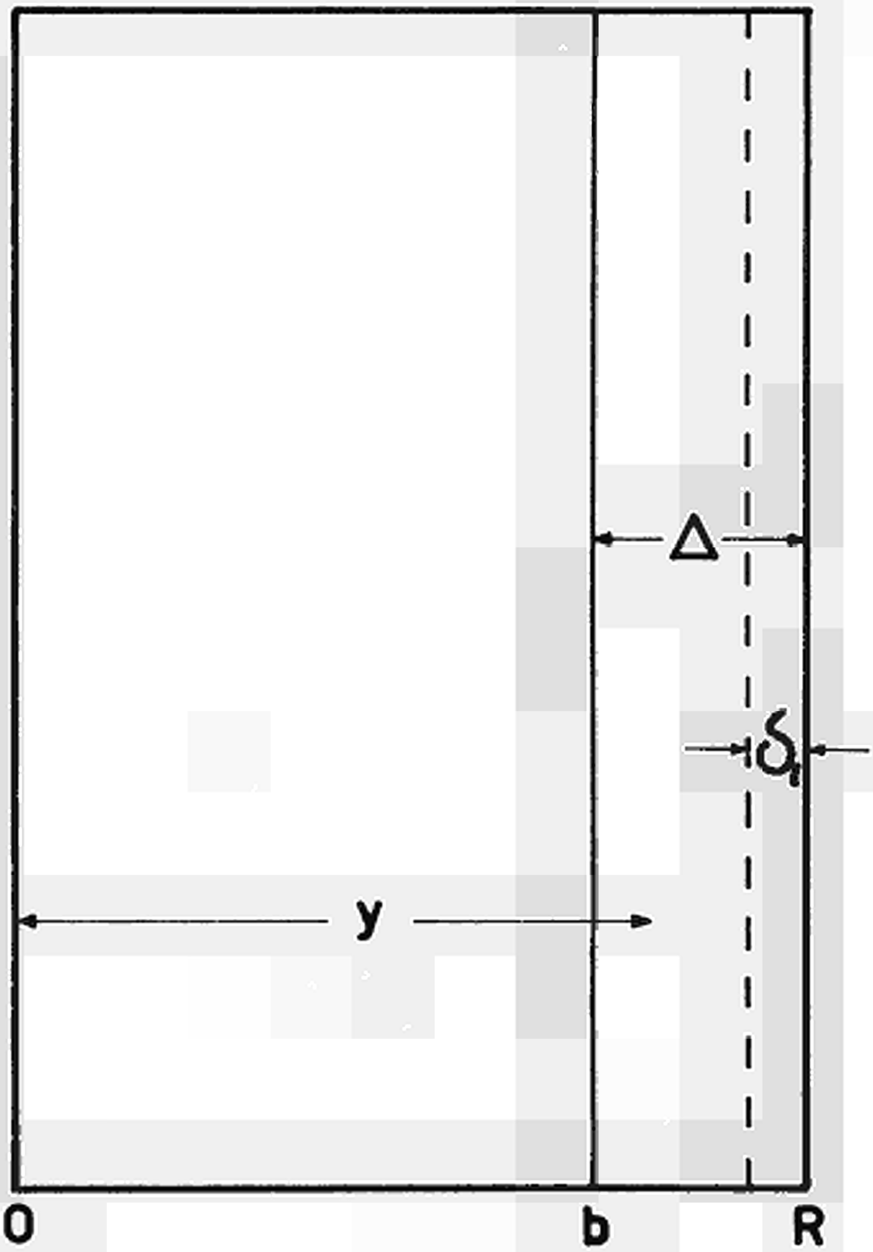

Fig. 3

By introducing the dimensionless variables

(11) $$y^+ = \left(1 - \frac{\tau_c}{\tau_0}\right)\frac{y-b}{\Delta} + \frac{\tau_c}{\tau_0}$$

(12) $$u^+ = \chi \sqrt{\frac{\rho}{\tau_0}} \quad u$$

and by using eq. (9) one has

(13) $$\frac{du^+}{dy^+} = -\frac{1}{\pm 2\sqrt{y^+} + c_1}$$

The costant C_1 is obtained by matching the turbulent region with the laminar sublayer of thickness $\delta_L = h\mu/(\chi\sqrt{\tau_0\rho}) \sim 12.1\mu/\sqrt{\tau_0\rho}$ With the usual assumptions(see for instance reference 3):

i) $\delta_L \ll R$ ii) $\left.\frac{du}{dy}\right|_{\delta_L^+} = \frac{1}{h}\left.\frac{du}{dy}\right|_{\delta_L^-}$

iii) $\tau(\delta_L) = \tau_0 = \rho\chi^2\delta_L^2\frac{d^2u}{dy^2}$

and the additional one

iiii) $$\frac{\delta_L}{\Delta}\left(1 - \frac{\tau_c}{\tau_0}\right) \ll 1$$

it is $C_1 = 2$. The sing minus of eq. (13) was taken because the velocity must decrease with y. Integrating eq. (13) and introducing the original variables, it is (for $b \leq y \leq R-\delta_L$)

(14) $$u = \frac{1}{\chi}\sqrt{\frac{\tau_0}{\rho}}\left\{ln\left|1 - \sqrt{(1-a)(y-b)/\Delta + a}\right| + \sqrt{(1-a)(y-b)/\Delta + a}\right\} + c_2$$

with $a = \tau_c/\tau_0$. In the laminar sublayer $R-\delta_L \leq y \leq R$ the expression of the velocity u is:

(15) $$u = \frac{\tau_L - \tau_0}{2\Delta\mu}(R-y)^2 + \frac{\tau_0}{\mu}(R-y) \sim \frac{\tau_0}{\mu}(R-y)$$

By equating eqs. (14) and (15) at $y = R - \delta_L$, C_2 becomes:

(16) $$C_2 = \frac{\tau_0 \delta_L}{\mu} - \frac{1}{\chi}\sqrt{\frac{\tau_0}{\rho}}\left\{ \ln\left| 1 - \sqrt{1 + (a-1)\frac{\delta_L}{\Delta}} \right| + \sqrt{1 + (a-1)\frac{\delta_L}{\Delta}} \right\}$$

As it seems reasonable to think that laminar sublayer is affected only by the liquid physical properties, whilst the gas affects the film turbulent region through a density variation, it is possible to set in eqs. (10) (14) (15) and (16) $\mu = \mu_L$; $\delta_L = 12.1\mu_L/\sqrt{\tau_0 \rho_L}$; $\rho = \bar{\rho}_f$. $\bar{\rho}_f$ is the average value of the film density, obtained by averaging eq. (4):

(17) $$\bar{\rho}_f = \rho_L + \alpha^*(\rho_G - \rho_L)\,\frac{R - 3/2 \cdot \Delta}{2R - \Delta}$$

With these substitutions the velocity profiles in the film become:

(18) $$u_f = \frac{\tau_0}{\mu_L}(R-y) \qquad R - \delta_L \le y \le R$$

(19) $$u_f = \frac{\tau_0 \delta_L}{\mu_L} + \frac{1}{\chi}\sqrt{\frac{\tau_0}{\bar{\rho}_f}}\left\{ \ln \frac{1 - \sqrt{(1-a)(y-b)/\Delta + a}}{1 - \sqrt{1 + (a-1)\delta_L/\Delta}} - \sqrt{1 + (a-1)\delta_L/\Delta} + \sqrt{(1-a)(y-b)/\Delta + a} \right\} \qquad b \le y \le R - \delta_L$$

In the above equations the only unknown quantity is the film thickness Δ (provided that the pressure drop and the average void fraction $\bar{\alpha}$ are known).

4 - AVERAGE FILM VELOCITY

The average value of the film velocity is obtained by integrating eqs. (18) and (19) over the film flow section

(20)
$$\bar{u}_f = \frac{2}{R^2-b^2}\left\{ \int_{R-\delta_L}^{R} \frac{\tau_0 (R^2-y^2)}{\mu_L R}\, y\,dy + C_1 \int_b^{R-\delta_L} y\,dy + \right.$$
$$\left. + \frac{1}{\chi}\sqrt{\frac{\tau_0}{\rho_f}} \int_b^{R-\delta_L} \left[\ln\left| 1-\sqrt{(1-a)\frac{y-b}{\Delta}+a} \right| + \sqrt{(1-a)\frac{y-b}{\Delta}+a} \right] y\,dy \right\}$$
$$= \frac{2}{R^2-b^2}\left\{ A_1 + A_2 + A_3 \right\}$$

A_1 and A_2 can be easily calculated:

(21) $$A_1 = \frac{\tau_0 \delta_L^2}{2\mu_L}\left(R - \delta_L + \frac{\delta_L^2}{4R} \right) \sim \frac{\tau_0 \delta_L^2 R}{2\mu_L}$$

(22) $$A_2 = C_1 \left\{ R(\Delta - \delta_L) - \frac{\Delta^2 - \delta_L^2}{2} \right\}$$

When $a>1$, introducing the new variable $t = \frac{y-b}{\Delta}$, A_3 becomes:

(23)
$$A_3 = \frac{1}{\chi}\sqrt{\frac{\tau_0}{\rho_f}}\left\{ \Delta^2 \int_0^{1-\frac{\delta_L}{\Delta}} t \ln\left| \sqrt{(1-a)t+a} - 1 \right| dt + \Delta b \int_0^{1-\frac{\delta_L}{\Delta}} \ln\left| \sqrt{(1-a)t+a} - 1 \right| dt \right.$$
$$\left. + \Delta^2 \int_0^{1-\frac{\delta_L}{\Delta}} t\sqrt{(1-a)t+a}\, dt + \Delta b \int_0^{1-\frac{\delta_L}{\Delta}} \sqrt{(1-a)t+a}\, dt \right\}$$
$$= \frac{1}{\chi}\sqrt{\frac{\tau_0}{\rho_f}} \left\{ \Delta^2 I_1 + \Delta b I_2 + \Delta^2 I_3 + \Delta b I_4 \right\}$$

Putting $X = \sqrt{(1-a)t+a} - 1$ and $Z = X + 1$, one obtains:

$$I_1 = \frac{2}{(1-a)^2}\left\{\left(\frac{x^4}{4}+x^3+\frac{3-a}{2}x^2+x-ax\right)\ln x-\left(\frac{x^4}{16}+\frac{x^3}{3}+\frac{3-a}{4}x^2+x-ax\right)\right\}\Bigg|_{\sqrt{a}-1}^{\sqrt{1+(a-1)\frac{\delta t}{\Delta}}-1}$$

$$I_2 = \frac{2}{(1-a)}\left\{\left(\frac{x^2}{2}+x\right)\ln x-\left(\frac{x^2}{4}+x\right)\right\}\Bigg|_{\sqrt{a}-1}^{\sqrt{1+(a-1)\frac{\delta t}{\Delta}}-1}$$

$$I_3 = \frac{2}{(1-a)^2}\left\{\frac{z^5}{5}-\frac{az^3}{3}\right\}\Bigg|_{\sqrt{a}}^{\sqrt{1+(a-1)\frac{\delta t}{\Delta}}}$$

$$I_4 = \frac{2}{(1-a)}\left\{\frac{z^3}{3}\right\}\Bigg|_{\sqrt{a}}^{\sqrt{1+(a-1)\frac{\delta t}{\Delta}}}$$

In the case $a < 1$, I_3 and I_4 have the same expression, whilst I_1 and I_2 become:

$$I_1' = -\frac{2}{(1-a)^2}\left\{\left(-\frac{x^4}{4}+x^3-\frac{3-a}{2}x^2+x-ax\right)\ln x-\left(-\frac{x^4}{16}+\frac{x^3}{3}-\frac{3-a}{4}x^2+x-ax\right)\right\}\Bigg|_{1-\sqrt{a}}^{1-\sqrt{1+(a-1)\frac{\delta t}{\Delta}}}$$

$$I_2' = -\frac{2}{(1-a)}\left\{\left(x-\frac{x^2}{2}\right)\ln x+\frac{x^2}{4}-x\right\}\Bigg|_{1-\sqrt{a}}^{1-\sqrt{1+(a-1)\frac{\delta t}{\Delta}}}$$

where $X = 1-\sqrt{(1-a)t+a}$.

The separation betweeen the two cases $a>1$ and $a<1$ is due to the presence of the modulus in A_3 . The case a=1 is a trivial one in which $\tau = \tau_0 = \tau_c$ and the solution can be obtained directly by eq. (9)

5 - LIQUID AND GAS FILM FLOWRATE

Neglecting the variation of the void fraction in the laminar sublayer, the liquid film-flowrate is given by

$$\Gamma_{LF} = 2\pi\rho_L\left\{\int_{R-\delta_L}^{R}\frac{\tau_0(R^2-y^2)}{2\mu_L R}\,y\,dy + C_1\int_{b}^{R-\delta_L}\left(1-\alpha^*\frac{R-y}{\Delta}\right)y\,dy\right.\tag{24}$$

$$\left.+\frac{1}{\chi}\sqrt{\frac{\tau_0}{\rho_f}}\int_{b}^{R-\delta_L}\left[\ln\left|\sqrt{(1-a)\frac{y-b}{\Delta}+a}-1\right|+\sqrt{(1-a)\frac{y-b}{\Delta}+a}\right]\left(1-\alpha^*\frac{R-y}{\Delta}\right)y\,dy\right\}$$

$$= 2\pi\rho_L\left\{A_1+A_4+A_5\right\}$$

where

$$A_4=\left\{R(\Delta-\delta_L)-\frac{\Delta^2-\delta_L^2}{2}-\alpha^*\frac{\Delta R}{2}+\alpha^*\frac{\Delta^2}{3}+\alpha^*\frac{\delta_L^2 R}{2\Delta}-\alpha^*\frac{\delta_L^3}{3\Delta}\right\}\tag{25}$$

$$A_5=(1-\alpha^*)A_3+\frac{\alpha^*}{\chi}\sqrt{\frac{\tau_0}{\rho_f}}\left\{\Delta^2(I_5+I_6)+\Delta(R-\Delta)(I_2+I_3)\right\}\tag{26}$$

In the case $a>1$

$$I_5=\frac{2}{(1-a)^3}\left\{\left[\frac{x^6}{6}+x^5+\frac{5-a}{2}x^4+\frac{10-6a}{3}x^3+\frac{5-6a+a^2}{2}x^2+(1-a)^2x\right]\ln x\right.$$

$$\left.\left.-\left[\frac{x^6}{36}+\frac{x^5}{5}+\frac{5-a}{2}x^4+\frac{10-6a}{9}x^3+\frac{5-6a+a^2}{4}x^2+(1-a)^2x\right]\right\}\right|_{\sqrt{a}-1}^{\sqrt{1+(a-1)\frac{\delta_L}{\Delta}}-1}$$

$$I_6=\frac{2}{(1-a)^3}\left\{\frac{x^7}{7}-\frac{2ax^5}{5}+\frac{a^2x^3}{3}\right\}\Bigg|_{\sqrt{a}}^{\sqrt{1+(a-1)\frac{\delta_L}{\Delta}}}$$

whilst in the case $a<1$ I_5 becomes

$$I_5 = -\frac{2}{(1-a)^3}\left\{\left(-\frac{x^6}{6}+x^5-\frac{a+5}{2}x^4+\frac{10+6a}{3}x^3-\frac{5+a^2+6a}{2}x^2+x+a^2x-2ax\right)\ln x -\right.$$

$$\left.-\left(-\frac{x^6}{36}+\frac{x^5}{5}-\frac{5+a}{8}x^4+\frac{10+6a}{9}x^3-\frac{5+a^2+6a}{4}x^2+(1-a)^2x\right)\right\}\Bigg|_{1-\sqrt{a}}^{1-\sqrt{1+(a-1)\frac{\xi}{\delta}}}$$

In the same way the gas film flowrate Γ_{GF} is given by

(27) $$\Gamma_{GF} = 2\pi\rho_L\left\{c_1\kappa^*\left[\frac{\Delta R}{2}\left(1-\frac{\xi^2}{R^2}\right)+\frac{\xi^3}{3\Delta}\right]+\frac{\kappa^*}{\chi}\sqrt{\frac{\tau_i}{\rho_L}}\cdot\right.$$

$$\left.\cdot\left[\Delta^2(I_1+I_3-I_5-I_6)+\Delta b(I_2+I_4-I_3-I_1)\right]\right\}$$

The integration limits have not been explicitely substituted in the above expressions since the solution requires a digital **computer.** The approximate **expressions** obtained through expansion in series are still too complex to be analytically treated owing to the presence of $(1-\bar{\alpha})$ and $(1-\alpha^*)$ which can be of the same order as Δ/R.

6 - CORE VELOCITY DISTRIBUTION

The shear stress in the core is obtained by eq. (6) with $K_1 = \frac{\tau_c}{b}$ and $K_2 = 0$:

$$\tau = \frac{\tau_c y}{b} \tag{28}$$

Substituting eq.(28) in eq. (10) and integrating it twice, one has:

$$u = \frac{1}{\chi}\sqrt{\frac{\tau_c}{\rho}}\left\{ ln\left(c_1' - \sqrt{\frac{y}{b}}\right) + \sqrt{\frac{y}{b}}\right\} + c_2' \qquad 0 \leq y \leq b \tag{29}$$

With the following assumptions

a) The core velocity distribution u_c is affected only by the gaseous phase density

b) The presence of dispersed liquid reduces the mixing length only through a lessening in the numerical value of the constant χ

the core velocity profile becomes

$$u_c = \frac{1}{\chi_c}\sqrt{\frac{\tau_c}{\rho_c}}\left\{ ln\left(c_1' - \sqrt{\frac{y}{b}}\right) + \sqrt{\frac{y}{b}}\right\} + c_2' \tag{30}$$

where χ_c is the new value of the constant χ taken equal to 0.27 according to some experimental results[2]. The two constants c_1' and c_2' can be determined by matching the velocities at the film-core interface and by giving the local core slip ratio between the two phases. Since there are no information about the value of the slip ratio, the latter condition is replaced by the assumption that the ratio of the velocity derivatives at the interface is equal to a function K depending only on the physical properties of the two phases:

$$\frac{du_c}{dy}\bigg|_{b^-} \bigg/ \frac{du_f}{dy}\bigg|_{b^+} = K \tag{31}$$

By eq. (10) it is

$$\tau(b) = \tau_c = \rho_f \chi^2 \left| \frac{(du_f/dy)^4}{(d^2u_f/dy^2)^2} \right|_{b^+} = \rho_G \chi_c^2 \left| \frac{(du_c/dy)^4}{(d^2u_c/dy^2)^2} \right|_{b^-} \tag{32}$$

and by eq. (29) and (31)

$$K^2 = \frac{\rho_f \chi^2}{\rho_G \chi_c^2} \tag{33}$$

On the basis of these two conditions one has

$$C_1' = 1 + \frac{\Delta\sqrt{a}}{R\sqrt{a} + b} \tag{34}$$

$$C_2' = \frac{\tau_0 \delta_L}{\mu_L} + \frac{1}{\chi}\sqrt{\frac{\tau_0}{\rho_f}}\left[\ln \frac{1-\sqrt{a}}{1-\sqrt{1+(a-1)\frac{\delta_L}{\Delta}}} - \sqrt{1+(a-1)\frac{\delta_L}{\Delta}} + \sqrt{a}\right] - \frac{1}{\chi_c}\sqrt{\frac{\tau_0}{\rho_f}}\left\{\ln(C_1'-1) + 1\right\} \tag{35}$$

Condition (31) can be replaced by stating the equality of the densities, velocities and costants χ, χ_c at the film-core interface. This condition seems to be reasonable as a local one even though the film density is approximated by its average value $\bar{\rho}_f$. It corresponds to the physical idea that all quantities must be continuous.

The core velocity distribution was also tested with Prandtl's universal distribution:

(36) $$u_c = u_m - \frac{1}{\chi_c}\sqrt{\frac{\tau_0}{\bar{\rho}_f}}\ \ln \frac{R}{R-y}$$

In this case there is only a constant, u_m, which can be obtained by matching like before the velocities at the film-core interface.

(37) $$u_m = \frac{1}{\chi_c}\sqrt{\frac{\tau_c}{\rho_G}}\ \ln\frac{R}{\Delta} + \frac{\tau_0\,\delta_L}{\mu_L} + \frac{1}{\chi}\sqrt{\frac{\tau_0}{\bar{\rho}_f}}\ \cdot$$

$$\left\{\ln \frac{1-\sqrt{a}}{1-\sqrt{1+(a-1)\frac{\delta_L}{\Delta}}} + \sqrt{a} - \sqrt{1+(a-1)\frac{\delta_L}{\Delta}}\right\}$$

7 - LIQUID AND GAS CORE FLOWRATE

The mean value of the core velocity is obtained by integrating eq. (30) on the core section:

$$(38) \quad \overline{u}_c = \frac{2}{b^2}\int_0^b u_c\, y\, dy = c_2' + \frac{1}{\chi_c}\sqrt{\frac{\tau_c}{\rho_G}}\int_0^b \left\{ \ln\left(c_1' - \sqrt{\frac{y}{b}}\right) + \sqrt{\frac{y}{b}} \right\} y\, dy$$

Setting $X = c_1' - \sqrt{\frac{y}{b}}$

$$\int_0^b y \ln\left(c_1' - \sqrt{y/b}\right) dy = 2b^2 \int_{c_1'}^{c_1'-1} \left(X^3 - 3c_1' X^2 + 3c_1' X - c_1'^3\right) \ln X \, dX =$$

$$2b^2 \left\{ \frac{c_1'^4}{4} \ln c_1' - \frac{(c_1'-1)(c_1'^3 + c_1'^2 + c_1' + 1)}{4} \ln(c_1'-1) - \frac{10c_1'^4 + 2c_1'^3 + 6c_1'^2 + 4c_1' + 3}{48} \right\}$$

and finally

$$(39) \quad \overline{u}_c = c_2' + \frac{1}{\chi_c}\sqrt{\frac{\tau_c}{\rho_G}} \left\{ \frac{4}{5} + c_1'^4 \ln c_1' + (c_1'-1)(c_1'^3 + c_1'^2 + c_1' + 1)\ln(c_1'-1) - \frac{10c_1'^4 + 2c_1'^3 + 6c_1'^2 + 4c_1' + 3}{12} \right\}$$

The gas flowrate in the core Γ_{GC} can be written

$$(40) \qquad \Gamma_{GC} = A_{GC}\, \overline{u}_c\, \rho_G$$

where $A_{GC} = \pi b^2 \alpha^*$ is the section of the core filled up with the gas. The liquid flowrate can be obtained by the balance equation on the total liquid flowrate Γ_L :

$$(41) \qquad \Gamma_{LC} = \Gamma_L - \Gamma_{Lf}$$

In the case of Prandtl's universal distribution eqs. (40) and (41) are still valid whilst the average core velocity becomes:

$$\text{(42)} \qquad \overline{u}_c = \frac{2}{b^2} \int_0^b \left\{ u_m - \frac{1}{\chi_c} \sqrt{\frac{\tau_c}{\rho_G}} \ln \frac{R}{R-y} \right\} y\,dy$$

$$= u_m + \frac{1}{\chi_c} \sqrt{\frac{\tau_c}{\rho_G}} \left\{ \frac{2\Delta}{R} \ln \frac{R}{\Delta} - \frac{3}{2} - \frac{\Delta}{R} \right\}$$

8 - FILM THICKNESS CALCULATION

The film thickness can be calculated through a balance equation of the gas flowrate

$$\Gamma_G = \Gamma_{Gf} + \Gamma_{Gc} \tag{43}$$

As Γ_G is equal to $\Gamma \cdot X$, where Γ is the total mass flowrate and X the quality, by introducing in eq. (43) the expression of Γ_{Gf} and Γ_{Gc} given by eqs. (27) and (40), it is obtained:

$$\begin{aligned} \Gamma \cdot X = {} & 2\pi \rho_G \left\{ c_1 \alpha^* \left[\frac{\Delta R}{2}\left(1 - \frac{\delta_L^2}{\Delta^2}\right) + \frac{\delta_L^3}{3\Delta} \right] + \frac{\alpha^*}{\chi} \sqrt{\frac{\tau_0}{\rho_f}} \left[\Delta^2 (I_1 + I_3 - I_5 - I_6) + b\Delta (I_2 + I_4 \right.\right. \\ & \left.\left. - I_1 - I_3) \right] \right\} + \pi b^2 \alpha^* \rho_G \left\{ c'_2 + \frac{1}{\chi_c} \sqrt{\frac{\tau_0}{\rho_f}} \left[\frac{4}{5} + c_1'^4 \ln c'_1 - (c'_1 - 1)(c_1'^3 + c_1'^2 + c'_1 + 1) \ln (c'_1 - 1) \right.\right. \\ & \left.\left. - \frac{10 c_1'^4 + 2 c_1'^3 + 6 c_1'^2 + 4 c'_2 + 3}{12} \right] \right\} \end{aligned} \tag{44}$$

The film thickness Δ is now the only unknown of eq. (44). This equation is too complex to be analytically solved and for this reason the calculation of Δ has been programmed on a digital computer.

9 - MODEL PREDICTIONS AND COMPARISON WITH EXPERIMENTAL RESULTS

The model predictions have been compared with a set of experimental results.
The data taken into account are mainly related to film thickness measurements, since only a few data about film flowrate or entrained liquid are available at present. The comparisons have been performed for water-inert gas systems and for various values of the physical and geometrical paramaters. The input data required by the computer program are:

σ = liquid surface tension
ρ_L, ρ_G = liquid and gas density
μ_L = liquid viscosity
G = total specific mass flowrate
X = quality
R =duct radius

As previously said, the knowledge of the total pressure drop and the void fraction is also required. The program can calculate these quantities by using available correlations or read them as input data.

10 - COMPARISON WITH CISE DATA

Several film thickness measurements [4,5], performed at CISE laboratories in different times, have been used for testing the model predictions. The pressure drop and the average void fraction have been calculated by the following correlations [7,8]:

$$\frac{dp}{dz} = \frac{0.43}{D^{1.2}} \left(\frac{\gamma}{73}\right)^{0.4} \left(\frac{\mu_L}{0.01}\right)^{0.04} \left(G^2 \bar{v}\right)^{0.35} \tag{45}$$

$$1-\bar{\alpha} = (1-X_v)\left\{ 1 + \frac{1.35\, X_v^{\,n}}{1+0.335\,\frac{G}{\gamma}\, D^{1.2}} \left(\frac{1}{\rho_G}\right)^{1/4} \right\} \tag{46}$$

where $n = 0.9 + 0.05\,\gamma$

$1-X_v$ = liquid volume flowrate quality

$\bar{v}$ = flowrate specific volume of the mixture

Preference was given to the pressure drop calculated by means of eq.(45) rather than to the measured one in order to avoid the experimental fluctuations.However both values have been used for a set of data. As one can see in table 1° the use of the correlation (45) does not bring about remarkable differences in the calculated values of the film thickness. In the same table the result obtained by using the distributions of Von Karman eq. (39) and of Prandtl eq. (42) are also compared. In this case as well the differences are not remarkable. All the other result are summarized in tables 2° - 6°. In these tables are repòrted the measured and predicted film thickness and the calculated liquid film flowrate in addition to the

physical paramenters describing the system. Some of the results are also shown in figs. 4 ÷ 31.
In figs. 4 ÷6 the film thickness values of table 1° are plotted as a function of the specific gas flowrate G_G for some values of the specific gas flowrate G_L . Here and in the following figures the full line joins the experimental points, whilst the dashed line the predicted ones.
Figs. 7 ÷ 11 show the data of table 2°, figs. 12 ÷ 15 those of table 4°, figs. 16 ÷ 18 those of table 5° and figs. 19 ÷ 20 those of table 6°. No data of table 3° are plotted since the experimental conditions are equal to those of table 5° which are more recent. As one can see by the diagrams and tables the model here presented predicts the film thickness trend as a function of G, X, P and R in a correct way. The agreement fails at the extreme values of X and G. This fact can be better seen in figs. 21 - 22 where some data of table 2° with the film thickness versus G_L at costant G_G are plotted.
At very low quality the substitution of the cylindrical geometry with the plane one and the matching of the laminar sublayer with the turbulent region, neglecting the buffer zone, is no longer justified. On the other hand for very high values of X the film and laminar sublayer thickness are comparable and therefore the assumption $\frac{\delta_L}{\Delta}\left(1-\frac{\tau_i}{\tau_0}\right) \ll 1$ fails.
In the range of validity of the introduced assumption, also the quantitative agreement seems to be satisfactory. It can be noted that the experimental measurements define an electrical film thickness, whilst the model predicts a thickness defined by purely fluidodynamics considerations.
As for the liquid flowrate a test of the model predictions is more difficult. In figs. 23 ÷ 25 the film and entrained liquid flowrate and the ratio Γ_{LF}/Γ_L are shown as a function of X

for three values of the total flowrate ($\Gamma = 491, 736, 982$ g./sec)
The data are related to water-argon mixture at 22 ata, for a duct with R = 1.25 cm. The trend with G and X seems to be reasonable and under some aspects similar to one experimentally observed at Harwell in steam-water system 9/. Fig. (26) shows the velocity profile for the case $\Gamma = 982$ g/sec ($X = 0.35$) as a function of the radial coordinate. Some results reported by Cravarolo Hassid are also shown (dashed line) in figs. (24) (25) and (26).

Some further comparisons have been obtained by plotting the quantity $\Gamma_{Lf} / \pi D \mu_L$ versus the dimensionless thickness $\Delta^+ = \frac{\Delta}{\mu_L}\sqrt{\tau_0 \rho_L}$. In ref.2 it is suggested that the experimental results are well correlated by the equation

$$\frac{\Gamma_{Lf}}{\pi D \mu_L} = 5.3 \, (\Delta^+)^{1.1} \tag{47}$$

Fig . 27 is a plot, in a log log chart, of eq. (47) together with the lines delimiting ± 10 % variations.

The dots represent the values predicted by the model for some values of table 5°. For a fully comparison the predictions of Levy Model and those of Dukler - Hewitt analysis derived from, reference 2, are also reported.

As one can see the values predicted by these theories are overestimated (40 - 60 %) in comparison with the experimental correlation. The prediction of the present model shows a very good agreement, except for the low region, where, as previously said, the hypothesis introduced are not completely satisfied.

In the authors' opinion the model predictions are best correlated by defining $\Delta^+ = \frac{\Delta}{\mu_L}\sqrt{\tau_0 \bar{\rho}_f}$, since $\bar{\rho}_f$ represent the true mean film density. Since it is always $\bar{\rho}_f < \rho_L$, with this definition the predicted values are no longer in agreement with eq. (47), as shown in fig. 28 where some data of table 1° are plotted. The relation between $\Gamma_{Lf}/\pi D\mu_L$ and the new Δ^+ becomes:

$$(48) \qquad \frac{\Gamma_{Lf}}{\pi D \mu_L} = 3.16\,(\Delta^+)^{1.26} \qquad \Delta^+ = \frac{\Delta}{\mu_L}\sqrt{\tau_0\,\bar{\rho}_f}$$

Figs. 29 ÷ 31 show the data of tables 1°,2°,6° and those corresponding to G = 100, 150, 200, gr/cm^2. sec with the line of eq. (48).

Trend of Δ as a function of G_G (data of table I)

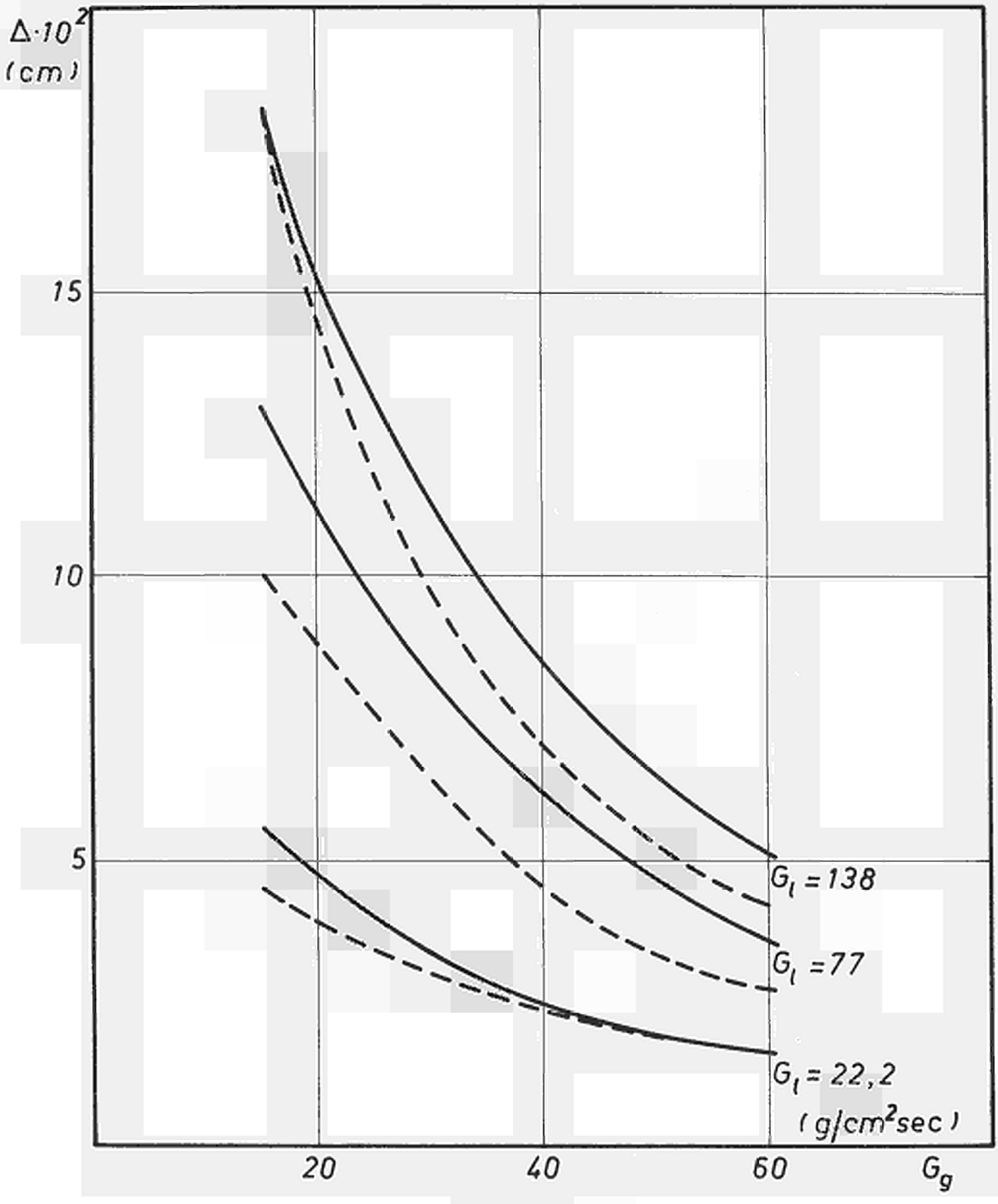

Fig. 4

Trend of Δ as a function of G_G (data of table I)

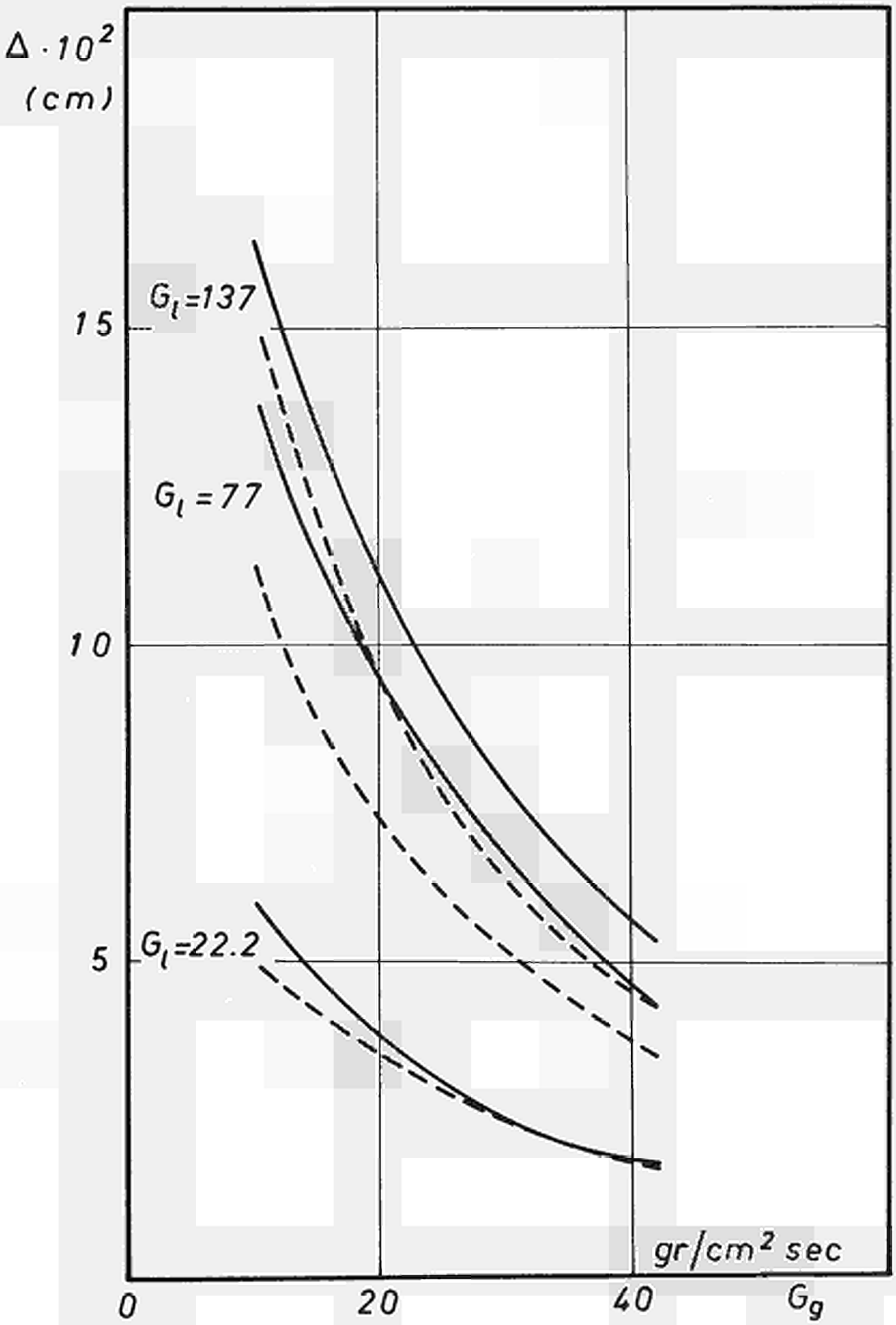

Fig. 5

Trend of Δ as a function of G_G (data of table I)

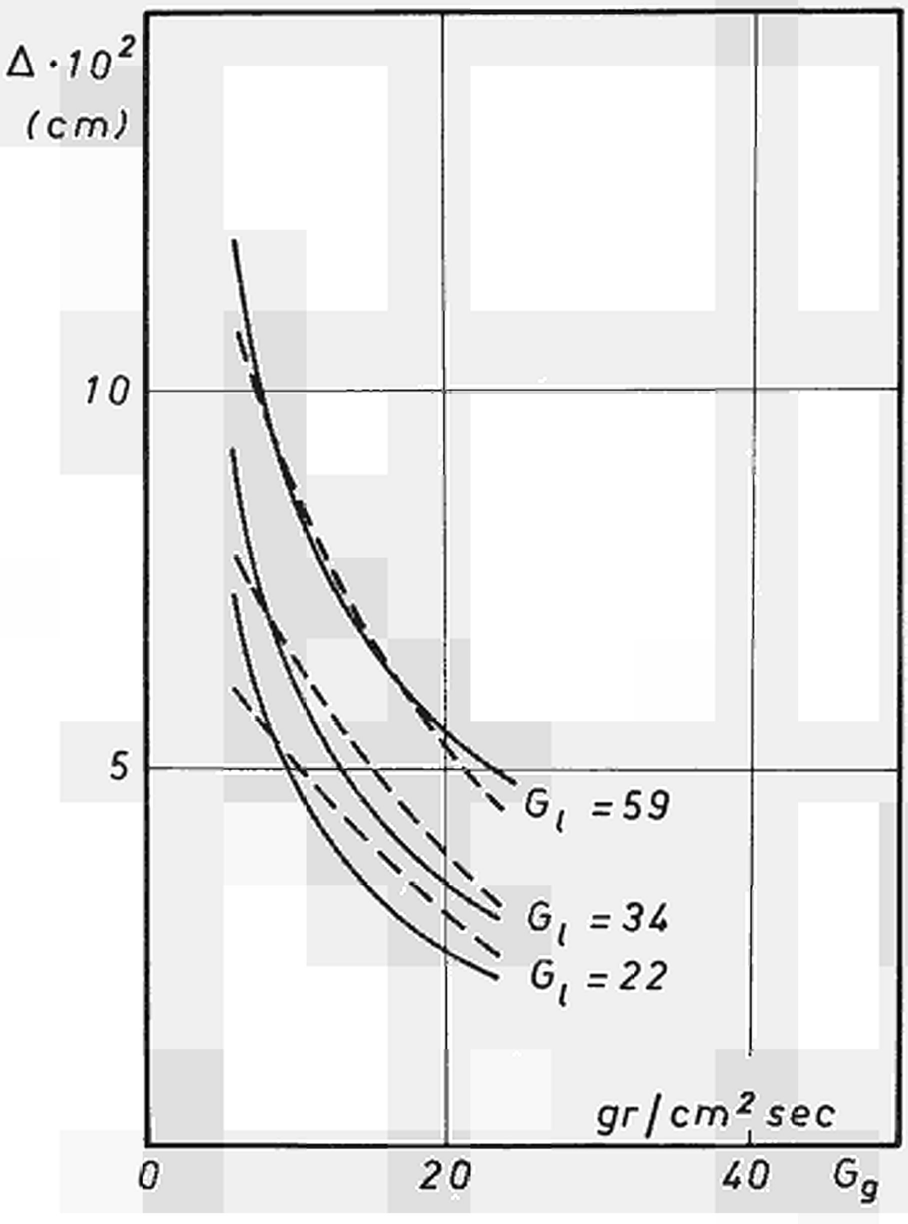

Fig. 6

Trend of Δ as a function of G_g (data of table II)

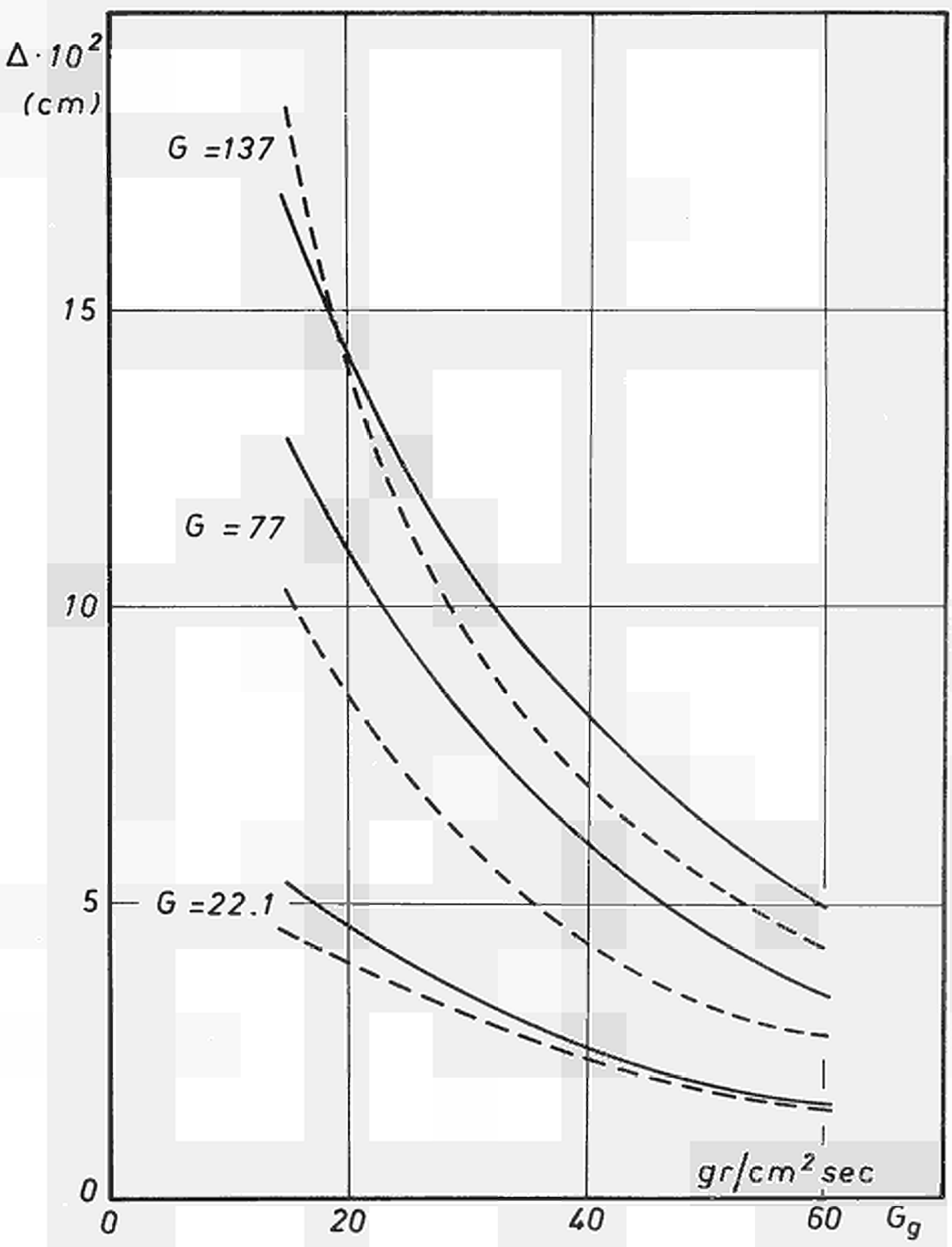

Fig. 7

Trend of Δ as a function of G_g (data of table II)

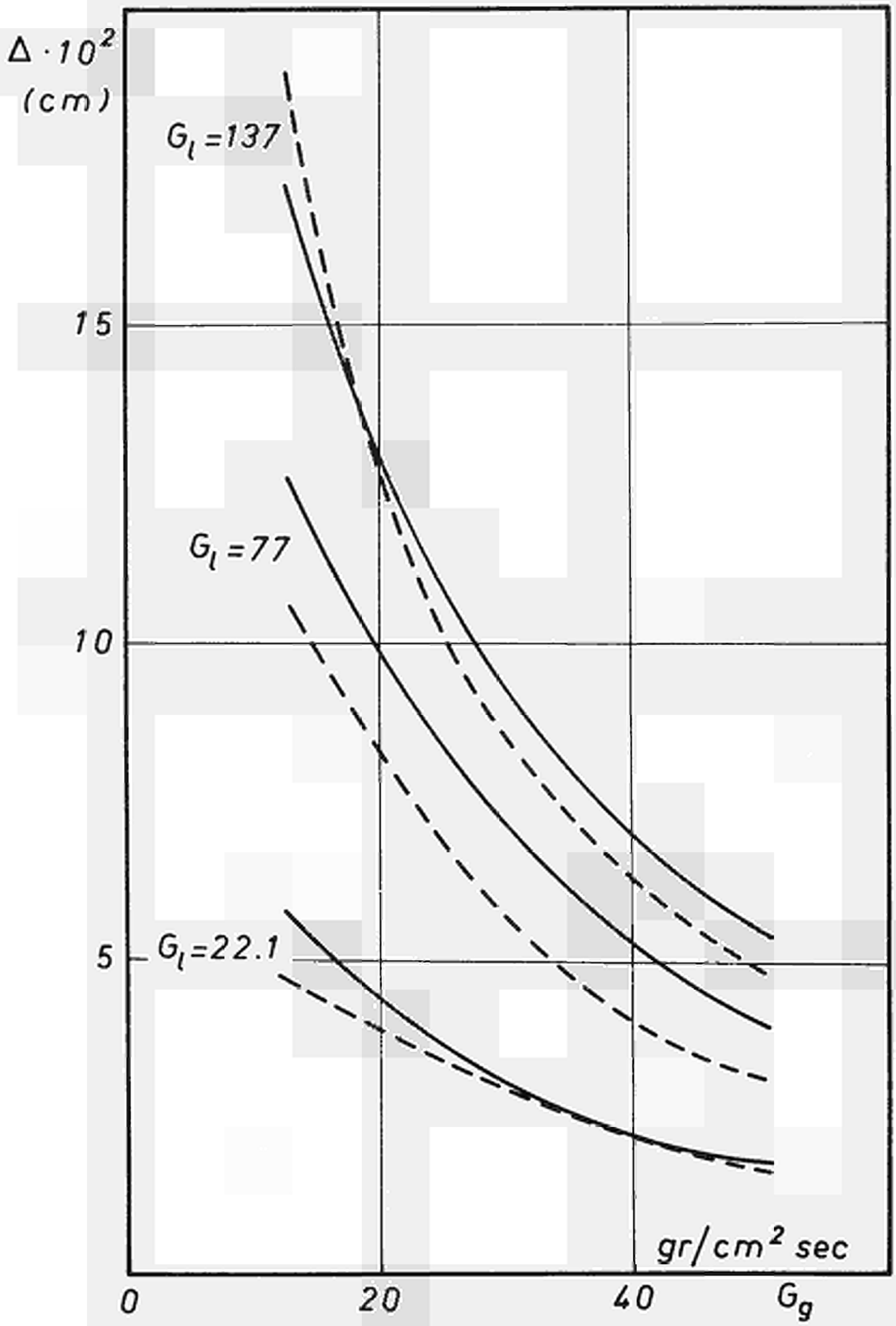

Fig. 8

Trend of Δ as a function of G_G (data of table II)

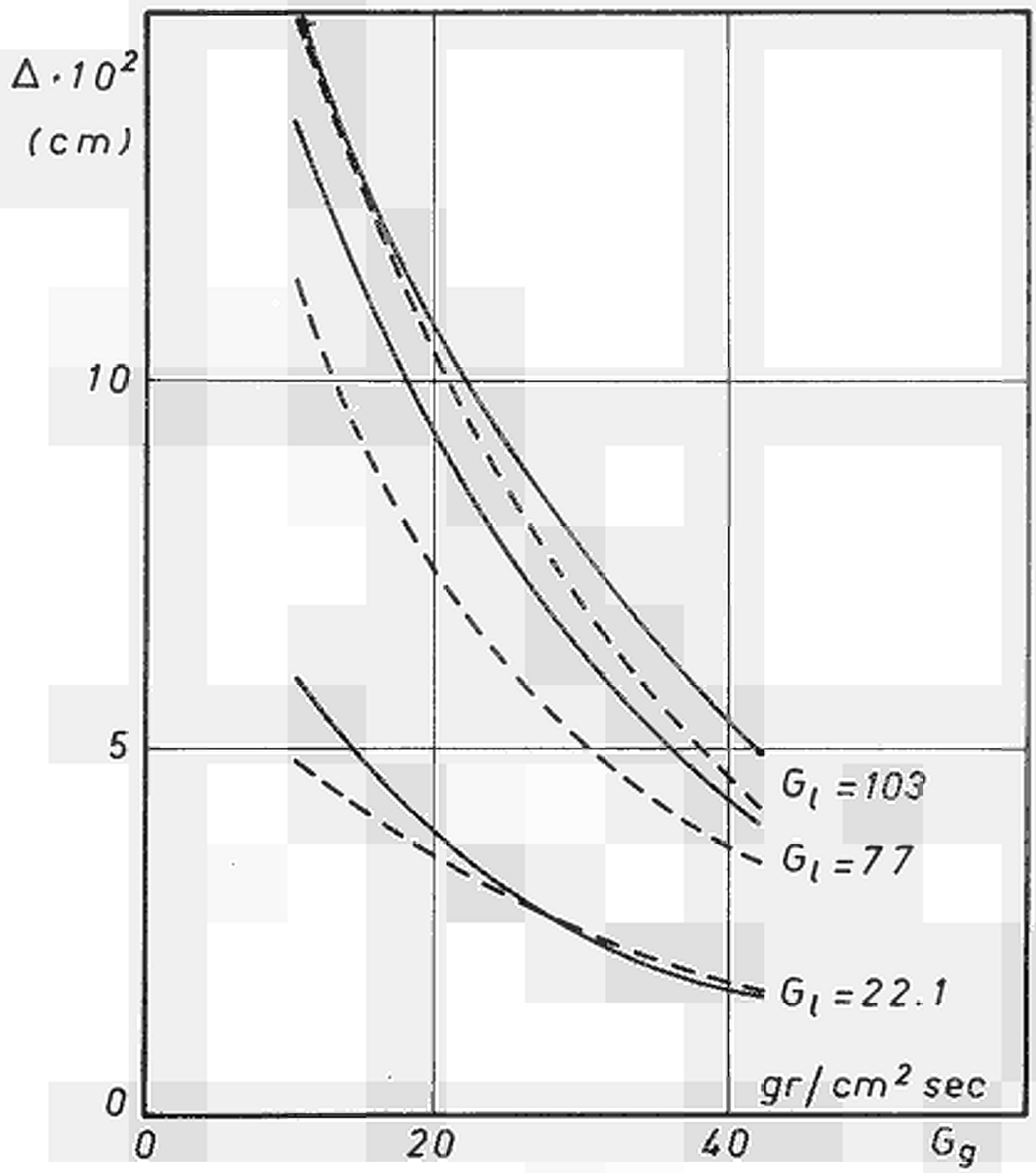

Fig. 9

Trend of Δ as a function of G_G (data of table II)

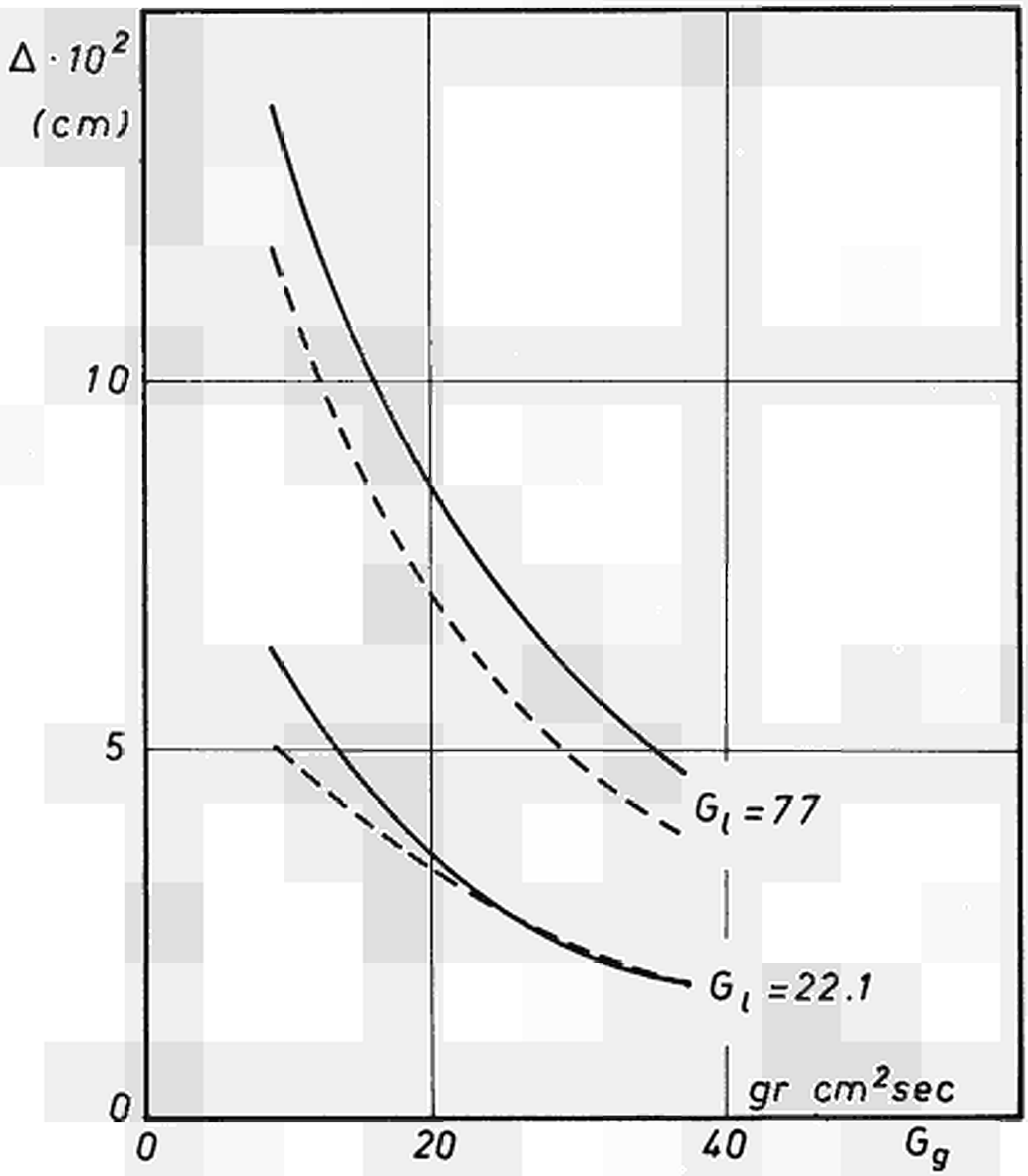

Fig. 10

Trend of Δ as a function of G_G (data of table II)

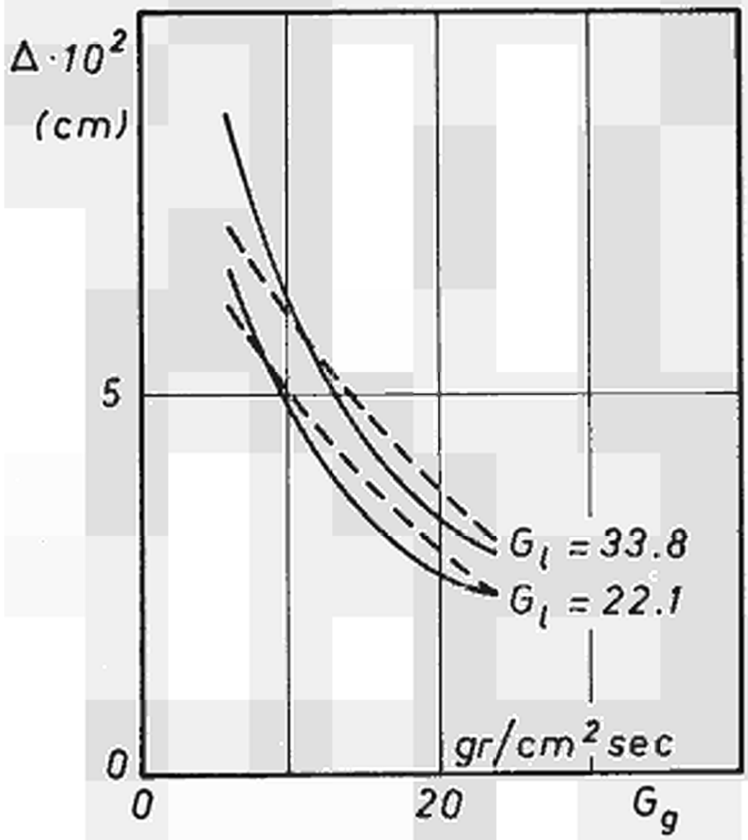

Fig. 11

Trend of Δ as a function of G_G (data of table IV)

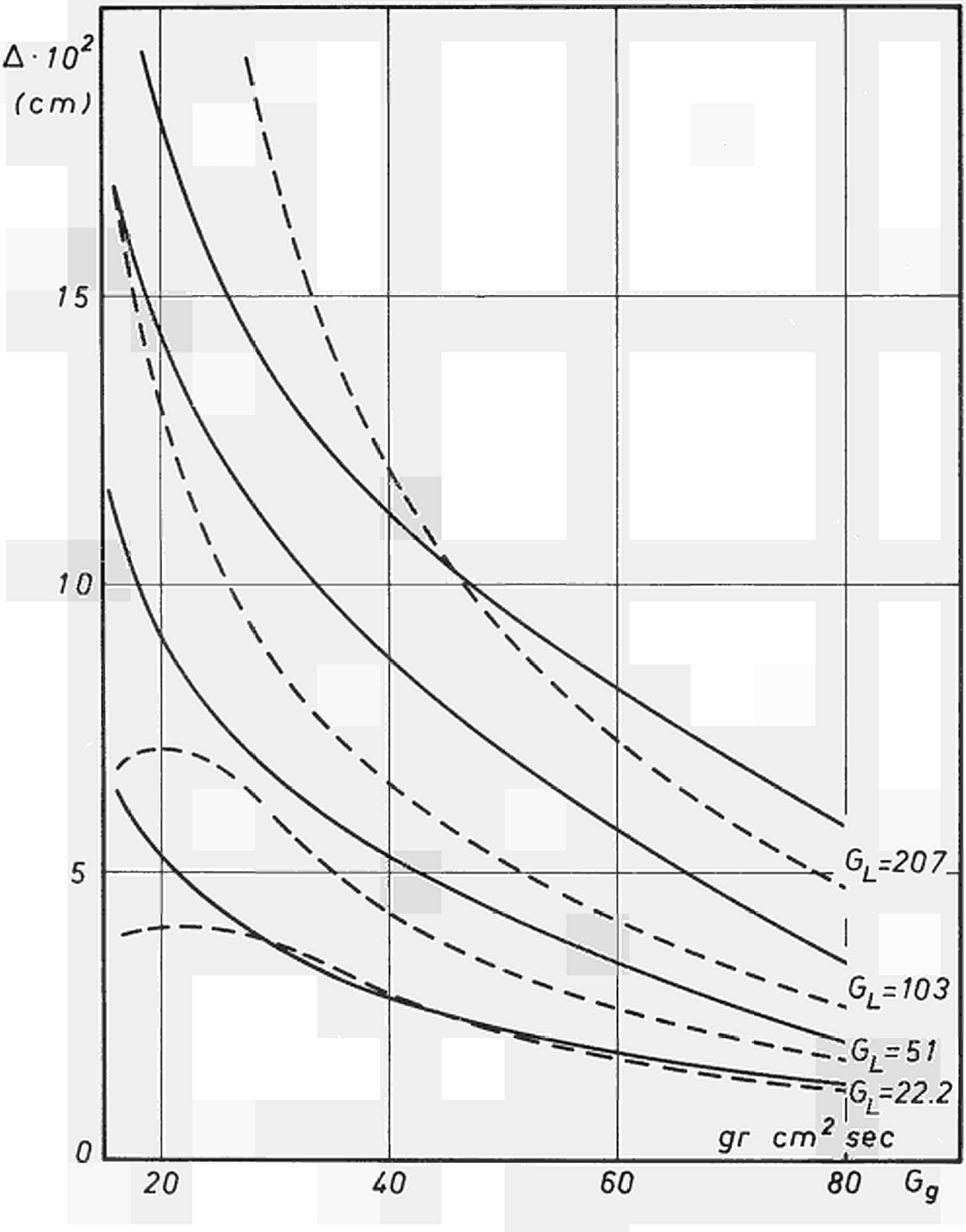

Fig. 12

Trend of Δ as a function of G_G (data of table V)

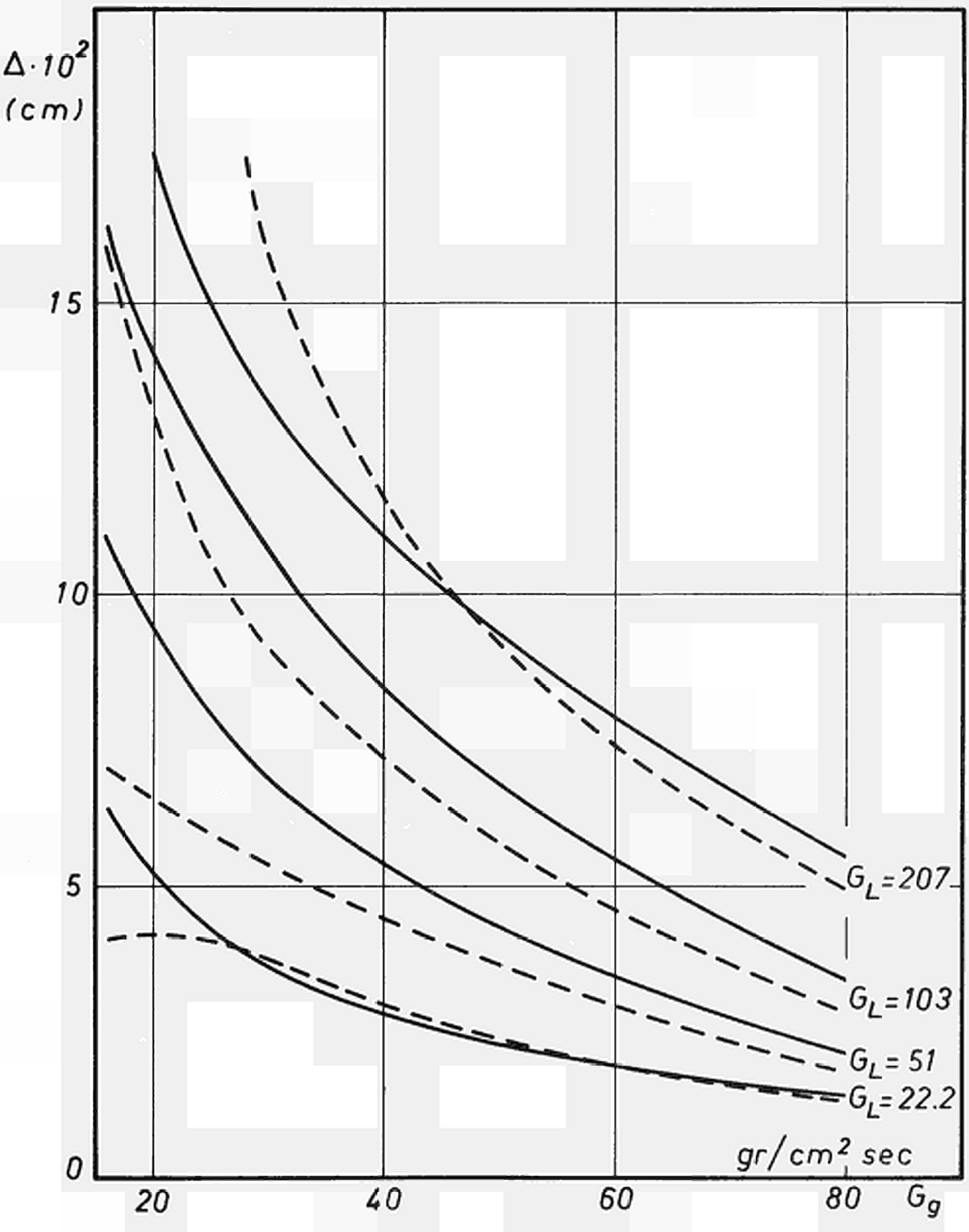

Fig. 13

Trend of Δ as a function of G_G (data of table IV)

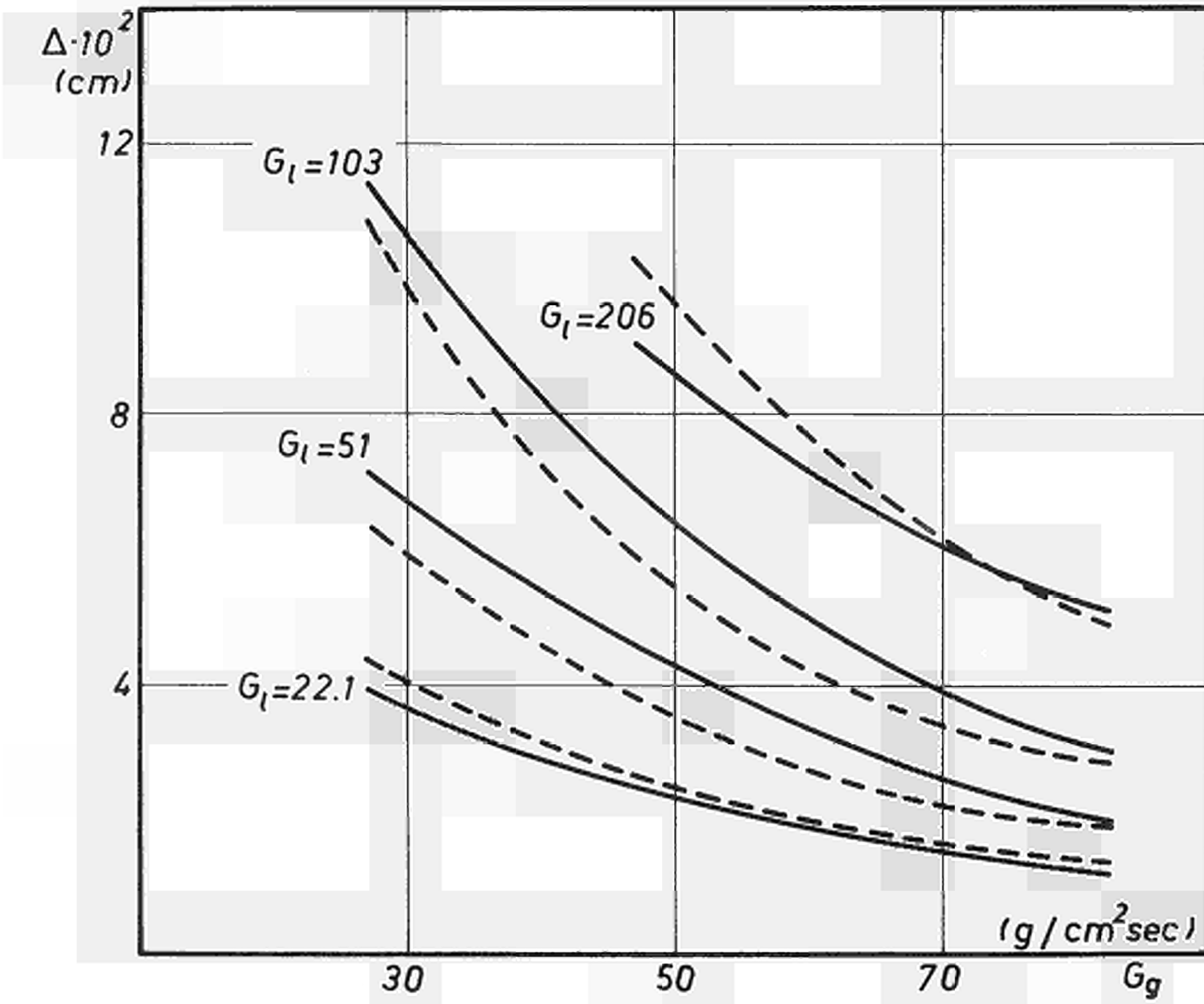

Fig. 14

Trend of Δ as a function of G_G (data of table IV)

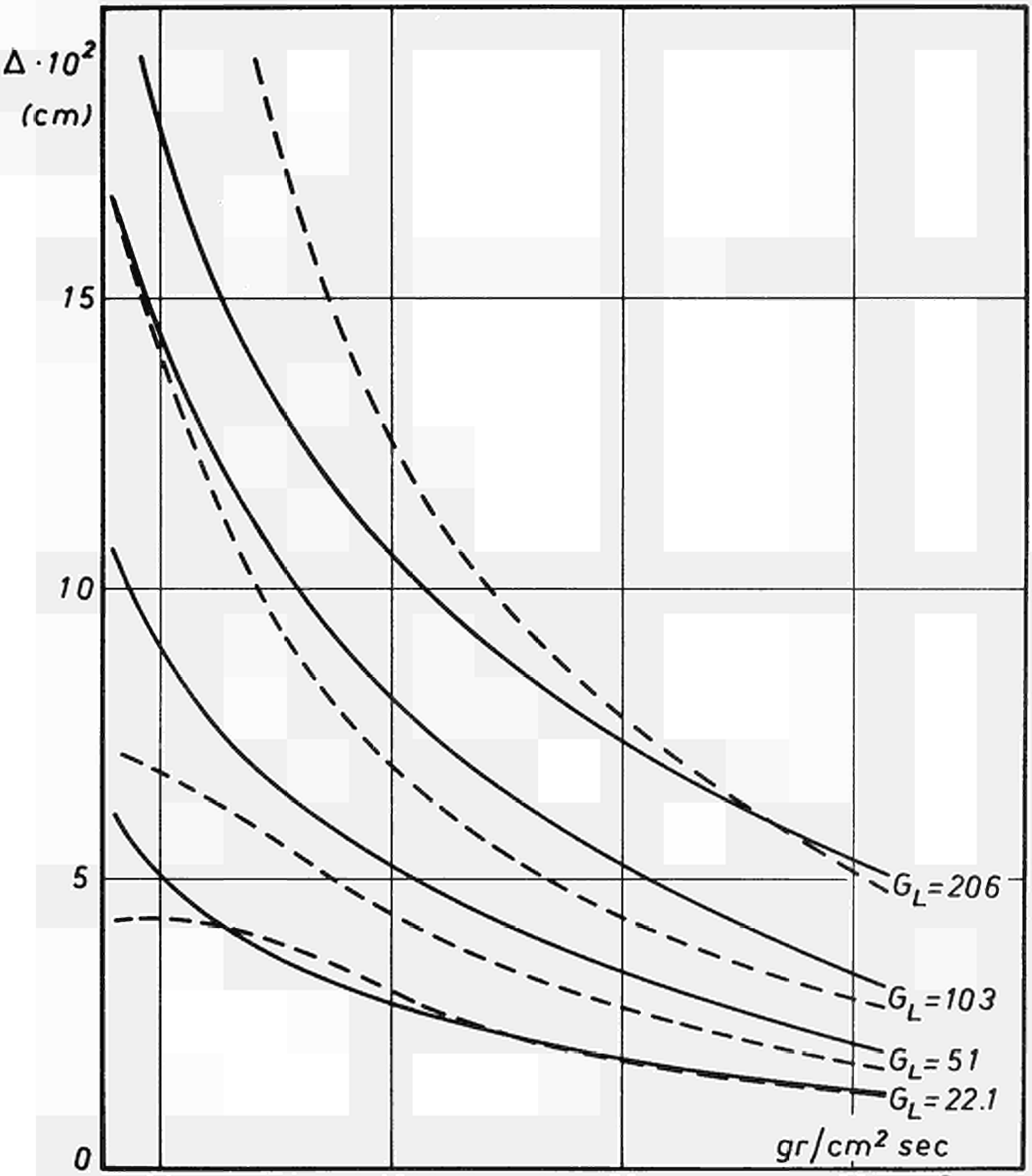

Fig. 15

Trend of Δ as a function of G_G (data of table V)

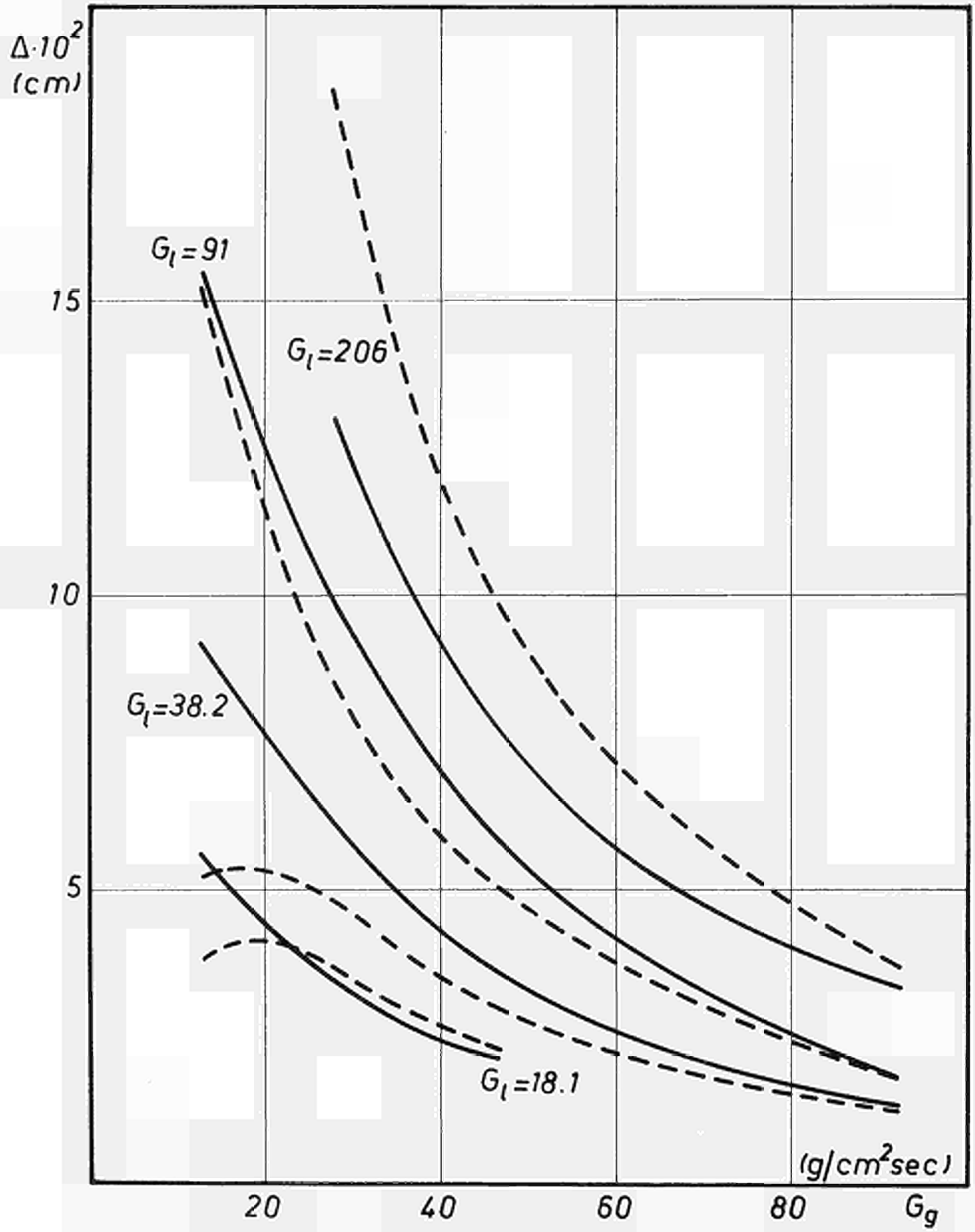

Fig. 16

Trend of Δ as a function of G_G (data of table V)

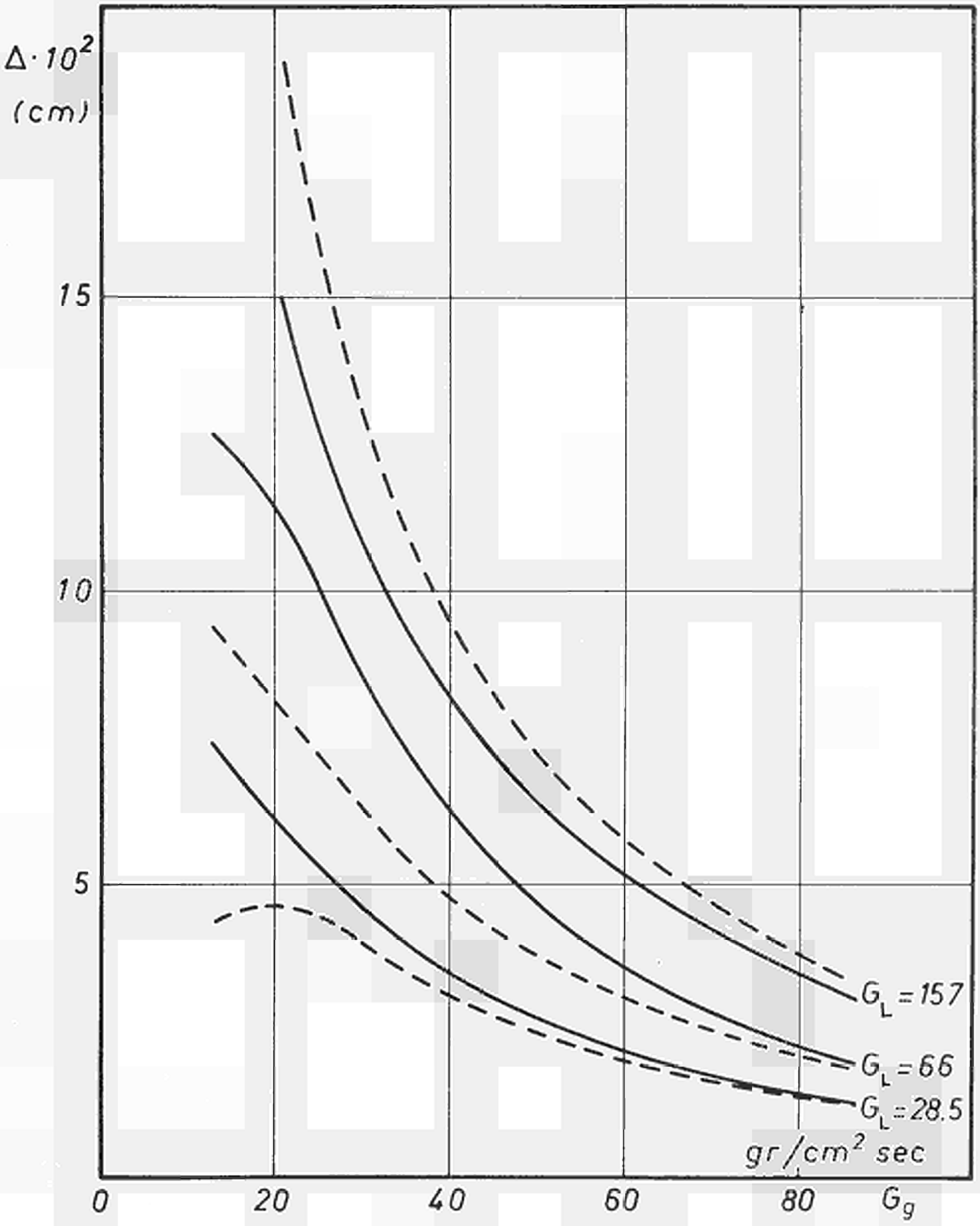

Fig. 17

Trend of Δ as a function of G_G (data of table V)

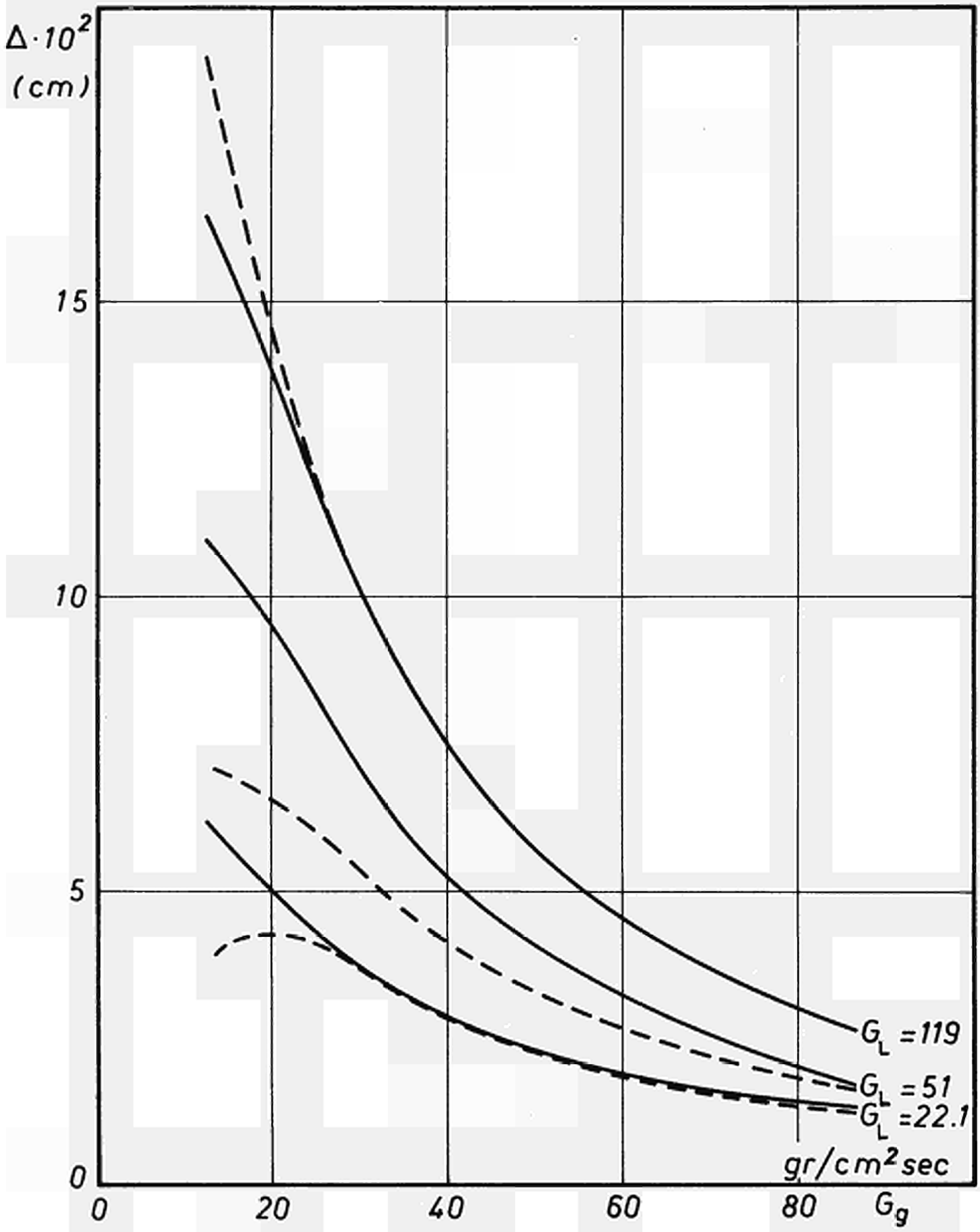

Fig. 18

Trend of Δ as a function of G_G (data of table VI)

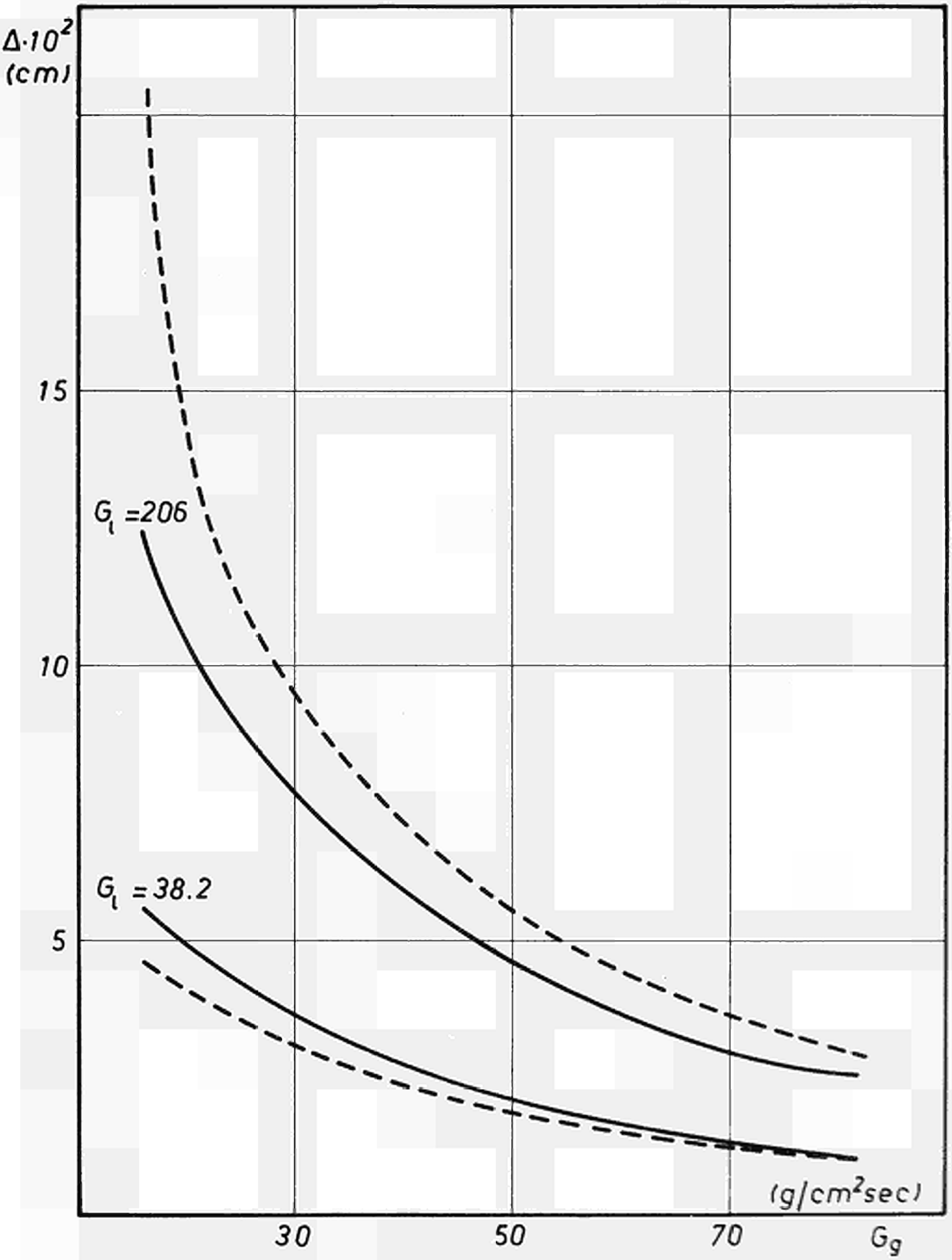

Fig. 19

Trend of Δ as a function of G_G (data of table VI)

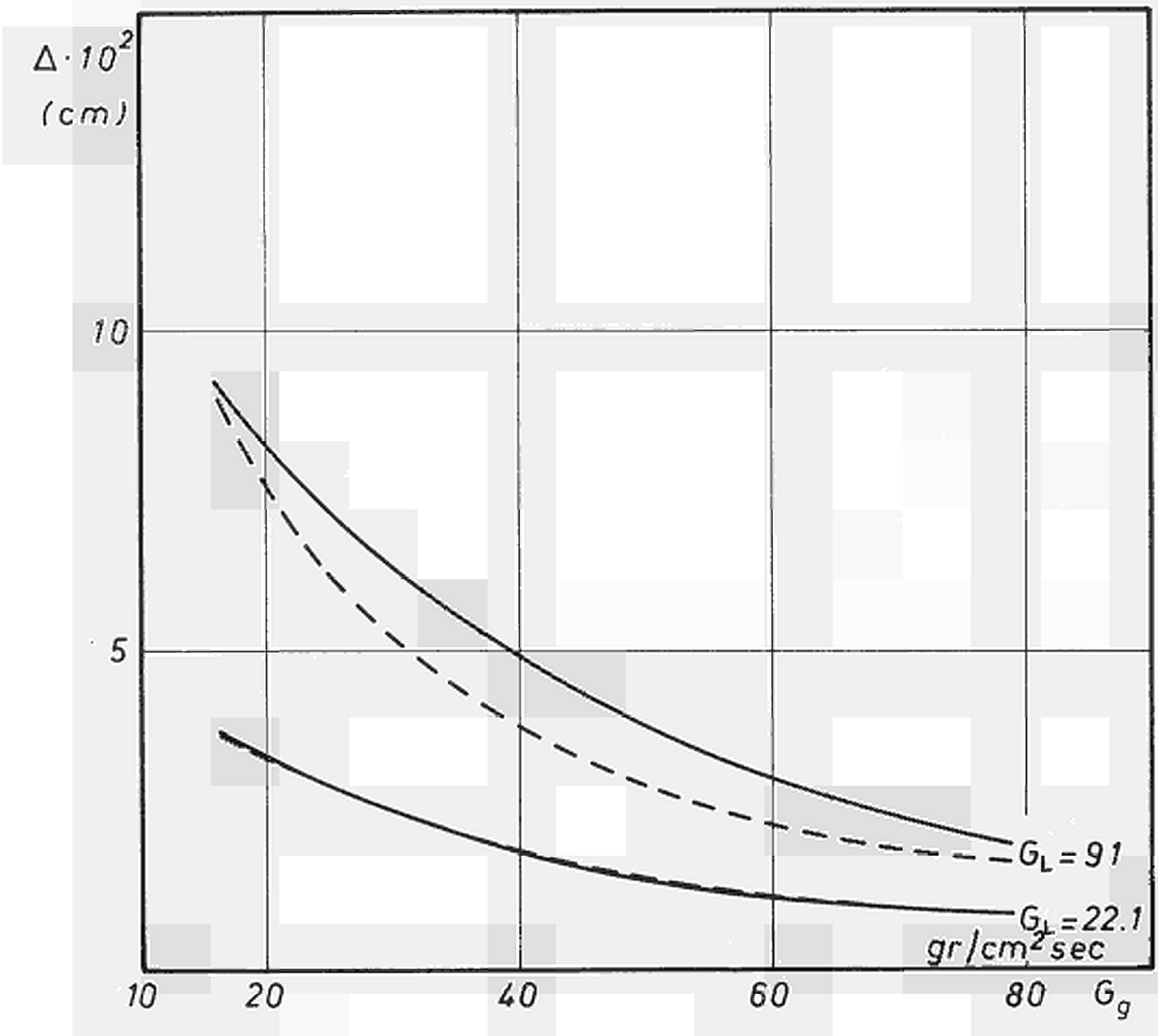

Fig. 20

Trend of Δ as a function of G_L (some data of table II)

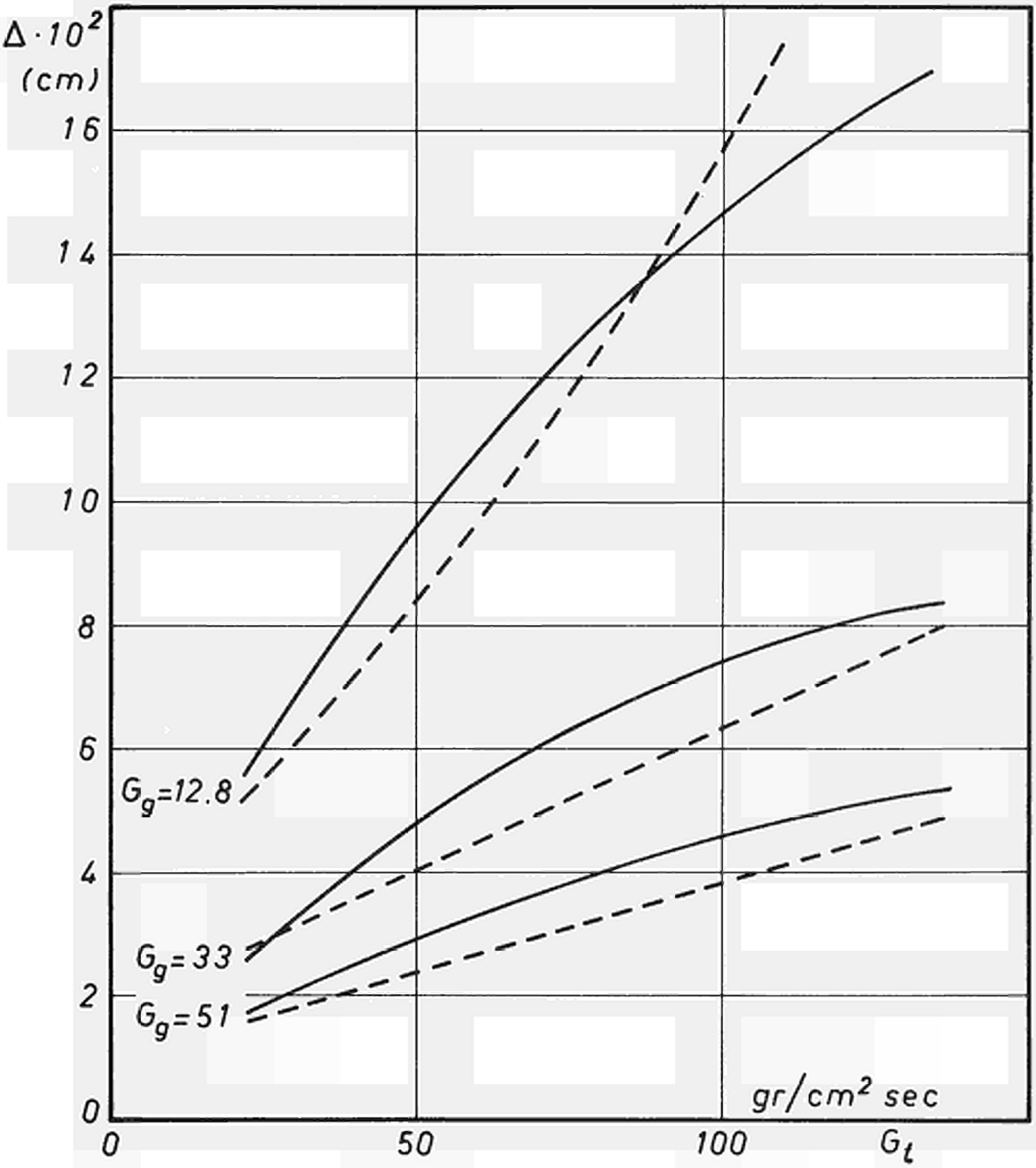

Fig. 21

Trend of Δ as a function of G_L (some data of table II)

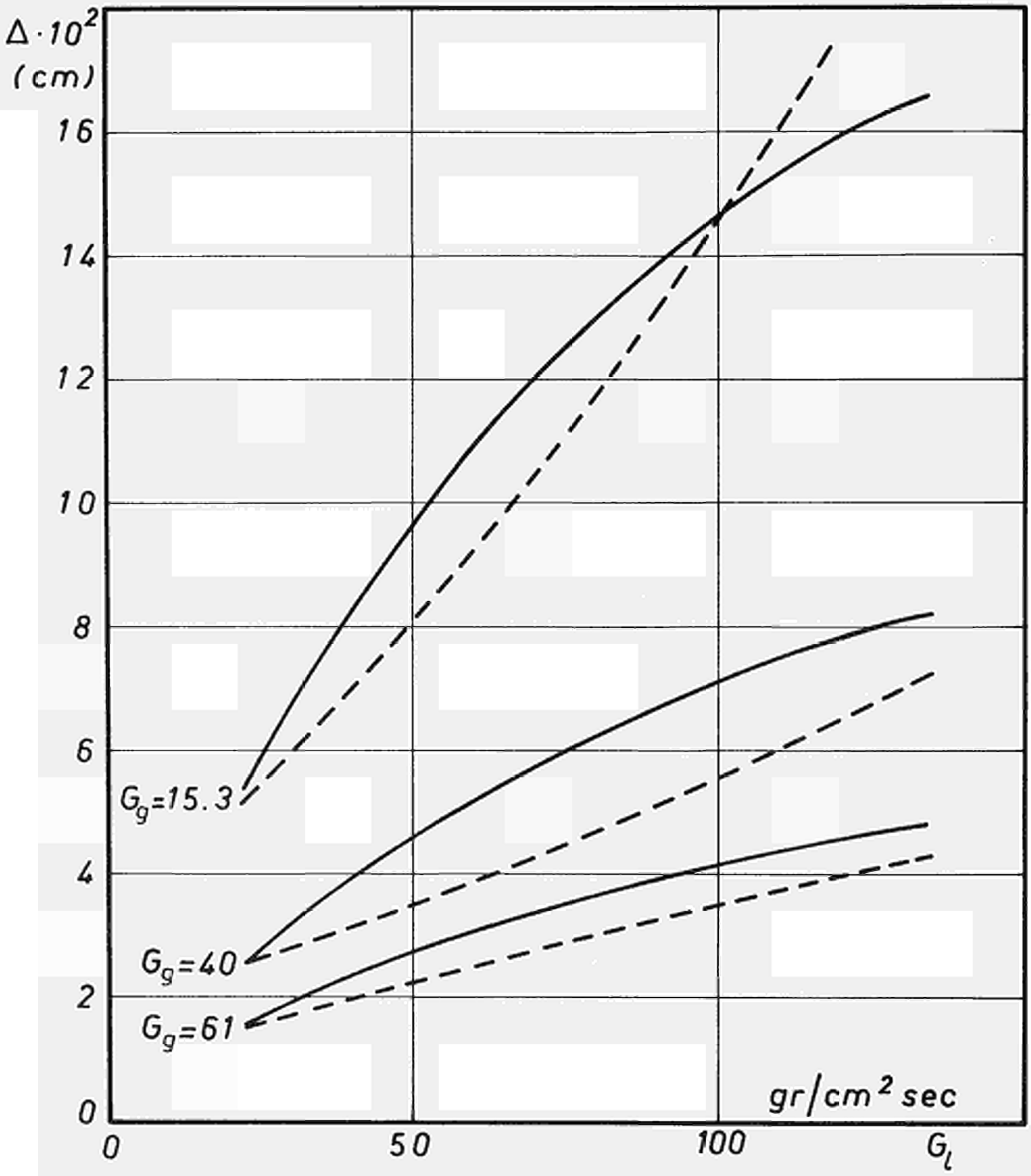

Fig. 22

Film flowrate as a function of quality X

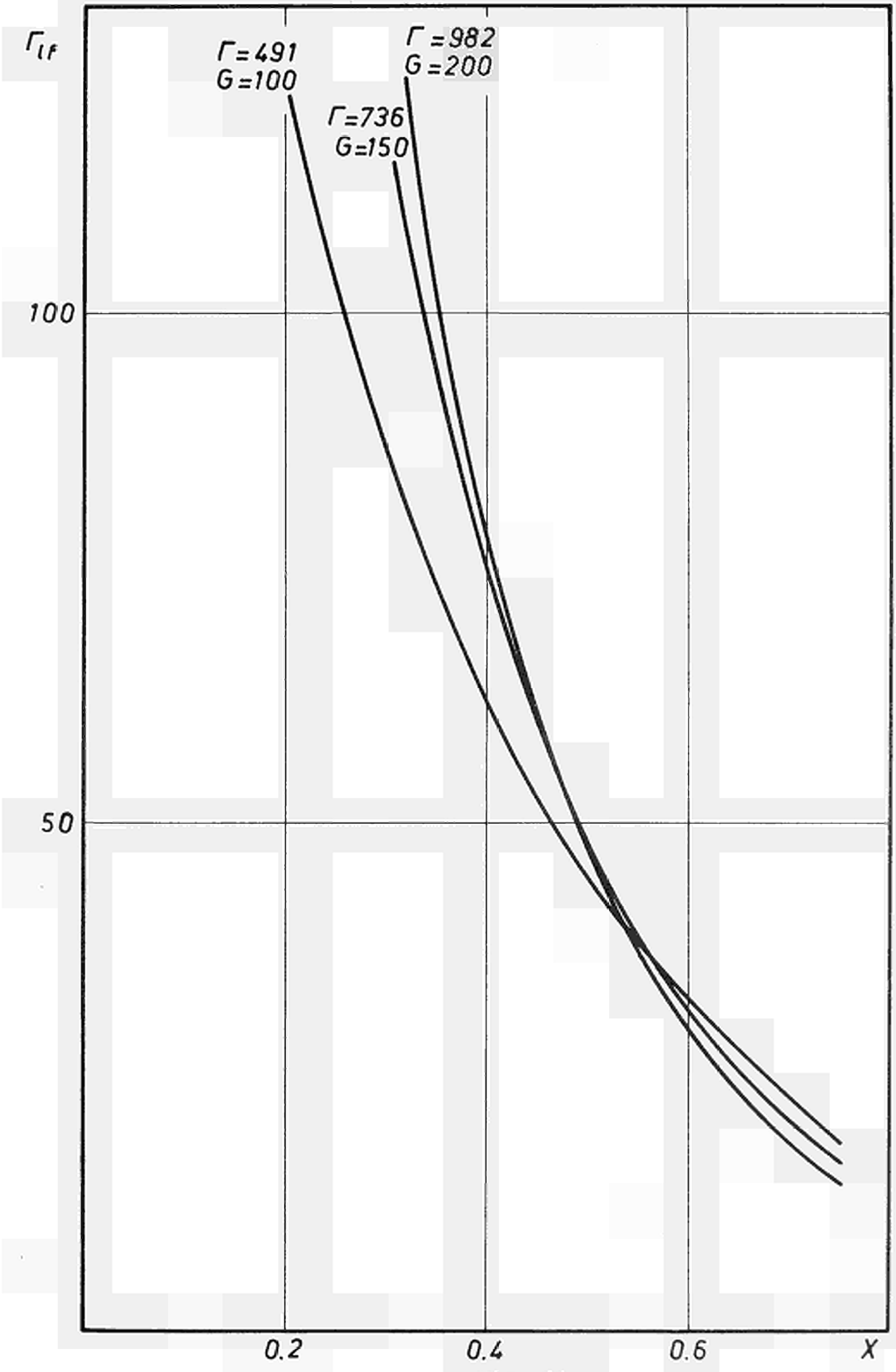

Fig. 23

Entrained liquid flowrate as a function of quality X

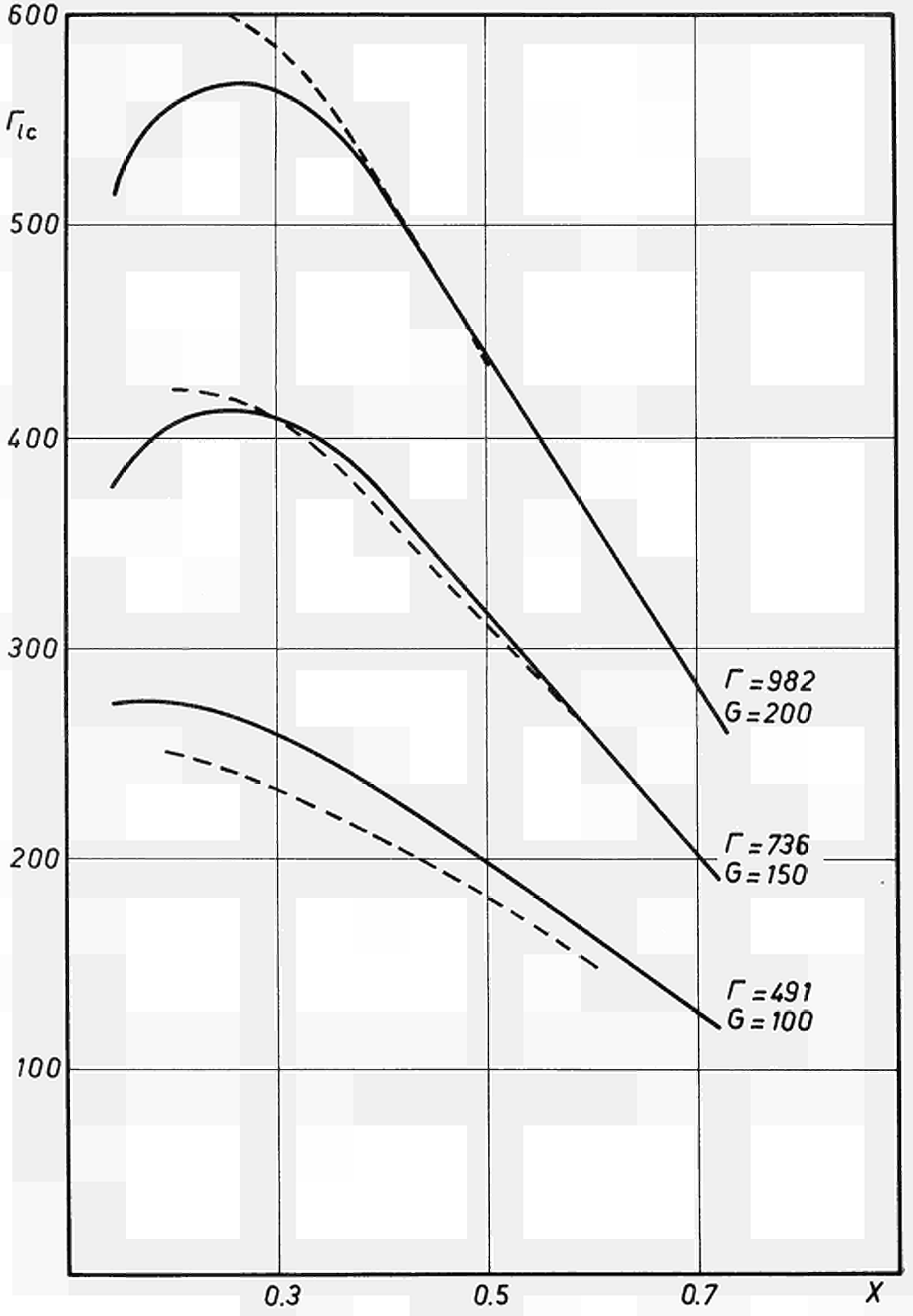

Fig. 24

Ratio Γ_{Lf}/Γ_L as a function of quality X

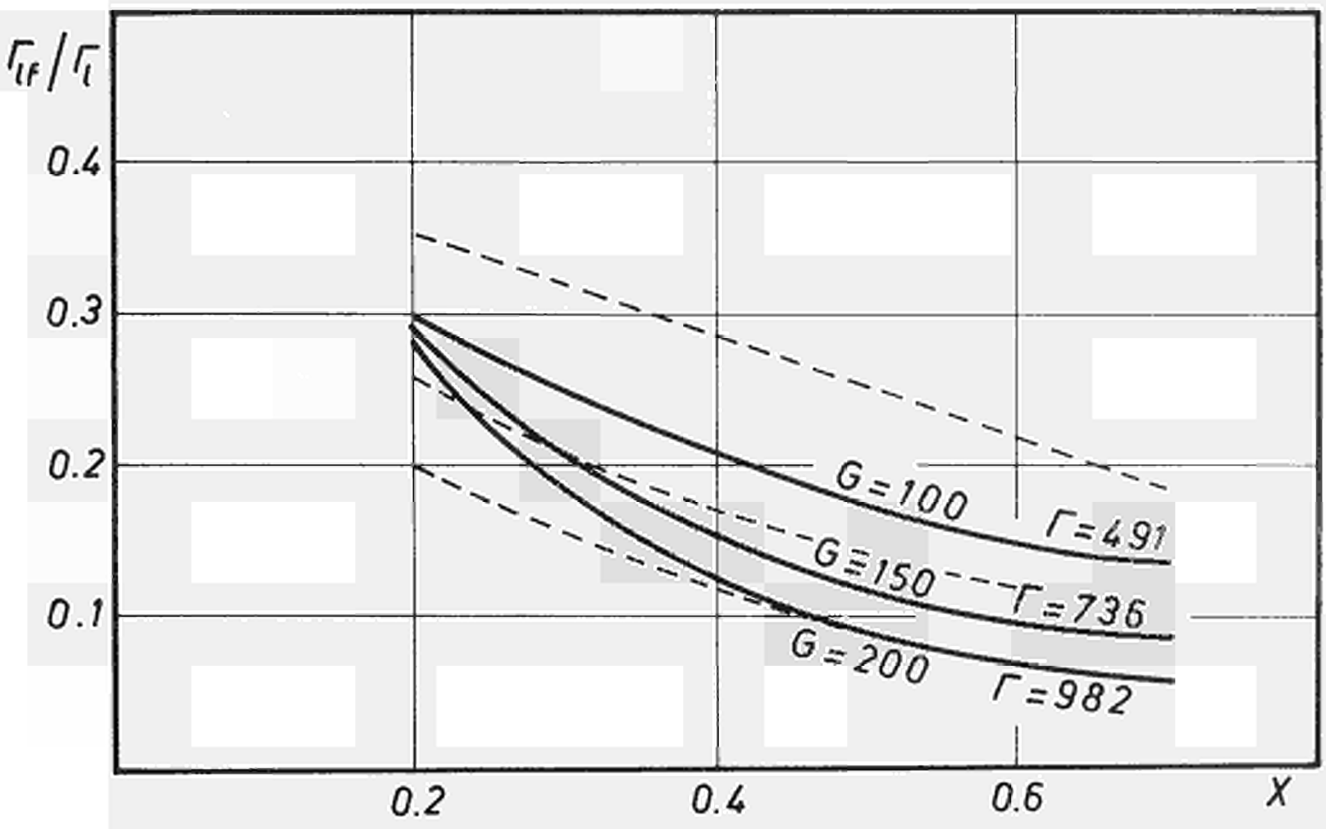

Fig. 25

Comparison between Velocity Profile of our model and the experimental one.

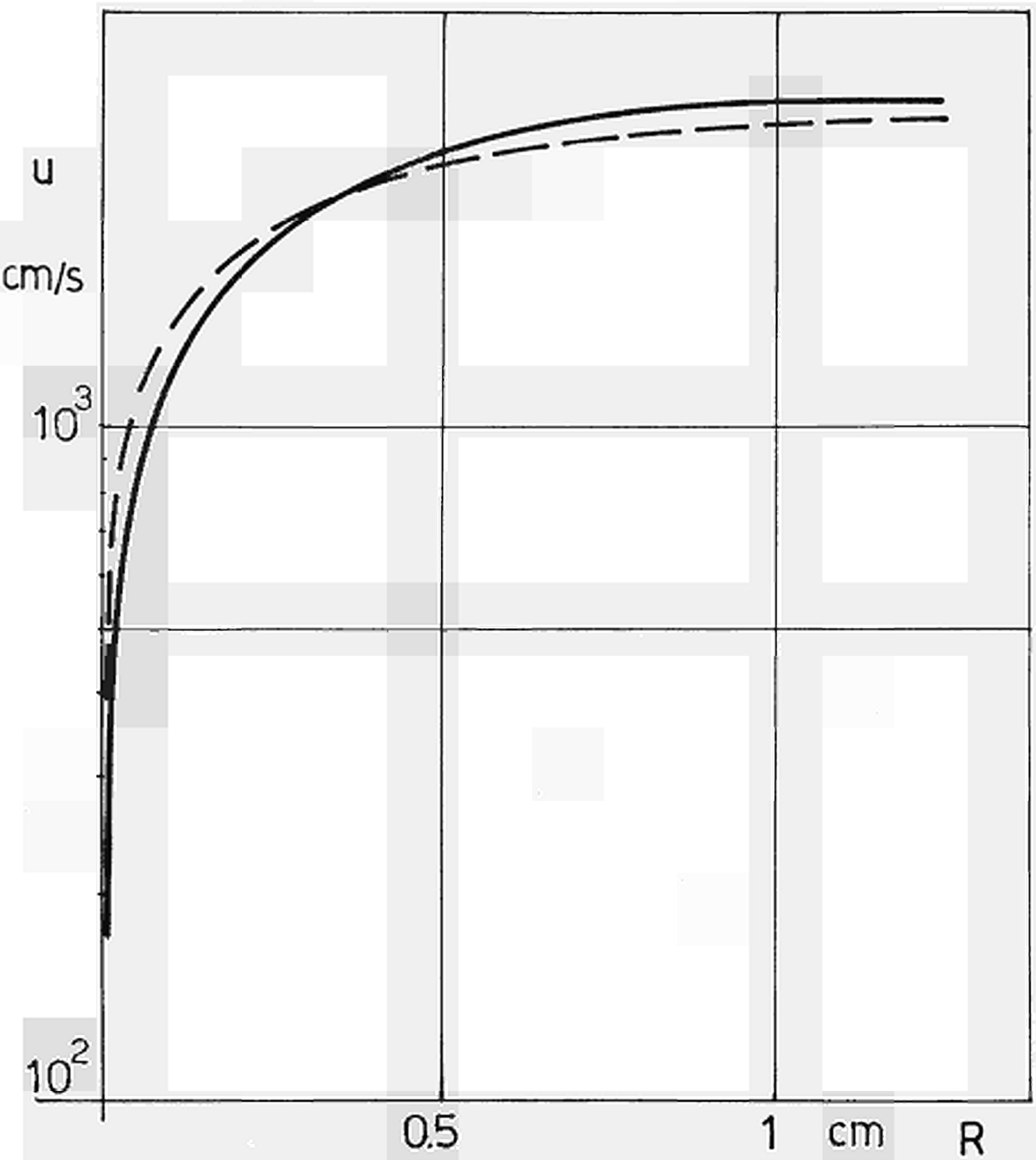

Fig. 26

Comparison amongst the predictions of Dukler - Hewitt-Levy and our model with respect to eq.47

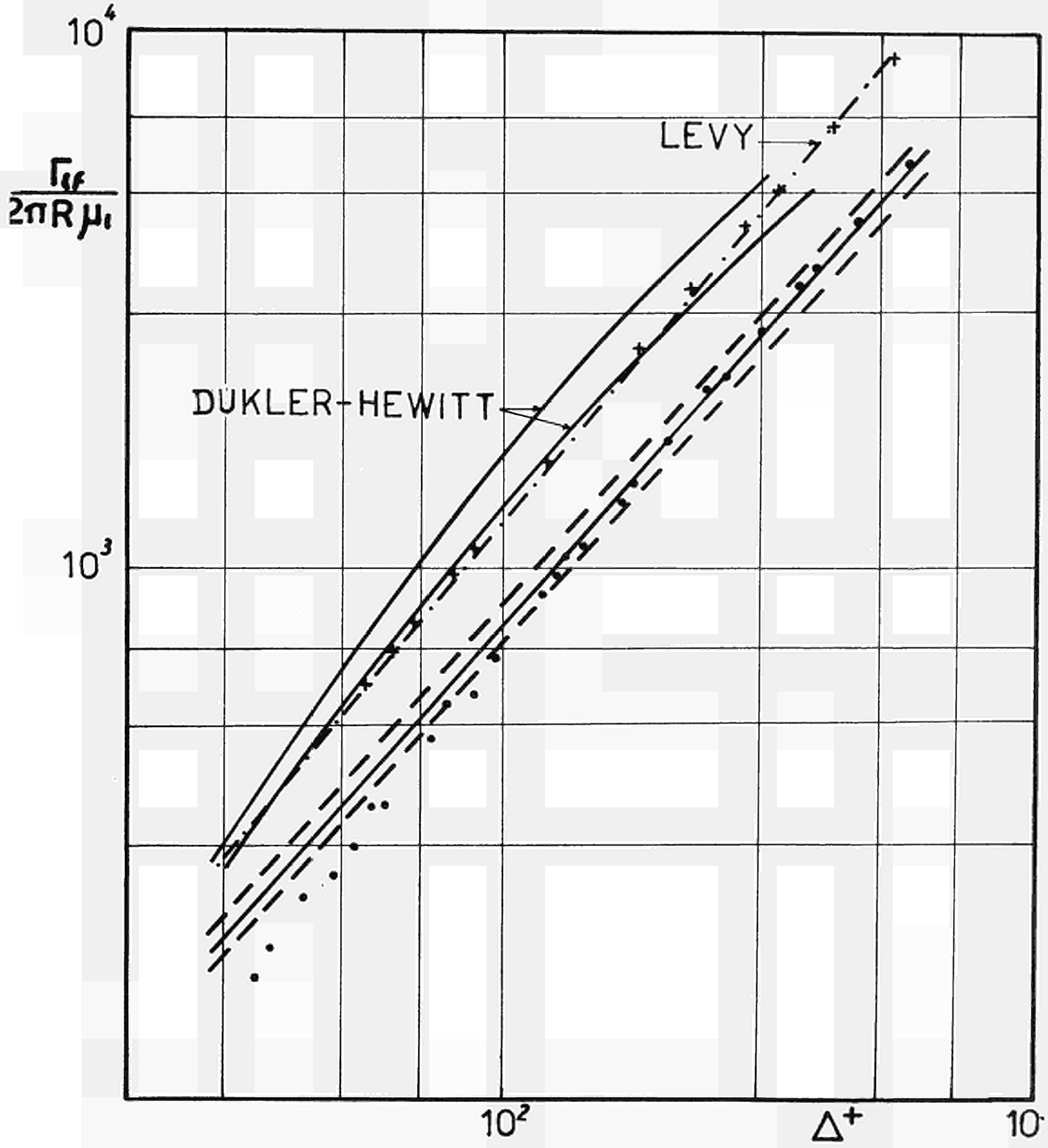

Fig. 27

Data of table I according to eq. 48

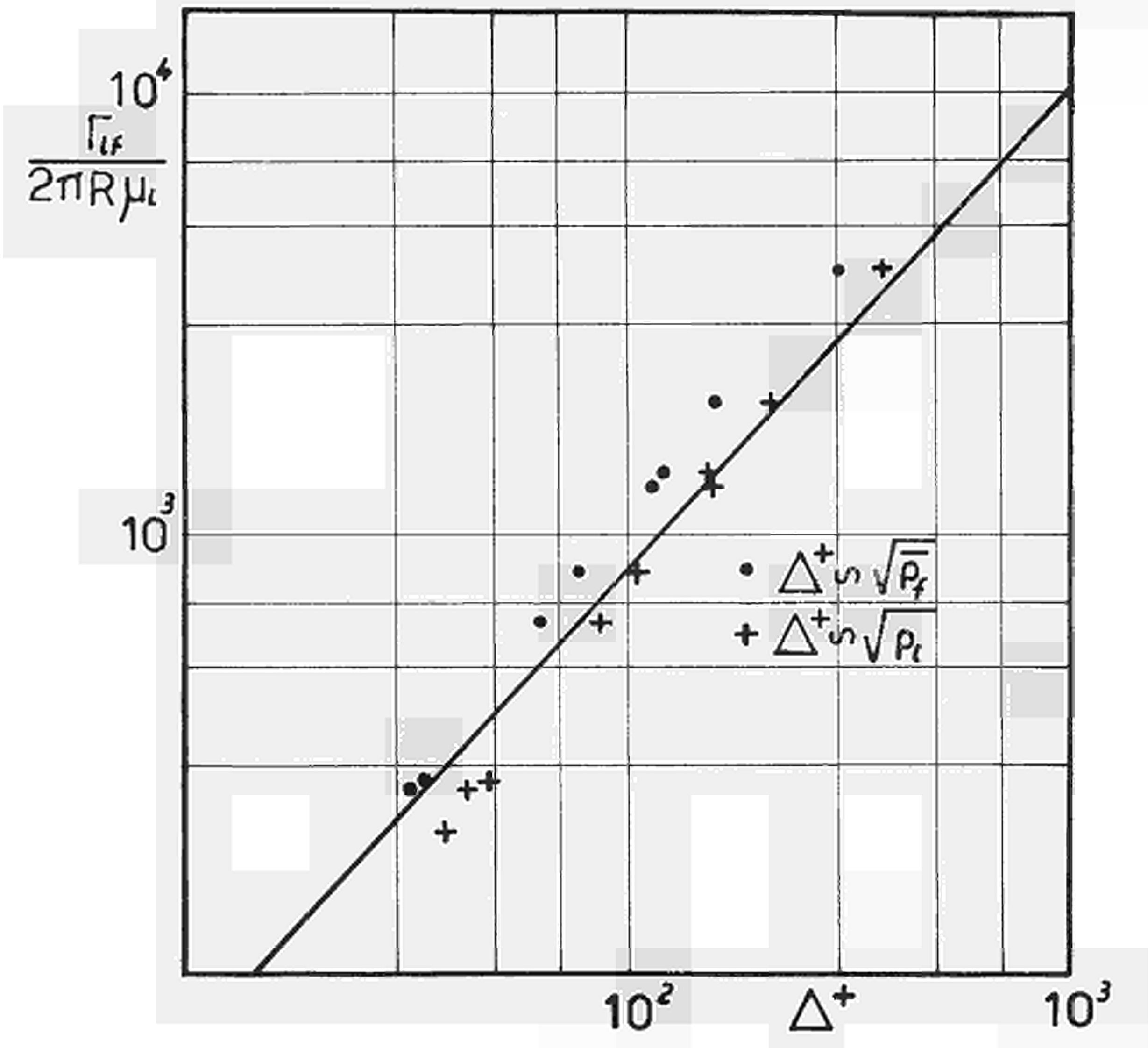

Fig. 28

Data of table II according to eq. 48

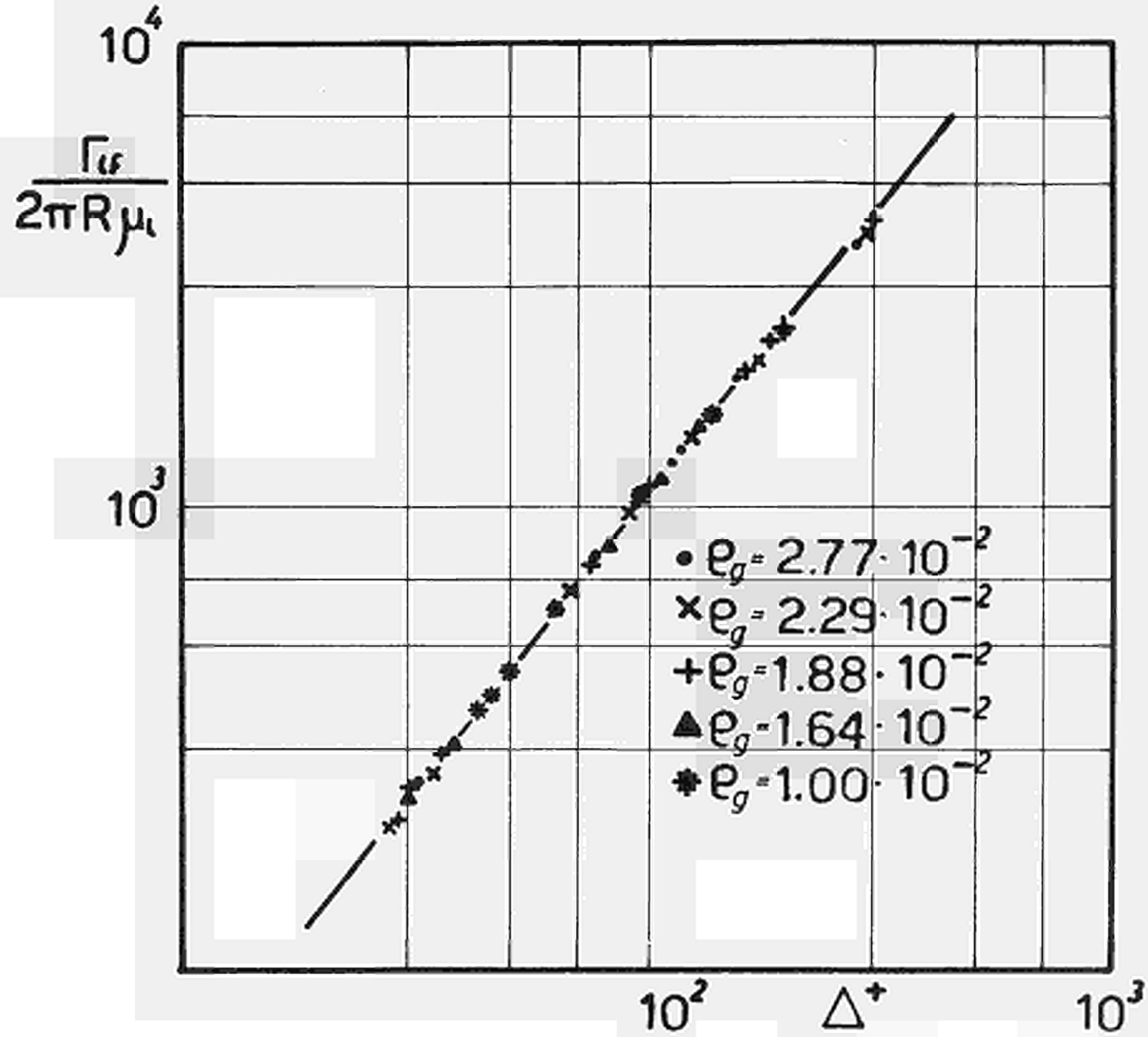

Fig. 29

Data of table VI according to eq. 48

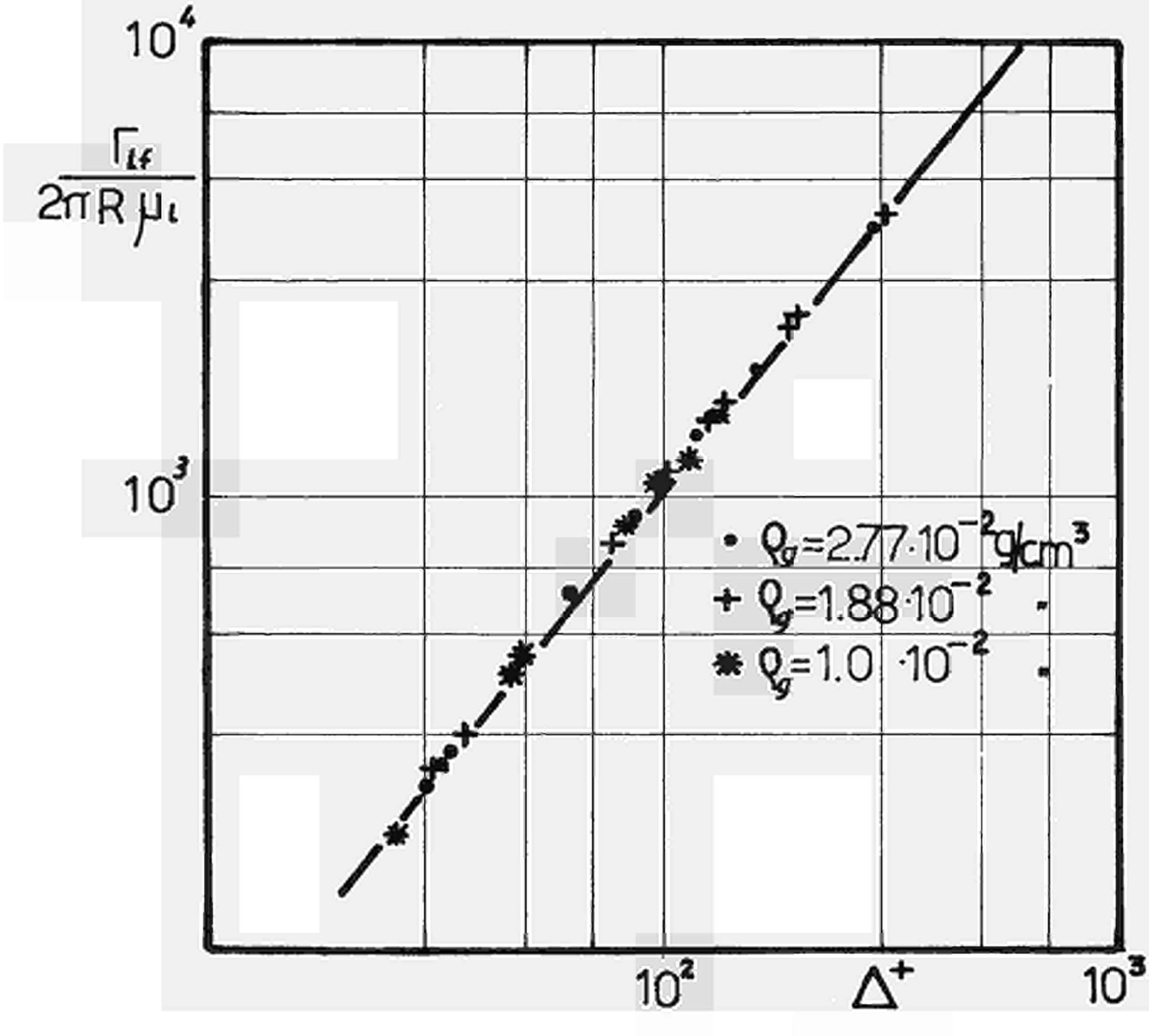

Fig. 30

Data corresponding to G = 100, 150, 200 according to eq. 48

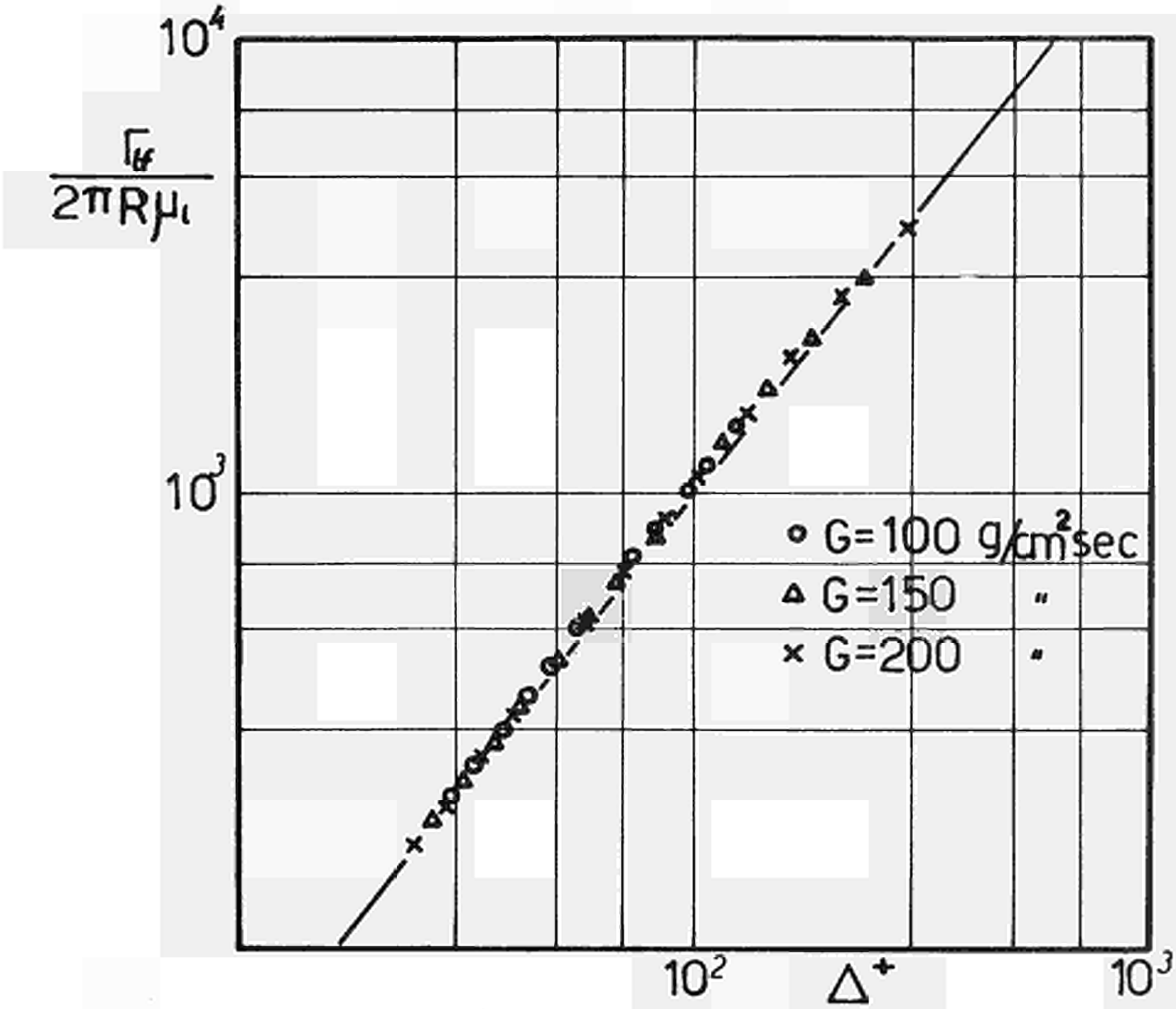

Fig. 31

11 - COMPARISON WITH HARWELL DATA

The model predictions have been compared with a set of measurements performed at Harwell [10].
Only some data have been taken into account, because many of them are external to the model validity range owing to the low mass flowrate. The values of the pressure drop and the average void fraction have been derived from experimental measurements. This leads to some fluctuations in the predicted values as it can be seen in fig. 32 where the film thickness is plotted versus G_L for $G_G \simeq 3 \div 20$ g./cm^2·sec . The original report gives three different film thicknesses obtained by the different measurement techniques. The first one (F_1) was calculated from the hold up assuming that all the liquid is present in the film.
The second one (F_2) was obtained by the CISE film conductance method and the third one (F_3) by conductance probe method.
In fig. 32 also the values corresponding to F_1 and F_2 are plotted. The model predictions seem to be well inside the strip defined by the values of F_1 and F_2 . All the examined data are reported in table 8°.

Comparison amongst Harwell predictions and our model.

o Model predictions + Holdup method ▲ Probe Conductance method

$\rho_g = 1.5\ 10^{-3}\ g/cm^3$ $G_g = 3.2\ g/cm^2.sec$

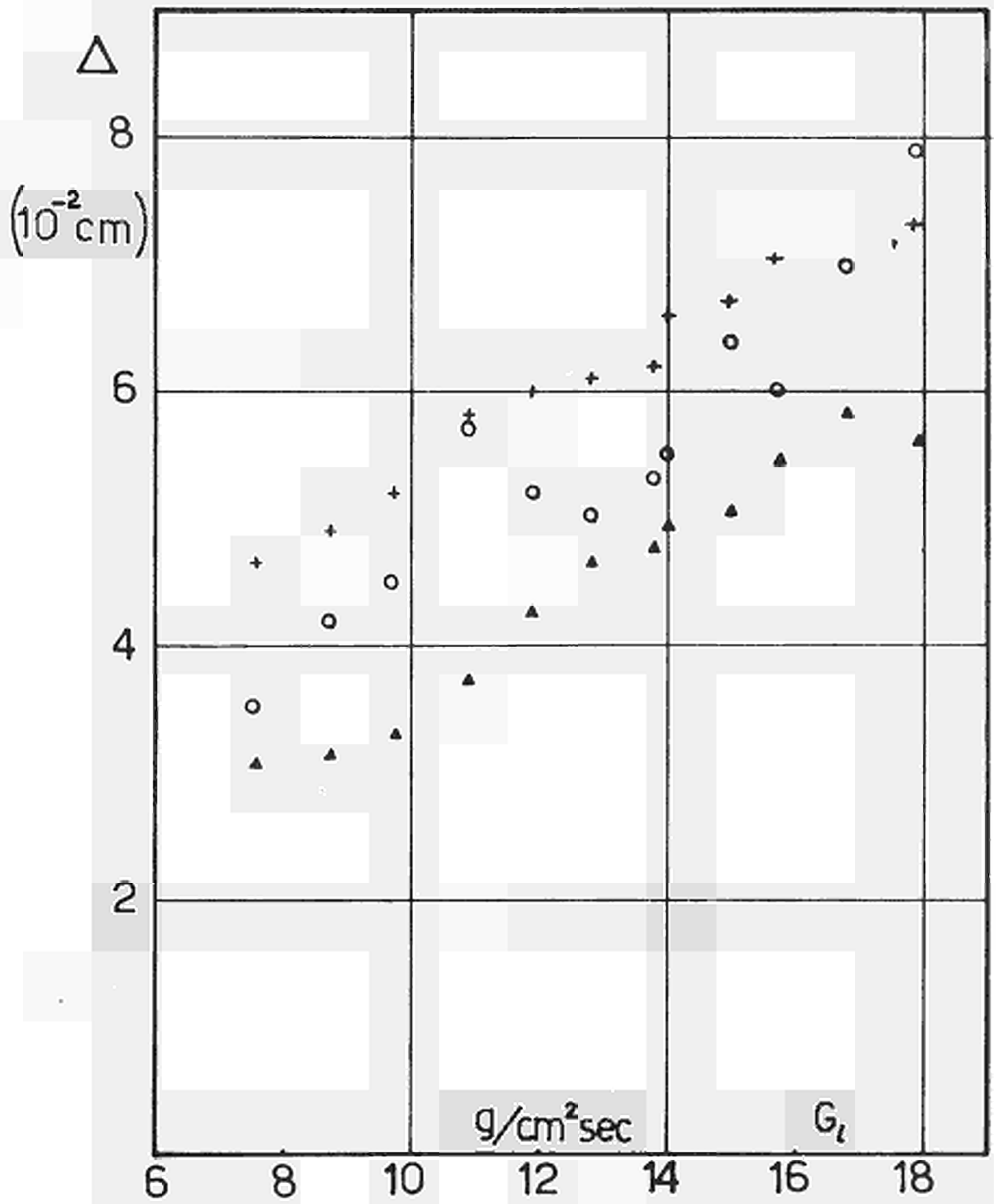

Fig. 32

12 - A SIMPLIFIED MODEL

In order to obtain some more simple analytical expressions to handle, it has also been tried to describe in another way the film and core velocity distribution. The basic assumptions of the this simplified model are the following:

a) Blasius velocity profile describes the film velocity, whilst Prandtl universal profile describes the core velocity.

b) the local void fraction exhibits a parabolic trend in the film, starting from zero at the duct wall up to a constant value α^* at the film-core boundary. In the core it is assumed as constant and equal to α^*.

c) The local slip ratio of the film is taken equal to 1

d) It has also been assumed in the core that the dispersed liquid travels with the gas mean velocity, except for the fraction corresponding to the one filled up with the gas in the film, which has a velocity equal to the film mean velocity. According to this, one has in the film:

(49) $$u_f = U \left(\frac{R-y}{\Delta} \right)^n \qquad b \leq y \leq R$$

(50) $$\alpha = \alpha^* \left(\frac{R-y}{\Delta} \right)^{1/m} \qquad b \leq y \leq R$$

where n is equal to 0.5 and m is a parameter introduced so as to take into account not only linear distributions. If $\bar{\alpha}$ is the mean value of the void fraction, obtaindd, as previously said, from an existing correlation, α^* can be determined by the following equation

$$(51)\quad \pi R^2 \bar{\alpha} = 2\pi \int_0^b \alpha^* y\,dy + 2\pi \int_b^R \alpha^* \left(\frac{R-y}{\Delta}\right)^{1/m} y\,dy$$

which gives

$$(52)\quad \alpha^* = \frac{R^2 (m+1)(2m+1)\bar{\alpha}}{(m+1)(2m+1) b^2 + 2\Delta\{m(2m+1)R - m(m+1)\Delta\}}$$

With this value of α^*, it is now possible to define the film local density ρ_f and the mean density $\bar{\rho}_f$

$$(53)\quad \rho_f = \rho_L\left\{1 - \left(\frac{R-y}{\Delta}\right)^{1/m}\alpha^*\right\} + \rho_G \left(\frac{R-y}{\Delta}\right)^{1/m}\alpha^*$$

$$(54)\quad \bar{\rho}_f = \frac{2}{R^2-b^2}\int_b^R \rho_f\, y\,dy = \rho_L\left\{1 - \frac{2\alpha^*}{2R-\Delta}\left(\frac{mR}{m+1} - \frac{m\Delta}{2m+1}\right)\right\} + \rho_G \frac{2\alpha^*}{2R-\Delta}\left(\frac{mR}{m+1} - \frac{m\Delta}{2m+1}\right)$$

The core velocity profile is given by

$$(55)\quad u_c = U_m - \frac{1}{\chi_c}\sqrt{\frac{\tau_0}{\rho_G}}\ \ln \frac{R}{R-y} \qquad\qquad 0 \le y \le b$$

Eqs. (49) and (55) contain two undetermined constants, U and U_m : the former can be obtained by matching the velocities at the film-core boundary, the latter by means of the balance equation on gas flowrate $\Gamma_G^M = \Gamma_{Gf}^M + \Gamma_{Gc}^M$. Thus it is

$$(56)\quad U = U_m - \frac{1}{\chi_c}\sqrt{\frac{\tau_0}{\rho_G}}\ \ln \frac{R}{\Delta}$$

On the other hand, Γ_{Gf}^M, Γ_{Gc}^M being given by

(57) $$\Gamma'_{Gf} = 2\pi\rho_G\left(U_m - \frac{1}{\chi_c}\sqrt{\frac{\tau_0}{\rho_G}}\ln\frac{R}{\Delta}\right)\int_b^R \alpha^*\left(\frac{R-y}{\Delta}\right)^{1/m}\left(\frac{R-y}{\Delta}\right)^m y\,dy =$$

$$2\pi\rho_G\left(U_m - \frac{1}{\chi_c}\sqrt{\frac{\tau_0}{\rho_G}}\ln\frac{R}{\Delta}\right)\alpha^*\Delta m\left(\frac{R}{1+m+m\cdot n} - \frac{\Delta}{1+2m+m\cdot n}\right)$$

(58) $$\Gamma'_{Gc} = 2\pi\rho_G\alpha^*\int_0^b\left(U_m - \frac{1}{\chi_c}\sqrt{\frac{\tau_0}{\rho_G}}\ln\frac{R}{R-y}\right)y\,dy = \pi\rho_G\alpha^* b^2\bar{u}_c$$

U_m becomes

(59) $$U_m = \left\{\frac{\Gamma'\cdot x}{\rho_G\alpha^*} + \frac{1}{\chi_c}\sqrt{\frac{\tau_0}{\rho_G}}\left[2\Delta m\left(\frac{R}{1+m+m\cdot n} - \frac{\Delta}{1+2m+m\cdot n}\right)\ln\frac{R}{\Delta} + b^2\left(1-\frac{R^2}{b^2}\right)\ln\frac{R}{\Delta} + Rb + \frac{b^2}{2}\right]\right\} \Big/ \left\{2\Delta m\left(\frac{R}{1+m+m\cdot m} - \frac{\Delta}{1+2m+m\cdot n}\right) + b^2\right\}$$

The film thiekness Δ can be calculated by the liquid flowrate balance equation:

(60) $$\Gamma'(1-x) = \Gamma'_{Lf} + \Gamma'_{Lc}$$

Now, the liquid film flowrate Γ'_{Lf} and the core liquid flowrate Γ'_{Lc} on the basis of the assumption d, are:

(61) $$\Gamma'_{Lf} = 2\pi\rho_L\int_b^R\left(U_m - \frac{1}{\chi_c}\sqrt{\frac{\tau_0}{\rho_G}}\ln\frac{R}{\Delta}\right)\left\{1-\left(\frac{R-y}{\Delta}\right)^{1/m}\alpha^*\right\}\left(\frac{R-y}{\Delta}\right)^m y\,dy$$

$$= 2\pi\rho_L\left(U_m - \frac{1}{\chi_c}\sqrt{\frac{\tau_0}{\rho_G}}\ln\frac{R}{\Delta}\right)\Delta\left\{\frac{b(m+1)+R}{(m+1)(m+2)} - \alpha^* m\left(\frac{R}{1+m+m\cdot n} - \frac{\Delta}{1+2m+m\cdot n}\right)\right\}$$

(62) $$\Gamma'_{Lc} = \rho_L\left\{\pi b^2(1-\alpha^*) - A_{Gf}\right\}\bar{u}_c + \rho_L A_{Gf}\bar{u}_f$$

where $\bar{u}_c$ and $\bar{u}_f$ are the mean velocity in the core and in the film respectively, A_{Gf} is the film section filled up with the gas. Their expressions are

(63) $$\bar{u}_c = U_m + \frac{1}{\chi_c}\sqrt{\frac{\tau_0}{\rho_G}}\left(\frac{2\Delta}{R}\ln\frac{R}{\Delta} - \frac{3}{2} - \frac{\Delta}{R}\right)$$

$$(64)\quad \overline{u}_f = 2\left(U_m - \frac{1}{\chi_c}\sqrt{\frac{\tau_0}{\rho_G}}\ln\frac{R}{\Delta}\right)\frac{(m+1)b+R}{(2R-\Delta)(m+1)(m+2)}$$

$$(65)\quad A_{Gf} = 2\pi\alpha^*\left(\frac{mR}{m+1} - \frac{m\Delta}{2m+1}\right)$$

Thus, substituing eqs. 61 - 65 in eq. (60) one obtaines

$$(66)\quad \begin{aligned} &\Gamma(1-x) - 2\pi\rho_L\left(U_m - \frac{1}{\chi_c}\sqrt{\frac{\tau_0}{\rho_G}}\ln\frac{R}{\Delta}\right)\Delta\left\{\frac{R+b(n+1)}{(n+1)(n+2)} - \alpha^*\left(\frac{mR}{1+m+m\cdot n} - \frac{m\Delta}{1+2m+m\cdot n}\right)\right\} \\ &- \rho_L\left\{\pi b^2(1-\alpha^*) - 2\pi\alpha^*\left(\frac{mR}{m+1} - \frac{m\Delta}{2m+1}\right)\Delta\right\}\cdot\left\{U_m - \frac{1}{\chi_c}\sqrt{\frac{\tau_0}{\rho_G}}\left(1-\frac{R^2}{b^2}\right)\ln\frac{R}{\Delta} + \frac{R}{b} + \frac{1}{2}\right\} = 0 \end{aligned}$$

in which the only unknown is the film thickness Δ. Also this equation is too complex to be analytically solved and therefore it has been programmed on a digital computer. The values of Δ and of the other quantities depending on it, predicted by this simplified model, are shown in table 8° for $m = 1/2$ and $m = 1$. As one can see from this table and from fig. 33, the qualitative and quantitative agreement between experimental data and predicted values is satisfaction for a wide range of values. The model predictions fail for high values of Δ; namely the trend of Δ as a function of G_L at G_G constant shows an inversion of the dependence on G_L for Δ values corresponding to a line in which X is ~ 0.2, whilst the trend for the other values of Δ is correct also in the region of low Δ.

Trend of Δ as a function of G_L for some data of table IV.

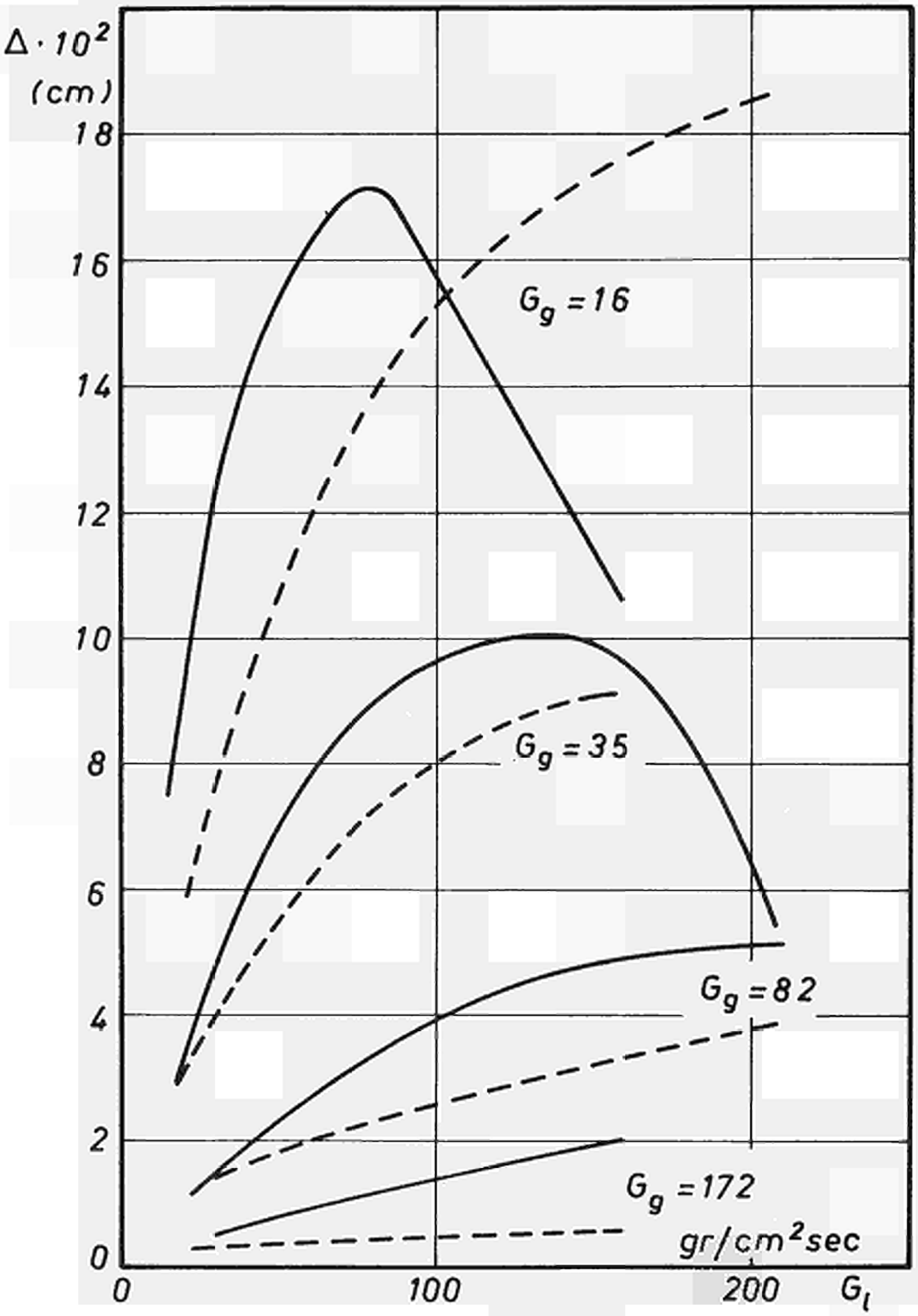

Fig. 33

APPENDIX

The shear stress τ in cylindrical geometry is

(1) $$\tau = \frac{c_1}{r} + \frac{c_2}{2} r$$

By the conditions $\tau = \tau_c$ for $r = b$ and $\tau = \tau_0$ for $r = R$ it is

(2) $$c_1 = \tau_c b - \frac{b^2(\tau_0 - \tau_c \cdot b/R)}{R - b^2/R}$$

(3) $$c_2 = 2 \frac{\tau_0 - \tau_c b/R}{R - b^2/R}$$

thus eq. (1) becomes

(4) $$\tau_{cyl} = \frac{1}{r} \frac{bR(\tau_c R - \tau_0 b)}{R^2 - b^2} + r \frac{\tau_0 R - \tau_c b}{R^2 - b^2}$$

or also

(5) $$\tau_{cyl} = \frac{R\tau_0 - b\tau_c}{R^2 - b^2} \frac{r^2 - b^2}{r} + \frac{\tau_c b}{r}$$

τ_{plane} being equal to $(\tau_0 - \tau_c)\frac{y - b}{R - b} + \tau_c$, eq. (5) can be rewritten

(6) $$\tau_{cyl} = \tau_{plane} + \Delta\tau$$

where :
$$\Delta\tau = \frac{r - b}{r} \frac{(R - r)(\tau_0 b - \tau_c R)}{R^2 - b^2}$$

Putting now $\Delta\tau = H(r) - \tau_{plane}$ one has

(7) $$\tau_{cyl} = \tau_{plane}\left\{1 + H(r)\right\}$$

where $$H(r) = \frac{ab - R}{R + b}\,\frac{(r-b)(R-r)}{r\left\{r(a-1) - (ab-R)\right\}}.$$

The average value of this function

$$\bar{H} = \frac{1}{R-b}\int_b^R H(r)\,dr = \frac{ab-R}{R^2-b^2}\left\{\frac{\Delta}{1-a} - \frac{\ln a}{1-a}\left[\frac{ab-R}{1-a} + R + b\right] - \frac{R\,b}{ab-R}\ln\frac{ab}{R}\right\}$$

As this value is $\approx 10^{-4}$, for a wide range of Δ values the substitution of τ_{cyl} with τ_{plane} is well justified.

TAB. I REF. 4 Water-Nitrogen R=1.25 cm

G	X	$\Delta_s \cdot 10^2$	$\Delta_c \cdot 10^2$ eq.(32)	$\Delta_c \cdot 10^2$ *eq.(32)	Γ_{lf}
$\rho_g = 2.77 \cdot 10^{-2}$ g/cm^3					
37.6	0.408	5.65	4.87	4.48	21.8
92.4	0.166	12.9	9.74	10.1	114.33
153	0.101	18.3	13.2	18.4	325
62.3	0.643	2.5	2.35	2.39	22.58
117.2	0.342	6.3	3.22	4.54	74.95
178.2	0.225	8.5	5.61	7	16.23
83.2	0.685	1.64	1.76	1.68	19.76
138	0.442	3.56	2.31	2.70	50.81
139	0.305	5.2	2.78	4.17	108
$\rho_g = 1.88 \cdot 10^{-2}$ g/cm^3					
32.8	0.324	6.05	5.68	4.84	21.39
87.6	0.121	13.8	12.3	11.2	123.77
113.6	0.094	16.4	15.3	11.2	207.95
148.6	0.0715	18.6	18.9	19.9	346.6
49.8	0.553	2.81	2.9	2.82	25.08
104.7	0.264	7.25	4.31	5.57	91.6
130.7	0.211	8.5	5.69	6.84	133.5
165.7	0.167	9.6	7.91	8.48	197.32
64.3	0.655	1.87	1.75	1.73	18.46
119.2	0.352	4.39	2.90	3.46	64.98
145.5	0.290	5.37	2.98	4.23	93.36

$\rho_g = 1.0 \cdot 10^{-2}$ g/cm^3					
G	X	$\Delta_S \cdot 10^2$	$\Delta_C \cdot 10^2$ eq. (32)	$\Delta_C \cdot 10^2$ * eq. (32)	Γ_{lf}
28	0.208	7.3	8.22	5.9	22.78
39.8	0.146	9.2	9.3	7.6	37.27
64.8	0.0895	12	12.5	10.7	20.19
37.2	0.403	3.25	4.04	3.97	33.63
48.9	0.307	4.42	4.80	4.93	51.48
3.9	0.203	6.65	6.07	6.50	91.00
45.2	0.510	2.21	2.62	2.48	25.34
57.1	0.404	3.04	3.3	3.11	38.04
82.1	0.280	4.98	4.01	4.41	72.8

* $\frac{dP}{dz}$ experimental

TAB. II REF. 2 Water-Argon R = 1.25 cm $\rho_G = 2.77 \cdot 10^{-2}$ g/cm^3

G	X	$\Delta_s \cdot 10^2$	$\Delta_c \cdot 10^2$	Γ_{lf}
57.4	0.410	2.99	3.05	
45.7	0.515	2.41	2.42	
73.7	0.206	6.4	6.10	
48.9	0.310	4.43	4.88	51.022
37.2	0.408	3.47	3.91	33.15
64.4	0.0915	12.9	10.61	90.08
39.65	0.148	8.7	7.19	37.72
27.96	0.258	6.65	6.18	30.81
113.9	0.327	4.71	3.71	69.80
58.7	0.625	1.91	1.87	20.36
161.4	0.151	3.34	8.99	209.01
100.8	0.242	7.41	5.94	97.51
45.9	0.330	2.91	2.95	25.62
146.2	0.063	16.9	20.46	352.40
86.2	0.106	13.7	11.76	127.75
31.2	0.295	6.31	4.97	20.80

G	X	$\Delta_S \cdot 10^2$	$\Delta_C \cdot 10^2$	Γ_{lf}
145.3	0.291	5	4.21	92.85
119.2	0.354	4.24	3.43	64.38
63.8	0.663	1.74	1.70	17.86
164.8	0.169	9.08	8.37	194.23
150.6	0.185	8.49	7.70	167.26
104.5	0.267	7.01	5.50	90.40
49.3	0.555	2.68	2.82	24.87
147.6	0.072	17.0	19.7	342.41
113.6	0.0935	15.0	14.91	208.47
87.6	0.121	13.6	11.23	123.77
32.6	0.325	6.00	4.82	21.15
188	0.270	5.41	4.65	120.10
127.7	0.401	3.94	3	56.00
72.4	0.705	1.72	1.55	17.08
170.3	0.196	8.43	7.65	176.60
110.1	0.302	6.42	5.02	82.34
55.2	0.600	2.59	2.58	23.51
149.8	0.855	17.2	19.1	332.15
89.7	0.142	12.6	10.66	118.39
34.8	0.405	5.76	4.587	22.95

TAB. III REF.(4) Water-Argon ρ_G = $3.61 \cdot 10^{-2}$ g/cm^3 R= 1.25 cm

G	X	$\Delta_S 10^2$	$\Delta_C 10^2$	Γ_{lf}
185	0.508	2.31	2.02	43.72
171	0.550	2.09	1.81	36.27
160	0.585	1.92	1.67	31.30
152	0.620	1.75	1.53	27.19
145	0.650	1.62	1.44	24.25
138	0.680	1.53	1.35	21.74
132	0.710	1.42	1.28	19.58
128	0.735	1.31	1.21	17.97
122	0.770	1.18	1.13	16.04
119	0.790	1.11	1.09	15.05
116	0.810	1.01	1.06	14.19
112	0.840	0.92	1.00	13.00
288	0.285	5.53	4.64	152.11
259	0.317	4.63	4.07	122.10
239	0.343	4.28	3.70	103.46
219	0.375	3.87	3.30	85.59
200	0.407	3.33	3.00	71.66
184	0.440	3.15	2.73	60.39
172	0.472	2.88	2.49	51.73
158	0.513	2.68	2.24	42.93
147	0.550	2.52	2.05	36.65
139	0.580	2.29	1.91	32.45
132	0.615	2.08	1.77	28.32
125	0.645	1.92	1.67	25.40
119	0.680	1.80	1.55	22.43
115	0.705	1.68	1.48	20.61

G	X	$\Delta_s 10^2$	$\Delta_C 10^2$	Γ_{LF}
109	0.740	1.49	1.39	18.39
103	1.760	1.40	1.34	16.31
99.1	0.820	1.12	1.21	14.47
278	0.259	6.43	5.60	178.34
249	0.289	5.80	4.91	143.14
229	0.314	5.10	4.45	120.94
209	0.345	4.64	3.97	99.66
191	0.337	4.20	3.56	82.31
175	0.413	3.91	3.18	68.34
163	0.443	3.51	2.92	58.80
149	0.485	3.37	2.61	48.36
138	0.521	3.02	2.40	41.33
130	0.550	2.85	2.26	36.68
122	0.585	2.55	2.09	31.99
116	0.620	2.28	1.94	28.11
110	0.650	2.14	1.84	25.32
105	0.680	2.04	1.73	22.86
100	0.715	1.83	1.63	20.42
97	0.735	1.66	1.58	19.22
93.6	0.760	1.52	1.52	17.81
89.6	0.800	1.35	1.41	15.90
269	0.234	7.4	6.73	208.15
240	0.263	7.25	5.87	165.78
220	0.285	6.43	5.37	141.08
200	0.313	5.84	4.80	116.33
182	0.345	4.98	4.29	96.13
166	0.378	4.68	3.86	79.76
153	0.407	4.50	3.54	68.49
139	0.449	4.10	3.16	55.89
129	0.484	3.64	2.90	47.74
121	0.520	3.45	2.65	41.09
113	0.545	3.16	2.55	37.10

G	X	$\Delta_s \cdot 10^2$	$\Delta_c \cdot 10^2$	Γ_{lf}
106	0.580	2.84	2.37	32.52
99.8	0.615	2.55	2.22	28.71
95.4	0.645	2.38	2.10	25.94
90.1	0.680	2.18	1.98	23.17
87.1	0.710	2.00	1.87	21.22
83.7	0.740	1.84	1.77	19.32
79.7	0.770	1.59	1.70	17.85
260	0.207	8.6	8.27	247.31
231	0.235	7.9	7.14	194.85
211	0.257	7.35	6.45	163.78
191	0.286	6.9	5.69	133.23
173	0.132	6.3	5.19	111.72
157	0.345	5.85	4.62	91.67
145	0.375	5.51	4.20	77.79
131	0.416	5.09	3.74	63.16
120	0.448	4.46	3.46	54.28
112	0.480	4.14	3.20	47.21
105	0.510	3.86	3.01	41.68
98.1	0.550	3.44	2.76	35.87
92.1	0.585	3.04	2.58	31.57
87.5	0.615	2.84	2.45	29.04
79.2	0.680	2.34	2.29	
75.8	0.710	2.10	2.10	
71.8	0.750	1.90	1.96	
253	0.186	9.7	9.81	285.53
224	0.213	9.31	8.40	221.21
204	0.229	8.6	7.80	190.38
184	0.254	8.5	6.94	156.10
166	0.283	8	6.14	129.96
150	0.311	7.1	5.55	105.39
138	0.339	6.7	5.04	89.14

G	X	$\Delta_s 10^2$	$\Delta_c 10^2$	Γ_{lf}
113	0.413	5.5	4.08	60.67
105	0.443	5.02	3.79	53.00
97.6	0.477	4.62	3.52	45.52
90.8	0.513	4.08	3.40	
84.8	0.550	3.72	3.00	
75.1	0.620	2.96	2.96	
72.1	0.650	2.72	2.55	25.25
68.7	0.680	2.45	2.44	23.14
247	0.171	10.7	11.15	313.42
219	0.190	10.2	9.97	254.55
198	0.208	9.9	9.03	213.76
178	0.232	9.3	8.00	173.76
160	0.257	8.9	7.14	142.54
144	0.286	8.4	6.35	116.22
132	0.310	7.8	5.83	99.12
120	0.350	6.8	5.10	79.67
108	0.383	6.2	4.67	66.39
99.7	0.417	5.6	4.27	
92.2	0.447	5.11	4.00	
85.4	0.482	4.66	3.70	
79.4	0.519	4.06	3.44	37.42
74.9	0.550	3.77	3.26	33.58
69.6	0.590	3.36	3.05	29.47
66.6	0.620	3.07	2.90	26.97
63.2	0.650	2.85	2.79	24.73
59.2	0.690	2.45	2.65	22.21
241	0.140	12	14.82	395.63
212	0.165	11.7	12.22	298.31
192	0.182	11	10.94	247.56
172	0.203	10.7	9.68	200.91
154	0.226	10.4	8.60	162.99

G	X	$\Delta_s 10^2$	$\Delta_c 10^2$	Γ_{lf}
138	0.250	10	7.71	133.45
126	0.277	9.2	6.83	110.46
112	0.312	8	6.05	
101	0.344	7.1	5.50	
93.2	0.372	6.4	5.10	63.10
85.7	0.405	5.8	4.69	53.93
78.9	0.440	5.4	4.33	46.37
72.9	0.476	4.67	4.04	40.28
68.5	0.506	4.32	3.82	36.15
63.2	0.550	3.77	3.55	31.35
60.2	0.580	3.46	3.40	28.71
56.8	0.610	3.21	3.27	26.33
52.7	0.660	2.76	3.06	23.17
238	0.134	13.6	15.76	413.32
209	0.153	12.7	13.56	323.43
189	0.174	12.8	11.65	258.48
169	0.188	12	10.74	217.28
151	0.211	11.5	9.43	174.21
135	0.235	10.7	8.37	140.76
123	0.258	10.1	7.57	117.68
109	0.292	8.9	6.60	
98.7	0.324	8	5.95	76.71
90.2	0.351	7.1	5.51	65.52
82.7	0.383	6.5	5.10	55.81
75.9	0.418	5.9	4.67	47.77
69.9	0.454	5.3	4.34	41.31
65.5	0.484	4.88	4.11	36.97
60.2	0.526	4.17	3.84	32.13
57.2	0.554	3.81	3.69	29.54
53.8	0.590	3.56	3.51	26.76
49.8	0.640	2.98	3.30	23.64

G	X	$\Delta_s 10^2$	$\Delta_c 10^2$	Γ_{lf}
234	0.118	14.4	18.72	472.29
205	0.135	14.1	16.04	379.94
185	0.148	13.6	14.44	306.08
165	0.168	12.8	12.44	241.65
147	0.188	12.5	10.92	193.01
130	0.211	12.0	9.57	153.49
119	0.233	11.3	8.50	123.70
105	0.264	10.00	7.47	98.98
93.8	0.293	9.1	6.70	80.59
86	0.320	8.1	6.13	68.25
78.5	0.350	7.4	5.63	57.71
71.7	0.384	6.7	5.17	48.49
65.7	0.419	6.1	4.80	42.05
61.3	0.449	5.35	4.53	37.44
56	0.491	4.76	4.23	32.40
53	0.520	4,43	4.06	29.73
49.5	0.555	3.94	3.90	26.91
45.5	0.600	3.43	3.71	24.04
230	0.105	16.0	21.85	531.75
201	0.121	15.7	18.52	410.37
181	0.133	15.4	16.5	337.72
161	0.150	14.6	14.35	268.22
143	0.169	13.8	12.43	211.13
127	0.190	13.3	13.52	
115	0.210	12.2	9.65	135.62
101	0.239	10.8	8.30	104.05
90.5	0.267	9.8	7.34	83.20
82.7	0.293	9	6.67	69.62
75.2	0.322	8.3	6.07	58.05
68.2	0.354	7.3	5.56	48.61
62.3	0.387	6.5	5.15	41.51
57.9	0.419	6	4.83	36.61

G	X	$\Delta_s \cdot 10^2$	$\Delta_c \cdot 10^2$	Γ_{lf}
52.6	0.458	5.15	4.53	31.57
49.6	0.486	4.97	4.53	28.93
46.2	0.522	4.43	4.17	26.23
42.2	0.570	3.74	3.98	23.36
227	0.093	17.3	25.61	599.03
188	0.106	17.2	21.92	444.67
178	0.118	16.1	19.30	379.77
158	0.133	15.5	16.64	298.70
140	0.151	15.1	14.20	231.29
124	0.170	14.4	12.23	179.33
112	0.188	13.2	10.78	144.26
98.1	0.215	11.9	9.15	108.19
87.4	0.241	11.3	8	84.59
79.6	0.265	10	7.2	69.56
72.2	0.293	9.2	6.47	57.01
65.3	0.323	8.3	5.87	46.93
59.3	0.356	7.3	5.38	39.33
54.8	0.383	6.5	5.06	34.34
49.5	0.424	5.7	4.70	29.25
46.5	0.452	5.4	4.52	26.74
43.1	0.487	4.93	4.34	24.19
39.1	0.538	4.30	4.15	21.64
224	0.081	19.3	30.61	683.38
195	0.094	18.5	25.64	523.91
175	0.104	17.4	22.63	427.46
155	0.118	17.1	19.26	331.02
137	0.133	16.2	16.41	254.89
121	0.151	15.6	13.88	193.95
109	0.167	14.5	12.10	153.36
95.3	0.192	12.7	10.4	111.37
84.6	0.216	11.9	8.61	84.51

G	X	$\Delta_s \cdot 10^2$	$\Delta_c \cdot 10^2$	Γ_{lf}
74.5	0.238	11.2	7.50	63.96
69.3	0.264	10	6.74	53.69
62.5	0.293	8.9	6.01	43.00
56.4	0.323	8.3	5.43	34.90
52	0.350	7.7	5.05	30.00
46.7	0.390	6.7	4.66	25.09
43.7	0.416	6.1	4.47	22.74
40.3	0.452	5.31	4.30	20.57
36.3	0.502	4.88	4.14	18.50
202	0.072	21	35.4	758.03
193	0.082	20.6	30.5	595.93
173	0.092	18.9	26.4	478.92
153	0.104	18.6	22.48	371.85
135	0.118	17.9	18.9	282.04
119	0.134	16.7	15.75	210.06
107	0.149	15.6	13.48	162.19
92.9	0.171	14.7	10.97	114.00
82.1	0.192	12.7	9.18	82.90
74.3	0.213	12.1	7.91	63.69
66.8	0.237	11.0	6.80	48.25
60	0.263	10.4	5.90	36.71
54	0.293	9.2	5.19	28.63
49.6	0.319	8.4	4.74	23.81
44.3	0.357	7.4	4.3	19.22
41.3	0.383	7.0	4.1	17.22
37.9	0.417	6.2	3.91	15.40
33.9	0.474	5.57	3.86	14.47

TABLE IV Ref. 6 Water–Argon R=1.25 cm ρ_g =3.61 10^{-2} g/cm^3

G	X	$\Delta_s \cdot 10^2$	$\Delta_c \cdot 10^2$	Γ_{lf}
34.1	0.47	5.6	3.85	14.45
38.1	0.42	6.2	3.96	15.91
44.5	0.36	7.4	4.34	19.82
54.2	0.296	9.2	5.23	29.31
67	0.209	11	7.03	46.46
84	0.190	12.7	9.42	87.7
107	0.133	15.5	15.2	178.6
135	0.118	16.5	18.9	282.46
173	0.0925	17.4	26.2	476.52
222	0.072	18.8	35.4	758.34
39.4	0.540	4.3	4.15	21.86
43.4	0.491	4.93	4.33	24.5
49.8	0.427	5.7	4.7	29.6
59.5	0.358	7.3	5.37	39.54
72.3	0.235	9.2	6.44	57
87.3	0.242	11.3	7.96	84.25
112.3	0.188	12.2	10.8	144.55
140.3	0.152	13.3	14.1	230
178.3	0.120	15	18.89	373.31
227.3	0.094	16.1	25.25	592.6
45.8	0.605	3.43	3.67	239.67
49.8	0.555	3.94	3.88	27.04
56.2	0.492	4.76	4.22	32.44
65.9	0.420	6.1	4.78	42.1
78.7	0.352	7.4	5.6	57.6
93.7	0.296	8.9	6.63	79.8

G	X	$\Delta_S \cdot 10^2$	$\Delta_C \cdot 10^2$	Γ_{LF}
118.7	0.233	10	8.14	118.36
146.7	0.189	10.7	10.85	192.14
184.7	0.150	11.7	14.2	301.6
233.7	0.118	13	18.7	472.47
53.1	0.660	2.76	3.05	23.24
57.1	0.614	3.21	3.23	26.14
63.5	0.550	3.77	3.55	31.42
73.2	0.478	4.67	4.00	40.17
86	0.407	5.8	4.65	53.75
101	0.346	7.35	5.44	73.40
126	0.278	8	6.85	110.30
154	0.227	8.5	8.54	162.36
192	0.181	9.3	11.02	249.43
241	0.145	11.7	14.1	379.54
65.1	0.723	2.14	2.28	20.60
69.1	0.680	2.45	2.43	23.11
75.5	0.623	2.96	2.65	27.44
85.2	0.550	3.72	2.95	45.6
98	0.480	4.47	3.47	45.6
113	0.416	5.2	4.03	60
138	0.340	5.9	5.01	89
166	0.283	6.2	6.13	127.1
204	0.230	6.95	7.74	189.47
253	0.186	7.7	9.81	283.63
84.1	0.738	1.88	1.77	19.76
100.2	0.608	2.55	2.26	29.43

G	X	$\Delta_s \cdot 10^2$	$\Delta_c \cdot 10^2$	Γ_{lf}
113	0.550	2.90	2.50	36.44
128	0.484	3.38	2.91	47.7
153	0.405	3.96	3.58	69.06
181	0.342	4.35	4.36	97.50
219	0.283	4.86	5.45	142.60
268	0.231	5.45	6.88	212.2
104.1	0.790	1.26	1.25	15.73
110.5	0.745	1.49	1.35	18.05
120.2	0.681	1.71	1.54	22.30
133	0.616	1.91	1.75	28.20
148	0.555	2.20	2	35.88
173	0.474	2.53	2.45	51.25
201	0.408	2.93	2.97	71.32
239	0.343	3.39	3.69	103.46
288	0.284	3.34	4.67	153.07
117.1	0.805	0.92	1.06	14.39
123.5	0.770	1.05	1.12	16
133.2	0.716	1.25	1.24	19.07
146	0.650	1.51	1.43	24.23
161	0.590	1.68	1.63	30.63
186	0.510	1.94	1.99	43.30
204	0.466	2.24	2.24	53.36
252	0.377	2.67	2.37	85.95
301	0.316	3.27	3.73	126

G	X	$\Delta_S \cdot 10^2$	$\Delta_C \cdot 10^2$	Γ_{lf}
149.1	0.854	0.56	0.69	10.95
155.5	0.817	0.61	0.75	12.28
165.2	0.769	0.69	0.82	14.36
178	0.714	0.78	0.95	17.46
193	0.658	0.83	1.05	21.66
218	0.582	1.00	1.27	29.85
246	0.517	1.12	1.52	40.33
284	0.447	1.35	1.88	57.77
194.1	0.887	0.360	0.45	8.55
200.1	0.86	0.377	0.48	9.2
210.2	0.82	0.395	0.51	10.37
223	0.77	0.417	0.58	12.32
238	0.723	0.450	0.65	14.66
263	0.654	0.460	0.77	19.48
291	0.59	0.57	0.92	26.05
329	0.523	0.66	1.12	36.25
222.1	0.9	0.286	0.36	76.24
228.1	0.877	0.309	0.37	80.75
238.2	0.84	0.314	0.41	89.52
251	0.797	0.325	0.45	10.26
266	0.752	0.343	0.50	12.05

TABLE V Ref.6 Argon-Water R = 0.75 cm ρ_g = 3.61 10^{-2} g/cm^3

G	X	$\Delta_S \cdot 10^2$	$\Delta_C \cdot 10^2$	Γ_{lf}
38.1	0.42	3.78	3.68	10.7
54.2	0.296	5.60	4.60	18.22
107	0.1495	9.20	8.95	67.32
222	0.072	12.40	20.56	264.52
49.9	0.557	2.57	2.71	11.31
66	0.422	3.79	3.31	17.9
118.8	0.236	6.60	5.44	48.44
233.8	0.119	8.10		
69.1	0.68	1.63	1.64	9.02
15.2	0.552	2.47	2.02	13.4
138	0.34	4.22	3.23	33.4
253	0.186	5.00	5.96	101.54
104.1	0.788	0.87	0.87	6.40
120.2	0.683	1.23	1.04	8.68
173	0.474	1.88	1.62	19.43
288	0.284	2.63	2.95	56.31

TABLE VI Ref. 4 Water-Argon R=1.25 cm ρ_s =3.61 10^{-2}g/cm^3

G	X	$\Delta_s \cdot 10^2$	$\Delta_c \cdot 10^2$	Γ_{lf}
T = 16 °C				
289	0.283	5.7	4.66	152.2
185	0.443	3.31	2.66	58.66
132	0.614	2.03	1.76	28
103.2	0.788	1.32	1.26	15.63
253.9	0.185	10.1	9.81	282.6
149.7	0.313	7.6	5.46	103
97.7	0.480	4.66	3.46	45
68.8	0.680	2.51	2.43	22.75
233.8	0.115	14.6	19.23	481
129.7	0.210	11.8	9.58	151
77.6	0.346	7.25	5.66	56.6
48.8	0.550	3.94	3.91	26.35
222.8	0.685	21.5	37.2	785.4
118.8	0.134	16.8	15.67	207.5
66.8	0.237	11.4	6.78	47.6
38.1	0.415	6.5	3.91	15.31
T = 24 °C				
290	0.286	5.35	4.72	160.4
184	0.450	3.19	2.67	61.47
131	0.634	2.08	1.71	28.27
103.2	0.805	1.30	1.24	16.06
264	0.216	9.8	7.92	247
234	0.092	14.2	26.8	654
130	0.209	11.8	9.96	164

G	X	$\Delta_s \cdot 10^2$	$\Delta_c \cdot 10^2$	Γ_{cf}
77.9	0.349	7.35	5.80	61.34
49.1	0.554	3.95	4.01	28.9
223.1	0.072	21.7	37.01	806
119.1	0.135	16.3	16.06	220
67	0.239	11.0	6.96	52
38.2	0.420	6.3	4.07	17.2
T = 30 °C				
289	0.287	5.15	4.79	167
186	0.446	3.06	2.75	66
133	0.620	2.00	1.79	31.52
104.1	0.790	1.20	1.30	17.85
253.4	0.188	9.0	10.17	309
150.1	0.317	6.95	5.65	114.6
98	0.484	4.59	3.58	50.62
69.1	0.690	2.50	2.48	25.76
233.2	0.117	13.8	20	525
130	0.209	11.4	10.2	172
77.9	0.348	7.15	5.93	64.77
49	0.565	3.92	4.01	30.16
222.1	0.0725	20.9	37.2	820
119.1	0.135	16.1	16.4	230
67	0.240	11.0	7.07	54.48
381	0.424	6.15	4.16	18.37
T = 37 °C				
289	0.288	5.10	4.87	174
185	0.446	3.13	2.82	69.56
132	0.629	2.03	1.79	32.32
104.1	0.790	1.31	1.32	19
253.3	0.186	9.4	10.6	328.65

G	X	$\Delta_S \cdot 10^2$	$\Delta_C \cdot 10^2$	Γ_{LF}
150.2	0.314	7.1	5.87	122.33
98	0.480	4.70	3.71	54.45
68.7	0.685	2.54	2.57	27.77
232	0.116	14.3	20.95	553
130	0.208	11.5	10.52	181
78	0.347	7.1	6.09	68.64
49.1	0.550	3.88	4.22	32.77
221.9	0.0720	21.6	39	882
118.9	0.134	16.6	16.94	242
67	0.249	10.7	7.14	57.95
38.2	0.421	6.25	4.23	19.38

TAB VII Ref. 10 Harwell - Acqua Aria $R = 1.59$ cm

G	X	$F_1 \cdot 10^2$	$F_2 \cdot 10^2$	$F_3 \cdot 10^2$	$\Delta_c \cdot 10^2$
$\rho_g = 1.40 \cdot 10^{-3}$ g/cm^3 $\mu_l = 9.43 \cdot 10^{-3}$ g/cm·sec					
11	0.29	4.67	3.09	3.98	3.53
12	0.27	4.90	3.17	4.11	4.23
$\rho_g = 1.50 \cdot 10^{-3}$ g/cm^3 $\mu_l = 9.43 \cdot 10^{-3}$ g/cm·sec					
13	0.25	5.18	3.32	4.24	4.53
13	0.24	5.38	3.55	4.47	5.58
14	0.22	5.81	3.73	4.62	5.78
15	0.21	5.89	4.26	4.74	5.28
16	0.20	6.09	4.64	4.95	5.05
17	0.19	6.19	4.74	5.05	5.36
17	0.18	6.65	4.95	5.15	5.54
18	0.17	6.68	5.05	5.33	6.47
19	0.17	7.03	5.43	5.43	6.03
$\rho_g = 1.60 \cdot 10^{-3}$ g/cm^3 $\mu_l = 9.43 \cdot 10^{-3}$ g/cm·sec					
20	0.16	7.03	5.58	5.56	7.19
21	0.15	7.29	5.58	5.58	7.91
$\rho_g = 1.70 \cdot 10^{-3}$ g/cm^3 $\mu_l = 1.06 \cdot 10^{-2}$ g/cm·sec					
13	0.35	3.32	2.56	3.27	4.54
14	0.33	3.42	2.71	3.35	4.61
15	0.32	3.81	2.84	3.42	3.77
16	0.3	4.47	3.10	3.68	3.89
17	0.29	4.52	3.20	3.75	3.82
17	0.27	4.52	3.35	3.83	5.09

G	X	$F_1 \cdot 10^2$	$F_2 \cdot 10^2$	$F_3 \cdot 10^2$	$\Delta_c \cdot 10^2$
$\rho_g = 1.80.10^{-3}$ g/cm^3			$\mu_l = 9.72.10^{-3}$ g/cm sec		
18	0.26	4.54	3.40	3.98	5.10
19	0.25	4.57	3.63	4.14	7.08
20	0.24	4.85	3.65	4.21	6.21
21	0.23	5.13	3.78	4.37	5.13
21	0.22	5.33	4.01	4.47	6.40
22	0.21	5.38	4.19	4.70	6.40
23	0.21	5.71	4.36	4.77	5.95
24	0.20	5.89	4.64	4.87	6.09
25	0.19	5.91	5.00	4.92	6.50
26	0.18	6.07	5.38	5.13	7.23
$\rho_g = 1.90.10^{-3}$ g/cm^3			$\mu_l = 9.72.10^{-3}$ g/cm sec		
24	0.20	5.89	4.64	4.87	6.09
25	0.19	5.91	5.00	4.92	6.50
26	0.18	6.07	5.38	5.13	7.23
27	0.18	6.85	5.53	8.23	5.74
28	0.17	6.95	5.84	5.28	7.23
29	0.17	7.13	6.19	5.36	6.86
$\rho_g = 2.10.10^{-3}$ g/cm^3			$\mu_l = 9.28.10^{-3}$ g/cm sec		
21	0.30	4.55	2.87	3.68	4.94
22	0.29	4.75	3.02	3.73	4.94
23	0.28	4.82	3.05	3.83	4.98
24	0.27	5.02	3.12	3.94	5.05
25	0.26	5.05	3.22	4.04	4.87
25	0.25	5.46	3.38	4.14	5.57
$\rho_g = 2.20.10^{-3}$ g/cm^3			$\mu_l = 9.28.10^{-3}$ g/cm sec		

G	X	$F_1 \cdot 10^2$	$F_2 \cdot 10^2$	$F_3 \cdot 10^2$	$\Delta_c \cdot 10^2$
26	0.24	5.61	3.43	4.26	5.97
27	0.24	5.36	3.32	4.36	5.58
28	0.23	5.56	3.43	4.47	5.83
29	0.22	6.04	3.53	4.52	5.65
30	0.21	6.17	3.76	4.72	6.16
$\rho_g = 2.20 \cdot 10^{-3}$ g/cm³			$\mu_l = 1.06 \cdot 10^{-2}$ g/cm sec		
22	0.32	4.44	2.60	3.66	4.44
23	0.31	4.44	2.66	3.78	4.42
24	0.30	4.52	2.79	3.86	4.44
$\rho_g = 2.30 \cdot 10^{-3}$ g/cm³			$\mu_l = 1.06 \cdot 10^{-2}$ g/cm·sec		
25	0.29	4.64	2.92	3.96	4.80
25	0.28	4.82	2.97	4.06	5.42
26	0.27	4.92	3.04	4.14	5.38
27	0.26	4.97	3.09	4.24	5.53
28	0.26	5.23	3.22	4.34	5.02
29	0.25	5.33	3.27	4.47	4.94
29	0.24	5.38	3.40	4.54	5.64
$\rho_g = 2.40 \cdot 10^{-3}$ g/cm³			$\mu_l = 1.06 \cdot 10^{-2}$ g/cm sec		
30	0.24	5.74	3.27	4.64	5.49
31	0.23	5.89	3.50	4.72	5.48

G	X	$\Delta_s \cdot 10^2$	$\Delta_c \cdot 10^2$	Γ_{lf}
198.1	0.308	4.88	4.12	105.91
137.7	0.442	3.61	2.71	50.80
82.6	0.730	1.58	1.48	17.10
177	0.226	8.26	6.97	160.76
109.8	0.364	6.05	4.26	66.57
61.9	0.645	2.57	2.39	22.44
152.3	0.101	16.7	18.4	322.41
92.2	0.166	12.8	10.14	113.90
37.3	0.410	5.43	4.46	21.52

TABLE VIII Some data of reference 6 R=1.25 cm ρ_G=3.61 10^{-2} g/cm^3

G	X	$\Delta_s \cdot 10^2$	$\Delta_c \cdot 10^2$	Γ_{lf}
34.1	0.470	5.6	7.80	45.17
38.1	0.420	6.2	9.36	61.62
44.5	0.360	7.4	11.52	86.99
54.2	0.236	9.2	13.97	119.8
67	0.209	11	18,21	167.5
84	0.190	12.7	16.9	167.4
107	0.133	15.5	16.57	149.1
135	0.118	16.5	13.91	117.1
173	0.093	17.4	10.64	55.26
222	0.072	18.8	7.24	=
53.1	0.660	2.76	2.81	14.87
57.1	0.614	3.21	3.43	22.41
63.5	0.550	3.77	4.40	35.67
73.2	0.478	4.67	5.64	55.37
86	0.407	5.8	6.99	79.92
101	0.346	7.35	8.25	105.2
126	0.278	8	9.36	130.2
154	0.227	8.5	9.90	142.3
192	0.181	9.3	9.64	132.3
241	0.145	11.7	8.62	100.6
104.1	0.790	1.26	1.12	9.001
110.5	0.745	1.49	1.44	15.41
120.2	0.681	1.71	1.94	26.65
133	0.616	1.31	2.49	40.89
148	0.-555	2.20	3.03	56.53
173	0.474	2.53	3.79	79.67
201	0.408	2.93	4.38	98.90
239	0.343	3.39	4.88	115
288	0.284	3.34	5.17	122
194	0.887	0.360	0.38	3.855
200.5	0.860	0.377	0.49	7.25
210.5	0.820	0.395	0.66	13.29
223	0.770	0.417	0.885	22.17
238	0.723	0.450	1.105	31.97
263	0.654	0.460	1.43	47.69
291	0.590	0.57	1.75	63.81
329	0.523	0.66	2.07	80.40

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Alfred Nobel

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