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SOME PROBLEMS OF STRESS WAVE PRODUCTION ENCOUNTERED IN THE STUDY OF PULSED FAST REACTOR DYNAMICS

by

J. RANDLES and R. JAARSMA

1967

Joint Nuclear Research Center
Ispra Establishment - Italy

Reactor Physics Department
Reactor Theory and Analysis

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KEYWORDS

Chapter I

REACTORS
PULSES
THERMAL STRESSES
FUEL ELEMENTS
FUEL CANS
TEMPERATURE
DIFFERENTIAL EQUATIONS

SORA, fast reactors, thermal shock

Chapter II

REACTORS
PULSES
REACTOR SAFETY
REACTOR CORE
LIQUID METAL COOLANT
ACCIDENTS
FAILURES
MECHANICAL STRUCTURES
PRESSURE
SHOCK WAVES

SORA, fast reactors, compression

CONTENTS

SOME PROBLEMS OF STRESS WAVE PRODUCTION ENCOUNTERED IN THE STUDY OF PULSED FAST REACTOR DYNAMICS+

INTRODUCTION

The pulsed fast reactor SORA[1,2], proposed by the reactor physics department at Ispra as a general research tool, has features which lead to some highly unusual problems in the safety studies of the system.

The first such feature of interest in this article is the very rapid temperature rise occuring in the fuel at each pulse. Because of the rapidity of this rise and the inertia of the medium (fuel), the axial thermal expansion of the fuel slugs lags behind the temperature. The internal stresses generated by this lag then propagate to all parts of the structure, in particular the fuel cladding, and the whole system is agitated by axial elastic vibrations. The effect of such a phenomenon on the neutron kinetics of the reactor has already been studied[3,4]. The objective of the present article is to focus more attention on the vibrations, to evaluate the manner in which the fuel slug "jumps" in response to its internal stresses and to predict the stresses induced by recoil in the cladding and structural materials. In this way, some idea will be obtained of the safety margin against cladding fatigue and rupture (due to the repetition of stressing with each pulse) and of the sort of temperature rise which might break the cladding in a single (accidentally large) pulse. All of these questions are considered in part 1 of the article.

The second feature of the SORA reactor of present concern is the existence of a heavy moving reflector system. In that part of the safety studies which deals with the so-called "worst hypothetical accident"[8], it is assumed that a part of this system breaks away while in motion and collides with the reactor in such a region that the core is vigorously compressed. In studying the mechanics of this collision it is very important to determine the pressure developed in the liquid metal coolant as the core volume is reduced. This would be easy if the coolant was rigidly contained in the core (so that the compressed mass is constant), but this is not the case. The core is

+Manuscript received on September 6, 1967.

naturally open at the bottom and top for the entry and exit of the coolant, while the thrust of the colliding fragment occurs from the side. Thus, as soon as the collision begins, the coolant starts to flow out of the core through both the entrance and exit of the system and the pressure developed then depends not only on the amplitude but also on the rapidity of core compression relative to the speed with which the coolant can leave the core. Since the rate at which the liquid metal can escape depends on the velocity of compression waves, the problem is essentially that of describing the formation and propagation of sound waves in the coolant channels. This problem and the accompanying one of the mechanics of the colliding fragment are considered in part 2 of the article.

1. RESPONSE OF A FUEL SLUG AND CLADDING TO A THERMAL SHOCK (Jack Randles)

1.1 Mathematical Description

For purposes of setting up a mathematical model, the configuration of fuel slug, cladding and supporting structure are visualised as in figure 1. The cladding is held at its upper end which is assumed to take the whole weight of the fuel element. The fuel slug is a uniform bar with its lower end resting in close contact with the closed bottom of the cladding but with its lateral boundary separated from the inner wall of the cladding by a small gap. The existence of this gap means that the fuel slug and cladding can be assumed to have no interaction except through their point of contact at the bottom end. Thus, we can formulate the dynamics of the system entirely in terms of axial particle displacments. In addition, it will be assumed that the contact which exists between the fuel and cladding can be broken without loss of energy as soon as the stress in the junction ceases to be compressive. Thus, we shall have a situation in which the fuel slug jumps free of cladding and suddenly eliminates their mutual interaction. Such a sudden break in the coupling between the two main components of the system provides the basis for a great simplification in the analysis, since it is clear that all effects of importance are determined during the short time interval leading up to the break. Therefore it becomes possible to obtain all required information by considering the dynamics of the system during this interval alone. One further feature of the fuel element which simplifies the description of the dynamic behaviour is the fact the length L_1 of cladding is considerably greater than the length L of the fuel.

If all axial distances, x , are measured relative to an origin placed at the bottom of the fuel slug when the system is in an unheated, unstressed state (before the commencement of the shock), then initially the fuel and cladding material is distributed uniformly along the x-axis with the fuel slug lying in the range $0 < x < L$ and the cladding in $0 < x < L_1$. Subsequent to some time point, t = 0, we suppose this state to be disturbed by the appearance of a temperature rise, $T(x,t)$, in the fuel. The effect of such a rise will be an axial particle displacement, not only in the fuel slug but also in the cladding, since the two components have a point of contact at $x = 0$. Thus, it is necessary to introduce two displacement fields, $\xi(x,t)$ and $\xi_1(x,t)$, describing the axial distortion of the fuel slug and that of the cladding respectively. The quantity $\xi(x,t)$ has the significance that if x is the initial position of a cross sectional element of the fuel slug, the position of the same element at time t is $x + \xi$. This definition applies also to the displacements, ξ_1 , of the cladding. If we suppose that no heat transfer occurs between the fuel and cladding, a reasonable assumption in view of the very short timescale of interest, then it can be shown[3,5] that ξ and ξ_1 satisfy the equations:

$$\frac{1}{c^2}\frac{\partial^2 \xi}{\partial t^2} = \frac{\partial^2 \xi}{\partial x^2} - \alpha \frac{\partial T}{\partial x} \tag{1.1}$$

and

$$\frac{1}{c_1^2}\frac{\partial^2 \xi_1}{\partial t^2} = \frac{\partial^2 \xi_1}{\partial x^2} \tag{1.2}$$

where $c = \sqrt{E/\rho}$ is the velocity of axial compression waves in the fuel slug, E and ρ being respectively the Young's modulus and density of the fuel material; $c_1 = \sqrt{E_1/\rho_1}$ is the velocity of axial compression waves in the cladding, E_1 and ρ_1 being respectively the Young's modulus and density of the cladding material and α is the coefficient of linear expansion of the fuel. The stress in the fuel slug is given by[3]

$$S_f(x,t) = E\left(\frac{\partial \xi}{\partial x} - \alpha T\right)$$

and that in the cladding by (1.3)

$$S_c(x,t) = E_1 \frac{\partial \xi_1}{\partial x}$$

Equations (1.1) and (1.2) both have the form of a classical one-dimensional wave equation except that (1.1) is slightly modified by a term depending on the temperature gradient.

The boundary conditions to equations (1.1) and (1.2) can be formulated from all the known facts about the system. In the first place, we know that at the moment of commencement of the temperature rise at t = 0, the whole system is stationary and undisturbed. Thus, we can write:

$$\left.\begin{array}{ll} \text{(a)} & \xi = 0 \\ \text{(b)} & \dot{\xi} = 0 \\ \text{(c)} & \xi_1 = 0 \\ \text{(d)} & \dot{\xi}_1 = 0 \end{array}\right\} \quad \text{at} \quad t = 0$$

At the junction between the fuel slug and cladding, we know that the two displacements must be equal :

$$\text{(e)} \qquad \xi_1 = \xi \qquad \text{at} \quad x = 0.$$

In addition, since the mass of the base part of the cladding (on which the slug rests) is negligible in comparison to the mass of the slug, the intertial effect of this part can be ignored. Thus, the total force exerted on the junction by the fuel slug can be assumed to be equal and opposite to that exerted by the cladding and we can write:

$$\text{(f)} \qquad AE\left(\frac{\partial \xi}{\partial x} - \alpha T\right) = -A_1 E_1 \frac{\partial \xi_1}{\partial x} \qquad \text{at} \quad x = 0,$$

where A and A_1 are the cross sectional areas of the fuel slug and cladding respectively. In formulating condition (f), use has been made of equations (1.3). A further item in our knowledge of the system is that the upper end of the fuel slug is free and the internal stress must therefore go to zero at this end:

(g) $$\frac{\partial \xi}{\partial x} = \alpha T \qquad \text{at} \quad x = L.$$

The condition at the upper end of the cladding is not so easy to write down since the response of the supporting structure is rather complicated. It is emphasised again here, however, that the dynamical effects of interest in this paper are the very rapid phenomena immediately following the onset of the temperature rise and, for these, the behaviour of the supporting structure is completely unimportant. During the initial time interval $0 < t < L_1/c_1$ required by a disturbance to travel up the length of the cladding, we know that, irrespective of the supporting structure, we must have

(h) $$\xi_1 = 0 \qquad \text{at} \quad x = L_1$$

It will be seen that this condition, though limited in the duration of its validity, will be sufficient to determine the required information.

Although equations (1.1), (1.2) and the boundary conditions (a)-(h) apply for a temperature rise, $T(x,t)$, with any type of spatial distribution and time dependence, we have solved the problem only for cases where T does not in fact depend on x. For such cases, where the temperature distribution is always uniform, we can write

$$T = T(t) \tag{1.4}$$

and the term, $\alpha\, \partial T/\partial x$, in equation (1.1) then vanishes identically. This small simplification in the theory at the outset yields a very large saving in the analysis later, since we now have a situation in which both displacement fields, ξ and ξ_1, are governed by the classical wave equation. In such a case, the functional form of ξ and ξ_1 satsifying equations (1.1) and (1.2) is well known. It is a superposition of forward and backward moving waves:

$$\xi(x,t) = f(ct-x) + g(ct+x) \tag{1.5}$$

and

$$\xi_1(x,t) = \phi(c_1 t-x) + \psi(c_1 t+x) \tag{1.6}$$

where f and ϕ are the forward and g and ψ the backward moving waves and the velocities of propagation c and c_1 are (as mentioned above) associated with the fuel slug and cladding respectively. The explicit form of the functions f, g, ϕ and ψ must be determined from the boundary conditions.

This type of formulation is a very common method of attacking dynamical problems in the theory of elasticity and appears in all good textbooks on the subject (see, for example, the well known treatise of LOVE[(6)], p. 431). In the present problem we have to satisfy 8 boundary conditions. The first four of these, (a), (b), (c) and (d), give respectively:

$$\left.\begin{aligned} f(-x) + g(x) &= 0 \\ f'(-x) + g'(x) &= 0 \end{aligned}\right\} \quad \text{for} \quad 0 < x < L$$

$$\left.\begin{aligned} \phi(-x) + \psi(x) &= 0 \\ \phi'(-x) + \psi'(x) &= 0 \end{aligned}\right\} \quad \text{for} \quad 0 < x < L_1$$

where f', g', ϕ' and ψ' are the first derivatives of f, g, ϕ and ψ respectively. On integrating the second and fourth of these equations with respect to x and setting the resulting two arbitrary constants equal to zero (they could be retained but carry no physical significance and cancel identically later in the analysis) we obtain:

$$\left.\begin{array}{llll} f(z) = 0 & \text{for} & -L < z < 0 & \text{(i)} \\ g(z) = 0 & \text{for} & 0 < z < L & \text{(ii)} \\ \phi(z) = 0 & \text{for} & -L_1 < z < 0 & \text{(iii)} \\ \Psi(z) = 0 & \text{for} & 0 < z < L_1 & \text{(iv)} \end{array}\right\} \qquad (1.7)$$

If we now put

$$\lambda = \frac{c_1}{c}$$

and

$$r = \frac{A_1 E_1 c}{A\ E\ c_1} \qquad (1.8)$$

which is a measure of the relative rigidity of the cladding, conditions (e) and (f) give respectively, for $z > 0$

$$f(z) + g(z) = \phi(\lambda z) + \Psi(\lambda z) \qquad (1.9)$$

and

$$-f'(z) + g'(z) - \alpha T(z/c) = -\lambda r \left\{ -\phi'(\lambda z) + \Psi'(\lambda z) \right\}$$

On integration with respect to z and application of equations (1.7), this equation gives for $z > 0$:

$$-f(z) + g(z) - \alpha \int_0^z T\left(\frac{z'}{c}\right) dz' = -r \left\{ -\phi(\lambda z) + \Psi(\lambda z) \right\} \qquad (1.10)$$

Finally, from conditions (g) and (h) and equations (1.7) we get

$$g(z) = f(z-2L) + \alpha \int_L^z T\left(\frac{z'-L}{c}\right) dz' \quad \text{for } z > L \tag{1.11}$$

and

$$\psi(z) = -\phi(z-2L_1) \qquad \text{for } L_1 < z < 2L_1 \tag{1.12}$$

the z variable being limited to values less than $2L_1$ because of the restriction of condition (h) to times within the wave transit time, L_1/c_1, along the cladding.

Equations (1.7), (1.9), (1.10), (1.11) and (1.12) now provide the basis for the progressive evaluation of the wave components f, g, ϕ and ψ within a sequence of intervals on the z-axis. Fortunately, as already mentioned, the dynamical events of interest in this paper all occur rapidly and we can restrict ourselves to a consideration of only the first three intervals. With such a restriction, the theory can be yet further simplified by using the fact that the length L_1 of the cladding is much longer than that of the fuel slug, L. Hence it is valid to assume that the transit time , L/c , of waves along the slug is less than the time L_1/c_1, required by waves to traverse the cladding. Hence we can write

$$\lambda L < L_1 \tag{1.13}$$

With this condition in mind, we resume the analysis.

From equation (1.11) we see that, for $L < z < 2L$,

$$g(z) = \alpha \int_L^z T\left(\frac{z'-L}{c}\right) dz' \tag{1.14}$$

since, by equation (1.7i), $f(z-2L)$ is zero in this interval. In addition, because of equation (1.7iv) and condition (1.13), $\Psi(\lambda z) = 0$ in the interval $0 < z < L$ and equations (1.9) and (1.10) reduce to

$$f(z) = \phi(\lambda z)$$

and

$$f(z) + r\,\phi(\lambda z) = -\alpha \int_0^z T\left(\frac{z'}{c}\right) dz'$$

Hence, for $0 < z < L$

$$f(z) = -\frac{\alpha}{1+r} \int_0^z T\left(\frac{z'}{c}\right) dz' \qquad (1.15)$$

Furthermore, from equations (1.12) and (1.7iii) we see that $\Psi(z) = 0$ for $L_1 < z < 2L_1$, so that, by condition (1.13) we have $\Psi(\lambda z) = 0$ for $L < z < 2L$. Therefore, in the range $L < z < 2L$, equations (1.9) and (1.10) take on, with the help of equation (1.14), the form:

$$f(z) - \phi(z) = -\alpha \int_L^z T\left(\frac{z'-L}{c}\right) dz'$$

and

$$f(z) + r\,\phi(z) = \alpha \int_L^z T\left(\frac{z'-L}{c}\right) dz' - \alpha \int_0^z T\left(\frac{z'}{c}\right) dz'$$

from which it follows that, in the range $L < z < 2L$:

$$f(z) = \frac{\alpha}{1+r} \left\{ (1-r) \int_L^z T\left(\frac{z'-L}{c}\right) dz' - \int_0^z T\left(\frac{z'}{c}\right) dz' \right\}. \qquad (1.16)$$

The last quantity to be derived in this way is the function $g(z)$ in the range $2L < z < 3L$. By equations (1.11) and (1.15), this is given by

$$g(z) = \alpha \int_L^z T\left(\frac{z'-L}{c}\right) dz' - \frac{\alpha}{1+r} \int_0^{z-2L} T\left(\frac{z'}{c}\right) dz'. \qquad (1.17)$$

It will be very convenient at this stage, before continuing with further developments, to summarise the above derived formulae for the wave-functions. We shall be interested only in the displacement field in the fuel slug and will therefore omit Φ and Ψ . Collecting formulae (1.7i), (1.15) and (1.16), we have for the wave component f(z) :

$$f(z) = \begin{cases} 0 & \text{for } z \leqslant 0 \\ -\dfrac{\alpha}{1+r} \displaystyle\int_0^z T\left(\frac{z'}{c}\right) dz' & \text{for } 0 \leqslant z \leqslant L \\ \dfrac{\alpha}{1+r} \left\{ (1-r) \displaystyle\int_L^z T\left(\frac{z'-L}{c}\right) dz' - \int_0^z T\left(\frac{z'}{c}\right) dz' \right\} & \text{for } L \leqslant z \leqslant 2L \end{cases} \qquad (1.18)$$

and from equations (1.7ii), (1.14) and (1.17), g(z) is given by:

$$g(z) = \begin{cases} 0 & \text{for } z \leqslant L \\ \alpha \displaystyle\int_L^z T\left(\frac{z'-L}{c}\right) dz' & \text{for } L \leqslant z \leqslant 2L \\ \alpha \displaystyle\int_L^z T\left(\frac{z'-L}{c}\right) dz' - \frac{\alpha}{1+r} \int_0^{z-2L} T\left(\frac{z'}{c}\right) dz' & \text{for } 2L \leqslant z \leqslant 3L \end{cases} \qquad (1.19)$$

It is interesting to note the very simple manner in which the relative cladding rigidity, r , appears in these results.

In order to evaluate the time point, t_j , when the fuel slug jumps free of the cladding, it is necessary to evaluate the stress at the junction, $S_o(t)$. According to the first of equations (1.3), this is given by

$$S_o(t) = E\left(\frac{\partial \xi}{\partial x} - \alpha T\right)_{x=0}$$

Using equation (1.5) and the fact that T is independent of x, this gives:

$$S_o(t) = E\left(-f'(ct) + g'(ct) - \alpha T(t)\right)$$

Introduction of (1.18) and (1.19) into this formula then yields:

$$S_o(t) = \begin{cases} -\dfrac{r}{1+r} E\alpha T(t) & \text{for } 0 \leqslant t \leqslant \dfrac{L}{c} \\ \dfrac{2r}{1+r} E\alpha T\left(t-\dfrac{L}{c}\right) - \dfrac{r}{1+r} E\alpha T(t) & \text{for } \dfrac{L}{c} \leqslant t \leqslant \dfrac{2L}{c} \end{cases} \qquad (1.20)$$

When $S_o(t) < 0$, the slug and cladding are pressed together and remain in close contact. Inspection of equation (1.20) shows clearly that this state of affairs persists for at least a time L/c after the onset of the disturbance. For some time in the range $L/c < t < 2L/c$, however, equation (1.20) reaches zero and the time at which this occurs is precisely the moment $t = t_j$ when the slug jumps free of the cladding. For $t > t_j$, $S_o(t) = 0$ and equation (1.20) breaks down. Clearly, t_j is the solution of

$$2\,T\left(t - \frac{L}{c}\right) - T(t) = 0 \qquad (1.21)$$

It is interesting to note that t_j is independent of all parameters except the wave transit time along the fuel slug.

The mean velocity, V_j, with which the fuel slug jumps can be derived quite easily. By definition we have

$$V_j = \frac{1}{L}\int_0^L \dot{\xi}(x, t_j)\,dx$$

Substitution from (1.5) leads to:

$$V_j = \frac{c}{L}\int_0^L \left\{ f'(ct_j - x) + g'(ct_j + x) \right\} dx$$

which, on evaluation of the integrals gives:

$$V_j = \frac{c}{L}\left\{ f(ct_j) - f(ct_j - L) + g(ct_j + L) - g(ct_j) \right\}. \tag{1.22}$$

Remembering that ct_j lies in the range $L \leqslant ct_j \leqslant 2L$, the manner of substituting for the wave functions from (1.18) and (1.19) is obvious. We obtain, after some manipulation:

$$V_j = \frac{r}{1+r}\,\frac{\alpha c^2}{L}\left\{ \int_0^{t_j} T(t)\,dt - 2\int_0^{t_j - L/c} T(t)\,dt \right\} \tag{1.23}$$

In the next two sections we shall apply these general formulae to explore in some detail the thermoelastic response of a fuel element to temperature shocks of a definite mathematical form.

1.2 Discontinuous Temperature Shock

The type of thermal shock which is easiest to consider, and which brings out the physical processes very clearly, is the case of a discontinuous temperature rise. For this, the function $T(t)$ will have the form

$$T(t) = \begin{cases} 0 & \text{for} \quad t < 0 \\ T_o & \text{for} \quad t \geq 0 \end{cases} \tag{1.24}$$

With such an input the wave components (1.18) and (1.19) reduce to

$$f(z) = \begin{cases} 0 & \text{for} \quad z \leq 0 \\ -\dfrac{\alpha T_o z}{1+r} & \text{for} \quad 0 \leq z \leq L \\ -\dfrac{\alpha T_o}{1+r}\left[rz + (1-r)L\right] & \text{for} \quad L \leq z \leq 2L \end{cases} \tag{1.25}$$

and

$$g(z) = \begin{cases} 0 & \text{for} \quad z \leq L \\ \alpha T_o (z-L) & \text{for} \quad L \leq z \leq 2L \\ \dfrac{r}{1+r}\alpha T_o z - \dfrac{1-r}{1+r}\alpha T_o L & \text{for} \quad 2L \leq z \leq 3L \end{cases} \tag{1.26}$$

while the stress in the junction, (1.20), becomes

$$S_o(t) = \begin{cases} -\dfrac{r}{1+r} E\alpha T_o & \text{for} \quad 0 \leq t \leq \dfrac{L}{c} \\ 0 & \text{for} \quad t > \dfrac{L}{c} \end{cases} \tag{1.27}$$

When $t = L/c$, the slug jumps free of the cladding in the manner discussed in the last section and we have therefore set $S_o = 0$ beyond this point. If the slug and cladding had been tightly fastened together, the stress would have been exactly reversed at the instant $t = L/c$, giving a tension of $rE\alpha T_o/(1+r)$ for a duration $L/c \leq t \leq 2L/c$. Setting $t_j = L/c$ in equation (1.23) we get for the jump velocity:

$$V_j = \frac{r}{1+r}\alpha T_o c \tag{1.28}$$

It is interesting to examine how the mechanical energy generated by the thermal shock is distributed among the different modes of motion in the system. Because of the discontinuous nature of the shock in the present case, a potential energy of

$$E_{tot} = \frac{1}{2} AEL(\alpha T_o)^2 \tag{1.29}$$

is suddenly created in the fuel slug at $t = 0$. Since there is no motion in the system at this instant, this also represents the total energy available for wave and stress production. We already know that the fuel slug eventually responds by jumping free of the cladding and therefore it is clear that the energy E_{tot} becomes divided into three parts:

(1) A part E_j representing the kinetic energy of jumping.

(2) A part E_f representing the total vibrational energy of the fuel slug.

(3) A part E_c representing the total (vibrational) energy transferred to the cladding.

By the law of energy conservation, we have

$$E_j + E_f + E_c = E_{tot} \tag{1.30}$$

The jumping energy, E_j, is very easy to evaluate. It is

$$E_j = \frac{1}{2}\rho AL V_j^2$$

ρAL being the total mass of the slug. Substituting for V_j from (1.28) and then for c^2 (= E/ρ), we get

$$E_j = \left(\frac{r}{1+r}\right)^2 E_{tot} \tag{1.31}$$

The vibrational energy, E_f , excited in the fuel slug presents a little more difficulty but its evaluation can be greatly simplified by noting, from the analysis leading to equation (1.28), that at the instant $t = L/c$ when jumping occurs, the particle velocity in the slug is everywhere equal to V_j . This means that at this instant, the whole kinetic energy of the slug is contained in E_j , the energy of the vibrational motion being instantaneously all in the potential form. From this it follows that we can equate E_f to the instantaneous strain energy in the slug at $t = L/c$. By using equations (1.3), (1.5), (1.25) and (1.26) it is easy to see that the stress and strain are both <u>uniform</u> at this moment, the stress, S_f , being equal to $E\alpha T_o/(1+r)$. The energy corresponding to this uniform stress is $ALS_f^2/2E$, so that we get finally for E_f:

$$E_f = \left(\frac{1}{1+r}\right)^2 E_{tot} \quad . \tag{1.32}$$

From equations (1.30), (1.31) and 1.32), it follows that the energy transferred to the cladding is given by

$$E_c = \frac{2r}{(1+r)^2} E_{tot} \quad . \tag{1.33}$$

The last three equations give a very clear summary of the dynamical behaviour resulting from the mutual interaction of the fuel slug and cladding. In the limit of very weak cladding, $r \to 0$ and we see that all of the energy goes into longitudinal vibrations in the fuel slug. In the limit of very rigid cladding, $r \to \infty$ and we see that all of the energy goes into jumping. The energy transferred to the cladding goes through a maximum of $E_{tot}/2$ when the fuel slug and cladding are of equal rigidity, i.e. when $r = 1$. In practice the cladding is usually a very thin tube of much smaller cross sectional area than the fuel slug. Thus, in general the relative rigidity r (see equation (1.8)) is small and we tend always to be near to the "weak cladding" limit. This fact does not help to reduce the stress, $S_c(t)$, in the cladding, however, since the large ratio of A to A_1 serves as an amplifying factor in this case. Such an effect is very easy to understand if we write down the force balance condition at the junction ($x = 0$):

$$A_1 S_c = A S_0$$

so that, by equations (1.8) and (1.27), the cladding stress at the junction is given by

$$S_c(t) = \begin{cases} \dfrac{c/c_1}{1+r} E_1 \alpha T_0 & \text{if } 0 \leq t \leq \dfrac{L}{c} \\ 0 & \text{if } \quad t > \dfrac{L}{c} \end{cases} \tag{1.34}$$

which is virtually independent of the cladding rigidity when $r \ll 1$. Subsequent to the time point L/c, the cladding will undergo damped oscillations in which the stress will at no time exceed that given by equation (1.34). Thus, for a discontinuous thermal shock, S_c will be the maximum stress which the cladding has to bear.

It should be emphasised in closing this section that the discontinuous thermal shock treated here is a much more severe disturbance than that occuring in an actual pulsed reactor. Usually, the temperature rise, though rapid, takes a definite time and the resulting thermoelastic response is softer than that described above. From the point of view of reactor safety analysis, the theory here is therefore pessimistic, predicting larger stresses in the fuel elements than actually occur. In order to correct for this, the theory is developed in the next section for a type of thermal shock very close to that expected in a real system. Later, in section 1.4, both approximations will be applied to the SORA reactor and the errors due to the assumption of a discontinuous temperature rise will be seen numerically.

1.3 Continuous Temperature Shock

The temperature rise which occurs in the fuel during an excursion of a pulsed fast reactor can be quite well approximated by the function:

$$T(t) = \begin{cases} T_o \dfrac{t}{\tau} & \text{for} \quad 0 \leqslant t \leqslant \tau \\ T_o & \text{for} \quad t \geqslant \tau \end{cases} \tag{1.35}$$

where T_o is the ultimate temperature rise and τ a time constant depending on the nuclear and geometrical properties of the reactor and the speed of the pulsation device. It may be helpful here to state that, for the typical case of the SORA reactor, τ is about 50 μsec as compared to about 90 μsec for the wave transit time, L/c, along the fuel slugs. Thus, it is clear that the real dynamical behaviour to be considered in this section is not greatly different from that resulting from a discontinuous shock.

By substituting equation (1.35) into (1.21) it can easily be shown that the time interval between the onset of the shock and jumping of the fuel slug is given by

$$t_j = \begin{cases} \dfrac{L}{c} + \dfrac{\tau}{2} & \text{for} \quad \tau \leqslant \dfrac{2L}{c} \\ \dfrac{2L}{c} & \text{for} \quad \tau \geqslant \dfrac{2L}{c} \end{cases} \tag{1.36}$$

Similarly, by introducing (1.35) into equation (1.20) we obtain for the fuel stress at the junction with the cladding the following formulae:

when $\tau < L/c$

$$S_o(t) = \begin{cases} -\dfrac{r}{1+r} \dfrac{E\alpha T_o}{\tau} t & \text{for} \quad 0 \leqslant t \leqslant \tau \\ -\dfrac{r}{1+r} E\alpha T_o & \text{for} \quad \tau \leqslant t \leqslant \dfrac{L}{c} \\ \dfrac{r}{1+r} E\alpha T_o \left\{ \dfrac{2}{\tau}\left(t - \dfrac{L}{c}\right) - 1 \right\} & \text{for} \quad \dfrac{L}{c} \leqslant t \leqslant t_j \\ 0 & \text{for} \quad t \geqslant t_j; \end{cases} \tag{1.37}$$

when $L/c \leq \tau \leq 2L/c$

$$S_o(t) = \begin{cases} -\frac{r}{1+r}\frac{E\alpha T_o}{\tau}t & \text{for } 0 \leq t \leq \frac{L}{c} \\ \frac{r}{1+r}\frac{E\alpha T_o}{\tau}\left(t-\frac{2L}{c}\right) & \text{for } \frac{L}{c} \leq t \leq \tau \\ \frac{r}{1+r}E\alpha T_o\left\{\frac{2}{\tau}\left(t-\frac{L}{c}\right)-1\right\} & \text{for } \tau \leq t \leq t_j \\ 0 & \text{for } t \geq t_j \ ; \end{cases} \tag{1.38}$$

when $\tau \geq 2L/c$

$$S_o(t) = \begin{cases} -\frac{r}{1+r}\frac{E\alpha T_o}{\tau}t & \text{for } 0 \leq t \leq \frac{L}{c} \\ \frac{r}{1+r}\frac{E\alpha T_o}{\tau}\left(t-\frac{2L}{c}\right) & \text{for } \frac{L}{c} \leq t \leq \frac{2L}{c} \\ 0 & \text{for } t \geq \frac{2L}{c} \ . \end{cases} \tag{1.39}$$

It is clear from these expressions that, irrespective of the rate of temperature rise, T_o/τ , the maximum stress, S_{omax} , on the fuel side of the junction always occurs at $t = L/c$ and that

$$S_{omax} = \begin{cases} -\frac{r}{1+r}E\alpha T_o & \text{for } \tau \leq \frac{L}{c} \\ -\frac{r}{1+r}\left(\frac{L}{\tau c}\right)E\alpha T_o & \text{for } \tau \geq \frac{L}{c} \end{cases} \tag{1.40}$$

Thus, the maximum stress obtained by assuming a discontinuous temperature rise (equation (1.27) is correct provided the rise time, τ , is less than the wave transit time, L/c , along the fuel slug. On the other hand, if $\tau > L/c$ the stress obtained from the discontinuous model is too large and must be reduced by a factor $L/\tau c$.

The jumping velocity of the slug can be evaluated by substituting (1.35) into equation (1.23) and using (1.36). The result is

$$V_j = \begin{cases} \frac{r}{1+r}\left(1-\frac{1}{4}\frac{\tau c}{L}\right)\alpha T_0 c & \text{for } \tau \leq \frac{2L}{c} \\ \frac{r}{1+r}\left(\frac{L}{\tau c}\right)\alpha T_0 c & \text{for } \tau \geq \frac{2L}{c} \end{cases} \tag{1.41}$$

Thus we see, by comparing this formula with equation (1.28), that the magnitude of the jumping effect is always smaller than that predicted by the discontinuous temperature model, the reduction factor being $(1 - \tau c/4L)$ when $\tau \leq 2L/c$ and $L/\tau c$ when $\tau \geq 2L/c$.

1.4 Application to the SORA Fuel Element

The formulae derived in sections 1.2 and 1.3 offer a very simple means of assessing the behaviour of actual pulsed reactor fuel elements. In this section we shall apply them to the SORA reactor where the fuel slug is a 24 cm long Uranium-Molibdenum alloy (10% by weight of Mo) clad in a long tube of stainless steel. The relevant parameters have been assumed to have the following values.

	Fuel Slug	Cladding
Young's modulus (dynes/cm^2)	$E=1.2\times10^{12}$	$E_1=2\times10^{12}$
Density (gm/cm^3)	$\rho=17.3$	$\rho_1=7.92$
Cross sectional area (cm^2)	$A=1.54$	$A_1=0.139$
Axial sound velocity (cm/sec)	$c=2.63\times10^5$	$c_1=5.03\times10^5$
Linear coef. of expansion ($^\circ C^{-1}$)	$\alpha=1.4\times10^{-5}$	-
Length (cm)	$L=24$	-

Using equation (1.8), we get for the relative cladding rigidity

$$r = 0.0787$$

If we assume a temperature rise time, τ , of 50 μsec then we shall be in the region $\tau < L/c$, since the value of the wave transit time along the slug is $L/c = 91.3$ μsec. Hence, the stress generated in the fuel and cladding at the junction between them is the same as that generated by a discontinuous temperature rise. By equation (1.40) (or (1.27)) the maximum value of this stress is, on the fuel side

$$S_{omax} = -1.23 \times 10^6 \, T_o \text{ dynes/cm}^2$$

while the maximum stress on the cladding side is, by equation (1.34)

$$S_{c\,max} = 1.36 \times 10^7 \, T_o \text{ dynes/cm}^2.$$

The former stress is compressive (negative) while the latter represents a tension.

If we assume, as criteria of safety, that these stresses must

(a) lie well below the fatigue stresses under normal pulsed operation and

(b) never exceed the unirradiated elastic limit during an accidentally large pulse,

then the limits to be imposed on the fuel temperature rise, T_o , both for normal and accidental circumstances are very easy to calculate. Assuming that the elastic limit and fatigue stress for the cladding are 2.5×10^9 dynes/cm^2 and 1.0×10^9 dynes/cm^2 respectively, while for the fuel they are 2.0×10^9 dynes/cm^2 and 0.8×10^9 dynes/cm^2 respectively, then the above criteria can be put in the form

$$\left.\begin{array}{lll} \text{(i)} & S_{cmax} & \ll 10^9 \\ \text{(ii)} & |S_{omax}| & \ll 0.8\times10^9 \end{array}\right\} \text{dynes/cm}^2 \text{ during normal pulsed operation}$$

and

$$\left.\begin{array}{lll} \text{(iii)} & S_{cmax} & < 2.5\times10^9 \\ \text{(iv)} & |S_{omax}| & < 2.0\times10^9 \end{array}\right\} \text{dynes/cm}^2 \text{ during an accidentally large pulse.}$$

From (i) and (iii) we see that the safety of the cladding requires that

$T_o \ll 74\ ^oC$ during normal pulsed operation

and

$T_o < 184\ ^oC$ during an accidentally large pulse,

while, from (ii) and (iv), the fuel retains its integrity provided that

$T_o \ll 650\ ^oC$ during normal pulsed operation

and

$T_o < 1600\ ^oC$ during an accidentally large pulse.

It is immediately obvious that the cladding is mechanically more vulnerable than the fuel although, with the proposed temperature rise of $\sim 1\ ^oC$, the safety margin during normal operation is enormous. In addition, the limit of 184 ^oC for an accidentally large excursion is still greater than that for modes of damage other than mechanical changes [(4)]. It is interesting to note that the vulnerability of the cladding as compareu to the fuel is due entirely to the smallness of the area ratio, A_1/A .

The fuel jumping effect can be calculate from equations (1.28) and 41). If the temperature rise is regarded as discontinuous, the jumping ocity is 0.269 T_o cm/sec. If, however, we allow for the continuity in temperature rise, the jumping velocity is reduced (equation 1.41) by

13.7% to 0.232 T_o cm/sec. It follows that the fuel jumping occurs only at about 0.2 cm/sec during normal pulsed operation but may rise to 20 cm/sec for an accidentally large rise of about 100 ^oC.

It is of interest to estimate the proportions of the total mechanical energy going into the three processes: fuelslug jumping, fuel slug vibrations and cladding vibrations. For this purpose we use the formulae derived in section (1.2) on the basis of the discontinuous temperature approximation. From equations (1.31), (1.32) and (1.33), we obtain for the ratios between the above three quantities:

$$E_j : E_f : E_c = 0.0053 : 0.8593 : 0.1354$$

Thus we see that very little of the energy goes into jumping or cladding vibrations and for many purposes,outside the present study,the fuel slugs could be regarded as free at both ends.

When times longer than $\sim$L/c are considered, however, it is important to remember that the fuel slug will eventually fall back onto the junction with the cladding and that, consequently, further energy transfer will occur. The stresses generated by such further interaction will be less than those evaluated above because the oscillations of the fuel slug will have been damped slightly by internal dissipative mechanisms. The process of falling back onto the cladding will provide strong assistance to these mechanisms in their important role of removing the oscillations in the fuel slug before the arrival of another pulse and the generation of another burst of oscillations. Looked at in this way, the process of falling back onto the cladding is seen to be an important safety mechanism helping to limit the amplitude of the oscillations imposed by recurrent thermal shocks.

From these considerations, there arises an important safety problem.If the slug for some reason becomes "stuck" and fails to fall back onto the cladding after jumping free (during the initial time interval of $\sim$L/c), then the only processes available to damp the vibrations of the slug are the internal dissipative mechanisms. If these are not sufficiently strong, the amplitude of the vibrations will rise to some large value due to the repetition of thermal shocks and the accidental fall of the slug into its seat in the cladding may then cause extensive damage. To calculate the extent of this danger necessitates a knowledge of the non-adiabatic and frictional effects occuring in the fuel material. A theoretical and experimental investigation of these effects is at present underway.

2. THE BEHAVIOUR OF A LIQUID METAL COOLANT DURING THE COLLISION OF A HEAVY MOVING BROKEN PART (OF THE PULSATION DEVICE) WITH THE CORE OF A SORA TYPE REACTOR (Jack Randles)

2.1 Mathematical Description

In a pulsed fast reactor of the SORA type, the worst hypothetical accident[8] takes the form of a fracture in the arm of the rotating pulsation device, the collision of the broken piece with the core and the insertion of a dangerous amount of reactivity as a result of the reduction in core volume. Thus, in order to obtain this reactivity input with reasonable accuracy, the mechanics of the core compression process have to be quite well understood and the core volume reduction $\Delta V(t)$, obtained explicitly as a function of the time, t. This would not be too difficult if the core was just a solid structure for which the elastic forces evoked by the impact could be written down fairly easily and the equation of motion of the broken piece solved directly. The core is not, however, such an elastically simply body. The whole volume not occupied by the fuel and structural components is filled by a liquid metal coolant which flows vertically upwards along the axis of the system. When a fragment of the pulsation arm strikes the core, it does so at right angles to this axis from the side. Thus, the force of the impact is able to eject liquid metal away from the compressed zone towards the inlet and exit of the coolant channels. The effect of such ejection is to weaken the elastic force which might otherwise slow down and repel the fragment before an appreciable volume change (and reactivity input) has occurred. This process is especially important in a SORA type reactor where the core volume must be reduced by 3-4% before a significant elastic force is evoked from the deformation of materials other than the coolant.

Let us state the problem in quantitative terms. We shall consider the impact of a mass m incident with a velocity v_0 and the consequent compression of a length ℓ of the core from the "side" direction (perpendicular to the core axis), ℓ being less than or equal to the core height, and, in order not to complicate the study of the ejection process, we shall ignore all stresses except those generated in the coolant. At the commencement of such a collision, the coolant will undergo compression at a rate depending on the incident velocity v_0 , but the process of ejection from the top and bottom of the compressed zone will be delayed by the time required by compression waves in the coolant to propagate along the compressed length ℓ . Assuming a typical value of 3×10^5 cm/sec for the speed of acoustic waves in liquid metal and 6 cm for the compressed length in a typical hypothetical accident, we see that we are dealing with delays of the order of 20 μsec. Because of the structure of the coolant channels the system behaves like a compressed assembly of small tubes containing liquid metal and to obtain the mean pressure which is "seen" by the colliding mass m it will be assumed that all of the channels are compressed simultaneously. This implies that we shall be neglecting the time taken by the shock of the impact to travel across the core and that the pressure will therefore be spatially constant at any given core cross section. Although somewhat inaccurate, this assumption is not restrictive since it in no way affects the phenomenon of present interest, i.e. the axial coolant flow. In any case the assumption leads to an overestimate of the volume and reactivity changes which, from the point of view of reactor safety analysis, is conservative, as required. Thus, the problem of evaluating the force acting on the colliding fragment reduces to that of evaluating the mean pressure, p , in any one of the coolant channels as if it were a small tube.

From the point of view of such a tube, the effect of the collision is to reduce its lateral dimension and raise the density of the liquid metal within it. For instance, if $\Delta V(t)/V$ is the relative reduction in core volume at time t (V = core volume), then, in the absence of any axial flow of coolant out of the tube, the change in density, $\Delta\rho$, is obviously given by

$$\frac{\Delta\rho(t)}{\rho_0} = \frac{\Delta V(t)}{f V} \qquad (2.1)$$

where ρ_0 is the initial density and fV the initial volume of the coolant in the compressed length. The quantity f is the ratio of the coolant volume in the compressed zone to the volume $\mathcal{V}$ of the core. In terms of the overall coolant volume fraction in the core, f_o , the compressed length, ℓ , and core length, L , we have

$$f = \frac{\ell}{L} f_o \tag{2.2}$$

If we now make an allowance for the ejection of coolant from the ends of the compressed length by solving the equations of inviscid, laminar flow to first order in the applied compression $\Delta\mathcal{V}/\mathcal{V}$, we obtain the following expression for the mean density change:

$$\frac{\Delta\rho(t)}{\rho_o} = \frac{1}{f\mathcal{V}}\left\{\Delta\mathcal{V}(t) - \int_0^t F(t')\,\Delta\dot{\mathcal{V}}(t-t')dt'\right\} \tag{2.3}$$

Here, the time dependent mean density change $\Delta\rho(t)$ is obtained by averaging the time and <u>space</u> dependent density <u>distribution</u> along the compressed length, ℓ . The function F(t') describes the delay in the ejection of the coolant.

Although equation (2.3) can be derived rigorously from the linearized equations of inviscid laminar flow, there exists a much more powerful and physically clear alternative. In this, we take equation (2.3) as a hypothesis in which the delay function F(t') is to be determined. It is very important to note that, in fact, equation (2.3) is the only general way of expressing the process under consideration since it is obvious that the mean density change at time t must be equal to that which would occur in the absence of ejection (1st term) <u>minus</u> a correction depending on the entire history of volume changes up to time t . In a linear theory, the only way of expressing such a history dependent effect is by means of the integral appearing in the second term of (2.3). Thus, without going into the detailed hydrodynamics, we can regard equation (2.3) as the obvious general solution to first order in the applied disturbance $\Delta\mathcal{V}/\mathcal{V}$.

The problem, then, is to determine $F(t')$. This can be achieved in the following very simple way. Let us imagine a sudden change in core volume:

$$\Delta V(t) = \begin{cases} 0 & \text{if} \quad t < \varepsilon \\ \Delta V_0 & \text{if} \quad t \geq \varepsilon \end{cases} \tag{2.4}$$

so that

$$\Delta \dot{V}(t) = \Delta V_0 \, \delta(t-\varepsilon) \tag{2.5}$$

where ε is an infinitesimal time introduced only to express the fact that the disturbance commences just after $t = 0$ and $\delta(t-\varepsilon)$ is the Dirac delta function. Substituting (2.4) and (2.5) into (2.3), we see that the response in the mean density, $\Delta\rho_0$, to a sudden reduction in the core volume is given by

$$\frac{\Delta\rho_0(t)}{\rho_0} = \frac{\Delta V_0}{f V}\left(1 - F(t)\right) \tag{2.6}$$

where the limit $\varepsilon \to 0$ was taken after integration. It follows immediately from (2.6) that if we are able to find the response $\Delta\rho_0$ for a sudden volume change, we shall automatically obtain also $F(t)$.

The prediction of $\Delta\rho_0(t)$ is a problem which can be treated entirely on a pictorial basis, since the response of the coolant to a sudden compression is very easy to visualize. Immediately after compression occurs, no coolant has had time to escape and the density is merely raised by an amount given by equation (2.1); i.e.

$$\frac{\Delta\rho_0}{\rho_0} = \frac{\Delta V_0}{f V} \qquad \text{for} \quad t = 0 \tag{2.7}$$

At the ends of the compressed length, however, there exist sharp discontinuities of density and pressure and it is clear that, from the moment these discontinuities are formed, they will behave like wave fronts propagating with the normal velocity c of acoustic waves. The situation may best be seen by referring to figure 2. In diagramme (a) of this figure is shown the initial distribution of density along the compressed length ℓ. Inside this length the density is everywhere equal to $\rho_0 + \Delta\rho_0(0)$ while outside, it has the unperturbed value ρ_0. Diagramme (b) shows the situation existing some time later when the discontinuities formed at $t = 0$ have divided into four wave fronts of amplitude $\Delta\rho_0(0)/2$. Two of these fronts propagate towards the mid-point, O, of the compressed zone and the other two propagate outwards. If τ is the transit time of waves along the compressed length, i.e.

$$\tau = \frac{\ell}{c} \tag{2.8}$$

then the situation in diagramme (b) applies only during the time interval $0 < t < \tau/2$. At $t = \tau/2$, the ingoing waves interact at the central point O and are transformed into a pair outgoing reflected waves which remain inside the compressed length only during the interval $\tau/2 < t < \tau$. This situation is shown in diagramme (c) of figure 2. When $t > \tau$, all wave fronts have left the compressed zone (diagramme (d)), the density there being restored to its undisturbed value ρ_0, and we can assume, with some accuracy in the present context, that these fronts are subsequently dissipated in the body of the external coolant circuit.

It is quite self evident from the above description that the mean density in the compressed zone will fall linearly with time according to the equation:

$$\Delta\rho_0(t) = \begin{cases} \Delta\rho_0(0)\left(1 - \frac{t}{\tau}\right) & \text{for } 0 \leqslant t \leqslant \tau \\ 0 & \text{for } t \geqslant \tau \end{cases}$$

Substituting for $\Delta \rho_0(0)$ from equation (2.7) and comparing the result with (2.6), we get immediately the required formula for F(t):

$$F(t) = \begin{cases} \frac{t}{\tau} & \text{for} \quad 0 \leq t \leq \tau \\ 1 & \text{for} \quad t \geq \tau \end{cases} \tag{2.9}$$

Thus, the expression (2.3) for the response of the mean density to a quite general volume contraction, $\Delta V(t)$, is seen to be remarkably simple.

In order to study the slowing down of the colliding fragment and evaluate the resulting core volume change $\Delta V(t)$ explicitly (on the assumption of no stresses other than those in the coolant), it remains only to write down the relationship between the mean pressure, p, "seen" by the fragment and the mean density change, $\Delta \rho(t)$. Since the theory as developed so far is restricted to a linear approximation, it suffices for this purpose to assume a normal elastic law:

$$p = \kappa \frac{\Delta \rho}{\rho_0} \tag{2.10}$$

According to the data of Bridgman[7] for sodium, such a law holds quite well for all pressures up to about 4×10^4 atmospheres, κ being equal to about 7×10^{10} dynes/cm^2.

Let us now write down the equation of motion of the fragment (mass m, initial velocity v_0) by assuming that the force of the collision is applied over a known area A (one dimension of which is the compressed length). If v is the instantaneous velocity of the fragment, then:

$$m\dot{v} = -Ap$$

and, by geometrical considerations:

$$\Delta\dot{V} = Av$$

Combining these equations with (2.3) and (2.10) and writing

$$y(t) = \frac{\Delta V(t)}{V} \tag{2.11}$$

for the relative volume change, the equation of motion becomes

$$\ddot{y}(t) = -\omega^2 \left\{ y(t) - \int_0^t F(t')\, y(t-t')\, dt' \right\} \tag{2.12}$$

where

$$\omega = \sqrt{\frac{\kappa A^2}{mfV}} \tag{2.13}$$

is the natural frequency of core compression-decompression in the absence of coolant ejection and the boundary conditions are:

$$y(0) = 0 \tag{2.14}$$

which says that the core is initially undeformed and

$$\dot{y}(0) = \frac{A v_0}{V} \tag{2.15}$$

which says that the initial velocity of the fragment is v_0 .

The equation of motion (2.12) can be transformed from an integro-differential equation into a purely differential equation of third order. By applying the method of partial integration and using equations (2.9) and (2.14), the time integral becomes

$$\int_0^t \frac{dF(t')}{dt'}\, y(t-t')\, dt'$$

If the whole equation is then differentiated once with respect to t and further use made of partial integration and equations (2.9) and (2.14), it is easy to show that

$$\dddot{y}(t) + \omega^2 \dot{y}(t) = \begin{cases} \frac{\omega^2}{\tau} y(t) & \text{if} \quad t \leqslant \tau \\ \\ \frac{\omega^2}{\tau}\left(y(t) - y(t-\tau)\right) & \text{if} \quad t \geqslant \tau \end{cases} \tag{2.16}$$

It will be somewhat more convenient for the numerical solution of this equation as well as informative from the physical viewpoint to introduce the dimensionless variable

$$x = \omega t \tag{2.17}$$

and to regard y as a function of this variable rather than the time. This procedure modifies equation (2.16) so that it reads:

$$\frac{d^3 y(x)}{dx^3} + \frac{dy(x)}{dx} = \begin{cases} \frac{1}{\omega\tau} \; y(x) & \text{if } x \leqslant \omega\tau \\ \\ \frac{1}{\omega\tau}\left(y(x) - y(x-\omega\tau)\right) & \text{if } x \geqslant \omega\tau \end{cases} \tag{2.18}$$

The boundary conditions to this equation follow immediately from (2.12), (2.14), (2.15) and (2.17):

$$\left.\begin{aligned} y &= 0 \\ \frac{dy}{dx} &= \frac{A v_0}{\omega V} \\ \frac{d^2 y}{dx^2} &= 0 \end{aligned}\right\} \quad \text{at} \quad x = 0 \tag{2.19}$$

From equation (2.18), we see that if the coolant ejection time τ is much greater than the natural period $1/\omega$ of core compression-decompression in the absence of ejection, i.e. if

$$\omega\tau \gg 1$$

then, for times up to the order of magnitude of $1/\omega$:

$$y = \frac{Av_0}{\omega V} \sin \omega t$$

and the collision process is insensitive to the ejection of coolant. On the other hand, if

$$\omega\tau \ll 1$$

then coolant expulsion occurs with such relative promptness that the colliding fragment retains its initial velocity v_0 for times of at least $O(1/\omega)$, during which:

$$y = \frac{Av_0}{V} t$$

These limiting cases are physically obvious and the only information which we have obtained from the theory are the conditions ($\omega\tau \gg 1$ or $\omega\tau \ll 1$) under which they apply. It is now clear, however, that the region of greatest interest for the study of the collision process is the region $\omega\tau \sim 1$. This is true not only from the purely "academic" viewpoint, but also for firm practical reasons: the circumstances surrounding a hypothetical fracture of the SORA pulsation arm and its collision with the core lead to values of $\omega\tau$ in the range $0.1 \lessapprox \omega\tau \lessapprox 2.0$. In order to assess the behaviour of the core volume in this range, it is necessary to obtain the full solution to equation (2.18).

This solution has been obtained numerically with the aid of a computer programme (see appendix) written for the purpose. Since (2.18) is linear, the solution is essentially unaffected by the initial derivative and the numerical analysis uses the simple boundary conditions

$$\left.\begin{aligned} y &= 0 \\ \frac{dy}{dx} &= 1 \\ \frac{d^2y}{dx^2} &= 0 \end{aligned}\right\} \quad \text{at} \quad x = 0 \tag{2.20}$$

instead of (2.19).

Figures 3,4 and 5 show some typical results. Although the solution is not interesting for the SORA accident studies beyond the point where d^2y/dx^2 changes sign (i.e. where the fragment "bounces" off the core) it has nevertheless been plotted over many cycles. Curves are given for $\omega\tau$ = 0.6, 1.0, 2, 2.5, 4.5, 5 and 10 and we see that in all cases, $y(x)$ is a damped oscillatory function converging to some limit $y(\infty)$.

The limit $y(\infty)$ can actually be evaluated analytically in a very simple manner by Laplace transforming equation (2.18) and applying boundary conditions (2.20). This procedure gives the general solution:

$$y(x) = \frac{1}{2\pi i} \int_{\Gamma} \frac{e^{px}\, p\, dp}{p^3 + p - \frac{1}{\omega\tau}(1 - e^{-\omega\tau p})} \tag{2.21}$$

where Γ is the usual path running parallel to the imaginary axis and lying to the right of all poles. Since $y(x)$ is bounded, the asymptotic value reached as $x \to \infty$ is given by the residue of the pole at $p = 0$. Taking $p \to 0$ in the integrand of (2.21), expanding $e^{-\omega\tau p}$ in a power series and neglecting all powers of p above the second, we get

$$y(\infty) = \frac{2/\omega\tau}{2\pi i} \lim_{x \to \infty} \int_{\Gamma} \frac{e^{px}\, dp}{p}$$

i.e.

$$y(\infty) = \frac{2}{\omega\tau} \tag{2.22}$$

The numerical results all conform to this expression.

On examination of the first maximum in each of the solutions $y(x)$ in figures 3,4 and 5, it will be noted that the effect of coolant ejection during the collision is to enhance the amplitude of the change in core volume by some factor $F(\omega\tau)$ which depends only on $\omega\tau$. The function $F(\omega\tau)$ has been calculated and is plotted in figure 6.

Figures 3,4 and 5 also indicate that, as $\omega\tau$ decreases, the tendency of the solution to oscillate weakens. It was found that for $\omega\tau \lessapprox 1.75$ the oscillations vanish altogether and $y(x)$ goes directly to its limit $y(\infty)$. Thus, for $\omega\tau \lessapprox 1.75$, the enhancement factor is given by (2.22):

$$F(\omega\tau) = \frac{2}{\omega\tau} \tag{2.23}$$

This behaviour in the solution corresponds to a collision in which the fragment strikes and compresses the core but does not "bounce", the core being left with a permanent deformation. Such a situation is purely academic since, in practice, forces other than that due to coolant compression come into play and ensure that the "bouncing" phenomenon occurs. Nevertheless, it is always possible to use the function $F(\omega\tau)$ as a measure of the effect of coolant ejection in a complete model of the collision process and this has been done elsewhere[8].

APPENDIX

(Reinder Jaarsma)

Solution of Equation (2.18) for Any Value of $\omega\tau$

(a) Times within the wave transit time: $x \leqslant \omega\tau$

For $x \leqslant \omega\tau$, the delay term in equation (2.18) is not operative and the solution can be derived in a simple analytical form by writing

$$y(x) = K_0 e^{rx} \tag{A1}$$

Substitution of this formula into (2.18) gives the following cubic equation for the parameter r:

$$r^3 + r - \frac{1}{\omega\tau} = 0 \tag{A2}$$

of which the roots r_1, r_2 and r_3 are given by

$$\left.\begin{aligned} r_1 &= a \\ r_2 &= -\frac{a}{2} + ib \\ r_3 &= -\frac{a}{2} - ib \end{aligned}\right\} \tag{A3}$$

where

$$a = \left[\left(\frac{1}{(2\omega\tau)^2} + \frac{1}{27}\right)^{1/2} + \frac{1}{2\omega\tau}\right]^{1/3} - \left[\left(\frac{1}{(2\omega\tau)^2} + \frac{1}{27}\right)^{1/2} - \frac{1}{2\omega\tau}\right]^{1/3} \tag{A4}$$

and

$$b = \left(1 + \frac{3a^2}{4}\right)^{1/2} \tag{A5}$$

The general solution of equation (2.18) for $x \leq \omega\tau$ is given by

$$y = K_1 e^{r_1 x} + K_2 e^{r_2 x} + K_3 e^{r_3 x}$$

i.e. using (A3)

$$y = Je^{ax} + e^{-\frac{1}{2}ax}(K\cos bx + L\sin bx) \tag{A6}$$

where the constants J, K and L (which are related to K_1, K_2 and K_3) are to be determined from the three boundary conditions (2.20). We obtain

$$\left.\begin{aligned} J = -K &= \frac{a}{3a^2+1} \\ L &= \frac{3a^2+2}{2b(3a^2+1)} \end{aligned}\right\} \tag{A7}$$

(b) Times greater than the wave transit time: $x \geq \omega\tau$

In this region, the solution is most conveniently obtained by a direct numerical method. Hence the Runga-Kutta method in the form very conveniently formulated by Zurmühl [9] has been used. The flow chart (see below) of the computer programme designed to carry out the necessary arithmetic embodies the essential steps of Zurmühl's formulation. The function y is evaluated, step by step, at a sequence of points $x_m = x_o + m\Delta x$, Δx being the step interval and m = 0, 1, 2, 3 .... In order that the finite difference representation shall give a good approximation to the differential equation (2.18), Δx must be made sufficiently small. The value of the delay terms $y(x_m - \omega\tau)$ at the points $x = x_m$ is calculated from equation (A6) as long as $(x - \omega\tau) \leq \omega\tau$, but when $(x - \omega\tau) > \omega\tau$, $y(x_m - \omega\tau)$ is taken from the previously stored numerical solution. In order to evaluate $Y_{n+\frac{1}{2}}$, the value of y in the middle of a step interval (as required in the Runga-Kutta method), the following cubic interpolation formula is used:

$$Y_{n+\frac{1}{2}} = \frac{1}{16}\left(-Y_{n-1} + 9Y_n + 9Y_{n+1} - Y_{n+2}\right)$$

The values of $y'(\omega\tau)$ and $y''(\omega\tau)$ are calculated from equation (A6).

The error in the Runga-Kutta method used here is of the order of $(\Delta x)^5$ and the results have been calculated with $\Delta x = 0.025$. By applying the numerical procedure to the region $x < \omega\tau$ and comparing the result with the known analytical solution (A6) in the same region, this value of Δx has been found to give very good accuracy. Another check is provided by the known asymptotic behaviour $y \to \frac{2}{\omega\tau}$ as $x \to \infty$.

The computer flow chart on the next page explains all steps in the numerical procedure for evaluating y(x). It is necessary only to explain the notation:

$y'''(x) = -y'(x) + \frac{1}{\omega\tau}\left[y(x) - y(x - \omega\tau)\right]$ is denoted by $f(x,y,y')$,

$h = \Delta x$, the step length,

$v' = y'h$,

$v'' = y''\frac{h^2}{2!}$,

$K_j = y'''\frac{h^3}{3!} = f(x,y,y')\frac{h^3}{6}$.

Intermediate results are denoted by y_c, y'_c, v'_c, v''_c.

For further explanation, the reader is referred to the book of Zurmühl[9].

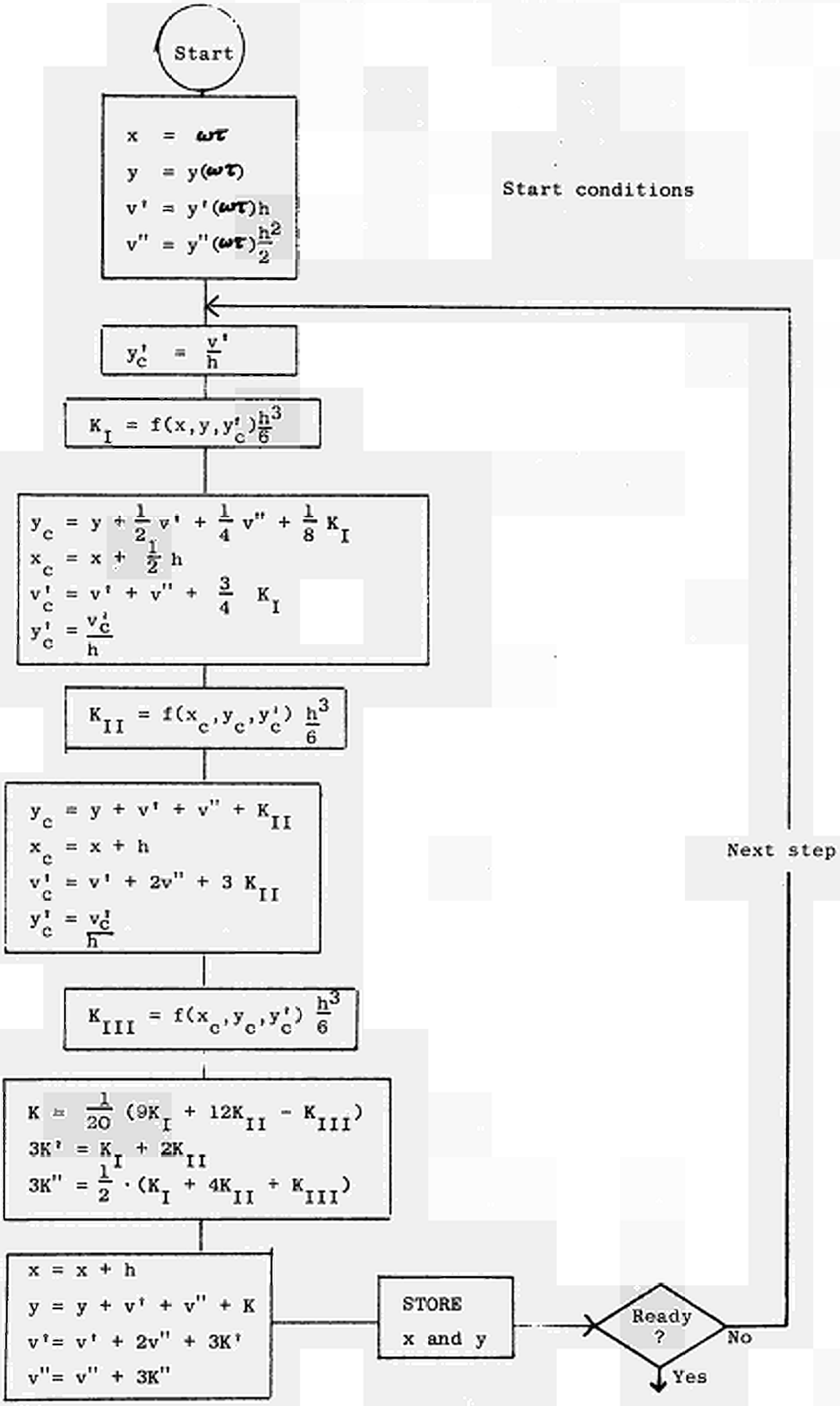

Start
$x = \omega\tau$
$y = y(\omega\tau)$
$v' = y'(\omega\tau)h$
$v'' = y''(\omega\tau)\frac{h^2}{2}$
Start conditions
$y_c' = \frac{v'}{h}$
$K_I = f(x, y, y_c')\frac{h^3}{6}$
$y_c = y + \frac{1}{2}v' + \frac{1}{4}v'' + \frac{1}{8}K_I$
$x_c = x + \frac{1}{2}h$
$v_c' = v' + v'' + \frac{3}{4}K_I$
$y_c' = \frac{v_c'}{h}$
$K_{II} = f(x_c, y_c, y_c')\frac{h^3}{6}$
$y_c = y + v' + v'' + K_{II}$
$x_c = x + h$
$v_c' = v' + 2v'' + 3K_{II}$
$y_c' = \frac{v_c'}{h}$
$K_{III} = f(x_c, y_c, y_c')\frac{h^3}{6}$
$K = \frac{1}{20}(9K_I + 12K_{II} - K_{III})$
$3K' = K_I + 2K_{II}$
$3K'' = \frac{1}{2}\cdot(K_I + 4K_{II} + K_{III})$
$x = x + h$
$y = y + v' + v'' + K$
$v' = v' + 2v'' + 3K'$
$v'' = v'' + 3K''$
STORE
x and y
Ready ?
No
Yes
Next step

Figure 1

CONFIGURATION OF FUEL SLUG, CLADDING AND SUPPORTING STRUCTURE IN THE SORA CORE

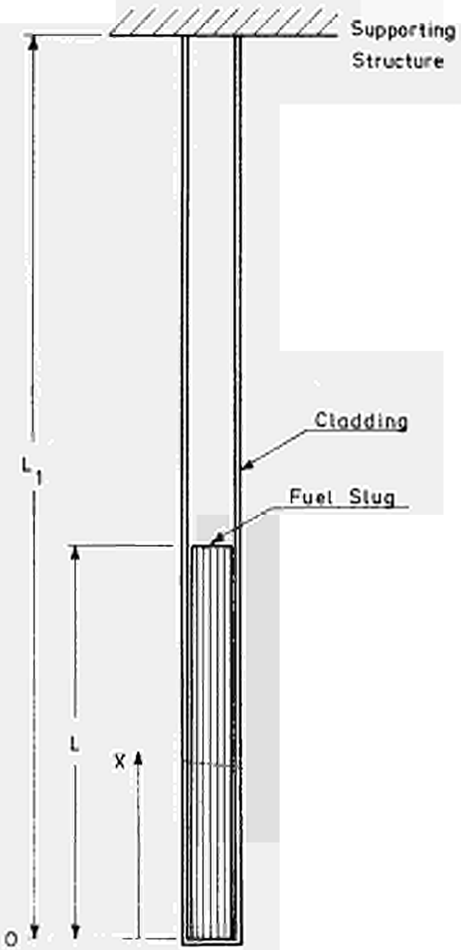
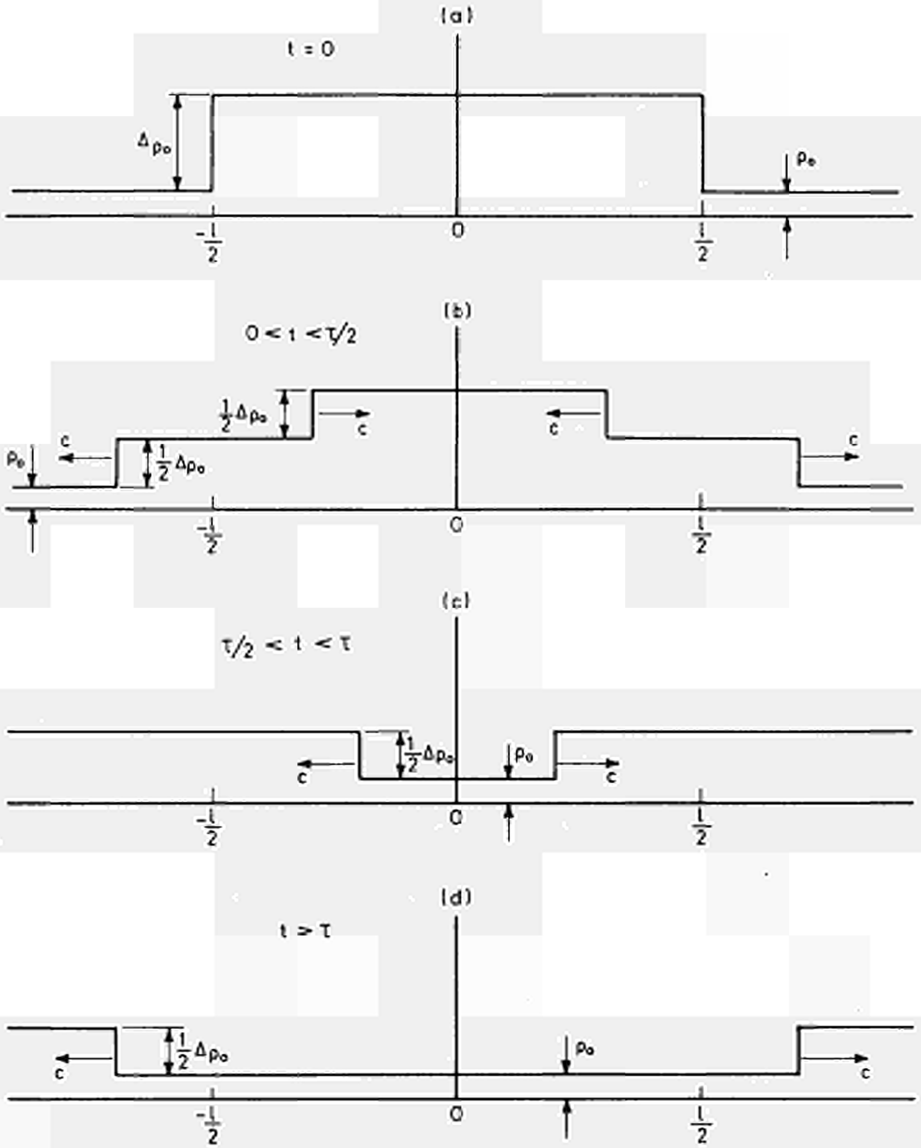


Figure 2

PROPAGATION OF COMPRESSION WAVES IN THE COOLANT DUE TO A HYPOTHETICAL SUDDEN CHANGE IN CORE VOLUME



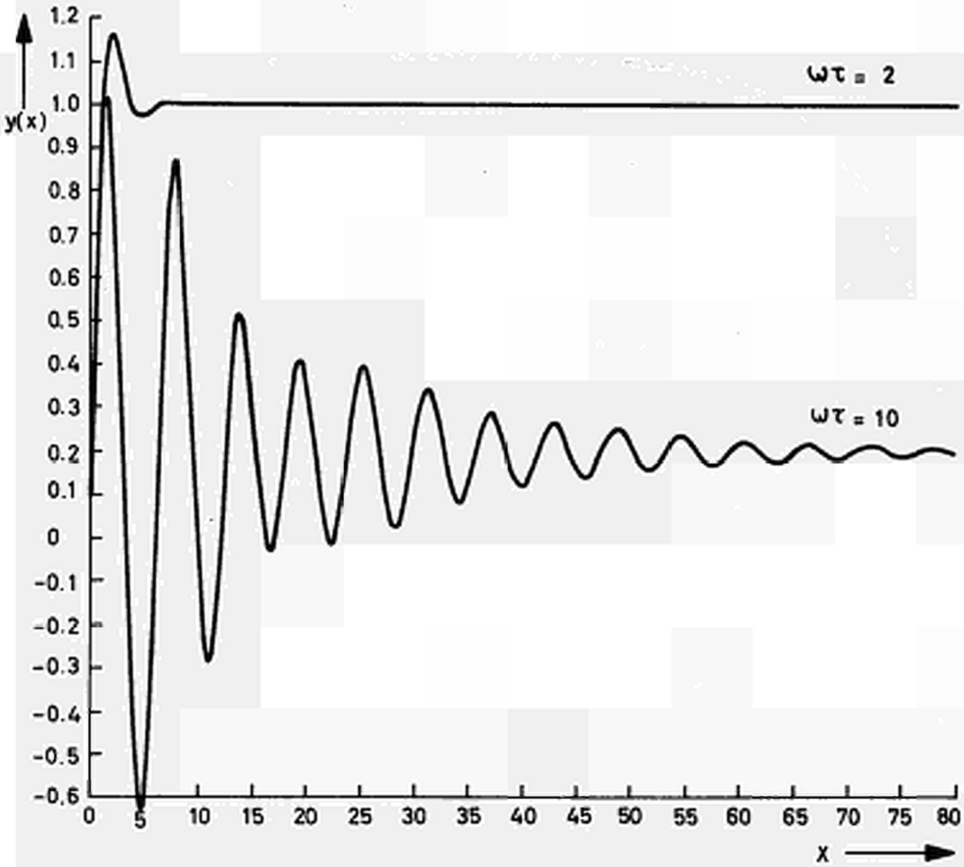


FIGURE 3 - SOLUTIONS FOR $\omega\tau = 2.0$ AND 10.0

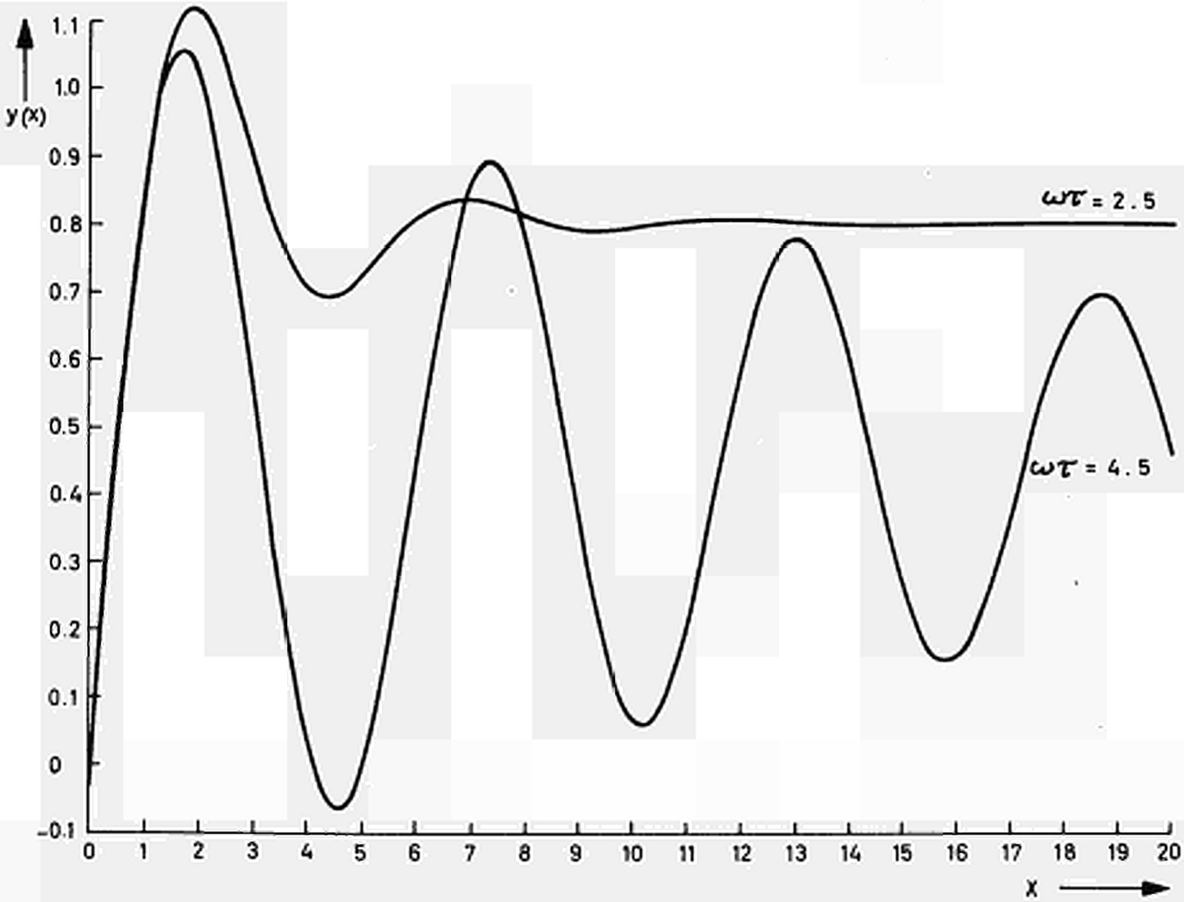


FIGURE 4 - SOLUTIONS FOR $\omega\tau = 2.5$ AND 4.5

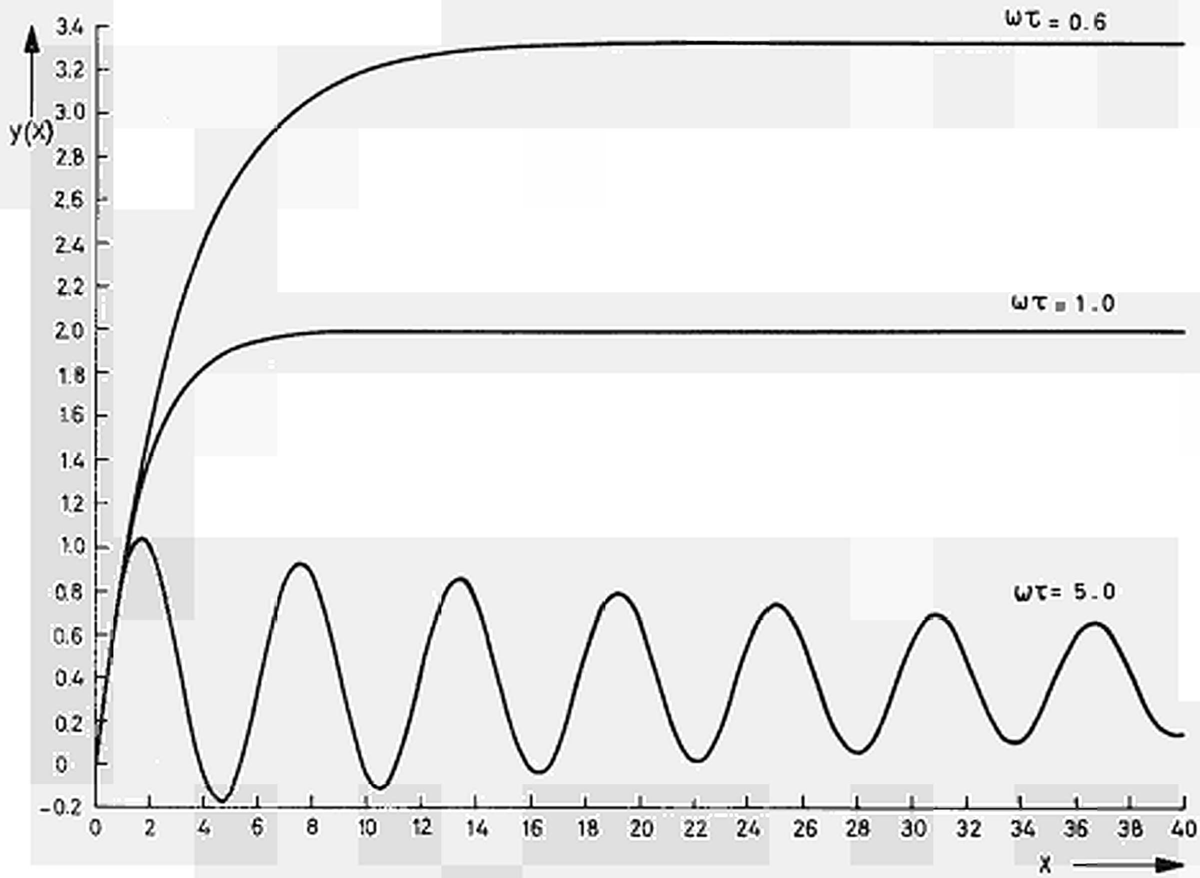


FIGURE 5 - SOLUTIONS FOR $\omega\tau = 0.6, 1.0$ AND 5.0

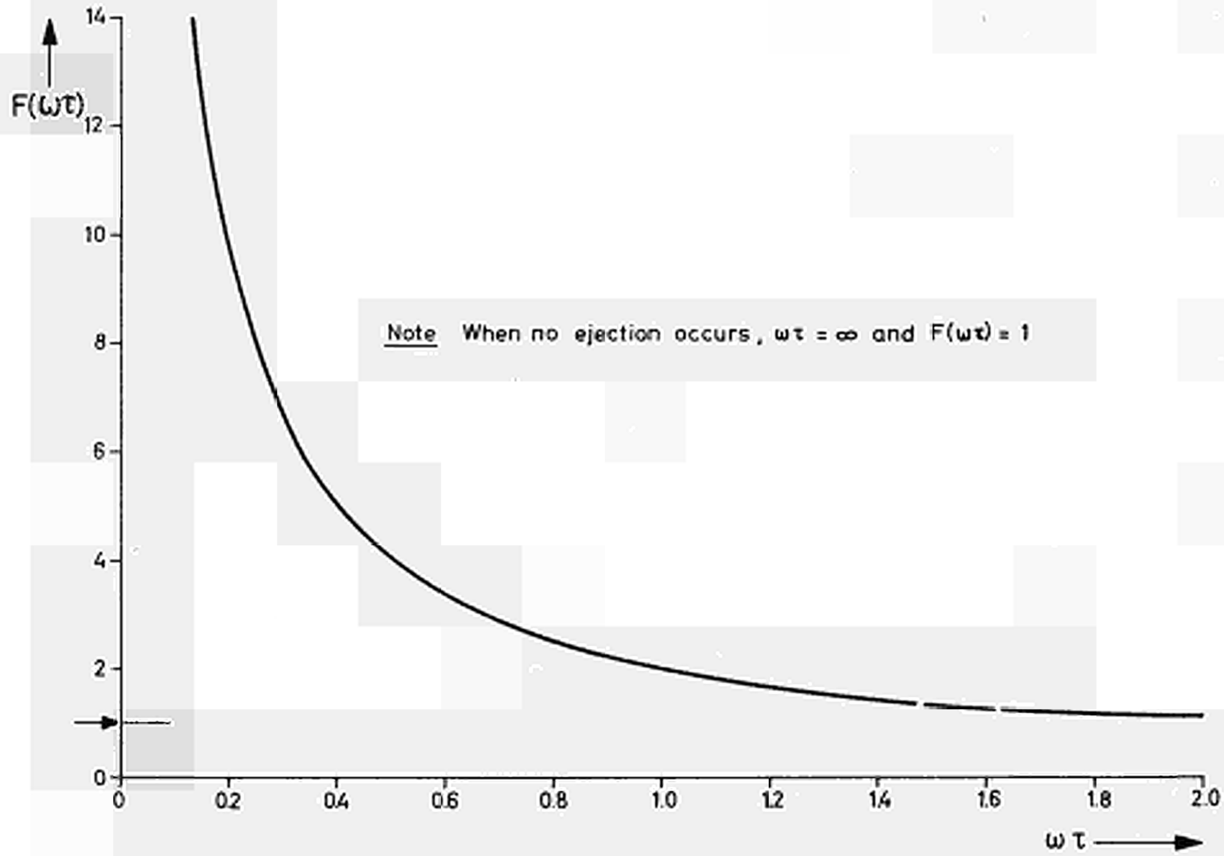


FIG. 6 ENHANCEMENT OF VOLUME CHANGE DUE TO COOLANT EJECTION

REFERENCES

1. Larrimore, J.A. et al., "The SORA reactor: Design Status Report", AEC-ENEA Seminar on Intense Neutron Sources, Santa Fe, 1966.

2. Raievski, V. et al, "The Pulsed Fast Reactor as a Source for Pulsed Neutron Experiments", paper SM62/34, IAEA conference, Karlsruhe, May 1965.

3. Randles, J., "Feedback due to Elastic Waves and Doppler Coefficient during the Excursions of a Pulsed Fast Reactor", Jour. of Nuclear Energy A/B, 20, 1, 1966.

4. Randles, J., "Accident and Self-Regulation Studies of Pulsed Fast Reactors", Jour. of Nuclear Energy A/B, 20, 713, 1966.

5. Michaels, J.E., "Thermally Induced Elastic Wave Propagation in Slender Bars", Proc. of the 3'rd U.S. National Congress of Applied Mechanics, Brown Univ., 1958.

6. Love, A.E.H., "A Treatise on the Mathematical Theory of Elasticity", 4'th edition, Dover Publications.

7. Bridgman, P.W., Reviews of Modern Physics, 1946

8. Randles, J., "Theoretical Analysis of Hypothetical Destructive Accidents in a Pulsed Fast Reactor", paper SM-101/46, Symp. on Fast React. Phys. and Safety, Karlsruhe, 1967.

9. Zurmühl, R., "Praktische Mathematik für Ingenieure und Physiker", Ch. 3.6, page 429, 3rd ed., Springer Verlag, 1961.

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Alfred Nobel

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