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SABINE
A ONE DIMENSIONAL BULK SHIELDING PROGRAM

by

C. PONTI*, H. PREUSCH** and H. SCHUBART***

* Euratom
** A.I.V. Büro Darmstadt
*** Technische Hochschule Hannover

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The removal diffusion model has been applied ; particular attention
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Neutron fluxes for 26 energy groups and gamma fluxes for 7 groups are calculated in plane, cylindrical and spherical geometry.

Experimentally determined removal c.s. are used for the more important shielding materials.

Gamma sources include the radiation emitted either by fission or neutron capture or inelastic neutron scattering. Three forms of region dependent build-up-factors may be used to determine the gamma fluxes : build-up-factors for 6 materials have been calculated with the BIGGI 3 gamma transport program.

Other quantities calculated by SABINE may be any neutron response function, gamma dose and energy deposition.

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EUROPEAN ATOMIC ENERGY COMMUNITY - EURATOM

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SUMMARY

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KEYWORDS

CONTENTS

S A B I N E
A ONE DIMENSIONAL BULK SHIELDING PROGRAM[(+)]

INTRODUCTION

This report describes the physical foundations, the mathematical methods, the structure and the use of the program Sabine.

The code is the result of an effort of formulating and applying the Removal-Diffusion model in a way as accurate as possible, of solving a wide class of shielding problems, taking into account several possible geometries of the source and of the shielding regions, and of providing the maximum amount of information concerning neutron and gamma penetration, heat deposition, or reaction rates.

The Removal-Diffusion model is a way of solving neutron penetration problems suggested about ten years ago [1]: it has been applied with more or less refiniments in several programs for shielding calculations [2, 3, 4]. On the basis of the experience made up to now on these programs, the authors think that this method of solution is satisfactory and efficient, at least when applied to massive hydrogeneous shields, and that the recourse to more sophisticated methods, does not give generally an increase in accuracy such as to compensate for the greater cost.

An experimental program of removal c. s. measurements, developed in connection with the Padova University [5], has provided the basic data; on the other hand special care was taken for applying the Removal-Diffusion method in a way as coherent and general as possible.

SABINE is a Fortran IV program for IBM 7090 or 360. It calculates the following quantities as functions of the distance from the core boundary:

a) Neutron fluxes for up to 35 groups

b). Total neutron dose rate

c) An integral over energy of the total neutron flux times

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an arbitrary response function, e.g. reaction rate, activation, etc.

d) Gamma fluxes for up to 7 energy groups, separating the contributions of the different source regions.

e) Gamma heating and dose rate.

The gamma flux is obtained as the product of the uncollided flux times a region dependent build-up-factor, which is interpolated from a proper table of values. Tables of build-up-factors for several materials and gamma groups have been calculated by the BIGGI 3 [6] gamma transport code.

The development of the program SABINE has been the object of a collaboration between the Shielding Group of EURATOM-Ispra, the A.E.G. Kernenergieanlagen in Frankfurt/Main, the A.I.V. BÜRO in Darmstadt, and the "Arbeitsgruppe für bautechnischen Strahlenschutz der T.H.Hannover".
The program is available through the E.N.E.A. Computer Program Library.

1. DEFINITION OF THE PROBLEM.

1.1 Sources of Radiation

The program SABINE determines the energy dependent neutron and gamma fluxes through a shield assembly composed of 1 to 20 homogeneous regions, which surround a source (core) composed of two regions.

The neutron source is a fission density distribution inside the 2 core regions; the gamma source is the sum of 3 terms:

a) gamma radiation emitted by fission and fission products at equilibrium,

b) neutron capture gamma rays,

c) radiation from inelastic neutron scattering.

In the core regions the gamma source may take into account all the 3 terms; in the shield, only items b and c are considered.

1.2 Geometry

The geometry of the core is defined by an index IGRC which may take four values, corresponding to the following possibilities:

Table 1

IGRC	Geometry of the core
0	Infinite plane slab
1	Finite or infinite cylinder radiating in radial direction
2	Sphere
3	Finite cylinder radiating along its axial direction (Disk geometry)

In the different cases, the two core regions may be respectively: 2 plane slabs, a cylinder surrounded by a coaxial cylindrical annulus of equal or unequal height, a sphere surrounded by a spherical shell, two coaxial cylinders with equal or unequal radii (Fig. 1). This last case is for instance that of the axial shield of a cylindrical reactor. A particular feature of the SABINE program is that the geometry of the shield can be approximated in different ways for the different calculations to be performed. This fact has mainly two reasons: with the aim of saving machine time, the real geometry can be approximated with a simpler one for a particular type of calculation for which this implies tolerable errors; furthermore, if our interest is focused over a given quantity or region of the assembly, we can choose the geometrical representation which is more convenient for that.

In what follows we will call "primary gamma" the radiation originated from sources inside the core, and "secondary gamma" the radiation produced by the gamma sources inside the shield; besides we note that in the frame of our model, the primary gamma flux and the removal neutron flux obey to equations which are formally equal.

The shield geometry for the different possible calculations is defined by the following indexes:

IGRS: for the Removal neutrons and primary gamma fluxes
IGDS: for the solution of the Diffusion equation
IGSS: for the calculation of the Secondary gamma fluxes.

These indexes may take the values:

0 : the shield regions are plane slabs
1 : the shield regions are cylindrical annuli
2 : the shield regions are spherical shells

Furthermore the case IGSS=3 is possible: in this case the shield regions are assumed to be cylinders, radiating along
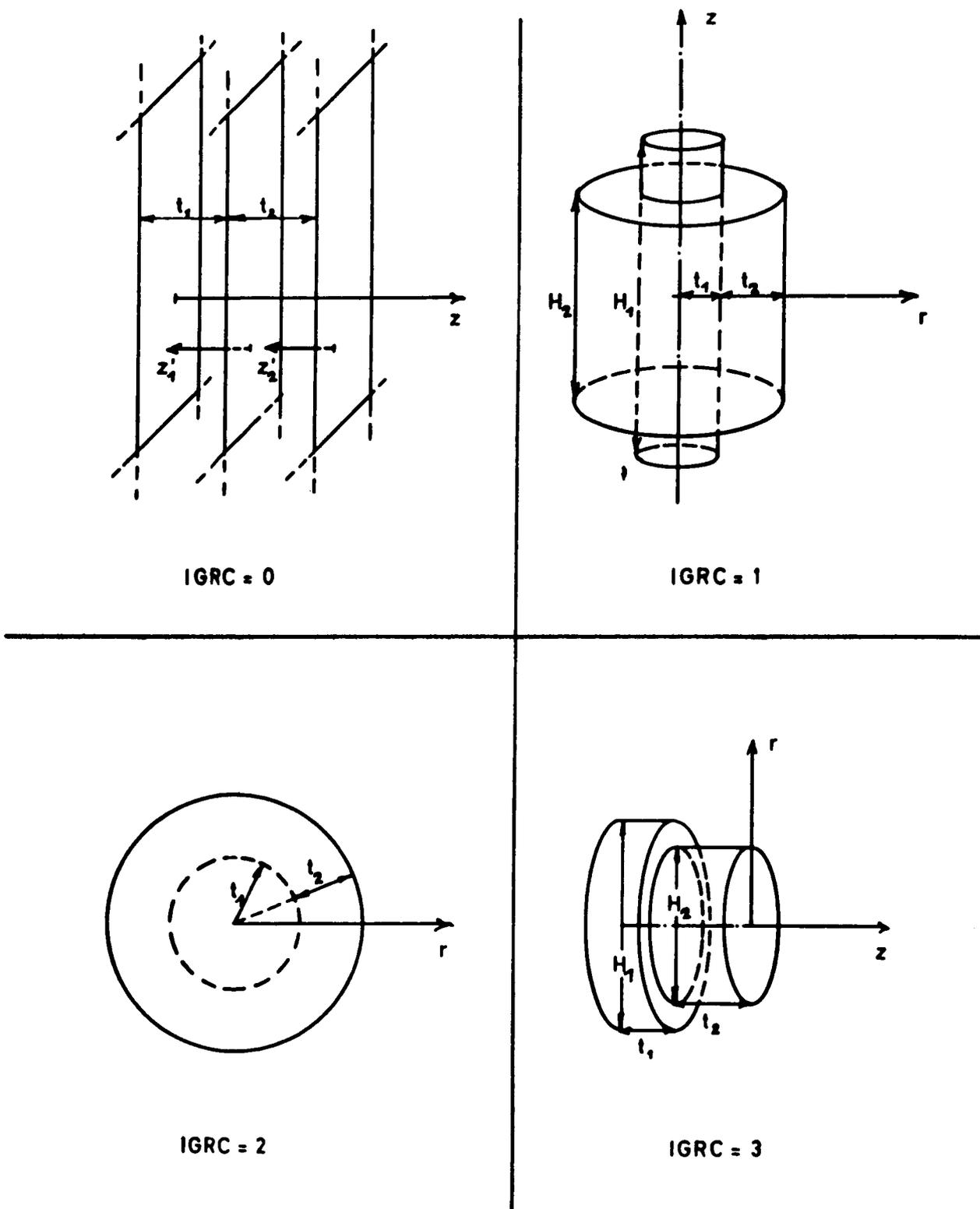
Fig. 1   Possible core geometries; neutron and gamma fluxes are calculated along the horizontal axis.

their axis, similarly to the case IGRC=3 (Fig. 1); this
situation will be called briefly "disk" geometry. Most
of the running time needed by SABINE is spent in calcula-
ting the removal neutron fluxes and the gamma fluxes, and
these calculations are more time consuming for spherical
and mainly for cylindrical geometry: when possible the
indexes IGRS=0 and IGSS=0 or 3 should be preferred.
Table 2 summarizes the possible combinations of the geome-
trical indexes; the index IGDS is not dependent upon the
others, and may be quite arbitrary. It happens frequently
that one has to solve problems for which a "disk" geometry
is preferable for the removal flux calculation and a sphe-
rical geometry for the solution of the diffusion equation.

Table 2

Combination n°	IGRC	IGRS	IGSS
1	0	0	0
✱ 2	1	0	0
✱ 3	1	0	3
4	1	1	0
5	1	1	1
6	1	1	3
✱ 7	2	0	0
✱ 8	2	0	3
9	2	2	2
10	2	2	0
11	2	2	3
12	3	0	0
13	3	0	3

In the cases indicated with *, the outer surface of the core and the inner surface of the shield are not coincident, but only tangent in one point or one line: the space between them is assumed to be filled with the material of the first shielding region, but no gamma sources are considered there.

1.3 The Shield Regions

The shield is composed of 1 to 20 regions numerated from 3 on: regions 1 and 2 are the core regions. Any region may contain a number of elements (or isotopes or compounds described in the library) smaller or equal to 10. Among the shielding regions there can be air gaps: inside the gaps the diffusion equation is not solved (see section 2.8) and no source of secondary gamma radiation is considered there. The first and last region of the shield must not be a gap; two air gaps must not be side by side.

2. CALCULATION OF THE NEUTRON FLUX.

2.1 Neutron Energy Group Structure

In what follows reference is made for sake of simplicity
to the group structure of the neutron data library prepa-
red for SABINE, that is presently in use; however the ar-
guments can be easily generalized for a different choice
of the energy groups.
The energy limits as well as other details concerning the
neutron groups are given in Table 3.
The energy range between 0.5 and 18 Mev has been divided
into 19 removal groups, having roughly constant energy
width; the calculation of the total neutron flux is per-
formed in a 26 groups scheme, that covers the energy range
between 0 and about 15 Mev, with lethargy intervals of 0.5
- 1. The number of these groups and their energy range have
been chosen mainly on the basis of the following conside-
rations:

1) The slowing down length of each group must be smaller
   than the relaxation length of the penetrating component,
   described by the removal flux.

2) The energy width of the groups should be narrow enough,
   in order that the dependence of the group averaged c.s.
   upon the weighting spectrum be not important.

3) The lethargy mesh should be more fine in the fast region
   than elsewhere because: a) this is the most important
   part of the spectrum to be determined; b) it is general-
   ly the most penetrating component; c) the presence of
   the inelastic scattering implies a detailed treatment
   of the transfer matrix.

4) The running time for the calculation of the neutron
   diffusion and slowing down in SABINE, using the maximum
   number of groups ,35, allowed by the program, is about
   the same as the time needed for the removal fluxes cal-

TABLE 3

Energy Structure of the Neutron Groups in the Present Library

i	E_H	Δu		n	E_H (MeV)	χ_i
0	18. MeV			1	18.	3.359,-6
				2	16.5	1.348,-5
1	14.918	.9		3	14.918	2.062,-5
				4	14.	5.080,-5
				5	13.	1.160,-4
				6	12.	2.630,-4
				7	11.	5.89 ,-4
				8	10.	1.310,-3
				9	9.	2.86 ,-3
				10	8.	6.15 ,-3
				11	7.	1.18 ,-2
2	6.065	.5		12	6.065	2.10 ,-2
				13	5.2	3.48 ,-2
				14	4.4	5.22 ,-2
3	3.68	.5		15	3.68	7.69 ,-2
				16	3.00	1.337,-1
4	2.23	.5		17	2.23	2.310,-1
5	1.35	.5		18	1.35	1.802,-1
6	821.keV	.5		19	.821	1.147,-1 r
7	498.	.5				
8	302.	.5				
9	183.	.75				
10	86.5	1.00				
11	31.8	1.00				
12	11.7	1.00				
13	4.31	1.00				
14	1.58	1.00				
15	583. eV	1.00				
16	214.	1.00				
17	78.9	1.00				
18	29.0	1.00				
19	10.7	.75				
20	5.04	.5				
21	3.06	.5				
22	1.85	.5				
23	1.12	.5				
24	.682	.5				
25	.414	.728				
26	.200	-				

i   index of the diffusion groups

E_H upper energy of the groups

Δu lethargy width

n   removal groups' index

χ_i fraction of fission neutrons emitted into the ith group

culation, in plane geometry, and smaller for cylindrical
and spherical geometry, and hence reducing the number of
groups does not mean, in general, an important saving.
The upper limit of the highest energy group has been cho-
sen higher than necessary: it could be reduced, but not
too much if one is interested in the knowledge of the
fast spectrum.

2.2 The Source Distribution for Removal Neutrons

The program considers as neutron sources the fission neu-
trons generated inside region 1 and 2. For both of these
regions the program computes the number of neutrons $S_n(Q)$
emitted with energy corresponding to the removal group n,
per unit volume and time at the point Q, as

$$S_n(Q) = S_o \chi_n \gamma F(Q)$$

S_o = fission density at the outer edge of the region
$\quad$ (fissions/cm^3.sec)

χ_n = fraction of fission neutrons released in the nth
$\quad$ group according to the Cranberg fission spectrum.

γ = average number of neutrons per fission = 2.46

$F(Q)$= function describing the space dependence of the fis-
$\quad$ sion density in the region considered.

For the different possible geometries of the source regions
(cfr. section 1.2 and Fig.1), $F(Q)$ may have the different
forms considered in the following.

2.2.1 Plane Geometry, IGRC=0

In this case the function depends upon one variable z';
$F(Q)=F(z')$. The z'-axis has the origin on the outer face of
the region and is oriented towards the inner of the region:
we have for instance (see Fig.1)

$\quad$ in region 1 $\qquad z_1' = t_1 - Z \qquad$ (1a)

$\quad$ and in region 2 $\quad z_2' = t_1 + t_2 - Z \qquad$ (1b)

F(z') must be normalized in such a way to have the value
1 on the outer boundary of the region: F(0)=1.
The function F(z') may be specified in two ways:

a) pointwise, that is providing (M+1) values F_m at
   equidistant points:

$$F_m = F(z'_m), \quad z'_m = (m-1)\frac{t}{M}, \quad m = 1,2,\ldots,M+1$$

where t is the thickness of the region.
Necessarily

$$F_1 = 1 \quad \text{and} \quad M \leqslant 50$$

In this case the program computes the J coefficients
of the polynomial of degree (J-1) which best fits,
in the least squares sense, the given space distri-
bution.

$$F(z') \cong \sum_{j=1}^{J} a_j z'^{j-1} \tag{2}$$

A value for $J \leqslant 10$ must be precised by the user.

b) the J coefficients of the polynomial can be given
   as input data: also in this case $J \leq 10$ and $a_1 = 1$
   $(F(0) = a_1 = 1)$.

2.2.2 Cylindrical Geometry, IGRC=1

We assume in this case

$$F(Q) = h(r').g(z)$$

In region 1 and 2 we have respectively (see Fig.1)

$$\begin{aligned} r'_1 &= t_1 - r \\ r'_2 &= t_1 + t_2 - r \end{aligned} \tag{3}$$

The geometry and the function F(Q) are supposed to be
symmetrical around the z-axis and the plane z=0: g(z)=
=g(-z), so that g(z) needs to be specified only for posi-
tive z. The function g(z) is given the form (2), and can
be determined either directly through the coefficients

$a_j (a_1 = 1)$ or providing a table of values $g_m = g(z_m)$ with $z_m = (m-1)H/2M$, $m = 1, 2, \ldots, M+1$, M 50, $g_1 = 1$.

The function $h(r')$ may take one of the forms:

$$h(r') = e^{-Kr'} \tag{4a}$$

$$h(r') = \sum_{j=1}^{J} b_j \, r'^{j-1} \quad , \quad b_1 = 1 \quad , \quad J \leqslant 10 \tag{4b}$$

If the form (4a) is chosen, the value of k has to be given; if the form (4b) is chosen, the user has to provide the coefficients b_j, or a set of values $h_m = h(r'_m)$, for $M+1$ equidistant radial points, with $r'_1 = 0$ (outer surface), r'_{M+1} on the inner surface, and $h_1 = 1$: in this case the coefficients b_j are calculated by the program.

2.2.3 Spherical Geometry, IGRC=2

In this case the source distribution is again a function $h(r')$; r' has the form (3) and $h(r')$ has the same form, description and limitations as the corresponding function in cylindrical geometry.

2.2.4 Disk Geometry, IGRC=3

As explained in section 1.2, this expression means that the source regions consist of two cylinders which are shielded in the direction of their axis; in this case the distribution function is

$$F(Q) = h(r)g(z')$$

r is the distance of the point Q from the z axis, and z' is the distance between Q and the right boundary of the region as in (1).

The possibilities and restrictions for h and g are the same as described in section 2.2.2.

2.3 Removal Flux Calculation

The removal flux is calculated (see for instance ref. 1) as the flux of neutrons which have not suffered "removal" collisions.

For each of the 19 groups the contribution to the removal flux in a point P, due to a differential volume element dV around the source point Q (for isotropic source) is

$$dF^r(P) = \frac{S(Q)K(P,Q)}{4\pi\ \overline{PQ}^2}\ dV \tag{5}$$

where

$S(Q)$ = source strength in Q for the group considered ($neutron/cm^3$ sec)

$K(P,Q)= \exp\left[-\int_{PQ} \Sigma^r(s)\,ds\right]$

Σ^r = region dependent macroscopic removal c.s.

The removal flux in P is the result of the integration of eq.(5) over the source volume: this is a numerical integration which is performed as reported in Part 3. The point P may move along the r or z axis shown in Fig. 1 for the possible geometries.

After the removal fluxes and the removal collision densities have been calculated for each of the 19 groups of Table 3, namely

$$F_n^r(P) \quad \text{and} \quad F_n^r\ \Sigma_n^r\ (P), \quad n = 1,2,\ldots 19$$

these are added to get removal fluxes and collision densities corresponding to the broad groups i.

For instance for the group i=2 one has

R. flux $\qquad \phi_2^r(P) = \sum_{n=12}^{14} F_n^r(P)$

R. collision dens. $\qquad C_2^r(P) = \sum_{n=12}^{14} F_n^r(P)\Sigma_n^r(P)$

Note that at the interface between two regions the program calculates two values of the collision density, because this function is discontinuous there.

2.4 The Source Terms of the Diffusion Equations.

The coupling of the removal neutrons into the diffusion equations, through the source terms, has to be considered carefully when applying the R.D. (Removal Diffusion) model, especially if one wishes to get from this simple description of the physical reality a good estimate of the neutron spectrum.

On one side we have a R. flux describing the fast neutron penetration- which strictly refers to the empirical idea of R. c.s., and on the other the set of multigroup eqs. that describe the neutron diffusion and slowing down, within the frame of the D. approximation. The slowing down of neutrons should be accounted for through a proper transfer matrix: the same matrix will be used either for the R or for the D neutrons, despite of the fact that their spectra are in general different; however they should not be so much different as to produce important deviations in the average of the c.s. over the energy interval of the same group. As shown in table 3 the lethargy width of the fast groups is about 0.5.

The calculation of the total transfer matrix will be considered later: it will require some particular remarks, when applied to the R. flux. The following notation is used:

$\phi_i^r (z)$ removal flux of group i at the point z

$c_i^r (z)$ removal collision density for the group i at z

Σ_i^a macroscopic absorption c.s. of the ith group

Σ_{ij} macroscopic total transfer c.s. from the ith to the jth energy group.

The following balance of the neutrons removed from the ith group, per unit volume and time at z holds:

$\phi_i^r(z)\,\Sigma_i^a$ are absorbed

$\phi_i^r(z)\,\Sigma_{ii}^*$ enter as diffusion neutrons into the ith group (the meaning of the $*$ is explained below)

$\phi_i^r(z)\,\Sigma_{ij}$ go into the jth group ($j > i$)

$\phi_i^r(z)\,\Sigma_i^r = C_i^r(z)$ are removed from the group i.

It is clear that the sum the first three terms, which gives the total number of neutrons removed from group i (per unit volume and time at z) must be equal to $C_i^r(z)$, otherwise the neutron balance is not saved.

This remark may seem to be obvious, but if we think that on one side the absorption and transfer c.s. are calculated from the basic c.s. under given assumptions for the elastic and inelastic scattering, and on the other side the R.c.s. is obtained largely, on an empirical basis, with arguments quite indipendent from those which determine, the calculation of the other c.s., then we realise that generally the neutron balance is not automaticly respected, but must be explicitly imposed. The sum $\Sigma_i^a + \Sigma_{ii} + \Sigma_{i,i+1} + ..$ is the total, and not the removal c.s. This discrepancy is originated from the fact that the calculation of the transfer c.s., takes into account as usual, the elastic and inelastic scattering, and impliedly accounts also for those collisions (which do not produce important energy loss or angular deflections) entering the term Σ_{ii}, which do not remove actually the neutron from the virgin beam.

In order to be coherent with the assumptions of the model, a new set of diagonal terms Σ_{ii}^* of the transfer matrix will be calculated, imposing that the balance of the removed neutrons be saved, namely:

$$\Sigma_i^r = \Sigma_i^a + \Sigma_{ii}^* + \Sigma_{i,i+1} + \Sigma_{i,i+2}$$

or

$$\Sigma_{ii}^* = \Sigma_i^r - \left(\Sigma_i^a + \Sigma_{i,i+1} + \Sigma_{i,i+2} + ...\right) = \qquad (6)$$

$$= \Sigma_i^r - \Sigma_i^{out}$$

The other (non-diagonal) terms of the transfer matrix ac-counting for collisions that imply important energy losses, will be calculated in the usual way (section 2.5.2).

The sum of the terms enclosed in parenthesis in (6) will be indicated with Σ_i^{out} because it accounts for all those collisions which produce the loss of a neutron from the ith energy group.

It is now possible to write the source terms of the diffusion equations, considering all possible neutron transfer as indicated in Table 3; the assumption is made that in the "zero" group (above ~ 15 Mev) there are only removal neutrons, and these may be scattered only into the first group. The source term of the first diffusion equation will hence be:

$$S_1(z) = c_0^r(z) - \Sigma_0^a \, \phi_0^r(z) + \phi_1^r(z) \, \Sigma_{11}^* \qquad (7)$$

The solution of this equation will be $\phi_1^d(z)$, that is the diffusion flux of the first group and $\phi_1(z) = \phi_1^d(z) + \phi_1^r(z)$ will be the total flux of the group.

Similarly for the following groups one has (omitting the space dependence):

$$S_2 = \phi_1 \Sigma_{1,2} + \phi_2^r \, \Sigma_{22}^*$$

$$S_3 = \phi_1 \Sigma_{1,3} + \phi_2 \Sigma_{2,3} + \phi_3^r \Sigma_{33}^*$$

$$\vdots$$

$$S_6 = \phi_1 \Sigma_{1,6} + \phi_2 \Sigma_{2,6} + \dots + \phi_5 \Sigma_{5,6} + \phi_6^r \Sigma_{66}^*$$

From the 7th group on, that is below about 0.5 Mev, the R. flux is neglected:

$$i \geqslant 7 \qquad S_i = \phi_1 \Sigma_{1,i} + \phi_2 \Sigma_{2,i} + \dots + \phi_{i-1} \Sigma_{i-1,i}$$

2.5 Group Cross-Sections Library

2.5.1 Data for the Removal Flux Calculation

These are of two kinds: the fission neutrons spectrum and the energy dependent removal c.s.

The fraction of fission neutrons emitted in the nth removal group has been calculated on the basis of the Cranberg fission spectrum, and are written in Table 3.

The energy dependent removal c.s. come from different sources of information: if available, measured values have been chosen, as for H_2O, Pb, C, Fe and Al, which have been measured at the 5,5 Mev accelerator of Padova [5], where other measurements for different materials are foreseen in the next future. These data will be included in the library as soon as they will become available. For the other elements or isotopes the compilations of Greenborg [7] and Avery [2] have been used.

2.5.2 Data for the Diffusion Calculation

The following microscopic group c.s. are needed for the diffusion calculation:

σ_0^a absorption c.s. for the "zero" group to be put into eq. (7)

$\sigma_{i,j}$ total transfer c.s. from the ith to the jth group.

σ_i^{out} this c.s. accounts for all the collisions that remove the neutron from the ith group either by slowing down or by absorption; it is $\sigma_i^{out} = \sigma_i^{tot} - \sigma_{i,i}$.

σ_i^{tr} transport c.s. of the ith group.

The above quantities have been calculated for the "elements" listed in Table 4, using the General Atomic basic c.s. library, through the GGC II [8] program that is the combination of GAM [9] and GATHER [10]. GAM calculates the fast spectrum

in the B-3 approximation, and GATHER the thermal spectrum
in the B-1 approximation, for an homogeneous medium. Group
(either micro or macro-scopic) c. s. are then averaged over
the calculated (or optionally provided as input) fast and
thermal spectra.

The elastic scattering kernels are correct to sixth order
for anisotropic scattering in the C-M system; inelastic
scattering is assumed to be isotropic in the L-system. The
energy degradation by inelastic scattering is calculated
considering the excitation energies when these are known,
or using the evaporation model when they are not known.
Further details are to be found in [9,10]. As pointed out
in section 2.1 the energy width of the groups is narrow
enough, so that the group averaged c. s. do not strongly
depend on the spectrum; for the c. s. of many elements this
has been checked, but nevertheless exceptions to this rule
are possible. The weighting spectrum used hitherto is the
slowing-down spectrum due to a fission source in water.
Other data can be added to the library for the same or
different elements using an arbitrary weighting spectrum.
The program SABLIB has been prepared to read the data pun-
ched by GGC, to rearrange them, to read some other libra-
ry data (e.g. the removal c. s.) and to write them on the
Library Tape for SABINE.

Data for the 37 "elements" listed in Table 4 are now availa-
ble for the energy group structure of Table 3. Data for
other elements or different energy arrangements or weighting
spectra can be provided on request.

Table 4

Materials included in the Library and corresponding
identification numbers.

Material	Id. no.	Material	Id. no.
Hydrogen	1	Iron	26
Deuterium	2	Cobalt	27
Lithium	3	Nickel	28
Beryllium	4	Copper	29
Boron	5	Zinc	30
Carbon	6	Zirconium	40
Oxigen	8	Molybdenum	42
Sodium	11	Cadmium	48
Magnesium	12	Indium	49
Aluminum	13	Tin	50
Silicon	14	Barium	56
Phosphorus	15	Dysprosium	66
Sulfur	16	Tungsten	74
Water	18	Gold	79
Potassium	19	Lead	82
Calcium	20	Uranium 235	235
Titanium	22	Uranium 238	238
Vanadium	23		
Chromium	24		
Manganese	25		

2.6 Solution of the Diffusion Equation

Once the removal flux has been calculated, the program
computes the source terms of the multigroup diffusion
equations as explained in section 2.4. Then for each neu-
tron group i, we have to solve an equation of the fol-
lowing kind

$$D \left[\phi''(r) + \frac{P}{r} \phi'(r) \right] - \Sigma \phi(r) + S(r) = 0 \qquad (8)$$

D is the diffusion coefficient calculated as $1/3 \Sigma^{tr}$

$\Sigma = \Sigma^{out} + DB^2$, B^2 is the buckling to account roughly for a
possible transversal leakage

Σ^{tr} and Σ^{out} are evaluated for any group and region from
the microscopic c.s. of the elements which are present
in the region: the microscopic c.s. are considered in
section 2.5.2

P is a geometry index (the same as IGDS in section 1.2)

$$P = 0 \text{ means plane geometry}$$
$$P = 1 \text{ means cylindrical geometry}$$
$$P = 2 \text{ means spherical geometry}$$

One has to find the solution of eq. (8) satisfying the
following boundary conditions:

$$a_0 D \phi' + a_1 \phi + a_2 = 0 \quad \text{at the inner boundary} \quad (8a)$$
$$b_0 D \phi' + b_1 \phi + b_2 = 0 \quad \text{at the outer boundary} \quad (8b)$$

Continuity of flux and current through the internal
boundaries is assumed.

Appendix A shows how the second order linear differential
equation (8) can be replaced by the following equivalent
system of three first order differential equations

$$U' = \frac{U^2}{D} - \frac{PU}{r} - \Sigma \qquad (9a)$$

$$V' = V\left(\frac{U}{D} - \frac{P}{Z}\right) + S \qquad (9b)$$

$$D\emptyset' + U\emptyset + V = 0 \qquad (9c)$$

This system has several advantages: the function $U(r)$ can be easily solved from eq. (9a) and inserted into (9b), which is a linear first order differential equation solvable through standard formulas; the functions U and V can then be put into (9c) which is similarly solved for $\emptyset(r)$; the continuity conditions for flux and current are easily satisfied by imposing continuity to the functions $U(r)$, $V(r)$ and $\emptyset(r)$, as it is shown by eq. (9c); the boundary conditions expressed in general form by eqs. (8a) and (8b), are easily converted into boundary conditions for the functions U, V and $\emptyset$, because (8a) and (8b) are formally similar to (9c).
Actually the outer boundary condition is satisfied if we put:

$$U(R_e) = b_1/b_0 \\ V(R_e) = b_2/b_0$$ at the outer boundary R_e of the shield $\qquad (10)$

With this starting values for the functions $U(r)$ and $V(r)$, the eqs. (9a and b), can be integrated (Appendix B) proceeding from outside to inside: once the values $U(R_i)$ and $V(R_i)$ at the inner boundary of the shield are known, the quantity:

$$\emptyset(R_i) = \frac{a_2 - a_0 \, V(R_i)}{a_0 U(R_i) - a_1} \qquad (11)$$

is taken as inner boundary value for the function $\emptyset(r)$ (see below), then eq. (9c) is integrated proceeding from inside to outside, as shown in Appendix B.
Eq.(11) is the result of writing (9c) at the inner boundary, and solving the system of this equation together with eq. (8a), to obtain the boundary values of flux and current.

2.7 Boundary Conditions

It is useful to make a few more remarks about this subject. The user has to provide a proper set of coefficients a_0, a_1, a_2 and b_1, b_2 to represent the particular boundary conditions of his problems.

2.7.1 Outer Boundary

The coefficient b_0 is not allowed to be zero (that would produce non limited values of U and V in eq. 10), so that it has been fixed once for ever $b_0 = 1$; only b_1 and b_2 need to be specified to the program. As a consequence of that, it is not allowed to specify the outer value of the flux (which is assumed to be a result and not a datum of the problem). The usual condition at the outer boundary is of zero incoming current, that is

$$J_- = \frac{\phi}{4} + \frac{D\phi'}{2} = 0$$

and this corresponds to the choice $b_1 = .5$, $b_2 = 0$, $(b_0 = 1)$. Similarly a linear extrapolation distance $d = \frac{-\phi}{\phi'}$ is expressed by $b_1 = D/d$, $b_2 = 0$, being D the diffusion coefficient of the external region.

2.7.2 Inner Boundary

Here are a few examples of inner boundary conditions. If one needs to define a boundary value of the flux

$$\phi(R_i) = \phi_0, \quad \text{then the set} \quad \begin{aligned} a_0 &= 0 \\ a_1 &= -1 \\ a_2 &= \phi_0 \end{aligned}$$

into eq. (8a) realizes the required condition. Similarly one can impose:

$$\phi'(R_i)=0 \quad \text{with} \quad a_0=1, \; a_1=0, \; a_2=0$$

$$\text{or} \; -D\phi'(R_i)=J_0 \quad \text{with} \quad a_0=1, \; a_1=0, \; a_2=J_0$$

All the above boundary conditions are applied to the
d i f f u s i o n flux, that is to the solution of the
diffusion eq.; and hence in those neutron groups (1 to 6
for the scheme of Table 3) where a D. flux and a R. flux
cohabit, the total flux will satisfy different boundary
conditions. The only exception to this rule is the case
$a_0=0$, which corresponds to a given boundary value of the
flux: in this case the program treats it as a total flux;
the boundary value for the D. flux is given in such a ca-
se either by the difference total flux minus the computed
value of the R. flux at the boundary, or it is set equal
zero if the difference were negative.

2.8 Air Gaps

SABINE can take account of the effect of air gaps inside
the shield through proper conditions connectig the va-
lues of flux and current on the two sides of the gap. Let
r_1, r_2 be the position coordinates and J_1, J_2 the values
of the net current at each side of the gap: $J(r)= -D\phi'(r)$.
Two different treatments of the gap are allowed.

2.8.1 P1 approximation

If the thickness of the gap is small compared to its
transversal dimensions, and hence the transversal leakage
is negligible, then the P1 approximation [11] states the
following equations for the flux and current through the
gap:

$$\frac{J_1}{J_2} = \left(\frac{r_2}{r_1}\right)^P \tag{12a}$$

$$\frac{\phi_1}{2} - J_1 = \frac{\phi_2}{2} - J_2 \, f_P(q) \qquad q = r_1/r_2 \tag{12b}$$

where $f_p(q)$ is a function depending on the geometry:

for plane geom.   (P=1) $f_0(q) = 1$

for cylindrical geom.   (P=2) $f_1(q) = \frac{2}{\pi}\left(\frac{\arcsin q}{q} + \sqrt{1-q^2}\right)$

for spherical geom.   (P=3) $f_2(q) = 1-(1-q^2)^{3/2}$

Note that in the first case flux and current are continuous through the gap.

2.8.2 A Different (Optional) Treatment

If the air gap is thick, relations different from (12) may be required in order, for instance, to take into account the transversal leakage in plane or cylindrical geometry. SABINE can take account of discontinuity conditions for flux and current through that region, in the form

$$\phi_2 = \alpha\, \phi_1 \; ; \; \alpha > 0$$
$$J_2 = \beta\, J_1 \; ; \; \beta \neq 0$$

In this case the user has to provide a couple of values α and β for each neutron group.

The presence of gaps in the problem to be solved is subject to two restrictions: namely the first and the last region of the shield cannot be air gaps, and two gaps cannot be side by side. No more than 3 gaps of this type may be present.

2.9 Response Functions

SABINE can optionally evaluate three kinds of Response functions.

a) $R(x) = \sum_{i=i_1}^{i_2} \phi_i(x)\, f_i \quad , \; j \quad 1 \le i_1 \le i_2 \le 26$ (13)

This is the integral over energy of the total neutron
flux times an arbitrary function of energy, which has
to be specified group-wise. In this way the program can
provide for instance dose rates, reaction rates, fast
or epithermal flux (f_i=1 for the proper groups) and so
on.

b) The response function f_i may be <u>region dependent</u> (such
   as that required to evaluate for instance heat deposi-
   tion or activation): in this case the user has to pro-
   vide a table of values for each region; then SABINE
   computes the function (13) inserting for f_i the table
   of the region to which x belongs.

c) A third possibility has been devised to calculate the
   reaction rate of <u>threshold detectors.</u> These are at pre-
   sent the tool most frequently applied to get information
   on the fast region of the neutron spectrum.
   Macroscopic c.s. for such detectors may be provided for
   each of the removal groups, that are finer than the
   diffusion ones (see Table 3). The program itself calcu-
   lates the broad group c.s., by averaging over the com-
   puted removal spectrum: for any region R

$$\Sigma_{i,R} = \frac{\sum_{n \in i} \Sigma_n \phi_n^r(x_R)}{\sum_{n \in i} \phi_n^r(x_R)}$$

Σ_n is the (input) value for the c.s. in the nth remo-
val group;

$\phi_n^r(x_R)$, n=1,2,...19 is the removal spectrum in the
middle point of the Rth region.
The sums are extended to those fine groups which lie in-
side the broad group i(i=1,2,...6)
Then at any point x belonging to region R the program
evaluates

$$T(x) = \sum_{i=1}^{6} \phi_i(x) \Sigma_{i,R}$$

Note that generally this function is discontinuous at
the interfaces between two regions.

3. CALCULATION OF THE GAMMA FLUXES.

   3.1 Group Structure

   SABINE can compute the space distribution of the gamma fluxes for 7 energy groups. This number is practically fixed by the gamma data library which is used: to change this number (or the number of neutron groups) a new library needs to be provided. The library presently in use implies the seven groups scheme shown in table 5. The subscript g, $g=1$ to 7, will be used to denote gamma energy groups.

   Table 5

Gamma group	Energy limits (Mev)
1	0. – 1.0
2	1.0 – 2.0
3	2.0 – 3.0
4	3.0 – 5.0
5	5.0 – 7.0
6	7.0 – 8.5
7	8.5 – 10.0

   3.2 Gamma Sources

   Inside the core regions, namely regions 1 and 2 three kinds of sources may be considered:

   a) Gamma rays from fission
   b) Capture gamma rays
   c) Inelastic scattering gamma rays.

Item a) includes prompt gammas emitted by fission and gamma radiation by equilibrium fission products.

In the shielding regions (region 3 and following) only sources of type b) and c) are considered.

3.2.1 Gamma Sources Inside the Core

The gamma source for the gth group (energy emitted in form of photons with energy belonging to the gth interval) at the point Q of a core region is given by

$$S_g(Q)= S_{o,g}G(Q) \qquad Mev/cm^3.sec$$

$S_{o,g}$ is the value of the source at the outer boundary of the region considered

$G(Q)$ is the dimensionless function describing the space behaviour of the source (the same for all groups),to be described in input with the same rules and restrictions as for the function

$F(Q)$ (section 2.2) describing the fission density.

If the gamma source is essentially a fission source, then $F(Q) \simeq G(Q)$, and the same input information may be repeated for both functions.

$S_{o,g}$ is calculated by the program as

$$S_{o,g}= S_o f_g + \sum_{i=L}^{26} \phi_{o,i}\, p_{i,g} + \sum_{i=L}^{N} \phi_{o,i}q_{i,g} \qquad (14)$$

where

S_o = boundary value of the fission density (section 2.2)

f_g = energy emitted by fission, and equilibrium fission products, in the gth group (Mev/fission)

$\phi_{o,i}$= total neutron flux of the ith group at the outer core boundary;

L is an input datum (section 4.2).

$$p_{ig} = \sum_e N_e \, \sigma_{e,i}(n, \gamma) C_{e,g}$$

$\sigma_{e,i}(n,\gamma)=$ microscopic (n,γ) c.s. of the ith neutron group and e^{th} element (barns)

$C_{e,g}$,   capture gamma spectrum of the eth element, i.e. gamma energy yield in the gth group per neutron capture (Mev/capture)

N_e = nuclear density of the eth element in the region considered (nuclei/barn.cm)

$$q_{i,g} = \sum_{j=0}^{M} \sum_e N_e \, \sigma_e^{in}(i,i+j) Q_e(i,j,g)$$

$\sigma_e^{in}(i,i+j)=$ micro. inelastic transfer c.s. from group i to i+j (barns) for the eth element

$Q_e(i,j,g)=$ gamma energy emitted into the gth group, as a result of the inelastic scattering of a neutron from group i to i+j, by the eth element.

The sums over e include all the elements of the region; the sum over j extends to a proper number of lower groups: in the present library M=7. The second sum in (14) extends to those neutron groups, from the Lth on which lie above 0.5 Mev: at present N=6; if $L > N$ this sum is neglected. For normal calculations it should be L=1, to account for all possible gamma sources; but if only low energy groups need to be considered, then a suitable value of L should be chosen; for instance if the inelastic scattering gammas are negligible, and only thermal neutron capture needs to be considered, set L=26. Finally if $L > 26$ no gamma source and no gamma flux is computed, and the calculation stops after the neutron results have been printed.

Formula (14) holds for region 1 and 2 but with some minor differences: in the second region it is exactly applied; in the first region the gamma source is evaluated only if S_0 (fission density at the outer boundary of the region) is positive; if $S_0 = 0$, no gamma source is considered within the first region; in this case the region behaves as a

normal shielding region (but in plane geometry its presence is neglected).

3.2.2 Gamma Sources Inside the Shield

In any of the shielding regions except possible air gaps, SABINE calculates the gamma source distribution in a number of points specified by the input data and then fits this distribution with the product of an exponential function, crossing the end-point values, and a polynomial with a number of coefficients NGCF (and degree NGCF-1) to be specified in input.

The gamma source vs. the distance x_1 from the core boundary for the gth group is

$$S_g(x_1) = \sum_{i=L}^{26} \phi_i(x_1) p_{i,g} + \sum_{i=L}^{N} \phi_i(x_1) q_{i,g} \qquad (15)$$

In plane or spherical geometry, where S_g is a function of one space variable (IGSS is equal zero and two respectively), the source is given by (15). In the case of cylindrical (IGSS=1) or disk geometry (IGSS=3), S_g is depending upon one more variable x_2 (along the transversal direction), indicated by z or r for the cylindrical or disk geometry respectively in Fig. 1. In such cases the program assumes

$$S_g(x_1,x_2) = S_g(x_1) f(x_2)$$

$f(x_2)$ is assumed to be a polynomial with coefficients given in input:

$$f(x_2) = \sum_{i=1}^{NCF} a_i x_2^{i-1} \qquad a_1 = 1 \quad , \qquad NCF \leq 6$$

However if the transversal dimension of the region is infinite, one should put N=1 and a_1=1.

3.3 Calculation of the Gamma Fluxes

Once the distributions of the sources for the different groups have been calculated, the program computes and prints for each required point P (of the shield) and each region r, the gamma flux $G_{r,g}(P)$ (Mev/cm^2 sec) due to the sources of the gth group, contained in the rth region:

$$G_{r,g}(P) = \int_{V_r} S_g(Q) \frac{B_g(PQ)\exp(-\tau(PQ))}{4\pi \ \overline{PQ}^2} \, dV(Q) \qquad (16)$$

$S_g(Q)$ = gamma energy emitted into the gth group per unit volume at Q (Mev/cm^3 sec)

$B_g(PQ)$ = Build-up factor (see section 3.4), dimensionless.

$\tau(PQ)$ = $\int_{\overline{PQ}} ds \ \mu_g(s)$

μ_g = linear attenuation coefficient of gth group (cm^{-1})

$dV(Q)$ = volume element around Q, which moves to cover the whole volume V_r of the rth region.

The following sections explain how the Build-up-factors are calculated and how the numerical integration of (16) is worked out.

3.4 Build-up-Factors

The gamma part of the data library of SABINE includes, for each gamma group, a table of "point isotropic dose Build-up-Factors" vs. the distance t (in m.f.p.) from the source point Q to the dose point P, for seven materials (see Table 6) ranging from air to lead.

All what follows refers to each of the gamma groups; the subscript g will be omitted for sake of simplicity.

Let $B_m(t)$ be the B.u.F. for a homogeneous medium of the mth material: these functions have been calculated by the BIGGI 3 [6] program, which solves numerically the transport equation for gamma radiation in plane geometry, and then tabulated for t values between 0 and 30 m.f.p. at steps of 5.

Now assume we need to calculate the B.u.F. to be inserted
in eq.(16), for a laminated medium such as shown in Fig.2:
let Q be a point source in region 1 containing the mate-
rial 1, and P the dose point in the Nth region containing
the Nth material, x_n the part of segment PQ inside region
n with linear attenuation coefficient μ_n. Now let

$$t_1 = \mu_1 x_1$$
$$t_2 = t_1 + \mu_2 x_2$$
$$t_N = t_{N-1} + \mu_N x_N$$

One can choose now among three different ways of calcula-
ting the B.u.F. vs. t, namely B(t), which provide the sa-
me result if the path t lies within a single region, but
different results in a laminated medium (Fig.3). The user
may choose which way he needs by specifying the proper in-
put value NBU:

NBU = 1 $B(t_N) = B_N(t_N)$

The B.u.F. is calculated by interpolation between the
values tabulated for the material of the region to which
the dose point P belongs.

NBU = 2   The calculation is done with the Broder's [14]
            formula

$$B(t_N) = \sum_{n=1}^{N} B_n(t_n) - \sum_{n=2}^{N} B_n(t_{n-1})$$

            which is easier to understand in the following
            recurrent form:

$$B(t_1) = B_1(t_1)$$
$$B(t_N) = B_N(t_N) + \left[B(t_{N-1}) - B_N(t_{N-1}) \right]$$

NBU = 3   This is a modification of Kitazume's formula [15]
            and in recurrent form reads:

$$B(t_1) = B_1(t_1)$$
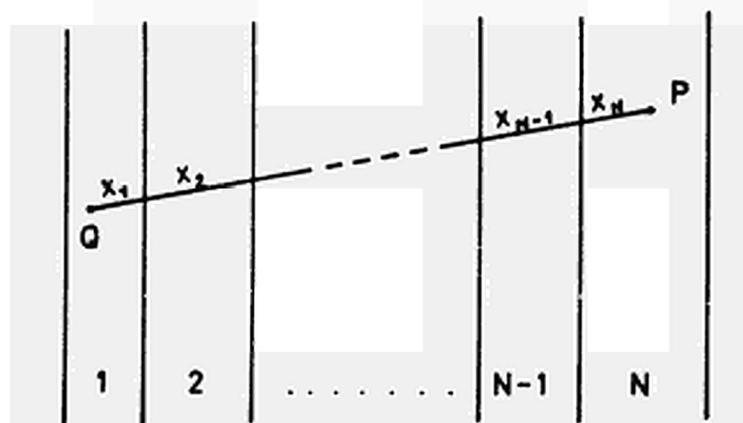
Fig. 2
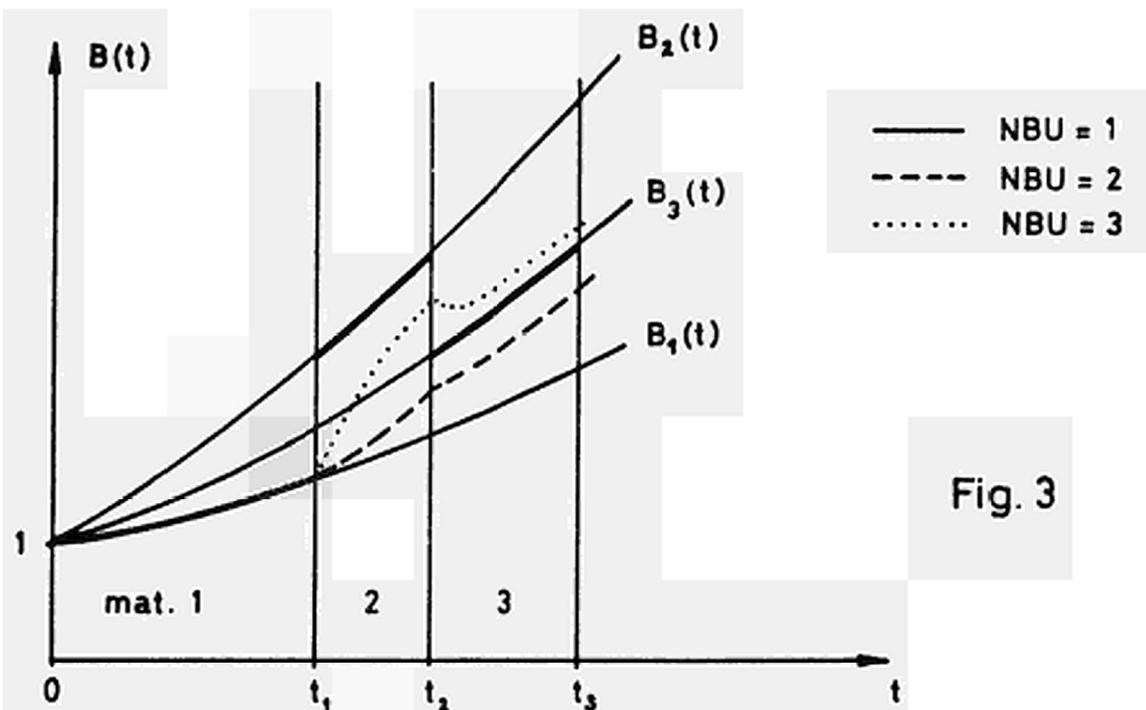
Fig. 3

Build-up-factors for laminated media resulting from the different approximations: $B_1(t)$, $B_2(t)$, $B_3(t)$, ... are the build-up-factors for material 1,2,3,... respectively.

$$B(t_N) = B_N(t_N) + \left[B(t_{N-1}) - B_N(t_{N-1})\right] Z_N$$

$$Z_N = (1 - \beta_{N,N-1}) \exp(-\alpha_{N,N-1} \mu_N x_N) + \beta_{N,N-1}$$

$\alpha_{i,j}$ and $\beta_{i,j}$ are coefficients, tabulated for each couple of materials i and j, that determine the "transient" part of the BUF when a boundary between material i and j is crossed; $|\beta| \leq 1$.

Fig. 3 shows the behaviour of the function $B(t)$ in the three different cases, represented respectively by the full line, the dashed line, and the dotted line.

Note that for NBU = 1 the function $B(t)$ is discontinuous at the interface. For NBU = 2, $B(t)$ is continuous and parallel inside any region n to the function $B_n(t)$.

When NBU = 3, we have to distinguish among three possible cases:

a) if $\beta = 1$ or $\alpha = 0$, this case reduces to the previous one of the Broder's formula (NBU = 2)

b) $\beta = 0$, this case is equivalent to the original formula of Kitazume; $B(t)$ is continuous and, if $\alpha > 0$, tends to $B_n(t)$ inside the nth region as x_n increases; if $\alpha < 0$ the two curves diverge.

c) $\beta < 1$, $\alpha > 0$ this is the more general case: $B(t)$ is continuous and tends to become parallel to $B_n(t)$ inside the nth region, as x_n increases.

The coefficients α and β have been determined by fitting $B(t)$ as expressed by the third form, to a number of calculations of gamma doses (performed with BIGGI 3) in laminated media composed of two (and sometime three) materials, with variable thickness. Actually they have been calculated only for 4 materials, namely H_2O, Al, Fe and Pb; normal and heavy concrete are assumed to have the same values as Al and Fe respectively; for air they are not used.

The choice among the three forms of B.u.F. is mainly a matter of experience. The second (NBU=2) and even more the third (NBU=3) form of B.u.F. are more expensive, in terms of machine time, than the first, and should be preferred only if necessary. NBU=1 may be suitable to know the gamma dose outside the shield, or at points lying beyond a thick (with respect to the m.f.p.) layer which is homogeneous or composed of similar materials. Similar materials are in practice - from the point of view of the gamma penetration - those with similar densities. NBU=2 may be suggested to investigate the effect of a set of thin layers; and NBU=3 if these layers are composed of very "different" materials like, for instance iron and water.

Furthermore SABINE allows the use of two different forms of B.u.F. for the radiation originated inside the core and for the secondary radiation.

3.5 Gamma Energy Deposition

Besides the gamma flux and dose, SABINE always provides the gamma Energy deposition (in $Watt/cm^3$) as the product of the gamma flux, the macroscopic energy absorption c.s., and the constant $1.60 \cdot 10^{-13}$ (Mev/cm^3 sec to $Watt/cm^3$). This implies the assumption that the dose B.u.F. equals the energy absorption B.u.F., which is not always true, mainly for low energies or at the interfaces.

3.6. The Numerical Integration

Equation (5) of section 2.3 and (16) of section 3.3 lead to integrations over two or three space variables which have to be performed numerically. Assume that as a step of this calculation we need to compute the value of the following integral

$$Y = \int_a^b f(x)\,dx \qquad\qquad (17)$$

a and b being the limits for the space variable x within a given source region. To choose the mesh path for the integration, the program calculates first the minimum relaxation length λ for the radiation considered in that region:

$$\lambda = \frac{1}{\underset{n}{Max}\left[\Sigma_n^r\right]} \qquad \text{for removal neutrons}$$

$$\lambda = \frac{1}{\underset{g}{Max}\left[\mu_g\right]} \qquad \text{for gamma radiation}$$

Then the mesh path is set approximately equal to λ/m, where m is an integer to be specified in input for each source region and space variable. There are two spatial variables for the integration in plane, spherical and disk geometry (r and ϑ), and three in cylindrical geometry (r, ϑ, ψ).

The interval (a,b) is divided in N equal steps, with

$N=$ Integer Part of $\frac{b-a}{\lambda/m}$.

However if this value is smaller than 2 or greater than 50, the program sets $N=2$ or $N=50$ respectively.

The integration is performed with the formulae of Newton-Cotes [13] with n points, taking $n=7$ where possible and smaller elsewhere.

The m's values should be chosen in order to achieve the needed accuracy for the numerical integration. Attention should be paid to the fact that the machine time required to perform the numerical integrations (which is most of the total time spent by SABINE), is proportional to the product $m_r \cdot m_\vartheta$ in plane, disk and spherical geometry, and to the product $m_r \cdot m_\vartheta \cdot m_\psi$ in cylindrical geometry. The tests which have been carried out hitherto show that $m_r = m_\vartheta = m_\psi = 1$ is generally sufficient to keep the numerical errors within a few per cent.

The integration (17) proceeds from the point a where
f(x) is expected to have its maximum value to the point
b where f(x) should have its minimum: this is in most
cases possible due to the regular and monotonic behaviour
of the integrands within each source region.
To save computer time the program stops the integration
if a point $\bar{x}$ is reached such that

$$\frac{f(\bar{x})}{f(a)} < \eta$$

One value of η for each region has to be provided in in-
put: $\eta \approx 10^{-3}$ is suggested for the most important source
regions, $\eta \approx 10^{-2}$ for the others.

3.7 Gamma Data Library

The data needed to calculate the gamma fluxes are of 3
kinds.

a) General data: they include (g=1 to 7)

f_g prompt plus equilibrium gamma fission spectrum

d_g conversion factors from MeV/cm^2.sec to mR/h

b) data for each element or mixture of elements:

μ_g mass attenuation coefficients (cm^2/gr)

μ_g^E energy absorption coefficients(cm^2/gr)

C_g capture gamma spectrum

$\sigma(n,\gamma)_i$ microscopic (n,γ) c.s. for neutrons of the ith
group, (barns). Furthermore for those elements
which are mainly responsible for the (n,n'γ) reac-
tions we need:

$\sigma^{in}(i,i+j)$ micro.c.s. for the transfer by inelastic
scattering from the ith to the (i+j)th group:
i=1 to 6; j=1 to 7.

Q(i,j,g), inelastic scattering gamma spectrum (see
section 3.21)

c) data for the build-up factors (see section 3.4): for each of the 7 materials of Table 6. and each of the 7 gamma groups, we need:

$B(t_i)$, $t_i = i.5$, $i = 0$ to 6; table of B.u.F. **vs.** distance in m.f.p.

$\alpha(m')$ and $\beta(m')$ coefficients which take into account the transition from material m' to the material considered; $m' = 1$ to 7.

All the data listed have been written on the library tape, behind the neutron data library.

Table 6

List of materials for which the Build-up-Factors have been calculated.

Identification no. (MBU)	Material
1	Air (B.u.F. =1)
2	Water
3	Aluminum
4	Ordinary Concrete
5	Heavy Concrete
6	Iron
7	Lead

4. USER'S MANUAL

4.1 Computer Requirements

SABINE is a FORTRAN IV program written for the IBM 7090.
It is an overlay job requiring a 32K memory and 7 tape
units, not including the system monitor tapes. The use
of tapes is described in the following table.

Table 7

Tape Configuration

Fortran Tape No.	Unit Designation at Ispra	Tape Use in SABINE
2	B2	Pool; temporary binary data storage
3	B3	Idem
4	A4	Idem
5	A2	Standard BCD input
6	A3	Standard printed output
9	A5	SABINE data library
10	B5	Same as B2

4.2 Input Specifications

The input specifications needed to run the program SABINE
are shown schematically in Table 8, and listed here be-
low. One or more problems may be solved during the same
run. Columns 71-80 are free for possible labels.

Repeat	card no.	columns 10	20	30	40	50	60	70	80
	1	N							
	2	Title card							
	3	IGRC	IGRS	IGDS	IGGS				
	4	NREG	IFGAM	NBUC	NBUS				
NREG times	5	J	ZR(J)	H(J)	T(J)	DEN(J)	IGAP(J) MBU(J)	NEMR(J)	
" "	6	J	$M_\Phi(J)$ $M_R(J)$	$M_\Psi(J)$	ETHA(J)	NDIF(J) NREM(J)	NPRT(J) NGS(J)	NGPR(J) NGCF(J)	
" "	7	J ID	FREM	ID FREM	ID FREM	ID FREM	ID FREM		
	7	J ID	FREM						
2	8	S(J)	ISR(J)	NCFR(J)	NNFR(J)	ISZ(J)	NCFZ(J)	NNFZ(J)	
	9	$AR_1(J)$	$AR_2(J)$	$AR_3(J)$	.....				
	10	$AZ_1(J)$	$AZ_2(J)$	$AZ_3(J)$	.....				
	11	BSQ(3)	BSQ(4)	BSQ(5)	.....				
	12	NTH NFRD							
NTH	13	name of the threshold detector							
	14	Σ_1	Σ_2	Σ_3	.....	Σ_{19}			(3 cards)
NFRD	15	I1 I2 name of the region dependent response function							
	16	F(3,I1)	F(3,I1+1)	F(3,I1+2)	.....	F(3,I2)			
	16	.....							
	16	F(NREG,I1)	F(NREG,I1+1)	.....	....	F(NREG,I2)			
26	17	α_1	β_1	α_2	β_2	.....			
	18	b_1	b_2	a_0	a_1	a_2			
NRIR	19	NRIR IFDOSE							
	20	I1 I2 name of the region independent response function							
	21	F(I1)	F(I1+1)	F(I1+2)	.....	F(I2)			
		If γ fluxes are not needed (IFGAM > 26) no more card for this problem							
2	22		ISR(J)	NCFR(J)	NNFR(J)	ISZ(J)	NCFZ(J)	NNFZ(J)	
	23	$AR_1(J)$	$AR_2(J)$	$AR_3(J)$	.....				
	24	$AZ_1(J)$	$AZ_2(J)$	$AZ_3(J)$	.....				
(NREG-2)	25	J NCF	$A_1(J)$	$A_2(J)$	$A_3(J)$	.....			
		Other problems (if any) continue with card 2							

TABLE 8

Card	Columns	Format	Name	Description
1	1-10	I10	N	Number of problems to be solved
2	2-70	14A5		Title card; the content of this card is printed as head-line in each page of the output.
3	1-10	I10	IGRC	Index for the core geometry (section 1.2)

0 for plane geometry
1 for cylindrical geometry
2 for spherical geometry
3 for disk geometry

| | 11-20 | I10 | IGRS | Index of the shield geometry, for the calculation of removal neutrons and gamma radiation from the core. |

0 for plane slabs
1 for cylindrical shells
2 for spherical shells

| | 21-30 | I10 | IGDS | Index of the shield geometry for the solution of the diffusion equation. |

Same possibilities as IGRS.

| | 31-40 | I10 | IGSS | Index of the shield geometry for the calculation of the (secondary) gamma flux originated inside the shield |

0 infinite plane slabs
1 cylindrical shells
2 spherical shells
3 disks

| 4 | 1-10 | I10 | NREG | Number of regions $\leq$ 22 (2 source regions and no more than 20 shielding regions) |
| | 11-20 | I10 | IFGAM | Controls the calculation of the gamma sources: if IFGAM $>$ 26 (number of neutron groups) no gamma calculation is performed. When IFGAM $\leq$ 26 the gamma sources are calculated considering besides the fission source, the reactions (n, γ) and $(n,n'\gamma)$ with neutrons of the groups $I \geq$ IFGAM. This is the parameter called L in eqs.(14) and (15) of section 3.2.1 and 3.2.2. |

21-30	I10	NBUC	Determine the form of the Build-up-Factor for the Core
31-40	I10	NBUS	and the Shield gamma radiation respectively (section 3.4)

One card 5 is needed for each region J, J=1, NREG.

5	1-10	I10	J	Index óf the region
	11-20	F10.0	ZR(J)	Thickness of the region (cm)
	21-30	F10.0	H(J)	For plane geometry not used For cylind. geometry: height for finite cyl., zero for infinite cyl. For sph. geometry not used For disk geometry diameter of the disk.
	31-40	F10.0	T(J)	Temperature of the region (°C)
	41-50	F10.0	DEN(J)	Density (gr/cm^3)
	51-55	I5	IGAP(J)	= 1 normal region 2 this region is an air gap to be treated in P1 approx. (section 2.8.1) 3 Air gap with α an β values in input (section 2.8.2)
	56-60	I5	MBU(J)	Code number of the material for the Build-up-Factor of this region.
	61-65	I5	NEMR(J)	No. of elements in the region, $\leq$ 10.

One card 6 is needed for each region J; the content of columns 41-70 need to be specified only for shield regions ($J \geqslant 3$).

6	1-10	I10	J	Index of the region
	11-15	I5	$M_\vartheta(J)$	Determine the fraction of the minimum relaxation lenght to be used as mesh interval for the numerical integration along ϑ, R, and ψ respectively. Use $M_\vartheta = M_R = M_\psi = 1$ as standard value (section 3.6)
	16-20	I5	$M_R(J)$	
	21-25	I5	$M_\psi(J)$	
	31-40	E10.0	ETHA(J)	Relative accuracy for numerical integration (section 3.6). Use .001 as standard value.

41-45	I5	NDIF(J)	Number of intervals for the calculation of neutron flux (and numerical integration of eqs. B15 and B16 of Appendix B). Recommanded values for the mesh path d=ZR/NDIF are given for several media in Table 9.

$$4 \leqslant NDIF(J) \leqslant 250$$

$$\sum_{J=3}^{NREG} NDIF(J) \leqslant 1000$$

46-50	I5	NREM(J)	The removal flux is calculated at each NREMth point (these points are separated by d.NREM) and interpolated logarithmically in the others. NDNI(J)/NREM(J) must be integer, and

$$NREM(J) \leqslant 50$$

$$\sum_{J=3}^{NREG} \frac{NDIF(J)}{NREM(J)} \leqslant 100$$

51-55	I5	NPRT(J)	Neutron fluxes are printed at any NPRTth mesh point, i.e. spacing for neutron output is d.NPRT.

NDIF(J)/NPRT(J) must be integer and

$$\sum_{J=3}^{NREG} \frac{NDIF(J)}{NPRT(J)} \leqslant 200$$

If no gamma calculation is required (IFGAM $>$ 26) no other information is needed on this card.

56-60	I5	NGS (J)	The spacing for gamma source calculation is d.NGS: no more than 50 source points per region and 500 for the whole shield,i.e.:

$$\frac{NDIF(J)}{NGS(J)} \leqslant 50 \quad \text{(must be integer)}$$

$$\sum_{J=3}^{NREG} \frac{NDIF(J)}{NGS(J)} \leqslant 500$$

61-65	I5	NGPR(J)	Gamma fluxes are calculated and printed only at points where the neutron fluxes have been printed; precisely at each NGPRth of them.

$$\frac{NDIF(J)}{NPRT(J).NGPR(J)} \quad \text{must be integer}$$

				if $NGPR(J) \leqslant 0$, there will be no gamma flux calculation for this region.
	66-70	I5	NGCF(J)	Number of coefficients for the polynomial which fits the gamma source distribution in the Jth region (see section 3.2.2 and 4.3) $NGCF(J) \leqslant 10, NGCF(J) \leqslant \dfrac{NDIF(J}{NGS(J)} +1$

In the case of an air gap put MBU=1; the quantities H,T, of card 5 and M_θ, M_R, M_γ, ETHA, NGS, NGCF of card 6 need not to be specified: set NDIF=NREM=NPRT=NGPR=1 if no gamma flux is required inside the gap. NREM must be positive, and card 7 describing the composition must be punched also for the gaps.

7	1-5	I5	J	Index of the region
	6-8	I3	ID	Identification no. for the first element or material (table 4) and
	9-18	F10.0	FREM	Corresponding weight fraction in the Jth region.
	19-21 32-34 45-47 58-60	I3	ID	A couple of value ID, FREM must be given for each element; NEMR(J) couples of numbers for the Jth region, 5 per card; if
	22-31 35-44 48-57 61-70	F10.0	FREM	NEMR(J) > 5, two cards 7 must be present for this region.

Cards 8-10 are needed for each core region, i.e. twice: they describe the fission source distribution (read section 2.2 before writing these cards).

8	1-10	E10.0	S(J)	Fission/cm^3.sec at the outer boundary of the Jth region. If $S(J) \leqslant 0$, no other information is needed for this region (omit cards 9-10)
	11-20	I10	ISR(J)	How the radial distribution is specified? 1 the coefficients of a polynom are given 2 point wise 3 exponential distribution $e^{-kr'}$;

21-30	I10	NCFR(J)	Number of coefficients of the polynomial to be given (if ISR(J)=1) of to be calculated (if ISR(J)=2); not used if ISR(J)=3.

$NCFR(J) \leq 10$

31-40	I10	NWFR(J)	If the radial distribution is given pointwise (ISR(J)=2), $NWFR(J) \leq 51$ values at equidistant points must be given. Not used if ISR(J)=1 or 3. The above three quantities refer to the radial distribution, and are not used in plane geometry (IGRC=0). The three following refer to the source distribution vs.z, and are not used in spherical geometry (IGRC=2).
41-50	I10	ISZ(J)	How the z-distribution is specified?

1 if the coeffic.of a polyn. are given
2 pointwise

51-60	I10	NCFZ(J)	Number of coefficients (≤ 10) to be given (if ISZ(J)=1) or calculated (if ISZ(J)=2); for infinite cylindrical geometry NCFZ(J) should be 1.
61-70	I10	NWFZ(J)	If ISZ(J)=2, $NWFZ(J) \leq 51$ values at equidistant points must be given. Not used if ISZ(J)=1, or for infinite cylinder.

Card 9 has to be omitted in the case of plane geometry (IGRC=0), and inserted in the other cases: it contains the coefficients or the tabulated values or the exponent for the radial distribution:

9	1-10	E10.0	$AR_1(J)$	First coefficient (b_1 in section 2.2) or first tabulated value (h_1) or value of k in the case of exponential radial distribution.
	21-30	E10.0	$AR_2(J)$	Other coefficients or tabulated values, no more than seven per card; use more cards if necessary.
	.	E10.0	$AR_3(J)$	
	:	E10.0	.	
	.		:	
			.	

If the core geometry is not spherical (IGRC $\neq$ 2) card 10 is necessary to provide the coefficients or tabulated values of the Z-distribution.

10	1-10	E10.0	$AZ_1(J)$	Coefficients or tabulated va-
	11-20	E10.0	$AZ_2(J)$	lues, seven per card; use as
	:	E10.0	:	many cards as necessary.

The next card contains the values of the transversal square buckling of the shielding regions.

11	1-10	E10.3	BSQ(3)	Transverse buckling of the
	11-20	E10.3	BSQ(4)	third and following regions.
	:	E10.3	:	

Card 12 refers to the calculation of Response Functions of type (b) and (c) of section 2.9.

12	1-5	I5	NTH	Number of reaction rates of Threshold detectors to be calculated.
	6-10	I5	NFRD	Number of Region Dependent response functions to be calculated.

Cards 13-14 are needed for each threshold detector; omit if NTH=0.

13	2-72	12A6		Name of the detector: the content of this card is printed before the calculated reaction rate.

14	1-10	E10.3	Σ_1	Macroscopic c.s. of the detector
	11-20	E10.3	Σ_2	for the energy range correspon-
	:	E10.3	:	ding to the removal group 1 to 19.
			Σ_{19}	19 values, seven per card must be punched.

Cards 15-16 are needed for each region dependent response function; omit if NFRD=0.

15	1-5	I5	I1	Indexes of the neutron groups which are the limits of the sum in eq.(13) section 2.9; this implies that for the other groups the corresponding terms are zeros.
	6-10	I5	I2	
	11-70	10A6	Label	To be printed as identification of the response function.

Card 16 is needed for each shielding region: J=3 to NREG.

16	1-10	E10.3	F(J,I1)	These are the f_i values of
	11-20	E10.3	F(J,I1+1)	eq.(13) section[1] 2.9. Give se-
		E10.3		ven values per card, and use
				as many cards as necessary.

One of these cards or sets of cards 16 is needed for each shielding region; repeat the set 15-16 for each region dependent R.F. (i.e. NFRD times).

Cards 17-18 contain the magnitudes that determine the boundary conditions for the neutron groups: a couple of cards for each of the 26 neutron groups is needed if some air gap to be treated in the way described in section 2.8.2 (with α and β values in input) is present, if not card 17 must be omitted.

17	1-10	E10.3	α_1	One couple of values for each
	11-20		β_1	of the air gaps for which
	$\vdots$		α_2	IGAP=3 in card 5. No more than
			β_2	three gaps of this type may be
	51-60		α_3	present.
			β_3	

18	1-10	E10.3	b_1	Coefficients that determine the
	11-20		b_2	neutron boundary conditions at
			a_0	the outer (b_1,b_2) and inner
			a_1	(a_0,a_1,a_2) boundary of the shield
			a_2	(sections 2.7.1 and 2.7.2).

We recall that the set of values corresponding to the more usual boundary conditions, of no incoming current from outside the shield, and a given flux $\emptyset_0$ at the core-shield interface, is in the order the following: 0.5, 0., 0., -1., $\emptyset_0$.

The next card refers to the calculation of the region independent Response Functions.

19	1-5	I5	NRIR	Number of region independent R.F. to be calculated (if any). If NRIR=0, cards 20-21 must be omitted.
	6-10	I5	IFDOSE	Should neutron dose rate be calculated?
				0 No
				1 Yes

If NRIR is positive, NRIR sets of cards 20-21 must be provided, one for each region independent R.F.

20	1-5	I5	I1	Limits of the sum in eq.(13)
	6-10	I5	I2	section (2.9)
	11-70	10A6		Label to be printed as identification for this R.F.
21	1-10	E10.3	F(I1)	Values to be specified group-
	11-20		F(I1+1)	wise of the terms f_i in eq.(13): use as many cards as necessary, with seven values per card.

If gammas fluxes are not needed, there are no other cards for this problem.

Cards 22-24 describe the space distribution of the gamma source (G(Q) of section 3.2.1) in each core region: two sets of these cards must be given for the two core regions. They are written in the same way as cards 8-10 (see Table 8), but the columns 1-10 in card 22 are not read. If the shape of the gamma source is the same as that of the neutron source, cards 22-24 are the copy of cards 8-10. Note that if S(1)- fission density at the outer edge of the first region- is zero, the cards for the first region must not be inserted; the source description for the second region must always be given.

For plane or spherical source geometries (IGSS=0 or 2) no other information is needed; in the other cases, for each shield region, the shape of the transversal distribution of the gamma sources, in the form of a polynomial (section 3.2.2) must be specified.

25	1-5	I5	J	Index of the region; J=3,4... NREG
	6-10	I5	NCF	no. of coefficients of the polynomial: NCF $\leq$ 6
	11-20	E10.0	$A_1(J)$	Coefficients of the polynomial
	21-30		$A_2(J)$	

Note that for infinite cylindrical regions only one coefficient $A_1=1$ should be given.

No other card for this problem; other problems (if $N > 1$ in the first card) continue with card 2.

4.3 Choice of Mesh Intervals

This section contains some suggestions for choosing the values of those input parameters which determine the mesh intervals for the different calculation; they are punched in columns 41-70 of card 6.

The mesh path d for the diffusion calculation is the ratio between the thickness of the region and the value of NDIF for that region; all the other intervals (see below) are expressed in terms of d.

Inside each region the removal flux is calculated at intervals $d_r = $ d.NREM; the total neutron fluxes are printed at intervals $d_p = $ d.NPRT; the gamma source are calculated at points separated by $d_s = $ d.NGS; the spacing for printing the gamma fluxes is d= d_p.NGPR= d.NPRT.NGPR (this means that the results of the gamma calculations are printed only at points where the neutron fluxes have also been written).

Note that the number of coefficients NGCF may not exceed the number of source values to be fitted, namely:

$$NGCF \leqslant \frac{NDIF}{NGS} + 1$$

must hold in each region.

The values of d should lie within the range suggested in Table 9 for several materials: finer mesh should be preferred where the neutron fluxes are less regular, e.g. within absorbing regions, or near boundaries between materials with different properties.

Table 9

Recommended values of the mesh path d.

Material	d(cm)
Water	.25 - .50
Heavy Water	.70 -1.50
Beryllium	.50 -1.00
Graphite	1.00 -2.00
Aluminum	1.50 -2.00
Concrete	1.50 -2.50
Iron	.50 -1.00
Lead	1.50 -3.00

d_r should be small enough so that the removal fluxes can
be approximated by exponentials within an interval d_r:
the range 5.0-10.0 cm is recommended for d_r.
The running time of SABINE is affected mainly by two num-
bers: the number N_1 of points where the removal fluxes
are calculated and the number N_2 of points where the gamma
fluxes are calculated; they are

$$N_1 = \sum_{J=3}^{NREG} \frac{NDIF(J)}{NREM(J)} \qquad N_2 = \sum_{J=3}^{NREG} \frac{NDIF(J)}{NPRT(J) \cdot NGPR(J)}$$

Sometime it may be convenient to divide a region in two
or more parts, in order to have there different mesh in-
tervals, to improve accuracy or speed. This happens mainly
for water regions; for them in particular, considerations
related to problems of under - and over-flow suggest to use
water regions no more than 60 cm thick, by subdividing if
necessary, in more regions.

4.4 Output of the Program

The output of the program does not require particular explanations: the content of the title card (card 2) is printed as headline of each page of the output; labels have been provided for any quantity which is printed so that no difficulty should arise for their understanding. The input values are printed out in a way quite similar to how they are written in the input sheet, to allow for an easy check of them.

The output consists mainly of three parts: the input data, the neutron fluxes and related magnitudes, and the results of the gamma part.

In the second part, after the total neutron fluxes vs. distance from the core boundary have been printed for the 26 groups, the results of each neutron Response Function (if any) are printed with the proper label.

The third part starts with a description of the gamma source distribution in the whole system: either given in input or calculated by the program. Then for any point P at which the gamma fluxes have been computed, the following quantities are printed:

- z distance of the point from the core boundary
- $G(IRS,IG)$, $IG=1$ to 7; $IRS=1$ to NREG; gamma flux at P due to the energy emitted inside the gamma group IG, in the region IRS.
- Gamma flux per source region (sum over IG of $G(IRS,IG)$) and per group (sum over IRS)
- Total gamma flux
- Total gamma dose, and the contributions of each group.
- Gamma energy deposition, and the contributions of each gamma group.

CONCLUSION

The SABINE program has been devised to provide an efficient tool for the solution of a wide class of practical shield design problems.

A comparison between the predictions of SABINE and the results of experimental measurements for the shield of different reactors is in progress. Water, Iron and Concrete shields are being considered: the result of these comparisons will be the subject of a separate report. However from our experience up to now the calculations show a satisfactory agreement with measurements also for deep penetrations (≈ 200 cm).

The running time for a neutron problem in plane geometry is normally in the range from 5' to 15' on the IBM 7090; the time for a problem with both the neutron and the gamma flux calculation is rougly the double.

The results of SABINE, as well as of many other similar programs, are in general rather sensitive to the way of representing a given problem through the input data (choice of the geometry indexes, description of the radiation sources, boundary conditions, parameters for the numerical integrations, etc.); the user should carefully read this report and pay attention to the input preparation in order to save man and machine time, and to get reliable results.

For questions concerning the understanding of the report, the data library, or the use and results of the program, the reader may apply to Dr. C. Ponti - EURATOM ,T.C.R., C.C.R. Ispra (Varese) Italy.

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APPENDIX A

The Factorization of the Diffusion Equation

The following procedure is different but equivalent to that suggested by E.C. Ridley 12

Eq. (8) may be rewritten in the form

$$D\phi''(r)+ \frac{P}{r} D\phi' - \phi(r)+ S(r) = 0 \qquad (A1)$$

D and are region dependent constants.

One looks for two functions U (r) and V(r) which satisfy the following equation

$$D\phi'(r)+ U(r)\phi(r)+V(r)=0 \qquad (A2)$$

when $\phi(r)$ is the solution of (A1).

By deriving eq. (A2) we have

$$D\phi''+U\phi'+U'\phi+V' = 0$$

Inserting here ϕ' as solved from eq. (A2) we get

$$D\phi'' = \phi(\frac{U^2}{D} -U') - V'+ \frac{UV}{D} \qquad (A3)$$

Now substitute $D\phi''$ from eq. (A3) and $D\phi'$ from eq. (A2) into (A1) and have an eq. in ϕ:

$$\phi(\frac{U^2}{D} - U' - \frac{P}{r} U-)+ \frac{UV}{D} - V' - \frac{P}{r} V+S= 0$$

In order that this becomes an identity the two following equations must hold:

$$\frac{U^2}{D} -U' - \frac{P}{r}U- = 0$$

$$\frac{UV}{D} -V' - \frac{PV}{r} + S=0$$

and these are the first order differential eqs. that determine U and V respectively.

APPENDIX B

The Solution of the System of Differential Equations (9)

In Appendix A it is shown how the diffusion eq.

$$D\phi''(r) + \frac{P}{r} D\phi'(r) - \phi(r) + S(r) = 0 \qquad (B1)$$

may be separated into the following equivalent system:

$$U' = \frac{U^2}{D} - \frac{P}{r} U - \qquad (B2)$$

$$V' = V(\frac{U}{D} - \frac{P}{r}) + S \qquad (B3)$$

$$D\phi' + U\phi + V = 0 \qquad (B4)$$

This system may be solved, integrating eqs. (B2, B3 and B4) one after the other.

Now let $t = Kr$, being $K = \sqrt{\Sigma/D} > 0$, and write

$$\dot{\phi} = d\phi(t)/dt$$

and similarly for the other functions; the above system becomes

$$\dot{U} = U^2/KD - PU/t - KD \qquad (B5)$$

$$\dot{V} = V(U/KD - P/t) + S/K \qquad (B6)$$

$$KD\dot{\phi} + U\phi + V = 0 \qquad (B7$$

Eq. (B5) is a Riccati equation, which can be transformed into a known second order differential equation through the substitution

$$U = -KD \frac{\dot{n}(t)}{n(t)} \qquad n(t) \neq 0 \qquad (B8)$$

Eq. (B5) becomes now

$$\ddot{n} + \frac{P}{t} \dot{n} - n = 0 \qquad (B9)$$

The general solution of (B9) is known for the three geometries; they are generally dependent from two coefficients, which are determined from the boundary conditions; but since

we are interested only in the ratio $\dot{n}/n$ that determines U, we do not care for a proportionality factor, so that the solutions of (B9) may be written respectively:

$$P = 0 \qquad n(t) = Be^{t} + e^{-t}$$
$$P = 1 \qquad n(t) = BI_0(t) + K_0(t) \qquad \qquad (B10)$$
$$P = 2 \qquad n(t) = \tfrac{1}{t}(Be^{t} + e^{-t})$$

$I_0(t)$ and $K_0(t)$ are the modified Bessel functions of zero order.

Now assume that the shield contains N regions with outer radii r_j, j=1 to N (Fig. B1), and let

$$t_{j1} = K_j r_{j-1}$$
$$t_{j2} = K_j r_j$$

$t_{j1} \le t \le t_{j2}$ is the interval of t corresponding to the jth region with inverse diffusion length $K_j = \sqrt{\Sigma_j / D_j}$
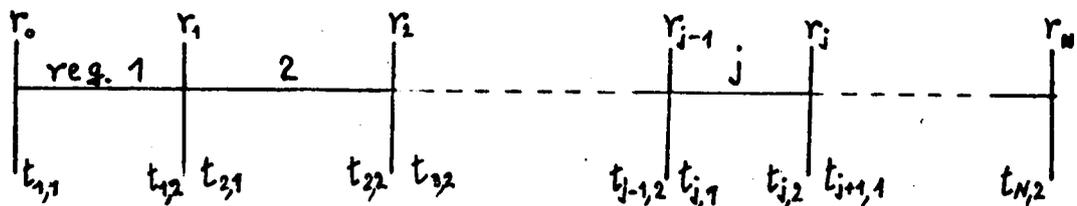
Fig. B1

Let the boundary condition for the function U(t) be

$$U(t_{N,2}) = U_N$$

then eq. (B8) at $t = t_{N,2}$ gives

$$- \frac{U_N}{K_N D_N} = \frac{\dot{n}(t_{N,2})}{n(t_{N,2})} \qquad \qquad (B11)$$

and this equation determines the coefficient B of (B10).
Once n(t) and U(t) are known for the last region, one com-
putes $U(t_{N,1})$ at the left boundary, and since U has to be
continuous

$$U(t_{N-1,2}) = U(t_{N,1})$$

Similarly one can evaluate a new coefficient B for the
function n (t) in region N-1.
This process is continued until n(t) has been determined
in the whole shield.
Now using (B8), eqs. (B6) and (B7) become

$$\dot{V} = -V(\frac{\dot{n}}{n} + \frac{P}{t}) + \frac{S}{K} \tag{B12}$$

$$\dot{\phi} = \phi \frac{\dot{n}}{n} - \frac{V}{KD} \tag{B13}$$

These are linear first order differential eqs. the integral
of which can be easily written.
If the initial value for V(t) is

$$V(t_{N,2}) = V_N \tag{B14}$$

then, as it may be directly checked, the solution of (B12)
for the Nth region is

$$V(t) = \frac{1}{t^P n} \left[V_N t_{N,2}^P n(t_{N,2}) - \frac{1}{K_N} \int_t^{t_{N,2}} S(x) x^P n(x) dx \right] \tag{B15}$$

Now being V continuous through the interfaces

$$V(t_{N,1}) = V(t_{N-1,2}) = V_{N-1}$$

and decreasing the index from N to 1 in (B15), the program
calculates the function V proceeding stepwise from the outer
to the inner regions.
Then the solution of (B13) may be written for the first
shielding region:

$$\phi(t) = n \left[-\frac{1}{K_1 D_1} \int_{t_{1,1}}^t \frac{V(x)}{n(x)} dx + \frac{\phi_1}{n(t_{1,1})} \right] \tag{B16}$$

being $\phi_1 = \phi(t_{1,1})$ the boundary value for the flux.
A boundary value for the second region is taken as

$$\phi_2 = \phi(t_{2,1}) = \phi(t_{1,2})$$

and increasing the index from 1 to N , the program computes finally the flux, proceeding stepwise from the inner to the outer regions.

The method outlined provides an analitical solution of the diffusion equation, except for the fact that the integrals appearing in eqs. (B15) and (B16) are evaluated numerically by the program.
The choice of the mesh path for the numerical integration (performed with the Simpson's rule) is less delicate than in the finite difference methods, and the mesh path required to get a given accuracy is now greater than that needed with finite difference method, with a corresponding saving of machine time.
Recommended values of mesh interval are given for some materials in Table 9; in general it should be chosen in such a way that the mesh interval t in terms of the variable t be

$$\Delta t \lesssim 5$$

All the above analysis holds if the following requirements are fulfilled:

a) $K > 0$
b) $t > 0$ when $P > 0$
c) $n(t) \neq 0$

Condition a) is satisfied for all media but vacuum, for which a particular treatment is needed (see section 2.8).
Condition b) implies that for cylindrical or spherical geometries the origin r_0 of the shield is positive, since the point $r = 0$ is a singularity.

Condition c) is verified in all the "physical" problems:
actually it is easy to see that the function n(r) is,
except for a proportionality factor, a solution of the ho-
mogeneous diffusion equation, satisfying the outer bounda-
ry condition

$$b_0 Dn'(r) + b_1 n(r) = 0 \qquad \text{at} \quad r = R_e$$

which is the homogeneous eq. corresponding to (8.b).
In other words n(r) is a solution of the homogeneous pro-
blem, that is the original problem, in which it is set
S(r)=0 in eq.(8), b_2=0 in eq. (8b), and neglecting eq. (8a).
More precisely one can see, by proceeding analitically,
that the function n(r) may never be equal to zero if b_1/b_0
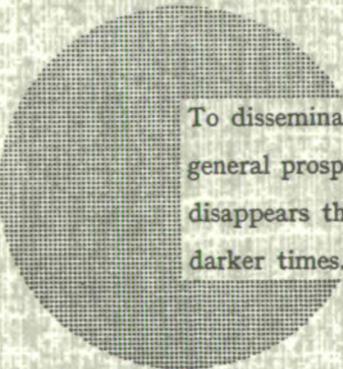is positive, which is always true at the outer boundary of
a shield.

To disseminate knowledge is to disseminate prosperity — I mean general prosperity and not individual riches — and with prosperity disappears the greater part of the evil which is our heritage from darker times.

Alfred Nobel

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