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# TEMPERATURE AND THERMAL STRESS DISTRIBUTION IN SMOOTH AND FINNED CANNINGS DUE TO AXIAL FLUX VARIATIONS

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J. REYNEN

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The theory has been programmed on a digital computer. The code, named ATEAS, handles empirical flux distributions defined at a number of discrete points by assuming a linear flux distribution between adjacent points.

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## J. REYNEN

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ORGEL Program

## Summary

Temperature and thermal stress distributions due to thermal neutron flux peaking near the end plugs of fuel rods are determined for both smooth and finned cannings.

The theory has been programmed on a digital computer. The code, named ATEAS, handles empirical flux distributions defined at a number of discrete points by assuming a linear flux distribution between adjacent points.

# CONTENTS

# TEMPERATURE AND THERMAL STRESS DISTRIBUTION IN SMOOTH AND FINNED CANNINGS DUE TO AXIAL FLUX VARIATIONS [+]

## 1. INTRODUCTION

Axial flux variations near the end plugs of fuel rods cause rather accentuated temperature variations, resulting in important thermal stresses. This paper deals with the determination of such temperature and thermal stress distributions. Special attention has been paid to numerical methods in order to handle empirical thermal neutron flux distributions. For that purpose the theory has been programmed on a digital computer (ATEAS [++]).

For the determination of the stresses, the classical theory of symmetrically loaded thin cylinders has been employed. In appendix 2 a correction for finned cannings is presented.

In chapter 6, the thermal stress distribution for an ORGEL type of canning (G·19), as calculated by ATEAS, is presented.

## 2. TEMPERATURE DISTRIBUTION FOR GIVEN FLUX DISTRIBUTION

The differential equation governing the axial heat transport in the canning can be written as (see appendix 1):

$$\frac{d^2T}{dx^2} - m^2 T = -\frac{m^2}{h} q \tag{1}$$

The homogeneous solution of (1) is given by:

$$T = A\, e^{mx} + B\, e^{-mx} \tag{2}$$

As can be seen from (2), the homogeneous solution only plays a part near boundaries. The perturbation of the temperature due to the end plug levels out very fast, and it has been verified that only the particular integral of (1) contributes to thermal stresses.

The particular integral of (1) is given by the following convolution integral (see appendix 1)

$$T(x) = \frac{m}{2h} \int_{-\infty}^{+\infty} q(y) \, e^{-m|x-y|} \, dy \qquad (3)$$

It is obvious that the contribution to the temperature at a place x of a flux at a place y levels out with increasing $|x-y|$ and the integral can be truncated at an appropriate value of $|x-y|$.

For later convenience the following abbreviations are introduced:

$$QM = \int_{-\infty}^{x} q(y) \, e^{-m(x-y)} \, dy \qquad (4)$$

$$QP = \int_{x}^{\infty} q(y) \, e^{-m(y-x)} \, dy \qquad (5)$$

Equation (3) can now be written as:

$$T = \frac{m}{2h} \, ( \, QM + QP) \qquad (6)$$

The integrals QM and QP can in some cases be calculated analytically. The purpose of this paper, however, is rather to develop a numerical method to handle empirical neutron flux distributions. In chapter 5 is explained, how by defining the flux level at discrete points $x_i$ the temperature at any point x can be computed from equation (6).

## 3. THERMAL STRESSES FOR A GIVEN TEMPERATURE DISTRIBUTION

In the classical theory for symmetrical loaded thin cylinders, the analogy of the elastic embedded beam is employed. This theory, which is given in handbooks on the subject (ref. 1 and 2), is repeated in appendix 2, including the correction for the temperature field and an approximate theory for finned cannings. It turns out that for finned cannings the differential equation governing the radial displacement of the canning wall is the same as for smooth cannings.

$$\frac{d^4 u}{dx^4} + 4\beta^4 u = 4\beta^4 \alpha aT \tag{1}$$

The homogeneous solution of this equation is dealt with in detail in handbooks on the subject (ref. 1 and 2). It can be written as:

$$u(x) = e^{-\beta x}(A \sin\beta x + B \cos\beta x) + e^{\beta x}(C \sin\beta x + D \cos\beta x) \tag{2}$$

As can be seen from (2), it is only related with boundary effects. The thermal stress problem due to axial temperature variations consists therefore rather in finding a particular integral of (1), taking care of the right-hand-side member. Eventually an homogeneous solution at the ends has to be added to satisfy the boundary conditions.

The particular integral of equation (1) is given by the convolution integral (see appendix 3):

$$u(x) = \alpha aT(x) + \frac{\alpha a}{4\beta} \int_{-\infty}^{+\infty} \frac{d^2 T(y)}{dy^2} \psi \left\{ \beta \, | \, x - y \, | \right\} dy$$

$$\tag{3}$$

$$\psi(\beta x) = e^{-\beta x}(\cos\beta x - \sin\beta x)$$

From the displacement field, the moment M, the shear force Q and the hoop force H are found by differentiating (see appendix 2).

For convenience the following abbreviations are introduced:

$$TCM = \int_{-\infty}^{x} \frac{d^2T(y)}{dy^2} \, e^{-\beta(x-y)} \, \cos\beta(x-y) \, dy \qquad (4)$$

$$TCP = \int_{x}^{\infty} \frac{d^2T(y)}{dy^2} \, e^{-\beta(y-x)} \, \cos\beta(y-x) \, dy \qquad (5)$$

$$TSM = \int_{-\infty}^{x} \frac{d^2T(y)}{dy^2} \, e^{-\beta(x-y)} \, \sin\beta(x-y) \, dy \qquad (6)$$

$$TSP = \int_{x}^{\infty} \frac{d^2T(y)}{dy^2} \, e^{-\beta(y-x)} \, \sin\beta(y-x) \, dy \qquad (7)$$

Furthermore, in order to facilitate the algebra, complex functions are introduced:

$$\psi(\beta x) = e^{-\beta x}(\cos\beta x - \sin\beta x)$$
$$= (Re - Im) \, e^{-\beta x(1-j)} \qquad (8)$$

$$\varphi(\beta x) = e^{-\beta x}(\cos\beta x + \sin\beta x)$$
$$= (Re + Im) \, e^{-\beta x(1-j)} \qquad (9)$$

$$TM = TCM + jTSM \qquad (10)$$

$$TP = TCP + jTSP \qquad (11)$$

The displacement field (3) can then be written as:

$$u = \alpha \, aT + \frac{\alpha a}{4\beta} (TCM + TCP - TSM - TSP) \qquad (12)$$

The derivatives become:

$$\frac{du}{dx} = \alpha \, a \, \frac{dT}{dx} - \frac{1}{2}\alpha \, a \, (TCM - TCP) \qquad (13)$$

$$\frac{d^2u}{dx^2} = \frac{1}{2} \alpha \, a \, \beta \, (\text{TCM} + \text{TCP} + \text{TSM} + \text{TSP}) \tag{14}$$

$$\frac{d^3u}{dx^3} = -\alpha a \beta^2 \, (\text{TSM} - \text{TSP}) \tag{15}$$

$$\frac{d^4u}{dx^4} = -\alpha a \beta^3 \, (\text{TCM} + \text{TCP} - \text{TSM} - \text{TSP}) \tag{16}$$

In chapter 5 is explained how by defining the temperature at discrete points $x_i$, the integrals TCM, TCP,...etc. can be computed for any value of x.

## 4. THERMAL STRESSES FOR GIVEN FLUX DISTRIBUTION

The relations derived in chapter 2 and 3 are combined. The displacement field is now given by the following convolution integral (see appendix 3):

$$
\begin{aligned}
u(x) = \;& \frac{\alpha a m}{2h} \int_{-\infty}^{+\infty} q(y) \, e^{-m|x-y|} \, dy \\
& - \frac{\alpha a m^2}{4\beta h} \; \frac{1}{1+4 \, (\frac{\beta}{m})^4} \left\{ 4(\frac{\beta}{m})^4 \int_{-\infty}^{+\infty} q(y) \, \psi( \, \beta|x-y| \, ) \, dy \right. \\
& \qquad\qquad - 2(\frac{\beta}{m})^2 \int_{-\infty}^{+\infty} q(y)\varphi \, ( \beta \, |x-y| \, )\,dy \\
& \qquad\qquad \left. + 2(\frac{\beta}{m}) \int_{-\infty}^{+\infty} q(y) \, e^{-m|x-y|} \, dy \right\}
\end{aligned} \tag{1}
$$

For convenience the following abbreviations are introduced:

$$\text{MQM} = \int_{-\infty}^{x} q(y) \, e^{-m(x-y)} \, dy \tag{2}$$

$$\text{MQP} = \int_{x}^{\infty} q(y) \, e^{-m(y-x)} \, dy \tag{3}$$

$$\text{QCM} = \int_{-\infty}^{x} q(y) \, e^{-\beta(x-y)} \cos\beta \, (x-y) \, dy \tag{4}$$

$$\text{QCP} = \int_{x}^{\infty} q(y) \, e^{-\beta(y-x)} \cos \beta \, (y-x) \, dy \tag{5}$$

$$QSM = \int_{-\infty}^{x} q(y)\, e^{-\beta(x-y)} \sin\beta\,(x-y)\, dy \qquad (6)$$

$$QSP = \int_{x}^{\infty} q(y)\, e^{-\beta(y-x)} \sin\beta\,(y-x)\, dy \qquad (7)$$

The displacement field can then be written as:

$$u = \frac{\alpha\,am}{2h}\,(MQM + MQP)$$

$$- \frac{\alpha\,am^2}{4\,\beta\,h}\,\frac{1}{1+4\left(\frac{\beta}{m}\right)^4}\left[\, 4\left(\frac{\beta}{m}\right)^4\,(QCM + QCP - QSM - QSP)\right.$$

$$- 2\left(\frac{\beta}{m}\right)^2\,(QCM + QCP + QSM + QSP) \qquad (8)$$

$$\left. + 2\left(\frac{\beta}{m}\right)\,(MQM + MQP)\right]$$

The derivatives become:

$$\frac{du}{dx} = -\frac{\alpha\,am^2}{2h}\,(MQM - MQP)$$

$$+ \frac{\alpha\,am^2}{2h}\,\frac{1}{1+4\left(\frac{\beta}{m}\right)^4}\left[\, 4\left(\frac{\beta}{m}\right)^4\,(QCM - QCP)\right.$$

$$- 2\left(\frac{\beta}{m}\right)^2\,(QSM - QSP) \qquad (9)$$

$$\left. + (MQM - MQP)\right]$$

$$\frac{d^2u}{dx^2} = \frac{\alpha\,am^3}{2h}\,(MQM + MQP)$$

$$- \frac{\alpha\,am^2}{2h}\,\frac{\beta}{1+4\left(\frac{\beta}{m}\right)^4}\left[\, 4\left(\frac{\beta}{m}\right)^4\,(QCM + QSM + QCP + QSP)\right.$$

$$+ 2\left(\frac{\beta}{m}\right)^2\,(QCM + QCP - QSM - QSP) \qquad (10)$$

$$\left. + \left(\frac{m}{\beta}\right)\,(MQM + MQP)\right]$$

$$\frac{d^3 u}{dx^3} = -\frac{\alpha\,am^4}{2h}\ (MQM - MQP)$$

$$+ \frac{\alpha am^2}{2h}\ \frac{2\,\beta^2}{1+4(\frac{\beta}{m})^4}\ \left[\ 4(\frac{\beta}{m})^4\ (QSM - QSP)\right.$$

$$+\ 2(\frac{\beta}{m})^2\ (QCM - QCP)$$

$$\left.-\ \frac{1}{2}(\frac{m}{\beta})^2\ (MQM - MQP)\ \right] \qquad (11)$$

$$\frac{d^4 u}{dx^4} = \frac{\alpha\,am^5}{2h}\ (MQM + MQP)$$

$$+ \frac{\alpha am^2}{2h}\ \frac{2\,\beta^3}{1+4(\frac{\beta}{m})^4}\ \left[\ 4(\frac{\beta}{m})^4\ (QCM + QCP - QSM - QSP)\right.$$

$$-\ 2(\frac{\beta}{m})^2\ (QCM + QSM + QCP + QSP)$$

$$\left.-\ \frac{1}{2}(\frac{m}{\beta})^3\ (MQM + MQP)\ \right] \qquad (12)$$

In chapter 5 is explained how by defining the flux level at discrete points $x_i$ the integrals QCM, QCP etc. can be computed for any value of x.

## 5. NUMERICAL METHODS

As shown in the previous chapters the determination of the temperature and thermal stress distribution can be reduced to the evaluation of the convolution integrals of the type:

$$I(y) = \int_{-\infty}^{+\infty} q(x)\ e^{-\beta\,|x-y|}\ \cos\beta\,|x-y|\ dx \qquad (1)$$

This kind of integrals can be evaluated analytically for a lot of functions, but the actual determination of the maximum stress and the place where it occurs remains a laborious algebraic computation.

It proved to be worthwhile to automize this kind of computations by means of a digital computer, enabling also to work directly with empirical flux distributions.

Consider for that a flux distribution defined in N discrete points:

$$(x_i, \ q_i) \qquad\qquad i = 1 \rightarrow N \qquad\qquad (2)$$
$$x_i < x_{i+1}$$

Assuming a linear distribution of the flux between two adjacent points, the function q is then represented by:

$$q(x) = A_i + B_i x$$

$$B_i = \frac{q_{i+1} - q_i}{x_{i+1} - x_i} \qquad\qquad (3)$$

$$A_i = q_i - B_i x_i$$

The flux level for $x < x_1$ and $x > x_N$ is taken equal to $q_1$ respectively $q_N$: Their contribution to the thermal stress at points between $x_1$ and $x_N$ decreases very rapidly with the distance to $x_1$ respectively $x_N$.

For $x_M < y < x_{M+1}$ the integral QCM, for example, can then be written as:

$$QCM = \int_{-\infty}^{x_1} q_1 \ e^{-\beta(y-x)} \ \cos\beta(y-x) \ dx$$

$$+ \ \sum_{i=1}^{M-1} \int_{x_i}^{x_{i+1}} (A_i + B_i x) \ e^{-\beta(y-x)} \ \cos\beta(y-x) \ dx \qquad\qquad (4)$$

$$+ \int_{x_M}^{y} (A_M + B_M x) \ e^{-\beta(y-x)} \ \cos\beta(y-x) \ dx$$

These integrals can be evaluated analytically, by partial integration or by introducing complex functions as in chapter 3. Introducing the abbreviations:

$$FC\ (x) = e^{-\beta x}\cos\beta\ x$$

$$FS\ (x) = e^{-\beta x}\sin\beta\ x \tag{5}$$

$$F\ (x) = e^{-mx}$$

the following expressions are obtained:

$$
\begin{aligned}
QCM = {} & \frac{1}{2\beta}\ A_o\ (\ FC(y-x_1) - FS(y-x_1)\ ) \\[6pt]
& + \frac{1}{2\beta}\ \sum_{i=1}^{M-1}\ (A_i+B_i x_{i+1})\ (\ FC(y-x_{i+1})-FS(y-x_{i+1})\ ) \\[6pt]
& - \frac{1}{2\beta}\ \sum_{i=1}^{M-1}\ (A_i+B_i x_i)\ (\ FC(y-x_i)-FS(y-x_i)\ ) \\[6pt]
& + \frac{1}{2\beta^2}\ \sum_{i=1}^{M-1}\ B_i\ (\ FS(y-x_{i+1}) - FS(y-x_i)\ ) \\[6pt]
& + \frac{1}{2\beta}\ (A_M + B_M\ y) \\[6pt]
& - \frac{1}{2\beta}\ (A_M + B_M\ x_M)\ (\ FC(y-x_M) - FS(y-x_M)\ ) \\[6pt]
& - \frac{1}{2\beta^2}\ B_M\ FS\ (y - x_M)
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
QSM = {} & \frac{1}{2\beta}\ A_o\ (\ FC(y - x_1) + FS(y - x_1)\ ) \\[6pt]
& + \frac{1}{2\beta}\ \sum_{i=1}^{M-1}\ (A_i+B_i x_{i+1})\ (\ FC(y-x_{i+1})+FS(y-x_{i+1})\ ) \\[6pt]
& - \frac{1}{2\beta}\ \sum_{i=1}^{M-1}\ (A_i + B_i x_i)\ (\ FC(y-x_i) + FS(y-x_i)\ ) \\[6pt]
& - \frac{1}{2\beta^2}\ \sum_{i=1}^{M-1}\ B_i\ (\ FC(y - x_{i+1}) - FC(y-x_i)\ ) \\[6pt]
& + \frac{1}{2\beta}\ (A_M + B_M\ y) \\[6pt]
& - \frac{1}{2\beta}\ (A_M + B_M x_M)\ (\ FC(y-x_M) + FS(y-x_M)\ ) \\[6pt]
& - \frac{1}{2\beta^2}\ B_M\ (\ 1 - FC\ (y - x_M)\ )
\end{aligned} \tag{7}
$$

$$QCP = -\frac{1}{2\beta} (A_M + B_M x_{M+1}) ( FC(x_{M+1} - y) - FS(x_{M+1} - y) )$$

$$+ \frac{1}{2\beta} (A_M + B_M y)$$

$$+ \frac{1}{2\beta^2} B_M \; FS(x_{M+1} - y)$$

$$- \frac{1}{2\beta} \sum_{i=M+1}^{N-1} (A_i + B_i x_{i+1}) ( FC(x_{i+1} - y) - FS(x_{i+1} - y) ) \qquad (8)$$

$$+ \frac{1}{2\beta} \sum_{i=M+1}^{N-1} (A_i + B_i x_i) \quad ( FC(x_i - y) - FS(x_i - y) )$$

$$+ \frac{1}{2\beta^2} \sum_{i=M+1}^{N-1} B_i ( FS(x_{i+1} - y) - FS(x_i - y) )$$

$$+ \frac{1}{2\beta} A_N ( FC(x_N - y) - FS(x_N - y) )$$

$$QSP = -\frac{1}{2\beta} (A_M + B_M x_{M+1}) ( FC(x_{M+1} - y) + FS(x_{M+1} - y) )$$

$$+ \frac{1}{2\beta} (A_M + B_M y)$$

$$- \frac{B_M}{2\beta^2} ( FC(x_{M+1} - y) - 1 )$$

$$- \frac{1}{2\beta} \sum_{i=M+1}^{N-1} (A_i + B_i x_{i+1}) ( FC(x_{i+1} - y) + FS(x_{i+1} - y) ) \qquad (9)$$

$$+ \frac{1}{2\beta} \sum_{i=M+1}^{N-1} (A_i + B_i x_i) \quad ( FC(x_i - y) + FS(x_i - y) )$$

$$- \frac{1}{2\beta^2} \sum_{i=M+1}^{N-1} B_i ( FC(x_{i+1} - y) - FC(x_i - y) )$$

$$+ \frac{1}{2\beta} A_N ( FC(x_N - y) + FS(x_N - y) )$$

$$MQM = \frac{1}{m} A_o F(y - x_1)$$

$$+ \frac{1}{m} \sum_{i=1}^{M-1} (A_i + B_i x_{i+1}) F(y - x_{i+1})$$

$$- \frac{1}{m} \sum_{i=1}^{M-1} (A_i + B_i x_i) F(y - x_i)$$

$$- \frac{1}{m^2} \sum_{i=1}^{M-1} B_i \left( F(y - x_{i+1}) - F(y - x_i) \right) \qquad (10)$$

$$+ \frac{1}{m} (A_M + B_M y)$$

$$- \frac{1}{m} (A_M + B_M x_M) F(y - x_M)$$

$$- \frac{1}{m^2} B_M \left( 1 - F(y - x_M) \right)$$

$$MQP = - \frac{1}{m} (A_M + B_M x_{M+1}) F(x_{M+1} - y)$$

$$+ \frac{1}{m} (A_M + B_M y)$$

$$- \frac{1}{m^2} B_M \left( F(x_{M+1} - y) - 1 \right)$$

$$- \frac{1}{m} \sum_{i=M+1}^{N-1} (A_i + B_i x_{i+1}) F(x_{i+1} - y) \qquad (11)$$

$$+ \frac{1}{m} \sum_{i=M+1}^{N-1} (A_i + B_i x_i) F(x_i - y)$$

$$- \frac{1}{m^2} \sum_{i=M+1}^{N-1} B_i \left( F(x_{i+1} - y) - F(x_i - y) \right)$$

$$+ \frac{1}{m} A_N F(x_N - y)$$

The value of M is defined by $x_M \leq y < x_{M+1}$

In cases where the temperature distribution is known from measurements, the equations derived in chapter 3 are to be used.

Consider for this a temperature distribution defined in N discrete points:

$$(x_i, T_i) \qquad i = 1 \to N \qquad x_i < x_{i+1} \tag{12}$$

Assuming a linear temperature distribution between two adjacent points $x_i$ and $x_{i+1}$, the change in gradient at a point $x_i$ is defined by

$$D2T_i = \frac{T_{i+1} - T_i}{x_{i+1} - x_i} - \frac{T_i - T_{i-1}}{x_i - x_{i-1}} \tag{13}$$

The integrals TCM, TCP etc. as defined in chapter 3 become:

$$TCM = \sum_{i=1}^{M} D2T_i \; FC \, (y - x_i) \tag{14}$$

$$TCP = \sum_{i=M+1}^{N} D2T_i \; FC \, (x_i - y) \tag{15}$$

$$TSM = \sum_{i=1}^{M} D2T_i \; FS \, (y - x_i) \tag{16}$$

$$TSP = \sum_{i=M+1}^{N} D2T_i \; FS \, (x_i - y) \tag{17}$$

M is defined by $\quad x_M < y < x_{M+1}$

Equations (6), (7).....(11) and (14), (15), (16) and (17) are the basic equations of the code ATEAS.

6. NUMERICAL APPLICATION TO A G 19 FINNED CANNING

In fig. 10 is given an enlarged photograph of the cross-section of a G 19 finned canning. Comparison with Fig. 6 yields the following values of the dimensions of the fin:

$$e_1 = 0.094 \text{ cm} \qquad\qquad b_1 = 0.042 \text{ cm}$$
$$e_2 = 0.194 \text{ cm} \qquad\qquad b_2 = 0.025 \text{ cm}$$
$$a = 0.78 \text{ cm} \qquad\qquad b_3 = 0.038 \text{ cm}$$

From these values ATEAS determined the parameters m and $\beta$

$\beta$ = 3.34 cm$^{-1}$

m = 2.611 cm$^{-1}$ (h = 1 Watt/cm$^2$ $^\circ$C, $\lambda$ = 1.8 Watt/cm $^\circ$C)

m = 3.692 cm$^{-1}$ (h = 2 Watt/cm$^2$ $^\circ$C, $\lambda$ = 1.8 Watt/cm $^\circ$C)

The flux distributions are chosen as given in Fig. 11:

Nominal value: q = 135 Watt/cm$^2$ on finned surface

Flux peak varying from 0% up to 50% over a length of 6 cm.

The corresponding temperature distributions, as calculated by ATEAS, are given in Fig. 11, respectively for h = 1 and h = 2 Watt/cm$^2$ $^\circ$C, and for the various flux peaks as mentioned before.

In Fig. 12 the temperature distribution for h = 1 Watt/cm$^2$ $^\circ$C is repeated on a larger scale. The bending stresses respectively at the top of the fin and at the inner canning wall, calculated for a Young's modulus E = 400,000 kg/cm$^2$, are also represented in Fig. 12.

In Fig. 13, the curves of Fig. 12 are repeated for the case of h = 2 Watt/cm$^2$ $^\circ$C.

It is noted that the stresses are proportional to the nominal value of the flux and also to Young's modulus, and can therefore easily be determined for other values of nominal flux and Young's modulus.

The equivalent stress at the corner of the fin, calculated according to the Huber deformation energy criterion, is small as compared to the bending stress (see Fig. 14). It can therefore be ignored in the design philosophy. This confirms that the deformation due to shear stresses is small as compared to the deformation due to bending stresses, which is one of the assumptions of the theory.

As can be seen from figures 12 and 13, the thermal bending stress phenomenon due to axial temperature variations is a local one, and it does not influence eventual bending stresses at the weld of canning to end plug, if the distance between fuel pellets and weld is bigger than about 1 cm (for G 19).

It might be of interest to compare the refined theory for finned cannings with the one for smooth cannings, and to try to determine an equivalent canning wall thickness for the homogenized finned canning. It will be shown that different equivalent thicknesses are to be applied for the temperature and for the stress calculation.

For $h = 1$ Watt/cm$^2$ $^{\circ}$C the parameter m for G 19 was equal to

$$m = 2.611 \text{ cm}^{-1}$$

According to equation (1) of appendix 1 the equivalent thickness for an homogenized G 19 canning becomes

$$e = \frac{h}{\lambda m^2} = 0.0815 \text{ cm.}$$

It turns out that the equivalent thickness is about 12% lower than the cylinder wall thickness of the finned canning ($e_1 = 0.094$ cm). Physically it means that the additional surface (wetted periphery) introduced by the fin has a greater influence on the axial conduction process than the additional cross-section, a phenomenon which verifies itself also for radial conduction.

As concerns the stress calculation, an equivalent thickness can be computed from equation (8) of appendix 2, in such a way that the elastic wave length is the same:

$$e = \sqrt{\frac{3(1 - \gamma^2)}{a \beta^2}} = 0.1892 \text{ cm}$$

$$( \beta = 3.34 \text{ cm}^{-1} )$$

This equivalent thickness is about 2% lower than the height of the fins ($e_2 = 0.194$). This is accidental and cannot be taken as a general law. The wave length $\beta$ is determined by the ratio of the circumferential rigidity (tension) to the axial rigidity (bending), both per unit length (equation 8, appendix 2). To obtain for the same wave length equal stresses, an equivalent thickness should be introduced such that the distances from neutral grain to grain of maximum bending stress

are equal:

$$e = 2 (e_2 - h_n)$$

For a G 19 canning $h_n = 0.0785$ cm

$$e = 0.231 \text{ cm.}$$

This equivalent thickness is about 20% higher than the height of the fins.

Summarizing, it seems not to be possible to homogenize a finned canning by introducing a kind of average thickness, but m, $\beta$ and $h_n$ are to be determined from the formulae of appendices 1 and 2.

## 7. CONCLUSION AND FINAL REMARKS

A theory has been developed to determine the temperature and thermal stress distribution due to axial flux variations for smooth and for finned cannings, assuming a linear stress-strain relation (Hook's law). The digital code ATEAS handles either empirical flux distributions or empirical temperature distributions. The latter is of interest for cases where the temperature distribution is calculated by taking into account the contribution of the fuel in the axial conduction (see ref. 3), which is neglected in this paper.

From the numerical example of chapter 6 follows that the phenomenon is local, and stresses vanish at about 1 cm distance of the flux perturbation (G 19).

For a G 19 finned canning with

| | |
|---|---|
| nominal flux | $q = 135$ Watt/cm$^2$ |
| peak | 20 % |
| heat transfer | $h = 2$ Watt/cm$^2$ $^\circ$C |
| Young's Modulus | $E = 4000$ kg/mm$^2$ |
| ($450^\circ$C) | |

the maximum bending stress at the top of the fin is equal

to

$$\sigma = 1.1 \ kg/mm^2$$

Shear stresses can be neglected in the design philosophy.

    If, like in other than G 19 ORGEL type of cannings, spacer fins are present, it is recommended to cut them in regions of accentuated temperature variations, in order to prevent excessive bending stresses at their outer grains.

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   "Etudes des perturbations circonférentielles et axiales
   de température dans une gaine cylindrique d'élément
   combustible (Méthodes mathématique et rhéoanalogique)"
   EUR 2480 f (1965)

# N O M E N C L A T U R E

$a$     radius of canning

$A_i$     constants in flux functions

$A$     integration constant

$b$     width

$b_1$   )
$b_2$   )   dimensions defining the finned canning wall cross-section
$b_3$   )

$B_i$     constant in flux functions

$e$     wall thickness of smooth canning

$e_1$     wall thickness of finned canning

$e_2$     height of fin

$h$     heat transfer coefficient

$h_n$     coordinate of neutral grain

$i$     index indicating axial position

$j$     $\sqrt{-1}$

$m$     thermal wave length

$p$     pressure

$q$     flux

$Q$     Fourier transform of flux

$s$     Fourier coordinate

$S$     cross-section of finned canning wall

$t$     imaginary coordinate in z plane

$T$     temperature

$u$     radial displacement

$U$     Fourier transform of radial displacement

$x$     axial coordinate

$y$     auxiliary axial coordinate

$z$     complex coordinate

$\alpha$     linear thermal **dilatation** coefficient

$\beta$     elastic wave length

$\xi$     auxiliary axial coordinate

$\theta$     Fourier transform of temperature

$\lambda$     conductivity of canning

## DIFFERENTIAL EQUATION GOVERNING THE AXIAL TEMPERATURE

## DISTRIBUTION AND ITS SOLUTION

### A. Canning without fins

A heat balance in axial direction yields (see Fig. 1):

$$- \lambda e \frac{d^2 T}{dx^2} = q - h T \qquad (1)$$

with $m = \sqrt{\frac{h}{\lambda e}}$ one obtains

$$\frac{d^2 T}{dx^2} - m^2 T = - \frac{m^2}{h} q \qquad (2)$$

### B. Canning with fins

Consider the cross section as given in Fig. 2.

$$- \lambda S \frac{d^2 T}{dx^2} = 2q(b_1 + b_2 + b_3) - 2h \left( b_1 + b_3 + \sqrt{b_2^2 + (e_2 - e_1)^2} \right) \qquad (3)$$

with

$$m = \sqrt{\frac{h}{\lambda} \frac{b_1 + b_3 + \sqrt{b_2^2 + (e_2 - e_1)^2}}{b_1 e_1 + b_3 e_2 + \frac{1}{2} b_2 (e_2 + e_1)}} \qquad (4)$$

and

$$FC = \frac{b_1 + b_3 + \sqrt{b_2^2 + (e_2 - e_1)^2}}{b_1 + b_2 + b_3} \qquad (5)$$

one obtains:

$$\frac{d^2 T}{dx^2} - m^2 T = - \frac{m^2}{h} \left( \frac{q}{FC} \right) \qquad (6)$$

Equation (6) is similar to equation (2), if for finned cannings the flux is divided by the finning coefficient FC.

## C. Solution of the heat transport equation

The Fourier transforms of T and q are defined by:

$$\theta(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} T(x) \, e^{jsx} \, dx \tag{7}$$

$$Q(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} q(x) \, e^{jsx} \, dx \tag{8}$$

The inverse transforms are defined by:

$$T(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \theta(s) \, e^{-jsx} \, ds \tag{9}$$

$$q(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} Q(s) \, e^{-jsx} \, ds \tag{10}$$

If $f_1(s)$ and $f_2(s)$ represent respectively the Fourier transform of the functions $F_1(x)$ and $F_2(x)$, then the convolution integral of $F_1$ and $F_2$ is defined by:

$$\int_{-\infty}^{+\infty} F_1(\xi) \, F_2(x-\xi) \, d\xi = \int_{-\infty}^{+\infty} F_1(x-\xi) \, F_2(\xi) \, d\xi =$$

$$= \int_{-\infty}^{\infty} f_1(s) \, f_2(s) \, e^{-jsx} \, ds \tag{11}$$

Inserting the transforms into the differential equation (2) yields:

$$\theta = \frac{m^2}{h} \frac{Q}{s^2 + m^2} \tag{12}$$

Applying the inverse transform gives:

$$T(x) = \frac{1}{\sqrt{2\pi}} \frac{m^2}{h} \int_{-\infty}^{+\infty} \frac{Q(s) \, e^{-jsx}}{s^2 + m^2} \, ds \tag{13}$$

Comparing (13) with (11) and calling $Q(s) = f_1(s)$ and $\frac{1}{s^2+m^2} = f_2(s)$, equation (13) becomes:

$$T(x) = \frac{1}{\sqrt{2\pi}} \frac{m^2}{h} \int_{-\infty}^{+\infty} q(\xi) \, F_2(x-\xi) \, d\xi \qquad (14)$$

The function $F_2(x)$ is defined from the inverse of $f_2(s)$

$$F_2(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{e^{-jsx}}{s^2+m^2} \, ds \qquad (15)$$

The latter integral is calculated by contour integration. Suppose x positive and consider the contour as presented in Fig. 3. For $R \to \infty$ the integrand becomes zero at the circle:

$$\oint_{R \to \infty} \frac{e^{-jzx}}{z^2+m^2} \, dz = \int_{+\infty}^{-\infty} \frac{e^{-jsx}}{s^2+m^2} \, ds \qquad (16)$$

The poles of the integrand are at $z = \pm jm$ and it is observed that only the pole $z = -jm$ is inside the contour. The residue at the pole $z = -jm$ becomes

$$\text{residue} = \frac{e^{-mx}}{-2jm} \qquad (17)$$

$$\int_{+\infty}^{-\infty} \frac{e^{-jsx}}{s^2+m^2} \, ds = 2\pi j \, \Sigma \text{ residue's} \qquad (18)$$

$$= 2\pi \left(\frac{e^{-mx}}{-2m}\right)$$

Finally, still for positive x, one gets

$$F_2(x) = \sqrt{2\pi} \, \frac{e^{-mx}}{2m} \qquad (19)$$

For physical reasons it is obvious that $F_2$ should be an even function and therefore:

$$F_2(x) = \sqrt{2\pi} \; \frac{e^{-m|x|}}{2m} \tag{20}$$

The temperature is then given by the following convolution integral:

$$T(x) = \frac{m}{2h} \int_{-\infty}^{+\infty} q(\xi) \; e^{-m|x-\xi|} \; d\xi \tag{21}$$

## THE DIFFERENTIAL EQUATION GOVERNING THE DISPLACEMENT

## OF A THIN CYLINDER WITH SYMMETRICAL LOADING

In order to derive a correction for finned cannings, the classical theory as presented in handbooks on the subject (ref. 1 and 2) is repeated.

A. Canning without fins

Consider an elementary part of the cylinder as depicted in Fig. 4. Due to symmetry its displacement is only in radial direction and denoted by u, positive when the radius tends to become bigger. The elementary part is regarded as a cantilever, loaded by forces and moments as given in Fig. 4. Deformations due to the shear force are neglected, which is by the way usual for cantilevers which are not high as compared to their length.

Application of the elementary bending theory yields:

$$\frac{E\ e^3}{12}\ \frac{d^2u}{dx^2} = M_{x\varphi} - \Upsilon\ M_{\varphi x}$$

$$\frac{E\ e^3}{12} \left(\frac{1}{a} - \frac{1}{a+u}\right) = -\Upsilon M_{x\varphi} + M_{\varphi x}$$

$$(1)$$

The change in curvature in tangential direction is small of the second order:

$$\frac{1}{a} - \frac{1}{a+u} \approx \frac{u}{a^2} \approx 0 \qquad\qquad (2)$$

Equations (1) become:

$$M_{\varphi x} = \Upsilon M_{x\varphi}$$

$$(3)$$

$$\frac{E\ e^3}{12}\ \frac{d^2u}{dx^2} = (1 - \Upsilon^2)\ M_{x\varphi}$$

The hoop force H is related to the displacement by

$$H = E e (\frac{u}{a} - \alpha T) \qquad (4)$$

Equations (3) and (4) are the stress-displacement relations of the problem.

The equilibrium equations follow from Fig. 4.

$$\frac{dQ}{dx} = -p + \frac{H}{a} \qquad (5)$$

$$\frac{dM}{dx} = -Q \qquad (6)$$

Combination of (3), (4), (5) and (6) yields:

$$\frac{d^4u}{dx^4} + 4\beta^4 u = 4\beta^4 (\alpha a T + \frac{pa^2}{e E}) \qquad (7)$$

$$4\beta^4 = \frac{12(1-\Upsilon^2)}{a^2 e^2} \qquad (8)$$

## B. Canning with fins

Consider the elementary part as given in Fig. 5. As compared to Fig. 4, the equilibrium equations are not changed. Only the relations between the displacement u and the forces and moments have to be changed.

As regards bending in axial direction, the inertia moment of the complicated cross-section (Fig 6) should be introduced. As in elementary bending theory a linear stress distribution can be assumed.

As regards bending in tangential direction (Fig. 7) the contribution of the fin is more difficult to calculate. In the canning without fin, the anti-clastical bending in tangential direction causes an increase of rigidity in axial direction of about 10% ($\frac{1}{1-\Upsilon^2} - 1 \approx 0.1$). It is therefore less important to know the exact elasticity in tangential direction. An easy way to estimate it, is to assume a linear stress distribution in every cross-section in $\varphi$ direction.

The hoop force H causes a displacement u, which can be calculated by assuming an uniform stress distribution in every cross-section in $\varphi$ direction (Fig. 8).

Adopting the above mentioned hypothesises yields:

$$\frac{d^2 u}{dx^2} = \frac{M_{x\varphi}}{E\ I_{x\varphi}} - \Upsilon \frac{M_{\varphi x}}{E\ I_{\varphi x}} \tag{9}$$

$$\frac{1}{a} - \frac{1}{a+u} = \frac{-\Upsilon M_{x\varphi}}{E\ I_{x\varphi}} + \frac{M_{\varphi x}}{E\ I_{\varphi x}} \approx 0 \tag{10}$$

$$M_{\varphi x} = \Upsilon \frac{I_{\varphi x}}{I_{x\varphi}} M_{x\varphi} \tag{11}$$

$$\frac{d^2 u}{dx^2} = \frac{(1-\Upsilon^2)\ M_{x\varphi}}{E\ I_{x\varphi}} \tag{12}$$

$$I_{x\varphi} = \iint (z-h_n)^2\ dA \qquad \text{(see Fig. 6)}$$

$$h_n = \frac{\iint z\ dA}{\iint dA} \qquad \text{(see Fig. 6)} \tag{13}$$

$$I_{x\varphi} = \frac{1}{3(b_1+b_2+b_3)}\left[ b_1\left( (e_1-h_n)^3 + h_n^3 \right) + b_3\left( (e_2-h_n)^3 + h_n^3 \right) \right.$$

$$\left. + b_2\left(\left((e_2-h_n)^2 + (e_1-h_n)^2\right)\left(\frac{e_1 + e_2 - 2h_n}{4}\right) + h_n^3\right) \right] \tag{14}$$

$$h_n = \frac{1}{2}\ \frac{b_1 e_1^2 + b_3 e_2^2 + \frac{1}{3}b_2(e_1^2 + e_1 e_2 + e_2^2)}{b_1 e + \frac{1}{2}b_2(e_1 + e_2) + b_3 e_2} \tag{15}$$

$$I_{\varphi x} = \frac{b_1+b_2+b_3}{12\int \frac{db}{e^3}} \qquad \text{(see Fig. 7)} \tag{16}$$

$$I_{\varphi x} = \frac{b_1 + b_2 + b_3}{12 \left( \dfrac{b_1}{e_1^3} + \dfrac{b_2(e_1 + e_2)}{2e_1^2 e_2^2} + \dfrac{b_3}{e_2^2} \right)} \tag{17}$$

$$H = E \left( \frac{u}{a} - \alpha T \right) h_1 \tag{18}$$

$$h_1 = \frac{b_1 + b_2 + b_3}{\displaystyle\int \frac{db}{e}} \qquad \text{(see Fig. 8)} \tag{19}$$

$$h_1 = \frac{b_1 + b_2 + b_3}{\dfrac{b_1}{e_1} + \dfrac{b_2}{e_2 - e_1} \ln\dfrac{e_2}{e_1} + \dfrac{b_3}{e_2}} \tag{20}$$

The differential equation for the displacement becomes:

$$\frac{d^4 u}{dx^4} + 4 \beta^4 u = 4 \beta^4 \left( \alpha aT + \frac{pa^2}{Eh_1} \right)$$

$$4 \beta^4 = \frac{(1 - \gamma^2) h_1}{I_{x\varphi} a^2} \tag{21}$$

## C. Stresses as function of the moments and forces

### 1. Canning without fins

Axial bending stress

$$\sigma_{bx} = \begin{matrix} - \\ + \end{matrix} \quad \frac{6 M_{x\varphi}}{e^2} \qquad \begin{matrix} - \text{ outer wall} \\ + \text{ inner wall} \end{matrix} \tag{22}$$

Tangential bending stress

$$\sigma_{b\varphi} = \begin{matrix} - \\ + \end{matrix} \quad \frac{6 \gamma M_{x\varphi}}{e^2} \qquad \begin{matrix} - \text{ outer wall} \\ + \text{ inner wall} \end{matrix} \tag{23}$$

Hoop stress

$$\sigma_{H\varphi} = \frac{H}{e} \tag{24}$$

Shear stress in midplane

$$\tau_{x\varphi} = \frac{3}{2} \frac{Q}{e} \qquad \text{(parabolic distribution according to elementary bending theory)} \qquad (25)$$

## 2. Canning with fins

Axial bending stress in outer grain of fin:

$$\sigma_{b,o,x} = -\frac{M}{I_{x\varphi}} (e_2 - h_n) \qquad (26)$$

Axial bending stress at interior canning wall:

$$\sigma_{b,i,x} = \frac{M}{I_{x\varphi}} h_n \qquad (27)$$

Tangential bending stress in cylinder wall:

$$\sigma_{b\varphi} = \frac{-}{+} \frac{6 M_{\varphi x}}{e_1^2} = \frac{-}{+} \frac{\Upsilon I_{\varphi x}}{I_{x\varphi}} M_{x\varphi} \frac{6}{e_1^2} \qquad (28)$$

- outer wall
+ inner wall

Tangential hoop stress

$$\sigma_{H,\varphi} = \frac{H}{e_1} \qquad (29)$$

It is noted that in the outer grain of the fin a maximum bending stress in axial direction occurs. In this grain no tangential bending stress exists. As other critical points should be mentioned the corner of the fin and the cylinder wall. The axial bending stress at that point is probably small:

$$\sigma = -E \frac{d^2 u}{dx^2} (e_1 - h_n) \qquad (30)$$

The tangential bending stress (28) and the hoop stress (29) might be rather big. Besides an important shear stress occurs due to the varying bending moment in axial direction. As in elementary bending theory this

shear stress is calculated from the equilibrium equation in axial direction (see Fig. 9).

$$(b_2+b_3)\tau = -(b_1+b_2+b_3) \int_0^{e_1} \frac{d\sigma}{dx} \, d\xi \tag{31}$$

$$\tau_{x\varphi} = -\frac{b_1+b_2+b_3}{b_1+b_2} E \frac{d^3u}{dx^3} \left(h_n \, e_1 - \frac{1}{2} \, e_1^2\right) \tag{32}$$

According to the Huber rupture hypothesis the equivalent stress at the corner of the fin becomes:

$$\sigma_e = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}$$

$$= \sqrt{(\sigma_1 + \sigma_2 + \sigma_3)^2 - 3(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)} \tag{33}$$

$$= \sqrt{(\sigma_x+\sigma_y+\sigma_z)^2 - 3(\sigma_x\sigma_y - \tau_{xy}^2 + \sigma_y\sigma_z - \tau_{yz}^2 + \sigma_z\sigma_x - \tau_{xz}^2)}$$

$$\sigma_e = \sqrt{(\sigma_a+\sigma_t+\sigma_H)^2 - 3\sigma_a(\sigma_t+\sigma_H) + 3\tau^2} \tag{34}$$

$\sigma_a$ = axial bending stress (30)

$\sigma_t$ = tangential bending stress (28)

$\sigma_H$ = tangential hoop stress (29)

$\tau$ = shear stress (32)

## SOLUTION OF THE DIFFERENTIAL EQUATION GOVERNING

## THE CYLINDER WALL DISPLACEMENT

1. Relation between displacement and temperature

The Fourier transform of the displacement field is
defined by:

$$U(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} U(x) \, e^{jsx} \, dx \tag{1}$$

From the differential equation one obtains:

$$U(s) = 4 \beta^4 \alpha a \frac{\theta}{s^4 + 4\beta^4} \tag{2}$$

(N.B. The pressure term is omitted for convenience, because
it is usually constant in x direction. It is noted, that a
varying pressure term can be treated in the same way as the
temperature term).

Application of the inverse transform yields:

$$u(x) = 4 \beta^4 \alpha a \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{\theta \, e^{-jsx}}{s^4 + 4\beta^4} \, ds \tag{3}$$

As in appendix 2 the integrand is regarded as the product
of two functions $f_1(s)$ and $f_2(s)$, which are the transforms
of two other functions $F_1(x)$ and $F_2(x)$.

$$u(x) = 4 \beta^4 \alpha a \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} T(\xi) \, F_2(x-\xi) \, d\xi \tag{4}$$

$$F_2(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{e^{-jsx}}{s^4 + 4\beta^4} \tag{5}$$

The latter integral is evaluated by contour integration of the integral:

$$\oint \frac{e^{-jzx}}{z^4 + 4\beta^4} = 2\pi j \; \Sigma \; \text{residue's} \tag{6}$$

For positive x the contour is taken as in Fig. 3. The integrand becomes zero on the circle, when $R \to \infty$:

$$\oint_{R \to \infty} \frac{e^{-jzx}}{z^4 + 4\beta^4} = \int_{\infty}^{-\infty} \frac{e^{-jsx}}{s^4 + 4\beta^4} \, ds \tag{7}$$

The poles are $z = \beta(1-j)$ and $z = -\beta(1+j)$

The function $F_2$ becomes:

$$F_2(x) = \sqrt{2\pi} \left( \frac{e^{j\beta(1+j)x}}{1+j} + \frac{e^{-j\beta(1-j)x}}{1-j} \right) \frac{1}{8\beta^3}$$

$$= \frac{1}{8\beta^3} \sqrt{2\pi} \; e^{-\beta x}(\cos\beta\,x + \sin\beta\,x) \tag{9}$$

$$= \frac{1}{8\beta^3} \sqrt{2\pi} \; \varphi(\beta\,x)$$

For physical reasons it is obvious that the function $F_2(x)$ is even

$$F_2(x) = \frac{1}{8\beta^3} \sqrt{2\pi} \; \varphi(|\beta x|) \tag{10}$$

The displacement field is thus given by the convolution integral:

$$u(x) = \frac{1}{2} \alpha\beta a \int_{-\infty}^{+\infty} T(\xi) \; \varphi(\beta\,|x-\xi|) \, d\xi \tag{11}$$

By partial integration (11) can be separated into a pure thermal dilatation displacement and into an elastic displacement.

$$u(x) = \alpha a T + \frac{\alpha a}{4\beta} \int_{-\infty}^{+\infty} \frac{d^2 T(\xi)}{d\xi^2} \psi(\beta|x-\xi|) \, d\xi$$

$$\psi(\beta x) = e^{-\beta x}(\cos\beta\,x - \sin\beta\,x) \tag{12}$$

## 2. Relation between displacement and fluxes

Combining the differential equations governing the axial heat transport and the radial displacement, and applying the Fourier transform yields:

$$U(s) = \frac{4\beta^4 \; \alpha \; am^2}{h} \; \frac{Q(s)}{(s^4+4\beta^4)(s^2+m^2)} \qquad (13)$$

The function $F_2(x)$ (see appendix 1) is now defined by:

$$F_2(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{e^{-jsx}}{(s^4+4\beta^4)(s^2+m^2)} \; ds \qquad (14)$$

This integral is calculated by contour integration :

$$\oint \frac{e^{-jzx}}{(z^4+4\beta^4)(z^2+m^2)} \; dz = 2\pi j \; \Sigma \; \text{residue's} \qquad (15)$$

For positive x the contour is taken as in Fig. 3 and for $R \to \infty$ the integrand becomes zero on the circle:

$$\oint \frac{e^{-jzx}}{(z^4+4\beta^4)(z^2+m^2)} \; dz = \int_{+\infty}^{-\infty} \frac{e^{-jsx}}{(s^4+4\beta^4)(s^2+m^2)} \; ds \qquad (16)$$

The poles are at $z = -jm$, $z = \beta(1-j)$ and $z = -\beta(1+j)$. The function $F_2(x)$ becomes (still for positive x):

$$F_2(x) = \sqrt{2\pi} \left[ \frac{e^{-mx}}{2m(m^4+4\beta^4)} + \frac{e^{j\beta(1+j)x}}{8\beta^3(1+j)(m^2+2j\beta^2)} + \right.$$

$$\left. + \frac{e^{-j\beta(1-j)x}}{8\beta^3(1-j)(m^2-2j\beta)} \right] \qquad (17)$$

$$= \frac{\sqrt{2\pi}}{m^4+4\beta^4} \left[ \frac{e^{-mx}}{2m} + \frac{m^2}{8\beta^3} \; \varphi(\beta x) - \frac{1}{4\beta} \; \psi(\beta x) \right]$$

Finally one obtains the following convolution integral for the displacement field $u(x)$:

$$u(x) = \frac{\alpha}{4} \blacksquare \frac{1}{1+4(\frac{\beta}{m})^4} \left\{ 2( \blacksquare \int_{-\infty}^{+\infty} q(\xi) \ \varphi \ (\beta \ |x-\xi| \ ) \ d\xi \right.$$

$$- 4( \blacksquare \int_{-\infty}^{+\infty} q(\xi) \ \phi \ (\acute{\beta} \ |x-\xi| \ ) \ d\xi \qquad (18)$$

$$\left. + \ \complement(\frac{\beta}{m})^5 \int_{-\infty}^{+\infty} \dot{q}(\xi) \ e^{-m| \ x-\xi|} \ d\xi \right\}$$

In formula 1 of chapter 4 the thermal dilatation displacement
has been separated:

$$u_{th} = \alpha \ a \ t = \blacksquare \int_{-\infty}^{+\infty} q(\xi) \ e^{-m|x-\xi|} \ d\xi \qquad (19)$$

Fig. 1



$$2hT\left[b_1+\sqrt{b_3^2+(e_2-e_1)^2}+b_3\right]$$

$$S=2b_1e_1+2b_3e_2+b_2(e_2-e_1)$$

Fig. 2



z - plane

$$z=s+jt$$
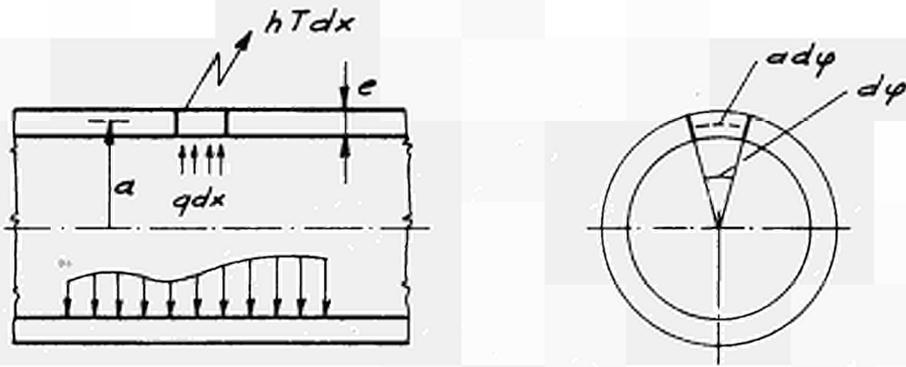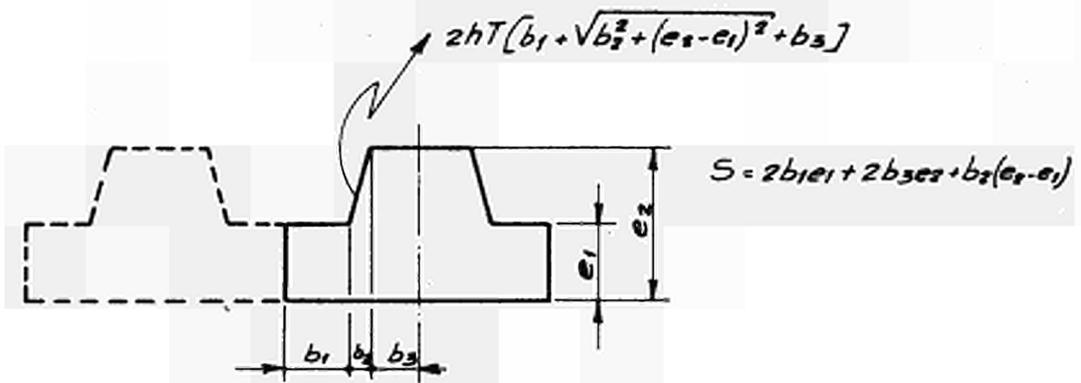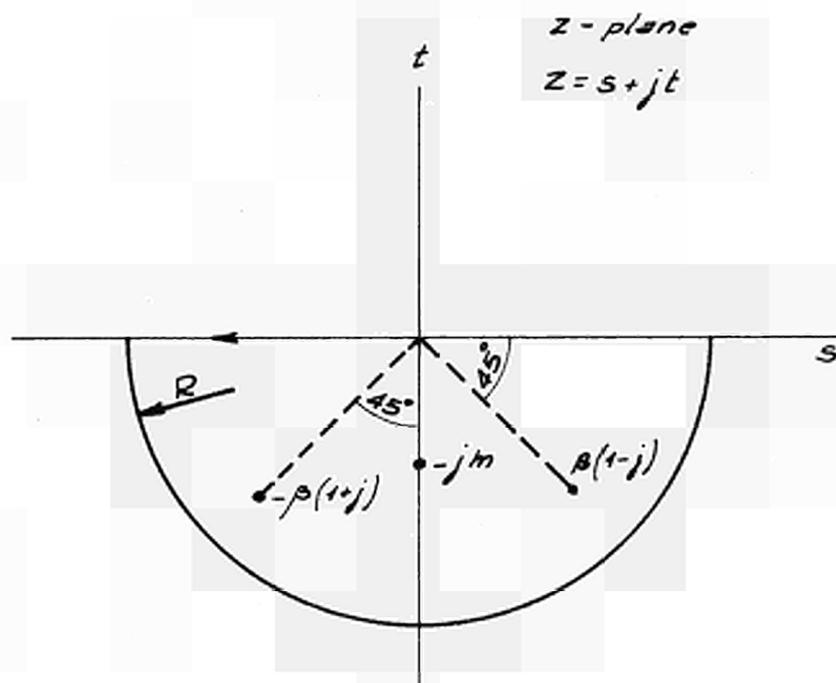
Fig. 3
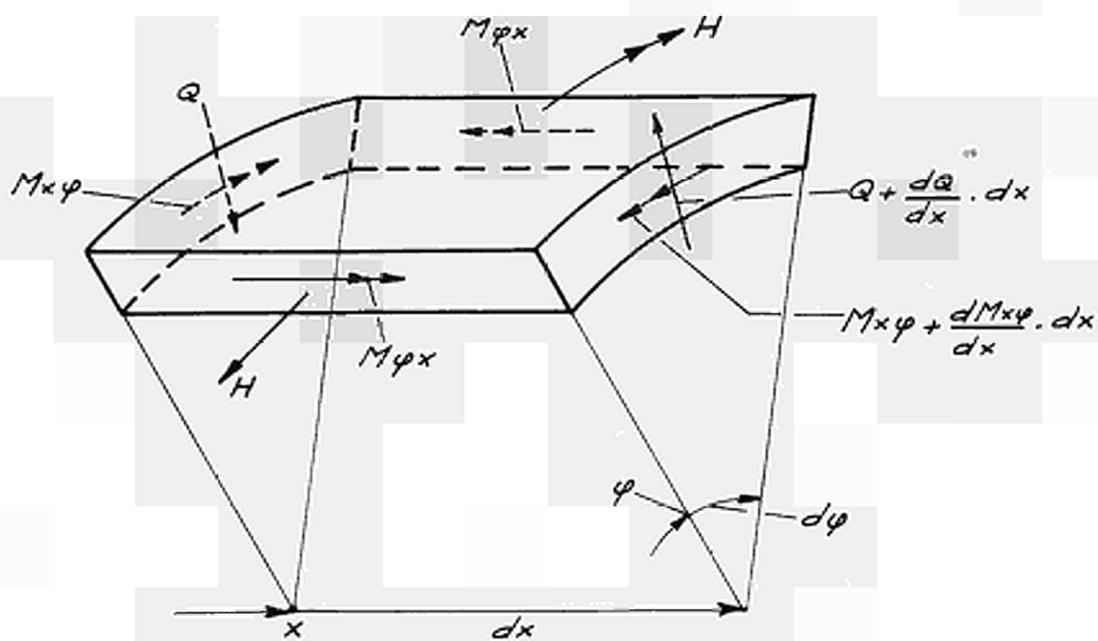
Fig. 4
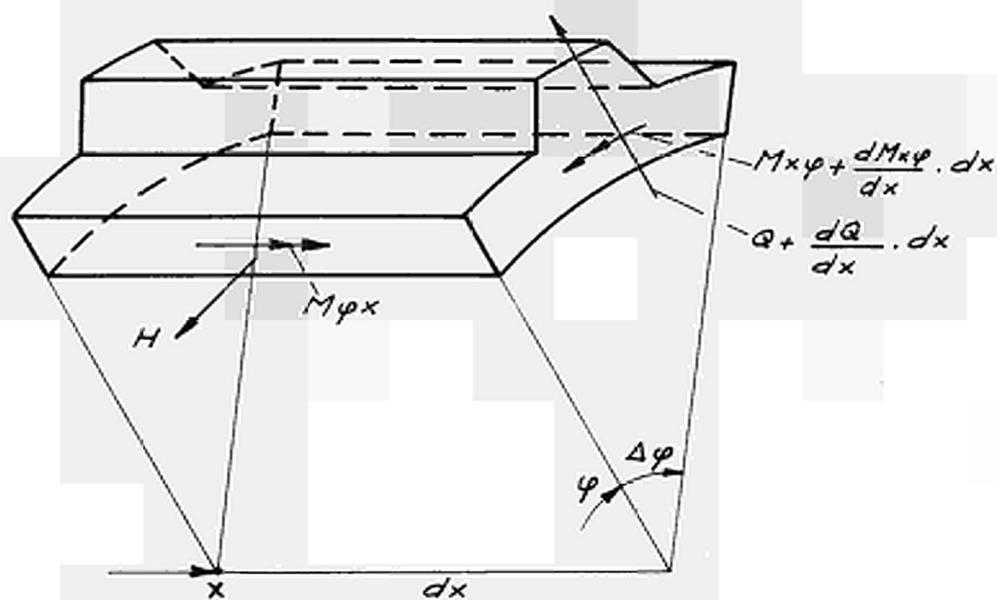


Fig. 5
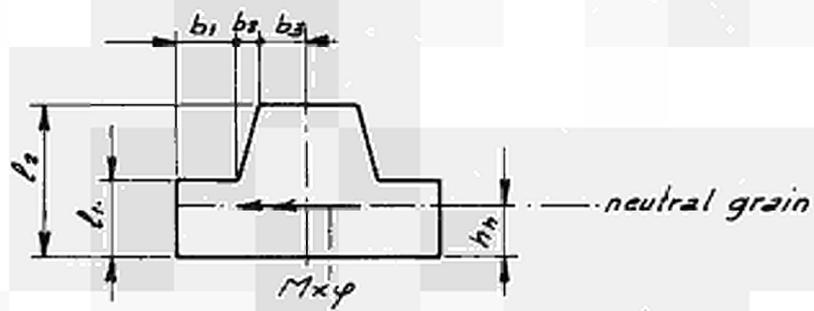
Fig. 6



Fig. 7



Fig. 8
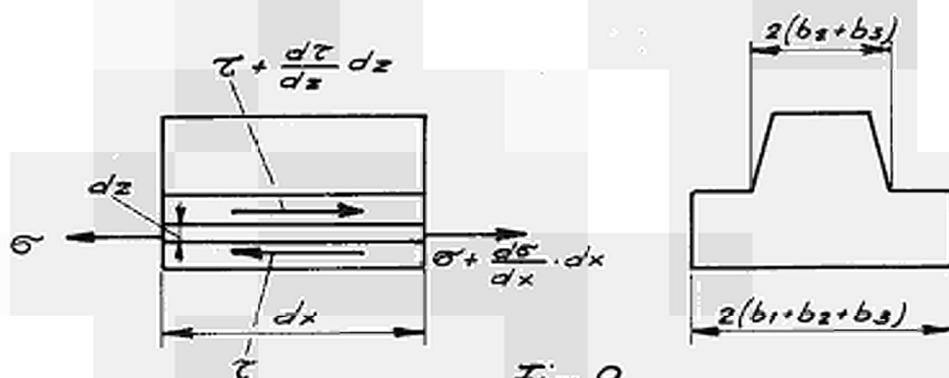


Fig. 9
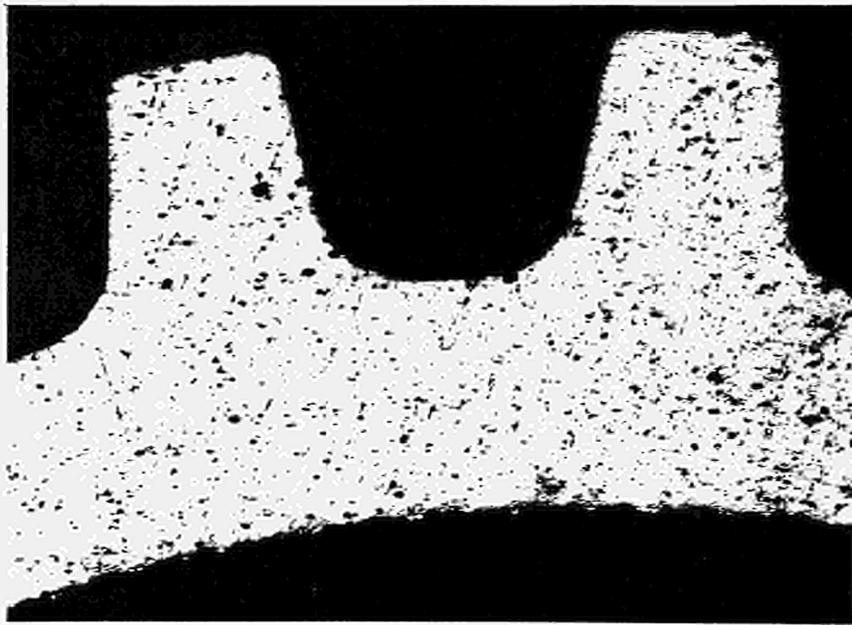
Fig. 10
enlarged photograph of a G 19
finned canning wall

Fig 11



q watt/cm²

peak

f = 50%
e = 40%
d = 30%
c = 20%
b = 10%
a = 0%

135

*Flux distribution with various peaks*

G 19

x cm

$m = 2.611\ cm^{-1}$
$h = 1.\ \dfrac{watt}{cm^2\ °C.}$

*Temperature for various flux peaks*

T °C

$m = 3,692\ cm^{-1}$
$h = 2.\ \dfrac{watt}{cm^2\ °C}$

*Temperature for various flux peaks*

T °C

Fig. 12

G1g

$m = 2,611 \text{ cm.}^{-1}$

$h = 1. \dfrac{\text{watt}}{\text{cm}^2 \ ^\circ C}$

Temperature for various flux peaks

Top of fin

Inner wall

$\beta = 3,34 \text{ cm}^{-1}$

$E = 400.000 \text{ kg/cm}^2$

Inner wall

Top of fin

Bending stress for various flux peaks

Temperature for various flux peaks

$m = 3,692 \ cm^{-1}$
$h = 2 \ \dfrac{watt}{cm^2 \, ℃}$

Bending stress for various flux peaks

$\beta = 3,34 \ cm^{-1}$
$E = 400.000 \ kg/cm^2$

Fig. 13

# Fig. 14

Equivalent stress in corner of fin according
to Huber deformation energy criterion

$$\sigma_e = \sqrt{\frac{1}{2}\left\{ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right\}}$$

$q = 135 \ watt/cm^2$

Peak 50 %

$h = 1 \ watt/cm^2 \ ^\circ C$

$m = 2,611 \ cm^{-1}$

$\beta = 3.34 \ cm^{-1}$

$E = 400.000$
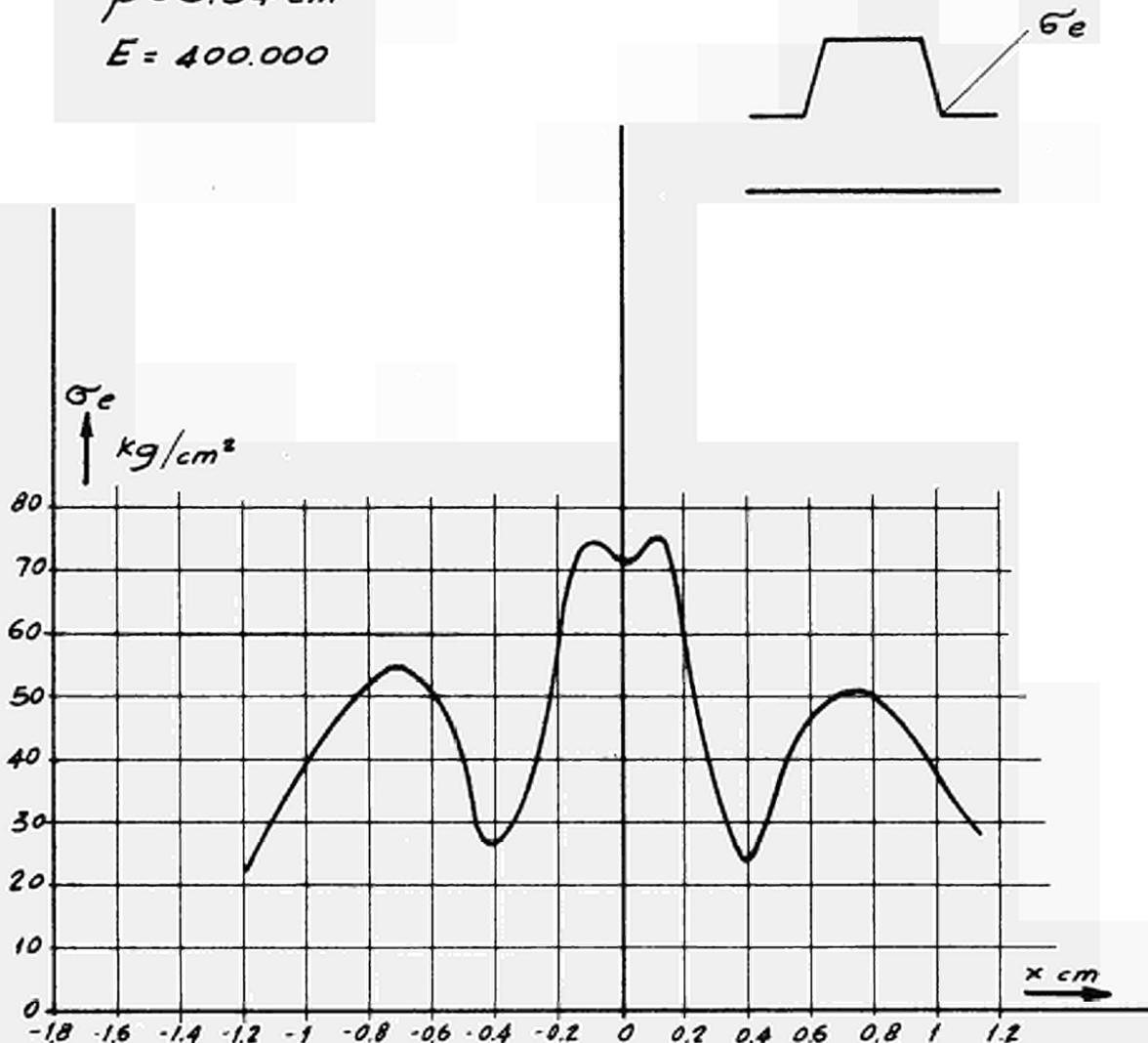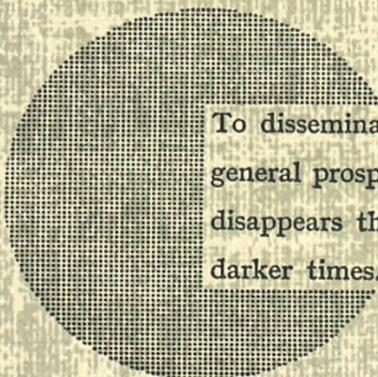
To disseminate knowledge is to disseminate prosperity — I mean general prosperity and not individual riches — and with prosperity disappears the greater part of the evil which is our heritage from darker times.

Alfred Nobel

# SALES OFFICES