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DETERMINING THE REGION OF ASYMPTOTIC STABILITY OF A COUPLED REACTOR WITH LINEAR POWER FEEDBACK

by

D. SCHWALM

1967

Joint Nuclear Research Center
Ispra Establishment - Italy

Reactor Physics Department
Research Reactors

Paper presented at the Conference on Coupled Reactor Kinetics
College Station, Texas, USA, January 23-24, 1967

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DETERMINING THE REGION OF ASYMPTOTOIC STABILITY OF A COUPLED REACTOR WITH LINEAR POWER FEEDBACK(*)

1. Introduction

As pointed out in [7], it seems convenient to study the asymptotic stability of a coupled reactor not in the large but in the small, since the dynamical equations are not valid in the whole of state space. According to [6] the kinetic equations of a two core system can be written, in commonly used symbols, as follows:

$$\dot{n}_1 = \frac{(1-\beta_1)k_1-1}{l_1} n_1 + \frac{\varepsilon_{21}}{l_2} n_2(t-\tau_{21}) + \sum_{(i)} \lambda_{i1} c_{i1} \tag{1.1}$$

$$\dot{n}_2 = \frac{(1-\beta_2)k_2-1}{l_2} n_2 + \frac{\varepsilon_{12}}{l_1} n_1(t-\tau_{12}) + \sum_{(i)} \lambda_{i2} c_{i2} \tag{1.2}$$

$$\dot{c}_{i1} = - \lambda_{i1} c_{i1} + \beta_{i1} \frac{k_1}{l_1} n_1 \tag{1.3}$$

$$\dot{c}_{i2} = - \lambda_{i2} c_{i2} + \beta_{i2} \frac{k_2}{l_2} n_2 . \tag{1.4}$$

The indices $_1$ and $_2$ refer to the first and the second core respectively, ε_{21} and ε_{12} are the coupling coefficients and τ_{12} and τ_{21} are time delay constants which represent the mean time required by a perturbation to travel from one core to the other. The influence of these time lags on the stability behaviour is not considered here since it has been studied intensively in [3].

The feedback reactivity is now written as a linear function of the power deviation:

$$\delta k_i^f = - \gamma_i (n_i - n_{io}) . \tag{1.5}$$

(*) Manuscript received on February 17, 1967.

Under this assumption the problem reduces, in the absence of delayed neutron precursors, to two dimensions. Therefore it is easy to plot the region of stability in state space. Even in the case of three dimensions the region of stability can be described plastically. Thus, in the last chapter the influence on the stability behaviour of one group of delayed neutrons in one core is investigated.

As mentioned in [7] it is necessary to distinguish between the parameter space (γ_1, γ_2) and the state space (n_1, n_2, c). The region of asymptotic stability in the small in the parameter space is given by the condition that the roots of the characteristic equation of the linearized system must have negative real parts. This condition is fulfilled if the parameters of the system satisfy the ROUTH-HURWITZ criterion.

As is well known, no estimation of the permissible extent of the deviation from the power equilibrium point of the state variables is possible in the linear theory. This can be done only if the nonlinear terms in the dynamical equations are also considered by means of LIAPUNOV's second method.

The domain of asymptotic stability (i.e. the bounds of the disturbances of the initial values) can be estimated on the basis of the fundamental theorem of LA SALLE [2] [4]:

THEOREM:
"Let V(x) be a scalar function with continuous first partial derivatives. Let Ω_l designate the region where $V(x) < 1$. Assume that Ω_l is bounded and that within Ω_l:

$$V(x) > 0 \text{ for } x \neq 0$$
$$\dot{V}(x) < 0 \text{ for } x \neq 0.$$

Then the origin is asymptotically stable, and above all, every solution in Ω_l tends to the origin as $t \to \infty$."

There exist two very similar ways to determine the region of stability in state space, proposed by SCHULTZ [5] and GEISS [4]. The procedure proposed by SCHULTZ is used here:

> A positive definite Liapunov function V with a negative definite time derivate $\dot{V}$ in the whole state space is chosen for the linearized system. This Liapunov function is then applied to the nonlinear system and a finite region about the origin in which $\dot{V}$ is negative is found by selecting the largest V = const. surface that fits into this region.

Since the theorem gives only sufficient conditions, one cannot expect to find in this way the complete region of stability. Thus it will be necessary to use several techniques for constructing Liapunov functions. A superposition of all possible surfaces can give a sufficiently good estimation of the region of stability.

There is a certain arbitrariness in choosing Liapunov functions for linear systems. Thus it seems convenient to use some constructing techniques.

2. Techniques for constructing Liapunov functions for linear systems

After the linearization the system (1.1-4) has the form

$$\dot{\vec{x}} = B\,\vec{x}, \qquad \vec{x}(0) = 0 \tag{2.1}$$

where the components of the vector $\vec{x}$ are the deviations of the state variables from the power equilibrium point and the elements of the matrix B are constant. Three different techniques for constructing Liapunov functions for a system of type (2.1) will now be discussed, which give necessary and sufficient conditions (in parameter space) for asymptotic stability. For a linear time invariant system this stability holds for all points of state space (complete stability).

2a. REISS - GEISS method [4]:

Starting from a linear time invariant system (where the variables x_i ($i = 1, \ldots, n$) are phase variables) which corresponds to a n-th order differential equation with constant coefficients, REISS and GEISS have shown that Liapunov's direct method gives necessary and sufficient conditions for the stability which are identical with the ROUTH-HURWITZ criterion.

If $\vec{x}' = (x_n, x_{n-1}, \ldots, x_1)$ and

$$B = \begin{pmatrix} -\frac{a_1}{a_0} & -\frac{a_2}{a_0} & \cdots & -\frac{a_{n-1}}{a_0} & -\frac{a_n}{a_0} \\ 1 & 0 & \cdots & 0 & 0 \\ & & & \vdots & \vdots \\ 0 & 1 & & \vdots & \vdots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & & 1 & 0 \end{pmatrix} \tag{2.2}$$

(2.1) is equivalent to

$$a_0 \frac{d^n x_1}{dt^n} + a_1 \frac{d^{n-1} x_1}{dt^{n-1}} + \dots + a_{n-1} \frac{dx_1}{dt} + a_n x_1 = 0 \qquad (2.3)$$

The chosen Liapunov function is the quadratic form:

$$V = \frac{1}{2} \vec{x}' P \vec{x} \qquad (2.4)$$

with

$$P = \begin{pmatrix} a_0 a_1 & 0 & a_0 a_3 & 0 \dots \\ 0 & a_1 a_2 - a_0 a_3 & 0 & \cdot \cdots \\ a_0 a_3 & 0 & a_0 a_5 + a_2 a_3 - a_1 a_4 & \cdot \cdots \\ 0 & a_1 a_4 - a_0 a_5 & 0 & \cdot \cdots \\ \vdots & \vdots & \vdots & \end{pmatrix} . \qquad (2.5)$$

It follows

$$\dot{V} = -(a_1 x_n + a_3 x_{n-2} + a_5 x_{n-4} + \dots)^2 \qquad (2.6)$$

Since $\dot{V}$ is a negative semidefinite function, (2.1-2) is asymptotically stable if P is positive definite and if $\dot{V}$ is not identically zero along a trajectory of (2.1-2). The last condition has been proved by REISS and GEISS, while RALSTON [4] showed that the requirement of positive definiteness of P is equivalent to the HURWITZ criterion:

The principal minors of

$$\begin{vmatrix} a_1 & a_0 & 0 & 0 \cdots 0 \\ a_3 & a_2 & a_1 & a_0 \cdots 0 \\ a_5 & a_4 & a_3 & a_2 \cdots 0 \\ \vdots & \vdots & \vdots & \vdots \ddots \vdots \\ 0 & 0 & 0 & 0 \quad a_n \end{vmatrix} \qquad (2.7)$$

must be positive.

2b. The SCHULTZ method [5]:

Another possibility is to compute the system eigenvalues and then to check that they have negative real parts. Since in general B has no diagonal form, system (2.1) can be changed by a linear transformation in such a way that the new coefficient matrix has a diagonal form. This approach has also been suggested by GEISS [4].

A linear transformation of coordinates

$$\vec{x} = P\vec{z} \qquad (2.8)$$

is made so that in the transformed equation

$$\dot{\vec{z}} = P^{-1}BP\vec{z} \qquad (2.9)$$

$P^{-1}BP$ is a diagonal matrix D,

$$D = \text{diag}\ (\lambda_1, \cdots \lambda_n), \qquad (2.10)$$

where $\lambda_1, \ldots, \lambda_n$ are the eigenvalues of the matrix B, i.e. roots of

$$\left| B - \lambda E \right| = 0. \qquad (2.11)$$

The Liapunov function is chosen as

$$V(\vec{z}) = \frac{1}{2} \vec{z}' \vec{z} \, . \tag{2.12}$$

It follows that

$$\frac{dV(\vec{z})}{dt} = \frac{1}{2} \vec{z}'(D' + D)\vec{z}$$

and with $D' = D$

$$\frac{dV(\vec{z})}{dt} = \sum_{i=1}^{n} \lambda_i z_i^2 . \tag{2.13}$$

The system is asymptotically stable if the eigenvalues are negative real, i.e. if the HURWITZ criterion is fulfilled.

In order to obtain the stability region in state space $\vec{x}$ it is necessary to write V in that coordinate system:

$$V(\vec{x}) = \frac{1}{2} \vec{x}' (P^{-1})' P^{-1} \vec{x}. \tag{2.14}$$

2c. <u>The GEISS method [4]:</u>

This method is straightforward. As B can now be a general matrix no transformation of coordinates is needed. This technique can be shortly described by a fundamental theorem proved by GEISS with the choice $V = \vec{x}' P\vec{x}$ and $\dot{V} = - \vec{x}' Q\vec{x}$.

THEOREM: "A necessary and sufficient condition for the complete stability of the trivial solution, $\vec{x}(t) = 0$, of the linear time invariant system

$$\dot{\vec{x}} = B\vec{x}$$

is: there exists a positive definite matrix, P, which is the solution of

$$B' P + PB = - Q \tag{2.15}$$

where Q is a n y positive definite matrix."

The matrix P is assumed to be symmetric. It is further possible to choose Q as a symmetric matrix. Hence the n^2 equations of (2.15) for the elements p_{ik} of P can be reduced to $\frac{n(n+1)}{2}$ equations.

Thus the stability is investigated by choosing a positive definite matrix Q (Q = I is opportune), computing the p_{ik} from $\frac{n(n+1)}{2}$ linear algebraic equations and checking P for positive definiteness by using the SYLVESTER criterion.

This method is very suited to the problem as will be seen in the following applications.

3. The two-dimensional case (no delayed neutrons)

These three methods will now be applied to the linearized equations of system (1.1-4) for the case when delayed neutrons are neglected. Thus the problem is reduced to two dimensions (x_1 and x_2) with the matrix B given by:

$$B = \begin{pmatrix} -\frac{1}{l_1}(\gamma_1 n_{10} + \Delta_1) & \frac{\varepsilon_{21}}{l_2} \\ \frac{\varepsilon_{12}}{l_1} & -\frac{1}{l_2}(\gamma_2 n_{20} + \Delta_2) \end{pmatrix} = \begin{pmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \end{pmatrix} \qquad (3.1)$$

if the power equilibrium state is denoted by (n_{10}, n_{20}) and the subcriticalities ($1 - k_{i0}$) by Δ_i.

At first the region in parameter space is derived from the characteristic equation

$$s^2 + s\left[\frac{\gamma_1}{l_1} n_{10} + \frac{\Delta_1}{l_1} + \frac{\gamma_2}{l_2} n_{20} + \frac{\Delta_2}{l_2}\right] + \frac{\gamma_1 \gamma_2}{l_1 l_2} n_{10} n_{20} + \frac{\gamma_1}{l_1} n_{10} \frac{\Delta_2}{l_2} + \frac{\gamma_2}{l_2} n_{20} \frac{\Delta_1}{l_1} = 0 . \qquad (3.2)$$

With

$$\varepsilon_i = \frac{\gamma_i}{l_i} n_{i0} \qquad (3.3)$$

the HURWITZ conditions are

$$\varepsilon_1 + \varepsilon_2 + \frac{\Delta_1}{l_1} + \frac{\Delta_2}{l_2} > 0 \qquad (3.4)$$

$$\varepsilon_1 \varepsilon_2 + \varepsilon_2 \frac{\Delta_1}{l_1} + \varepsilon_1 \frac{\Delta_2}{l_2} > 0 \qquad (3.5)$$

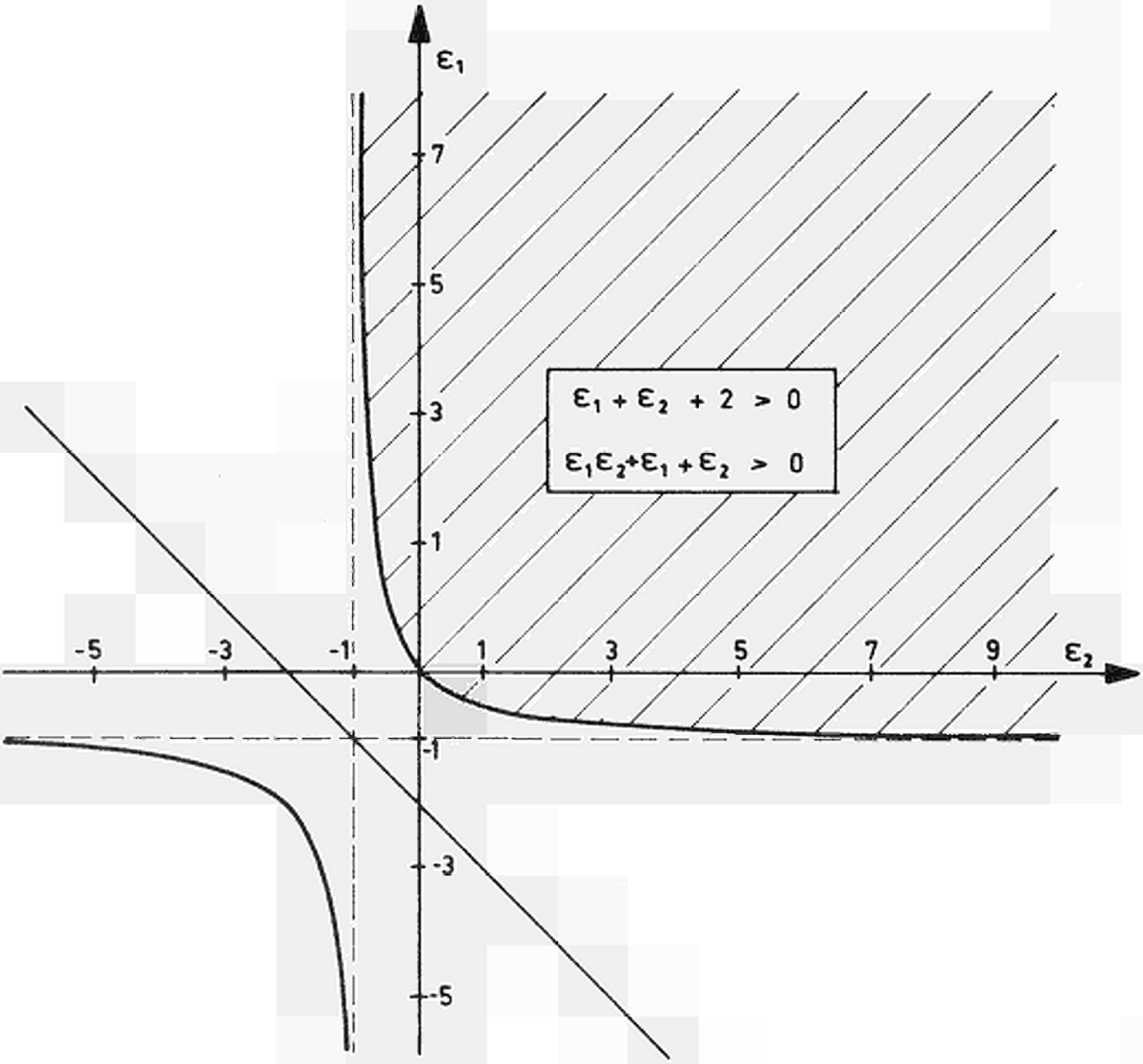


Fig. 1: Region of asymptotic stability in the parameter space $(\varepsilon_1, \varepsilon_2)$ with $\frac{\Delta_1}{l_1} = \frac{\Delta_2}{l_2} = 1$.

The region given by these two inequalities is plotted in Fig. 1 for the special values $\frac{\Delta_1}{I_1} = \frac{\Delta_2}{I_2} = 1$.

The nonlinear system has the simple form

$$\dot{x}_1 = b_{11}x_1 + b_{21}x_2 - \frac{\gamma_1}{I_1} x_1^2 \tag{3.6}$$

$$\dot{x}_2 = b_{12}x_1 + b_{22}x_2 - \frac{\gamma_2}{I_2} x_2^2 \ . \tag{3.7}$$

As mentioned in chapter 1 the Liapunov function for the linearized system must be applied to system (3.6-7). To begin with, the method 2a is used.

3a. Application of the REISS-GEISS method

In order to avoid confusion between state and phase variables the phase variables are now denoted by y_i while the symbols x_i are reserved for the state variables.

System (2.1) with (3.1) can easily be transformed into phase variable form:

$$\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= - a_1 y_2 - a_2 y_1 \end{aligned} \tag{3.8}$$

with

and

$$\begin{aligned} a_1 &= -(b_{11} + b_{22}) \\ a_2 &= b_{11}b_{22} - b_{12}b_{21} \ . \end{aligned} \tag{3.9}$$

According to (2.4-5) it follows that

$$V_1(y_1, y_2) = \frac{1}{2}(a_1 a_2 y_1^2 + a_1 y_2^2) \tag{3.10}$$

and

$$\dot{V}_1 = -a_1^2 y_2^2 . \tag{3.11}$$

Since $\dot{V}_1$ is a semidefinite function in state space it is modified to a definite function by adding a term:

$$\dot{V}_2 = -a_1^2 y_2^2 - \eta y_1^2 . \tag{3.12}$$

This function is negative definite in phase and state space if $\eta > 0$. From (3.12) V_2 can be calculated by integration by parts [4]

$$V_2 = \int \dot{V}_2 dt = -\int (a_1^2 y_2^2 + \eta y_1^2) dt$$

$$V_2 = \frac{1}{2}(a_1 a_2 y_1^2 + a_2 y_2^2) - \eta \int y_1^2 dt .$$

It follows that

$$\int y_1^2 dt = -\frac{a_1}{2a_2} y_1^2 - \frac{1}{a_2} \int y_1 \dot{y}_2 dt$$

$$\int y_1 \dot{y}_2 dt = y_1 y_2 + \frac{a_2}{2a_1} y_1^2 + \frac{1}{2a_1} y_2^2$$

and finally

$$V_2 = \frac{1}{2}\left(a_1 a_2 + \frac{a_1}{a_2}\eta + \frac{\eta}{a_1}\right) y_1^2 + \frac{\eta}{a_2} y_1 y_2 + \frac{1}{2}\left(a_1 + \frac{\eta}{a_1 a_2}\right) y_2^2 \tag{3.13}$$

which is positive definite in state and phase space if (3.10) is positive definite (i.e. if the HURWITZ conditions are fulfilled) and $\eta > 0$.

For a case with the numerical values:

$$n_{10} = n_{20} = 1, \frac{\varepsilon_{12}}{I_1} = \frac{\varepsilon_{11}}{I_2} = 1, \frac{\Delta_1}{I_1} = \frac{\Delta_2}{I_2} = 1, \frac{\gamma_1}{I_1} = \frac{\gamma_2}{I_2} = 1 \tag{3.14}$$

(3.13) becomes in state variables

$$\frac{24}{48+\eta} V_2(x_1, x_2) = 7x_1^2 + 4\left(\frac{\eta-48}{\eta+48}\right) x_1 x_2 + x_2^2 \tag{3.15}$$

Now η is chosen to be 48.

In order to obtain the stability region, $\dot{V} = 0$ of the non-linear system is plotted in Fig. 2. The largest ellipse $V = K$ that contacts the curve $\dot{V} = 0$ gives a finite region about the equilibrium point in which $\dot{V}$ is negative. The point of contact is the intersection point of the curve $\dot{V} = 0$ and the curve

$$\left.\frac{dx_1}{dx_2}\right|_{\dot{V}=0} = \left.\frac{dx_1}{dx_2}\right|_{V=K}.$$

Thus all solutions starting from any point in the interior of the ellipse $V = K$ tend asymptotically to the power equilibrium state. As will be seen later on, this region of stability is very small in comparison to other estimations based on the techniques 2b and 2c.

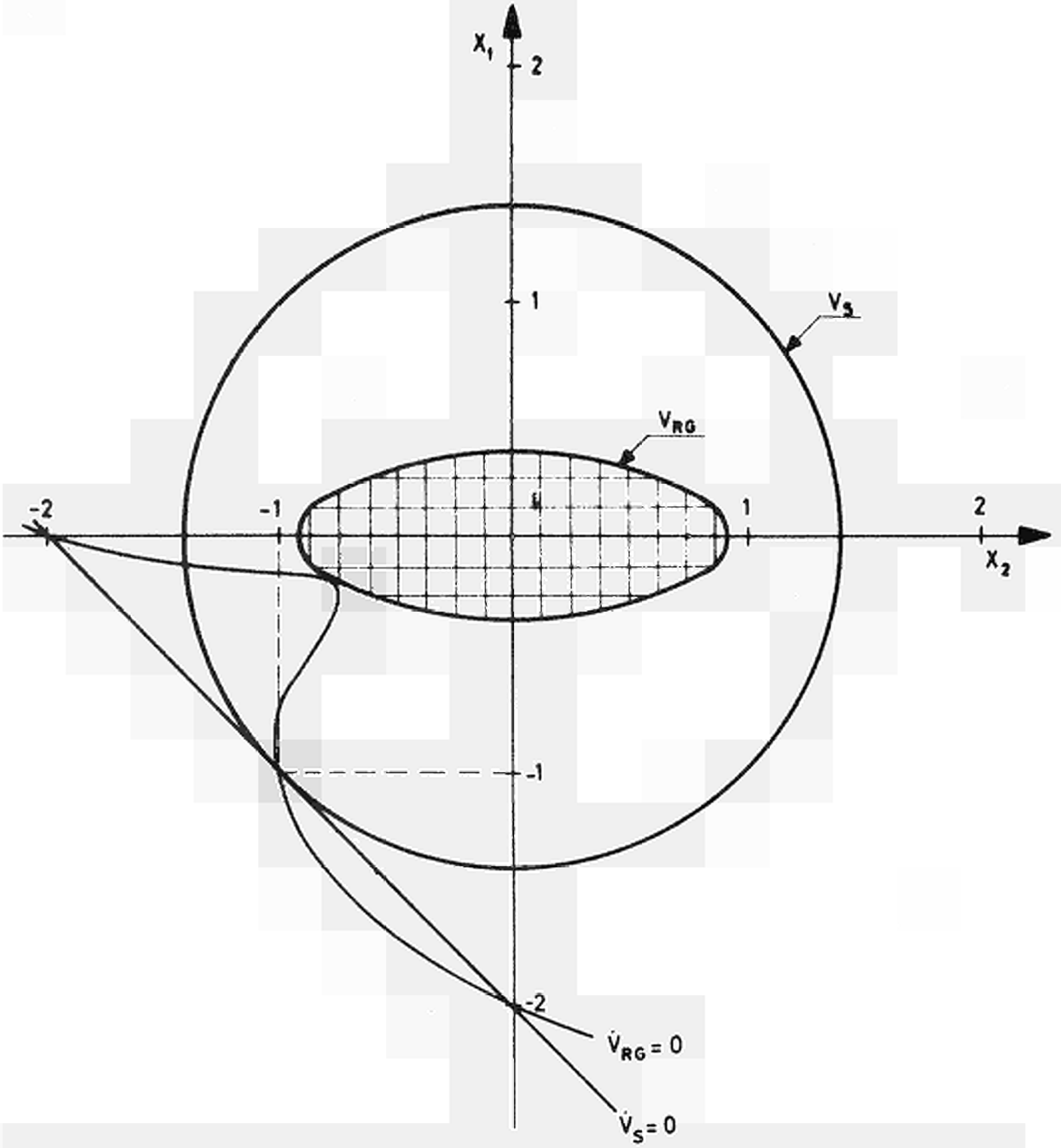


Fig. 2: Region of asymptotic stability in the state space (x_1, x_2) calculated with the REISS-GEISS (RG) and the SCHULTZ (S) method in case of equal coupling coefficients.

$$\left(n_{10} = n_{20} = 1 \; ; \; \frac{\varepsilon_{12}}{l_1} = \frac{\varepsilon_{21}}{l_2} = 1 \; ; \; \frac{\gamma_1}{l_1} = \frac{\gamma_2}{l_2} = 1 \right)$$

V_{RG}: $7x_1^2 + x_2^2 = 0.8425$; V_S: $x_1^2 + x_2^2 = 2$

3b. Application of the SCHULTZ method

As has been shown in [7] the Liapunov function (2.14) becomes with $D^2 = \left[\frac{(\lambda_1 - b_{22})(\lambda_2 - b_{11})}{b_{12}b_{21}} - 1\right]^2$

$$2D^2V(x_1,x_2) = x_1^2\left[1 + \left(\frac{\lambda_2-b_{11}}{b_{21}}\right)^2\right] - 2x_1x_2\left[\frac{\lambda_2-b_{11}}{b_{21}} + \frac{\lambda_1-b_{22}}{b_{12}}\right] + x_2^2\left[1 + \left(\frac{\lambda_1-b_{22}}{b_{12}}\right)^2\right] \tag{3.16}$$

which reduces, in the case of equal coupling factors $b_{12}=b_{21}$ $\left(\frac{\varepsilon_{12}}{l_1} = \frac{\varepsilon_{21}}{l_2}\right)$, to a circle.

The time derivative of (3.16) with respect to the nonlinear system of (3.6-7) is

$$D^2\dot{V} = \lambda_1\left[\frac{\lambda_1-b_{11}}{b_{21}}x_1-x_2\right]^2 + \lambda_2\left[\frac{\lambda_1-b_{22}}{b_{12}}x_2-x_1\right]^2 - \frac{\gamma_1}{l_1}\left[1+\left(\frac{\lambda_2-b_{11}}{b_{21}}\right)^2\right]x_1^3 - \frac{\gamma_2}{l_2}\left[1+\left(\frac{\lambda_1-b_{22}}{b_{12}}\right)^2\right]x_2^3 + \frac{\gamma_1}{l_1}\left[\frac{\lambda_2-b_{11}}{b_{21}} + \frac{\lambda_1-b_{22}}{b_{12}}\right]x_1^2x_2 + \frac{\gamma_2}{l_2}\left[\frac{\lambda_2-b_{11}}{b_{21}} + \frac{\lambda_1-b_{22}}{b_{12}}\right]x_1x_2^2. \tag{3.17}$$

For the example (3.14), $\dot{V} = 0$ is a straight line through the shut down state ($n_{10}=0$, $n_{20}=0$, $x_1=-1$, $x_2=-1$).

The stability region is given by a circle around the origin with a radius equal to the distance between the shut down state and the power equilibrium state (Fig. 2).

It has been shown generally in [7] that, in the case of equal coupling coefficients, the region of asymptotic stability in state space is bounded by a circle around the origin with the radius $(n_{10}^2 + n_{20}^2)^{0,5}$, i.e. the distance of the two singular points, if the feedback parameters are such that

$$\frac{\gamma_1}{l_1} n_{10} = \frac{\gamma_2}{l_2} n_{20}. \qquad (3.18)$$

If this relation is not fulfilled, the radius decreases. It has been verified that the parameters ε_i have only a small influence on the extent of the stability region.

As can be seen from (3.16), in the case of unequal coupling coefficients, the stability region is an ellipse. For the special example

$$n_{10} = 1,\ n_{20} = 2,\ \frac{\varepsilon_{12}}{l_1} = 1,\ \frac{\varepsilon_{21}}{l_2} = 2,\ \frac{\Delta_1}{l_1} = 4,\ \frac{\Delta_2}{l_2} = \frac{1}{2},$$

$$\frac{\gamma_1}{l_1} = 0.2,\ \frac{\gamma_2}{l_2} = 0.4 \qquad (3.19)$$

the curves $\dot{V}_s = 0$ and $V_s = K$ are plotted in Fig. 4 which is also mentioned later on. More details are discussed in [7].

3c. Application of the GEISS method

With the choice Q = I the elements p_{ik} of the matrix P must be calculated from the following system according to (2.15)

$$\begin{aligned} 2b_{11}p_{11} + 2b_{12}p_{12} &= -1 \\ b_{21}p_{11} + (b_{11}+b_{22})p_{12} + b_{12}p_{22} &= 0 \qquad (3.20) \\ 2b_{21}p_{12} + 2b_{22}p_{22} &= -1. \end{aligned}$$

By use of (3.9) it follows that

$$P = \frac{1}{2a_1a_2}\begin{vmatrix} (a_2+b_{22}^2+b_{12}^2) & -(b_{11}b_{12}+b_{22}b_{21}) \\ -(b_{11}b_{12}+b_{22}b_{21}) & (a_2+b_{11}^2+b_{21}^2) \end{vmatrix}. \qquad (3.21)$$

P is positive definite if

$$\begin{aligned} &\text{i)} \quad (a_2+b_{22}^2+b_{12}^2) > 0 \\ &\text{ii)} \quad (a_2+b_{22}^2+b_{12}^2)(a_2+b_{11}^2+b_{21}^2)-(b_{11}b_{12}+b_{22}b_{21})^2 > 0 . \end{aligned} \qquad (3.22)$$

After some recasting condition ii) has the simple form

$$a_2[(b_{11}+b_{22})^2 + (b_{12}-b_{21})^2] > 0.$$

Thus P is positive definite if

$$a_1 > 0 \quad \text{and} \quad a_2 > 0.$$

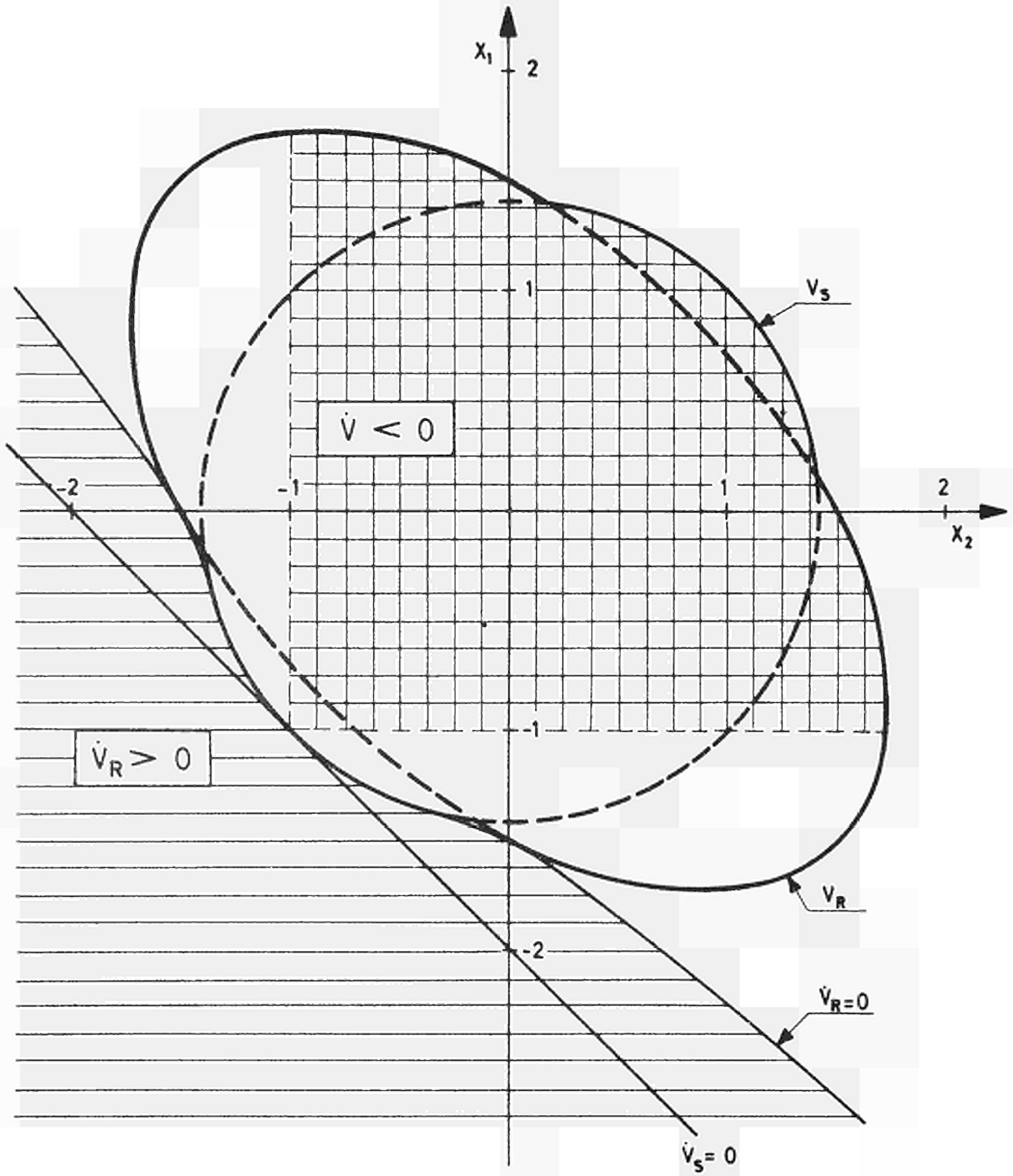
These conditions are identical with the HURWITZ criterion (2.7). By this the theorem of GEISS has been verified for $Q = I$ and $n = 2$.

The time derivative of V with respect to system (3.6-7) is

$$\frac{1}{2}\dot{V} = -x_1^2-x_2^2 - \frac{\gamma_1}{l_1}\frac{a_2+b_{22}^2+b_{12}^2}{a_1a_2}x_1^3 - \frac{\gamma_2}{l_2}\frac{a_2+b_{11}^2+b_{21}^2}{a_1a_2}x_2^3 +$$

$$(b_{11}b_{12}+b_{22}b_{21})\left(\frac{\gamma_1}{l_1}x_1^2x_2 + \frac{\gamma_2}{l_2}x_1x_2^2\right). \qquad (3.23)$$

For the example (3.14) with equal coupling coefficients, $\dot{V}_R = 0$ and $V_R = K$ are plotted in Fig. 3. The superposition of V_S and V_R gives the stability region. This region is bounded by the straight lines $x_1 = -1$, and $x_2 = -1$, since the power values can never become negative.

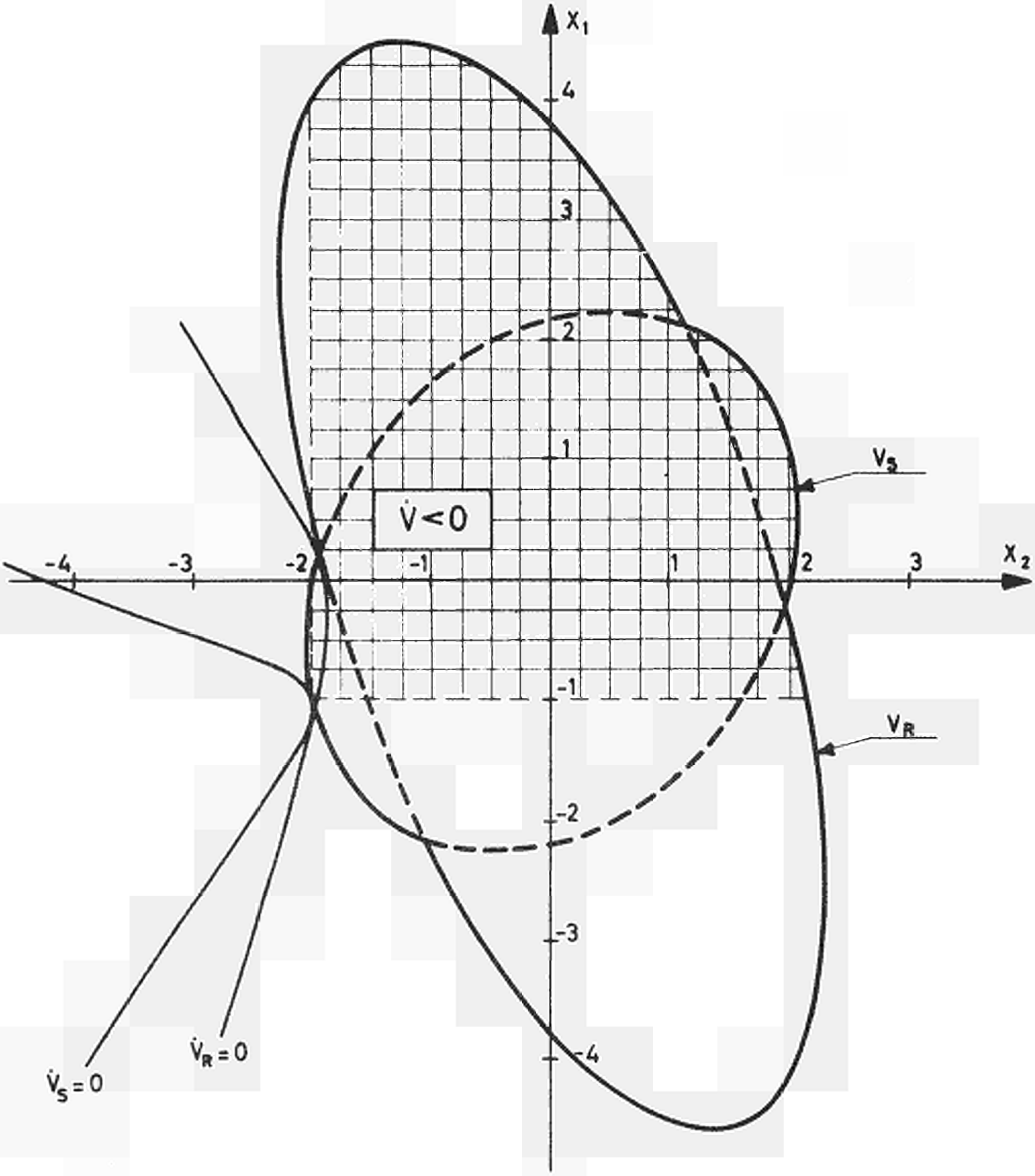


$$\left(n_{10} = n_{20} = 1 \; ; \; \frac{\varepsilon_{12}}{l_1} = \frac{\varepsilon_{21}}{l_2} = 1 \; ; \; \frac{\gamma_1}{l_1} = \frac{\gamma_2}{l_2} = 1 \right)$$

Fig. 3: Region of asymptotic stability in the state space (x_1, x_2) calculated with the SCHULTZ (S) and the GEISS (R) method in case of equal coupling factors.
V_R: $x_1^2 + x_1 x_2 + x_2^2 = 2.25$; V_S: $x_1^2 + x_2^2 = 2$

In the case of unequal coupling coefficients and with the parameter values of example (3.19) the stability region is plotted in Fig. 4. For these two special problems, it is seen from Fig. 3 and Fig. 4 that the GEISS method gives the largest surface, but that it is also necessary to use the SCHULTZ method in order to get a larger region by superposition. As mentioned before, this is not the complete region of stability. Up to now the problem of how to choose P and Q in order to obtain the best estimate is unresolved.

In contrast to the results of the last section, using the SCHULTZ method, the extent of the stability region is now much influenced by the parameters γ_1/l_1 and γ_2/l_2. This can be illustrated by assuming the values: $\gamma_1/l_1 = 0$, $\gamma_2/l_2 = 1$ or $\gamma_1/l_1 = 1$, $\gamma_2/l_2 = 0$ in the numerical example (3.14). These assumptions give the surfaces plotted in Fig. 5. In comparison to Fig. 3 the stability region is now much smaller.

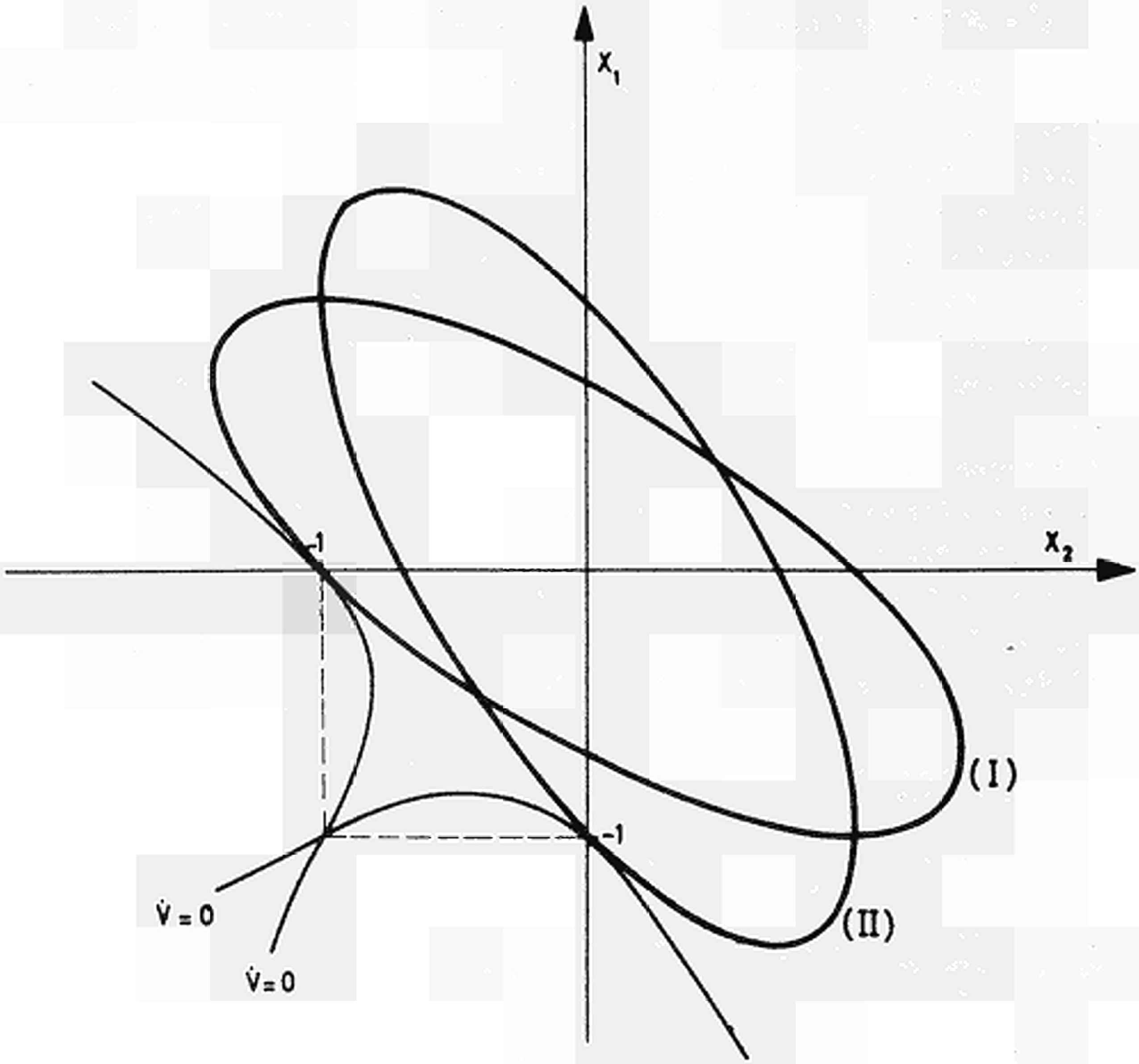


$$\left(n_{10} = 1,\ n_{20} = 2\ ;\ \frac{\varepsilon_{12}}{l_1} = 1,\ \frac{\varepsilon_{21}}{l_2} = 2\ ;\ \frac{\Delta_1}{l_1} = 4,\ \frac{\Delta_2}{l_2} = \frac{1}{2}\ ;\ \frac{\gamma_1}{l_1} = 0.2,\ \frac{\gamma_2}{l_2} = 0.4\right)$$

Fig. 4: Stability region in (x_1, x_2) in case of unequal coupling factors, calculated with the SCHULTZ (S) and GEISS (R) method.

V_R: $6.15x_1^2 + 13.6x_1x_2 + 25.1x_2^2 = 90.13$

V_S: $x_1^2 - 0.53x_1x_2 + 1.23x_2^2 = 4.84$



$$(I): \frac{\gamma_1}{l_1} = 0 \; ; \; \frac{\gamma_2}{l_2} = 1$$

$$(II): \frac{\gamma_1}{l_1} = 1 \; ; \; \frac{\gamma_2}{l_2} = 0$$

$$\left(n_{10} = n_{20} = 1 \; ; \; \frac{\varepsilon_{12}}{l_1} = \frac{\varepsilon_{21}}{l_2} = 1 \right)$$

Fig. 5: Stability region in (x_1, x_2) in case of equal coupling factors calculated with the GEISS method.

$$V_I: x_1^2 + x_1 x_2 + 0.5 x_2^2 = 0.5$$

$$V_{II}: 0.5 x_1^2 + x_1 x_2 + x_2^2 = 0.5$$

4. The three-dimensional case (delayed neutrons in one core)

In chapter 3 the effect of delayed neutrons has been neglected. As is known from the point dynamics model the delayed neutrons can improve or worsten the stability of a power reactor if the feedback transfer function is a lagging or leading function of real frequencies [1]. The influence of delayed neutrons will now be investigated in the two point model. According to eqs. (1.1-4) the problem is a four-dimensional one. However, in order to get a concrete picture of the stability region the problem is reduced to the three dimensions. This is achieved by considering delayed neutrons in one of the two cores. In the work to follow only the analysis of GEISS (see 2c and 3c) is applied. To obtain results which are comparable with those of chapter 3, Q = I is choden again. Two different problems are studied. To begin with, the simplest one is discussed.

4α) $\gamma_1 = 0, \gamma_2 \neq 0, \lambda_2 = \beta_2 = 0, \lambda_1 \neq 0, \beta_1 \neq 0$:

System (1.1-4) reduces to

$$\dot{x}_1 = -\left(\frac{\Delta_1}{l_1} + \beta \frac{k_{10}}{l_1}\right) x_1 + \frac{\varepsilon_{21}}{l_2} x_2 + \lambda x_3 \qquad (4.1)$$

$$\dot{x}_2 = \frac{\varepsilon_{12}}{l_1} x_1 - \left(\frac{\Delta_2}{l_2} + \frac{\gamma_2}{l_2} n_{20}\right) x_2 - \frac{\gamma_2}{l_2} x_2^2 \qquad (4.2)$$

$$\dot{x}_3 = \beta \frac{k_{10}}{l_1} x_1 - \lambda x_3 . \qquad (4.3)$$

The conditions for asymptotic stability in the parameter space (2.3) can easily be derived from (4.1-3):

i) $\frac{\Delta_1}{l_1} + \frac{\Delta_2}{l_2} + \beta \frac{k_{10}}{l_1} + \varepsilon_2 + \lambda > 0$

ii) $\varepsilon_2 \lambda \frac{\Delta_1}{l_1} > 0$

$$\text{iii)}\ \left(\frac{\Delta_1}{l_1} + \frac{\Delta_2}{l_2} + \varepsilon_2\right)\left[\frac{\Delta_2}{l_2}\,\beta\,\frac{k_{10}}{l_1} + \varepsilon_2\left(\frac{\Delta_1}{l_1} + \beta\,\frac{k_{10}}{l_1}\right) + \lambda\left(\frac{\Delta_1}{l_1} + \frac{\Delta_2}{l_2} + \beta\,\frac{k_{10}}{l_1} + \varepsilon_2 + \lambda\right)\right] + \beta\,\frac{k_{10}}{l_1}\left[\frac{\Delta_2}{l_2}\,\beta\,\frac{k_{10}}{l_1} + \varepsilon_2\left(\frac{\Delta_1}{l_1} + \beta\,\frac{k_{10}}{l_1}\right) + \lambda\left(\frac{\Delta_2}{l_2} + \varepsilon_2\right)\right] > 0.$$

Conditions i) and iii) are fulfilled if ii) is satisfied. A comparison with (3.4-5) shows that the delayed neutrons do not change the region of stability in parameter space.

The region in state space is estimated for the special problem:

$$n_{10} = n_{20} = c_{10} = 1,\ \frac{\Delta_1}{l_1} = \frac{\Delta_2}{l_2} = 1,\ \frac{\varepsilon_{12}}{l_1} = \frac{\varepsilon_{21}}{l_2} = 1,\ \varepsilon_2 = 1,\ \lambda = \beta\,\frac{k_{10}}{l_1} = 0.1\ . \tag{4.4}$$

According to 2c) the Liapunov function has the form

$$V = x_1^2 + \frac{1}{2}x_2^2 + 6x_3^2 + x_1x_2 + 2x_1x_3 + x_2x_3 = K. \tag{4.5}$$

The time derivative $\dot V$ with respect to system (4.1-3) with $\gamma_2/l_2 = 1$, is

$$\dot V = -x_1^2 - x_2^2 - x_3^2 - x_1x_2^2 - x_2^2x_3 - x_2^3\ . \tag{4.6}$$

The point of contact between $\dot V = 0$ and $V = K$ is now found by the following procedure:

Introduce $F_1 = \dot V = 0$ and $F_2 = V - K = 0$, so that the tangential plane in the point O (x_{10}, x_{20}, x_{30}) is given by

$$\left.\frac{\partial F_i}{\partial x_1}\right|_0 (x_1-x_{10}) + \left.\frac{\partial F_i}{\partial x_2}\right|_0 (x_2-x_{20}) + \left.\frac{\partial F_i}{\partial x_3}\right|_0 (x_3-x_{30}) = 0$$

for $i = 1,2$ (4.7)

or

$$u_i(x_{10}, x_{20}, x_{30})x_1 + v_i(x_{10}, x_{20}, x_{30})x_2 +$$
$$w_i(x_{10}, x_{20}, x_{30})x_3 + m_i(x_{10}, x_{20}, x_{30}) = 0. \quad (4.8)$$

These two planes are identical if

$$u_1 = u_2\alpha \quad (4.9)$$
$$v_1 = v_2\alpha \quad (4.10)$$
$$w_1 = w_2\alpha \quad (4.11)$$
$$m_1 = m_2\alpha \quad (4.12)$$

with α a real constant.

This is a system of 4 nonlinear algebraic equations with the unknowns x_{10}, x_{20}, x_{30}, α . Since there exists no general solution, it is solved for the special functions F_1 and F_2 (4.5+6) by eliminating the unknown x_{10} and graphical representation of the remaining three functions for some α - values in the x_{20}, x_{30} plane. System (4.9-12) has the special form

$$- 2x_{10} - x_{20}^2 = (2x_{10} + x_{20} + 2x_{30})\alpha \quad (4.13)$$

$$- 2x_{20} - 2x_{10}x_{20} - 2x_{20}x_{30} - 3x_{20}^2 = (x_{10} + x_{20} + x_{30})\alpha \quad (4.14)$$

$$- 2x_{30} - x_{20}^2 = (12x_{30} + 2x_{10} + x_{20})\alpha \quad (4.15)$$

$$x_{10}x_{20}^2 + x_{20}^3 + x_{20}^2 x_{30} = -2(x_{10}^2 + x_{10}x_{20} + \frac{1}{2} x_{20}^2 + 6x_{30}^2 + x_{20}x_{30})\alpha. \qquad (4.16)$$

The final system is obtained by eliminating x_{10}:

$$\text{(I)} \qquad x_{30} = -\frac{x_{20}^2 + \alpha x_{20}}{(12\alpha + 2 + 10\alpha^2 + 2\alpha)}$$

$$\text{(II)} \qquad x_{30} = \frac{(4\alpha + \alpha^2)x_{20} + 3\alpha x_{20}^2 - 2x_{20}^3}{(2\alpha + 10\alpha^2) + (4 + 20\alpha)x_{20}}$$

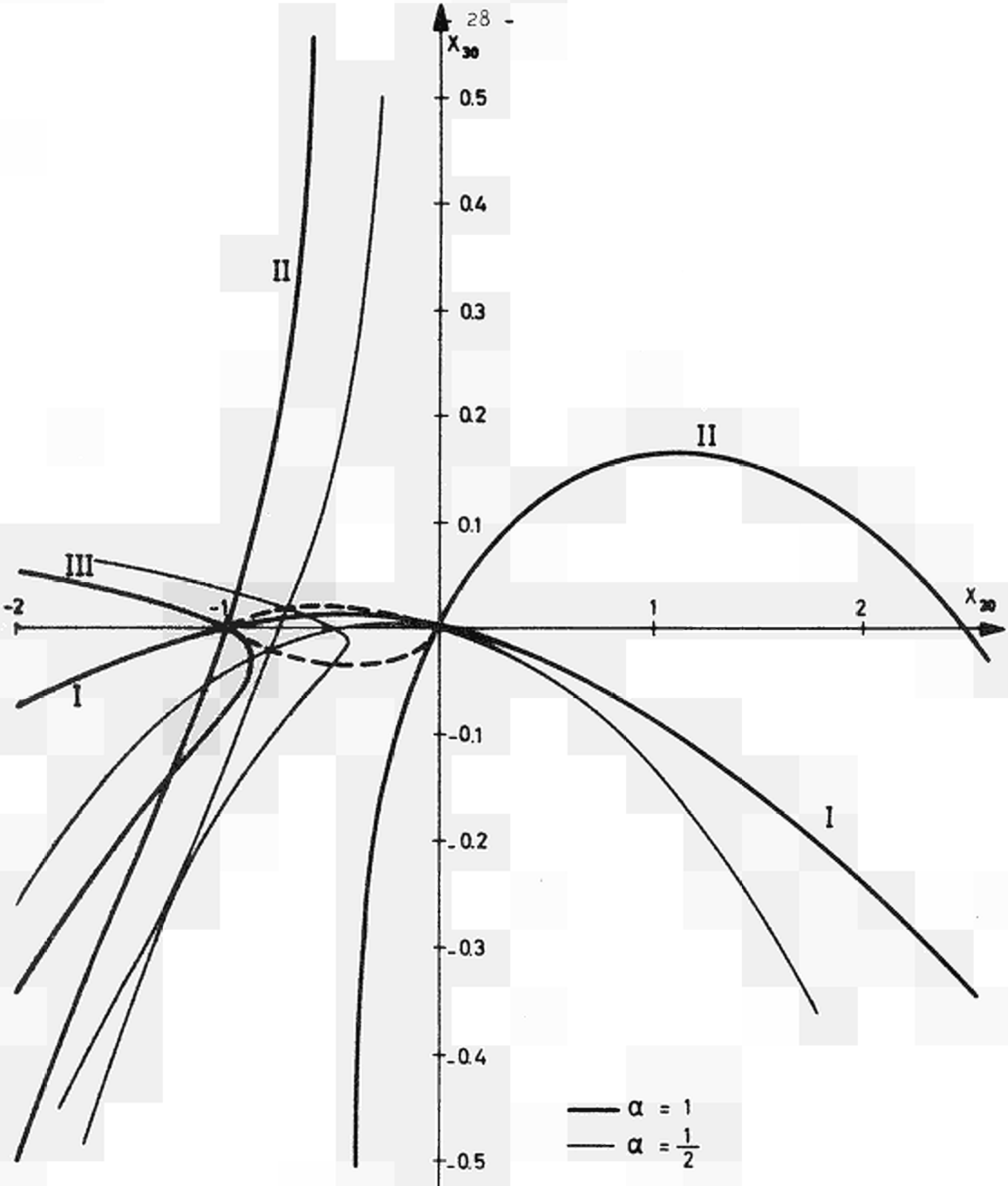
$$\text{(III)} \qquad \alpha x_{20}^3 + (2 + 10\alpha)x_{20}^2 x_{30} + \alpha^2 x_{20}^2 + (4 + 40\alpha + 120\alpha^2)x_{30}^2 = 0.$$

The required point of contact is the intersection point of all three curves. As known from the theory of nonlinear algebraic equations, system (4.13–16) can have $2 \cdot 2 \cdot 2 \cdot 3 = 24$ solution points. Obviously only the intersection point near the origin is of interest for $\alpha \neq 0$. As shown in Fig. 6 this special problem has its first intersection point in the vicinity of the origin, at ($x_{10} = 0$, $x_{20} = -1$, $x_{30} = 0$, $\alpha = 1$). The threefold intersection point is split up into the two dotted curves for $0 < \alpha < 1$.

Thus the stability region is enclosed by the ellipsoid

$$x_1^2 + x_1x_2 + \frac{1}{2} x_2^2 + 2x_1x_3 + x_2x_3 + 6x_3^2 = \frac{1}{2}. \qquad (4.17)$$

x_3 is the deviation of the delayed neutron precursors from the equilibrium state c_{10}. If no perturbations of the initial value c_{10} are considered ($x_3 = 0$), (4.17) reduces to the ellipse (I) of Fig. 5. Thus it has been proved for the special problem (4.4) that the delayed neutrons do not change the region of stability in the state space (x_1, x_2).



$$\left(n_{10} = n_{20} = 1 \; ; \; \frac{\varepsilon_{12}}{l_1} = \frac{\varepsilon_{21}}{l_2} = 1 \; ; \; \frac{\gamma_1}{l_1} = 0 \; , \; \frac{\gamma_2}{l_2} = 1 \right)$$

Fig. 6: Determination of the point of contact between $V_R = K$ and $V_R = 0$.

4β) $\gamma_1 = 0, \gamma_2 \neq 0, \lambda_1 = \beta_1 = 0, \lambda_2 \neq 0, \beta_2 \neq 0$

System (1.1-4) becomes now

$$\dot{x}_1 = - \frac{\Delta_1}{l_1} x_1 + \frac{\varepsilon_{21}}{l_2} x_2 \qquad (4.18)$$

$$\dot{x}_2 = \frac{\varepsilon_{12}}{l_1} x_1 - \left(\frac{\Lambda_2}{l_2} + \frac{\gamma_2}{l_2} n_{20} + \beta \frac{k_{20}}{l_2} \right) x_2 - \frac{\gamma_2}{l_2} x_2^2 + \lambda x_3 \qquad (4.19)$$

$$\dot{x}_3 = \beta \frac{k_{20}}{l_2} x_2 - \lambda x_3 \ . \qquad (4.20)$$

As with (4.4), the numbers

$$c_{20} = 1, \quad \frac{k_{20}}{l_2} = 0.1 \quad \text{and} \quad \frac{\gamma_1}{l_1} = 1, \qquad (4.21)$$

are assumed. This change in the values of (4.4) leads to another Liapunof function

$$V = x_1^2 + x_1 x_2 + \frac{1}{2} x_2^2 + x_1 x_3 + x_2 x_3 + 5.5 x_3^2 \qquad (4.22)$$

but to the same point of contact and hence to the same stability region in the x_1, x_2 plane.

Thus the extent of the stability region in the x_1, x_2, x_3 state space depends on whether the delayed neutrons are considered in the first or the second core.

As with 4α) it can be shown that the delayed neutrons do not influence the extent of the stability region in parameter space.

4γ) $\underline{\gamma_1 \neq 0,\ \gamma_2 \neq 0,\ \lambda_2 = \beta_2 = 0,\ \lambda_1 \neq 0,\ \beta_1 \neq 0}$

The differential equations of these problems are

$$\dot{x}_1 = -\left(\frac{\Delta_1}{l_1} + \beta\,\frac{k_{10}}{l_1} + \varepsilon_1\right)x_1 + \frac{\varepsilon_{21}}{l_2}\,x_2 + \lambda x_3 - \frac{\gamma_1}{l_1}\,x_1^2 \tag{4.23}$$

$$\dot{x}_2 = -\left(\frac{\Delta_2}{l_2} + \varepsilon_2\right)x_2 + \frac{\varepsilon_{12}}{l_1}\,x_1 - \frac{\gamma_2}{l_2}\,x_2^2 \tag{4.24}$$

$$\dot{x}_3 = -\,\lambda x_3 + \beta\,\frac{k_{10}}{l_1}\,x_1 \quad . \tag{4.25}$$

The HURWITZ conditions are now

i) $\quad \dfrac{\Delta_1}{l_1} + \dfrac{\Delta_2}{l_2} + \dfrac{k_{10}}{l_1} + \varepsilon_1 + \varepsilon_2 + \lambda > 0$

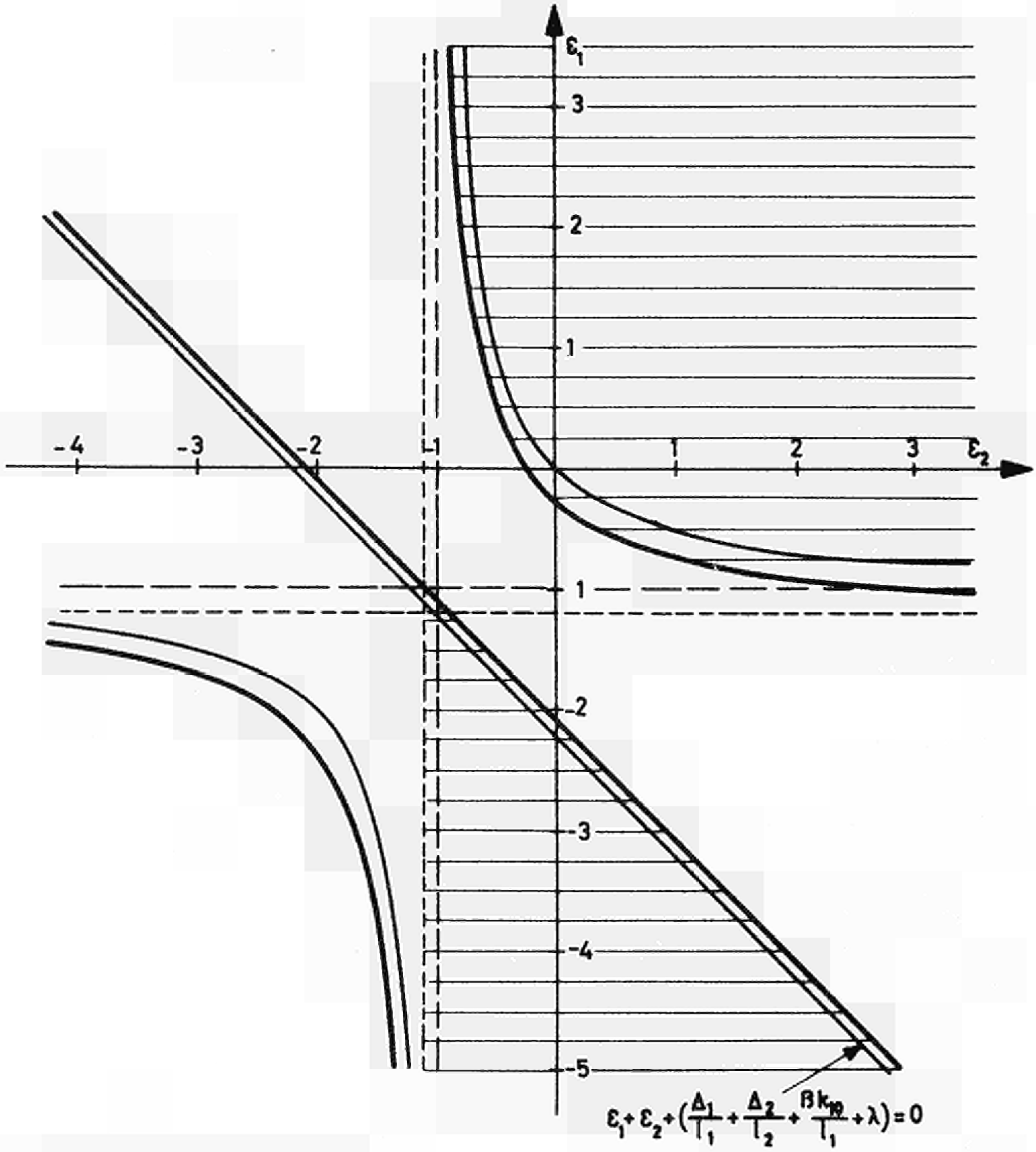
ii) $\quad \dfrac{\Delta_1}{l_1}\,\varepsilon_2 + \dfrac{\Delta_2}{l_2}\,\varepsilon_1 + \varepsilon_1\varepsilon_2 > 0$

iii) $$\left(\frac{\Delta_1}{l_1} + \frac{\Delta_2}{l_2} + \varepsilon_1 + \varepsilon_2\right)\cdot\left[\frac{\Delta_1}{l_1}\,\varepsilon_2 + \frac{\Delta_2}{l_2}\,\varepsilon_1 + \varepsilon_1\varepsilon_2 + \right.$$

$$\left.\beta\,\frac{k_{10}}{l_1}\left(\frac{\Delta_2}{l_2} + \varepsilon_2\right) + \lambda\left(\frac{\Delta_1}{l_1} + \frac{\Delta_2}{l_2} + \varepsilon_1 + \varepsilon_2 + \beta\,\frac{k_{10}}{l_1} + \lambda\right)\right] +$$

$$\beta\,\frac{k_{10}}{l_1}\left[\left(\lambda + \beta\,\frac{k_{10}}{l_1}\right)\left(\frac{\Delta_2}{l_2} + \varepsilon_2\right) + \frac{\Delta_1}{l_1}\,\varepsilon_2 + \frac{\Delta_2}{l_2}\,\varepsilon_1 + \varepsilon_1\varepsilon_2\right] > 0.$$

As shown in Fig. 7, condition iii) covers the shaded region. Thus it follows that the stability region in parameter space remains unchanged by the delayed neutrons as is seen by comparison with Fig. 1. This can also be proved by considering the different paranthesis terms of iii).



—— (thin line): $\frac{\Delta_1}{l_1}\varepsilon_2 + \frac{\Delta_2}{l_2}\varepsilon_1 + \varepsilon_1 \varepsilon_2 = 0$

—— (thick line): $$\left(\frac{\Delta_1}{l_1} + \frac{\Delta_2}{l_2} + \varepsilon_1 + \varepsilon_2\right)\left[\frac{\Delta_1}{l_1}\varepsilon_2 + \frac{\Delta_2}{l_2}\varepsilon_1 + \varepsilon_1 \varepsilon_2 + \frac{\beta k_{10}}{l_1}\left(\frac{\Delta_2}{l_2} + \varepsilon_2\right) + \lambda\left(\frac{\Delta_1}{l_1} + \frac{\beta k_{10}}{l_1} + \lambda + \frac{\Delta_2}{l_2} + \varepsilon_1 + \varepsilon_2\right)\right]$$

$$+ \frac{\beta k_{10}}{l_1}\left[\frac{\Delta_1}{l_1}\varepsilon_2 + \frac{\Delta_2}{l_2}\varepsilon_1 + \varepsilon_1 \varepsilon_2 + \left(\frac{\beta k_{10}}{l_1} + \lambda\right)\left(\frac{\Delta_2}{l_2} + \varepsilon_2\right)\right] = 0$$

Fig. 7: The HURWITZ conditions in case of $\lambda_2 = \beta_2 = 0$

and $\left(\frac{\Delta_1}{l_1} = \frac{\Delta_2}{l_2} = 1 ;\ \frac{\beta k_{10}}{l_1} = \lambda = 0.1\right)$

The Liapunov function is calculated by the GEISS method with the parameters (4.4):

$$3\,V = x_1^2 + x_1x_2 + x_2^2 + 2x_1x_3 + x_2x_3 + 16x_3^2 \qquad (4.26)$$

and its time derivative with respect to system (4.23-25) with $\gamma_1/l_1 = \gamma_2/l_2 = 1$ is given by

$$3\,\dot{V} = -\,3x_1^2 - 3x_2^2 - 3x_3^2 - 2x_1^3 - 2x_2^3 - x_1^2x_2 - x_1x_2^2 - 2x_1^2x_3 - x_2^2x_3. \qquad (4.27)$$

The points of contact between the curves V = K and $\dot{V} = 0$ are found by the same procedure as used in 4α). System (4.9-12) becomes

$$-6x_{10}-6x_{10}^2-2x_{10}x_{20}-x_{20}^2-4x_{10}x_{30} = (2x_{10}+x_{20}+2x_{30})\alpha \qquad (4.28)$$

$$-6x_{20}-6x_{20}^2-x_{10}^2-2x_{10}x_{20}-2x_{20}x_{30} = (x_{10}+2x_{20}+x_{30})\alpha \qquad (4.29)$$

$$-6x_{30}-2x_{10}^2-x_{20}^2 = (2x_{10}+x_{20}+32x_{30})\alpha \qquad (4.30)$$

$$2x_{10}^3+2x_{20}^3+x_{10}^2x_{20}+x_{10}x_{20}^2+2x_{10}^2x_{30}+x_{20}^2x_{30} = -\,(2x_{10}^2+2x_{10}x_{20}+2x_{20}^2+4x_{20}x_{30}+2x_{20}x_{30}+32x_{30}^2)\alpha \qquad (4.31)$$

Now it is convenient to eliminate x_{30}:

$$\text{(I)}\quad \frac{8}{6+32\alpha}\,x_{10}^3 + \left[\frac{12\alpha}{6+32\alpha} - 6\right]x_{10}^2 + \left[\frac{4\alpha^2}{6+32\alpha} - 2\alpha - 6\right]x_{10} =$$

$$\left[2 - \frac{4\alpha}{6+32\alpha}\right]x_{10}x_{20} - \frac{4}{6+32\alpha}\,x_{10}x_{20}^2 + \left[\alpha - \frac{2\alpha^2}{6+32\alpha}\right]x_{20} +$$

$$\left[1 - \frac{2\alpha}{6+32\alpha}\right]x_{20}^2 .$$

(II) $[6+30]x_{10}^2 + [6\alpha+30\alpha^2]x_{10} =$

$$2x_{20}^3+4x_{10}^2x_{20} - [12+60\alpha]x_{10}x_{20} - [36+189\alpha]x_{20}^2 - [63\alpha^2+204\alpha+36]x_{20}$$

(III) $-24x_{10}^4 + [2(6+32\alpha)^2-72\alpha-128\alpha^2]x_{10}^3 + [2\alpha(6+32\alpha)^2 -$

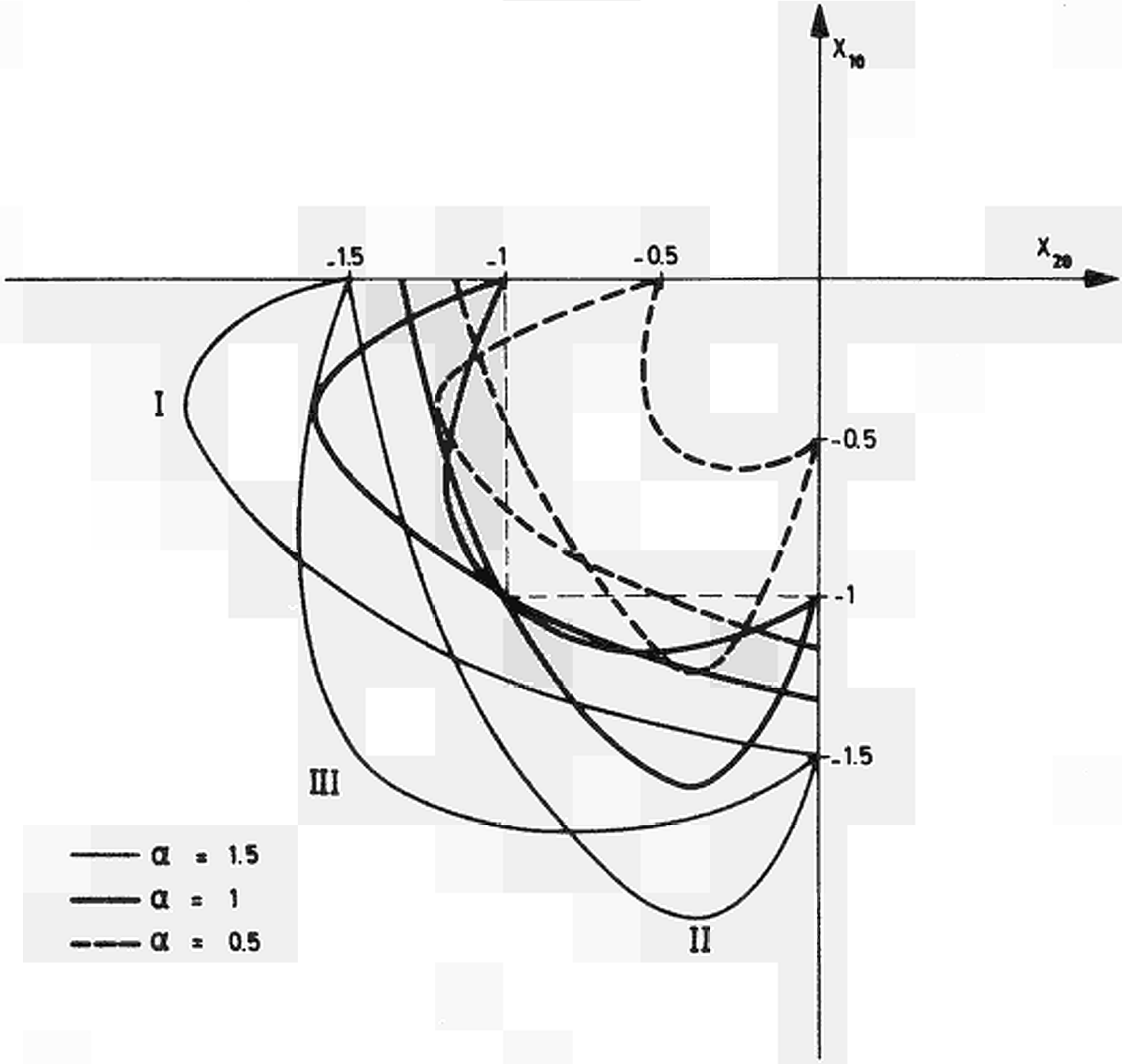
$$48\alpha^2-128\alpha^3]x_{10}^2 + [2\alpha(6+32\alpha)^2-48\alpha^2-128\alpha^3]x_{10}x_{20} + [(6+32\alpha)^2-36\alpha-64\alpha^2]x_{10}x_{20}^2 + [(6+32\alpha)^2-36\alpha - 64\alpha^2]x_{10}^2x_{20} - 24x_{10}^2x_{20}^2 =$$

$$6x_{20}^4 + [-2(6+32\alpha)^2+18\alpha+32\alpha^2]x_{20}^3 + [-2\alpha(6+32\alpha)^2 + 12\alpha^2+32\alpha^3]x_{20}^2.$$

These three curves are plotted in Fig. 8 for $\alpha = 1.5$, 1 and 0.5. It will be noted that there are 3 threefold intersection points of the three curves (−1.5, 0, 0), (−1, −1, 0) and (0, −1.5, 0) near the origin. As can be seen from Fig. 8, no further threefold intersection of the three curves is possible for $0 < \alpha < 1.5$. This can also be verified by calculating the roots along the axis x_{10} and x_{20}. Only for $\alpha = 1$ is the intersection point of curves I and II also an intersection point of curve III for $x_{10} \neq 0$ and $x_{20} \neq 0$. Thus it is proved that from the 24 possible solution points only these three are in the vicinity of the origin. The smallest ellipsoid goes through the points (−1.5, 0, 0) and (0, −1.5, 0). Hence the stability region is given by

$$x_1^2 + x_1x_2 + x_2^2 + 2x_1x_3 + x_2x_3 + 16x_3^2 = 2.25 \,. \qquad (4.32)$$

If no perturbation of the delayed neutron precursors is considered, (4.32) reduces to the ellipse in Fig. 3.



$$\left(n_{10} = n_{20} = 1 \;;\; \frac{\varepsilon_{12}}{l_1} = \frac{\varepsilon_{21}}{l_2} = 1 \;;\; \frac{\gamma_1}{l_1} = \frac{\gamma_2}{l_2} = 1 \right)$$

Fig. 8: Determination of the points of contact between $V_R = K$ and $\dot{V}_R = 0$.

Thus it has been shown in this chapter for some special problems that the delayed neutrons have no influence on the stability region in parameter space (ε_1, ε_2) and also no influence on the stability region in state space (n_1-n_{10}, n_2-n_{20}).

This is in accordance with the statements of GYFTOPOULOS [1] for point reactor dynamics, as the feedback transfer functions are here constants (γ_1 and γ_2).

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Alfred Nobel

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