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A TEST-PROGRAM FOR PSEUDO-RANDOM NUMBERS WITH UNIFORM DISTRIBUTION

by

G. FASSONE and S. ORTHMANN

1967

Joint Nuclear Research Center
Ispra Establishment - Italy

Scientific Information Processing Center - CETIS

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Printed by Smeets
Brussels, May 1967

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SUMMARY

In this report is presented a test program for pseudo-random sequences with uniform distribution in view of their applications to Monte-Carlo methods.

The distribution of pseudo-random sequences is studied in the (X, Y) plane and in the (X, Y, Z) space and the distribution of simple functions of n-tuples of pseudo-random numbers is compared with that predicted by probability theory.

The results obtained have been controlled with the α^2-test and with Kolmogorov's test.

A TEST-PROGRAM FOR PSEUDO-RANDOM NUMBERS WITH UNIFORM DISTRIBUTION(*)

Introduction

In this report is developed a complete test series of pseudo-random sequences in view of their applications to Monte-Carlo methods.

A classical test system has been studied by Kendall - B.Smith [10,11,12] and consists of four tests: the frequency test, the serial test, the gap test and the poker test. Usually the gap test is replaced by a frequency test of the length of runs.

In this paper we have retained only the frequency test and the runs test where these are more significant in the Monte-Carlo method applications. The frequency test involves counting the number of pseudo-random numbers that are found in subintervals of the range of generation (0,1).
Besides testing the randomness by the method of runs we must compare the theoretical and the empirical distribution of up and down runs **and of above and below the mean runs.**
The two others have been omitted because they only give information for binary digit distributions.

We have as well studied the distribution of pseudo-random sequences in the (X,Y) plane and in the (X,Y,Z) space with the frequency test. Lastly we have considered any simple functions of n-tuples of pseudo-random numbers (maximum, minimum and sum) **to** see if their distribution is according to the distribution that probability theory predicts.

The results obtained from several tests have been evaluated with the χ^2 test and with Kolmogorov's test.
The considered series of tests is more restrictive than those usually applied, therefore the information obtained is particularly significant.

(*) Manuscript received on February 20, 1967.

1. The χ^2 test

1.1. Statement of the Problem

Let X be a random continuous variable defined in the k-dimensional space R_k and we suppose that X assumes values X_j belonging to a given set $I \subset R_k$, where

$I = \bigcup_{r=1}^{\nu} I_r$ and I_r have no common points.

We suppose that X has a distribution

$$P(S) = Pr\left\{X \subset S\right\} \tag{1.1.1}$$

where $S \subset I$, and hence we shall obtain the probabilities

$$p_r = P(I_r) \tag{1.1.2}$$

Consider N independent trials, each of which has an outcome X_j and let f_r be the number of trials in which $X_j \subset I_r$.

We have

$$\sum_{r=1}^{\nu} p_r = 1$$
$$\sum_{r=1}^{\nu} f_r = N \tag{1.1.3}$$

Now f_r has a binomial distribution with mean

$$\mu = N\, p_r \tag{1.1.4}$$

and variance

$$\sigma^2 = N\, p_r(1 - p_r) \tag{1.1.5}$$

that is asymptotically normal.

Let be

$$\chi^2 = \sum_{r=1}^{\nu} \frac{(f_r - Np_r)^2}{Np_r} \qquad (1.1.6)$$

it is known [2] that the "percentage" is defined by the following formula:

$$\mathbf{Pr}\left\{\chi^2 \leq x\right\} = \frac{1}{2^{n/2}\,\Gamma(n/2)} \int_0^x t^{n/2-1}\, e^{-t/2}\, dt \qquad (1.1.7)$$

where n is the number of degrees of freedom and is in our case

$$n = \nu - 1 \qquad (1.1.8)$$

The probability (1.1.7) is given in extensive tables, for example [7], for $1 \leq n \leq 100$ with step 1. See also App.1.

We consider now a sequence of pseudo-random numbers $\xi_1, \xi_2, \ldots, \xi_s$, given by any generators, and determine the vectors

$$\begin{aligned} \eta_1 &= (\xi_1, \ldots, \xi_k) \\ \eta_2 &= (\xi_{k+1}, \ldots, \xi_{2k}) \\ &\cdots\cdots\cdots\cdots\cdots \\ \eta_M &= (\xi_{(M-1)k+1}, \ldots, \xi_{kM}) \end{aligned} \qquad (1.1.9)$$

Let $\left\{X_1, X_2, \ldots, X_N\right\}$ be the values that the random variable X assumes, we set

$$X_j = F(\eta_{M_1}, \eta_{M_1+1}, \ldots, \eta_{M_2}) \qquad (1.1.10)$$

with $1 \leq M_1 \leq M_2 \leq M$

If we want to built a test series for pseudo-random numbers, it is sufficient suitably vary the function F and the parameters k, ν, M_1, M_2, N and calculate the probability (1.1.7), that gives the confidence with which we must accept the pseudo-random sequence.

1.2. Linear Uniformity Test

Let $I \subset R_1$ be the unit interval $(0,1)$ and let I_r be the interval $\left(\frac{r-1}{\nu}, \frac{r}{\nu}\right)$ with $(r=0,1,2,\dots,\nu)$ and putting $k=1$, $M_1 = M_2 = \dot{j}$ we have

$$X_j = F(\xi_j) \tag{1.2.1}$$

We consider now F as unit function, i.e.

$$X_j = \xi_j \tag{1.2.2}$$

As well we have

$$p_{i_1} = p_{i_2} = \frac{1}{\nu} \quad \text{for every } i_1,\ i_2 \tag{1.2.3}$$

i.e.

$$\chi^2 = \frac{N}{\nu} \sum_{r=1}^{\nu} \left(f_r - \frac{N}{\nu}\right) \tag{1.2.4}$$

The test has been applied with $N = 10^4$ and with the following values of ν :

$\nu = 4$ because it gives useful information for applications of the Monte Carlo method to the solution of partial differential equations.

$\nu = 10$ because it gives useful information about distribution of the first decimal digit.

$\nu = 100$ because it gives sufficient information about numbers distribution.

1.3. Pairs Uniformity Test

Let $I \subset R_2$ be the unit square of the (x,y) plane, $(0 \leq x \leq 1 \ , \ 0 \leq y \leq 1)$ and let $I_r \equiv I_{r_1,r_2}$ be the square

$$\left(\frac{r_1-1}{\nu} \leq x \leq \frac{r_1}{\nu} \ , \ \frac{r_2-1}{\nu} \leq y \leq \frac{r_2}{\nu}\right)$$

where varying r the pair (r_1, r_2) is changed in conformity with the sequence $(1,1 \ ; \ 1,2;\ldots; \ 1,\nu; \ 2,1;\ldots; \ \nu,\nu)$.

Putting k=2, $M_1 = M_2 = j$ and F unit function, we have:

$$X_j = \eta_j \tag{1.3.1}$$

As well we have

$$p_{i_1} = p_{i_2} = \frac{1}{\nu^2} \text{ for every } i_1, \ i_2 \tag{1.3.2}$$

The test has been applied with $N = 5 \cdot 10^4$, $\nu = 10$

1.4. Test for Uniformity of Triples

This test is similar to that for pairs except that $I \subset R_3$ is the unit cube of the (x,y,z) space, and $M_1 = M_2 = j$, F unit function, $N = 3 \cdot 10^4$, $\nu = 10$.

1.5. Test for Uniformity of Maximums

Let $I \subset R_1$ be the unit interval (0,1) and let I_r be the interval $\left(\frac{r-1}{\nu}, \frac{r}{\nu}\right)$ with $(r=0,1,\ldots,\nu)$ and putting k=1, $M_1 = (j-1)m+1$, $M_2 = jm+1$ we have

$$X_j = F\ (\xi_{M_1}, \xi_{M_1+1}, \ldots, \xi_{M_2}) \tag{1.5.1}$$

Let

$$F(\xi) = \left[\operatorname*{Max}_{1 \leq l \leq m} (\xi_1, \xi_2, \ldots, \xi_m)\right]^m \tag{1.5.2}$$

we have

$$X_j = \left[\operatorname{Max}\left(\xi_{(j-1)m+1}, \ldots, \xi_{jm+1}\right)\right]^m \tag{1.5.3}$$

If we consider m random variables, independent and uniformly distributed over (0,1) $X_1, \ldots, X_m$, and let $X = \operatorname*{Max}_{1 \leq i \leq m} (X_1, \ldots, X_m)$ be, we have

$$\Pr\left\{X < k\right\} = \Pr\left\{X_1 < k, \ldots, X_m < k\right\} =$$

$$= \Pr\left\{X_1 < k\right\} \ldots\ldots. \Pr\left\{X_m < k\right\} = k^m \tag{1.5.4}$$

Hence the distribution function

$$F(X) = X^m \tag{1.5.5}$$

should be uniform.

As well we have

$$p_{i_1} = p_{i_2} = \frac{1}{\nu} \quad \text{for every } i_1,\ i_2 \qquad (1.5.6)$$

The test has been applied with $N=10^4$, $\nu = 100$ and with $m = 2,5,10$.

1.6. Test for Uniformity of Minimums

The test is similar to that for the maximums, putting

$$F(\xi) = \left[1 - \min_{1\leq l\leq m} (\xi_1,\xi_2,\ldots,\xi_m)\right]^m \qquad (1.6.1)$$

i.e.

$$X_j = \left[1 - \text{Min}\,(\xi_{(j-1)m+1},\ldots,\xi_{jm+1})\right]^m \qquad (1.6.2)$$

1.7. Test for Uniformity of Sums

This test is similar to that for maximums and minimums, putting

$$X_j = \sum_{l=(j-1)m+1}^{jm+1} \xi_l \quad (\text{mod. } 1) \qquad (1.7.1)$$

and we have

$$p_{i_1} = p_{i_2} = \frac{1}{\nu} \text{ for every } i_1,\ i_2 \qquad (1.7.2)$$

This follows by induction from formulas of the sum of m independent random variables uniformly distributed [2 pag. 245]. See also [6].

2. The Kolmogorov's Test

Let us suppose that the set $I \subset R_k$, in which is defined the random variables X, is made up of ν^k sets $E_{r_1,r_2,\ldots,r_k}$, such that

$$E_{r_1,r_2,\ldots,j_1,\ldots,r_k} \subset E_{r_1,r_2,\ldots,j_2\ldots,r_k} \text{ for } j_1 < j_2$$
$$E_{\nu,\ldots,\nu} = I \qquad (2.1)$$

Let

$$P_{r_1,\ldots,r_k} = P(E_{r_1,\ldots,r_k}) = \mathbf{Pr}(X \subset E_{r_1,\ldots,r_k}) \qquad (2.2)$$

Consider N independent trials with outcome X_j and let $F_{r_1,\ldots,r_k}$ be the number of trials in which $X_j \subset E_{r_1,\ldots,r_k}$

We have

$$P_{\nu,\ldots,\nu} = 1$$
$$F_{\nu,\ldots,\nu} = N \qquad (2.3)$$

If in particular $E_{r_1,r_2\ldots,r_k}$ is the ipercube in the $(X_1,X_2,\ldots,X_k)$ space defined from

$$\left(X_1 \leq \frac{r_1}{\nu}\ ,\ X_2 \leq \frac{r_2}{\nu}\ ,\ldots,\ X_k \leq \frac{r_k}{\nu}\right)$$

we have

$$P_{r_1,\ldots,r_k} = \frac{r_1 \cdot r_2 \cdot \ \ldots\ \cdot r_k}{\nu^k} \qquad (2.4)$$

Putting

$$D = \max_{\substack{1 \leq r_i \leq \nu \\ i=1,2,\ldots,k}} \left| P_{r_1,\ldots,r_k} - F_{r_1,\ldots,r_k} \right| \qquad (2.5)$$

we have for $N \to \infty$

$$\Pr\left\{D\sqrt{N} < \lambda\right\} \to 1-2\sum_{\eta=1}^{\infty} (-1)^{\eta-1} e^{-2\eta^2\lambda^2} = k(\lambda) \qquad (2.6)$$

The probability (2.6) is given, in extensive tables, for example [15], for $0{,}28 \leq \lambda \leq 2.75$ with step 0.01. See also App.2

Kolmogorov's criterion has been applied to all uniformity tests.

3. The Runs Tests

3.1. Runs Up and Down

This criterion applies to the case in which the pseudo-random numbers should be divided in two classes C_1, C_2. Let consider a sequence U_o, $U_1,\dots,U_k,\dots$ we say that the number $U_k \in C_1$ if $U_k - U_{k+1} < 0$ and that $U_k \in C_2$ if $U_k - U_{k+1} > 0$. A run is the set of numbers following each other and belonging to the same class. The number of pseudo-random numbers in a run is called its length. Let consider N pseudo-random numbers and let f_l be the runs numbers of length l, we compare the values f_l with the theoretical values φ_l. For the case of N = 1024 we have [16]

$$\varphi_1 = 213.4$$

$$\varphi_2 = 93.7$$

$$\varphi_3 = 26.9$$

$$\varphi_4 = 5.8$$

$$\varphi_h = 1.2 \qquad \text{for } h \geq 5$$

for the class C_1(up).
The same values are good for the numbers of class C_2(down)

3.2. Runs Above and Below the Mean

Let consider a sequence U_o, $U_1,\dots,U_k,\dots$ we say that $U_k \in C_1$ if $U_k \leq 0.5$ and that $U_k \in C_2$ if $U_k > 0.5$. We compare the values f_l with the theoretical values φ_l. For N = 1024 we have [16]

$$\varphi_1 = 128.0$$
$$\varphi_2 = 64.0$$
$$\varphi_3 = 32.0$$
$$\varphi_4 = 16.0$$
$$\varphi_5 = 8.0$$
$$\varphi_6 = 4.0$$
$$\varphi_7 = 2.0$$
$$\varphi_8 = 1.0$$
$$\varphi_9 = 0.5$$
$$\varphi_h = 0.5 \qquad \text{for } h \geq 10$$

for the class C_1 (above).
The same values are good for the numbers of class C_2 (below).

4. Uniform Random Number Generators

The test program has been applied to a series of uniform random number generators : moltiplicative and mixed congruential generators, two combining two congruential generators, and one alternative generator. We have considered binary and decimal generators. Numerical results are presented in App. 3

4.1. Moltiplicative Conguential Generators

Consider the iterative process

$$U_{k+1} = a\, U_k \pmod{M} \qquad (4.1.1)$$

with $U_o = b$

where a, b, M are integer numbers and it is convenient to choose $M = 2^m$ for a binary machine and $M = 10^n$ for a decimal machine. Barnett [1] has shown that for $M = 2^n$, a, b odd and $a \equiv 3 \pmod 8$ or $a \equiv 5 \pmod 8$ the period P of the generator is $P = 2^{n-2}$.

We have considered the following generators

RMTB $\quad U_{k+1} = (2^{17}+3)U_k \quad (\text{mod. } 2^{35})\ U_o = 1$

RMTD $\quad U_{k+1} = (2^5+3)\ U_k \quad (\text{mod. } 10^{10})U_o = 1$

RMTDL $\quad U_{k+1} = 23\ U_k \quad (\text{mod } 10^8+1)U_o = 47594118$

RMTDL was the first conguential generator and was proposed by Lehmer . Its period is P = 58882352.

To test RMTD on a 35 bits binary machine we have considered the following iterative process

$$U_{k+1} = 3\,U_k + 10^5(U_k - n\cdot 10^5) \pmod{10^{10}}$$

$$n = \left[\frac{10^5 + 3}{10^{10}}\,U_k\right] \qquad (4.1.2)$$

where [X] is the integral part of X.

4.2. Mixed Congruential Generators

Consider the iterative process

$$U_{k+1} = a\,U_k + c \pmod{M} \qquad (4.2.1)$$

with $U_o = b$

where a, b, c, M are integer numbers and $M = 2^n$ for binary machines and $M = 10^n$ for decimal machines.

Rotenberg [14] has shown that for $M = 2^n$, $a = 2^r + 1$ with $r \geqslant 2$ and c odd the full period P = M is obtained. Moreover in [4] it is shown that for $M = 10^n$ $a = 10^r+1$, c prime to 10 the full period P of generator P = M is obtained.

We have considered the following generators

RMB1	$U_{k+1} = (2^{12} + 1)\,U_k+1$	$\pmod{2^{35}}$	$U_o = 20867350019$
RMB2	$U_{k+1} = (2^7 + 1)\,U_k+.788\cdot 2^{35}$	$\pmod{2^{35}}$	$U_o = 18575103187$
RMB3	$U_{k+1} = (2^7 + 1)\,U_k+1$	$\pmod{2^{35}}$	$U_o = 0$
RMD	$U_{k+1} = 101\,U_k + 1$	$\pmod{10^{10}}$	$U_o = 0$

To test RMD on a 35-bits binary machine we have considered the following iterative process

$$U_{k+1} = U_k + 100\,(U_k - n.10^8) + 1 \qquad (\text{mod } 10^{10})$$
$$n = \left[\frac{101}{10^{10}}\, U_k\right] \qquad (4.2.2)$$

where [X] is the integral part of X

4.3. Combination of Two Congruential Generators

Consider two pseudo-random generators

$$U_{k+1} = a\, U_k + c \qquad (\text{mod } M)$$
$$U_o = b \qquad (4.3.1)$$

$$V_{k+1} = a'\, V_k + c' \qquad (\text{mod } M)$$
$$V_o = b' \qquad (4.3.2)$$

Let $U[U_1, U_2, \ldots, U_{10}]$ be the vector whose ten components are obtained in accordance with (4.3.1).

Let

$$n = [10\, V_{k+1}] + 1 \qquad (4.3.3)$$

where [X] is the integral part of X.
We have evidently $0 < n \leqslant 10$ and we consider the nth component in the vector U.

Let

$$\xi_{k+1} = U_n \qquad (4.3.4)$$

ξ_{k+1} gives the desiderated pseudo-random sequence.

Lastly with the generator (4.3.1) we calculate a new component U_n.

We have considered the following generators

RANMX1
$$\begin{cases} U_{k+1} = (2^{17}+3)U_k & (\mathrm{mod}\ 2^{35})\quad U_o = 1 \\ V_{k+1} = (2^{7}+1)V_{k+1} & (\mathrm{mod}\ 2^{35})\quad V_o = 1 \end{cases}$$

RANMX2
$$\begin{cases} U_{k+1} = (2^{17}+3)U_k & (\mathrm{mod}\ 2^{35})\quad U_o = 1 \\ V_{k+1} = (2^{12}+1)V_{k+1}+1 & (\mathrm{mod}\ 2^{35})\quad V_o = 20867350019 \end{cases}$$

4.4. Generator of Generalized Fibonacci Type

Consider the iterative process

$$U_{k+1} = a\, U_k + U_{k-1} \quad (\mathrm{mod}\ M)$$
$$U_o = b \qquad U_1 = c \tag{4.4.1}$$

In [4] it is shown that for $M = 10^n$, $a = 10^r+1$, $b = 0$, $c = 1$, the period is

$$P = \mathrm{l{\cdot}c{\cdot}m}\ \ (3{\cdot}2^{n-1},\ 4{\cdot}5^n) \tag{4.4.2}$$

We have considered the following generator

RFBD
$$U_{k+1} = (10^4+1)U_k+U_{k-1} \qquad (\mathrm{mod}\ 10^8)$$
$$U_o = 0 \qquad U_1 = 1$$

To test RFBD on a 35-bits binary machine we have considered the following iterative process

$$U_{k+1} = U_k + U_{k-1} + 10^4(U_k - n.10^4) \quad (\text{mod } 10^8)$$

$$n = \left[\frac{10^4+1}{10^8} U_k + \frac{U_{k-1}}{10^8}\right] \qquad (4.4.3)$$

where [X] is the integral part of X.

5. Program Description

It is possible to execute eight tests on a pseudo-random numbers sequence. There are:

5.1. Linear Uniformity

It is called with IFTEST = 1 and NSET = 0
As well it is necessary to specify

N total number of pseudo-random numbers to generate

ν number of partial intervals

U_o initial value of sequence

In output we should have:

- table of distribution function f_r (1.1)
- distribution's mean μ
- distribution's variance σ^2
- degree of freedom $n = \nu - 1$ (1.1.8)
- percentage (1.1.7)
- χ^2 (1.1.6)
- table of cumulative distribution function F(pgf 2)
- maximum D (2.5)
- as result of Kolmogorov's test:

GOOD if $1.0 < D\sqrt{n} < 1.5$ or
$D\sqrt{n} < 0.5$

VERY GOOD if $0.7 < D\sqrt{n} < 1.0$
REJECTED for other values of $D\sqrt{n}$

If $n > 100$ the program calculates the corresponding values of χ^2 with the relation:

$$\chi^2 = \frac{1}{2}\left(\sqrt{2n-1} + \mu_i\right)^2$$

because for $n > 30$ the χ^2 distribution is approximately normal with mean $\sqrt{2n-1}$ and variance 1. Therefore the μ_i values are obtained from a table of normal distribution [5] for the values of desired percentage.

The program calculates for the percentage 50.0 the χ^2 value, comparing it with χ^2_c obtained from (1.1.6). If $\chi^2 < \chi^2_c$ the program calculates a new χ^2 for probabilities greater than 50.0. If $\chi^2 > \chi^2_c$ is used the relation

$$\chi^2 = \frac{1}{2}\left(\sqrt{2n-1} - \mu_i\right)^2$$

and a new χ^2 for probabilities less than 50.0 is calculated.

The considered probabilities are:

50.0, 60.0, 70.0, 80.0, 90.0, 95.0, 97.5, 99.0, 99.5, 99.75, 99.9, 99.95

50.0, 40.0, 30.0, 20.0, 10.0, 5.0, 2.5, 1.0, 0.5, 0.25, 0.1, 0.05

The corresponding values of μ_i are:

0.0, 0.253, 0.524, 0.842, 1.282, 1.645, 1.960, 2.326, 2.576, 2.807, 3.090, 3.291

5.2. Pairs

It is called with IFTEST = 2 and NSET = 0. As well it is necessary to specify:

N total number of pseudo-random numbers to generate

ν number of partial intervals for each axis (1.3)

U_o initial value of sequence.

For the output the procedure is similar to that for Linear Uniformity.

5.3. Triples

It is called with IFTEST = 3 and NSET = 0. As well it is necessary to specify:

N total number of pseudo-random numbers to generate

ν number of partial intervals for each axis (1.4)

U_o initial value of sequences

The output is similar to that for Linear Uniformity.

5.4.-6. Maximum, Minimum and Sum of m Numbers

These three tests being similar, have been put together. They are called with IFTEST = 10 and NSET = 0. As well it is necessary to specify:

N total number of pseudo-random numbers to generate

ν number of partial intervals

U_o initial value of sequence

$m \leq 10$ because each m-tuple is formed with the first m pseudo-random numbers of a vector with length 10.

The output is similar to that for Linear Uniformity.

5.7.-8. Runs Up and Down - Runs Above and Below the Mean

These two tests, being similar, have been put together. They are called with NSET $\leq$ 64 where NSET is the number of sets. As well it is necessary to specify

N = 1024 total number of pseudo-random numbers to generate for each set. Using N$\neq$1024 it is necessary to change the theoretical values in the main program.

U_o initial value of sequence.

In output we should have:

- observed average numbers of runs (up and down) with the expected average numbers.
- χ^2 value for each set
- table of obtained runs number.

The same output is obtained in the Runs Above and Below the Mean.

The program in the first six cases makes three successive tests each one starting at the point where the previous test finished.

It is necessary to specify IFTW = 0 to have the χ^2 table in output (only for degree of freedom n $\leq$ 100).

The subroutine QKHI (NU, QK, IFTW) with the BLOCK DATA <<TABLE>> should be used independently of the program, for example, to amplify the χ^2 table (See also the program).

6. Note

6.1. Time

The execution time of this program is dependent on the total number of generated pseudo-random numbers and on the number of partial intervals. By experience we are able to say that to execute all the tests with the data of "Sample Input", the program employed

~ 11,5′	for binary whether mixed or moltiplicative conguential generators
~ 13′ ÷ 13,5′	for decimal whether mixed or moltiplicative generators
~ 16′	for Lehmer's generator (RMTDL)
~ 13′ ÷ 13,5′	for binary combination of two conguential generators (RANMX1, RANMX2)

6.2. Space

Routine	Locations	
	decimal	octal
MAIN PROGRAM (TEGUS)	7219	16063
KOLM	266	412
QKHI	3250	6262
ARUN	55	67
ASSIN	123	173
OUT	74	112
TABLE	BLOCK DATA	
TOTAL OF LOCATIONS	10987	25353

6.3. Usage

The program, written in FORTRAN IV for IBM 7090, requires that the pseudo-random numbers generator has been programmed as a FUNCTION.

The cards that eventually would have to be modified, have an asterisk in the columns 73-80.

The input description and a sample input are in the following three pages.

Word	1	2	3	4	5	6
Column	1 ÷ 12	13 ÷ 24	25 ÷ 36	37 ÷ 48	49 ÷ 51	52 ÷ 54
Format	I 12	I 12	I 12	I 12	I 3	I 3
Card 1	total of generated pseudo-random numbers if zero: call exit	number of partial intervals	start value of sequence	total of sets for the "RMNS TESTS"	If: 1 - UNIFORMITY TEST 2 - PAIRS TEST 3 - TRIPLES TEST 10 - MAXIMUM, MINIMUM AND SUM of "m" generated number TEST	value of "m" (only if IFTEST = 10)
Symbol	N	NU	INPV	NSET	IFTEST	IFN

[continue]

Word	7					
Column	55 ÷ 57					
Format	I 3					
Card 1	If zero the sub-program QKHI write the χ^2 table					
Symbol	IFTW					

Word	1
Column	2 - 72
Format	12A6
Card 2	Title of the test
Symbol	TITLE

Word						
Column						
Format						
Card						
Symbol						

SAMPLE INPUT

```
10000             100 20867350019                    1     1
     UNIFORMITY  TEST          FOR   THE   FUNCTION RMB1(INPV)  NU=100
10000              10 20867350019                    1     1
     UNIFORMITY  TEST          FOR   THE   FUNCTION RMB1(INPV)  NU= 10
10000               4 20867350019                    1     1
     UNIFORMITY  TEST          FOR   THE   FUNCTION RMB1(INPV)  NU=  4
50000              10 20867350019                    2     1
    PAIRS  TEST                FOR   THE   FUNCTION RMB1(INPV)
30000              10 20867350019                    3     1
TRIPLES  TEST FOR  THE FUNCTION RMB1(INPV)
100000            100 20867350019                 10 10    1
MAXIMUM,MINIMUM,SUM  TESTS  FOR THE FUNCTION RMB1(INPV)      M= 10
100000            100 20867350019                 10  5    1
MAXIMUM,MINIMUM,SUM  TESTS  FOR THE FUNCTION RMB1(INPV)      M=  5
100000            100 20867350019                 10  2    1
MAXIMUM,MINIMUM,SUM  TESTS  FOR THE FUNCTION RMB1(INPV)      M=  2
 1024                 20867350019           32
     RUNS   TESTS              FOR   THE   FUNCTION RMB1(INPV)
———— BLANK   CARD ————————————————
```

6.4. Output

The annexed example is the output of generator RMB1. Only the first case of each test is reported.

UNIFORMITY TEST FOR THE FUNCTION RMB1(INPV) NU=100

1	89.	102.	103.	92.	84.	77.	88.	108.	113.	117.
11	91.	97.	98.	98.	100.	95.	114.	101.	118.	104.
21	93.	73.	100.	100.	90.	110.	99.	102.	113.	101.
31	88.	103.	105.	103.	95.	96.	110.	100.	99.	95.
41	108.	111.	105.	74.	94.	105.	116.	94.	110.	110.
51	84.	102.	117.	88.	105.	97.	120.	107.	96.	86.
61	94.	80.	101.	106.	97.	96.	93.	93.	99.	110.
71	92.	116.	85.	100.	112.	97.	108.	101.	104.	99.
81	93.	105.	98.	90.	97.	111.	116.	110.	80.	102.
91	98.	99.	114.	102.	95.	95.	110.	112.	105.	92.

OBSERVED MEAN = 0.5027378

OBSERVED VARIANCE = 0.0830587   EXPECTED VARIANCE = 0.083333

DEGREE OF FREEDOM = 99

PERCENTAGE = 0.5000E 02

KHI-SQUARED = 0.97760E 02

KOLMOGOROV S TEST

MAX = 6.500000E-03

CUMULATIVE DISTRIBUTION FUNCTION

1	0.00890	0.01910	0.02940	0.03860	0.04700	0.05470	0.06350	0.07430	0.08560	0.09730
11	0.10640	0.11610	0.12590	0.13570	0.14570	0.15520	0.16660	0.17670	0.18850	0.19890
21	0.20820	0.21550	0.22550	0.23550	0.24450	0.25550	0.26540	0.27560	0.28690	0.29700
31	0.30580	0.31610	0.32660	0.33690	0.34640	0.35600	0.36700	0.37700	0.38690	0.39640
41	0.40720	0.41830	0.42880	0.43620	0.44560	0.45610	0.46770	0.47710	0.48810	0.49910
51	0.50750	0.51770	0.52940	0.53820	0.54870	0.55840	0.57040	0.58110	0.59070	0.59930
61	0.60870	0.61670	0.62680	0.63740	0.64710	0.65670	0.66600	0.67530	0.68520	0.69620
71	0.70540	0.71700	0.72550	0.73550	0.74670	0.75640	0.76720	0.77730	0.78770	0.79760
81	0.80690	0.81740	0.82720	0.83620	0.84590	0.85700	0.86860	0.87960	0.88760	0.89780
91	0.90760	0.91750	0.92890	0.93910	0.94860	0.95810	0.96910	0.98030	0.99080	1.00000

THE HYPOTHESIS OF UNIFORMITY IS
GOOD
WHILE THE RESULT IS 6.500E-01

UNIFORMITY TEST FOR THE FUNCTION RMB1(INPV) NU= 10

```
1            973. 1016.  981.  994. 1027. 1002.  969. 1014. 1002. 1022.
```


OBSERVED MEAN = 0.5027378

OBSERVED VARIANCE = 0.0830587 EXPECTED VARIANCE = 0.083333

DEGREE OF FREEDOM = 9

PERCENTAGE = 0.1000E 02

CHI-SQUARED = 0.37600E 01

KOLMOGOROV S TEST

MAX = 3.800012E-03

CUMULATIVE DISTRIBUTION FUNCTION

1	0.09730	0.19890	0.29700	0.39640	0.49910	0.59930	0.69620	0.79760	0.89780	1.00000

THE HYPOTHESIS OF UNIFORMITY IS
GOOD
WHILE THE RESULT IS 3.800E-01

UNIFORMITY TEST FOR THE FUNCTION RMB1(INPV) NU= 4

```
1          2445. 2546. 2476. 2533.    0.    0.    0.    0.    0.    0.

   OBSERVED  MEAN   =   0.5027378
   OBSERVED VARIANCE =   0.0830587          EXPECTED VARIANCE =   0.083333

DEGREE OF FREEDOM =     3
PERCENTAGE  =     0.6000E 02
KHI-SQUARED  =    0.27224E 01
```

KOLMOGOROV S   TEST

MAX =   5.500002E-03

CUMULATIVE DISTRIBUTION FUNCTION

1	0.24450	0.49910	0.74670	1.00000	0.	0.	0.	0.	0.	0.

THE HYPOTHESIS OF UNIFORMITY IS
GOOD
WHILE THE RESULT IS   5.500E-01

PAIRS TEST FOR THE FUNCTION RMB1(INPV)

1	252.	215.	247.	255.	258.	260.	273.	233.	247.	246.
2	220.	260.	262.	291.	258.	267.	269.	276.	220.	267.
3	267.	253.	231.	235.	245.	217.	261.	248.	241.	252.
4	263.	259.	255.	209.	224.	252.	237.	246.	262.	255.
5	268.	258.	239.	244.	255.	265.	263.	270.	244.	232.
6	244.	235.	225.	245.	250.	273.	249.	262.	243.	267.
7	252.	229.	235.	260.	236.	261.	262.	241.	264.	231.
8	262.	239.	252.	266.	266.	252.	262.	262.	268.	231.
9	240.	242.	253.	220.	255.	244.	267.	244.	227.	256.
10	233.	261.	253.	244.	242.	244.	261.	250.	262.	252.

DEGREE OF FREEDOM = 99

PERCENTAGE = 0.4000E 02

KHI-SQUARED = 0.91976E 02

KOLMOGOROV S TEST

MAX = 5.520016E-03

CUMULATIVE DISTRIBUTION FUNCTION

1	0.01008	0.01868	0.02856	0.03876	0.04908	0.05948	0.07040	0.07972	0.08960	0.09944
2	0.01888	0.03788	0.05824	0.08008	0.10072	0.12180	0.14348	0.16384	0.18252	0.20304
3	0.02956	0.05868	0.08828	0.11952	0.14996	0.17972	0.21184	0.24212	0.27044	0.30104
4	0.04008	0.07956	0.11936	0.15896	0.19836	0.23820	0.27980	0.31992	0.35872	0.39952
5	0.05080	0.10060	0.14996	0.19932	0.24892	0.29936	0.35148	0.40240	0.45096	0.50104
6	0.06056	0.11976	0.17812	0.23728	0.29688	0.35824	0.42032	0.48172	0.54000	0.60076
7	0.07064	0.13900	0.20676	0.27632	0.34536	0.41716	0.48972	0.56076	0.62960	0.69960
8	0.08112	0.15904	0.23688	0.31708	0.39676	0.47864	0.56168	0.64320	0.72276	0.80200
9	0.09072	0.17832	0.26628	0.35528	0.44516	0.53680	0.63052	0.72180	0.81044	0.89992
10	0.10004	0.19808	0.29616	0.39492	0.49448	0.59588	0.70004	0.80132	0.90044	1.00000

THE HYPOTHESIS OF UNIFORMITY IS

VERY GOOD

WHILE THE RESULT IS 8.728E-01

TRIPLES TEST FOR THE FUNCTION RMB1(INPV)

1	1	12.	10.	6.	11.	9.	10.	15.	8.	9.	13.
1	2	9.	10.	5.	8.	8.	9.	11.	12.	7.	9.
1	3	10.	11.	12.	10.	9.	12.	8.	8.	7.	9.
1	4	6.	12.	6.	3.	7.	13.	10.	8.	7.	6.
1	5	8.	12.	8.	10.	8.	8.	10.	5.	3.	7.
1	6	9.	5.	3.	8.	12.	8.	5.	8.	6.	5.
1	7	6.	11.	3.	11.	7.	11.	8.	7.	13.	8.
1	8	3.	9.	4.	8.	12.	9.	9.	8.	8.	7.
1	9	7.	13.	13.	6.	4.	12.	13.	9.	7.	5.
1	10	6.	19.	9.	11.	12.	10.	8.	7.	10.	6.
2	1	12.	11.	10.	8.	11.	10.	13.	10.	6.	10.
2	2	8.	8.	4.	4.	8.	10.	10.	12.	4.	12.
2	3	10.	11.	7.	9.	4.	7.	14.	8.	5.	8.
2	4	8.	7.	5.	8.	9.	7.	11.	12.	7.	11.
2	5	7.	7.	14.	8.	10.	10.	14.	12.	8.	8.
2	6	7.	10.	13.	5.	9.	11.	13.	9.	10.	10.
2	7	4.	3.	8.	7.	8.	8.	11.	4.	16.	12.
2	8	8.	8.	9.	10.	3.	9.	5.	10.	8.	4.
2	9	4.	4.	8.	9.	10.	7.	10.	5.	9.	14.
2	10	4.	5.	7.	12.	6.	6.	14.	12.	5.	4.
3	1	7.	9.	10.	10.	6.	11.	9.	10.	15.	7.
3	2	6.	14.	6.	14.	7.	8.	8.	11.	9.	6.
3	3	9.	2.	6.	11.	4.	9.	7.	11.	10.	8.
3	4	11.	8.	9.	13.	4.	11.	12.	3.	5.	9.
3	5	8.	3.	8.	11.	7.	13.	9.	11.	8.	10.
3	6	8.	14.	11.	5.	6.	7.	5.	10.	9.	14.
3	7	8.	6.	12.	10.	5.	11.	11.	13.	11.	5.
3	8	8.	10.	7.	10.	9.	13.	10.	14.	10.	5.
3	9	10.	7.	5.	10.	8.	6.	9.	8.	12.	3.
3	10	14.	2.	6.	12.	13.	7.	11.	8.	7.	11.

4	1	7.	9.	6.	9.	16.	6.	6.	14.	6.	10.
4	2	7.	8.	8.	14.	7.	2.	11.	7.	9.	16.
4	3	7.	5.	9.	6.	18.	11.	10.	4.	9.	13.
4	4	7.	13.	6.	4.	10.	8.	5.	11.	17.	8.
4	5	7.	8.	6.	6.	6.	13.	9.	7.	11.	8.
4	6	8.	7.	8.	11.	11.	8.	12.	11.	11.	4.
4	7	10.	6.	9.	14.	11.	5.	8.	7.	8.	11.
4	8	10.	6.	12.	16.	11.	9.	9.	7.	9.	8.
4	9	6.	10.	7.	10.	8.	10.	13.	11.	12.	7.
4	10	4.	8.	8.	6.	12.	9.	10.	8.	4.	12.
5	1	6.	8.	12.	7.	12.	9.	4.	8.	10.	17.
5	2	7.	9.	7.	9.	8.	12.	8.	15.	8.	15.
5	3	12.	8.	10.	8.	4.	7.	4.	8.	12.	11.
5	4	9.	14.	8.	7.	8.	13.	6.	9.	12.	15.
5	5	9.	7.	9.	10.	4.	12.	6.	14.	11.	11.
5	6	12.	8.	7.	10.	6.	10.	8.	15.	4.	16.
5	7	14.	3.	7.	7.	8.	9.	4.	11.	10.	15.
5	8	7.	8.	10.	9.	6.	5.	8.	12.	11.	16.
5	9	10.	6.	12.	9.	9.	9.	7.	8.	6.	10.
5	10	10.	7.	7.	11.	6.	13.	11.	6.	16.	14.
6	1	12.	13.	7.	9.	9.	6.	17.	8.	12.	6.
6	2	9.	10.	4.	10.	7.	8.	8.	9.	7.	11.
6	3	13.	10.	11.	4.	5.	8.	8.	7.	11.	10.
6	4	7.	4.	7.	9.	4.	12.	6.	7.	13.	6.
6	5	15.	11.	9.	7.	7.	5.	10.	4.	11.	7.
6	6	13.	5.	10.	12.	8.	11.	18.	4.	10.	11.
6	7	9.	14.	10.	10.	5.	9.	10.	7.	13.	9.
6	8	8.	7.	12.	12.	6.	9.	11.	12.	10.	9.
6	9	10.	15.	5.	9.	6.	8.	10.	4.	8.	6.
6	10	13.	15.	8.	13.	5.	10.	8.	7.	7.	7.

7	1	9.	13.	9.	4.	10.	6.	7.	10.	6.	4.
7	2	7.	1.	8.	6.	9.	10.	6.	14.	8.	11.
7	3	5.	13.	12.	7.	8.	5.	4.	15.	5.	13.
7	4	15.	10.	8.	8.	12.	5.	11.	9.	14.	9.
7	5	10.	16.	9.	7.	6.	8.	3.	8.	12.	11.
7	6	6.	15.	8.	11.	7.	10.	10.	17.	9.	11.
7	7	8.	11.	7.	6.	11.	11.	8.	9.	9.	10.
7	8	11.	10.	8.	3.	10.	7.	8.	8.	9.	10.
7	9	6.	6.	8.	11.	9.	9.	12.	10.	5.	9.
7	10	12.	9.	6.	8.	10.	5.	7.	11.	7.	15.
8	1	8.	11.	10.	13.	11.	8.	11.	10.	8.	7.
8	2	8.	9.	14.	12.	8.	10.	10.	7.	13.	8.
8	3	7.	17.	11.	17.	9.	8.	9.	11.	10.	4.
8	4	4.	11.	15.	11.	13.	10.	11.	10.	11.	6.
8	5	14.	13.	10.	15.	9.	13.	10.	3.	10.	10.
8	6	11.	9.	11.	14.	7.	12.	4.	11.	13.	10.
8	7	12.	8.	13.	13.	7.	8.	6.	9.	12.	11.
8	8	10.	5.	13.	12.	6.	8.	9.	6.	13.	7.
8	9	7.	12.	12.	11.	4.	8.	8.	7.	7.	11.
8	10	8.	12.	10.	15.	4.	7.	8.	9.	9.	7.
9	1	5.	7.	13.	10.	8.	9.	10.	9.	6.	10.
9	2	9.	8.	5.	11.	20.	5.	10.	7.	12.	9.
9	3	6.	7.	11.	9.	8.	10.	14.	12.	9.	8.
9	4	6.	5.	9.	6.	13.	12.	4.	8.	11.	10.
9	5	7.	13.	10.	7.	12.	8.	11.	13.	14.	12.
9	6	7.	10.	9.	10.	11.	9.	12.	6.	6.	9.
9	7	7.	10.	7.	10.	8.	11.	11.	6.	5.	11.
9	8	7.	11.	6.	11.	10.	5.	11.	5.	11.	10.
9	9	11.	7.	9.	10.	9.	5.	6.	11.	4.	7.
9	10	11.	10.	13.	8.	5.	6.	12.	3.	12.	9.

10	1	13.	4.	14.	11.	12.	9.	7.	5.	10.	13.
10	2	10.	9.	8.	10.	14.	13.	13.	16.	7.	8.
10	3	8.	7.	8.	8.	10.	10.	7.	9.	12.	10.
10	4	9.	7.	12.	7.	9.	6.	7.	8.	8.	10.
10	5	11.	4.	12.	13.	12.	11.	3.	6.	7.	6.
10	6	11.	7.	9.	9.	11.	8.	6.	4.	10.	7.
10	7	16.	8.	12.	6.	13.	11.	7.	5.	7.	9.
10	8	10.	6.	12.	7.	9.	11.	11.	6.	13.	14.
10	9	15.	11.	10.	6.	13.	10.	8.	12.	8.	8.
10	10	9.	14.	5.	6.	11.	8.	8.	11.	5.	6.

DEGREE OF FREEDOM = 999
PERCENTAGE = 0.5000E 02
KHI-SQUARED = 0.99850E 03

DEGREE OF FREEDOM = 999
PERCENTAGE = 0.4000E 02
KHI-SQUARED = 0.98723E 03

DEGREE OF FREEDOM = 999
PERCENTAGE = 0.3000E 02
KHI-SQUARED = 0.97522E 03
KHI-SQUARED CALCULATED = 0.97978E 03

KOLMOGOROV S TEST

MAX = 1.833341E-02

CUMULATIVE DISTRIBUTION FUNCTION

1	1	0.00133	0.00244	0.00311	0.00433	0.00533	0.00644	0.00811	0.00900	0.01000	0.01144
1	2	0.00233	0.00456	0.00578	0.00789	0.00978	0.01189	0.01478	0.01700	0.01878	0.02122
1	3	0.00344	0.00689	0.00944	0.01267	0.01556	0.01900	0.02278	0.02589	0.02844	0.03189
1	4	0.00411	0.00889	0.01211	0.01567	0.01933	0.02422	0.02911	0.03311	0.03644	0.04056
1	5	0.00500	0.01111	0.01522	0.01989	0.02444	0.03022	0.03622	0.04078	0.04444	0.04933
1	6	0.00600	0.01267	0.01711	0.02267	0.02856	0.03522	0.04178	0.04722	0.05156	0.05700
1	7	0.00667	0.01456	0.01933	0.02611	0.03278	0.04067	0.04811	0.05433	0.06011	0.06644
1	8	0.00700	0.01589	0.02111	0.02878	0.03678	0.04567	0.05411	0.06122	0.06789	0.07500
1	9	0.00778	0.01811	0.02478	0.03311	0.04156	0.05178	0.06167	0.06978	0.07722	0.08489
1	10	0.00844	0.02089	0.02856	0.03811	0.04789	0.05922	0.07000	0.07889	0.08744	0.09578
2	1	0.00267	0.00500	0.00678	0.00889	0.01111	0.01333	0.01644	0.01844	0.02011	0.02267
2	2	0.00456	0.00889	0.01167	0.01511	0.01911	0.02344	0.02889	0.03356	0.03644	0.04133
2	3	0.00678	0.01356	0.01844	0.02400	0.02944	0.03589	0.04378	0.05022	0.05444	0.06122
2	4	0.00833	0.01722	0.02333	0.03011	0.03733	0.04600	0.05622	0.06489	0.07067	0.07933
2	5	0.01000	0.02100	0.02956	0.03833	0.04756	0.05822	0.07111	0.08167	0.08867	0.09900
2	6	0.01178	0.02444	0.03478	0.04500	0.05656	0.06933	0.08422	0.09667	0.10544	0.11744
2	7	0.01289	0.02711	0.03867	0.05089	0.06411	0.07900	0.09600	0.10967	0.12167	0.13589
2	8	0.01411	0.03022	0.04322	0.05744	0.07233	0.08922	0.10778	0.12344	0.13722	0.15267
2	9	0.01533	0.03333	0.04867	0.06456	0.08100	0.10000	0.12111	0.13833	0.15389	0.17144
2	10	0.01644	0.03711	0.05422	0.07267	0.09111	0.11189	0.13544	0.15478	0.17200	0.19067
3	1	0.00344	0.00678	0.00967	0.01289	0.01578	0.01922	0.02333	0.02644	0.02978	0.03311
3	2	0.00600	0.01289	0.01744	0.02356	0.02900	0.03544	0.04278	0.04978	0.05533	0.06167
3	3	0.00922	0.01878	0.02611	0.03556	0.04289	0.05244	0.06300	0.07300	0.08100	0.09011
3	4	0.01200	0.02456	0.03411	0.04622	0.05578	0.06878	0.08300	0.09556	0.10567	0.11767
3	5	0.01456	0.02956	0.04244	0.05778	0.07011	0.08656	0.10444	0.12011	0.13233	0.14711
3	6	0.01722	0.03544	0.05133	0.06867	0.08400	0.10333	0.12378	0.14244	0.15744	0.17544
3	7	0.01922	0.03967	0.05811	0.07856	0.09611	0.11878	0.14256	0.16389	0.18333	0.20411
3	8	0.02133	0.04478	0.06544	0.08900	0.10922	0.13533	0.16178	0.18667	0.20900	0.23156
3	9	0.02367	0.04978	0.07333	0.09967	0.12233	0.15122	0.18122	0.20856	0.23400	0.25900
3	10	0.02633	0.05533	0.08133	0.11156	0.13767	0.16911	0.20278	0.23311	0.26100	0.28833

4	1	0.00422	0.00856	0.01211	0.01633	0.02100	0.02511	0.02989	0.03456	0.03856	0.04300
4	2	0.00756	0.01633	0.02244	0.03111	0.03911	0.04644	0.05567	0.06500	0.07222	0.08144
4	3	0.01156	0.02356	0.03344	0.04611	0.05800	0.06967	0.08322	0.09600	0.10667	0.12011
4	4	0.01511	0.03156	0.04433	0.06011	0.07533	0.09133	0.10911	0.12567	0.14033	0.15756
4	5	0.01844	0.03822	0.05500	0.07467	0.09333	0.11422	0.13667	0.15711	0.17511	0.19600
4	6	0.02200	0.04578	0.06644	0.08933	0.11222	0.13689	0.16322	0.18789	0.20989	0.23444
4	7	0.02511	0.05178	0.07600	0.10356	0.12989	0.15844	0.18900	0.21711	0.24444	0.27300
4	8	0.02833	0.05867	0.08644	0.11889	0.14911	0.18211	0.21633	0.24878	0.28000	0.31122
4	9	0.03133	0.06544	0.09689	0.13322	0.16678	0.20367	0.24289	0.27900	0.31467	0.34911
4	10	0.03444	0.07233	0.10711	0.14800	0.18633	0.22678	0.27078	0.31078	0.34933	0.38744
5	1	0.00489	0.01011	0.01500	0.02000	0.02600	0.03111	0.03633	0.04189	0.04700	0.05333
5	2	0.00900	0.01967	0.02789	0.03833	0.04856	0.05822	0.06878	0.08067	0.08989	0.10267
5	3	0.01433	0.02911	0.04222	0.05756	0.07211	0.08689	0.10222	0.11844	0.13244	0.15067
5	4	0.01889	0.03967	0.05656	0.07578	0.09456	0.11511	0.13533	0.15633	0.17567	0.19933
5	5	0.02322	0.04811	0.07000	0.09422	0.11689	0.14367	0.16922	0.19567	0.21956	0.24811
5	6	0.02811	0.05789	0.08444	0.11300	0.14056	0.17222	0.20256	0.23489	0.26322	0.29722
5	7	0.03278	0.06578	0.09657	0.13067	0.16256	0.19911	0.23411	0.27111	0.30589	0.34556
5	8	0.03678	0.07433	0.10989	0.14978	0.18622	0.22778	0.26733	0.31000	0.34989	0.39400
5	9	0.04089	0.08289	0.12344	0.16822	0.20900	0.25544	0.30078	0.34800	0.39300	0.44144
5	10	0.04511	0.09167	0.13633	0.18689	0.23311	0.28456	0.33589	0.38767	0.43733	0.49100
6	1	0.00622	0.01289	0.01856	0.02456	0.03156	0.03733	0.04444	0.05089	0.05733	0.06433
6	2	0.01133	0.02456	0.03400	0.04656	0.05856	0.06978	0.08311	0.09689	0.10822	0.12289
6	3	0.01811	0.03656	0.05211	0.07000	0.08689	0.10411	0.12311	0.14200	0.15933	0.18056
6	4	0.02344	0.04833	0.06844	0.09122	0.11278	0.13711	0.16167	0.18611	0.21022	0.23756
6	5	0.02944	0.05967	0.08578	0.11433	0.14056	0.17167	0.20267	0.23300	0.26289	0.29589
6	6	0.03578	0.07144	0.10333	0.13756	0.16956	0.20678	0.24456	0.28122	0.31667	0.35633
6	7	0.04144	0.08189	0.11922	0.16000	0.19689	0.24000	0.28356	0.32567	0.36900	0.41533
6	8	0.04633	0.09211	0.13544	0.18344	0.22556	0.27467	0.32400	0.37311	0.42267	0.47444
6	9	0.05156	0.10344	0.15233	0.20622	0.25333	0.30822	0.36444	0.41856	0.47411	0.53089
6	10	0.05722	0.11533	0.16922	0.23033	0.28344	0.34444	0.40756	0.46700	0.52800	0.59078

7	1	0.00722	0.01533	0.02200	0.02844	0.03656	0.04300	0.05089	0.05844	0.06556	0.07300
7	2	0.01311	0.02789	0.03922	0.05289	0.06700	0.08000	0.09478	0.11122	0.12411	0.14044
7	3	0.02044	0.04189	0.06067	0.08044	0.10033	0.11989	0.14078	0.16400	0.18344	0.20778
7	4	0.02744	0.05644	0.08067	0.10622	0.13211	0.15933	0.18700	0.21678	0.24456	0.27600
7	5	0.03456	0.07067	0.10139	0.13400	0.16522	0.20011	0.23456	0.27111	0.30600	0.34433
7	6	0.04156	0.08478	0.12267	0.16167	0.19944	0.24156	0.28389	0.32867	0.37011	0.41633
7	7	0.04811	0.09733	0.14144	0.18767	0.23156	0.28078	0.32978	0.38100	0.43133	0.48533
7	8	0.05422	0.10989	0.16089	0.21467	0.26489	0.32089	0.37656	0.43567	0.49322	0.55378
7	9	0.06011	0.12256	0.18000	0.24089	0.29711	0.35989	0.42378	0.48900	0.55311	0.61967
7	10	0.06711	0.13678	0.19989	0.26889	0.33222	0.40167	0.47322	0.54500	0.61533	0.68956
8	1	0.00811	0.01744	0.02522	0.03311	0.04244	0.04978	0.05889	0.06756	0.07556	0.08378
8	2	0.01489	0.03189	0.04589	0.06233	0.07856	0.09356	0.11067	0.12900	0.14422	0.16222
8	3	0.02300	0.04856	0.07122	0.09567	0.11867	0.14111	0.16533	0.19167	0.21456	0.24100
8	4	0.03044	0.06478	0.09456	0.12600	0.15644	0.18767	0.21989	0.25389	0.28633	0.32056
8	5	0.03911	0.08200	0.11989	0.15956	0.19633	0.23667	0.27678	0.31789	0.35856	0.40078
8	6	0.04733	0.09833	0.14411	0.19222	0.23633	0.28522	0.33367	0.38422	0.43289	0.48411
8	7	0.05522	0.11311	0.16656	0.22333	0.27433	0.33122	0.38700	0.44500	0.50389	0.56411
8	8	0.06244	0.12733	0.18911	0.25478	0.31278	0.37733	0.44078	0.50733	0.57489	0.64244
8	9	0.06911	0.14211	0.21167	0.28567	0.35011	0.42233	0.49489	0.56833	0.64322	0.71800
8	10	0.07700	0.15856	0.23489	0.31867	0.39067	0.47033	0.55144	0.63244	0.71456	0.79778
9	1	0.00867	0.01878	0.02800	0.03700	0.04722	0.05556	0.06578	0.07544	0.08411	0.09344
9	2	0.01644	0.03511	0.05111	0.06989	0.08922	0.10578	0.12511	0.14522	0.16244	0.18256
9	3	0.02522	0.05322	0.07911	0.10689	0.13389	0.15900	0.18700	0.21644	0.24233	0.27178
9	4	0.03333	0.07067	0.10467	0.14011	0.17600	0.21122	0.24767	0.28567	0.32233	0.36067
9	5	0.04278	0.09011	0.13333	0.17778	0.22133	0.26656	0.31211	0.35867	0.40511	0.45278
9	6	0.05178	0.10833	0.16044	0.21444	0.26656	0.32133	0.37656	0.43322	0.48833	0.54600
9	7	0.06044	0.12500	0.18556	0.24933	0.30922	0.37322	0.43700	0.50178	0.56767	0.63556
9	8	0.06844	0.14122	0.21078	0.28467	0.35267	0.42489	0.49756	0.57144	0.64722	0.72356
9	9	0.07633	0.15800	0.23633	0.31967	0.39511	0.47556	0.55800	0.64000	0.72356	0.80789
9	10	0.08544	0.17678	0.26333	0.35733	0.44089	0.52944	0.62178	0.71167	0.80378	0.89756

10	1	0.01011	0.02067	0.03144	0.04167	0.05322	0.06256	0.07356	0.08378	0.09356	0.10433
10	2	0.01900	0.03911	0.05756	0.07867	0.10089	0.11989	0.14144	0.16389	0.18300	0.20544
10	3	0.02867	0.05889	0.08811	0.11911	0.15011	0.17878	0.20978	0.24256	0.27167	0.30456
10	4	0.03778	0.07811	0.11678	0.15622	0.19711	0.23656	0.27678	0.31900	0.35978	0.40267
10	5	0.04844	0.09922	0.14844	0.19833	0.24822	0.29889	0.34856	0.40000	0.45133	0.50422
10	6	0.05867	0.11944	0.17856	0.23900	0.29867	0.35978	0.41978	0.48178	0.54289	0.60656
10	7	0.06911	0.13878	0.20757	0.27856	0.34744	0.41900	0.48833	0.55900	0.63167	0.70656
10	8	0.07822	0.15678	0.23600	0.31778	0.39578	0.47678	0.55622	0.63667	0.72067	0.80556
10	9	0.08778	0.17644	0.26556	0.35744	0.44433	0.53467	0.62478	0.71467	0.80733	0.90111
10	10	0.09789	0.19778	0.29557	0.39889	0.49511	0.59444	0.69533	0.79433	0.89611	1.00000

THE HYPOTHESIS OF UNIFORMITY IS
REJECTED
WHILE THE RESULT IS 1.739E 00

XIMUM,MINIMUM,SUM TESTS FOR THE FUNCTION RMB1(INPV) M= 10

MAXIMUM TEST

1	124.	134.	130.	104.	98.	110.	108.	109.	114.	97.
11	99.	88.	92.	93.	99.	90.	96.	102.	81.	87.
21	96.	81.	88.	104.	100.	102.	98.	94.	97.	101.
31	105.	108.	107.	94.	107.	99.	109.	101.	108.	98.
41	105.	104.	94.	108.	91.	103.	98.	104.	90.	84.
51	88.	103.	99.	102.	98.	95.	109.	119.	95.	88.
61	96.	92.	100.	91.	83.	99.	113.	117.	96.	121.
71	89.	109.	87.	100.	97.	104.	83.	108.	107.	111.
81	84.	103.	112.	92.	73.	102.	101.	105.	110.	107.
91	114.	98.	91.	97.	122.	87.	102.	84.	90.	93.

DEGREE OF FREEDOM = 99

PERCENTAGE = 0.9000E 02

CHI-SQUARED = 0.11410E 03

KOLMOGOROV S TEST

MAX = 1.310000E-02

CUMULATIVE DISTRIBUTION FUNCTION

1	0.01240	0.02580	0.03880	0.04920	0.05900	0.07000	0.08080	0.09170	0.10310	0.11280
11	0.12270	0.13150	0.14070	0.15000	0.15990	0.16890	0.17850	0.18870	0.19680	0.20550
21	0.21510	0.22320	0.23200	0.24240	0.25240	0.26260	0.27240	0.28180	0.29150	0.30160
31	0.31210	0.32290	0.33360	0.34300	0.35370	0.36360	0.37450	0.38460	0.39540	0.40520
41	0.41580	0.42620	0.43560	0.44640	0.45550	0.46580	0.47560	0.48600	0.49500	0.50340
51	0.51220	0.52250	0.53240	0.54260	0.55240	0.56190	0.57280	0.58470	0.59420	0.60300
61	0.61260	0.62180	0.63180	0.64090	0.64920	0.65910	0.67040	0.68210	0.69170	0.70380
71	0.71270	0.72360	0.73230	0.74230	0.75200	0.76240	0.77070	0.78150	0.79220	0.80330
81	0.81170	0.82200	0.83320	0.84240	0.84970	0.85990	0.87000	0.88050	0.89150	0.90220
91	0.91360	0.92340	0.93250	0.94220	0.95440	0.96310	0.97330	0.98170	0.99070	1.00000

THE HYPOTHESIS OF UNIFORMITY IS
GOOD
WHILE THE RESULT IS 1.310E 00

MINIMUM TEST

1	151.	102.	120.	106.	98.	105.	98.	89.	81.	83.
11	110.	104.	96.	84.	82.	78.	109.	97.	90.	89.
21	82.	89.	98.	114.	91.	108.	85.	98.	84.	92.
31	102.	91.	97.	83.	124.	122.	95.	97.	74.	99.
41	112.	100.	117.	107.	106.	88.	119.	98.	89.	110.
51	94.	93.	98.	108.	95.	96.	85.	109.	128.	95.
61	104.	98.	106.	99.	97.	107.	99.	111.	98.	112.
71	99.	76.	101.	108.	105.	91.	90.	96.	93.	102.
81	109.	104.	112.	108.	107.	108.	93.	99.	112.	106.
91	88.	97.	102.	120.	100.	94.	92.	105.	118.	90.

DEGREE OF FREEDOM = 99
PERCENTAGE = 0.9990E 02
CHI-SQUARED = 0.14458E 03

KOLMOGOROV S TEST

MAX = 1.140003E-02

CUMULATIVE DISTRIBUTION FUNCTION

1	0.01510	0.02530	0.03730	0.04790	0.05770	0.06820	0.07800	0.08690	0.09500	0.10330
11	0.11430	0.12470	0.13430	0.14270	0.15090	0.15870	0.16960	0.17930	0.18830	0.19720
21	0.20540	0.21430	0.22410	0.23550	0.24460	0.25540	0.26390	0.27370	0.28210	0.29130
31	0.30150	0.31060	0.32030	0.32860	0.34100	0.35320	0.36270	0.37240	0.37980	0.38970
41	0.40090	0.41090	0.42260	0.43330	0.44390	0.45270	0.46460	0.47440	0.48330	0.49430
51	0.50370	0.51300	0.52280	0.53360	0.54310	0.55270	0.56120	0.57210	0.58490	0.59440
61	0.60480	0.61460	0.62520	0.63510	0.64480	0.65550	0.66540	0.67650	0.68630	0.69750
71	0.70740	0.71500	0.72510	0.73590	0.74640	0.75550	0.76450	0.77410	0.78340	0.79360
81	0.80450	0.81490	0.82510	0.83690	0.84760	0.85840	0.86770	0.87760	0.88880	0.89940
91	0.90820	0.91790	0.92810	0.94010	0.95010	0.95950	0.96870	0.97920	0.99100	1.00000

THE HYPOTHESIS OF UNIFORMITY IS

GOOD

WHILE THE RESULT IS 1.140E 00

SUM TEST

1	95.	105.	100.	115.	96.	74.	87.	91.	97.	93.
11	98.	120.	92.	117.	115.	109.	110.	97.	98.	123.
21	89.	108.	113.	98.	100.	97.	100.	98.	92.	120.
31	120.	96.	113.	95.	104.	107.	114.	92.	98.	104.
41	90.	86.	98.	114.	88.	92.	109.	103.	112.	104.
51	103.	83.	112.	94.	106.	97.	107.	105.	99.	94.
61	92.	99.	102.	90.	90.	91.	107.	103.	112.	106.
71	96.	111.	85.	109.	95.	82.	87.	87.	100.	104.
81	90.	104.	109.	87.	93.	100.	105.	92.	111.	83.
91	80.	104.	87.	107.	106.	90.	92.	116.	117.	93.

DEGREE OF FREEDOM = 99

PERCENTAGE = 0.7000E 02

KHI-SQUARED = 0.10434E 03

KOLMOGOROV S TEST

MAX = 9.599961E-03

CUMULATIVE DISTRIBUTION FUNCTION

1	0.00950	0.02000	0.03000	0.04150	0.05110	0.05850	0.06720	0.07630	0.08600	0.09530
11	0.10510	0.11710	0.12630	0.13800	0.14950	0.16040	0.17140	0.18110	0.19090	0.20320
21	0.21210	0.22290	0.23420	0.24400	0.25400	0.26370	0.27370	0.28350	0.29270	0.30470
31	0.31670	0.32630	0.33760	0.34710	0.35750	0.36820	0.37960	0.38880	0.39860	0.40900
41	0.41800	0.42660	0.43640	0.44780	0.45660	0.46580	0.47670	0.48700	0.49820	0.50860
51	0.51890	0.52720	0.53840	0.54780	0.55840	0.56810	0.57880	0.58930	0.59920	0.60860
61	0.61780	0.62770	0.63790	0.64690	0.65590	0.66500	0.67570	0.68600	0.69720	0.70780
71	0.71740	0.72850	0.73700	0.74790	0.75740	0.76560	0.77430	0.78300	0.79300	0.80340
81	0.81240	0.82280	0.83370	0.84240	0.85170	0.86170	0.87220	0.88140	0.89250	0.90080
91	0.90880	0.91920	0.92790	0.93860	0.94920	0.95820	0.96740	0.97900	0.99070	1.00000

THE HYPOTHESIS OF UNIFORMITY IS

VERY GOOD

WHILE THE RESULT IS 9.600E-01

XIMUM,MINIMUM,SUM TESTS FOR THE FUNCTION RMB1(INPV) M= 5

MAXIMUM TEST

1	91.	90.	102.	90.	95.	98.	112.	102.	99.	119.
11	100.	100.	106.	113.	106.	88.	97.	91.	107.	94.
21	95.	113.	104.	102.	103.	90.	99.	114.	125.	92.
31	100.	96.	93.	80.	104.	94.	86.	103.	105.	99.
41	107.	82.	94.	98.	101.	111.	88.	109.	88.	124.
51	111.	111.	96.	95.	109.	91.	99.	92.	93.	84.
61	100.	103.	96.	103.	116.	119.	101.	89.	111.	96.
71	83.	107.	96.	98.	98.	93.	100.	90.	105.	93.
81	86.	108.	112.	104.	108.	94.	99.	91.	112.	99.
91	93.	96.	99.	103.	108.	111.	94.	100.	108.	98.

DEGREE OF FREEDOM = 99

PERCENTAGE = 0.2000E 02

KHI-SQUARED = 0.82420E 02

KOLMOGOROV S TEST

MAX = 4.499983E-03

CUMULATIVE DISTRIBUTION FUNCTION

1	0.00910	0.01810	0.02830	0.03730	0.04680	0.05660	0.06780	0.07800	0.08790	0.09980
11	0.10980	0.11980	0.13040	0.14170	0.15230	0.16110	0.17080	0.17990	0.19060	0.20000
21	0.20950	0.22080	0.23120	0.24140	0.25170	0.26070	0.27060	0.28200	0.29450	0.30370
31	0.31370	0.32330	0.33260	0.34060	0.35100	0.36040	0.36900	0.37930	0.38980	0.39970
41	0.41040	0.41860	0.42800	0.43780	0.44790	0.45900	0.46780	0.47870	0.48750	0.49990
51	0.51100	0.52210	0.53170	0.54120	0.55210	0.56120	0.57110	0.58030	0.58960	0.59800
61	0.60800	0.61830	0.62790	0.63820	0.64980	0.66170	0.67180	0.68070	0.69180	0.70140
71	0.70970	0.72040	0.73000	0.73980	0.74960	0.75890	0.76890	0.77790	0.78840	0.79770
81	0.80630	0.81710	0.82830	0.83870	0.84950	0.85890	0.86880	0.87790	0.88910	0.89900
91	0.90830	0.91790	0.92780	0.93810	0.94890	0.96000	0.96940	0.97940	0.99020	1.00000

THE HYPOTHESIS OF UNIFORMITY IS
GOOD
WHILE THE RESULT IS 4.500E-01

MINIMUM TEST

1	119.	105.	97.	75.	89.	70.	116.	92.	78.	96.
11	99.	93.	111.	98.	97.	106.	93.	112.	105.	106.
21	121.	82.	101.	118.	119.	97.	97.	103.	83.	78.
31	85.	94.	113.	95.	111.	101.	110.	103.	102.	77.
41	97.	88.	114.	95.	113.	91.	87.	94.	104.	100.
51	105.	99.	112.	77.	91.	115.	93.	89.	104.	117.
61	93.	95.	100.	111.	105.	103.	127.	89.	107.	95.
71	96.	99.	104.	103.	95.	92.	115.	110.	106.	94.
81	102.	104.	116.	113.	81.	104.	91.	94.	99.	104.
91	119.	97.	89.	114.	102.	89.	118.	101.	92.	100.

DEGREE OF FREEDOM = 99

PERCENTAGE = 0.9900E 02

KHI-SQUARED = 0.12968E 03

KOLMOGOROV S TEST

MAX = 8.900091E-03

CUMULATIVE DISTRIBUTION FUNCTION

1	0.01190	0.02240	0.03210	0.03960	0.04850	0.05550	0.06710	0.07630	0.08410	0.09370
11	0.10360	0.11290	0.12400	0.13380	0.14350	0.15410	0.16340	0.17460	0.18510	0.19570
21	0.20780	0.21600	0.22610	0.23790	0.24980	0.25950	0.26920	0.27950	0.28780	0.29560
31	0.30410	0.31350	0.32480	0.33430	0.34540	0.35550	0.36650	0.37680	0.38700	0.39470
41	0.40440	0.41320	0.42460	0.43410	0.44540	0.45450	0.46320	0.47260	0.48300	0.49300
51	0.50350	0.51340	0.52460	0.53230	0.54140	0.55290	0.56220	0.57110	0.58150	0.59320
61	0.60250	0.61200	0.62200	0.63310	0.64360	0.65390	0.66660	0.67550	0.68620	0.69570
71	0.70530	0.71520	0.72560	0.73590	0.74540	0.75460	0.76610	0.77710	0.78770	0.79710
81	0.80730	0.81770	0.82930	0.84060	0.84870	0.85910	0.86820	0.87760	0.88750	0.89790
91	0.90980	0.91950	0.92840	0.93980	0.95000	0.95890	0.97070	0.98080	0.99000	1.00000

THE HYPOTHESIS OF UNIFORMITY IS
VERY GOOD
WHILE THE RESULT IS 8.900E-01

SUM TEST

1	82.	110.	96.	111.	96.	105.	92.	118.	89.	99.
11	94.	100.	93.	103.	108.	113.	106.	97.	93.	99.
21	82.	87.	106.	90.	100.	98.	90.	94.	104.	93.
31	105.	100.	114.	86.	104.	85.	110.	101.	106.	106.
41	105.	119.	116.	100.	99.	91.	79.	93.	108.	94.
51	98.	87.	108.	103.	98.	131.	92.	105.	91.	108.
61	94.	90.	96.	111.	98.	102.	94.	97.	99.	98.
71	95.	115.	109.	95.	97.	95.	78.	95.	99.	117.
81	115.	104.	107.	80.	102.	108.	112.	94.	89.	119.
91	122.	88.	88.	102.	114.	108.	85.	101.	86.	112.

DEGREE OF FREEDOM = 99

PERCENTAGE = 0.7000E 02

CHI-SQUARED = 0.10500E 03

KOLMOGOROV S TEST

MAX = 5.800035E-03

CUMULATIVE DISTRIBUTION FUNCTION

1	0.00820	0.01920	0.02880	0.03990	0.04950	0.06000	0.06920	0.08100	0.08990	0.09980
11	0.10920	0.11920	0.12850	0.13880	0.14960	0.16090	0.17150	0.18120	0.19050	0.20040
21	0.20860	0.21730	0.22790	0.23690	0.24690	0.25670	0.26570	0.27510	0.28550	0.29480
31	0.30530	0.31530	0.32670	0.33530	0.34570	0.35420	0.36520	0.37530	0.38590	0.39650
41	0.40700	0.41890	0.43050	0.44050	0.45040	0.45950	0.46740	0.47670	0.48750	0.49690
51	0.50670	0.51540	0.52620	0.53650	0.54630	0.55940	0.56860	0.57910	0.58820	0.59900
61	0.60840	0.61740	0.62700	0.63810	0.64790	0.65810	0.66750	0.67720	0.68710	0.69690
71	0.70640	0.71790	0.72880	0.73830	0.74800	0.75750	0.76530	0.77480	0.78470	0.79640
81	0.80790	0.81830	0.82900	0.83700	0.84720	0.85800	0.86920	0.87860	0.88750	0.89940
91	0.91160	0.92040	0.92920	0.93940	0.95080	0.96160	0.97010	0.98020	0.98880	1.00000

THE HYPOTHESIS OF UNIFORMITY IS
GOOD
WHILE THE RESULT IS 5.800E-01

XIMUM,MINIMUM,SUM TESTS FOR THE FUNCTION RMB1(INPV) M= 2

MAXIMUM TEST

1	111.	116.	97.	118.	101.	103.	88.	103.	87.	99.
11	88.	122.	100.	89.	124.	105.	91.	96.	112.	99.
21	100.	88.	82.	96.	101.	93.	103.	113.	78.	98.
31	89.	107.	107.	96.	115.	101.	98.	116.	91.	110.
41	102.	90.	95.	110.	95.	106.	101.	98.	90.	107.
51	111.	110.	97.	93.	103.	89.	116.	92.	99.	92.
61	96.	89.	103.	103.	91.	100.	97.	95.	107.	89.
71	95.	101.	110.	97.	92.	123.	104.	95.	119.	109.
81	93.	95.	104.	108.	140.	81.	85.	88.	112.	79.
91	91.	104.	101.	102.	88.	102.	88.	100.	98.	99.

DEGREE OF FREEDOM = 99

PERCENTAGE = 0.9000E 02

KHI-SQUARED = 0.11094E 03

KOLMOGOROV S TEST

MAX = 8.199826E-03

CUMULATIVE DISTRIBUTION FUNCTION

1	0.01110	0.02270	0.03240	0.04420	0.05430	0.06460	0.07340	0.08370	0.09240	0.10230
11	0.11110	0.12330	0.13330	0.14220	0.15460	0.16510	0.17420	0.18380	0.19500	0.20490
21	0.21490	0.22370	0.23190	0.24150	0.25160	0.26090	0.27120	0.28250	0.29030	0.30010
31	0.30900	0.31970	0.33040	0.34000	0.35150	0.36160	0.37140	0.38300	0.39210	0.40310
41	0.41330	0.42230	0.43180	0.44280	0.45230	0.46290	0.47300	0.48280	0.49180	0.50250
51	0.51360	0.52460	0.53430	0.54360	0.55390	0.56280	0.57440	0.58360	0.59350	0.60270
61	0.61230	0.62120	0.63150	0.64180	0.65090	0.66090	0.67060	0.68010	0.69080	0.69970
71	0.70920	0.71930	0.73030	0.74000	0.74920	0.76150	0.77190	0.78140	0.79330	0.80420
81	0.81350	0.82300	0.83340	0.84420	0.85820	0.86630	0.87480	0.88360	0.89480	0.90270
91	0.91180	0.92220	0.93230	0.94250	0.95130	0.96150	0.97030	0.98030	0.99010	1.00000

THE HYPOTHESIS OF UNIFORMITY IS
VERY GOOD
WHILE THE RESULT IS 8.200E-01

MINIMUM TEST

1	100.	113.	106.	97.	87.	91.	93.	111.	90.	106.
11	95.	104.	108.	94.	106.	99.	108.	105.	99.	98.
21	106.	90.	112.	100.	90.	87.	81.	103.	108.	98.
31	89.	71.	80.	96.	121.	95.	98.	98.	101.	96.
41	103.	95.	102.	116.	112.	102.	101.	101.	84.	94.
51	95.	105.	104.	90.	104.	93.	103.	97.	103.	105.
61	86.	92.	83.	105.	114.	111.	119.	107.	107.	102.
71	90.	96.	117.	78.	98.	106.	113.	99.	97.	99.
81	91.	111.	95.	93.	118.	116.	111.	112.	92.	100.
91	105.	102.	111.	97.	88.	99.	103.	110.	104.	84.

DEGREE OF FREEDOM = 99

PERCENTAGE = 0.4000E 02

KHI-SQUARED = 0.92500E 02

KOLMOGOROV S TEST

MAX = 1.000009E-02

CUMULATIVE DISTRIBUTION FUNCTION

1	0.01000	0.02130	0.03190	0.04160	0.05030	0.05940	0.06870	0.07980	0.08880	0.09940
11	0.10890	0.11930	0.13010	0.13950	0.15010	0.16000	0.17080	0.18130	0.19120	0.20100
21	0.21160	0.22060	0.23180	0.24180	0.25080	0.25950	0.26760	0.27790	0.28870	0.29850
31	0.30740	0.31450	0.32250	0.33210	0.34420	0.35370	0.36350	0.37330	0.38340	0.39300
41	0.40330	0.41280	0.42300	0.43460	0.44580	0.45600	0.46610	0.47620	0.48460	0.49400
51	0.50350	0.51400	0.52440	0.53340	0.54380	0.55310	0.56340	0.57310	0.58340	0.59390
61	0.60250	0.61170	0.62000	0.63050	0.64190	0.65300	0.66490	0.67560	0.68630	0.69650
71	0.70550	0.71510	0.72680	0.73460	0.74440	0.75500	0.76630	0.77620	0.78590	0.79580
81	0.80490	0.81600	0.82550	0.83480	0.84660	0.85820	0.86930	0.88050	0.88970	0.89970
91	0.91020	0.92040	0.93150	0.94120	0.95000	0.95990	0.97020	0.98120	0.99160	1.00000

THE HYPOTHESIS OF UNIFORMITY IS

GOOD

WHILE THE RESULT IS 1.000E 00

SUM TEST

1	106.	95.	100.	113.	106.	93.	113.	97.	96.	102.
11	102.	110.	90.	98.	106.	97.	81.	103.	106.	101.
21	91.	106.	96.	96.	86.	94.	102.	113.	96.	108.
31	103.	105.	90.	107.	115.	93.	80.	108.	111.	93.
41	108.	107.	104.	100.	79.	101.	105.	106.	97.	84.
51	109.	105.	105.	107.	96.	107.	112.	93.	110.	85.
61	98.	95.	100.	109.	96.	115.	91.	100.	101.	89.
71	90.	80.	93.	97.	103.	101.	110.	109.	111.	79.
81	93.	95.	99.	87.	103.	114.	93.	87.	103.	113.
91	112.	98.	110.	91.	106.	97.	97.	105.	97.	105.

DEGREE OF FREEDOM = 99
PERCENTAGE = 0.1000E 02
KHI-SQUARED = 0.77100E 02

KOLMOGOROV S TEST

MAX = 4.299901E-03

CUMULATIVE DISTRIBUTION FUNCTION

1	0.01060	0.02010	0.03010	0.04140	0.05200	0.06130	0.07260	0.08230	0.09190	0.10210
11	0.11230	0.12330	0.13230	0.14210	0.15270	0.16240	0.17050	0.18080	0.19140	0.20150
21	0.21060	0.22120	0.23080	0.24040	0.24900	0.25840	0.26860	0.27990	0.28950	0.30030
31	0.31060	0.32110	0.33010	0.34080	0.35230	0.36160	0.36960	0.38040	0.39150	0.40080
41	0.41160	0.42230	0.43270	0.44270	0.45060	0.46070	0.47120	0.48180	0.49150	0.49990
51	0.51080	0.52130	0.53180	0.54250	0.55210	0.56280	0.57400	0.58330	0.59430	0.60280
61	0.61260	0.62210	0.63210	0.64300	0.65260	0.66410	0.67320	0.68320	0.69330	0.70220
71	0.71120	0.71920	0.72850	0.73820	0.74850	0.75860	0.76960	0.78050	0.79160	0.79950
81	0.80880	0.81830	0.82820	0.83690	0.84720	0.85860	0.86790	0.87660	0.88690	0.89820
91	0.90940	0.91920	0.93020	0.93930	0.94990	0.95960	0.96930	0.97980	0.98950	1.00000

THE HYPOTHESIS OF UNIFORMITY IS

GOOD

WHILE THE RESULT IS 4.300E-01

RUNS TESTS FOR THE FUNCTION RMB1(INPV)

RUNS UP AND DOWN BASED ON 32 SETS OF 1024

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LENGTH OF RUN	OBSERVED AVERAGE NUMBER OF RUNS UP	DOWN	EXPECTED AVERAGE NUMBER OF RUNS
1	209.6	210.2	213.4
2	99.6	100.6	93.7
3	22.8	20.8	26.9
4	5.0	4.2	5.8
CE.5	2.5	3.5	1.2

SET	KHI-SQUARE UP	KHI-SQUARE DOWN
1	2.5008E 01	3.0299E 01
2	2.6735E 01	2.7172E 01
3	2.8281E 01	5.9320E 01
4	2.9213E 01	1.8832E 01
5	2.5188E 01	4.1680E 01
6	2.5452E 01	2.3695E 01
7	3.3568E 01	7.5315E 01
8	2.6954E 01	2.7644E 01
9	2.9165E 01	5.9426E 01
10	2.6298E 01	2.1598E 01
11	3.6355E 01	4.8149E 01
12	2.6746E 01	3.1040E 01
13	2.9836E 01	3.7432E 01
14	2.4363E 01	1.8560E 01
15	3.0698E 01	5.8694E 01
16	3.0896E 01	3.2367E 01
17	2.5511E 01	2.6394E 01
18	2.1469E 01	2.0018E 01
19	2.0856E 01	3.0789E 01
20	2.8558E 01	2.5245E 01
21	1.8543E 01	2.6357E 01
22	2.6962E 01	2.4694E 01
23	2.6260E 01	3.0116E 01
24	1.8210E 01	2.7519E 01
25	2.1934E 01	1.9896E 01
26	3.0284E 01	2.5310E 01
27	2.8305E 01	4.0823E 01
28	2.6616E 01	2.5029E 01
29	3.4741E 01	2.6791E 01
30	3.2954E 01	2.1859E 01
31	2.3003E 01	3.9364E 01
32	2.0658E 01	2.5506E 01

NUMBER OF SET	LENGTH OF RUN — UP 1	UP 2	UP 3	UP 4	UP GE. 5	DOWN 1	DOWN 2	DOWN 3	DOWN 4	DOWN GE. 5
1	198.	79.	31.	9	6.	203	81.	26.	6.	7
2	249.	114.	11.	0	0.	246	117.	11.	0.	0
3	174.	88.	31.	8	6.	175	86.	32.	5.	9
4	229.	125.	10.	0	0.	232	117.	16.	0.	0
5	196.	74.	32.	12	5.	198	83.	23.	7.	8
6	251.	112.	12.	0	0.	249	112.	13.	0.	0
7	181.	69.	39.	11	5.	172	85.	33.	4.	10
8	252.	112.	11.	0	0.	249	116.	11.	0.	0
9	165.	83.	41.	10	4.	171	90.	28.	5.	9
10	243.	117.	11.	0	0.	241	115.	14.	0.	0
11	177.	88.	27.	9	7.	181	79.	31.	9.	8
12	233.	122.	11.	0	0.	227	128.	10.	0.	0
13	165.	87.	38.	9	5.	171	93.	25.	8.	7
14	240.	117.	12.	0	0.	241	112.	16.	0.	0
15	180.	78.	34.	9	6.	176	85.	31.	6.	9
16	235.	124.	9.	0	0.	232	127.	9.	0.	0
17	176.	80.	37.	12	4.	182	85.	27.	9.	6
18	231.	119.	14.	0	0.	231	118.	15.	0.	0
19	175.	81.	39.	7	4.	171	91.	28.	10.	6
20	236.	122.	10.	0	0.	237	120.	12.	0.	0
21	201.	79.	28.	13	4.	215	66.	26.	14.	4
22	237.	121.	11.	0	0.	237	119.	12.	0.	0
23	179.	72.	41.	9	4.	174	79.	34.	12.	5
24	228.	117.	16.	0	0.	223	127.	12.	0.	0
25	179.	88.	32.	10	5.	191	79.	29.	10.	5
26	264.	107.	11.	0	0.	265	101.	15.	0.	0
27	178.	82.	27.	13	5.	174	85.	28.	11.	7
28	238.	120.	11.	0	0.	236	120.	12.	0.	0
29	152.	98.	36.	9	5.	163	90.	34.	8.	5
30	226.	129.	9.	0	0.	230	120.	14.	0.	0
31	183.	78.	32.	10	5.	182	80.	28.	11.	7
32	255.	104.	16.	0	0.	250	113.	12.	0.	0

****

LENGTH OF RUN	OBSERVED AVERAGE NUMBER OF RUNS ABOVE	BELOW	EXPECTED AVERAGE NUMBER OF RUNS
1	125.1	124.3	128.0
2	70.0	71.0	64.0
3	30.7	30.9	32.0
4	14.3	14.8	16.0
5	8.9	9.2	8.0
6	2.7	2.1	4.0
7	2.2	2.3	2.0
8	1.0	1.0	1.0
9	0.6	0.5	0.5
GE.10	0.9	0.3	0.5

SET	KHI-SQUARE ABOVE	KHI-SQUARE BELOW
1	4.7266E 00	4.8047E 00
2	6.7422E 00	1.2891E 01
3	8.5469E 00	2.5625E 00
4	3.6227E 01	1.9156E 01
5	7.1172E 00	3.2578E 00
6	1.2367E 01	5.6719E 00
7	7.5078E 00	1.2844E 01
8	1.0609E 01	2.1445E 01
9	4.3438E 00	1.6797E 01
10	2.5242E 01	1.9195E 01
11	1.0437E 01	3.8281E 00
12	1.5016E 01	2.0977E 01
13	7.1016E 00	3.4219E 00
14	8.8750E 00	1.9508E 01
15	1.1750E 01	4.5469E 00
16	2.1102E 01	3.7109E 00
17	1.0211E 01	9.7500E 00
18	8.5469E 00	1.0375E 01
19	1.2328E 01	8.6406E 00
20	1.4375E 01	1.9453E 01
21	8.1641E 00	1.3289E 01
22	7.8047E 00	2.9453E 00
23	9.6641E 00	7.8828E 00
24	1.7937E 01	2.0031E 01
25	1.1070E 01	1.3664E 01
26	1.3578E 01	6.4844E 00
27	1.0211E 01	7.7109E 00
28	1.4797E 01	2.4164E 01
29	7.7109E 00	5.8750E 00
30	8.8984E 00	1.0625E 01
31	9.1094E 00	3.0313E 00

ABOVE THE MEAN

NUMBER OF SET	LENGTH OF RUN 1	2	3	4	5	6	7	8	9	GE 10
1	127	72	33	19	5	2	2	1	1	0
2	131	63	25	22	11	3	1	1	0	0
3	130	75	26	12	10	2	4	1	0	0
4	101	72	47	6	8	1	6	0	0	2
5	129	79	27	13	8	3	2	0	1	0
6	135	75	31	9	6	2	5	1	0	1
7	125	62	24	18	9	4	2	1	0	2
8	124	77	29	13	6	2	1	1	2	1
9	124	72	29	17	9	5	3	0	0	0
10	103	65	31	15	10	1	5	1	3	1
11	118	62	37	13	8	7	2	0	0	2
12	132	73	22	6	12	5	2	2	1	0
13	125	72	27	20	8	3	4	0	0	1
14	140	70	38	11	4	2	3	1	0	0
15	126	66	25	22	11	3	1	0	0	2
16	113	68	33	15	10	2	1	1	3	2
17	123	75	24	22	11	3	1	0	0	1
18	128	61	43	12	11	2	1	1	0	1
19	112	69	32	21	9	3	0	0	1	2
20	132	78	34	9	4	2	2	3	1	0
21	145	52	33	20	7	4	1	0	1	0
22	127	71	35	8	8	3	3	2	0	1
23	125	70	33	14	6	5	0	0	1	2
24	116	68	30	7	11	3	6	2	1	0
25	139	72	26	12	12	2	0	2	1	1
26	146	77	27	8	7	2	3	2	1	0
27	123	73	24	18	6	4	2	2	0	2
28	124	71	23	12	15	1	2	2	0	1
29	121	75	30	21	6	1	2	1	0	1
30	117	69	36	15	12	2	0	0	0	1
31	126	65	30	15	11	1	1	1	1	2
32	115	71	40	12	14	0	3	3	0	0

BELOW THE MEAN

NUMBER OF SET	LENGTH OF RUN 1	2	3	4	5	6	7	8	9	GE 10
1	139.	59.	31.	19.	9.	3.	3.	0.	0.	0
2	126.	69.	37.	7.	9.	4	5.	0.	1.	0
3	128.	70.	32.	16.	6.	4.	3.	1.	1.	0.
4	118.	68.	32.	8.	5.	2.	4.	4.	1.	1.
5	127.	70.	30.	17.	10.	4.	2.	0.	0.	1.
6	136.	65.	29.	22.	9.	2.	1.	1	0.	0
7	110.	78.	22.	20.	11.	4.	2.	0.	1.	0.
8	123.	70.	36.	7.	15.	0.	2.	1.	0.	2.
9	124.	79.	21.	18.	13.	0.	2.	0.	1.	0
10	109.	62.	34.	9.	13.	0.	2.	2.	1.	2
11	116.	71.	30.	17.	8.	3.	1.	1.	1.	0.
12	109.	80.	41.	12.	7.	0.	1.	2.	2.	1.
13	132.	71.	31.	14.	6.	5.	1.	1.	0.	0.
14	125.	84.	26.	19.	14.	0	0.	1.	0.	0.
15	128.	67.	31.	12.	11.	3.	2.	0.	1.	0.
16	127.	65.	30.	11.	8.	2.	2.	1.	1.	1.
17	130.	78.	21.	16.	11.	3.	1.	1.	0.	0.
18	132.	74.	30.	11.	6.	2.	3.	3.	0.	0.
19	118.	71.	27.	19.	12.	1.	1.	1.	0.	0.
20	140.	69.	26.	9.	12.	1.	5.	1.	2.	0.
21	115.	80.	39.	15.	11.	1.	0.	1.	0.	0.
22	131.	60.	36.	18.	7.	3.	1.	1.	0.	1.
23	117.	72.	38.	19.	8.	1.	2.	0.	0.	0.
24	110.	64.	40.	10.	12.	1.	6.	1.	0.	0.
25	135.	68.	29.	26.	6.	0.	2.	0.	0.	0.
26	142.	75.	26.	13.	7.	5.	2.	1.	1.	0.
27	117.	77.	26.	20.	8.	2.	2.	1.	0.	0
28	115.	74.	39.	6.	8.	0.	5.	1.	1.	2.
29	122.	78.	27.	18.	8.	4.	1.	1.	0.	0.
30	120.	72.	26.	16.	8.	2.	4.	3.	1.	0.
31	122.	66.	32.	17.	9.	2.	3.	1.	0.	0.
32	134.	65.	34.	13.	8.	2.	2.	0.	0.	0.

APPENDIX 1

χ^2 Table

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TABLE OF KHI-SQUARED

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FROM

* STATISTICAL TABLES AND FORMULAS * BY A. HALD

WILEY PUBLICATIONS IN STATISTICS (JANUARY,1960)

F	P =0.05	0.1	0.5	1.0	2.5	5.0	10.0	20.0	30.0	40.0	50.0
1	3.93E-07	1.57E-06	3.93E-05	1.57E-04	9.82E-04	3.93E-03	1.58E-02	6.42E-02	1.48E-01	2.75E-01	4.55E-01
2	1.00E-03	2.00E-03	1.00E-02	2.01E-02	5.06E-02	1.03E-01	2.11E-01	4.46E-01	7.13E-01	1.02E 00	1.39E 00
3	1.53E-02	2.43E-02	7.17E-02	1.15E-01	2.16E-01	3.52E-01	5.84E-01	1.00E 00	1.42E 00	1.87E 00	2.37E 00
4	6.39E-02	9.08E-02	2.07E-01	2.97E-01	4.84E-01	7.11E-01	1.06E 00	1.65E 00	2.19E 00	2.75E 00	3.36E 00
5	1.58E-01	2.10E-01	4.12E-01	5.54E-01	8.31E-01	1.15E 00	1.61E 00	2.34E 00	3.00E 00	3.66E 00	4.35E 00
6	2.99E-01	3.81E-01	6.76E-01	8.72E-01	1.24E 00	1.64E 00	2.20E 00	3.07E 00	3.83E 00	4.57E 00	5.35E 00
7	4.85E-01	5.98E-01	9.89E-01	1.24E 00	1.69E 00	2.17E 00	2.83E 00	3.82E 00	4.67E 00	5.49E 00	6.35E 00
8	7.10E-01	8.57E-01	1.34E 00	1.65E 00	2.18E 00	2.73E 00	3.49E 00	4.59E 00	5.53E 00	6.42E 00	7.34E 00
9	9.72E-01	1.15E 00	1.73E 00	2.09E 00	2.70E 00	3.33E 00	4.17E 00	5.38E 00	6.39E 00	7.36E 00	8.34E 00
10	1.26E 00	1.48E 00	2.16E 00	2.56E 00	3.25E 00	3.94E 00	4.87E 00	6.18E 00	7.27E 00	8.30E 00	9.34E 00
11	1.59E 00	1.83E 00	2.60E 00	3.05E 00	3.82E 00	4.57E 00	5.58E 00	6.99E 00	8.15E 00	9.24E 00	1.03E 01
12	1.93E 00	2.21E 00	3.07E 00	3.57E 00	4.40E 00	5.23E 00	6.30E 00	7.81E 00	9.03E 00	1.02E 01	1.13E 01
13	2.31E 00	2.62E 00	3.57E 00	4.11E 00	5.01E 00	5.89E 00	7.04E 00	8.63E 00	9.93E 00	1.11E 01	1.23E 01
14	2.70E 00	3.04E 00	4.07E 00	4.66E 00	5.63E 00	6.57E 00	7.79E 00	9.47E 00	1.08E 01	1.21E 01	1.33E 01
15	3.11E 00	3.48E 00	4.60E 00	5.23E 00	6.26E 00	7.26E 00	8.55E 00	1.03E 01	1.17E 01	1.30E 01	1.43E 01
16	3.54E 00	3.94E 00	5.14E 00	5.81E 00	6.91E 00	7.96E 00	9.31E 00	1.12E 01	1.26E 01	1.40E 01	1.53E 01
17	3.98E 00	4.42E 00	5.70E 00	6.41E 00	7.56E 00	8.67E 00	1.01E 01	1.20E 01	1.35E 01	1.49E 01	1.63E 01
18	4.44E 00	4.90E 00	6.26E 00	7.01E 00	8.23E 00	9.39E 00	1.09E 01	1.29E 01	1.44E 01	1.59E 01	1.73E 01
19	4.91E 00	5.41E 00	6.84E 00	7.63E 00	8.91E 00	1.01E 01	1.17E 01	1.37E 01	1.54E 01	1.69E 01	1.83E 01
20	5.40E 00	5.92E 00	7.43E 00	8.26E 00	9.59E 00	1.09E 01	1.24E 01	1.46E 01	1.63E 01	1.78E 01	1.93E 01

F	P =0.05	0.1	0.5	1.0	2.5	5.0	10.0	20.0	30.0	40.0	50.0
21	5.90	6.45	8.03	8.90	10.3	11.6	13.2	15.4	17.2	18.8	20.3
22	6.40	6.98	8.64	9.54	11.0	12.3	14.0	16.3	18.1	19.7	21.3
23	6.92	7.53	9.26	10.2	11.7	13.1	14.8	17.2	19.0	20.7	22.3
24	7.45	8.08	9.89	10.9	12.4	13.8	15.7	18.1	19.9	21.7	23.3
25	7.99	8.65	10.5	11.5	13.1	14.6	16.5	18.9	20.9	22.6	24.3
26	8.54	9.22	11.2	12.2	13.8	15.4	17.3	19.8	21.8	23.6	25.3
27	9.09	9.80	11.8	12.9	14.6	16.2	18.1	20.7	22.7	24.5	26.3
28	9.66	10.4	12.5	13.6	15.3	16.9	18.9	21.6	23.6	25.5	27.3
29	10.2	11.0	13.1	14.3	16.0	17.7	19.8	22.5	24.6	26.5	28.3
30	10.8	11.6	13.8	15.0	16.8	18.5	20.6	23.4	25.5	27.4	29.3
31	11.4	12.2	14.5	15.7	17.5	19.3	21.4	24.3	26.4	28.4	30.3
32	12.0	12.8	15.1	16.4	18.3	20.1	22.3	25.1	27.4	29.4	31.3
33	12.6	13.4	15.8	17.1	19.0	20.9	23.1	26.0	28.3	30.3	32.3
34	13.2	14.1	16.5	17.8	19.8	21.7	24.0	26.9	29.2	31.3	33.3
35	13.8	14.7	17.2	18.5	20.6	22.5	24.8	27.8	30.2	32.3	34.3
36	14.4	15.3	17.9	19.2	21.3	23.3	25.6	28.7	31.1	33.3	35.3
37	15.0	16.0	18.6	20.0	22.1	24.1	26.5	29.6	32.1	34.2	36.3
38	15.6	16.6	19.3	20.7	22.9	24.9	27.3	30.5	33.0	35.2	37.3
39	16.3	17.3	20.0	21.4	23.7	25.7	28.2	31.4	33.9	36.2	38.3
40	16.9	17.9	20.7	22.2	24.4	26.5	29.1	32.3	34.9	37.1	39.3
41	17.5	18.6	21.4	22.9	25.2	27.3	29.9	33.3	35.8	38.1	40.3
42	18.2	19.2	22.1	23.7	26.0	28.1	30.8	34.2	36.8	39.1	41.3
43	18.8	19.9	22.9	24.4	26.8	29.0	31.6	35.1	37.7	40.0	42.3
44	19.5	20.6	23.6	25.1	27.6	29.8	32.5	36.0	38.6	41.0	43.3
45	20.1	21.3	24.3	25.9	28.4	30.6	33.4	36.9	39.6	42.0	44.3
46	20.8	21.9	25.0	26.7	29.2	31.4	34.2	37.8	40.5	43.0	45.3
47	21.5	22.6	25.8	27.4	30.0	32.3	35.1	38.7	41.5	43.9	46.3
48	22.1	23.3	26.5	28.2	30.8	33.1	35.9	39.6	42.4	44.9	47.3
49	22.8	24.0	27.2	28.9	31.6	33.9	36.8	40.5	43.4	45.9	48.3
50	23.5	24.7	28.0	29.7	32.4	34.8	37.7	41.4	44.3	46.9	49.3
51	24.1	25.4	28.7	30.5	33.2	35.6	38.6	42.4	45.3	47.8	50.3
52	24.8	26.1	29.5	31.2	34.0	36.4	39.4	43.3	46.2	48.8	51.3
53	25.5	26.8	30.2	32.0	34.8	37.3	40.3	44.2	47.2	49.8	52.3
54	26.2	27.5	31.0	32.8	35.6	38.1	41.2	45.1	48.1	50.8	53.3
55	26.9	28.2	31.7	33.6	36.4	39.0	42.1	46.0	49.1	51.7	54.3
56	27.6	28.9	32.5	34.3	37.2	39.8	42.9	47.0	50.0	52.7	55.3
57	28.2	29.6	33.2	35.1	38.0	40.6	43.8	47.9	51.0	53.7	56.3
58	28.9	30.3	34.0	35.9	38.8	41.5	44.7	48.8	51.9	54.7	57.3
59	29.6	31.0	34.8	36.7	39.7	42.3	45.6	49.7	52.9	55.6	58.3
60	30.3	31.7	35.5	37.5	40.5	43.2	46.5	50.6	53.8	56.6	59.3

F	P =0.05	0.1	0.5	1.0	2.5	5.0	10.0	20.0	30.0	40.0	50.0
61	31.0	32.5	36.3	38.3	41.3	44.0	47.3	51.6	54.8	57.6	60.3
62	31.7	33.2	37.1	39.1	42.1	44.9	48.2	52.5	55.7	58.6	61.3
63	32.5	33.9	37.8	39.9	43.0	45.7	49.1	53.4	56.7	59.6	62.3
64	33.2	34.6	38.6	40.6	43.8	46.6	50.0	54.3	57.6	60.5	63.3
65	33.9	35.4	39.4	41.4	44.6	47.4	50.9	55.3	58.6	61.5	64.3
66	34.6	36.1	40.2	42.2	45.4	48.3	51.8	56.2	59.5	62.5	65.3
67	35.3	36.8	40.9	43.0	46.3	49.2	52.7	57.1	60.5	63.5	66.3
68	36.0	37.6	41.7	43.8	47.1	50.0	53.5	58.0	61.4	64.4	67.3
69	36.7	38.3	42.5	44.6	47.9	50.9	54.4	59.0	62.4	65.4	68.3
70	37.5	39.0	43.3	45.4	48.8	51.7	55.3	59.9	63.3	66.4	69.3
71	38.2	39.8	44.1	46.2	49.6	52.6	56.2	60.8	64.3	67.4	70.3
72	38.9	40.5	44.8	47.1	50.4	53.5	57.1	61.8	65.3	68.4	71.3
73	39.6	41.3	45.6	47.9	51.3	54.3	58.0	62.7	66.2	69.3	72.3
74	40.4	42.0	46.4	48.7	52.1	55.2	58.9	63.6	67.2	70.3	73.3
75	41.1	42.8	47.2	49.5	52.9	56.1	59.8	64.5	68.1	71.3	74.3
76	41.8	43.5	48.0	50.3	53.8	56.9	60.7	65.5	69.1	72.3	75.3
77	42.6	44.3	48.8	51.1	54.6	57.8	61.6	66.4	70.0	73.2	76.3
78	43.3	45.0	49.6	51.9	55.5	58.7	62.5	67.3	71.0	74.2	77.3
79	44.1	45.8	50.4	52.7	56.3	59.5	63.4	68.3	72.0	75.2	78.3
80	44.8	46.5	51.2	53.5	57.2	60.4	64.3	69.2	72.9	76.2	79.3
81	45.5	47.3	52.0	54.4	58.0	61.3	65.2	70.1	73.9	77.2	80.3
82	46.3	48.0	52.8	55.2	58.8	62.1	66.1	71.1	74.8	78.1	81.3
83	47.0	48.8	53.6	56.0	59.7	63.0	67.0	72.0	75.8	79.1	82.3
84	47.8	49.6	54.4	56.8	60.5	63.9	67.9	72.9	76.8	80.1	83.3
85	48.5	50.3	55.2	57.6	61.4	64.7	68.8	73.9	77.7	81.1	84.3
86	49.3	51.1	56.0	58.5	62.2	65.6	69.7	74.8	78.7	82.1	85.3
87	50.0	51.9	56.8	59.3	63.1	66.5	70.6	75.7	79.6	83.0	86.3
88	50.8	52.6	57.6	60.1	63.9	67.4	71.5	76.7	80.6	84.0	87.3
89	51.5	53.4	58.4	60.9	64.8	68.2	72.4	77.6	81.6	85.0	88.3
90	52.3	54.2	59.2	61.8	65.6	69.1	73.3	78.6	82.5	86.0	89.3
91	53.0	54.9	60.0	62.6	66.5	70.0	74.2	79.5	83.5	87.0	90.3
92	53.8	55.7	60.8	63.4	67.4	70.9	75.1	80.4	84.4	88.0	91.3
93	54.5	56.5	61.6	64.2	68.2	71.8	76.0	81.4	85.4	88.9	92.3
94	55.3	57.2	62.4	65.1	69.1	72.6	76.9	82.3	86.4	89.9	93.3
95	56.1	58.0	63.2	65.9	69.9	73.5	77.8	83.2	87.3	90.9	94.3
96	56.8	58.8	64.1	66.7	70.8	74.4	78.7	84.2	88.3	91.9	95.3
97	57.6	59.6	64.9	67.6	71.6	75.3	79.6	85.1	89.2	92.9	96.3
98	58.4	60.4	65.7	68.4	72.5	76.2	80.5	86.1	90.2	93.8	97.3
99	59.1	61.1	66.5	69.2	73.4	77.0	81.4	87.0	91.2	94.8	98.3
100	59.9	61.9	67.3	70.1	74.2	77.9	82.4	87.9	92.1	95.8	99.3

F	P =60.0	70.0	80.0	90.0	95.0	97.5	99.0	99.5	99.9	99.95	F
1	70.8E-02	10.7E-01	16.4E-01	27.1E-01	38.4E-01	50.2E-01	66.3E-01	78.8E-01	10.8E 00	12.1E 00	1
2	18.3E-01	24.1E-01	32.2E-01	46.1E-01	59.9E-01	73.8E-01	92.1E-01	10.6E 00	13.8E 00	15.2E 00	2
3	29.5E-01	36.7E-01	46.4E-01	62.5E-01	78.1E-01	93.5E-01	11.3E 00	12.8E 00	16.3E 00	17.7E 00	3
4	40.4E-01	48.8E-01	59.9E-01	77.8E-01	94.9E-01	11.1E 00	13.3E 00	14.9E 00	18.5E 00	20.0E 00	4
5	51.3E-01	60.6E-01	72.9E-01	92.4E-01	11.1E 00	12.8E 00	15.1E 00	16.7E 00	20.5E 00	22.1E 00	5
6	62.1E-01	72.3E-01	85.6E-01	10.6E 00	12.6E 00	14.4E 00	16.8E 00	18.5E 00	22.5E 00	24.1E 00	6
7	72.8E-01	83.8E-01	98.0E-01	12.0E 00	14.1E 00	16.0E 00	18.5E 00	20.3E 00	24.3E 00	26.0E 00	7
8	83.5E-01	95.2E-01	11.0E 00	13.4E 00	15.5E 00	17.5E 00	20.1E 00	22.0E 00	26.1E 00	27.9E 00	8
9	94.1E-01	10.7E 00	12.2E 00	14.7E 00	16.9E 00	19.0E 00	21.7E 00	23.6E 00	27.9E 00	29.7E 00	9
10	10.5E 00	11.8E 00	13.4E 00	16.0E 00	18.3E 00	20.5E 00	23.2E 00	25.2E 00	29.6E 00	31.4E 00	10
11	11.5E 00	12.9E 00	14.6E 00	17.3E 00	19.7E 00	21.9E 00	24.7E 00	26.8E 00	31.3E 00	33.1E 00	11
12	12.6E 00	14.0E 00	15.8E 00	18.5E 00	21.0E 00	23.3E 00	26.2E 00	28.3E 00	32.9E 00	34.8E 00	12
13	13.6E 00	15.1E 00	17.0E 00	19.8E 00	22.4E 00	24.7E 00	27.7E 00	29.8E 00	34.5E 00	36.5E 00	13
14	14.7E 00	16.2E 00	18.2E 00	21.1E 00	23.7E 00	26.1E 00	29.1E 00	31.3E 00	36.1E 00	38.1E 00	14
15	15.7E 00	17.3E 00	19.3E 00	22.3E 00	25.0E 00	27.5E 00	30.6E 00	32.8E 00	37.7E 00	39.7E 00	15
16	16.8E 00	18.4E 00	20.5E 00	23.5E 00	26.3E 00	28.8E 00	32.0E 00	34.3E 00	39.3E 00	41.3E 00	16
17	17.8E 00	19.5E 00	21.6E 00	24.8E 00	27.6E 00	30.2E 00	33.4E 00	35.7E 00	40.8E 00	42.9E 00	17
18	18.9E 00	20.6E 00	22.8E 00	26.0E 00	28.9E 00	31.5E 00	34.8E 00	37.2E 00	42.3E 00	44.4E 00	18
19	19.9E 00	21.7E 00	23.9E 00	27.2E 00	30.1E 00	32.9E 00	36.2E 00	38.6E 00	43.8E 00	46.0E 00	19
20	21.0E 00	22.8E 00	25.0E 00	28.4E 00	31.4E 00	34.2E 00	37.6E 00	40.0E 00	45.3E 00	47.5E 00	20

F	P =60.0	70.0	80.0	90.0	95.0	97.5	99.0	99.5	99.9	99.95	F
21	22.0	23.9	26.2	29.6	32.7	35.5	38.9	41.4	46.8	49.0	21
22	23.0	24.9	27.3	30.8	33.9	36.8	40.3	42.8	48.3	50.5	22
23	24.1	26.0	28.4	32.0	35.2	38.1	41.6	44.2	49.7	52.0	23
24	25.1	27.1	29.6	33.2	36.4	39.4	43.0	45.6	51.2	53.5	24
25	26.1	28.2	30.7	34.4	37.7	40.6	44.3	46.9	52.6	54.9	25
26	27.2	29.2	31.8	35.6	38.9	41.9	45.6	48.3	54.1	56.4	26
27	28.2	30.3	32.9	36.7	40.1	43.2	47.0	49.6	55.5	57.9	27
28	29.2	31.4	34.0	37.9	41.3	44.5	48.3	51.0	56.9	59.3	28
29	30.3	32.5	35.1	39.1	42.6	45.7	49.6	52.3	58.3	60.7	29
30	31.3	33.5	36.3	40.3	43.8	47.0	50.9	53.7	59.7	62.2	30
31	32.3	34.6	37.4	41.4	45.0	48.2	52.2	55.0	61.1	63.6	31
32	33.4	35.7	38.5	42.6	46.2	49.5	53.5	56.3	62.5	65.0	32
33	34.4	36.7	39.6	43.7	47.4	50.7	54.8	57.6	63.9	66.4	33
34	35.4	37.8	40.7	44.9	48.6	52.0	56.1	59.0	65.2	67.8	34
35	36.5	38.9	41.8	46.1	49.8	53.2	57.3	60.3	66.6	69.2	35
36	37.5	39.9	42.9	47.2	51.0	54.4	58.6	61.6	68.0	70.6	36
37	38.5	41.0	44.0	48.4	52.2	55.7	59.9	62.9	69.3	72.0	37
38	39.6	42.0	45.1	49.5	53.4	56.9	61.2	64.2	70.7	73.4	38
39	40.6	43.1	46.2	50.7	54.6	58.1	62.4	65.5	72.1	74.7	39
40	41.6	44.2	47.3	51.8	55.8	59.3	63.7	66.8	73.4	76.1	40
41	42.7	45.2	48.4	52.9	56.9	60.6	65.0	68.1	74.7	77.5	41
42	43.7	46.3	49.5	54.1	58.1	61.8	66.2	69.3	76.1	78.8	42
43	44.7	47.3	50.5	55.2	59.3	63.0	67.5	70.6	77.4	80.2	43
44	45.7	48.4	51.6	56.4	60.5	64.2	68.7	71.9	78.7	81.5	44
45	46.8	49.5	52.7	57.5	61.7	65.4	70.0	73.2	80.1	82.9	45
46	47.8	50.5	53.8	58.6	62.8	66.6	71.2	74.4	81.4	84.2	46
47	48.8	51.6	54.9	59.8	64.0	67.8	72.4	75.7	82.7	85.6	47
48	49.8	52.6	56.0	60.9	65.2	69.0	73.7	77.0	84.0	86.9	48
49	50.9	53.7	57.1	62.0	66.3	70.2	74.9	78.2	85.4	88.2	49
50	51.9	54.7	58.2	63.2	67.5	71.4	76.2	79.5	86.7	89.6	50
51	52.9	55.8	59.2	64.3	68.7	72.6	77.4	80.7	88.0	90.9	51
52	53.9	56.8	60.3	65.4	69.8	73.8	78.6	82.0	89.3	92.2	52
53	55.0	57.9	61.4	66.5	71.0	75.0	79.8	83.3	90.6	93.5	53
54	56.0	58.9	62.5	67.7	72.2	76.2	81.1	84.5	91.9	94.8	54
55	57.0	60.0	63.6	68.8	73.3	77.4	82.3	85.7	93.2	96.2	55
56	58.0	61.0	64.7	69.9	74.5	78.6	83.5	87.0	94.5	97.5	56
57	59.1	62.1	65.7	71.0	75.6	79.8	84.7	88.2	95.8	98.8	57
58	60.1	63.1	66.8	72.2	76.8	80.9	86.0	89.5	97.0	100.1	58
59	61.1	64.2	67.9	73.3	77.9	82.1	87.2	90.7	98.3	101.4	59
60	62.1	65.2	69.0	74.4	79.1	83.3	88.4	92.0	99.6	102.7	60

F	P =60.0	70.0	80.0	90.0	95.0	97.5	99.0	99.5	99.9	99.95	F
61	63.2	66.3	70.0	75.5	80.2	84.5	89.6	93.2	100.9	104.0	61
62	64.2	67.3	71.1	76.6	81.4	85.7	90.8	94.4	102.2	105.3	62
63	65.2	68.4	72.2	77.7	82.5	86.8	92.0	95.6	103.4	106.6	63
64	66.2	69.4	73.3	78.9	83.7	88.0	93.2	96.9	104.7	107.9	64
65	67.2	70.5	74.4	80.0	84.8	89.2	94.4	98.1	106.0	109.2	65
66	68.3	71.5	75.4	81.1	86.0	90.3	95.6	99.3	107.3	110.5	66
67	69.3	72.6	76.5	82.2	87.1	91.5	96.8	100.6	108.5	111.7	67
68	70.3	73.6	77.6	83.3	88.3	92.7	98.0	101.8	109.8	113.0	68
69	71.3	74.6	78.6	84.4	89.4	93.9	99.2	103.0	111.1	114.3	69
70	72.4	75.7	79.7	85.5	90.5	95.0	100.4	104.2	112.3	115.6	70
71	73.4	76.7	80.8	86.6	91.7	96.2	101.6	105.4	113.6	116.9	71
72	74.4	77.8	81.9	87.7	92.8	97.4	102.8	106.6	114.8	118.1	72
73	75.4	78.8	82.9	88.8	93.9	98.5	104.0	107.9	116.1	119.4	73
74	76.4	79.9	84.0	90.0	95.1	99.7	105.2	109.1	117.3	120.7	74
75	77.5	80.9	85.1	91.1	96.2	100.8	106.4	110.3	118.6	121.9	75
76	78.5	82.0	86.1	92.2	97.4	102.0	107.6	111.5	119.9	123.2	76
77	79.5	83.0	87.2	93.3	98.5	103.2	108.8	112.7	121.1	124.5	77
78	80.5	84.0	88.3	94.4	99.6	104.3	110.0	113.9	122.3	125.7	78
79	81.5	85.1	89.3	95.5	100.7	105.5	111.1	115.1	123.6	127.0	79
80	82.6	86.1	90.4	96.6	101.9	106.6	112.3	116.3	124.8	128.3	80
81	83.6	87.2	91.5	97.7	103.0	107.8	113.5	117.5	126.1	129.5	81
82	84.6	88.2	92.5	98.8	104.1	108.9	114.7	118.7	127.3	130.8	82
83	85.6	89.2	93.6	99.9	105.3	110.1	115.9	119.9	128.6	132.0	83
84	86.6	90.3	94.7	101.0	106.4	111.2	117.1	121.1	129.8	133.3	84
85	87.7	91.3	95.7	102.1	107.5	112.4	118.2	122.3	131.0	134.5	85
86	88.7	92.4	96.8	103.2	108.6	113.5	119.4	123.5	132.3	135.8	86
87	89.7	93.4	97.9	104.3	109.8	114.7	120.6	124.7	133.5	137.0	87
88	90.7	94.4	98.9	105.4	110.9	115.8	121.8	125.9	134.7	138.3	88
89	91.7	95.5	100.0	106.5	112.0	117.0	122.9	127.1	136.0	139.5	89
90	92.8	96.5	101.1	107.6	113.1	118.1	124.1	128.3	137.2	140.8	90
91	93.8	97.6	102.1	108.7	114.3	119.3	125.3	129.5	138.4	142.0	91
92	94.8	98.6	103.2	109.8	115.4	120.4	126.5	130.7	139.7	143.3	92
93	95.8	99.6	104.2	110.9	116.5	121.6	127.6	131.9	140.9	144.5	93
94	96.8	100.7	105.3	111.9	117.6	122.7	128.8	133.1	142.1	145.8	94
95	97.9	101.7	106.4	113.0	118.8	123.9	130.0	134.2	143.3	147.0	95
96	98.9	102.8	107.4	114.1	119.9	125.0	131.1	135.4	144.6	148.2	96
97	99.9	103.8	108.5	115.2	121.0	126.1	132.3	136.6	145.8	149.5	97
98	100.9	104.8	109.5	116.3	122.1	127.3	133.5	137.8	147.0	150.7	98
99	101.9	105.9	110.6	117.4	123.2	128.4	134.6	139.0	148.2	151.9	99
100	102.9	106.9	111.7	118.5	124.3	129.6	135.8	140.2	149.4	153.2	100

APPENDIX 2

Kolmogorov's Table

TABLE FOR KOLMOGOROV'S TEST [15]

λ	$1-k(\lambda)$	λ	$1-k(\lambda)$
0.30	0.000009	1.55	0.983622
0.35	0.000303	1.60	0.988048
0.40	0.002808	1.65	0.931364
0.45	0.012590	1.70	0.993828
0.50	0.036055	1.75	0.995625
0.55	0.077183	1.80	0.996932
0.60	0.135718	1.85	0.997870
0.65	0.207987	1.90	0.998536
0.70	0.288765	1.95	0.999004
0.75	0.372833	2.00	0.999329
0.80	0.455857	2.05	0.999562
0.85	0.534682	2.10	0.999705
0.90	0.607270	2.15	0.999806
0.95	0.672516	2.20	0.999874
1.00	0.730000	2.25	0.999920
1.05	0.779794	2.30	0.999949
1.10	0.822282	2.35	0.999968
1.15	0.858038	2.40	0.999980
1.20	0.887750	2.45	0.999988
1.25	0.912132	2.50	0.9999925
1.30	0.931908	2.55	0.9999956
1.35	0.947756	2.60	0.9999974
1.40	0.960318	2.65	0.9999984
1.45	0.970158	2.70	0.9999990
1.50	0.977782	2.75	0.9999994

APPENDIX 3

The following tables report the numerical results of explained χ^2 and Kolmogorov's tests. The used symbols are:

R	for rejected sequences (See the 5.1.)
G	for good sequences
VG	for very good sequences
*	for probabilities less than 0.05

TEST TYPE / NAME OF FUNCTION	LINEAR UNIFORMITY									PAIRS			TRIPLES			MAXIMUM OF m NUMBERS								
	ν = 4			ν = 10			ν = 100									m = 2			m = 5			m = 10		
RMB1	40 G	60 G	80 G	90 G	50 VG	90 G	50 G	40 VG	10 G	60 VG	10 G	97.5 G	70 R	2.5 G	0.05 G	10 VG	10 G	20 G	80 G	50 VG	60 G	10 G	0.05 R	0.5 R
RMB2	10 G	10 G	40 G	20 G	80 G	10 G	30 G	30 G	80 G	50 VG	20 G	30 G	20 G	10 G	70 G	0.1 G	1 G	40 G	* G	* VG	* G	* G	* G	* G
RMB3	20 VG	70 G	70 G	10 VG	50 G	90 G	30 VG	40 G	70 G	10 G	70 G	10 VG	10 G	97.5 G	95 G	5 G	10 VG	5 G	0.1 G	* G	* VG	* VG	* VG	* G
RMD	60 G	97.5 G	30 VG	20 VG	95 G	20 VG	80 VG	30 G	80 VG	0.5 G	20 VG	1 VG	60 VG	60 G	30 G	* VG	* R	2.5 VG	* VG	* G	1 G	* VG	* VG	* G
RMTB	80 G	30 G	20 VG	70 G	30 G	40 VG	50 G	2.5 VG	20 VG	40 G	5 G	50 G	10 R	10 VG	5 G	10 VG	80 G	80 G	50 VG	5 G	10 G	80 G	40 G	40 VG
RMTD	10 G	80 G	2.5 VG	5 VG	80 G	50 VG	1 VG	20 VG	30 G	40 G	20 G	50 G	60 R	70 G	20 G	5 R	30 VG	1 VG	90 G	5 G	60 VG	50 G	50 VG	10 G
RMTDL	10 G	50 G	10 VG	10 G	80 G	10 G	70 G	50 G	2.5 G	* R	* R	* R	* R	* R	* G	* VG	* G	* G	* R	* G	* G	* R	* R	* G
RANMX1	80 G	30 G	20 VG	70 G	30 G	40 VG	95 G	2.5 VG	20 VG	20 VG	20 VG	40 G	40 G	70 VG	40 R	80 G	60 VG	97.5 G	40 VG	50 VG	1 G	20 VG	10 G	60 G
RANMX2	80 G	30 G	10 VG	70 G	30 G	40 VG	90 G	2.5 G	20 VG	20 G	20 VG	5 G	20 G	40 VG	90 G	60 G	70 VG	70 VG	80 G	40 VG	70 VG	10 VG	30 G	50 VG
RFB	50 G	30 G	60 G	80 G	5 VG	60 G	40 G	60 VG	30 VG	60 G	10 G	2.5 VG	30 G	1 G	40 VG	30 G	40 G	10 VG	60 VG	20 G	50 G	50 VG	60 VG	5 G

TEST TYPE / NAME OF FUNCTION	MINIMUM OF m NUMBERS									SUM OF m NUMBERS								
	$m = 2$			$m = 5$			$m = 10$			$m = 2$			$m = 5$			$m = 10$		
RMB1	60 G	80 VG	90 G	1 VG	80 G	30 G	0.1 G	2.5 R	* R	90 G	90 G	80 G	30 G	60 VG	70 G	30 VG	40 G	1 VG
RMB2	0.1 G	40 G	10 VG	* G	* G	0.05 G	* VG	* G	* VG	50 VG	10 G	80 G	50 G	20 G	50 G	80 G	20 VG	20 G
RMB3	10 G	50 G	5 G	* G	0.05 G	* G	* G	0.1 VG	* G	30 VG	10 VG	70 G	10 R	0.5 G	30 G	60 G	50 VG	50 G
RMD	10 G	0.5 VG	* VG	* G	* G	* G	* VG	* G	* G	95 G	10 G	30 VG	50 G	50 G	90 G	10 G	5 G	30 G
RMTB	90 G	70 G	50 VG	40 VG	20 VG	20 VG	30 G	20 G	20 G	90 G	40 G	30 G	50 G	30 G	70 G	2.5 G	80 G	99 G
RMTD	0.5 G	10 VG	2.5 VG	* VG	2.5 G	1 R	* G	* R	* R	60 VG	80 G	50 VG	50 VG	10 G	2.5 R	10 G	90 G	20 G
RMTDL	* G	* G	* G	* VG	* G	* G	* R	* R	* R	80 VG	20 VG	10 VG	10 G	60 G	90 G	30 G	70 VG	50 G
RANMX1	40 VG	90 VG	40 VG	5 G	20 VG	90 VG	30 VG	70 VG	10 G	50 VG	70 G	70 G	50 VG	2.5 VG	10 G	10 G	80 VG	60 VG
RANMX2	20 VG	50 VG	80 G	20 G	50 VG	80 VG	50 VG	20 G	20 G	20 G	97.5 G	60 VG	5 VG	60 G	30 G	20 G	70 G	80 G
RFB	60 G	2.5 VG	70 VG	20 G	10 G	50 VG	70 VG	0.1 VG	50 VG	50 VG	5 G	30 G	60 VG	70 VG	30 G	2.5 G	80 G	70 G

TEST TYPE / NAME OF FUNCTION	RUNS UP AND DOWN						RUNS ABOVE AND BELOW THE MEAN										
		213.4	93.7	26.9	5.8	1.2		128.	64.	32.	16.	8.	4.	2.	1.	0.5	0.5
RMB1	UP	209.6	99.6	22.8	5.0	2.5	ABOVE	125.1	70.0	30.7	14.3	8.9	2.7	2.2	1.0	0.6	0.9
	DOWN	210.2	100.6	20.8	4.2	3.5	BELOW	124.3	71.0	30.9	14.8	9.2	2.1	2.3	1.0	0.5	0.3
RMB2	UP	213.6	92.8	26.8	7.0	1.0	ABOVE	126.7	64.2	31.6	16.8	8.5	3.5	1.9	1.3	0.6	0.5
	DOWN	213.6	94.1	27.2	5.5	1.0	BELOW	130.2	62.9	29.7	15.7	8.7	4.2	2.0	1.1	0.5	0.4
RMB3	UP	212.8	90.3	29.3	5.7	1.3	ABOVE	126.5	64.2	31.8	15.6	7.5	4.5	2.0	1.0	0.5	0.6
	DOWN	209.5	96.4	26.4	5.6	1.5	BELOW	126.3	63.1	32.7	14.5	9.4	3.9	2.1	1.4	0.4	0.5
RMD	UP	212.7	91.6	27.2	6.3	1.7	ABOVE	126.8	63.7	31.7	16.2	7.9	4.0	1.8	1.1	0.5	0.6
	DOWN	212.7	91.6	27.6	6.3	1.4	BELOW	125.5	64.1	32.7	14.9	8.4	4.1	2.1	1.2	0.5	0.7
RMTB	UP	218.7	93.7	25.6	4.8	1.3	ABOVE	131.7	65.0	31.0	15.4	7.0	4.5	1.9	0.7	0.4	0.7
	DOWN	217.0	93.0	27.2	5.9	1.1	BELOW	131.6	62.7	31.4	16.1	8.2	4.2	2.4	0.8	0.4	0.6
RMTD	UP	213.5	93.7	25.9	5.7	1.3	ABOVE	129.3	63.2	32.8	15.6	7.5	4.2	2.6	1.1	0.3	0.6
	DOWN	209.8	96.2	26.2	6.4	1.6	BELOW	129.6	65.8	31.4	14.4	9.2	3.3	1.9	1.2	0.2	0.5
RMTDL	UP	200.4	95.2	27.4	6.5	1.8	ABOVE	117.5	63.4	31.8	15.5	8.8	4.0	2.0	1.2	0.6	0.6
	DOWN	200.0	92.2	29.6	7.6	2.0	BELOW	115.9	60.8	32.8	17.6	8.2	4.7	2.6	1.3	0.8	0.7
RANMX1	UP	211.4	95.9	27.3	5.3	1.4	ABOVE	126.7	64.3	32.9	15.9	8.0	4.1	2.2	0.9	0.6	0.6
	DOWN	214.7	93.0	27.1	5.8	0.8	BELOW	129.4	63.5	32.1	15.4	8.2	4.1	2.0	0.7	0.4	0.3
RANMX2	UP	213.6	92.3	27.2	6.3	1.0	ABOVE	126.3	63.3	32.0	16.7	8.4	4.7	2.1	0.9	0.4	0.6
	DOWN	212.0	93.8	27.2	6.2	1.3	BELOW	128.3	63.5	31.3	16.1	8.3	4.4	1.6	0.9	0.4	0.3
RFB	UP	211.7	91.7	27.0	7.3	0.9	ABOVE	129.2	64.4	31.7	15.3	7.4	4.2	1.8	1.2	0.6	0.7
	DOWN	208.5	95.7	26.6	6.4	1.5	BELOW	126.9	65.6	33.3	14.9	8.2	4.0	2.0	0.9	0.5	0.3

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Acknowledgment

We desire to thank Dr. Ilio Galligani for his many suggestions and his continuous aid.

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