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EUROPEAN ATOMIC ENERGY COMMUNITY — EURATOM

**COMPUTATION OF THE COMPRESSION
OF A MAGNETIC FIELD
BY MEANS OF A LINER DRIVEN BY AN EXPLOSION**

by

L. GUERRI, P. STELLA and A. TARONI

1967



Joint Nuclear Research Center
Ispra Establishment - Italy

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Brussels, April 1967 - 32 Pages - FB 40

This report describes the numerical solution of the following problem. A cylindrical liner, surrounded by an explosive ring, bounds a magnetic field. This field exerts a pressure on the internal wall of the liner, while the diffusion of the field on the interior of the liner is neglected. A detonation front, started on the external side of the explosive ring, reaches the liner and pushes it concentrically towards the axis. First the propagation of the detonation front and the state of the gas behind it is computed. Afterwards the interaction liner-gas is treated. The computation is carried out under the hypothesis of axial symmetry.

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Summary

This report describes the numerical solution of the following problem. A cylindrical liner, surrounded by an explosive ring, bounds a magnetic field. This field exerts a pressure on the internal wall of the liner, while the diffusion of the field on the interior of the liner is neglected. A detonation front, started on the external side of the explosive ring, reaches the liner and pushes it concentrically towards the axis. First the propagation of the detonation front and the state of the gas behind it is computed. Afterwards the interaction liner-gas is treated. The computation is carried out under the hypothesis of axial symmetry.

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1. Introduction

We consider the following problem:

Given a cylindrical liner surrounded by a ring of explosive. The internal wall of the liner is subject to a pressure:

$$p = \frac{B_0^2}{8\pi} \left[\left(\frac{X_e(0)}{X_e(t)} \right)^4 - 1 \right]$$

X_e being the internal radius of the liner. This pressure is due to a magnetic field bounded by the liner. It is assumed that the diffusion of the magnetic field in the liner is negligible. At a given instant a detonation is initiated on the external wall of the explosive. The detonation wave reaches the liner and compresses it. The liner moves concentrically towards the axis. The pressure on the internal wall of the liner increases first slowly and afterwards very rapidly so that the liner is strongly decelerated. Because the pressure in the interior of the liner has very high values, the liner is treated as an ordinary hydrodynamic medium.

The problem can be considered also with slab symmetry. The pressure on the wall is then:

$$p = \frac{B_0^2}{8\pi} \left[\left(\frac{X_e(0)}{X_e(t)} \right)^2 - 1 \right]$$

A detailed description and analysis of this problem is carried out in (7) of bibliography. The aim of this report is the description of the method used to solve numerically the problem.

This problem has been studied on behalf of the Euratom Group on Fusion (Frascati, Roma).

2. The two phases of the problem

We can distinguish two phases in the problem:

- a) The detonation propagates in the explosive and at a certain instant t reaches the external wall of the liner.

(*) Manuscript received on January 24, 1967.

b) For $t > \bar{t}$ a shock wave propagates in the liner and in the detonation gas. The liner moves towards the center and the shock wave in the liner reflects successively between the liner's walls.

The propagation of a detonation wave in an homogeneous medium at rest has been studied by Taylor, Zeldovich, Stanyukovich and other authors. In the slab case the Chapman-Jouguet hypothesis holds, as a consequence of which the front velocity is constant, the value of u , p , ρ on the front is constant and the flow behind the front is defined by a simple rarefaction wave centered in the origin of the detonation (Taylor's wave).

In the cylindrical or spherical case Zeldovich has shown that the Chapman-Jouguet hypothesis holds good only at the beginning of the detonation and the pressure on the front increases as the front approaches the axis or respectively the center.

To treat numerically the detonation in the cylindrical or spherical case, the solution is approximated for a short time with the corresponding slab solution. The solution is then continued by means of a implicit finite difference scheme.

The instant when the detonation wave reaches the liner is taken as the initial time of the second phase of the problem. In this phase two hydrodynamic media with given initial states are in contact. This phase is followed by means of an explicit lagrangean scheme.

3. Detonation phase

We have the usual symbols:

t	time		
h	lagrangean coordinate		
X	eulerian	"	
x	"	"	at initial time
u	velocity		

ρ density
 p pressure
 e specific internal energy
 c sound speed
 S entropy

u_0, ρ_0, p_0, \dots (uniform) velocity, density, pressure...
 of the unreacted explosive

u_1, ρ_1, p_1, \dots values of u, ρ, p, \dots of the gas (reacted
 explosive) on the detonation front.

D detonation front velocity. $D < 0$ since
 the detonation is convergent.

$$u_0 = 0$$

$$p_0 = 0$$

p_0 is set equal to zero, because it is negligible compared to p_1 .

The relation between X and h is :

$$\frac{h^{\nu+1}}{\nu+1} = \frac{1}{\rho_0} \int_{X(0,t)}^{X(h,t)} \rho(\xi,t) \xi^\nu d\xi + \frac{x_i^{\nu+1}}{\nu+1} \quad (3.1)$$

$\nu = 0, 1, 2$ according to whether we consider slab, cylindrical
 or spherical geometry.

x_i and x_e are respectively the internal and external
 radius of the ring of explosive.

From (3.1) it follows that :

$$h = x = X(h, 0)$$

With regard to the state equation of the detonation gas
 it is usually assumed that p is dependent only on ρ and not on S :

$$p = p(\rho)$$

This is equivalent to the hypothesis that the internal energy is separable:

$$e(\rho, S) = e^{(1)}(\rho) + e^{(2)}(S)$$

The usual assumption is:

$$p = A\rho^\gamma \tag{3.2}$$

A, γ constants and $\gamma = 3$.

The flow equations in lagrangean form are:

$$\left. \begin{aligned} \frac{1}{\rho} &= \frac{1}{\rho_0} \left(\frac{X}{h}\right)^\nu X_h \\ X_t &= u \\ u_t &= -\frac{1}{\rho_0} \left(\frac{X}{h}\right)^\nu p_h \\ e_t &= \frac{p}{\rho^2} \rho_t \\ p &= p(\rho) \end{aligned} \right\} \tag{3.3}$$

As a consequence of the hypothesis that the energy is separable the first three equations do not depend on the fourth. We have then the system:

$$\left. \begin{aligned} \frac{1}{\rho} &= \frac{1}{\rho_0} \left(\frac{X}{h}\right)^\nu X_h \\ X_t &= u \\ u_t &= -\frac{1}{\rho_0} \left(\frac{X}{h}\right)^\nu c^2 p_h \\ p &= p(\rho) \end{aligned} \right\} \tag{3.4}$$

i.d. we can determine the flow with regard to X , u , ρ , p independently from the energy equation. This equation can be taken into account if we are interested in the energy distribution. System (3.4) may be put into the equivalent form:

$$\left. \begin{aligned} \frac{1}{\rho} &= \frac{1}{\rho_0} \left(\frac{x}{h}\right)^\nu x_h \\ x_{tt} &= K \left(\frac{x}{h}\right)^\nu \left[\left(\frac{x}{h}\right)^\nu x_h \right]_h \end{aligned} \right\} \quad (3.5)$$

with:

$$\left. \begin{aligned} u &= x_t \\ p &= p(\rho) \\ K &= c^2 \rho^2 / \rho_0^2 \end{aligned} \right\} \quad (3.6)$$

In particular, if
then:

$$\begin{aligned} p &= A\rho^\gamma \\ K &= \gamma p / \rho_0^2 \end{aligned}$$

System (3.5) must be solved in the region:

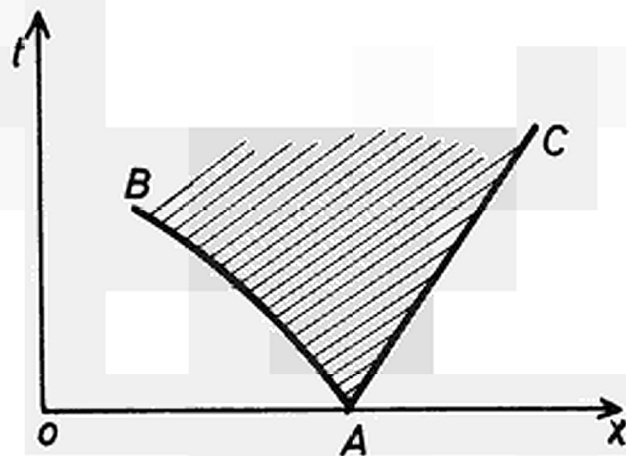


Fig. 1

- A origin of the detonation
- AB front trajectory
- AC free trajectory of last particle

In AC we have the condition:

$$p = 0$$

This condition can be substituted with the equivalent condition:

$$u = \text{constant}$$

In fact, from :

$$p = 0 \quad \text{on AC}$$

it follows

$$c = 0 \quad \text{on AC}$$

and from :

$$u_t = - \frac{1}{\rho_0} \left(\frac{x}{h} \right)^y c^2 \rho_h$$

it follows

$$u_t = 0 \quad \text{i.d.} \quad u = \text{constant} \quad \text{on AC}$$

Hence the particle trajectory AC is a straight line. In lagrangean coordinates the region in which the flow must be determined is :

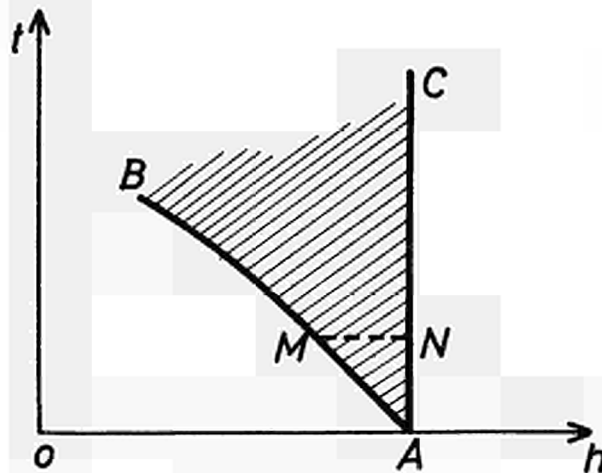


Fig. 2

In the region AMN the cylindrical or spherical solution is approximated with the slab solution. In the region BMNC the solution is computed by means of an implicate finite difference scheme. The initial data on MN are those provided by the slab approximation.

4. Taylor's wave

In the slab case the flow of the detonation gas is isentropic. For this reason on the front we need only to consider the Rankine-Hugoniot relations for the conservation of mass and momentum:

$$\rho_0 D = \rho_1 (D - u_1) \quad (4.1)$$

$$\rho_0 D^2 = p_1 + \rho_1 (D - u_1)^2 \quad (4.2)$$

For a backward-facing detonation front the Chapman-Jouguet condition is:

$$u_1 - c_1 = D \quad (4.3)$$

and the front speed D is constant. The state equation of the detonation gas is:

$$p = A \rho^\gamma \quad \gamma = 3 \quad (4.4)$$

Hence :

$$c^2 = \gamma p / \rho \quad (4.5)$$

From (4.2) and (4.3) it follows:

$$\rho_0 D^2 = p_1 + \rho_1 c_1^2$$

From this relation and from (4.5) it follows:

$$p_1 = \frac{\rho_0 D^2}{\gamma + 1} \quad (4.6)$$

From (4.1) and (4.2) it follows:

$$\rho_0 D^2 = p_1 + \rho_0 D (D - u_1)$$

i.d. $p_1 = \rho_0 D u_1$

From this relation and from (4.6) it follows:

$$u_1 = \frac{D}{\gamma+1} \quad (4.7)$$

From (4.3) and (4.7):

$$c_1 = \frac{-\gamma D}{\gamma+1} \quad (4.8)$$

From (4.1) and (4.7) :

$$\rho_1 = \frac{\gamma+1}{\gamma} \rho_0 \quad (4.9)$$

and substituting (4.6) and (4.9) into:

$$p_1 / \rho_1^\gamma = A$$

we obtain:

$$A = \frac{D^2}{\gamma+1} \left(\frac{\gamma}{\gamma+1} \right)^\gamma \rho_0^{1-\gamma} \quad (4.10)$$

In the slab case the flow behind the detonation front is defined by a simple rarefaction wave centered in the origin of the detonation (Taylor's wave). This wave is defined by the equations:

$$X - x_e = (u - c)t \quad (4.11)$$

$$u - \frac{2c}{\gamma+1} = u_1 - \frac{2c_1}{\gamma-1} \quad (4.12)$$

The first equation is the equation of the C^- characteristics issuing from the origin of the detonation.

The second equation means that the Riemann invariant R_- is constant in the region behind the front.

The Taylor's wave is a complete wave since $p = 0$ on the line $h = h_e$ i.d. there is no external resistance to the expanding detonation gas.

The escape velocity u_e is then:

$$u_e = u_1 - \frac{2c_1}{\gamma+1}$$

If we substitute (4.7) and (4.8) for u_1 and c_1 in (4.11) and (4.12) we obtain:

$$u = - \frac{D+2c}{\gamma-1} \quad (4.13)$$

$$X = x_e - \frac{D+(\gamma+1)c}{\gamma-1} t \quad (4.14)$$

Moreover, from:

$$c^2 = \gamma p / \rho = \gamma A \rho^{\gamma-1}$$

it follows:

$$\rho = \left(\frac{c^2}{\gamma A} \right)^{\frac{1}{\gamma-1}} \quad (4.15)$$

and

$$p = A \left(\frac{c^2}{\gamma A} \right)^{\frac{\gamma}{\gamma-1}} \quad (4.16)$$

The state behind the front is then completely defined if c is given as function of h , t . To do this we first remark that in the slab case:

$$\rho_0 h = \int_{X(0,t)}^{X(h,t)} \rho dX + \rho_0 x_i$$

and from this:

$$\rho_0 (h_e - h) = \int_X^{X_e} \rho dX = \int_c^{c_e} \rho \frac{\partial X}{\partial c} dc$$

$c_e = 0$ hence:

$$\rho_0 (h_e - h) = - \int_0^c \rho \frac{\partial X}{\partial c} dc \quad (4.17)$$

Differentiating (4.14) with respect to c we obtain:

$$\frac{\partial X}{\partial c} = - \frac{\gamma+1}{\gamma-1} t \quad (4.18)$$

and substituting (4.14) and (4.17) in (4.16):

$$\rho_0 (h e^{-h}) = c^{\frac{\gamma+1}{\gamma-1}} \left(\frac{1}{\gamma A} \right)^{\frac{1}{\gamma-1}} t$$

from which :

$$c = \left[\frac{\rho_0 (h e^{-h})}{t} \right]^{\frac{\gamma-1}{\gamma+1}} (\gamma A)^{\frac{1}{\gamma+1}} \quad (4.19)$$

From (4.10) :

$$\begin{aligned} (\gamma A)^{\frac{1}{\gamma+1}} &= |D|^{\frac{2}{\gamma+1}} \left(\frac{\gamma}{\gamma+1} \right) \rho_0^{-\frac{\gamma-1}{\gamma+1}} \\ &= \frac{|D|\gamma}{\gamma+1} |D|^{-\frac{\gamma-1}{\gamma+1}} \rho_0^{-\frac{\gamma-1}{\gamma+1}} \end{aligned}$$

Substituting this expression into (4.18) we obtain:

$$c = \frac{-\gamma D}{\gamma+1} \left[\frac{h e^{-h}}{-Dt} \right]^{\frac{\gamma-1}{\gamma+1}} \quad (4.20)$$

5. Finite difference scheme

Let us rewrite the flow equations for the gas:

$$\left\{ \begin{array}{l} \frac{1}{\rho} = \frac{1}{\rho_0} \left(\frac{X}{h} \right)^\gamma X_h \\ X_{tt} = K \left(\frac{X}{h} \right)^\gamma \left[\left(\frac{X}{h} \right)^\gamma X_h \right]_h \\ u = X_t \\ e_t = \frac{p}{\rho^2} \rho_t \\ p = A \rho^\gamma \\ K = \gamma p \rho / \rho_0^2 \end{array} \right. \quad (5.1)$$

with
$$A = \frac{D_0^2}{\gamma+1} \left(\frac{\gamma}{\gamma+1} \right)^\gamma \rho_0^{1-\gamma} \quad (\gamma=3)$$

and D_0 initial velocity of the front.

The energy equation has been added since in the second phase of the problem (liner-gas interaction) the momentum equation is no more independent from the energy equation. Hence, in the second phase we must consider the complete system (3.3). However, in the detonation phase X, u, p, e are determined independently from e .

System (5.1) is approximated with the following implicit scheme:

$$\begin{aligned} \rho_{j+1/2}^n &= \rho_0 / \left[\alpha_{j+1/2}^n \frac{X_{j+1}^n - X_j^n}{\Delta h} \right] & \alpha_{j+1/2}^n &= \left(\frac{X_j^n + X_{j+1}^n}{2h_{j+1/2}} \right)^\nu \\ p_{j+1/2}^n &= A \left(\rho_{j+1/2}^n \right)^\nu \\ K_j^n &= \frac{\gamma}{\rho_0^2} \frac{p_{j-1/2}^n \rho_{j-1/2}^n + p_{j+1/2}^n \rho_{j+1/2}^n}{2} \\ \frac{X_j^{n+1} - X_j^n}{\Delta t_n} - \frac{X_j^n - X_j^{n-1}}{\Delta t_{n-1}} &= \frac{K_j^n \left(\frac{X_j^n}{h_j} \right)^\nu \left[\alpha_{j+1/2}^n (X_{j+1}^{n+1} - X_j^{n+1}) - \alpha_{j-1/2}^n (X_j^{n+1} - X_{j-1}^{n+1}) \right]}{2(\Delta h)^2} \\ &+ (1-\theta) \frac{\alpha_{j+1/2}^n (X_{j+1}^n - X_j^n) - \alpha_{j-1/2}^n (X_j^n - X_{j-1}^n)}{(\Delta h)^2} \\ u_j^{n+1} &= \frac{X_j^{n+1} - X_j^n}{\Delta t_n} \\ e_{j+1/2}^{n+1} - e_{j+1/2}^n &= \frac{2(p_{j+1/2}^{n+1} + p_{j+1/2}^n)}{(\rho_{j+1/2}^{n+1} + \rho_{j+1/2}^n)} \left(\rho_{j+1/2}^{n+1} - \rho_{j+1/2}^n \right) \end{aligned} \quad (5.2)$$

This scheme is unconditionally stable if $\phi \geq 1/2$. Advantage is taken of this fact in order to set up a point mesh with constant step Δh and variable step Δt_n .

If $t_0 = 0$

and $t_n = t_{n-1} + \Delta t_{n-1} = \Delta t_0 + \Delta t_1 + \dots + \Delta t_{n-1}$

and $h_F = h_F(t)$

is the front trajectory in the h, t coordinates the steps Δt_n are chosen in such a way that

$$h_F(t_n)$$

be always coincident with a value h_j of the mesh (see fig. 3)

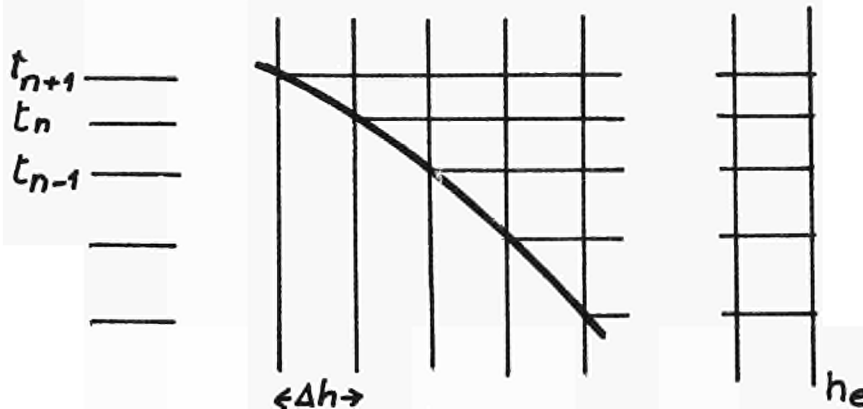


Fig. 3

On the line $h = h_e$

we have the condition $u = u_e$

It follows that:

$$X_e(t) = x_e + u_e t$$

or equivalently:

$$X_e(t+\Delta t) = X_e(t) + u_e \Delta t$$

The finite difference scheme (5.2) is applied to the mesh points internal to the region BMNC. The front trajectory is computed by means of the formula:

$$X_F(t_{n+1}) = X_F(t_n) + D(t_n)\Delta t_n$$

or :

$$X_F(t_{n+1}) = X_F(t_{n-1}) + D(t_n) (\Delta t_{n-1} + \Delta t_n)$$

This second formula has higher precision under the assumption that $\Delta t_n \approx \Delta t_{n-1}$.

It is:

$$X_F = h_F$$

In fact, until the moment a particle X is reached by the front, we have:

$$X(t) = x = h.$$

Δt_n is chosen in such a way that :

$$X(t_{n+1}) - X(t_n) = \Delta X = \Delta h$$

Hence:

$$\Delta t_n = \frac{\Delta h}{D(t_n)}$$

The finite difference scheme cannot be applied directly to the point adjacent to the front since we lack a point at the (n-1)-th level as shown in fig.4

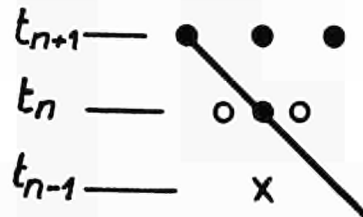


Fig. 4

The value of X at point (x) can be determined by linear or quadratic extrapolation on line $t = t_{n-1}$; in the same way

we obtain the values $\alpha_{j \pm 1/2}^n$ at the points (O) on line $t = t_n$. Then the finite difference scheme can be used in a totally implicit form with $\theta = 1$.

Another possibility is schematically shown in fig. 5

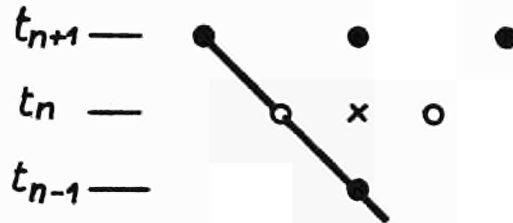


Fig. 5

The value of X at point (x) is extrapolated:

$$X(t_{n+1/2}) = X(t_n) + u(t_n) \frac{\Delta t_n}{2}$$

$X(t_n)$ and $u(t_n)$ are relative to point (.) on line $t = t_n$. The values of $\alpha_{j \pm 1/2}^{n+1/2}$ at points (O) are obtained by interpolation and the difference scheme is applied in a totally implicit form. The value of $X(t_{n+1})$ at the point adjacent to the front could be obtained by direct extrapolation:

$$X(t_{n+1}) = X(t_n) + u(t_n) \Delta t_n$$

However, the results obtained in this way are not satisfactory since a pressure peak builds up behind the front. A possible explanation could be that with this direct extrapolation the values of X on the front and at the adjacent point are weakly related to the other values of X behind the front. This is unsatisfactory on physical grounds since the hydrodynamic quantities on the front depend on the flow behind the front. This drawback is avoided if $\rho_{j+1/2}^{n+1}$ is computed by means of the first equation (5.2) with the exception of the half-point near the front. For this we set:

$$\rho_{F-1/2}^{n+1} = \rho_{F-3/2}^{n+1}$$

(see also Fig. 6)

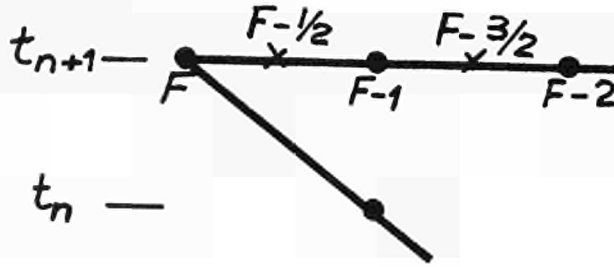


Fig. 6

In this way a good connexion is reestablished between the value of X on the front and the values of X behind the front. The profile of ρ is slightly flattened behind the front but this loss of precision in the computation of ρ is progressively reduced since ρ_h behind the front decreases as the time increases.

A possible variation of the finite difference scheme (5.2) would be to consider the h -differentiation for $t = t_{n+1}$ and $t = t_{n-1}$, together with suitable modifications for the two points near to the front.

Given X at time t_{n+1} we can compute u, ρ, p, e . In order to determine D, u_1, ρ_1, p_1 on the front at time t_{n+1} we use the Rankine-Hugoniot relations (conservation of mass and momentum only).

$$\rho_0 D = \rho_1 (D - u_1) \quad (5.3)$$

$$\rho_0 D^2 = p_1 + \rho_1 (D - u_1)^2 \quad (5.4)$$

From these relations it follows:

$$\rho_0 D^2 = p_1 + \rho_0 D (D - u_1)$$

hence:

$$D = \frac{p_1}{\rho_0 u_1} \quad (5.5)$$

Introducing this expression into (5.3) we obtain:

$$u_1^2 = p_1 \left(\frac{1}{\rho_0} - \frac{1}{\rho_1} \right)$$

and, more specifically:

$$u_1 = - \sqrt{p_1 \left(\frac{1}{\rho_0} - \frac{1}{\rho_1} \right)} \quad (5.6)$$

The sign - has been chosen since in (5.5) D is negative and ρ_0, p_1 positive. To conclude, on the front we have:

$$p_1 = A \rho_1^\gamma$$

$$u_1 = - \sqrt{p_1 \left(\frac{1}{\rho_0} - \frac{1}{\rho_1} \right)}$$

$$D = \frac{p_1}{\rho_0 u_1}$$

Actually, we have three unknowns D, u_1 , ρ_1 and two equations given by the Rankine-Hugoniot conditions. A third relation is supplied by the characteristic equation along the C^+ characteristic. However, to avoid the numerical difficulties connected with this last equation, it is better to find an approximate value for ρ_1 and to determine subsequently D, p_1 , u_1 .

ρ_1 is approximated with the formula:

$$\rho_1(t_{n+1}) = \rho_0 \left/ \left[\left(\frac{x_F^{n+1}}{h_F} \right) \frac{x_F^{n+1} - x_{f-1}^{n+1}}{\Delta h} \right] \right.$$

F being the h-index on the front.

As already said, in case we choose to compute x_{F-1}^{n+1} by direct extrapolation it is best to set

$$\rho_{F-3/2}^{n+1} = \rho_{F-3/2}^{n+1}$$

$\rho_{F-3/2}^{n+1}$ being computed with the first of equations (5.2).

The number of points to be considered increases with each iteration. In order not to increase unduly this number the step Δh can be periodically doubled, so that the corresponding number of points is halved. When Δh is doubled the $(n-2)$ th time level must be substituted for the $(n-1)$ th level.

6. Liner-gas interaction

Let \bar{t} be the time at which the detonation front reaches the liner. If we take \bar{t} as initial time we have then two hydrodynamic media which are going to interact. The initial state of the liner is uniform, the initial state of the gas is that defined by the flow behind the front at time \bar{t} .

We introduce the lagrangean coordinate h :

$$\begin{aligned} \frac{h^{\nu+1}}{\nu+1} &= \frac{1}{\rho_0} \int_{X(0,t)}^{X(h,t)} \rho(\xi,t) \xi^\nu d\xi + \frac{x_e^{\nu+1}}{\nu+1} \\ &= \frac{1}{\rho_0} \int_{x_e}^{\bar{x}} \bar{\rho} \xi^\nu d\xi + \frac{x_e^{\nu+1}}{\nu+1} \end{aligned}$$

\bar{x} being the eulerian coordinate at time \bar{t} .

Hence $\bar{x} = x$ for the liner and $\bar{x} = X(h,\bar{t})$ for the gas.

x_e is the internal wall of the liner

x_i the interface liner-gas

For $\bar{x} < x_i$ ρ_0 , ρ are relative to the liner and $\bar{\rho} = \rho_0$

For $\bar{x} > x_i$ ρ_0 is the density of the unreacted explosive and $\bar{\rho}$ the gas density at time \bar{t} .

In this case we have

$$\frac{h^{\nu+1}}{\nu+1} = \frac{1}{\rho_e^0} \int_{x_i}^{\bar{x}} \bar{\rho} \xi^\nu d\xi + \frac{x_i^{\nu+1}}{\nu+1}$$

(ρ_e^0 constant density of the unreacted explosive)

Hence for the gas we obtain the same h as defined by (3.1).

$h = x$ both for the liner and the gas at time $t = 0$.

The differential system to be solved, in lagrangean form, is:

$$\left. \begin{aligned} \frac{1}{\rho} &= \frac{1}{\rho_0} \left(\frac{x}{h}\right)^{\nu} x_h \\ x_t &= u \\ u_t &= -\frac{1}{\rho_0} p_h \\ e_t &= \frac{p}{\rho^2} \rho_t \\ p &= f(\rho, e) \end{aligned} \right\} \quad (6.1)$$

the boundary conditions are

$$\begin{aligned} p &= \frac{B_0}{8\pi} \left[\left(\frac{x_e(0)}{x_e(t)}\right)^4 - 1 \right] & h &= h_e \\ u &= u_e & h &= h_e \end{aligned}$$

This last condition can be substituted with the equivalent condition :

$$p = 0 \quad h = h_e$$

h_e, h_i, h_e are respectively the values of h corresponding to the internal wall of the liner, the interface, the external boundary of the expanding gas.

For $h < h_i$ we have the liner

For $h > h_i$ we have the gas

The state equation of the liner is:

$$p(\rho, e) = p_c(\rho) + \rho\gamma(\rho) (e - e_c(\rho))$$

with

$$p_c(\rho) = \sum_{j=1}^6 a_j (\rho/\rho_{OK})^{1+j/3}$$

$$\text{and } e_c(\rho) = \int_{\rho_{OK}}^{\rho} \frac{p_c(\xi)}{\xi^2} d\xi = \frac{1}{\rho_{OK}} \sum_{j=1}^6 \frac{3}{j} a_j (\rho/\rho_{OK})^{j/3}$$

$$\gamma(\rho) = \frac{1}{3} + \frac{\rho}{2} \frac{d^2 p_c / d\rho^2}{dp_c / d\rho}$$

ρ_{OK} is the density at absolute temperature $T = 0$ and pressure $p = 0$.

This state equation is derived from the formulae:

$$p(\rho, T) = p_c(\rho) + \rho\gamma(\rho)c_v (T - T_0 + \frac{E_0}{c_v})$$

$$e(\rho, T) = e_c(\rho) + c_v(T - T_0 + \frac{E_0}{c_v})$$

The state equation of the gas is:

$$p = A\rho^\gamma \quad (\gamma = 3)$$

The differential system (6.1) is approximated with the following finite difference scheme:

$$\left. \begin{aligned} \frac{u_j^{n+1} - u_j^n}{\Delta t} &= -\frac{1}{\rho_0} \left(\frac{p_{j+1/2}^n - p_{j-1/2}^n}{\Delta h} + \frac{q_{j+1/2}^n - q_{j-1/2}^n}{\Delta h} \right) \\ \frac{x_j^{n+1} - x_j^n}{\Delta t} &= u_j^{n+1} \\ \rho_{j+1/2}^{n+1} &= \rho_0 / \left[\left(\frac{x_j^{n+1} + x_{j+1}^{n+1}}{2h_{j+1/2}} \right)^\gamma \frac{x_{j+1}^{n+1} - x_j^{n+1}}{\Delta h} \right] \\ e_{j+1/2}^{n+1} - e_{j+1/2}^n &= \frac{p_{j+1/2}^n}{(\rho_{j+1/2}^n)^2} \left(\rho_{j+1/2}^n - \rho_{j+1/2}^{n+1} \right) \\ p_{j+1/2}^{n+1} &= f(\rho_{j+1/2}^{n+1}, e_{j+1/2}^{n+1}) \\ q_{j+1/2}^n &= \begin{cases} a^2 \rho_{j+1/2}^n (u_{j+1}^n - u_j^n) & \text{if } u_{j+1}^n - u_j^n < 0 \\ 0 & \text{if } u_{j+1}^n - u_j^n > 0 \end{cases} \end{aligned} \right\} (6.2)$$

with the stability condition:

$$\Delta t < \frac{\Delta x}{c}$$

q is the pseudo-viscosity term with $a=2$.

The liner-gas interface should be coincident with a h -line of the mesh having integer index. Because the liner has greater density than the gas, it may be convenient to employ different steps $\Delta_e h$ and $\Delta_g h$ in the liner and in the gas, $\Delta_g h$ being the greater.

This change of step may be done in different ways. For instance, the step Δh in the gas can be lengthened gradually.



Fig. 7

In this case the formulae of the finite difference scheme must be modified with regard to the gas, in order to take into account the fact that the intervals Δh are unequal.

A second solution is schematically represented in fig.8: the index of the interface

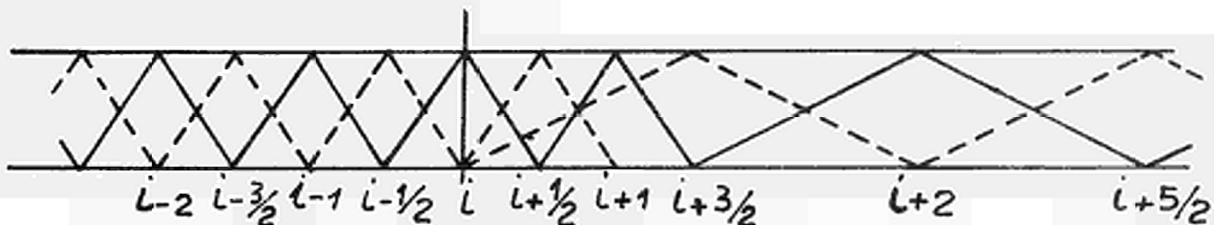


Fig. 8

Given $\Delta_e h$ and an odd integer (in the figure $m=3$) we define:

$$\Delta_g h = m \Delta_e h$$

and take m points after the i -th with step $\frac{1}{2} \Delta_e h$. The following

points are taken with step $\frac{1}{2} \Delta_g h$.

X, u, ρ, p, e, q are given on the n -th line. We compute X_j^{n+1} and u_j^{n+1} for

$$j = \dots i-2, i-1, i, i+1, i+2, \dots$$

The computation for each point is schematically represented by a triangle with the base on the n -th line, vertex on the $(n+1)$ th line and continuous sides. The state equation is that of the liner or of the gas according to whether $j \pm 1/2 \neq i$.

We then compute

$$\rho_{j \pm 1/2}^{n+1}, e_{j \pm 1/2}^{n+1}, p_{j \pm 1/2}^{n+1}, q_{j \pm 1/2}^{n+1}$$

(triangles with dashed sides)

A criterion for the choice of m could be:

$$m = \text{odd integer near to } \frac{\rho_e^0}{\rho_e}$$

(ρ_e^0 constant density of the unreacted explosive).

This criterion is based on the following considerations:

From the definition of h it follows:

$$h^\nu dh = \frac{1}{\rho_0} X^\nu dX = \frac{1}{\rho_0} dm$$

m mass and ρ_0 equal to ρ_e^0 or ρ_g^0 according to whether the particle X belongs to the liner or to the gas.

For two contiguous layers of liner and gas we have:

$$\Delta m_e = \rho_e^0 h_e^\nu \Delta h_e$$

$$\Delta m_g = \rho_g^0 h_g^\nu \Delta h_g$$

It should be:

$$\Delta m_e \approx \Delta m_g$$

since the layers are contiguous. Also, for the same reason:

$$h_e \approx h_g$$

Hence:

$$\rho_e^0 \Delta h_g \approx \rho_e^0 \Delta h_e$$

7. Numerical results

When $\nu = 0$ we have the slab case. It is then possible to compare the exact solution as given by formulae (4.13)...(4.19) with the numerical solution obtained by means of the finite difference scheme. The exact and numerical solution at a given time are presented in Table 1 and 2.

Table 3 gives the gas flow (cylinder geometry) behind the front at time $t = \bar{t}$.

Table 4 gives the trajectory of the two faces X_e and X_i of the liner.

Tables 5 and 6 give the numerical results for two different times.

8. Acknowledgments

We are indebted to Dr. J.P. Somon of Euratom Group on Fusion (Frascati, Rome) who suggested the problem to us and had a stimulating influence on our work.

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PLANE DETONATION * FINITE DIFFERENCE SCHEME

NH = 217 T = 2.6993E-06 D = -8.0045E 05 DH = 10.0000E-03

Table with 6 columns: H, X, U, P, RH. The table contains numerical data for various time steps from 00 to 100. The H column shows values from 9.0000 to 6.8400. The X column shows values from 1.0080 to 6.8400. The U column shows values from 4.0000 to -1.9194. The P column shows values from 0.0000 to 2.6118. The RH column shows values from 0.0000 to 2.2362.

TABLE 2

TABLE 4

t	X_l	X_i
6.483	3.5	3.8
6.765	3.5	3.729
6.913	3.490	3.706
7.005	3.446	3.693
7.223	3.349	3.659
7.390	3.285	3.621
7.537	3.234	3.574
7.701	3.179	3.510
7.83	3.137	3.453
7.964	3.090	3.390
8.104	3.037	3.326
8.249	2.970	3.270
8.402	2.890	3.213
8.620	2.764	3.132
8.857	2.628	3.037
9.050	2.524	2.953
9.257	2.420	2.857
9.405	2.344	2.785
9.642	2.223	2.665
9.812	2.133	2.576
1.009	1.980	2.431
1.039	1.802	2.291
1.060	1.672	2.198
1.082	1.525	2.097
1.107	1.348	1.981
1.138	1.127	1.841
1.201	6.742	1.560
1.246	3.524	1.365
1.271	2.507	1.260
1.292	2.453	1.181
1.313	2.670	1.104
1.337	2.965	1.057
1.360	3.224	1.120
1.377	3.440	1.193
1.398	3.694	1.275
1.424	4.173	1.390
1.451	4.957	1.50
1.478	5.924	1.591
1.504	6.910	1.667
1.529	7.883	1.730
1.553	8.883	1.784

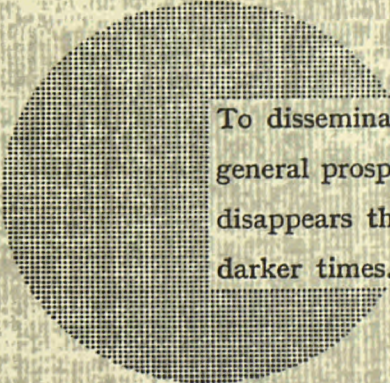
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To disseminate knowledge is to disseminate prosperity — I mean general prosperity and not individual riches — and with prosperity disappears the greater part of the evil which is our heritage from darker times.

Alfred Nobel

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