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**COMPUTATION OF THE MOTION OF A LINER UNDER THE  
IMPACT OF A UNIFORM GAS FLOW**

by

L. GUERRI, P. STELLA and A. TARONI

**1966**



Joint Nuclear Research Center  
Ispra Establishment - Italy

Scientific Information Processing Center - CETIS



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In the initial phase we have a Riemann problem the solution of which determines the states of the liner and the gas. Also the first reflexion of the shock on the free wall of the liner can be determined directly. This direct solution is used to test the efficiency of a finite difference scheme to be used in more general situations.

In the initial phase we have a Riemann problem the solution of which determines the states of the liner and the gas. Also the first reflexion of the shock on the free wall of the liner can be determined directly. This direct solution is used to test the efficiency of a finite difference scheme to be used in more general situations.

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## **SUMMARY**

This problem is treated in plane geometry and represents a simplified model of the motion of an explosively driven liner. The liner-gas interaction gives rise to a shock wave in both the media and we are particularly interested in the shock propagation in the liner.

In the initial phase we have a Riemann problem the solution of which determines the states of the liner and the gas. Also the first reflexion of the shock on the free wall of the liner can be determined directly. This direct solution is used to test the efficiency of a finite difference scheme to be used in more general situations.

## 1. Introduction

We consider the following problem:

Given a plane metallic liner at rest with constant density and pressure. At time  $t = 0$  a detonation wave hits the liner, this detonation being generated in an explosive in contact with the liner. We make the simplifying hypothesis that the detonation gas is in a uniform state with constant velocity, pressure and density. The pressure of the liner is negligible compared to that of the gas. Under the impact of a strong detonation wave the metallic liner behaves like an hydrodynamic medium.

We have then a Riemann problem and with the given initial data, a shock wave is generated in both the media. As is well known, the uniform states behind the shock in the liner and the gas are defined by the Rankine-Hugoniot conditions on the shock fronts and the supplementary condition that the velocity and pressure in the two media behind the shock fronts be the same.

The thickness of the gaseous medium is supposed to be much greater than that of the liner, so that we must take into account the reflexion of the shock on the other wall of the liner. This reflexion is represented by a centered rarefaction wave and may be easily determined.

We do not consider here the successive phase of reflexion of the rarefaction wave on the wall of the liner in contact with the detonation gas.

Along these lines we can determine the solution of the problem. The same problem is then solved step by step with a finite difference method. The good agreement between the solutions determined in two different ways permits to apply with confidence the numerical scheme to more general situations.

In fact the problem here described has been set up as a simplified model for the following problem to be studied in a later report: to determine the motion of an explosively driven liner in cylindrical geometry. The hypothesis of the uniform state of the detonation products is not made. This problem must be solved numerically and to test the efficiency of the numerical scheme to be employed the simplified version has been considered.

2. The state equation of the gas and the liner (\*)

We have the usual symbols:

- u velocity
- p pressure
- $\rho$  density
- e specific internal energy
- S entropy
- T temperature
- X eulerian coordinate
- h lagrangean coordinate
- t time

The detonation products are treated as a polytropic gas with state equation:

$$p = (\gamma - 1) e \rho \quad \gamma = 3$$

The metallic liner is treated as a hydrodynamic medium.  
e is equal to:

$$(2.1) \quad e(\rho, T) = e_c(\rho) + e_{th}(\rho, T) + e_f(\rho, T)$$

- $e_c$  potential energy to which is due the cohesion of the metal
- $e_{th}$  contribution due to the thermal energy of the atoms
- $e_f$  contribution due to the motion of free electrons.

It is:

$$p(\rho, T) = -\rho^2 T \int_0^T \frac{\partial e(\rho, \xi)}{\partial \rho} \frac{d\xi}{\xi^2}$$

Hence the pressure too has the additive form:

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(\*) For this paragraph see (2) of bibliography.



$$(2.2) \quad p(\rho, T) = p_c(\rho) + p_{th}(\rho, T) + p_f(\rho, T)$$

with :

$$p_c = \rho^2 \frac{de_c}{d\rho} \quad e_c = \int \frac{p_c}{\rho^2} d\rho$$

In particular the following formulae are used for  $e$  and  $p$  :

$$(2.3) \quad e = e_c(\rho) + e_v(T - T_0 + \frac{E_0}{c_v}) + \frac{\beta}{2} \left(\frac{\rho_{ok}}{\rho}\right)^{1/2} T^2$$

$$(2.4) \quad p = p_c(\rho) + \rho \chi(\rho) e_v(T - T_0 + \frac{E_0}{c_v}) + \rho_{ok} \frac{\beta}{4} \left(\frac{\rho_{ok}}{\rho}\right)^{1/2} T^2$$

$c_v, \beta, E_0, \rho_{ok}$  constants

$\rho_{ok}$  density at absolute temperature  $T = 0$  and pressure  $p = 0$

For  $p_c(\rho)$  and  $e_c(\rho)$  various analytical expressions have been proposed, suggested by theoretical considerations and experimental results.

The following formulae have been used in the numerical computations:

$$(2.5) \quad p_c(\rho) = \sum_{j=1}^7 a_j \left(\frac{\rho}{\rho_{ok}}\right)^{1+j/3}$$

which holds good for pressures up to  $10^{15}$  dyne/cm<sup>2</sup> ( $\sim 10^9$  atm) and:

$$(2.6) \quad e_c(\rho) = \int_{\rho_{ok}}^{\rho} \frac{p_c(\xi)}{\xi^2} d\xi = \frac{1}{\rho_{ok}} \sum_{j=1}^7 \frac{3}{j} a_j \left(\frac{\rho}{\rho_{ok}}\right)^{j/3}$$

With (2.5) and (2.6) the following expression for  $\chi(\rho)$  has been adopted:

$$(2.7) \quad \chi(\rho) = \frac{1}{3} + \frac{\rho}{2} \frac{d^2 p_c / d\rho^2}{dp_c / d\rho}$$

J.P. Somon has proposed the following simple form for  $p_c$  and  $e_c$  valid for pressures up to  $10^{12}$  dyne/cm<sup>2</sup> :

$$(2.8) \quad p_c = e_0^2 \rho_{ok} \left(\frac{\rho}{\rho_{ok}}\right)^2 \left(\frac{\rho}{\rho_{ok}} - 1\right)$$

$$(2.9) \quad e_c = \frac{e_0^2}{2} \left(\frac{\rho}{\rho_{ok}} - 1\right)^2$$

$\rho_0$  and  $c_0$  are normal density and sound speed. With (2.8) and (2.9) the following expression for  $\chi$  has been adopted:

$$(2.10) \quad \chi(\rho) = \frac{2}{3} + \frac{\rho}{2} \frac{d^2(p_c \rho^{-2/3})/d\rho^2}{d(p_c \rho^{-2/3})/d\rho}$$

$$= \frac{2}{3} \frac{14(\rho/\rho_{ok}) - 5}{7(\rho/\rho_{ok}) - 4}$$

A good approximation is given by :

$$(2.11) \quad \chi \approx \chi(\rho_{ok}) = 2$$

The term with  $T^2$  in the expressions of  $e$  and  $p$  may be neglected for pressures less than  $10^{13}$  dyne/cm<sup>2</sup>.

The numerical tests carried out show a good agreement between the results obtained with and without the term  $T^2$ . Neglecting this term we obtain:

$$(2.12) \quad e(\rho, p) = e_c(\rho) + \frac{1}{\rho \chi(\rho)} (p - p_c(\rho))$$

$$(2.13) \quad p(\rho, e) = p_c(\rho) + \rho \chi(\rho) (e - e_c(\rho))$$

### 3. The Riemann problem for the liner and the detonation products

We consider the following problem:

At the initial time  $t = 0$  the liner and the gas, both of them at constant state, are in contact. The liner is at rest, the gas has uniform velocity directed toward the liner. Moreover the thickness of the liner is much less than that of the gas. We want to follow the evolution of the phenomenon

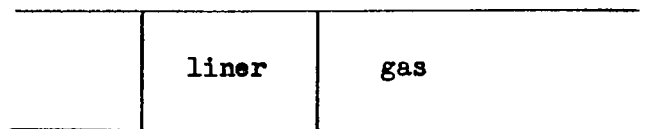


FIG. 1



Until the perturbation generated in the liner does not reach the free surface of the liner we have a Riemann problem. The solution is defined by four constant states  $S_1^0, S_1^1, S_g^0, S_g^1$  :  $S_1^0, S_g^0$  are the initial states of the liner and the gas.  $S_1^1, S_g^1$  are the states determined by the interaction between the two media.

In our case  $S_1^1$  and  $S_g^1$  are related to  $S_1^0$  and  $S_g^0$  by means of shock waves respectively backward and forward.

$S_1^1$  and  $S_g^1$  are determined by the Rankine-Hugoniot conditions plus the conditions:

$$(3.1) \quad \begin{aligned} u_1^1 &= u_g^1 \\ p_1^1 &= p_g^1 \end{aligned}$$

To determine  $u^1, p^1, \rho_1^1, \rho_g^1$  we proceed in the following way:

We consider the Hugoniot equation for each of the two media:

$$(3.2) \quad H(\rho, p) = 0$$

with :

$$H(\rho, p) = e(\rho, p) - e(\rho_0, p_0) + \frac{1}{2} (p + p_0) \left( \frac{1}{\rho_0} - \frac{1}{\rho} \right)$$

This equation defines all the states which can be connected to the state  $S^0$  by a shock. When an analytical expression is not available or too complicated to obtain (as in the case of the liner) the curve  $H = 0$  may be computed pointwise by some numerical procedure.

The  $u$  corresponding to  $\rho, p$  which satisfy equation (3.2), may be computed by means of formula

$$(3.3) \quad (u - u_0)^2 = \left( \frac{1}{\rho_0} - \frac{1}{\rho} \right) (p - p_0)$$

from which

$$(3.4) \quad u = u_0 \pm \sqrt{\left( \frac{1}{\rho_0} - \frac{1}{\rho} \right) (p - p_0)}$$

The velocities on the two sides of a shock front satisfy the relation

$$u_{\text{left}} > u_{\text{right}}$$

independently on which side the shock faces.

Hence in (3.4) there is the sign - for the liner and the sign + for the gas.

We consider now the Hugoniot curves for the liner and the gas:

$$H_1 = 0 \quad H_g = 0$$

in the  $\rho, p$  plane. We must find a point  $P_1 = (\rho_1, p)$  on  $H_1$  and a point  $P_g = (\rho_g, p)$  on  $H_g$  such that the corresponding  $u_1$  and  $u_g$ , computed by means of (3.3) are equal:

$$(3.5) \quad u_1 = u_g$$

To solve numerically this problem we put the equations

$$\begin{aligned} H_1 = 0 \quad \text{and} \quad H_g = 0 \quad \text{in the form} \\ \rho = \rho_1(p) \quad \text{and} \quad \rho = \rho_g(p) \end{aligned}$$

the functions  $\rho_1$  and  $\rho_g$  are tabulated or given by means of a formula.

We explore the  $p$ -axis with a sufficiently small step  $\Delta p$  (starting from  $p_g^0$  since  $p_g^1 > p_g^0$ ), interpolating if necessary and compute

$$\begin{array}{l} \nearrow \rho_1 \rightarrow u_1 \\ p \searrow \rho_g \rightarrow u_g \end{array}$$

We choose that value of  $p$  corresponding to which relation (3.5) is satisfied.



4. Reflexion of the shock wave on the wall of the liner

The Riemann solution is modified when the shock front reaches the other face of the liner. The reflexion of the shock on the other face of the gas is not considered since by hypothesis the thickness of the gas is much greater than that of the liner, and may be considered semi-infinite.

We determine the state of the liner in this new phase with the aid of some simple considerations.

- a) The solution is unique.
- b) A state bounded by a straight characteristic along which  $u$  and  $c$  are constant is a constant state or a simple wave.
- c) The liner being treated as an ordinary hydrodynamic medium, we cannot have a rarefaction shock wave.

A discontinuity of this kind is immediately resolved as a rarefaction waves in the origin of the discontinuity.

When the shock wave in the liner reaches the wall, the liner is in a constant compressed state  $S_1^1$  while at this free wall there is a negligible pressure  $p_1^0 \approx 0$ . Let  $x_1$  be the abscissa of the wall and  $t_1$  the time at which the shock hits the wall. The discontinuity in this point is resolved by c) into a centered rarefaction wave connecting the state  $S_1^1$  with the state  $S_1^2$  adjacent to the wall.

The state  $S_1^2$  is bounded to the right by the tail characteristic of the rarefaction wave and by b) is a constant state. It follows that the trajectory of the wall is a straight line

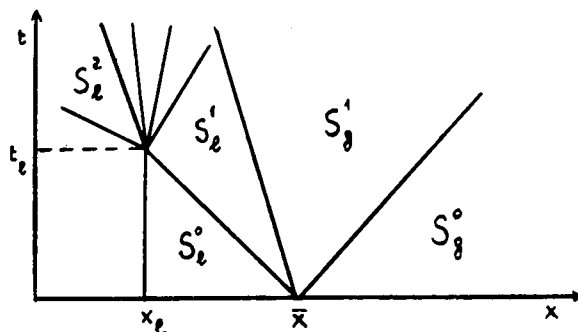


Fig. 2

The  $C^+$  characteristics of the centered wave pass through  $(x_1, t_1)$  and have direction:

$$(4.1) \quad \frac{dx}{dt} = u + c$$

$u$  and  $c$  have constant values along each characteristic.

The head characteristic has direction:

$$\frac{dx}{dt} = u_1 + c_1$$

The direction of the tail of the rarefaction wave is computed by means of the Riemann invariants.

We put: 
$$I(\rho) = \int_{\rho_0}^{\rho} \frac{c}{\rho} d\rho$$

and 
$$R^{\pm} = u \pm I$$

The quantity  $R^-$  is constant through the three regions  $S_1^1$ , rarefaction wave,  $S_1^2$ . On the tail characteristic  $C^*$  :

$$(4.3) \quad u^* - I^* = u_1 - I_1$$

and

$$(4.4) \quad I^* = I_0 = I(\rho_0) = 0$$

Hence the direction is :

$$\frac{dx}{dt} = u^* + c^*$$

and the state  $S_1^2$  is :

$$u_2 = u^*$$

$$c_2 = c^* = c_0$$

$$\rho_2 = \rho^* = \rho_0$$

$$p_2 = p_0 = 0$$

For the cases which have been treated the shock is the liner was a weak one.



The variation of  $R^+$  when passing from  $S_1^0$  to  $S_1^1$  is negligible and we have, with good approximation:

$$(4.5) \quad u_0 + I_0 = I_0 \simeq u_1 + I_1 \quad (u_0 = 0)$$

$R^-$  is invariant through the simple wave, hence:

$$u_2 - I_2 = u_1 - I_1$$

Beside:  $I_2 = I^* = I_0$

Hence:

$$(4.6) \quad u_2 - I_0 = u_1 - I_1$$

From (4.5) and (4.6) it follows:

$$(4.7) \quad u_2 \simeq 2u_1$$

To conclude:

We can determine directly:

$$S_1^1, S_g^1, x_1, t_1, S_1^2$$

the centered rarefaction wave, and in particular the direction of the head and tail characteristics of the wave. In first approximation:

$$u_2 \simeq 2u_1$$

In the computation of  $R^\pm$  the quantity:

$$I = \int^{\rho} \frac{c}{\rho} d\rho$$

has sometime been substituted with the quantity:

$$I_c = \int^{\rho} \frac{c_c}{\rho} d\rho$$

where: 
$$\rho_c = \frac{dp_c}{d\varphi}$$

i.d. in the state equation of the liner only the term  $p_c$  has been considered. In this way the integral  $I_c$  is easier to calculate.

Until  $p$  is less than  $10^6$  atm. the error due to the substitution of  $p$  with  $p_c$  is less than 5% (see Somon pp. 38).  
Hence:

$$I \approx I_c$$

### 5. Numerical solution of the problem

We introduce the lagrangean coordinate  $h$  :

$$(5.1) \quad h = \frac{1}{\rho_0} \int_{X(0,t)}^{X(h,t)} \rho(\xi,t) d\xi = x - x_1$$

where  $x = X(h,0)$

$x_1$  is the free wall of the liner

$\bar{x}$  is the interface liner-gas

Differentiating (5.1) with respect to  $h$  we obtain:

$$1 = \frac{\rho}{\rho_0} \frac{\partial X}{\partial h}$$

with (5.2) 
$$\rho = \begin{cases} \rho_l & h < \bar{h} = \bar{x} \\ \rho_g & h > \bar{h} \end{cases}$$

and (5.3) 
$$\rho_0 = \begin{cases} \rho_l^0 & h < \bar{h} \\ \rho_g^0 & h > \bar{h} \end{cases}$$

The system to be solved, in lagrangean form, is:

$$\begin{aligned}
 1 &= \frac{\rho}{\rho_0} \frac{\partial X}{\partial h} \\
 u &= \frac{\partial X}{\partial t} \\
 (5.4) \quad \frac{\partial u}{\partial t} &= - \frac{1}{\rho_0} \frac{\partial p}{\partial h} \\
 \frac{\partial e}{\partial t} &= \frac{p}{\rho} \frac{\partial \rho}{\partial t} \\
 p &= F(\rho, e)
 \end{aligned}$$

When  $h < \bar{h}$  we have the liner, hence:

$$\rho_0 = \rho_1^0 \quad \rho = \rho_1 \quad p = p_1 \quad e = e_1$$

When  $h > \bar{h}$  we have the gas, hence

$$\rho_0 = \rho_g^0 \quad \rho = \rho_g \quad p = p_g \quad e = e_g$$

In order to obtain the state equation of the liner, we consider the relations:

$$(5.5) \quad e = e_c(\rho) + c_v(T - T_0 + \frac{E_0}{c_v}) + \left(\frac{\rho_{ok}}{\rho}\right)^{1/2} \frac{\beta}{2} T^2$$

$$(5.6) \quad p = p_c(\rho) + \rho \chi(\rho) c_v (T - T_0 + \frac{E_0}{c_v}) + \rho_{ok} \left(\frac{\rho_{ok}}{\rho}\right)^{1/2} \frac{\beta}{4} T^2$$

(see § 2).

If we neglect the terms in  $T^2$  we get:

$$(5.7) \quad p = p_c(\rho) + \rho \chi(\rho) (e - e_c(\rho))$$

In the general case we proceed as follows:

From (5.5) we get:

$$(5.8) \quad \left(\frac{\rho_{ok}}{\rho}\right)^{1/2} \frac{\beta}{2} T^2 + c_v T + (e_c - e + E_o - c_v T_o) = 0$$

Since:  $e_c, E_o, c_v, \beta, \gamma, T-T_o, T_o - \frac{E_o}{c_v}$

are positive, it follows that  $e > e_c$  and the last term of (5.8) is negative. Thus we have only one positive root:

$$T = \frac{-c_v + \sqrt{c_v^2 + 2\left(\frac{\rho_{ok}}{\rho}\right)^{1/2} \beta (e - e_c + c_v T_o - E_o)}}{\left(\frac{\rho_{ok}}{\rho}\right)^{1/2} \beta}$$

We substitute this expression in (5.6) and we obtain  $p$  as function of  $e, \rho$

$$(5.9) \quad p = p(e, \rho)$$

In the same way we could express  $e$  as function of  $p, \rho$ .

The state equation of the gas is:

$$(5.10) \quad p = (\gamma - 1) e \rho \quad \gamma = 3$$

The differential system (5.4) must be solved with given initial and boundary conditions. In our case the initial conditions at time  $t = 0$  are:

$$\begin{aligned} p &= p_1^o = 0 \\ u &= u_1^o = 0 \\ \rho &= \rho_1^o \end{aligned} \quad h < \bar{h} \quad (\text{liner})$$

$$\begin{aligned} p &= p_g^o \\ u &= u_g^o \\ \rho &= \rho_g^o \end{aligned} \quad h > \bar{h} \quad (\text{gas})$$

The boundary conditions ( $t = 0$ ) are:

$$p = 0 \text{ when } h = 0 \quad p = p_g^o \text{ when } h = h_g$$



$h_g$  lagrangean coordinate of a gas particle, great enough so that we don't have to consider the shock reflexion in the gas. The second boundary condition may be substituted with:

$$u = u_g^0 \quad \text{or} \quad \rho = \rho_g^0$$

In lagrangean coordinate the path of the interface liner-gas is the vertical line:

$$h = \bar{h}$$

Thus the distinction between the two media is automatic. In eulerian coordinates this trajectory is more difficult to compute since it cuts across the net on which we apply the numerical scheme. Another advantage of the lagrangean coordinates is the possibility to use different  $\Delta h$ -steps for different media if necessary. It is quite difficult to do so in eulerian coordinates. To solve numerically the differential system (5.4) use has been made of the following scheme (see Richtmyer):

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} = - \frac{1}{\rho_0} \left( \frac{p_{j+1/2}^n - p_{j-1/2}^n}{\Delta h} + \frac{q_{j+1/2}^n - q_{j-1/2}^n}{\Delta h} \right)$$

$$\frac{X_j^{n+1} - X_j^n}{\Delta t} = u_j^{n+1}$$

$$\frac{1}{\rho_{j+1/2}^{n+1}} = \frac{1}{\rho_0} \frac{X_{j+1}^{n+1} - X_j^{n+1}}{\Delta h}$$

(5.10)

$$\frac{e_{j+1/2}^{n+1} - e_{j+1/2}^n}{\Delta t} = \frac{p_{j+1/2}^n}{(\rho_{j+1/2}^n)^2} \frac{\rho_{j+1/2}^{n+1} - \rho_{j+1/2}^n}{\Delta t}$$

$$p_{j+1/2}^{n+1} = F \left( \rho_{j+1/2}^{n+1}, e_{j+1/2}^{n+1} \right)$$

$$q_{j+1/2}^n = \begin{cases} a^2 \rho_{j+1/2}^n (u_{j+1}^n - u_j^n)^2 & \text{if } u_{j+1}^n - u_j^n < 0 \\ 0 & \text{if } u_{j+1}^n - u_j^n > 0 \end{cases}$$

with the stability condition:

$$\Delta t < \frac{\Delta X}{c}$$

The Richtmyer's pseudo-viscosity term  $q$  is introduced in order to deal with the shock discontinuities. For the problem treated  $a = 2$ .

We employ a staggered net. In the points with integer index  $j$  we compute  $X_j, u_j$  in the other points we compute  $p_{j+1/2}, \rho_{j+1/2}, e_{j+1/2}, q_{j+1/2}$ . In this scheme it is convenient that the interface liner-gas be coincident with a point  $\bar{j}$ .

With regard to the boundary condition:

$$p = p_1^0 = 0 \quad \text{at} \quad h = 0$$

we remark that we can exchange the role of the indices  $j$  and  $j \pm 1/2$  and compute  $X_{j+1/2}, u_{j+1/2}$  and  $p_j, \rho_j, e_j, q_j$ . Hence we can attribute to  $h = 0$  the index  $j = 0$  or  $j = 1/2$ .

At  $h = h_g$  (last gas particle) we can choose one of the conditions

$$u = u_g^0 \quad \rho = \rho_g^0 \quad p = p_g^0$$

## 6. Numerical results

We give here the results for a typical case. The data are given in the C.G.S. system.

$x_1 = 3$	first wall of liner
$\bar{x} = 3.8$	interface liner-gas
$x_g = 9$	second face of gaseous medium

Initial state  $S_1^0$  of liner:

$$\begin{aligned}u_0 &= 0 \\p_0 &= 10^6 \\ \rho_0 &= 8.93 \\c_i &= 3.97 \cdot 10^5\end{aligned}$$

Initial state  $S_g^0$  of gas:

$$\begin{aligned}u_0 &= -2 \cdot 10^5 \\p_0 &= 2.72 \cdot 10^{11} \\ \rho_0 &= 1.7 \\c_0 &= 7 \cdot 10^5\end{aligned}$$

The state equation is (2.13) i.d. the term  $T^2$  has been dropped.

$p_c$ ,  $e_c$ ,  $\gamma$  are given respectively by (2.5) (2.6) (2.7). In connexion with these formulae we have:

$$\begin{aligned}c_v &= 3.926 \cdot 10^6 \\T_0 &= 300 \text{ } ^\circ\text{K} \\E_0 &= 7.87 \cdot 10^8 \\ \rho_{0K} &= 9.024\end{aligned}$$

$$\begin{aligned}\beta &= 2 \\ a_1 &= 4.0283 \cdot 10^{12} \\ a_2 &= -3.0889 \cdot 10^{13} \\ a_3 &= 7.1612 \cdot 10^{13} \\ a_4 &= -7.7928 \cdot 10^{13} \\ a_5 &= 4.0096 \cdot 10^{13} \\ a_6 &= -6.9194 \cdot 10^{12} \\ a_7 &= 0\end{aligned}$$

The direct solution is first computed according to paragraphs 3 and 4. The results are summed up in Tables 1 and 2.

As already said the comparison between the direct solution and the solution computed by means of a finite difference scheme is carried out only for  $t < \bar{t}$  time at which the rarefaction front reaches the interface liner-gas. When using a finite difference scheme with a pseudo-viscosity term the shock discontinuities are not represented by sharp discontinuities but rather as zones of rapid variation of the hydrodynamic quantities. For this reason and due also to the effects of round off and approximation errors the shock position can be given only with a certain approximation. The same is true with regard to the head and tail characteristics of the rarefaction wave.

The shock trajectory in liner and gas, the interface trajectory, the head and tail characteristics of the rarefaction wave obtained by means of the direct method (D.M.) and the finite difference scheme (F.D.) are given in Tables 3, ..., 7. Subsequently the results of the finite difference scheme at various times are presented.



TABLE 1

States of liner and gas

	$s_1^0$	$s_1^1$	$s_1^2$	$s_g^0$	$s_g^1$
u	0	$-9.025 \cdot 10^4$	$-1.805 \cdot 10^4$	$-2 \cdot 10^5$	$-9.025 \cdot 10^4$
p	$10^6$	$4.23 \cdot 10^{11}$	$10^6$	$2.72 \cdot 10^{11}$	$4.23 \cdot 10^{11}$
$\rho$	8.93	1.0787.10	8.93	1.7	1.966
c	$3.97 \cdot 10^5$	$5.37 \cdot 10^5$	$3.97 \cdot 10^5$	$6.93 \cdot 10^5$	$8.03 \cdot 10^5$

TABLE 2

Shock fronts and rarefaction wave

Shock speed in liner :	$U_1 = -5.244 \cdot 10^5$
" " " gas :	$U_g = -6.112 \cdot 10^5$
Trajectory of shock front in liner :	$X = \bar{x} + U_1 t$
" " " " " gas :	$X = \bar{x} + U_g t$
" " liner-gas interface :	$X = \bar{x} + u_1 t$
	$(u_1 = u_1^1 = u_g^1)$
Origin of rarefaction wave : $(x_1, t_1)$	$x_1 = 3 \quad t_1 = 1.526 \cdot 10^6$
Head charact. of rarefaction wave	$X = x_1 + (t-t_1) (u_1^1 + c_1^1)$
Tail " " " "	$X = x_1 + (t-t_1) (u_1^2 + c_1^2)$
Intersection of head charact. of rarefaction wave and interface trajectory:	
	$X = 3.551 \quad \bar{t} = 2.759 \cdot 10^{-6}$

TABLE 3

Shock trajectory in liner

t	X(D.M.)	X(F.D.)
$1.424 \cdot 10^{-7}$	3.725	3.70
2.673 "	3.660	3.66
3.891 "	3.596	3.61
5.093 "	3.533	3.54
6.271 "	3.471	3.48
7.434 "	3.410	3.42
8.590 "	3.35	3.37
9.736 "	3.290	3.30
$1.088 \cdot 10^{-6}$	3.229	3.24
1.203 "	3.169	3.17
1.318 "	3.109	3.11

TABLE 4

Shock trajectory in gas

t	X(D.M.)	X(F.D.)
$2.673 \cdot 10^{-7}$	3.963	3.94
5.093 "	4.111	4.15
7.434 "	4.254	4.21
9.736 "	4.395	4.36
$1.203 \cdot 10^{-6}$	4.535	4.50
1.433 "	4.676	4.65
1.663 "	4.816	4.85
2.125 "	5.099	5.11
2.356 "	5.240	5.24
2.592 "	5.380	5.37

TABLE 5

Interface trajectory

t	X(D.M.)	X(F.D.)
$2.673 \cdot 10^{-7}$	3.776	3.781
5.093 "	3.754	3.758
7.434 "	3.733	3.737
9.736 "	3.712	3.716
$1.203 \cdot 10^{-6}$	3.691	3.695
1.433 "	3.670	3.674
1.663 "	3.650	3.654
1.894 "	3.629	3.633
2.125 "	3.608	3.612
2.356 "	3.587	3.591
2.592 "	3.566	3.569



.....  
TABLE 6

.....  
Head characteristic of rarefaction wave

t	X(D.M.)	X(F.D.)
$1.779 \cdot 10^{-6}$	3.113	3.14
1.894 "	3.164	3.18
2.009 "	3.216	3.24
2.125 "	3.267	3.28
2.240 "	3.319	3.34
2.356 "	3.371	3.39
2.473 "	3.423	3.44
2.592 "	3.476	3.49
2.712 "	3.53	3.55

.....  
TABLE 7

.....  
Tail characteristic of rarefaction wave

t	X(D.M.)	X(F.D.)
$1.779 \cdot 10^{-6}$	3.055	2.99
1.894 "	3.080	3.02
2.009 "	3.105	3.05
2.125 "	3.130	3.07
2.240 "	3.155	3.10
2.356 "	3.180	3.13
2.473 "	3.205	3.16
2.592 "	3.231	3.18
2.712 "	3.257	3.20

Acknowledgments

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Bibliography

- (1) Courant - Friedrichs : Supersonic Flow and Shock Waves  
Interscience Publishers 1948
- (2) J.P. Somon : L'Equation d'Etat des Solides  
Rapport Interne L.G.I. 64/3 (1964)
- (3) R.D. Richtmyer : Difference Method for Initial Value Problems  
Interscience Publishers 1957

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EURATOM - C. C. R. ISFRA - CETIS

POZZONI - CISANO BERG.



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1.4586E 01	10.8921E 00	-8.5251E 04	1.9622E 00	4.2000E 11	5.6500E 05
1.4732E 01	11.0380E 00	-8.5146E 04	1.9622E 00	4.2000E 11	5.6000E 05
1.4878E 01	11.1839E 00	-8.5041E 04	1.9622E 00	4.2000E 11	5.5500E 05
1.5024E 01	11.3298E 00	-8.4936E 04	1.9622E 00	4.2000E 11	5.5000E 05
1.5170E 01	11.4757E 00	-8.4831E 04	1.9622E 00	4.2000E 11	5.4500E 05
1.5316E 01	11.6216E 00	-8.4726E 04	1.9622E 00	4.2000E 11	5.4000E 05
1.5462E 01	11.7675E 00	-8.4621E 04	1.9622E 00	4.2000E 11	5.3500E 05
1.5608E 01	11.9134E 00	-8.4516E 04	1.9622E 00	4.2000E 11	5.3000E 05
1.5754E 01	12.0593E 00	-8.4411E 04	1.9622E 00	4.2000E 11	5.2500E 05
1.5899E 01	12.2052E 00	-8.4306E 04	1.9622E 00	4.2000E 11	5.2000E 05

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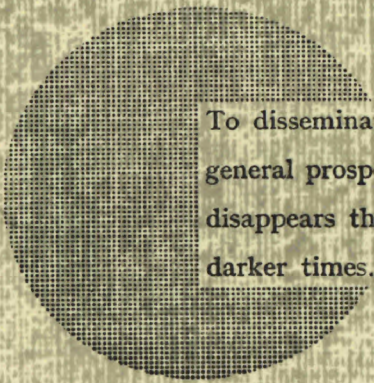
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Alfred Nobel



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