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EUROPEAN ATOMIC ENERGY COMMUNITY - EURATOM

STUDIES OF ^{240}Pu RESONANCE ABSORPTION IN HETEROGENEOUS SYSTEMS

by

F. BEONIO-BROCCHIERI and E. DIANA

1966

Joint Nuclear Research Center
Ispra Establishment - Italy

Reactor Physics Department
Reactor Theory and Analysis

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CONTENTS

SUMMARY

Usual methods for determining the effective resonance integral of the most common fertile materials introduce some important approximations. In order to investigate how the large practical width of the ^{240}Pu resonance at 1.05 eV may invalidate these approximations a basic study has been performed by comparing, for different heterogeneous systems, the results obtained with sophisticated methods to the results given by the more usual ones (Nordheim, Chernick and Vernon, Cohen and Goldstein). The same calculation methods have been tested against the experimental values measured by Nichols at Hanford.

Introduction

As is well known, the influence of the Pu-240 build-up at large irradiations on the reactivity curve of uranium fuels is large and this requires an accurate calculation of its absorption. The absorption cross section of Pu-240 presents eleven resolved resonances in the energy range below 120 eV: the most important resonance has its maximum at 1.05 eV and is responsible of more than 95 % of the total resonance absorption at infinite dilution.

If one extends directly to the calculation of the Pu-240 resonance absorption the usual methods employed for the resonance integral of the most common fertile materials (U-238 and Th-232), the principal approximations introduced are the following:

- neglect of the detailed spatial distributions in the different regions (only mean fluxes in fuel and moderator)
- neglect of the energy depression in the moderator (narrow resonance approximation)
- neglect of the moderator thermal motion.

This last approximation, in particular, may introduce important errors in the calculation of the Pu-240 resonance integral for systems of rather high moderator temperature, as we have shown in a previous paper /1/. In the following a basic study of the effect of the first two approximations will be performed by comparing the results obtained with sophisticated methods to the results given by the more usual ones (Nordheim, Chernick and Vernon, Cohen and Goldstein).

The same calculation methods will be tested against the experimental values measured by Nichols at Hanford.

Manuscript received on August 16, 1966.

Standard methods of resonance absorption calculation

In this analysis of the methods employed for the resonance integral calculations we represent the heterogeneous systems by an unit cell of volume V consisting of two regions: one occupied by the absorbing material (fuel) and the other one by the moderator.

The transport equation for the collision density per unit lethargy $F(\underline{r},u)$, in the approximation of **isotropic** scattering in the laboratory system, is

$$(1)\qquad F(\underline{r},u) = \Sigma_t(\underline{r},u)\int_V d\underline{r}'\,\frac{\exp[-\tau(\underline{r},\underline{r}';u)]}{4\pi|\underline{r}-\underline{r}'|^2}\int du'\,\frac{\Sigma_s(\underline{r}';u'\to u)}{\Sigma_t(\underline{r}';u')}F(\underline{r}',u')$$

where $\tau(\underline{r},\underline{r}',u)$ is the optical distance.

In order to make this equation more easy to handle, the authors, who treated the problem of resonance absorption, eliminated the spatial dependence considering only the mean collision densities in the fuel and moderator. The equations for these two quantities are obtained by integrating equation (1) separately over the two regions

$$(2)\qquad \begin{cases} F_f(u) = [1-P_f(u)]\int du'\,\dfrac{\Sigma_s^f(u'\to u)}{\Sigma_t^f(u')}F_f(u') + \dfrac{V_m}{V_f}P_m(u)\int du'\,\dfrac{\Sigma_s^m(u'\to u)}{\Sigma_t^m(u')}F_m(u') \\ F_m(u) = [1-P_m(u)]\int du'\,\dfrac{\Sigma_s^m(u'\to u)}{\Sigma_t^m(u')}F_m(u') + \dfrac{V_f}{V_m}P_f(u)\int du'\,\dfrac{\Sigma_s^f(u'\to u)}{\Sigma_t^f(u')}F_f(u') \end{cases}$$

where

$$P_f(u) = \frac{\Sigma_t^m(u)\int du'\,\dfrac{\Sigma_s^f(u'\to u)}{\Sigma_t^f(u')}\int_{V_m}d\underline{r}\int_{V_f}d\underline{r}'\,\dfrac{\exp[-\tau(\underline{r},\underline{r}';u')]}{4\pi|\underline{r}-\underline{r}'|^2}F_f(\underline{r}';u')}{V_f\int du'\,\dfrac{\Sigma_s^f(u'\to u)}{\Sigma_t^f(u')}F_f(u')}$$

$$P_m(u) = \frac{\Sigma_t^f(u)\int du'\,\dfrac{\Sigma_s^m(u'\to u)}{\Sigma_t^m(u')}\int_{V_f}d\underline{r}\int_{V_m}d\underline{r}'\,\dfrac{\exp[-\tau(\underline{r},\underline{r}';u')]}{4\pi|\underline{r}-\underline{r}'|^2}F_m(\underline{r}';u')}{V_m\int du'\,\dfrac{\Sigma_s^m(u'\to u)}{\Sigma_t^m(u')}F_m(u')}$$

At this stage spatial elimination is only formal, because the determination of P_f and P_m requires the solution of the original equation; the elimination becomes actual if one makes the usual hypothesis of flat flux. In this case there is another simplification because P_m and P_f are connected by the reciprocity relation

$$P_m V_m \Sigma_t^m = P_f V_f \Sigma_t^f$$

These mean escape probabilities must be calculated numerically; for some simple geometries it is possible to use the values tabulated in the book of Case et al. /2/.

Other assumptions usually made concern the dependence from lethargy:

i) resonance energies belong to the epithermal range, i.e. the scattering kernel is the simple slowing down kernel

$$\int du' \frac{\Sigma_s^{(i)}(u' \to u)}{\Sigma_t(u')} F(u') = \int_{u-\Delta_i}^{u} du' \frac{\exp[-(u-u')]}{\alpha_i} \frac{\Sigma_s^{(i)}(u')}{\Sigma_t(u')} F(u')$$

per collision with nuclide j

ii) resonances are well separated and may be treated as isolated

$$F(u) = F_{AS} \qquad \text{for} \quad u \ll u_{res}$$

iii) scattering collisions with the moderator produce large mean lethargy changes as compared to the practical width of the resonance

$$\int du' \frac{\Sigma_s^m(u' \to u)}{\Sigma_t^m(u')} F_m(u') \simeq F_{AS}. \qquad \text{(Narrow Resonance approximation)}$$

These assumptions allow to reduce the system (1) to a single integral equation for the mean collision density in the fuel

$$(3) \qquad F_f(u) = [1 - P_f(u)] \left\{ \sum_j \int_{u-\Delta_j}^{u} du' \frac{\exp[-(u-u')]}{\alpha_i} \frac{\Sigma_s^{(i)}(u')}{\Sigma_t^f(u')} F_f(u') \right\} + P_f(u) F_{AS}.$$

(where, now, index j refers only to nuclides in the fuel).
The methods usually employed in the resonance integral calculations for U-238 and Th-232 are based on equation (3) introducing sometimes other additional approximations in the evaluation both of the slowing-down operator and of the mean escape probability from the fuel $P_f(u)$.

A. Intermediate Resonance method

If the mean lethargy gain for a collision with nuclide j is much greater than the practical width of the resonance (Narrow-Resonance approximation) it is reasonable to put

$$\int_{u-\Delta_j}^{u} du' \frac{\exp[-(u-u')]}{\alpha_j} \Sigma_s^{(j)}(u') \Phi(u') = \Sigma_p^{(j)} \Phi_{As.}$$

In the other extreme case when the mean lethargy gain for a collision with nuclide j is much smaller than the practical width of the resonance (Infinite-Mass approximation) it is reasonable to put

$$\int_{u-\Delta_j}^{u} du' \frac{\exp[-(u-u')]}{\alpha_j} \Sigma_s^{(j)}(u') \Phi(u') = \Sigma_s^{(j)}(u) \Phi(u) .$$

In all other cases which, as it is physically evident, are intermediate between these two we may put

$$(4) \qquad \int_{u-\Delta_j}^{u} du' \frac{\exp[-(u-u')]}{\alpha_j} \Sigma_s^{(j)}(u') \Phi(u') = \lambda_j \Sigma_p^{(j)} \Phi_{As.} + (1-\lambda_j) \Sigma_s^{(j)}(u) \Phi(u) \qquad 0 \leq \lambda_j \leq 1$$

as originally suggested by R. Goldstein and E.R. Cohen /3/.
This Intermediate-Resonance approximation includes the two preceeding ones, which correspond, as may be easily seen, to the extreme values of the parameter λ_j . The general expression for the flux in the fuel is obtained combining equations (3) and (4)

$$(5) \qquad \Phi(\lambda, u) = \frac{[1-P(u)]\cdot[\sum_j \lambda_j \Sigma_p^{(j)}] + P(u) Z_e(u)}{\Sigma_t(u) - [1-P(u)]\cdot[\sum_j (1-\lambda_j) \Sigma_s^{(j)}(u)]} \Phi_{As.}$$

(we have dropped the index f because now we consider only quantities refering to fuel).

The choice for the values to assign to the parameters λ_j may be based on the following considerations. Equation (3) is an integral equation of the Volterra type, therefore the sequence of functions $\{ \Phi_{(n)}(\lambda, u) \}$ defined by the recurrence relations

(6) $$\Phi_{(n)}(\lambda,u) = [1 - P(u)] \cdot \left[\sum_j \int_{u-\Delta_j}^{u} du' \frac{\exp[-(u-u')]}{\alpha_j} \frac{\Sigma_s^{(j)}(u')}{\Sigma_t(u')} \Phi_{(n-1)}(\lambda,u') \right] + P(u)\, \Phi_{As}$$

$$(n = 2, 3, 4 \ldots)$$

$$\Phi_{(1)}(\lambda,u) = \Phi(\lambda,u)$$

converges to the solution $\Phi(u)$ of equation (3) and the sequence $\{I^{\lambda}_{(n)}\}$ of resonance integrals defined by

$$I^{\lambda}_{(n)} = \int_{res} du'\, \sigma_a(u') \frac{\Phi_{(n)}(\lambda,u)}{\Phi_{As.}}$$

converges to the correct resonance integral

$$I = \int_{res} du'\, \sigma_a(u') \frac{\Phi(u')}{\Phi_{As.}}$$

whatever are the parameters λ_j .

Goldstein and Cohen propose that the set of values to be assigned to parameters λ_j be such to satisfy the condition that the resonance integrals calculated in first and second approximation be equal.

The Intermediate-Resonance approximation is really interesting when it is possible to arrive at analytical expressions for the resonance integrals in first and second approximation. In Appendix 1 we show that this is possible using the rational approximation for the escape probability

$$P(u) = 1 \Big/ \left[1 + \bar{l} \cdot \Sigma_t(u) \right] \qquad (\bar{l} \text{ mean chord length})$$

and neglecting the Doppler broadening of the resonance and the scattering interference.

B. Nordheim's Method

In the method proposed by Nordheim /4/ the integral equation (3) is solved by direct numerical integration; in this case it is not necessary to introduce any approximation for escape probability, Doppler broadening and scattering interference.

Pu-240 Resonance Absorption

A. Comparison of the different calculation methods

In the treatment of Pu-240 resonance absorption and, more specifically, in the calculation of the resonance integral at 1.05 eV, not negligible errors may be introduced by the employ of the calculation methods above described. In a preceeding work /1/ it was shown that, in systems with high moderator temperatures, the approximation (i) (concerning the scattering kernel) may lead to underestimate the resonance integral by some ten per cent. It is our intention, now, to investigate how the large practical width of this resonance may invalidate the other approximations introduced in the derivation of equation (3); namely, the approximation of flat flux in the calculation of $P_f(u)$ and $P_m(u)$ and the approximation of Narrow Resonance for scattering in the moderator.

We made a series of calculations for heterogeneous systems, heavy water moderated, considering unit cells in which the fuel (rods or tubes) was of two different compositions (see table 1). The volume ratio of moderator to fuel was kept constant to a value of twenty-four. The temperature was 300°K.

Table 1

	Atom density (atoms/barn·cm)	
	Composition A	Composition B
U-235	$1.19 \cdot 10^{-4}$	$1.19 \cdot 10^{-4}$
U-238	$4.78 \cdot 10^{-2}$	$4.78 \cdot 10^{-2}$
Pu-239	$1.15 \cdot 10^{-4}$	$5.74 \cdot 10^{-4}$
Pu-240	$2.87 \cdot 10^{-5}$	$3.61 \cdot 10^{-5}$

As reference values for the resonance integral to which to compare the results obtained with the standard methods we took the values given by the code WDSN /5/. This code solves the neutron transport equation by using a multigroup scheme and the S_n approximation in cylindrical geometry.

For the present calculations the S_4 approximation was adopted in a thirty-group structure covering the energy range from zero to 2.2 eV.

All the necessary cross sections were obtained with the code GATHER-II /6/; the scattering kernels for moderator atoms were calculated using the free gas model, which seems to be adequate when one is interested in a small energy range around 1 eV.
In the calculation of the resonance integral by the standard methods the code ARES-II /7/ was chosen as representative of the Intermediate-Resonance approximation and the code ZUT /8/ for the Nordheim's method.

Table 2 and 3 give the values of the Pu-240 resonance integral at 1.05 eV as obtained by the different codes for the two compositions considered. The tables give also the geometrical characteristics of the systems.

Table 2 - Resonance integrals for composition A (barn)

Cylindrical fuel r = fuel radius [cm]		WDSN	ZUT	ARES-II
r = 0.9		2905	3137	2983
r = 1.5		2200	2424	2307
r = 2.		1860	2088	2018
r = 2.25		1722	1963	1909
r = 2.5		1602	1859	1816
Annular fuel* r_i = inner radius r_o = outer radius [cm]		WDSN	ZUT	ARES-II
r_i = 0.5	r_o = 2.06	1963	2159	2099
r_i = 3	r_o = 3.6	3436	3562	3429
r_i = 1	r_o = 2.69	1836	2051	2008
r_i = 4	r_o = 4.72	3209	3385	3236

* innerly and outerly moderated

Table 3 - Resonance integrals for composition B (barn)

Cylindrical fuel r = fuel radius [cm]	WDSN	ZUT	ARES-II
r = 2	1025	1170	1172
r = 2.25	946	1098	1097
r = 2.5	879	1038	1033

Annular fuel* r_i = inner radius r_o = outer radius [cm]		WDSN	ZUT	ARES-II
r_i = 0.5	r_o = 2.06	1079	1220	1227
r_i = 3	r_o = 3.6	1938	2071	2016
r_i = 1	r_o = 2.69	1001	1148	1165
r_i = 4	r_o = 4.72	1791	1940	1903

* innerly and outerly moderated

These same results are plotted in fig. 1 and 2 as a function of the fuel mean chord length (**).

The following conclusions may be drawn from the results above reported:

a) The flat flux approximation both in moderator and in fuel and the Narrow Resonance approximation in the moderator, which are used in the standard methods, lead to an overestimation of the resonance integral when compared with the reference values calculated by WDSN. This disagreement is an increasing function both of the fuel mean chord length and of the absorber density.
In the range of fuel mean chord lengths and of absorber densities considered in this work the overestimation varies from 3 % to 17 %.
To illustrate the error introduced by the flat flux approximation in the moderator, the spatial distribution of the slowing down

(**) When this work was practically terminated, we had the possibility to employ the code THERMOS /9/ with the same group structure and cross-sections already used in the WDSN calculations. The agreement between the results obtained with these two codes is good as may be seen in fig. 1.

sources and fluxes in the moderator for the energy group from 1.05 eV to 1.07 eV are reported in figures 3 and 4 for two typical cases. The curves, deduced from the WDSN results, refer to a cylindrical fuel of composition A and B and a radius of 2.25 cm.

b) The resonance integral values calculated by ARES-II lie between those calculated by WDSN and ZUT. This is due mainly to the fact that this code uses for the escape probability the rational approximation which underestimates the passage of neutrons from moderator to fuel (see fig. 5). In the limit of great values for $\ell \cdot \Sigma_t$ the rational approximation tends to be correct and the ARES-II and ZUT results tend to coincide.

c) The WDSN results show that it is not possible to describe with sufficient accuracy the Pu-240 resonance integral dependence from the absorber density and the fuel geometry in terms of the single parameter $\sqrt{\frac{S}{M}}$ (see fig. 6).

To conclude we may give an indication of the machine time requested by the different codes on our computer IBM-7090: for ARES-II 1.5 seconds, for ZUT 34 seconds and for WDSN 10 minutes.

C. Comparison with experimental results

The experimental measurements of Pu-240 resonance integral at our disposal were only those made by Nichols at Hanford /10/. The measures were performed with cylindrical rods of a cadmium-covered plutonium-aluminium alloy. The geometrical characteristics of the rods (length of 31.0 in. and diameter of 0.65 in.) and the total mass of aluminium (430 gm) were kept constant, while the total mass of plutonium changed together with its isotopic composition.

To be more precise, two different isotopic compositions (see table 4) were used; the reason of this was that the resonance integral values were deduced from the reactivity differences, measured in the PCTR where two rods which differed only for their Pu-240 content were put successively in the testing region. The factor connecting the resonance integral changes with the reactivity changes was determined by measurements on rods at nearly infinite dilution. All the resonance integral

values given in Nichols' work were normalized to a value at infinite dilution of 8607 barns, including the 1/v contribution above the cadmium cut-off.

Table 4 - Isotopic compositions of Plutonium (atom percentages)

	Pu-239	Pu-240	Pu-241	Pu-242
Lx plutonium	93.45	6.01	0.48	0.02
Hx plutonium	80.73	15.94	2.83	0.50

In the comparison between experimental results and calculations a certain number of the systems considered by Nichols (see table 5) were selected.

Table 5

Set	Rod	Mass Pu (gm)		N-240 (nuclei/barn cm)
1	A	0.0521	Lx	0.050×10^{-6}
	B	0.0568	Hx	0.142×10^{-6}
5	A	3.510	Lx	3.35×10^{-6}
	B	3.826	Hx	9.59×10^{-6}
8	A	82.556	Lx	78.0×10^{-6}
	B	90.000	Hx	220.0×10^{-6}

The result of this comparison is shown in fig. 7. All the calculated and experimental values were normalized to the same value of 8607 barns at infinite dilution. The Pu-240 absorption in the region of unresolved resonances was calculated in all cases using the code GAM-II /11/. In the case of WDSN calculations in which the upper energy limit was 2.2 eV, the absorption in the other resolved resonances was taken to be the same as that calculated by ZUT. As far as the 1/v absorption above the cadmium cut-off is concerned, the same value was taken in all cases for the absorption cross section at 2.200 m/sec (σ_{2200}= 286 ba

As conclusion of this comparison we may say that, due to the great experimental errors, these measurements do not allow a choice to be made among the different calculation methods. It seems to us that more accurate measures would be welcome.

Apperdix

Calculation of the parameters λ_j in the intermediate resonance method

When in equation (5) we use the rational approximation and neglect the Doppler broadening and the scattering interference, we obtain

$$(A\text{-}1) \qquad \Phi_{(1)}(\lambda,u) = \frac{\sigma_m \Phi_{As.}}{\sigma_a(u) + \lambda_o \sigma_{r_o}(u) + \sigma_m}$$

where $\sigma_m = \sigma_{ex} + \lambda_o \sigma_p + \sum_\kappa \lambda_\kappa \sigma_s^{(\kappa)}$

Therefore the resonance integral in first approximation is

$$(A\text{-}2) \qquad I^{\lambda}_{(1)} = \frac{\Gamma}{2E_o} \int_{-\infty}^{+\infty} dx \; \sigma_a(x) \frac{\Phi_{(1)}(\lambda,x)}{\Phi_{As.}} = \frac{I_\infty}{\beta_\lambda}$$

$$x = \frac{2}{\Gamma}(E - E_o) \quad ; \quad I_\infty = \frac{\pi}{2} \sigma_o \frac{\Gamma_\gamma}{E_o} \quad ; \quad \beta^2_\lambda = 1 + \frac{\sigma_o}{\sigma_m} \frac{\Gamma_\gamma + \lambda_o \Gamma_n}{\Gamma}$$

Using the general relation (6) the flux in second approximation is

$$(A\text{-}3) \qquad \Phi_{(2)}(\lambda,u) = \frac{\Phi_{As.}}{\sigma_{ex} + \sigma_t(u)} \left[\sigma_{ex} + \sum_j \left(\int_{u-\Delta_j}^{u} du \; \frac{\exp[-(u-u')]}{\alpha_j} \; \frac{\sigma_o^{(j)}(u') \, \sigma_m}{\sigma_a(u') + \lambda_o \sigma_{r_o}(u') + \sigma_m} \right) \right] .$$

Putting as a reasonable approximation

$$\int_{u-\Delta_j}^{u} du' \frac{\exp[-(u-u')]}{\alpha_j} \Omega(u') = \frac{\exp[-(u-u_o)]}{\delta_j} \int_{x}^{\left(\frac{x}{1-\alpha_j}+\delta_j\right)} dx' \Omega(x')$$

$$\delta_j = \frac{1-\alpha_j}{\alpha_j} \frac{\Gamma}{2E_o}$$

one obtains

$$(A\text{-}3') \qquad \Phi_{(2)}(\lambda,x) = \frac{\Phi_{As.}}{\sigma_{ex}+\sigma_e(x)} \left\{ (\sigma_{ex}+\sigma_p+\sum_\kappa \sigma_s^{(\kappa)}) + \right.$$

$$\left. + \frac{\Gamma}{2E_o} \exp[-(u-u_o)] \cdot \sum_j \left[\frac{1-\alpha_j}{\alpha_j} \frac{\xi_j^2-\beta_\lambda^2}{\beta_\lambda} \sigma_s^{(j)} \left(\operatorname{arctg}\left(\frac{x}{\beta_\lambda(1-\alpha_j)} + \frac{\delta_j}{\beta_\lambda} \right) - \operatorname{arctg} \frac{x}{\beta_\lambda} \right) \right] \right\}$$

where $\xi_o^2 = 1 + \frac{\sigma_m \Gamma_n}{\sigma_p^\circ \Gamma}$; $\xi_{j\neq o}^2 = 1$

Using the identity

$$\int_{-\infty}^{+\infty} dy \frac{\operatorname{arctg}(y+Q)}{P^2+y^2} = \frac{\pi}{P} \operatorname{arctg} \frac{Q}{1+P} \qquad [P \neq o]$$

and putting $\exp\left[-(u-u_o)\right] = 1$ the resonance integral in second approximation is given by

$$(A\text{-}4) \qquad I_{(2)}^{\lambda} = \frac{\Gamma}{2E_o} \int_{-\infty}^{+\infty} dx \; \sigma_a(x) \frac{\Phi_2(\lambda,x)}{\Phi_{As.}} =$$

$$= \frac{I_\infty}{\beta_1} \left\{ 1 + \sum_j \left[\frac{\Gamma(1-\alpha_j)}{2E_o\alpha_j} \frac{\xi_j^2-\beta_\lambda^2}{\beta_\lambda} \frac{\sigma_s^{(j)}}{\sigma_{ex}+\sigma_p^\circ+\sum_\kappa \sigma_s^{(\kappa)}} \operatorname{arctg} \frac{2E_o\alpha_j}{\Gamma[\beta_1+(1-\alpha_j)\beta_\lambda]} \right] \right\}$$

where

$$\beta_1^2 = 1 + \frac{\sigma_o}{\sigma_{ex}+\sigma_p^\circ+\sum_\kappa \sigma_s^{(\kappa)}}$$

By equating the resonance integrals in first and second approximation one obtains as shown by Hill and Schaefer /12/ the following set of coupled equations

$$(A\text{-}5) \qquad \lambda_j = 1 - \frac{\operatorname{arctg} x_j}{y_j}$$

where $x_j = \frac{2E_o\alpha_j}{\Gamma[\beta_1+(1-\alpha_j)\beta_\lambda]}$ and $y_j = \frac{2E_o\alpha_j}{\Gamma(1-\alpha_j)(\beta_1+\beta_\lambda)}$

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Symbols

$\Phi(\underline{r}, u)$	Neutron flux per unit lethargy
$F(\underline{r}, u)$	Neutron collision density per unit lethargy
Σ_t	Total macroscopic cross section
Σ_s	Scattering macroscopic cross section
Σ_a	Absorption macroscopic cross section
Σ_p , Σ_{rs} , Σ_i . . .	Potential, resonance and interference macroscopic scattering cross sections
$N^{(j)}$	Numer of atoms for isotope (j) per unit volume
$\sigma = \Sigma/N$ . . .	Microscopic cross section
V_m , V_f	Moderator and fuel volume
S	Fuel effective surface
$\bar{l} = 4V_f/S$ . . .	Fuel mean chord length

$A^{(j)}$ . . . . . Atomic mass for nuclide (j)

$\alpha^{(j)} = 4A^{(j)}/(1+A^{(j)})^2$

$\Delta_j = \ln\left[1/(1-\alpha^{(j)})\right]$

E_o . . . . . Resonance energy

Γ . . . . . Total level width of the resonance

Γ_γ , Γ_n . . . . Radiative capture and neutron widths

σ_o . . . . . Peak value of the total resonance

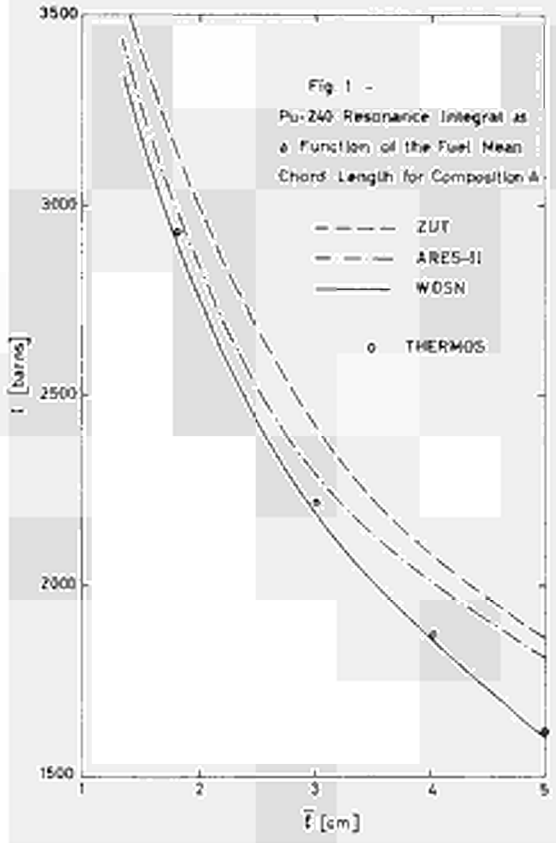


Fig. 1 - Pu-240 Resonance Integral as a Function of the Fuel Mean Chord Length for Composition A

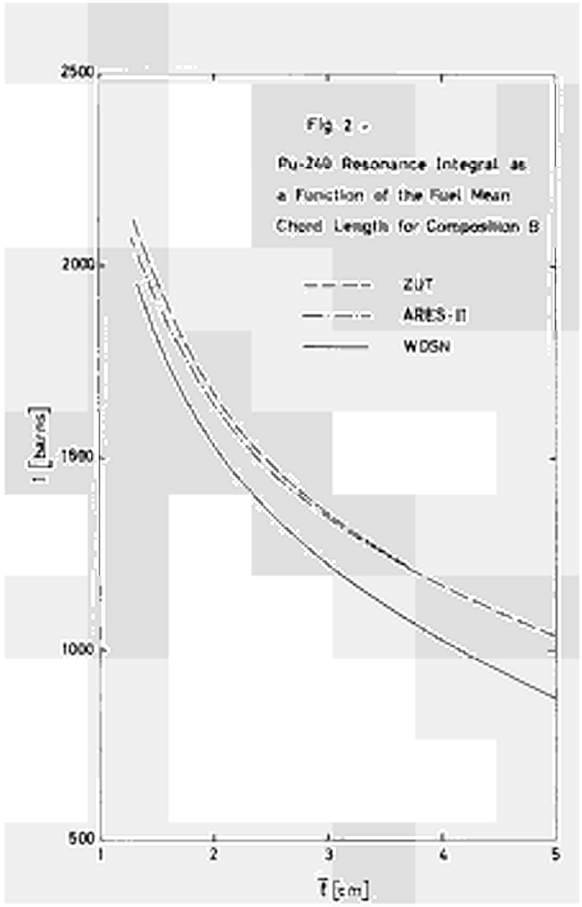


Fig. 2 -
Pu-240 Resonance Integral as a Function of the Fuel Mean Chord Length for Composition B

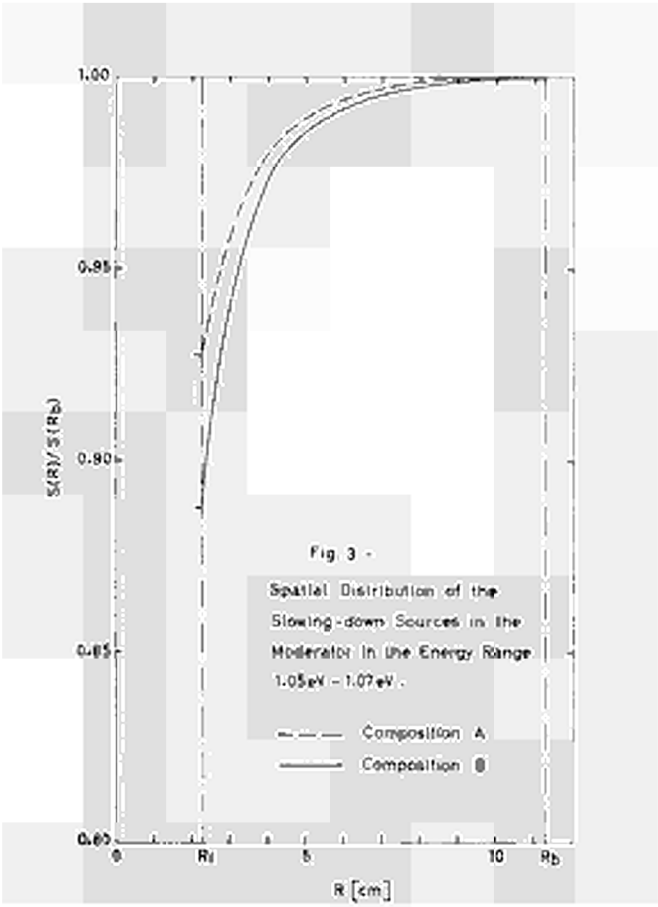


Fig. 3 -
Spatial Distribution of the Slowing-down Sources in the Moderator in the Energy Range 1.05eV – 1.07eV.

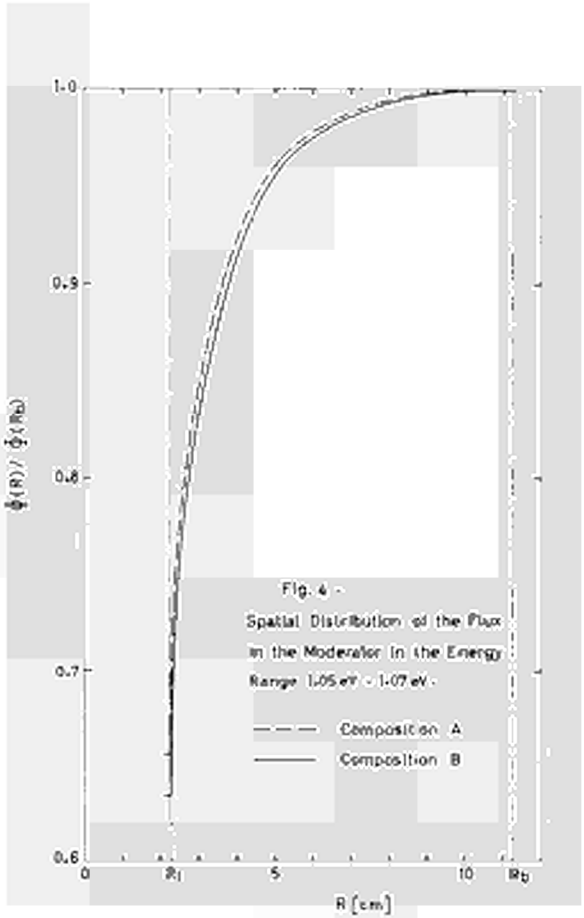


Fig. 4 -
Spatial Distribution of the Flux in the Moderator in the Energy Range 1.05 eV - 1.07 eV -

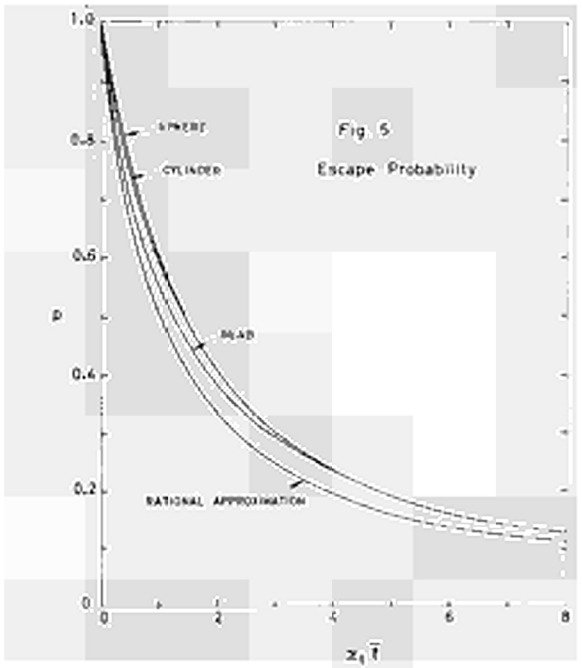
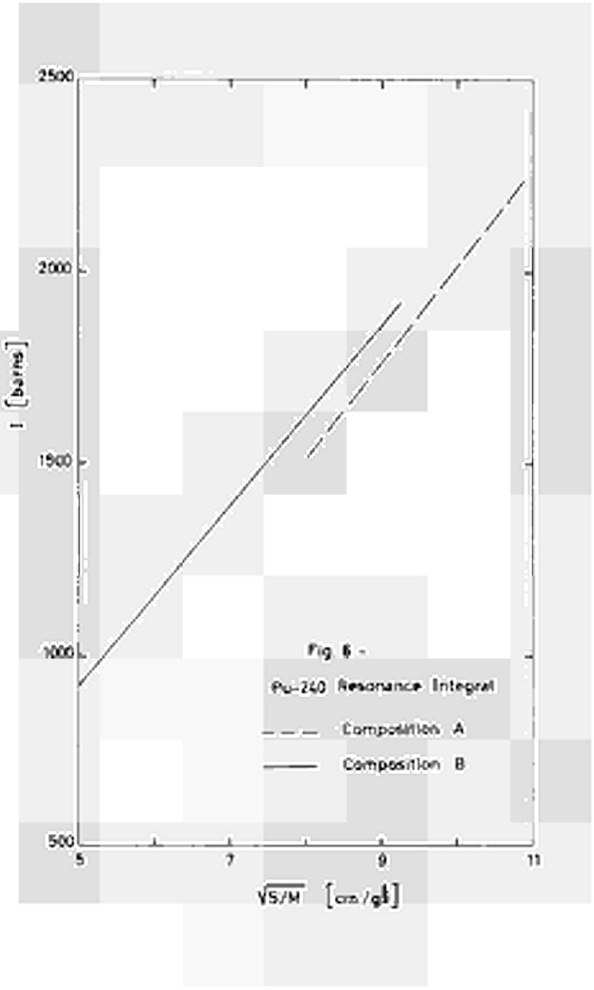
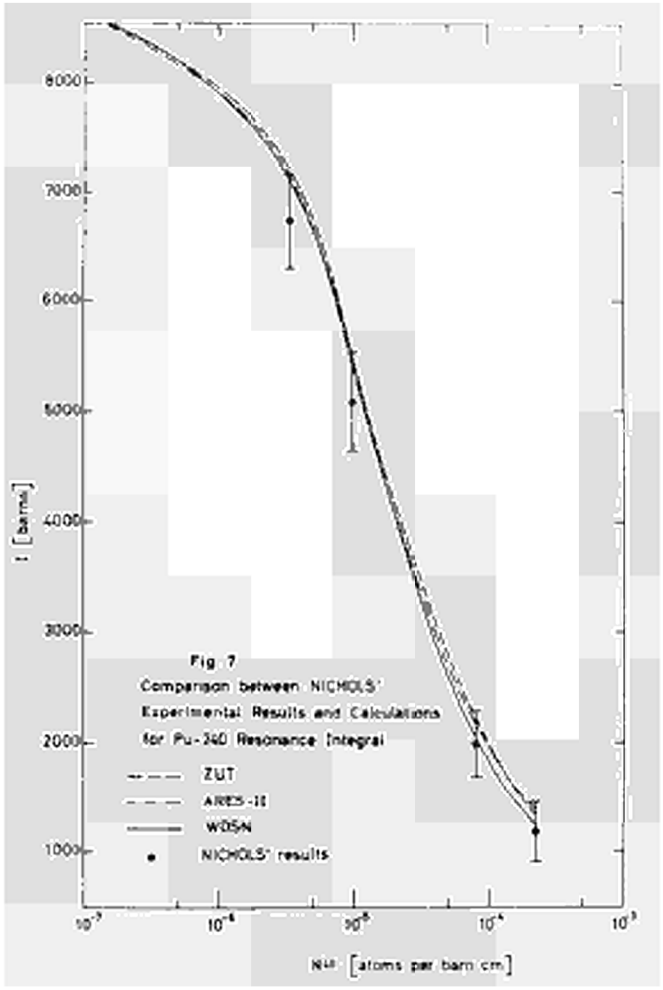
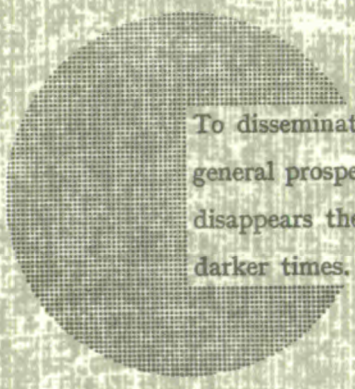


Fig. 5
Escape Probability


2500
2000
1500
1000
500
I [barns]
5
7
9
11
√S/M [cm/g½]
Fig. 6 -
Pu-240 Resonance Integral
Composition A
Composition B


8000
7000
6000
5000
4000
3000
2000
1000
I [barns]
10⁻³
10⁻⁴
10⁻⁵
10⁻⁶
10⁻⁷
N/σ [atoms per barn cm]
Fig. 7
Comparison between NICHOLS'
Experimental Results and Calculations
for Pu-240 Resonance Integral
ZUT
ARES-II
WOSN
NICHOLS' results



To disseminate knowledge is to disseminate prosperity — I mean general prosperity and not individual riches — and with prosperity disappears the greater part of the evil which is our heritage from darker times.

Alfred Nobel

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