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FIRST FLIGHT COLLISION PROBABILITIES IN PIN CLUSTERS AND ROD LATTICES

by

L. AMYOT (Euratom) and P. BENOIST (CEA, Saclay)

1966

Joint Nuclear Research Center
Ispra Establishment - Italy

Reactor Physics Department
Reactor Theory and Analysis

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SUMMARY

A new method of calculation is presented for directional probabilities of interaction in pin clusters and rod lattices. The theory, in principle rigorous from a geometrical point of view, has been programmed for the IBM 7090 as the computer code PROCOPE. Calculation results for Dancoff correction factors and collision probabilities are compared with values obtained with other methods. The validity of the technique is also checked by evaluating the fine structure of the thermal flux distributions in various gas-cooled clusters upon which measurements have been made at Saclay. As a practical illustration, a series of curves describe the behaviour of the various independent fuel-to-fuel collision probabilities in 19 (or 7) pin gas-cooled clusters. Finally, approximate formulæ are given for the pin-to-pin collision probability at both the white and the black limits.

1. Introduction

The probabilities of interaction between the various media composing a fuel element or a rod lattice have been the object of several studies in the recent past. In particular, a rigorous method of evaluating directional first collision probabilities in a cell made up of any number of concentric cylindrical annuli has been developed by Di Pasquantonio (1). The purpose of the present paper is the calculation of the directional probabilities of interaction in pin clusters and rod lattices, starting from the formalism presented in an earlier publication by one of the authors (2, annex I), without any approximation. Similar problems have been treated by other authors: Fukai (3) has given exact expressions for the first-flight collision probabilities in a regular lattice of cylindrical rods; and interactions in a geometry consisting of an array of circular rods surrounded by a number of concentric cylindrical annuli have been treated by Carlvik (4). However, both these authors have restricted themselves to the evaluation of ordinary probabilities.

2. Position of the problem and applications

The system considered is an array of homogeneous cylinders, infinite in height, plunged in a homogeneous medium within a cylindrical channel. The number and geometrical configuration of the rods are quite arbitrary, radius and physical composition are allowed to vary from rod to rod and the neutron mean free path in the surrounding medium may be finite or not. Both directional and ordinary first collision probabilities between the different media of the system are calculated.

The directional probabilities appear in the theory of the diffusion coefficient (2) and are closely related to the ordinary probabilities. Their knowledge allows the immediate calculation of radial and axial coefficients. This is of particular interest in the treatment of cells containing voided regions: in such assemblies, the effects of diffusion anisotropy are often important.

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In addition, the ordinary probabilities allow the determination of the factors ε , f and p in a cluster-fueled cell. In the case of f, the uniform and isotropic scattering density approximation may be assumed, either in the whole fuel element, or only in each individual pin, the first method often being sufficient. The second method requires not only the use of the global first collision probability P_{FF} in the fuel, but also the use of individual interaction from pin to pin.

Knowing these interactions it is easy to determine the pin-to-pin flux ratio; curves given in the present paper (Fig.3.7) allow the immediate calculation of these ratios in some classical types of clusters.

If the coolant in which the fuel pins are immersed is a liquid, a drawback of the model appears, since this medium is treated as a single region; the flat scattering density assumption in it may them lead in some cases to serious errors, particularly if the coolant is a hydrogeneous material (5).

The cladding regions may not be taken discreetly into account in the present formalism, but a simple method of correcting for their effect has recently been suggested by Bal Raj Sehgal (18).

A different type of problem which can be approached by the same formalism is the treatment of an infinite regular lattice of rods immersed in a moderator. The purpose here is to find the interaction between rods belonging to different cells, in the case of closely spaced (v.g. light water) lattices. The knowledge of the first collision probability P_{FF} allows the calculation of ε , f, p (in a multigroup scheme) and diffusion coefficients; however it seems that the diffusion anisotropy in light water lattices is very small (2, annex VIII). As far as the calculation of f is concerned, the flat scattering density assumptions both in fuel and moderator (which are generally good in light water lattices) lead to a formula where only P_{FF} enters (see Newmarch, 17). The great advantage of this method is to avoid the cylindrical cell approximation, which may lead to important errors in closely spaced lattices.

3. Relations between directional probabilities

Let $P_{ij,k}$ the probability relative to the direction k (k = z for axial and k = r for radial) for a neutron born uniformly in the medium i to suffer its first collision in the medium j . We will use a general formalism, assigning the index k = o to the ordinary probabilities

$$P_{ij,o} = P_{ij} = \frac{P_{ij,z} + 2P_{ij,r}}{3}$$

The conservation and reciprocity theorems which exist between ordinary probabilities are still valid without change for directional probabilities. The only modification concerns the equality:

$$P_{sj,k} = \frac{4V_j\Sigma_j}{S\xi_k} P_{js,k}$$

where V_j is the volume of a medium j in the channel, Σ_j its total cross section, S the outer surface of the channel, $P_{js,k}$ the probability for a neutron born in j to escape without collision from the channel and $P_{sj,k}$ the probability for a neutron entering the channel to suffer its first collision in j ; in this last probability, the neutron is supposed to enter the channel with a particular angular density (isotropic for k = o, equal to $\frac{1}{\pi S\xi_k}\Omega_k^2$ for $k \neq o$, where Ω_k is the projection of the direction considered on the axis k); the parameter k depends of the shape of the outer surface of the channel; here it is assumed cylindrical and we have $\xi_z = \frac{3}{4}$, $\xi_r = \frac{2}{3}$ and $\xi_o = 1$.

With the help of these different relations, it will thus be sufficient to calculate directly the independent probabilities $P_{ij,k}$ (where i and j refer to fuel pins), $P_{is,k}$ and the probability $P_{ss,k}$ that a neutron entering the channel will leave it without having suffered a collision.

4. Method of calculation

(1) Calculation of $P_{ij,k}$

Consider the situation illustrated in fig. 1, where, in general, any number of rods n may lie between a given pair i, j.

We may write

$$P_{ij,k} = \frac{1}{2\pi^2 \Sigma_i a_i} \cdot \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\phi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \, \frac{\frac{a_j}{d_{ij}} \cos\theta \cos\phi}{\sqrt{1-\left(\frac{a_j}{d_{ij}} \sin\theta - \frac{a_i}{d_{ij}} \sin\phi\right)^2}} \int_0^{\frac{\pi}{2}} \sin^2\psi \, d\psi \left[1 - e^{-\frac{2\Sigma_i a_i \cos\phi}{\sin\psi}} \right] \cdot$$

$$\cdot \left[1 - e^{-\frac{2\Sigma_j a_j \cos\theta}{\sin\psi}} \right] \cdot \exp\left[- \frac{\Sigma_c R_c + 2 \sum_m \Sigma_m a_m \cos\theta_m}{\sin\psi} \right] \cdot 3\Omega_{tc}^2 \qquad (1)$$

where d_{ij} is the center-to-center distance between pins i and j; a_i, Σ_i a_j, Σ_j are the radius and total cross section of pins i and j respectively and the path length through the surrounding medium c is given by

$$R_c = d_{ij} \sqrt{1-\left(\frac{a_j}{d_{ij}} \sin\theta - \frac{a_i}{d_{ij}} \sin\phi\right)^2} - a_i \cos\phi - a_j \cos\theta - 2 \sum_m a_m \cos\theta_m$$

The angle θ_m is related to θ and ϕ through the relationship

$$\frac{a_m}{d_{im}} \sin\theta_m - \frac{a_i}{d_{im}} \sin\phi = \sin(\gamma_m - \gamma_j) \sqrt{1-\left(\frac{a_j}{d_{ij}} \sin\theta - \frac{a_i}{d_{ij}} \sin\phi\right)^2} + \cos(\gamma_m - \gamma_j) \left[\frac{a_j}{d_{ij}} \sin\theta - \frac{a_i}{d_{ij}} \sin\phi \right]$$

In terms of the Bickley functions

$$K_{im}(x) = \int_0^{\frac{\pi}{2}} e^{-\frac{x}{\sin\psi}} \sin^{m-1}\psi \, d\psi$$

the ordinary collision probability P_{ij} and its radial component $P_{ij,r}$ may be written as

$$P_{ij} = \frac{1}{2\pi^2 \Sigma_i a_i} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\phi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \cdot \frac{\frac{a_j}{d_{ij}} \cos\theta \cos\phi}{\sqrt{1-\left(\frac{a_j}{d_{ij}} \sin\theta - \frac{a_i}{d_{ij}} \sin\phi\right)^2}} \cdot \left[Ki_3(x_{ij}) - Ki_3(x_{2ij}) - Ki_3(x_{3ij}) + Ki_3(x_{4ij}) \right] \qquad (2)$$

$$P_{ij,r} = \frac{3}{2} \cdot \frac{1}{2\pi^2 \Sigma_i a_i} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\phi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \cdot \frac{\frac{a_j}{d_{ij}} \cos\theta \cos\phi}{\sqrt{1-\left(\frac{a_j}{d_{ij}} \sin\theta - \frac{a_i}{d_{ij}} \sin\phi\right)^2}} \cdot \left[Ki_5(x_{ij}) - Ki_5(x_{2ij}) - Ki_5(x_{3ij}) + Ki_5(x_{4ij}) \right]$$

where

$$x_{1ij} = \Sigma_c R_c + 2 \sum_m \Sigma_m a_m \cos\theta_m$$

$$x_{2ij} = x_{1ij} + 2\Sigma_j a_j \cos\theta$$

$$x_{3ij} = x_{1ij} + 2\Sigma_i a_i \cos\phi$$

$$x_{4ij} = x_{1ij} + 2\Sigma_j a_j \cos\theta + 2\Sigma_i a_i \cos\phi$$

The axial component $P_{ij,z}$ is then

$$P_{ij,z} = 3P_{ij} - 2P_{ij,r}$$

In practical problems, the need will be felt for criteria that will permit the automatic exclusion of all rods n which cannot possibly lie on any neutron path going from i to j . This is done most simply by imposing the following conditions[x]:

(1) In order to be considered in equation (1), the position of a rod n must be such that the geometrical projection of the center-to-center distance d_{in} on the neutron path (θ , ϕ) is positive and shorter than the projection on the same axis of the center-to-center distance d_{ij}, i.e.

$$0 < d_{im}\cos\psi_m < d_{ij}\sqrt{1-\left(\frac{a_j}{d_{ij}}\sin\theta - \frac{a_i}{d_{ij}}\sin\phi\right)^2}$$

where

$$\cos\psi_m = \cos(\gamma_m-\gamma_j)\sqrt{1-\left(\frac{a_j}{d_{ij}}\sin\theta - \frac{a_i}{d_{ij}}\sin\phi\right)^2} - \sin(\gamma_m-\gamma_j)\left[\frac{a_j}{d_{ij}}\sin\theta - \frac{a_i}{d_{ij}}\sin\phi\right]$$

(2) The distance from the center of rod n to the neutron path (θ , ϕ) must be shorter than the radius of rod n, i.e.

$$\left|d_{im}\sin\psi_m + a_i\sin\phi\right| < a_m$$

where

$$\sin\psi_m = \sin(\gamma_m-\gamma_j)\sqrt{1-\left(\frac{a_j}{d_{ij}}\sin\theta - \frac{a_i}{d_{ij}}\sin\phi\right)^2} + \cos(\gamma_m-\gamma_j)\left[\frac{a_j}{d_{ij}}\sin\theta - \frac{a_i}{d_{ij}}\sin\phi\right]$$

[x] In the problem of the calculation of $P_{ij,k}$ in a cell composed of a number of concentric media, such conditions became very simple. A code using this procedure is now being established at Saclay.

(2) Calculation of $P_{is,k}$

The probabilities $P_{is,k}$ are determined in analogous fashion. We have

$$P_{is,k} = \frac{1}{2\pi^2 \Sigma_i a_i} \cdot \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\phi \cos\phi \int_0^{2\pi} d\omega \int_0^{\frac{\pi}{2}} \sin^2\psi \, d\psi \cdot \left[1 - e^{-\frac{2\Sigma_i a_i \cos\phi}{\sin\psi}} \right] \cdot$$

$$\cdot \exp\left[- \frac{\Sigma_c R_{cs} + 2\sum_m \Sigma_m a_m \cos\theta_m}{\sin\psi} \right] \cdot 3\Omega_{in}^*$$

where the total path length through c along (ω, ϕ) is given by

$$R_{cs} = \sqrt{c^2 - \left[x_i \sin(\omega+\phi) - y_i \cos(\omega+\phi) + a_i \sin\phi \right]^2} - a_i \cos\phi - x_i \cos(\omega+\phi) - y_i \sin(\omega+\phi) - 2\sum_m a_m \cos\theta_m$$

and the angle θ_m is related to ω and ϕ through the relationship

$$d_{im} \sin(\gamma_m - \omega - \phi) = a_m \sin\theta_m - a_i \sin\phi$$

The symbol c represents the radius of the channel in which the system is contained. The summation over n extends over all rods which are crossed by the neutron path (ω , ϕ).
In terms of Bickley functions, the ordinary collision probability P_{is} and its radial component $P_{is,r}$ may be expressed as

$$P_{is} = \frac{1}{2\pi^2 \Sigma_i a_i} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\phi \cos\phi \int_0^{2\pi} d\omega \left[Ki_3(\zeta_{1i}) - Ki_3(\zeta_{2i}) \right]$$

$$P_{is,r} = \frac{3}{2} \cdot \frac{1}{2\pi^2 \Sigma_i a_i} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\phi \cos\phi \int_0^{2\pi} d\omega \left[Ki_5(\zeta_{1i}) - Ki_5(\zeta_{2i}) \right]$$

where

$$\zeta_{1i} = \Sigma_c R_{cs} + 2 \sum_m \Sigma_m a_m \cos\theta_m$$

$$\zeta_{2i} = \zeta_{1i} + 2\Sigma_i a_i \cos\phi$$

The axial component $P_{is,z}$ is then

$$P_{is,z} = 3P_{is} - 2P_{is,r}$$

The rods n lying across a neutron path (ω , ϕ) all satisfy the following conditions:

$$d_{im}\cos(\gamma_m-\omega-\phi)>0$$

$$|d_{im}\sin(\gamma_m-\omega-\phi)+a_i\sin\phi|<a_m$$

(3) Calculation of $P_{ss,k}$

Let α be the azimuth of the point where a given neutron leaves the channel; ß , the angle between the neutron path and the channel radius at the same point. For the particular entering angular density defined in section 3, we may write

$$P_{ss,k}=\frac{1}{\pi^2\Sigma_{tk}}\int_0^{2\pi}d\alpha\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}d\beta\cos\beta\int_0^{\frac{\pi}{2}}d\psi\sin^2\psi\cdot e^{-\frac{\Sigma_0 Q_c+\sum_m \Sigma_m r_m}{\sin\psi}}\cdot 3\Omega_{tk}^2$$

where

$$Q_c=2c\cos\beta-\sum_m r_m$$

and r_m, the path length in rod n is given by

$$r_m=\sqrt{a_m^2-H_m^2}$$

$$H_m=|x_m\sin(\alpha+\beta)-y_m\cos(\alpha+\beta)-c\sin\beta|$$

In terms of Bickley functions, the ordinary collision probability P_{ss} and its radial component $P_{ss;r}$ may be expressed as

$$P_{ss}=\frac{1}{\pi^2}\int_0^{2\pi}d\alpha\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}d\beta\cos\beta\cdot Ki_3(y)$$

$$P_{ss,r}=\frac{1}{3\pi^2}\int_0^{2\pi}d\alpha\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}d\beta\cos\beta\cdot Ki_5(y)$$

where

$$y=\sum_m \Sigma_m r_m+\Sigma_0 R_c$$

The axial component $P_{ss,z}$ is given by

$$P_{ss,z} = 3P_{ss} - 2P_{ss,r}$$

(4) Other probabilities

The probability that a neutron born in fuel rod i suffers its next collision in medium c is easily found by making use of the conservation law

$$P_{ic,k} = 1 - \sum_j P_{ij,k} - P_{is}$$

Inversely, the probability that a neutron born in medium c suffers its next collision in rod i is

$$P_{ci,k} = \frac{\Sigma_i V_i}{\Sigma_c V_c} \cdot P_{ic,k}$$

The probability that a neutron born in medium c escapes from the channel without having suffered a collision is

$$P_{cs,k} = \frac{S\varepsilon_k}{4\Sigma_c V_c}\left[1 - P_{ss,k} - \sum_i \frac{4\Sigma_i V_i}{S\varepsilon_k} P_{is,k}\right]$$

where S is the external surface area of the channel and the summation indicated extends over all rods in the system.

Finally, the probability that a neutron born in medium c suffers its next collision in medium c is

$$P_{cc,k} = 1 - P_{cs,k} - \sum_i P_{ci,k}$$

5. The code PROCOPE

The above system of equations has been programmed in FORTRAN for the IBM-7090 as the computer code PROCOPE. The code will yield directional and ordinary probabilities for any geometrical configuration of cylindrical rods. However, in most clusters and lattices of practical interest, the pattern is fairly symmetrical. A large reduction in machine-time can usually be obtained by defining the various groups of symmetry into which the interactions may be distributed. To take advantage of this situation, provision has been made, in PROCOPE, to supply as input a matrix of symmetry. For many designs in frequent use, namely the 4, 7, 12, 19, 31, 37 pin clusters, as well as rectangular and hexagonal infinite lattices, this matrix of symmetry has been incorporated in the code.

Interactions in the infinite lattice are obtained by drawing a series of circles centred on a reference rod and tangent to all the rods belonging to a given group of symmetry. The cumulative fuel-to-fuel collision probabilities are thus determined for systems of increasing radii until the added contributions become negligible.

6. Comparison with existing theories

A direct test of the theory was provided by comparing the values of calculated escape probabilities and Dancoff coefficients in lattices and clusters with the results derived from other methods.

The fuel escape probability for an N-rod bundle is simply $(1-\frac{1}{N}\sum_i\sum_j P_{ij})$. Fukai (3) has derived an alternate and also exact expression for this quantity which he has applied to a number of infinite rectangular lattices. These calculations were repeated with the present formalism, using 20 integration points for each of the variables θ and ϕ. The results are presented in table 1. The numerical nature

of both Fukai's and the present formulation seems to be a sufficient explanation of the slight discrepancies. The reader is referred to Fukai's paper for a comparison with and detailed discussion of various approximate techniques, such as Nordheim's (7), Wigner's (8), Roe's (9) and Takahashi (10).

Table 1.- Escape Probability in Rectangular Lattices:-
(Radius of fuel rod = 0.183 in. Moderator-to-fuel volume ratio = 1
Moderator cross section = 1.4916 cm^{-1})

$\Sigma_p a_p$	FUKAI [3]	Equ. (2)
0.1	0.8074	0.8059
0.3	0.5799	0.5769
0.5	0.4474	0.4449
0.8	0.3297	0.3272
1	0.2792	0.2766
2	0.1549	0.1523
4	0.0804	0.0784
10	0.0326	0.0316

In a sequel (11) to this work on infinite lattices, Fukai has also evaluated, again using exact expressions, the escape probability and the Dancoff correction for several clusters, including a hexagonal 19-pin cluster of the type shown in Fig. 2. The cross section Σ_p and the radius a_p did not vary from pin to pin. Near perfect agreement was found between the values obtained from equ. (2) and Fukai's results. The Dancoff correction for an N-rod cluster is here defined as suggested by Pennington (12)

$$1-C = \lim_{\Sigma_p a_p \to \infty} 2\Sigma_p a_p \left(1 - \frac{1}{N} \sum_i \sum_j P_{ij}\right) \qquad (3)$$

Both sets of results are listed in table 2.

Table 2. - Escape Probability and Dancoff Correction Factor (1-C) in a Hexagonal 19-pin Cluster ($\frac{d}{a_f}$ = 2.6935)

	$\Sigma_f a_f$	$\Sigma_c a_f = 0.5$		$\Sigma_c a_f = 1.0$		$\Sigma_c a_f = 2.0$	
		FUKAI [11]	Equ. (3),(2)	FUKAI [11]	Equ. (3),(2)	FUKAI [11]	Equ. (3),(2)
Escape Proba-bility	0.1	.8214	.8214	.8541	.8541	.8752	.8751
	0.25	.6434	.6434	.6977	.6977	.7348	.7348
	0.5	.4659	.4659	.5278	.5278	.5727	.5727
	1.0	.2923	.2923	.3448	.3448	.3853	.3853
	2.0	.1617	.1618	.1952	.1952	.2218	.2219
	5.0	.0669	.0669	.0814	.0814	.0930	.0929
Dancoff Correct.	∞	.6732	.6729	.8196	.8193	.9367	.9366

Finally, Dancoff corrections for an infinite square lattice were computed with the help of equ. (3) and compared with the values obtained by Carlvik (2) (see table 3). Both here and in table 2 , $\Sigma_f a_f$ was taken as equal to 100. The number of integration points over each variable θ , ϕ was 20.

Table 3 . - Dancoff Correction (1-C) in Square Lattices (d=lattice pitch)

$\Sigma_c d$	$\frac{a_f}{d} = 0.25$		$\frac{a_f}{d} = 0.5$	
	Carlvik (2)	Equ. (3)	Carlvik(2)	Equ. (3)
0.25			.063	.060
0.5	.534	.650 x	.118	.116
1.0	.733	.771 x	.208	.207
2.0	.891	.896	.336	.335
4.0	.975	.974		

x These discrepancies remained even with $\Sigma_f a_f$ = 1000 and 30 points of integration over θ, ϕ ; they are still unexplained.

7. Comparison with Experiments

The validity of the technique outlined above has also been tested by calculating the fine structure of the thermal flux distributions in various gas-cooled clusters upon which measurements have been made at Saclay. Modified diffusion theory was assumed to apply in the moderator (6) while the fuel element was treated by considering the following collision balance equation:

$$\Sigma_j V_j \phi_j = \sum_{i=1}^{N} (\Sigma_{a_i} V_i \phi_i + Q_i)(P_{ij} + \frac{P_{is}}{P_s} P_{sj}) + Q_m \frac{P_{sj}}{P_s} \qquad (4)$$

where

$$P_s = \sum_i P_{is}$$

$$P_{si} = \frac{4\Sigma_i V_i}{S} P_{is}$$

and Q_i is the total source in region i .

Three cases were investigated:

(1) A 19-rod cluster within a channel of 5.3 cm internal radius (15) where the pins were symmetrically distributed in three concentric layers: a central pin, an inner ring of 6 pins at a radius of 1.9 cm, and an outer ring of 12 pins at a radius of 3.7 cm. All pins had a radius of 0.6 cm and were sheathed in two concentric layers of aluminium cladding with the following internal and external diameters: 12.3, 12.5 mm, 13.15 mm. The cross sections and sources were as follows:

	Σ_a	Σ_s	$\varepsilon\Sigma_s$
Fuel (UO_2)	.1697	.3298	.0119
Cladding (Al)	.1228	.0747	.00554

(2) A 19-rod cluster within a channel of 3.9 cm internal radius (16) where the pins were symmetrically distributed in four concentric layers: a central pin, an inner ring of 6 pins at a radius of 1.55 cm, an intermediate ring of 6 pins at a radius of 2.9 cm, an outer ring

of 6 pins at a radius of 3.025 cm. All pins had a radius of 0.55 cm and were sheathed in a stainless steel cladding with the following internal and external diameters: 11.07, 11.87 mm. The cross sections and sources were as follows:

	Σ_a	Σ_s	$\xi\Sigma_s$
Fuel (UO_2 1.35 % enriched)	.236	.330	.0119
Cladding (SS)	.2255	.8270	.0292

(3) A 19-rod cluster with exactly the same geometry as in Case (2) but where the fuel was natural uranium oxide as in Case (1) (16).

As stated previously, the theory, in its present state, will not take into account in an exact manner the presence of a cladding material around the fuel pins. Some approximate treatment must therefore be resorted to. In Case (1), the aluminium was assumed to be completely transparent to thermal neutrons and consequently the fuel pins were considered as unclad. In Cases (2) and (3), because of the high absorption and scattering cross sections of stainless steel, it was thought preferable to homogenize, by straightforward volume-weighting, the cladding and fuel regions in the volume of the clad pins. The results for all three cases are given in table 4. The central pin is taken as ring 1 and the ring number increases as we move outwards.

Table 4. Fine Flux Distribution in 19-Pin Clusters

Ring No.	Case (1)		Case (2)		Case (3)	
	ϕ_{th}	ϕ_{sup} (±1%)	ϕ_{th}	ϕ_{sup} (±1%)	ϕ_{th}	ϕ_{sup} (±1%)
1	1.000	1.000	1.000	1.000	1.000	1.000
2	1.029	1.024	1.061	1.064	1.041	1.043
3	1.120	1.114	1.253	1.248	1.173	1.171
4			1.280	1.271	1.191	1.201

8. Practical calculation

In most practical designs, all pins in a given fuel element are identical. The flux is then exactly the same in all pins located in symmetrical positions with respect to the center of the cluster provided the configuration is geometrically regular. The number of linear equations of the type (4) which must be used in the calculation of the fine flux distribution is thus considerably reduced with a consequent saving in machine-time.

In Figs.(3-7), a series of curves illustrate the behaviour of the various independent fuel-to-fuel collision probabilities in 19 (or 7) pin gas-cooled-clusters. Through the reciprocity theorem and the law of conservation of probabilities, all probabilities not shown here are easily derived.

The curves shown in fig. 3 are of quite general interest. They represent the first collision probability between pins not mutually shadowed by intermediate fuel pencils and may be used for instance to describe the interactions between the central pin in the rod cluster shown in fig.2 and the pins lying in ring a.

The probability P_{FF} shown in Fig.4 is the average fuel-to-fuel collision probability in a 19-rod cluster, assuming that the source is uniformly distributed throughout the bundle.
This is simply given by

$$P_{FF} = \frac{1}{N} \sum_i \sum_j P_{ij}$$

The fuel escape probability is immediately deduced.

In many problems, e.g. in the calculation of the thermal flux hyperfine structure, it will be desirable to obtain separately the fluxes in the fuelpins belonging to successive concentric rings. In Fig. 2, three fuel regions, a, b, c are thus defined, for each of which a balance equation of type (4) may be written. If the surrounding medium is void, it will be sufficient to evaluate the interaction probabilities P_{bb}, P_{bc}, P_{cc}. These are shown

in Figs.(5-7). The probability P_{aa} is given by the classical formulae for an isolated cylinder, while P_{ab} is easily obtained with the help of Fig. 3. Finally, P_{ac} may be derived from a knowledge of P_{FF}.

9. Approximate Formulae

Bonalumi (13) has shown that integrals of the type

$$I = \int_0^{\frac{\pi}{2}} f(t)\, Ki_m(z)\, dt$$

are remarkably well approximated by

$$I = F\, Ki_m(\bar{z}) \qquad (5)$$

where

$$F = \int_0^{\frac{\pi}{2}} f(t)\, dt$$

$$\bar{z} = \frac{1}{F} \int_0^{\frac{\pi}{2}} f(t)\, z(t)\, dt$$

Applying this recipe to the evaluation of the pin-to-pin collision probability P_{ij} as given by equ. (2) it is readily found that, in the absence of intermediate pencils,

$$P_{ij} \propto \frac{N F}{2\pi^2 \Sigma_F a_F} \left\{ Ki_3\left[\Sigma_C a_F \left(\frac{1}{F} - \frac{\pi}{2}\right)\right] - 2Ki_3\left[\Sigma_C a_F \left(\frac{1}{F} - \frac{\pi}{2}\right) + \frac{\pi}{2}\Sigma_F a_F\right] + Ki_3\left[\Sigma_C a_F \left(\frac{1}{F} - \frac{\pi}{2}\right) + \pi \Sigma_F a_F\right] \right\} \qquad (6)$$

where

$$F = \arcsin \frac{2a_F}{d_{ij}} - tg\left(\frac{1}{2} \arcsin \frac{2a_F}{d_{ij}}\right)$$

As shown in table 5 and fig. 8, equ.( ) is quite accurate for large values of $\Sigma_F a_F$ but less and less successful as the white limit is approached.

For small values of $\Sigma_F a_F$ it is preferable to use an alternate formulation (14) of P_{ij}, also exact in the case of identical pins i and j:

$$P_{ij} = \frac{2}{\pi^2}\Sigma_F a_F \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\phi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \cdot \frac{\frac{a_F}{d_{ij}}\cos^2\phi\cos^2\theta}{\sqrt{1-\left(\frac{a_F}{d_{ij}}\right)^2(\sin\theta-\sin\phi)^2}} \cdot \int_0^{\frac{\pi}{2}} d\psi \cdot \exp\left[-\frac{x_{1ij}+x_{2ij}+x_{3ij}+x_{4ij}}{4\sin\psi}\right] \cdot$$

$$\cdot \left\{1 + \sum_{m=2}^{\infty} \frac{1}{(2m)!}\left(\frac{\Sigma_F a_F}{4\sin\psi}\right)^{2m-2} \frac{(\cos\phi+\cos\theta)^{2m}-(\cos\phi-\cos\theta)^{2m}}{2\cos\phi\cos\theta}\right\}$$

$$= \frac{2}{\pi^2}\Sigma_F a_F \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\phi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \cdot \frac{\frac{a_F}{d_{ij}}\cos^2\phi\cos^2\theta}{\sqrt{1-\left(\frac{a_F}{d_{ij}}\right)^2(\sin\theta-\sin\phi)^2}} \left\{K_0\left(\frac{x_{1ij}+x_{2ij}+x_{3ij}+x_{4ij}}{4}\right) + \right.$$

$$\left. + \frac{1}{2}\left(\Sigma_F a_F\right)^2\left(\cos^2\phi+\cos^2\theta\right) K_1\left(\frac{x_{1ij}+x_{2ij}+x_{3ij}+x_{4ij}}{4}\right) + \ldots\right\}$$

Using again the approximate formula (5), it is found that, in the absence of intermediate pencils,

$$P_{ij} \simeq \frac{2}{\pi^2}\Sigma_F a_F \cdot A\left(\frac{a_F}{d_{ij}}\right)\left\{K_0(\tau) + \frac{1}{4}\left(\Sigma_F a_F\right)^2 K_1(\tau)\right\} \qquad (7)$$

where

$$\tau = 1.6970\left(\Sigma_F - \Sigma_C\right)a_F + \frac{\pi^2}{4}\frac{\Sigma_C a_F}{A\left(\frac{a_F}{d_{ij}}\right)}$$

$$A(K) = \pi K\left\{\frac{E(K)}{K'^2}\left[\frac{K'^2}{K^2} + \frac{1}{8} + \frac{1}{192}\cdot\frac{K^2}{K'^4}\left(8-7K'^2\right)\right] - F(K)\left[\frac{K'^2}{K^2} + \frac{1}{8} + \frac{1}{192}\cdot\frac{K^2}{K'^4}\left(4-3K'^2\right)\right]\right\}$$

$$K'^2 = 1-K^2$$

F(K), E(K) are the complete elliptic integrals of the first and second kind.

Very good at the white limit, equ.(7) fails badly towards the black limit. A look at Table 5 and Fig. 8 shows that P_{ij} is always underestimated by equ.(7) and overestimated by equ.(6). Near the maximum of the function P_{ij} a simple arithmetic average of approximation (6) and (7) would yield acceptable results in most practical cases.

Table 5 Comparison of Exact and Approximate Expressions of P_{ij}

$\Sigma_c d_{ij}$	$\Sigma_c a_c$	$\frac{a_j}{d_{ij}}$	P_{ij} (exact)	P_{ij} (eqn. 7)	P_{ij} (eqn. 6)
0.0	0.1	0.1	.005494	.005480	
		0.2	.011077	.011047	
		0.3	.016859	.016804	
		0.4	.023010	.022917	
		0.5	.029938	.029759	
	0.2	0.1	.008504	.008456	
		0.2	.017148	.017045	
		0.3	.026108	.025930	
		0.4	.035655	.035361	
		0.5	.046456	.045921	
	0.5	0.1	.011557	.011248	.012198
		0.2	.023315	.022669	.024656
		0.3	.035525	.034461	.037708
		0.4	.048590	.046881	.051948
		0.5	.063537	.060340	.069394
	1.0	0.1	.010616	.009599	.011360
		0.2	.021419	.019349	.022962
		0.3	.032671	.029434	.035118
		0.4	.044710	.040140	.048380
		0.5	.058754	.052126	.064627
	2.0	0.1	.007144		.007545
		0.2	.014431		.015251
		0.3	.022039		.023325
		0.4	.030276		.032133
		0.5	.040051		.042924
	10.0	0.1	.001592		.001597
		0.2	.003218		.003227
		0.3	.004921		.004936
		0.4	.006779		.006801
		0.5	.009050		.009084
	100.0	0.1	.000160		.000160
		0.2	.000323		.000323
		0.3	.000494		.000494
		0.4	.000680		.000680
		0.5	.000908		.000908

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Fig. 1 - COORDINATE SYSTEM FOR CALCULATION OF Pij

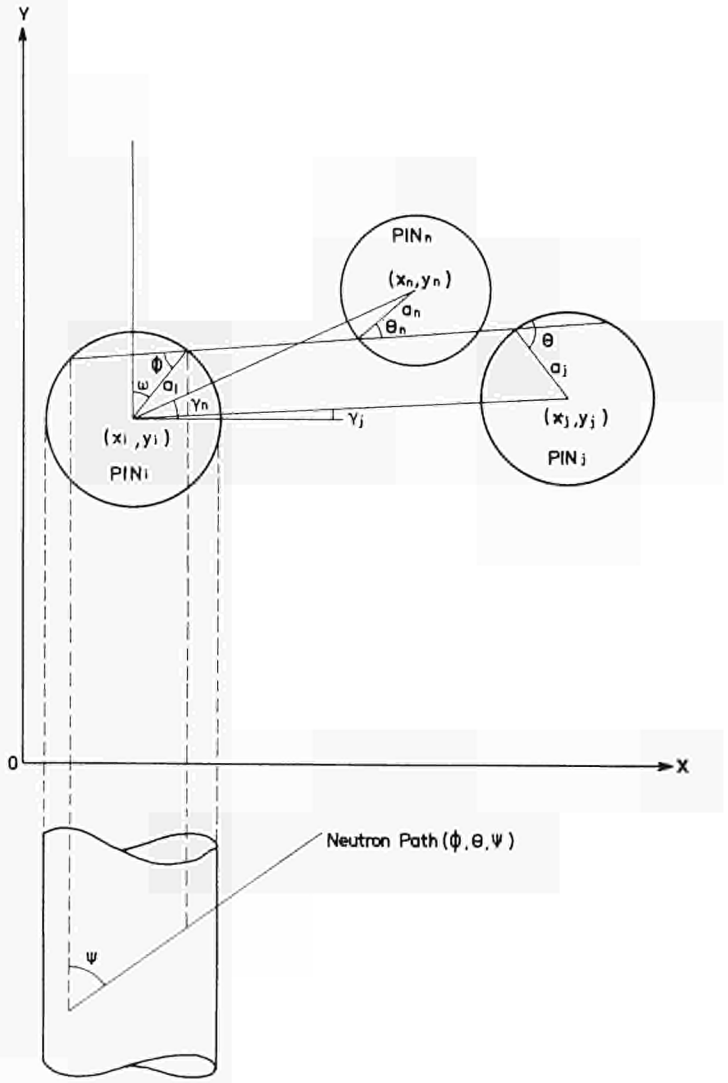


Fig. 2 - 19 ROD CLUSTER (CIRCULAR)

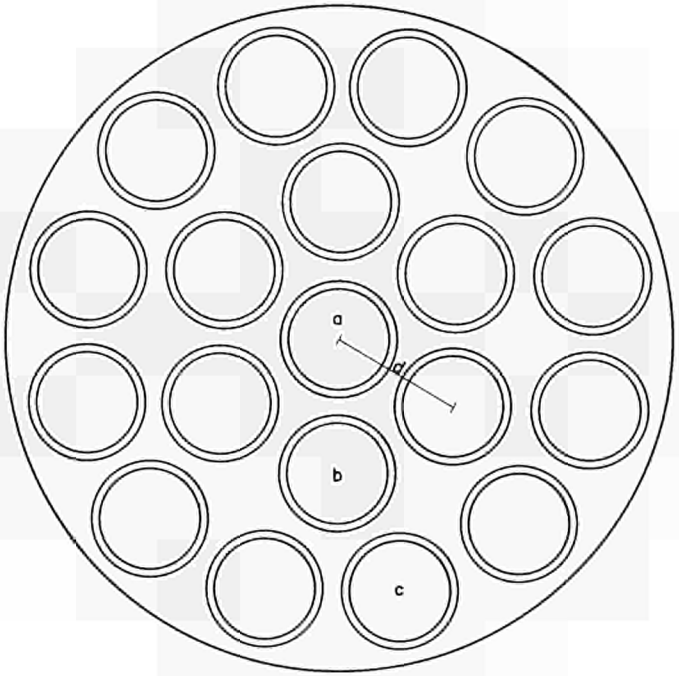


Fig. 3 – PIN-TO-PIN INTERACTION (NO INTERMEDIATE PINS)

($\Sigma_C = 0$)

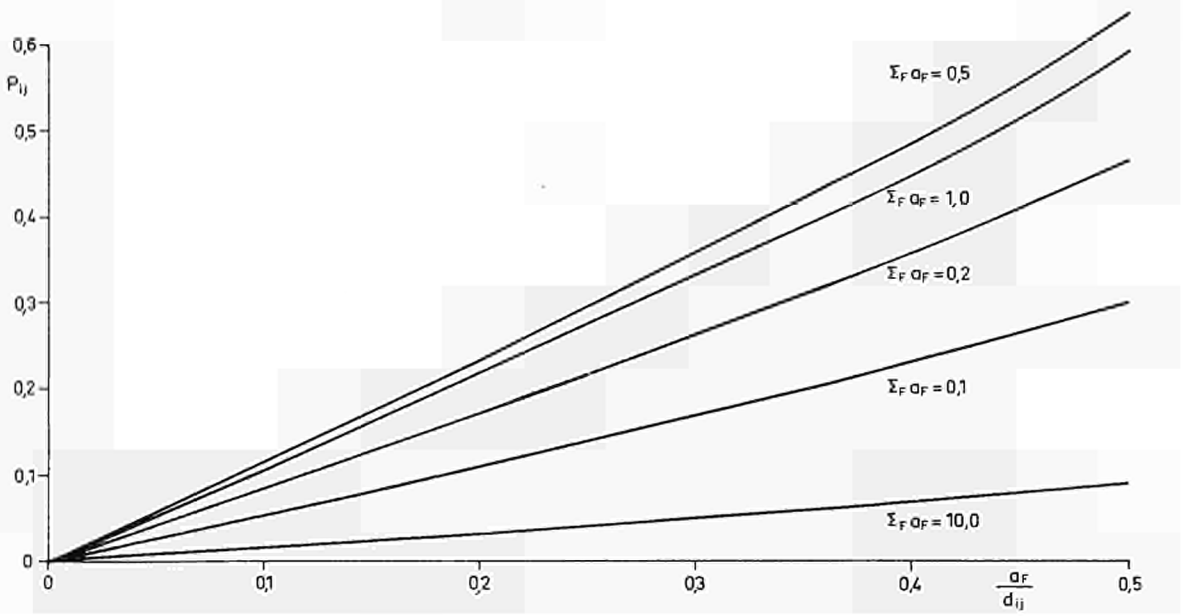


Fig. 4 – FUEL-TO-FUEL COLLISION PROBABILITY IN THE 19 ROD CLUSTER SHOWN FIG. 2

($\Sigma_C = 0$)

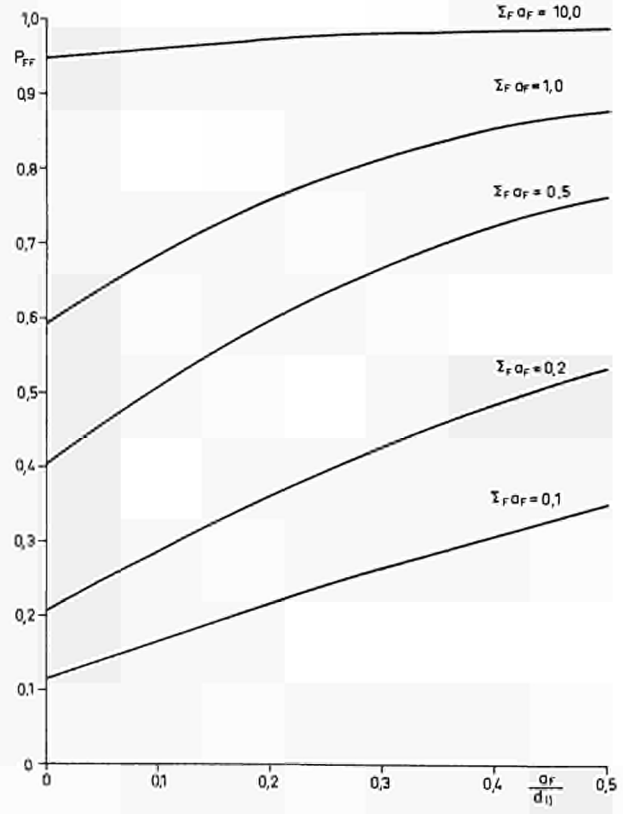


Fig. 5 – FUEL-TO-FUEL COLLISION PROBABILITY IN RING b OF A19– (OR 7–) ROD CLUSTER OF THE TYPE SHOWN IN Fig. 2 ($\Sigma_C = 0$)

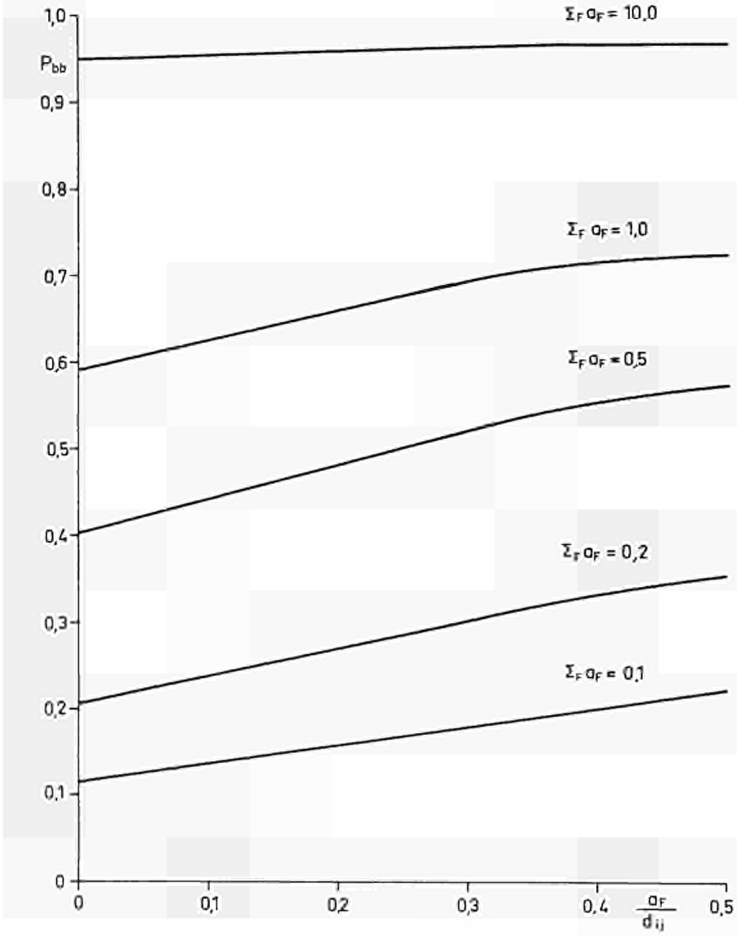


Fig. 6 FUEL-TO-FUEL COLLISION PROBABILITY IN RING c OF A19-ROD CLUSTER OF THE TYPE SHOWN IN Fig. 2 ($\Sigma_C = 0$)

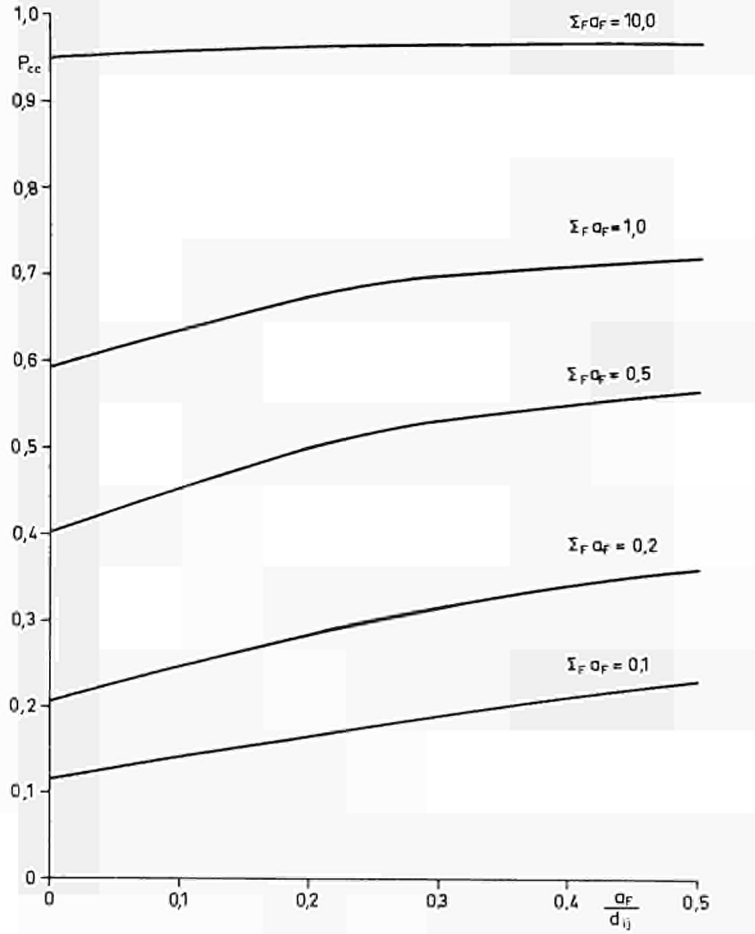


Fig. 7 – COLLISION PROBABILITY IN THE FUEL OF RING c
FOR A NEUTRON BORN IN THE FUEL OF RING b
(see Fig. 2) ($\Sigma_c = 0$)

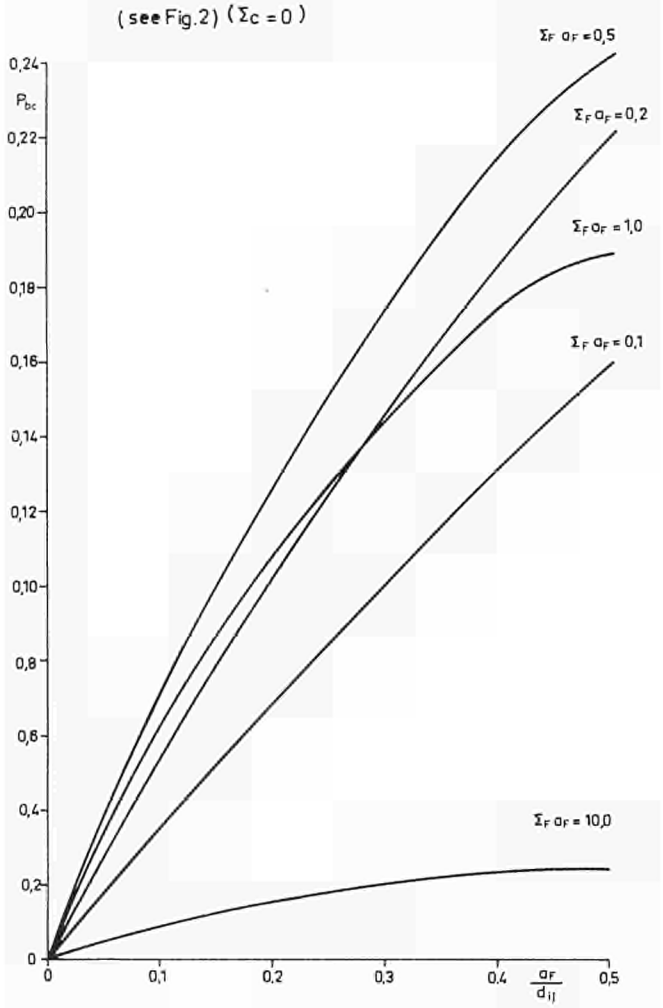
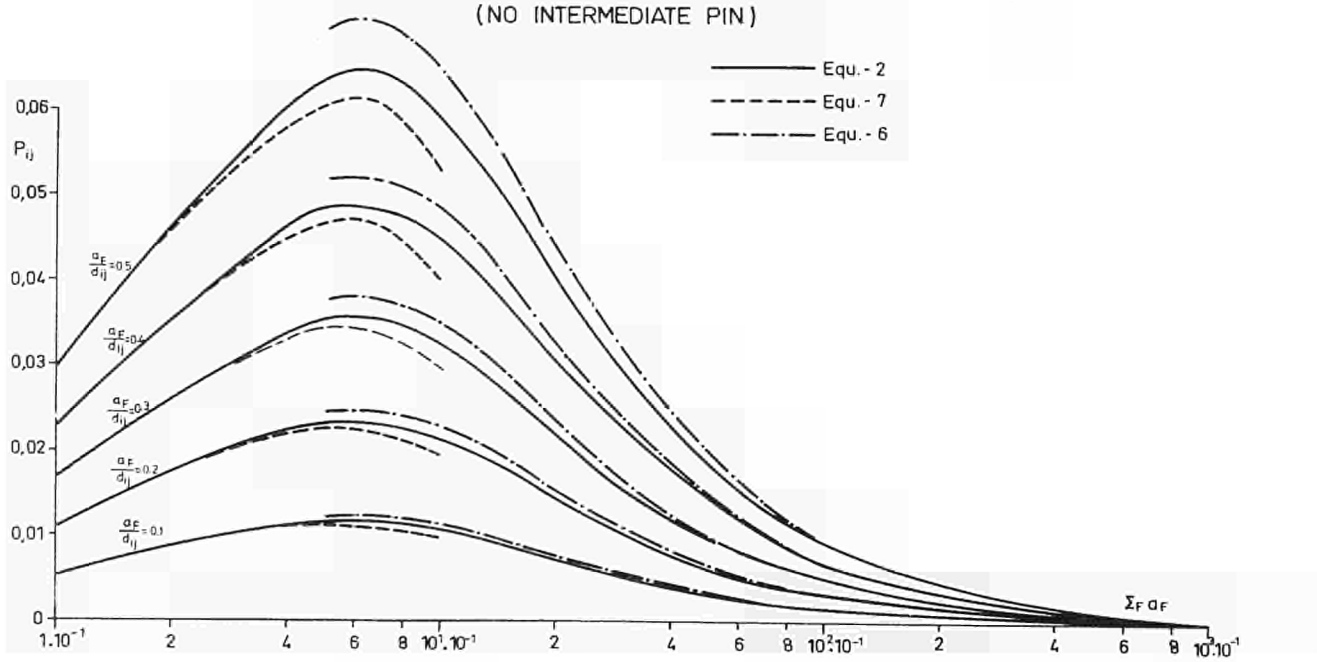
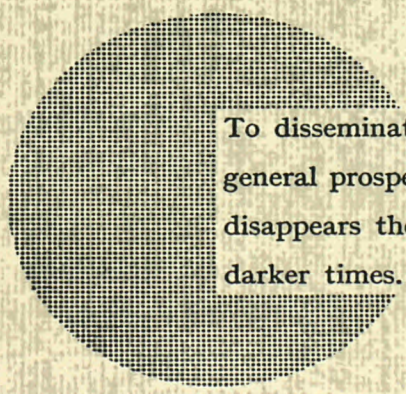


Fig. 8 – COMPARISON OF EXACT AND APPROXIMATE EXPRESSIONS OF P_{ij} ($\Sigma_c = 0$)
(NO INTERMEDIATE PIN)





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Alfred Nobel

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