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EUROPEAN ATOMIC ENERGY COMMUNITY - EURATOM

ON SOME PROBLEMS IN QUASI-NEURAL NETWORKS

by

L. VERBEEK

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Joint Nuclear Research Center
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ON SOME PROBLEMS IN QUASI-NEURAL NETWORKS (*)

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1 – Preface.

This preface aims at giving an indication of the general philosophical background of research work concerned with logic and the foundations of mathematics as well as a few remarks on the recent history of such endeavors because, and only in so far as, these are considered to be relevant for the subsequent discussion.

The philosophy of science occupies itself from time immemorial, among other things, with the limits of the knowledge of the human intellect. This is closely related to the perennial epistemological questions about the way in which we know our world. How is it possible that we know and understand the environment that from early childhood on operates on us and is manipulated by us? Can we abstract ourselves from this interaction so as to be able to form a collection of ideas describing sufficiently precise our world? Instead of continuing along such more or less contemplative lines which fall largely outside our competence it is deemed better to sketch quickly and rather superficially some of the forms in which such questions have been posed during the last century. For a concise but good review of the subject of the following remarks the reader is referred to chapter 3 of [1].

The development of an algebra of logic, especially after BOOLE's and JEVONS's work around 1850, was followed by attempts to show that all mathematics can be based on symbolic logic. Investigations aimed at proving the consistency and completeness of certain parts of mathematics led to the development of formal systems. These are axiomatic theories concerning a well-defined collection of symbols, of certain axioms concerning these symbols, and of rules for combining these symbols to form valid sequences of symbols. Such an axiomatic system is often called a formalized language in which sentences, or expressions, or formulae, are constructed from the symbols, or letters of the *alphabet*, according to the rules of the *grammar*.

In such a formal system meaning and truth are irrelevant notions. A formula is provable if it can be deduced from the axioms and the rules of the system. The system is consistent if there is no formula such that it, as well as its negation, can be proved, and the system is complete if the consistency can be proved with arguments allowed in the system. In 1931 GÖDEL [2] published his incompleteness theorem which says

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(*) To be presented at the International Spring School of Physics at the University of Naples in April/May 1962.

that one needs to introduce in a formal system something from the outside in order to prove that the system is consistent.

Five years later, the English logician TURING [3] wrote a paper dealing with the completeness problem and introduced a certain imaginary device, which he called a machine, that could carry out the operations defined in a formal system. Since then the name *Turing machine* is used to indicate a device that operates, according to rules inherent in its internal structure, on a tape that carries symbols and is presented to the machine. Changing the tape gives of course a change in the result of the operation of the machine. By now this method of distinction between operator and operand is common practice in digital computers which are *universal* by virtue of the possibility to feed it all sorts of programs because these are physically separate from the computer.

Contemporary research activities in the areas sketched above are often published under the heading automata theory.

2 – Introduction.

MCCULLOCH and PITTS published in 1943 an important paper [4] on theoretical neurophysiology. In this paper they made the assumption that as far as the logical properties of the nervous system are concerned its constituents, the neurons, can be regarded as *all-or-none* elements which on receiving a sufficient stimulation deliver a pulse. Considering the possible actions of interconnected formal neurons working in a discrete time scale they showed that all operations of the logic of propositions can be realized by such a network. It should be noted that their formal neurons are abstractions of real biological neurons in that they take one important function, well established for the peripheral nervous system of vertebrates, and, neglecting other known activities of the nervous system, show that this *all-or-none* effect is sufficient to constitute logical operations in a discrete time scale. Their result that a network of formal neurons is equivalent to Turing machines was an amazing achievement because it states that anything that can completely and unambiguously be stated in words is realizable by a finite neural network. The relevance of automata theory for theoretical neurophysiology and also the value of neurophysiological research for the synthesis of automata may be clear from these remarks.

It is interesting to note that, in 1956, KLEENE [5] developed a theory for neural networks along the lines of the McCulloch and Pitts paper and stated that certain parts of this last were obscure to him. In 1958 COPI, ELGOT and WRIGHT published a paper [6] to simplify and elucidate this work of KLEENE.

Many other authors have worked in the last two decennia on problems related to formal systems, automata or nets of formal neurons. The value of the theoretical work of MCCULLOCH and PITTS for experimental neurophysiology is much less obvious than the stimulus they have given to mathematical investigations and to epistemic development. The art of cybernetics and to a lesser degree also certain parts of information theory have profited from this interrelation of biological and mathematical disciplines. For a very interesting and stimulating comparison of the nervous system and existing computing machines as well as an outline of promising directions of research in this field we refer to [7].

3 – Quasi-neural networks.

A network is understood to be an interconnected set of elements which act on each other. The elements composing the network are often called neurons or formal neurons because they are units whose function is derived from the operation of biological neurons. Because it may avoid confusion we want to stress the fact that the elements are only abstract images of real neurons and we will indicate this by talking about quasi-neural networks. For an account of the development of basic ideas concerning actual neurons we refer the reader to [8].

The elements of the networks discussed here will be called threshold elements. They function with a discrete time scale with fixed intervals. Each threshold element has one or more input lines and one output line. Each line is at each instant of time in one of two possible states, *on* or *off*, indicated by 1 or 0, respectively. With each input line is associated a weight represented by an integer. An input line on the *on* state gives to the threshold element a signal with strength equal to its weight w. An input line in the *off* state gives a signal with strength zero to the threshold element. We denotet his by saying that the i^{th} input line x_i, where the value of x_i is 0 or 1, is arithmetically multiplied by its weight w_i and the signal given to the threshold element is $w_i x_i$.

The threshold element sums all the received signals at instant t and compares this sum with a parameter, its threshold

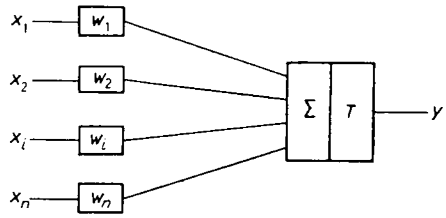


FIG. 1 – Operation of a threshold element:

$$y(t+1) = 1 \; if \sum_{i=1}^{n} w_i x_i(t) \geq T$$

$$y(t+1) = 0 \; if \sum_{i=1}^{n} w_i x_i(t) < T$$

T. If the sum equals or exceeds the threshold the state of the output line at instant $(t+1)$ is *on*, the output $y=1$; if the sum is smaller than the threshold $y=0$. This operation of a threshold element is pictured in Fig. 1.

Such threshold elements are capable to carry out logical functions. For example the disjunction of two variables $(x_1 \vee x_2)$, the conjunction of three variables $(x_1 \wedge x_2 \wedge x_3)$, and the Sheffer stroke function $\overline{(x_1 \wedge x_2)}$ are represented in Fig. 2-*a*, 2-*b*, and 2-*c*. It will be clear that

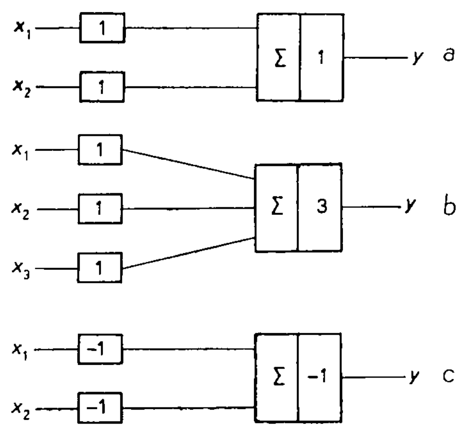


FIG. 2 – Examples of logical functions carried out by threshold elements.

⟨*a*⟩ $y(t+1) = x_1(t) \vee x_2(t)$

⟨*b*⟩ $y(t+1) = x_1(t) \wedge x_2(t) \wedge x_3(t)$

⟨*c*⟩ $y(t+1) = \overline{x_1(t) \wedge x_2(t)}$

interconnection of several threshold elements into a network makes it possible to execute logical operations on the binary signals of the input lines. Fig. 3 gives a simple example of such a network. In this example

$$y_1(t+1) = x_1(t) \vee x_2(t);$$
$$y_2(t+1) = \overline{x_1(t) \wedge x_2(t)},$$

and

$$y(t+2) = y_1(t+1) \wedge y_2(t+1) =$$
$$= [x_1(t) \wedge \overline{x_2(t)}] \vee [\overline{x_1(t)} \wedge x_2(t)].$$

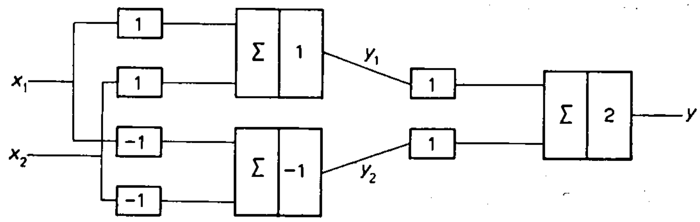


FIG. 3 – Examples of a network of threshold elements for the execution of a logical operation.

It should be remarked that by the representation of the logical operations with these threshold organs time enters into the description; this is a very important property as it makes it possible to construct physical devices to carry out the operations. Later on we will take up this point again.

Another necessary remark is that the operation of the threshold elements as defined here is not the only one considered in the literature. For instance, MCCULLOCH and PITTS [4] and also KLEENE [5] use formal neurons with only positive integral weights w and infinite negative weight (absolute inhibition).

Apart from many digital computer simulations several types of artificial neurons have been constructed in real hardware, see e.g. reference [9]. It is interesting to note that computer simulations of formal neurons are mostly used to study experimentally the activity of an ensemble of a large number of threshold elements, often interconnected in some *random* manner. On the other hand, transistorized threshold elements are used as components for networks carefully designed to carry out well-defined operations (on laboratory scale) or also as useful and truthful components to form models for neurophysiological research. The difference of these last two purposes is of course also reflected in a difference of the detailed operation of the threshold elements constructed.

4 – The Venn-diagram representation.

A convenient symbolism for the representation of the activity of a quasi-neural network was published by MCCULLOCH in reference [10]. Because this Venn-diagram symbolism proves to be a very helpful method of notation we will introduce it here.

Formulae in symbolic logic can be written in the form of a truthtable and Venn-diagrams are representations of truthtables.

A logical function of n two-valued variables is a dichotomy of the 2^n possible configurations of the variables as each of these is either 1 or else 0 and the function has for each configuration the value 1 or else 0. Hence in two-valued logic there are $2^{(2^n)}$ functions of n variables possible. Fig. 4 gives an example of a logical function of two binary variables, x_1 and x_2, which give rise to $2^2 = 4$ input configurations as indicated in the four spaces of Fig. 4-*a*. A logical function of x_1 and x_2 is represented by putting a 1 in those spaces of the Venn-diagram for which the function equals 1 and 0 in the spaces corresponding with an input configuration resulting in an output 0. With this interpretation Fig. 4-*b* represents the logical function $(x_1 \vee x_2)$, whereas Fig. 4-*c* represents the function $\overline{(x_1 \wedge x_2)}$. A logical function of three variables can be represented by a Venn-diagram as given in Fig. 5-*a*, indicating the $2^3 = 8$ spaces with the input configurations of the binary variables x_1, x_2 and x_3. Fig. 5-*b* represents hence the logical function $(x_3 \wedge x_2 \wedge x_3)$.

Note that the Fig. 4-*b*, 5-*b* and 4-*c* represent the logical functions realized by the threshold elements of Fig. 2-*a*, 2-*b*, and 2-*c*, respectively.

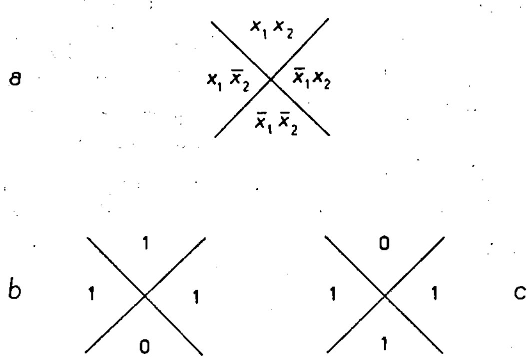


FIG. 4 – Venn-diagramm representation of logical functions of two variables, x_1 and x_2.
⟨*a*⟩ The four possible input configurations.
⟨*b*⟩ $x_1 \vee x_2$
⟨*c*⟩ . $\overline{x_1 \wedge x_2}$

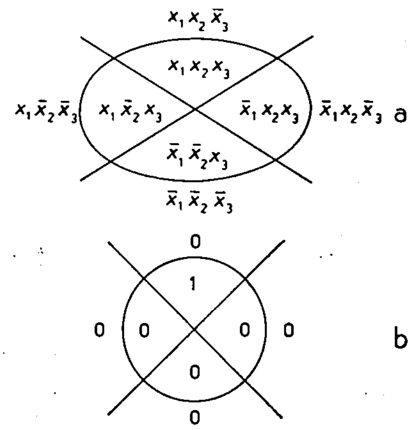


FIG. 5 – Venn-diagram representation of logical functions of three variables, x_1, x_2 and x_3.
⟨*a*⟩ The eight possible input configurations.
⟨*b*⟩ $x_1 \wedge x_2 \wedge x_3$.

A network of three threshold elements as given in Fig. 3 can be represented by three Venn-diagrams as in Fig. 6. Two of the threshold elements receive their input from the variables x_1 and x_2 and their output y_1 and y_2 form the input for the third Venn-diagram the output of which is y. It is possible to represent the operation of a network like this by a single Venn-diagram as is also indicated in Fig. 6. The rules for this sort of reduction are simple but we will not

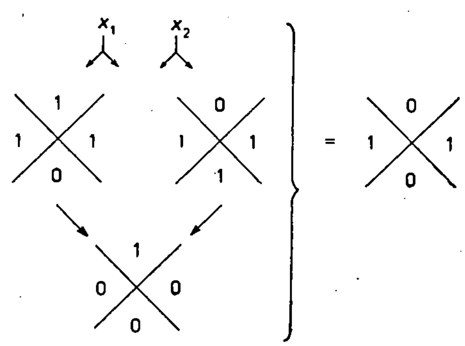


FIG. 6 – Venn-diagram representation of a network of three threshold organs and reduction of its operation.

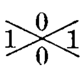
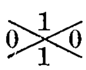
elaborate on them here as we will not use this in our further discussion.

The interested reader is referred to [10] and to [11] for more complete information on the properties and utilities of the Venn-diagram notation.

5 – Some theoretical problems.

For the relation of the theories concerning Turing machines, finite automata and quasi-neural nets we refer to [12] and the literature mentioned in that paper. With this remark we have disposed of a number of interesting mathematical questions which do not lend themselves for discussion in this context.

One question which has stimulated much discussion deals with the realizability of logical functions by networks of threshold elements. Through the manner in which these elements work it is inherently impossible to realize certain functions by means of only one element. For two input variables there are $2^{(2^2)} = 16$ possible functions of which two, to wit

 and 

are not realizable; these two can be realized by a network of three elements as in Fig. 3. For three variables there are $2^{(2^3)} = 256$ possible functions of which only 104 are realizable by a single threshold element. With increasing number of input variables an increasing ratio of the possible functions becomes unrealizable. For a discussion of this subject and the derivation of an upper bound of the number of realizable functions we refer to [13], further references can be found in [14]. Apart from the strange stubborness of this problem it is of interest because of possible utilization of threshold element logical circuitry instead of switching circuits in applications of Boolean algebra.

Quite another problem deals with the reliability of quasi-neural networks if the realistic assumption is made that the threshold elements are not ideal but subject to errors. Also the nervous system is remarkably reliable and this feature needs an explanation at least in principle.

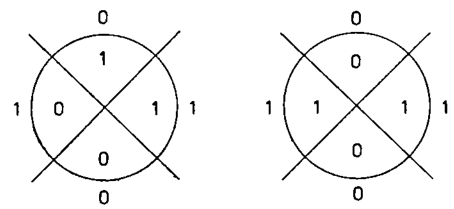


FIG. 7 – Examples of unrealizable functions of three variables.

Such principles could then be elaborated for use in digital computer systems and control systems with logical circuitry which need this quality for reliability badly. It seems rather evident that the large number of neurons in the nervous system has something to do with the reliable functioning of the system also under conditions in which the normal operation of the neurons is disturbed by changes in their metabolic environment as well as under conditions in which a part of the nervous system is dead, i.e. no longer active. This can only come about by a rather large degree of redundancy with which in normal circumstances many neurons are strictly superfluous for the correct operation of the ensemble. In order to investigate a possible method for putting redundancy in a network of threshold elements such that the overall operation is not disturbed by malfunction of part of the elements, VON NEUMANN [15] proposed certain schemes. He attributed to each element a given probability of error and formed a network of elements to replace a single element. He computed the number of elements necessary to obtain a negligible low probability of error of the total network operation. This work gave somewhat disappointing results in that an extremely large number of threshold elements was necessary to achieve a reasonably low probability of error of the total network. The same procedure with threshold elements more complicated than those used by VON NEUMANN in his investigation gave however a much more acceptable outcome as is shown in reference [16]. In connection with this reliability problem it should be remarked that an analogous problem was treated by MOORE and SHANNON [17]. The difference between their work dealing with errors in relay contacts, equivalent to connections in threshold elements, and VON NEUMANN's work with errors in logical operations, not contacts, is responsible for the great difference in numerical results. For further discussion of reliability of networks the reader is referred to [18], reference [19] gives, among other things, also a discussion of this subject.

In connection with the work by VON NEUMANN on reliability of quasi-neural networks it is worth while to note that he used the name probabilistic logic. This name refers to the assumption that the logical operation of a threshold element is not error-free but has a certain probability of error associated with it. Physically one can attribute this to fluctuations in the level of the threshold and in the strength of the signals. The difference between this point of view and consideration of errors in logical variables is essential. Hence the introduction of probabilistic logics in distinction to a logic of arguments which have a probabilistic character. Development of probabilistic logic has been restricted until now mainly to questions concerning reliability.

As has been mentioned already in the previous section on quasi-neural networks, time is inherent as independent variable as soon as one speaks about logical functions carried out by physical mechanisms. Hence in quasi-neural networks the study of the logical operations is inherently connected with time. It should be noted that by discretizing the time in connection with unit logical operations, as is done in the definition of threshold elements, the number of sequential operations is equivalent to the number of units of time. For networks in which simultaneous operations are carried out in a parallel manner it is however necessary to distinguish time and number of operations. As soon as one introduces feedback in a network, using the results of operations of a collection of elements as part of the input signals of the collection, it is clearly unavoidable to have an explicit way to describe logical time functions. Another point related to the time in quasi-neural networks deals with the variability of networks. One may want to intervene in the operation of a network on the basis of its outcome, i.e. one may want to control the operations, in order to influence the reliability or also to supply a flexibility to a given network. In such cases it is necessary to have an adequate description of the operation and hence of the time dependence of the activity of the network. These same remarks hold for a study of the possibilities and realizations of networks that change their operation on the basis of previous activities, i.e. networks that learn, are adaptive, have self-organizing properties.

6 – Some practical problems.

A general remark, often justly made concerning quasi-neural networks as well as automata theory in general, says that these theoretical constructs have not given rise to the realization of remarkably clever and useful machines to do much of the chores still done by mathematicians and *decision-makers* in several fields of human society. Of course it is rather foolish to expect such *practical* results and utilities from a theoretical game within a relatively short period of time. On the other hand there are numerous areas of technical problems in control engineering and information processing equipment which would profit very much from realizations of some of the theoretically possible schemes.

Instead of elaborating these points and indicating special achievements and expectations we will only describe some specific practical problems we have encountered in connection with the realization of quasi-neural networks. As a matter of fact they boil down to the lack of a technically feasible method to change logical operations. In terms of the threshold element in Fig. 1 it would be necessary to have a simple device functioning as the weight w on the input lines. In principle this should be a three-terminal gadget with one input, one output and one control terminal. Activating the control terminal should change the transfer, or the feed-forward values from the input to the output terminal. This control should be symmetric, that is, it should have the possibility to increase and to decrease the transfer by two opposite signals given to the control terminal. There seems to be a realization of such a device on the basis of reversible electro-chemical coating by a current so as to change a resistance. It is also possible to use photo-chemical properties of material to realize such a device. But up till now we do not know of a complete technical solution of this problem.

The realization of such a device with, if possible, not one but many input terminals so as to be able to carry out at the same time some operations, is however to be expected pretty soon.

As the transfer of such a device would be built up during operation it is at the same time a sort of memory of past events, hence makes it feasible to synthesize learning networks.

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SUMMARY

On some problems in quasi-neural networks.

This mainly tutorial paper gives a definition of quasi neural networks as logical circuits consisting of interconnected threshold elements. This network notion was originally conceived to represent in an abstract manner the logical operations of the central nervous system and became also of interest for automata theory. The Venn-diagram notation for two-valued logic is described. Discussed are problems concerning realizability of logical functions by threshold elements and the reliability of networks built from unreliable elements. The concepts of probabilistic logic and of logical time functions are given. A problem connected with physical realization of quasi-neural networks is pointed out.

To disseminate knowledge is to disseminate prosperity — I mean general prosperity and not individual riches — and with prosperity disappears the greater part of the evil which is our heritage from darker times.

Alfred Nobel