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EUROPEAN ATOMIC ENERGY COMMUNITY - EURATOM
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CONTRIBUTION TO THE THEORY OF THE PULSED NEUTRONS TECHNIQUE APPLIED TO FAST MULTIPLYING SYSTEMS

by

F. STORRER (Association Euratom - C. E. A.)
M. STIEVENART (Belgonucléaire)

1964

Contract No 006-62-1 RAAF
Paper presented at the « Colloque sur les Progrès en Théorie des Réacteurs »
Karlsruhe, Germany, April 23-24, 1963

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CONTENTS

CONTRIBUTION TO THE THEORY OF THE PULSED NEUTRONS TECHNIQUE APPLIED TO FAST MULTIPLYING SYSTEMS

1 — PURPOSE AND PROBLEMS OF PULSED EXPERIMENTS

Pulsed experiments can be used for measuring integral quantities which are of direct interest, such as β/l or the reactivity. They can also be used as clean and flexible integral experiments which, by comparison with theoretical predictions, can be used as a check of fundamental nuclear data or of methods of calculation. In all cases one is interested in the measure of quantities which are characteristic of the fundamental decay mode, such as its decay constant α or the corresponding spectrum. Unfortunately this fundamental decay mode can be obtained only asymptotically in time, since initially it is obscured by transient modes. In some cases one cannot even define a fundamental mode. A main purpose of this theory is to determine whether a fundamental mode exists and, if it exists, to find out if it can be experimentally separated from the transient modes.

2 — VARIOUS TYPES OF TRANSIENT MODES

We limit ourselves in this paper to the treatment of bare systems. Assume a burst of finite duration but of arbitrary spectrum and spatial distribution. Since the system is linear there is no loss of generality if we assume the burst to be a delta function in time. Since the spatial and spectral distributions of the burst neutrons are arbitrary and in general differ from the fundamental ones, spatial and spectral transient modes will be excited.

If it can be assumed that the flux always vanishes at the same extrapolated boundary for all energies, the spatial problem can be treated in a straightforward manner : the spatial distribution of the burst can be developed into the discrete set of spatial eigenfunctions of the system, each component of this development exciting the corresponding flux eigenfunction. The problem being linear the eigenfunctions can be treated independently. From now on we thus limit ourselves to one spatial mode, for example the fundamental one.

For each mode one has then :

$$\varnothing\ (\bar{r},\ \mathrm{E},\ t) = f\ (\bar{r})\ \varphi\ (\mathrm{E},\ t) \tag{1}$$

where $f\ (\bar{r})$ satisfies the equation :

$$\Delta f\,(\bar{r}) + \mathrm{B}^2 f\,(\bar{r}) = 0 \tag{2}$$

B^2 being the geometric buckling for that spatial mode.

The spectral problem can thus be described by the function $\varphi\ (\mathrm{E},\ t)$ which no longer contains the spatial variable.

In small fast assemblies the assumption that the extrapolation length is independent of energy is not very justified. One can as a first approximation take the energy dependence of the extrapolation length into account by using an energy dependent buckling, as suggested by Inönu [1].

3 — BASIS OF THE METHOD

Let us assume that the problem is solved for a non-multiplying medium. When a burst (*) of arbitrary spectrum is injected in such a medium one can then calculate φ (E, t) for that case. We show now that the solution for a multiplying medium can be simply derived from it.

When a burst is injected in a multiplying medium, we choose to look first at the first generation of neutrons, i.e. at those neutrons coming *directly* from the burst, not via a fission. In this « non-multiplying » process one can calculate the evolution of the flux φ (E, t) and thus the source of first fissions :

$$S_1(t) = \int_o^\infty \nu(E)\, \Sigma_f(E)\, \varphi(E, t)\, dE \tag{3}$$

$S_1(t)$ is of course dependent on the spectrum of the burst. Delayed neutrons are always neglected. For a burst having the fission spectrum χ (E), one has similarly a particular case of the function $S_1(t)$ which we denote by K (t).

This function K (t) allows us to calculate the n^{th} generation fission source in terms of the $(n\text{-}1)^{th}$ generation fission source :

$$S_n(t) = \int_o^\infty S_{n-1}(t\text{-}t')\, K(t')\, dt' \tag{4}$$

In Laplace notation this becomes

$$s_n(p) = s_{n-1}(p)\, k(p) \tag{5}$$

Clearly $k(p)$ is the transfer function linking (prompt) fission sources of successive generations, and

$$k(o) = \int_o^\infty K(t)\, dt = k_{\text{eff},\, p} \tag{6}$$

p refers to prompt neutrons.

The total fission source is given by

$$S(t) = S_1(t) + S_2(t) + S_3(t) + \ldots \tag{7}$$

and its Laplace transform by

$$s(p) = s_1(p)\, [1 + k(p) + k_2(\rho) + \ldots] = \frac{s_1(p)}{1 - k(p)}$$

We thus have the desired result for a multiplying medium :

$$\boxed{s(p) = \frac{s_1(p)}{1 - k(p)}} \tag{8}$$

where s_1 and k have been obtained using the theory of a non-multiplying medium, plus of course the simple integration (3).

(8) is a generalization of the expression giving the amplification of a static source in a subcritical assembly.

(*) We always assume a burst with total intensity normalized to unity.

The flux can of course be obtained from the total source (including the initial burst) in a straightforward manner.

Having solved the multiplying problem in terms of the non-multiplying one, we now have to solve the latter, and investigate its implications for the former.

4 — MULTIGROUP TREATMENT OF THE NON-MULTIPLYING PROBLEM

Using the n-group treatment the n-component flux vector $\bar{\varphi}(t)$ will behave as

$$\bar{\varphi}(t) = \sum_{i=1}^{n} \bar{A}_i e^{\gamma_i t} \tag{9}$$

The γ's can be obtained by setting equal to zero the determinant of the multigroup matrix, where they appear in each diagonal element. The advantage of the non-multiplying system lies in the fact that the matrix is then triangular (no up-scattering) and that the determinant reduces simply to the product of the diagonal elements. The n values of γ are then obtained by setting equal to zero each diagonal element. We thus have :

$$\gamma_i = -v_i \ (D_i B^2 + \Sigma_{ai} + \Sigma_{ri}) = -v_i \Sigma_i \tag{10}$$

a refers to absorption and r to removal by scattering to other groups. In (10) we also define Σ_i.

The Laplace transform of the flux vector has poles at $p = \gamma_i$, and the A 's are the corresponding residues. Only the magnitude of the A 's depends on the spectrum of the burst, and not their « direction ». In this paper we are mainly interested in the poles ; the residues can be found in a classical manner.

5 — MULTIGROUP TREATMENT OF THE MULTIPLYING MEDIUM

Consider equation (8). Both $s_1(p)$ and $k(p)$ have the same poles at γ_i, given by (10), where Σ_{ai} includes the fission cross-section. Since these functions appear respectively in the numerator and in the denominator, the function $s(p)$ does not have poles at the γ 's. The poles of $s(p)$ are given by the n solutions

$$\alpha_i \text{ of } \boxed{k(p) = 1} \tag{11}$$

This equation can be solved very easily. One could have also found the α_i 's by looking directly for the eigenvalues of the complete matrix corresponding to the multiplying system, but this last method is much more involved. Another advantage of equation (11) is that it gives a better insight into the localization of the α_i 's.

Indeed let us look at fig. 1, which shows a plot of $k(p)$ versus real values of p, as calculated with eleven groups for a 93 % enriched sphere of uranium having a diameter of 16,3 cm. $k(p)$ tends to zero when p tends to $+\infty$ or $-\infty$, and becomes infinite at its poles $\gamma_i = -v_i \Sigma_i$, which are the decay constants for the non-multiplying system. In this particular case there are n-1 values of α_i, one between each pair of poles γ_i, and the last α_1 is well separated. α_1 is on the right of all the poles γ_i, and hence of all the other α_i 's. It is negative or positive depending on whether $k(o) = k_{eff,\ p}$ is smaller or larger than unity.

It is clear that α_1 corresponds to the fundamental or persistent mode. If the number of groups increases the number of γ' s and α' s increases accordingly. The γ' s, and consequently also n-1 values of α bunch up closer and closer together, while α_1 does not change significantly. Of course when the groups become very narrow the medium can no longer be considered

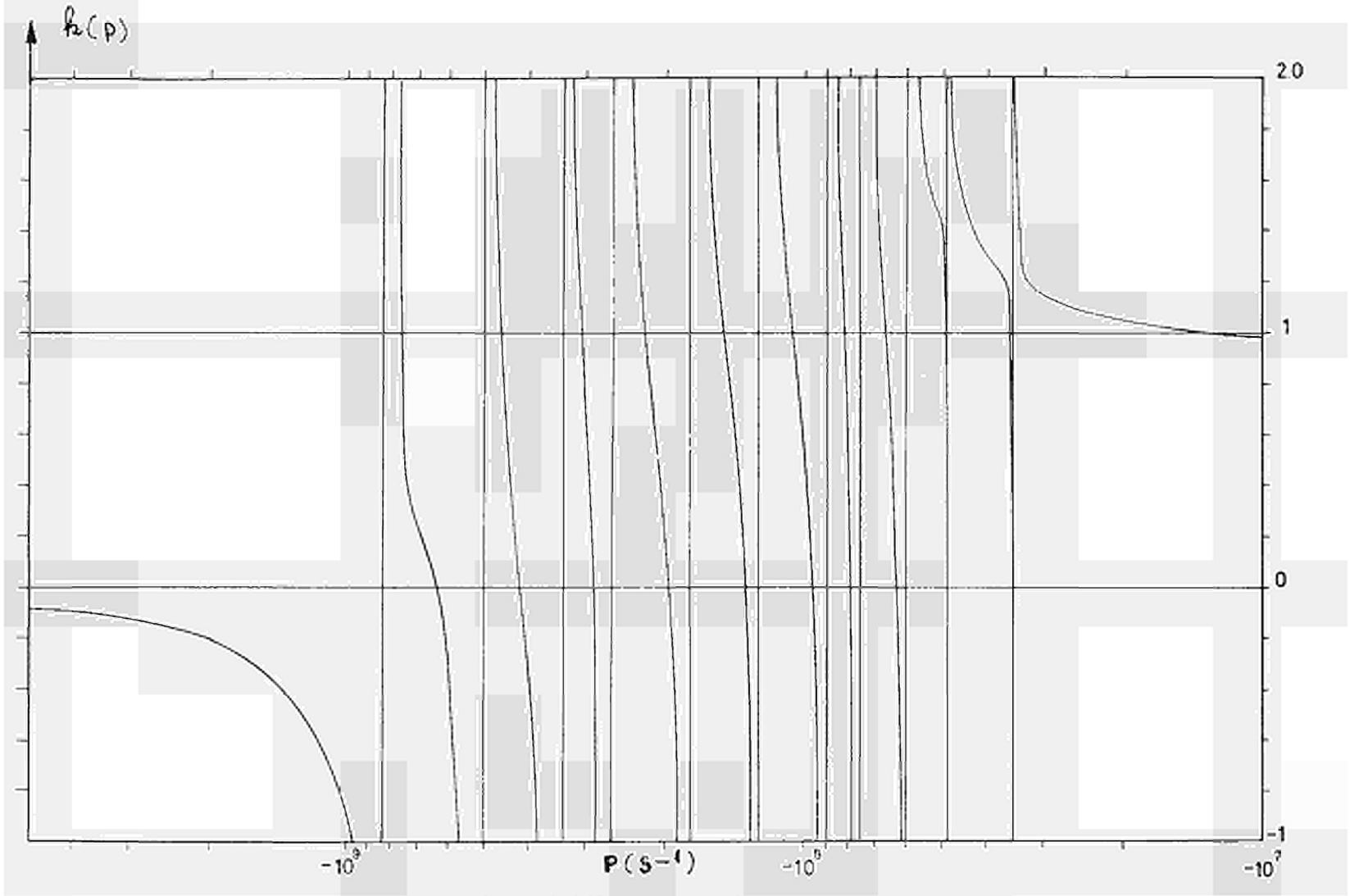


Fig. 1 — 93% enriched U sphere (D = 16,3 cm) ANL 11 group set.

« weakly absorbing », and our multigroup model does no longer hold. One can show that in that case γ_i tends in absolute value to an upper limit $v_i\Sigma_{ti}$, where Σ_{ti} is the total cross-section, unless the linear dimensions of the assembly are small compared to the mean free path.

The multigroup treatment is of course not very suitable to an extrapolation to a continuous energy model, but by inference from what has been said one can expect that in the realistic continuous energy model one would have only one true mode, corresponding to α_1, while the sum of all the other modes has to be replaced by an integral over « continuum » modes.

Going back to the multigroup model, let us look how γ_i varies with energy. Fig. 2 shows a plot of $-\gamma_i = v_i\Sigma_i$ as a function of i in an eleven group model, for similar spheres but with diameters of 13 and 17.6 cm. One can see that the smallest values of $v_i\Sigma_i$ come from the lowest energies, essentially because of the factor v. Hence it is clear that the transient α which is closest to α_1, is of the order of $- v_i\Sigma_i$ for the lowest energy neutrons.

Looking at fig. 3, which shows a plot of $k(p)$ for a sphere of 13 cm diameter, one sees that α_1 is practically equal to γ_n, the decay constant for the lowest energy group. Obviously this calculated value of γ_n, and hence of α_1, is very sensitive to the group structure which has been chosen. Hence this value of α_1 is practically meaningless. We will show now that in some cases some other α_i may correspond to a measurable and meaningful « pseudo » persistent mode.

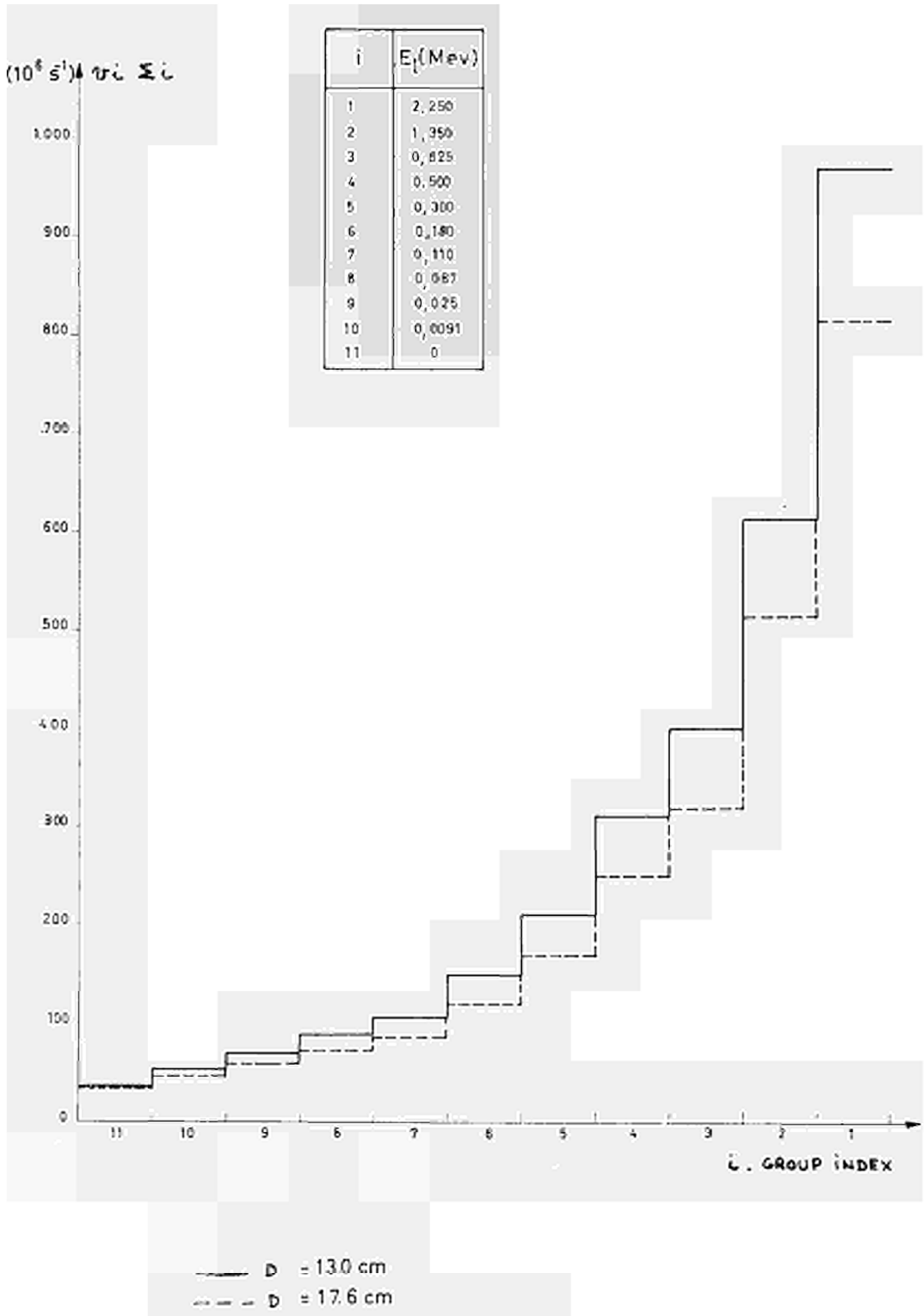
Imagine a pulsed assembly in which one can show that only a very small fraction of the neutrons — say one in 10^9 — ever slows down below some energy — say 10 ev. Imagine further that an adequate group structure has been chosen, extending down to only 10ev, such that no group is too wide, particularly at energies just above 10 ev. Assume that one has calculated an α_1 well on the right of all the γ' s, and hence of all the other α 's. One can expect that this α_1 corresponds to a true persistent mode, and that its value is quite insensitive to a change in the group structure. If one whishes to take into account the very few neutrons which slow down below 10 ev, one may want to add another group below this energy. Assume that for this new group the $\gamma_n = -v_n\Sigma_n$ is on the right of the previously obtained α_1. If we denote the new α 's by primed quantities, we show now that :

$$\begin{aligned} \alpha'_1 &\approx \gamma_n \\ \alpha'_2 &\approx \alpha_1 \end{aligned} \tag{12}$$

Indeed as a result of the addition of this new group, $k(p)$ has a new pole at $p = \gamma_n$, but the corresponding residue is very small since only a very small fraction of the neutrons is involved. Thus the function $k(p)$ is appreciably modified only when p is very close to γ_n. The new pole gives rise to a new value α_1, very close to γ_n (as is the case on fig. 3), and the old α'_1 being far enough from γ_n remains unchanged, but since it is now the second one it becomes α'_2. Looking now at the function $s(p)$ as given by (8), we see that the residue at any pole α_i, which gives the intensity of the corresponding mode, is given by :

$$r(\alpha_i) = - \frac{s_1(\alpha_i)}{\left[\dfrac{dk(p)}{dp}\right] p = \alpha_i} \tag{13}$$

At the pole α'_1 which we just considered the denominator is very large, while the numerator is not large, except if the burst spectrum is very poorly chosen. If for example the initial burst spectrum is the fission spectrum, the numerator is $s_1(\alpha_1) = k(\alpha_1) = 1$. Thus we see that the residue at α_1 is very small, and thus that this mode, even though it decays slower than the mode corresponding to $\alpha'_2 \approx \alpha_1$, has a very small amplitude. This could be expected since very few neutrons slow down to energies which excite this mode.



i	E_i (Mev)
1	2, 250
2	1, 350
3	0, 825
4	0. 500
5	0, 300
6	0, 180
7	0, 110
8	0, 067
9	0, 025
10	0, 0091
11	0

Fig. 2

More work should be done in this field, including numerical evaluations of residues involving the adjoint fluxes, but we feel that pseudo persistent modes could be defined and measured, which decay faster than other « parasitic » modes which are too small to be measurable. These last modes would probably be excited for dilute systems by neutrons with

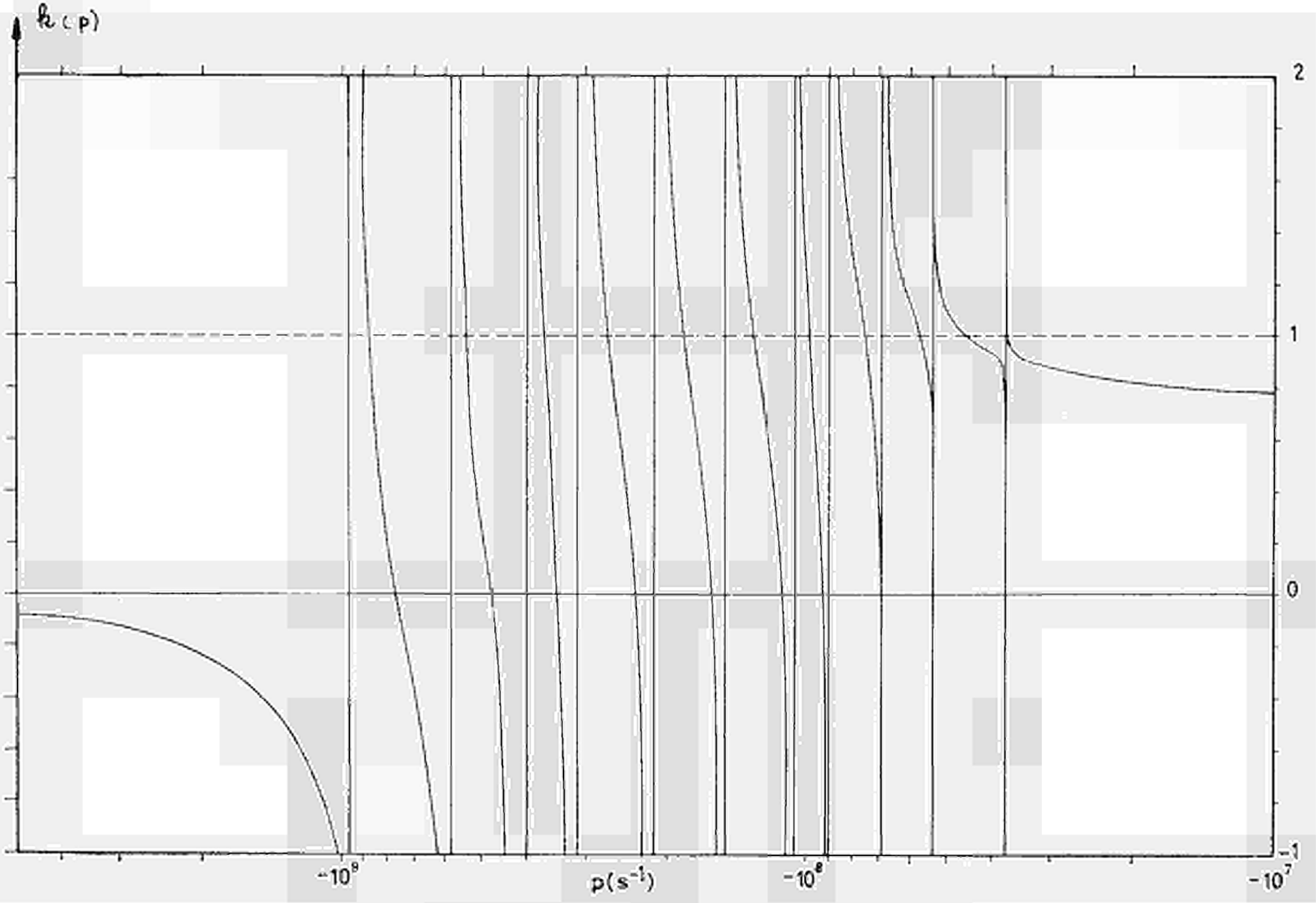


Fig. 3 — 93 % enriched U sphere (D = 13,0 cm) ANL 11 group set

energies between the thermal and resonance regions, where $v\Sigma$ has a minimum but where almost no neutrons are present. The calculation of the pseudo persistent α would best be done by using a group structure with a lower bound somewhere at the lower edge of the resonance region. For more concentrated systems, such as the 93 % enriched uranium sphere, having a harder spectrum, this lower bound should be chosen at a higher energy. If the « parasitic » modes give trouble in the experiments, they could probably be minimized by the addition of a poison having a large capture cros-section for the « parasitic » neutrons.

Figures 4 to 7 further illustrate what has just been discussed.

Fig. 4 shows $k(p)$ for three different group structures and for 2 assembly diameters. The data for each assembly have been normalized so as to give the same $k(o) = k_{eff,\,p}$. For the 13 cm diameter case, we see that the six group model gives a pole α_1 with a large residue, since $-\left(\frac{dk}{dp}\right)_{p\,=\,\alpha_1}$ is small. The three group model gives a different α_1 because it does not give a good enough description of the process. The eleven group model gives an α'_1 with a very small residue, since $-\left(\frac{dk}{dp}\right) p = \alpha'_1 \approx \infty$ and an α'_1 identical to the α_1 of the six group model, and of course with the same residue. This seems to indicate that the eleventh group, extending from 0 to 9.1 kev, gives rise to a parasitic mode. Fig. 5 shows α_1 as a function of diameter for the three group structures. The circles indicate experimental values of the persistent α for Godiva. Except for some inaccuracies in either calculations or experiments it seems that using more groups does not always yield better results, if these groups are poorly chosen. The interpretation is easy with the help of fig. 6. For diameters larger than 14.5 cm, α_1 is sufficiently on the right of γ_n for the eleventh group (denoted by $-\gamma$ (0-9.1 kev) on fig. 6) to be a true persistent mode with a large residue. (Example D = 17.6 cm of fig. 4 or D = 16.3 cm of fig. 1). For smaller values of D (D = 13 cm of fig. 3 and 4) the « pseudo »-persistent α is on the left of γ_n and $\alpha_1 \approx \gamma_n$ corresponds to a parasitic mode with a very small residue. The six and three group models approximate better the experimental results, but a still better approximation could be obtained by using more and *better chosen* groups. Fig. 7 shows α-curves for the same system for another 11-group set. Because the residues are very small for the last few groups, the corresponding α-curves practically line up to give a curve approximating the pseudo-persistent α. Such a curve could be obtained directly as an α_1 curve by choosing a different group structure (for instance no groups below 9.1 kev)

As a conclusion to this part of the paper we can say that :

1 — If α_1 is well on the right of the minimum value of $v\Sigma$ for all energies down to thermal it indicates a true persistent mode. This condition however is satisfied only for assemblies which are not subcritical by more than a few percent.

2 — If it is desired to extend the pulsed technique to less reactive systems one can probably define and measure pseudo-persistent modes.

3 — For very small or very subcritical systems, discrete modes probably no longer exist. The same is true for non-multiplying systems whatever their size, except in the thermal range where the upscattering caused by thermal agitation plays a role similar to the fission process.

We have given some qualitative considerations suggesting that there is a range in which pseudo persistent modes can be defined, as was said under 2. However we have not proven and hence we are not sure that what we denoted by pseudo persistent mode corresponds to a true discrete mode. This problem should be tackled using a continuous energy model, probably with approximate analytic functions to represent the energy dependence of the cross-sections. We suspect that the existence of a discrete pseudo-persistent mode requires a sharp

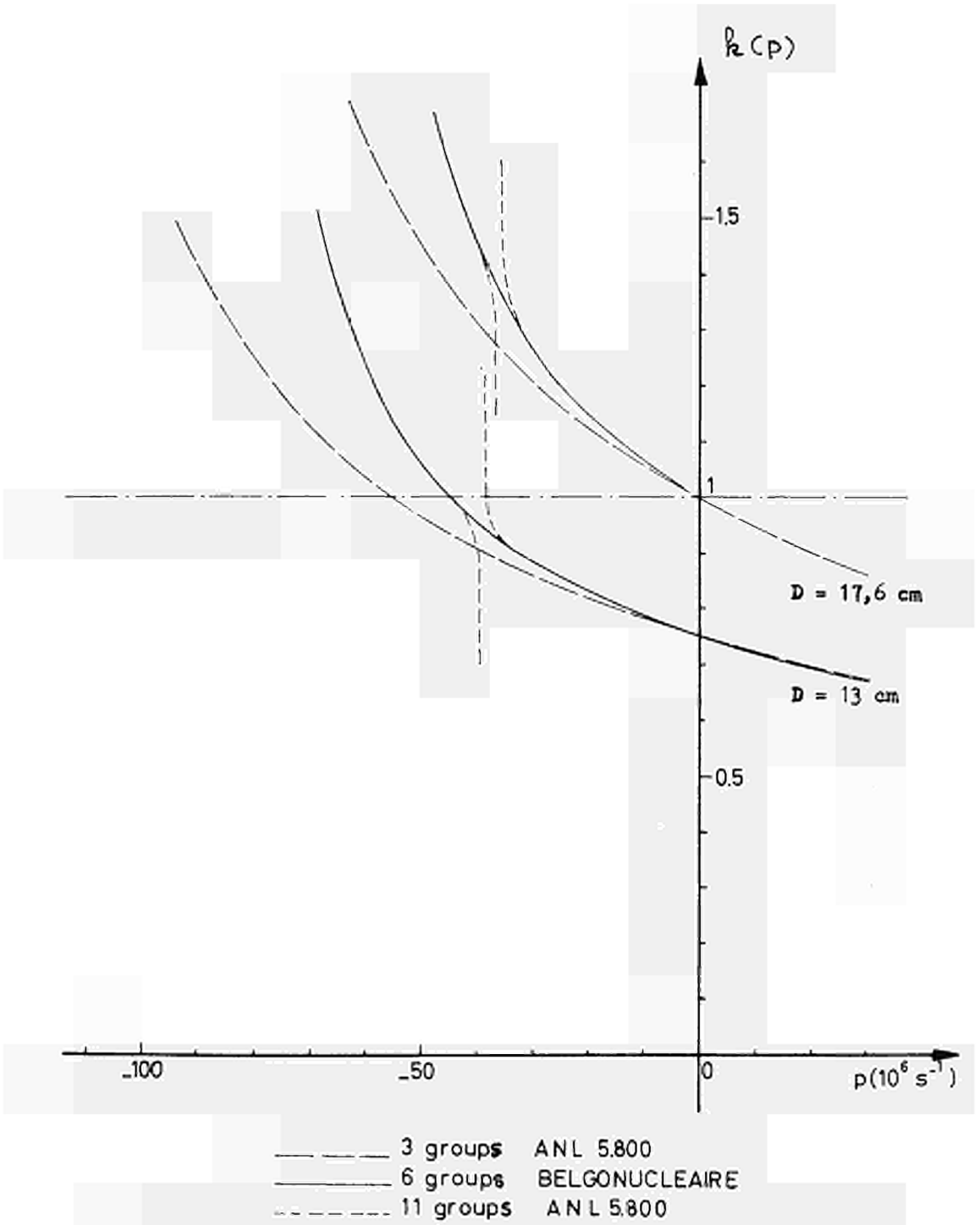


Fig. 4 — $k(p)$ for two 93 % enriched U spheres, using three different group structures

drop of $\nu\Sigma$ by going down from the energy range where most of the fissions occur to the range where $\nu\Sigma$ reaches a minimum.

The existence of a pseudo-persistent mode was also envisaged by G. de Saussure [2] in the case of thermalization in a moderating medium. There also a sharp variation of cross-section was invoked. M. Nelkin [3] also investigated this problem and remarks that observed decay constants which exceed the minimum collision rate « presumably do not correspond to proper fundamental mode solutions of the transport equation ».

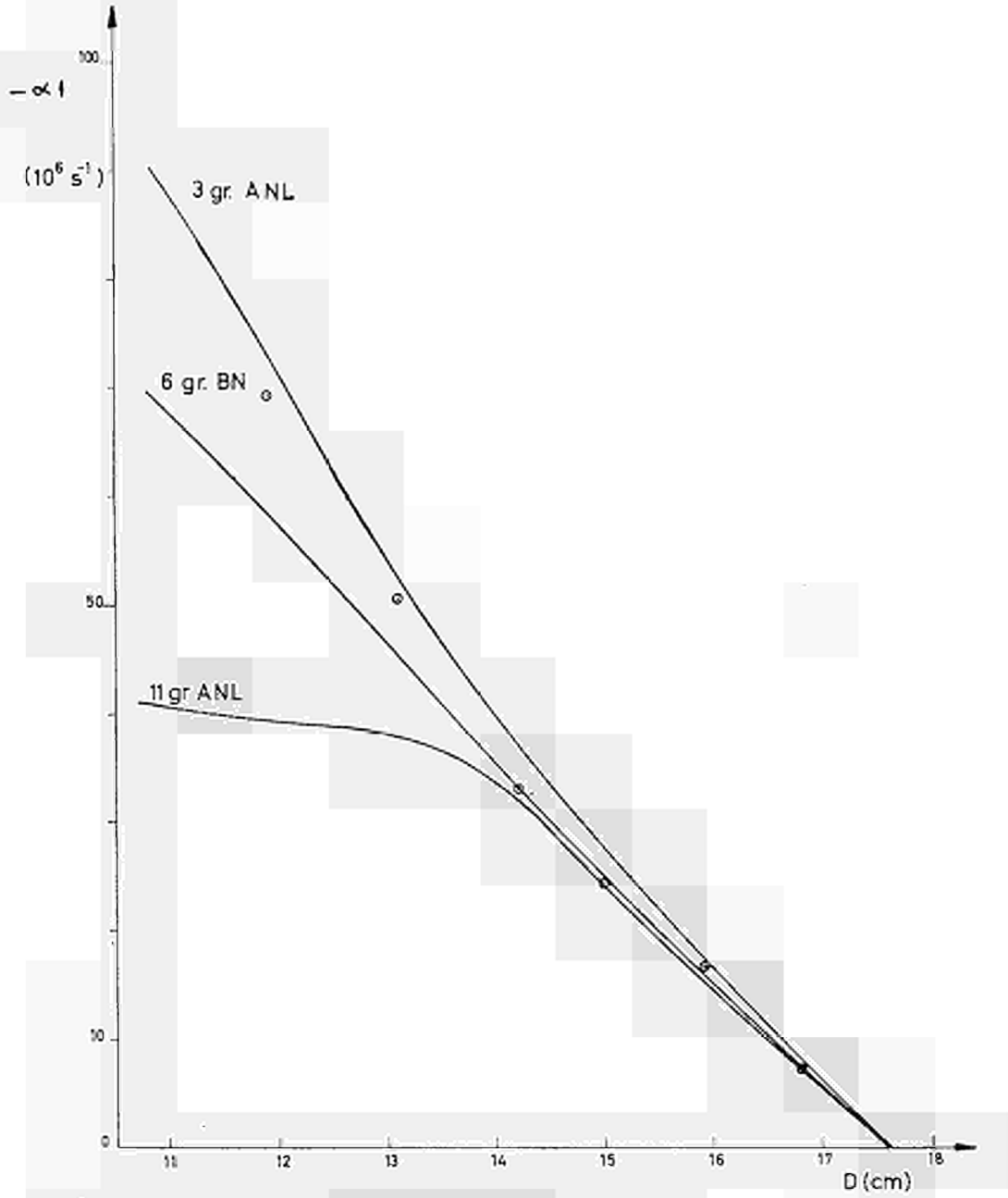


Fig. 5 — 93% enriched U spheres

α_1 versus D using three different group structures

⊙ experimental results

We feel it is essential to settle this question if the pulsed technique for fast neutron multiplying systems is not to be restricted to very slightly subcritical assemblies.

In order to have a rough idea of the range of $k_{eff,\ p}$ for which $|\alpha_1| < (v\Sigma)_{min}$ we use the approximate expression

$$\alpha_1 = -\frac{1 - k_p}{l} = -(1 - k_p)\, v\, (\Sigma_a + DB^2) \qquad (14)$$

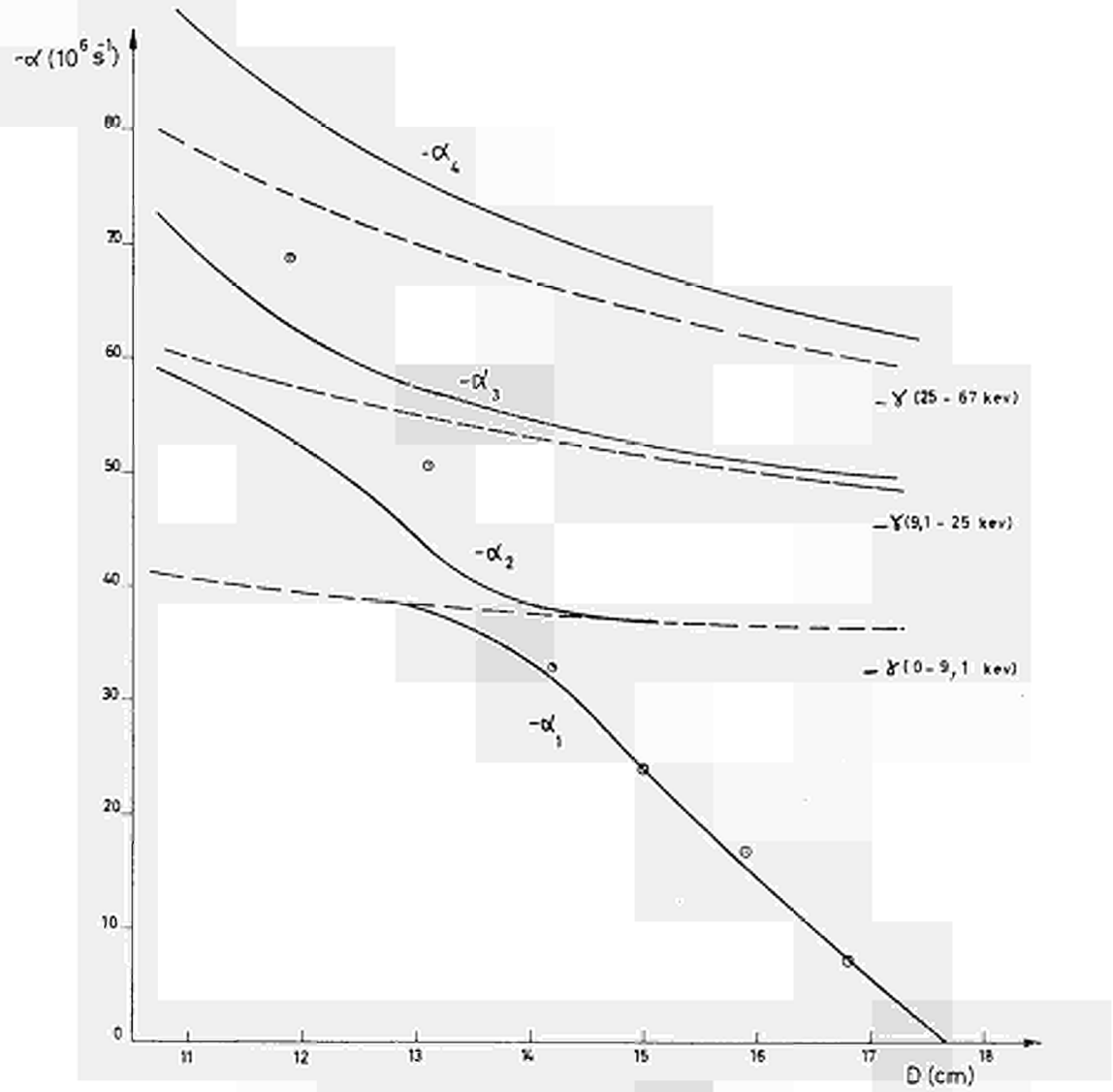


Fig. 6 — 93 % enriched U spheres
α_1, α_2, α_3, α_4 versus D, using ANL 11 group set

where the bar represents some average over the energy. The desired range is thus given approximately by

$$(1 - k_p) \frac{\overline{v(\Sigma_a + DB^2)}}{(v\Sigma)_{min}} < 1 \tag{15}$$

For any energy $v\Sigma > v(\Sigma_a + DB^2)$ (16)

but unfortunately the numerator refers to an average value while the denominator refers to a minimum value.

As a guide to the interpretation of the previous figures, figure 8 shows a plot of $k_{eff,\ p}$ versus diameter for 93 % enriched assemblies.

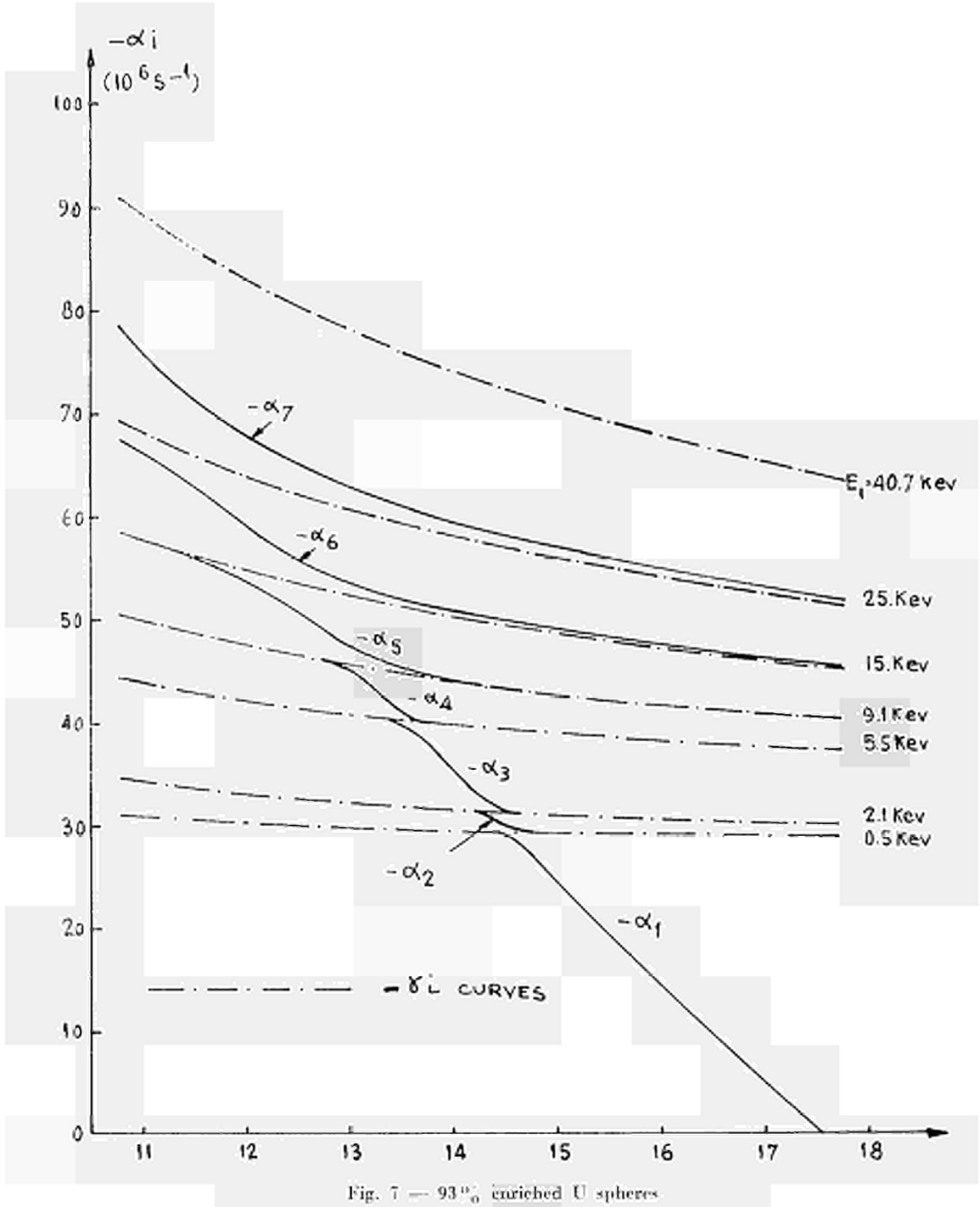


Fig. 7 — 93% enriched U spheres
α_i $(i = 1, 2 \ldots 7)$ versus D, using modified 11 group set

6 — FURTHER ILLUSTRATIONS OF THE METHOD

Figures 9, 10 and 11 show eleven group plots of $k(p)$ for three *natural uranium* spheres of different diameters. As can be seen α_1 for these very highly subcritical (*) systems is identi-

(*) The value of $k_{eff,\,p}$ can be read from the graph by extrapolating $k(p)$ to $p = 0$.

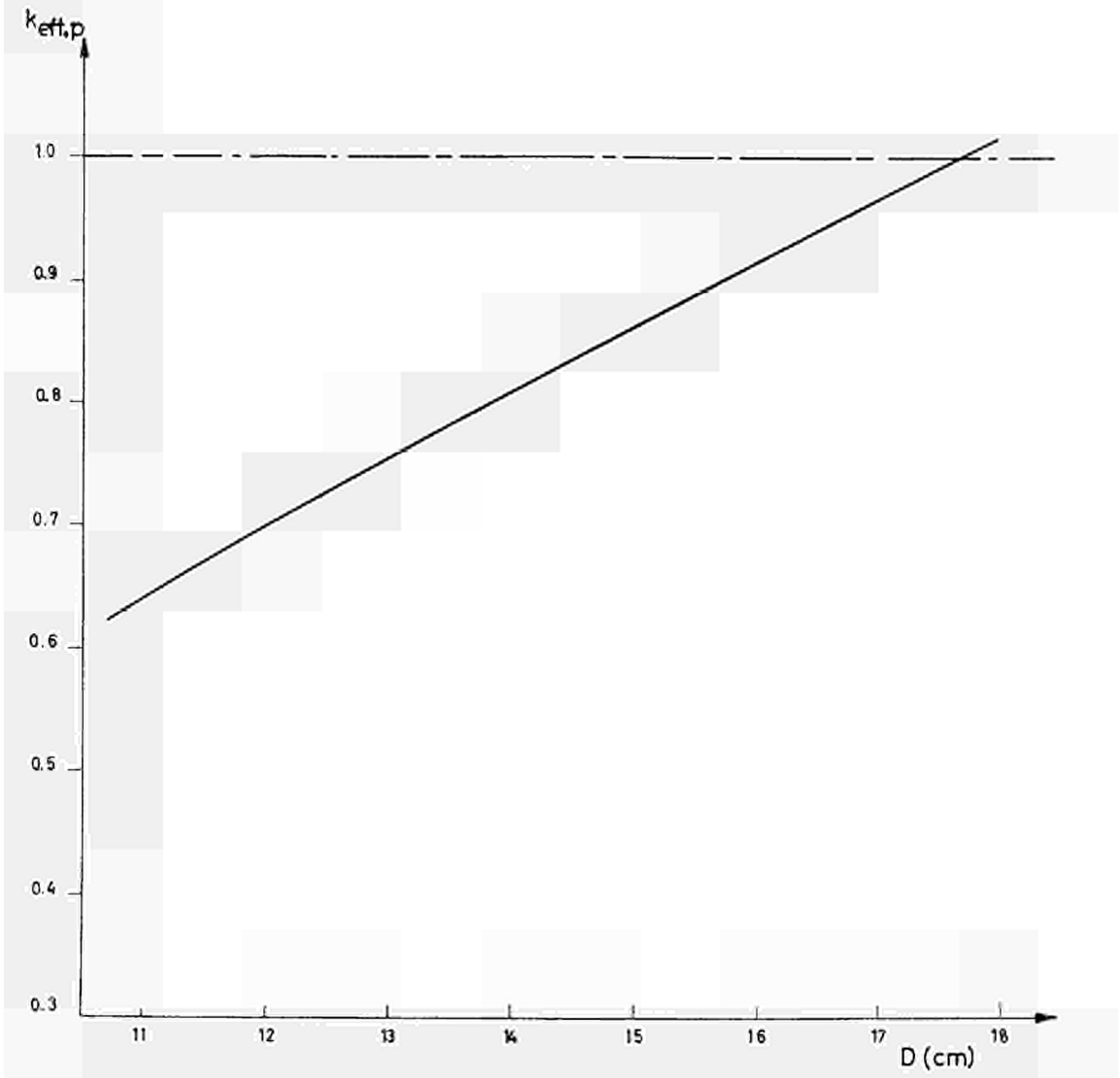


Fig. 8 — 93 % enriched U sphere
$k_{eff,\,p} = k(0)$ versus D, using ANL 11 group set

cal to γ_n, and the corresponding residue is negligible. It of course does not correspond to a persistent mode, and we doubt that one could define a pseudo persistent mode for such systems. Here again a better group structure should be tried.

The interesting feature of these plots is the fact that the residue at some poles of $k\,(p)$ is negative.

As a result of this, one has in some cases two values of α in an interval between two successive values of γ, and no α in an adjacent interval. On fig. 9 this happens twice, all α 's being real. On fig. 10 the curve $k\,(p)$ is at one point tangent to $k\,(p) = 1$, and this leads to a double root. On fig. 11 one has two complex conjugate α 's. Thus modes appear which tend to zero in a damped oscillation. It cannot be said at this time whether such oscillations could actually occur in a pulsed assembly or whether they are introduced artificially by the multigroup model.

Fig. 12 shows a plot of $k\,(p)$ for a 112.2 cm diameter sphere of *pure U*-238, using the same eleven group model. Because of the fission threshold, only three groups appear in the calculation of $k\,(p)$, which thus exhibits only three poles.

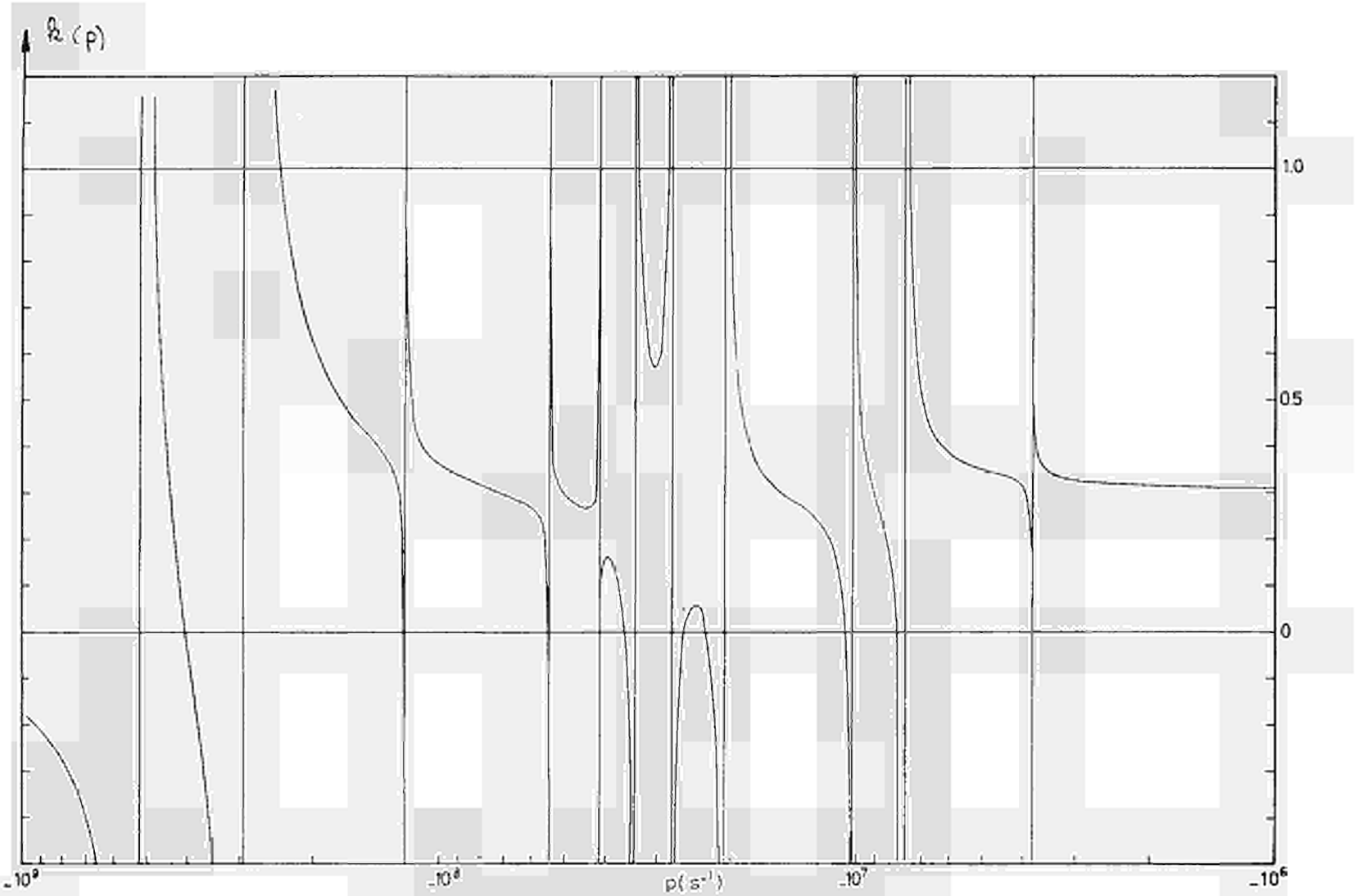


Fig. 9 — NAT U sphere (D = 52,08 cm) ANL 11 group set

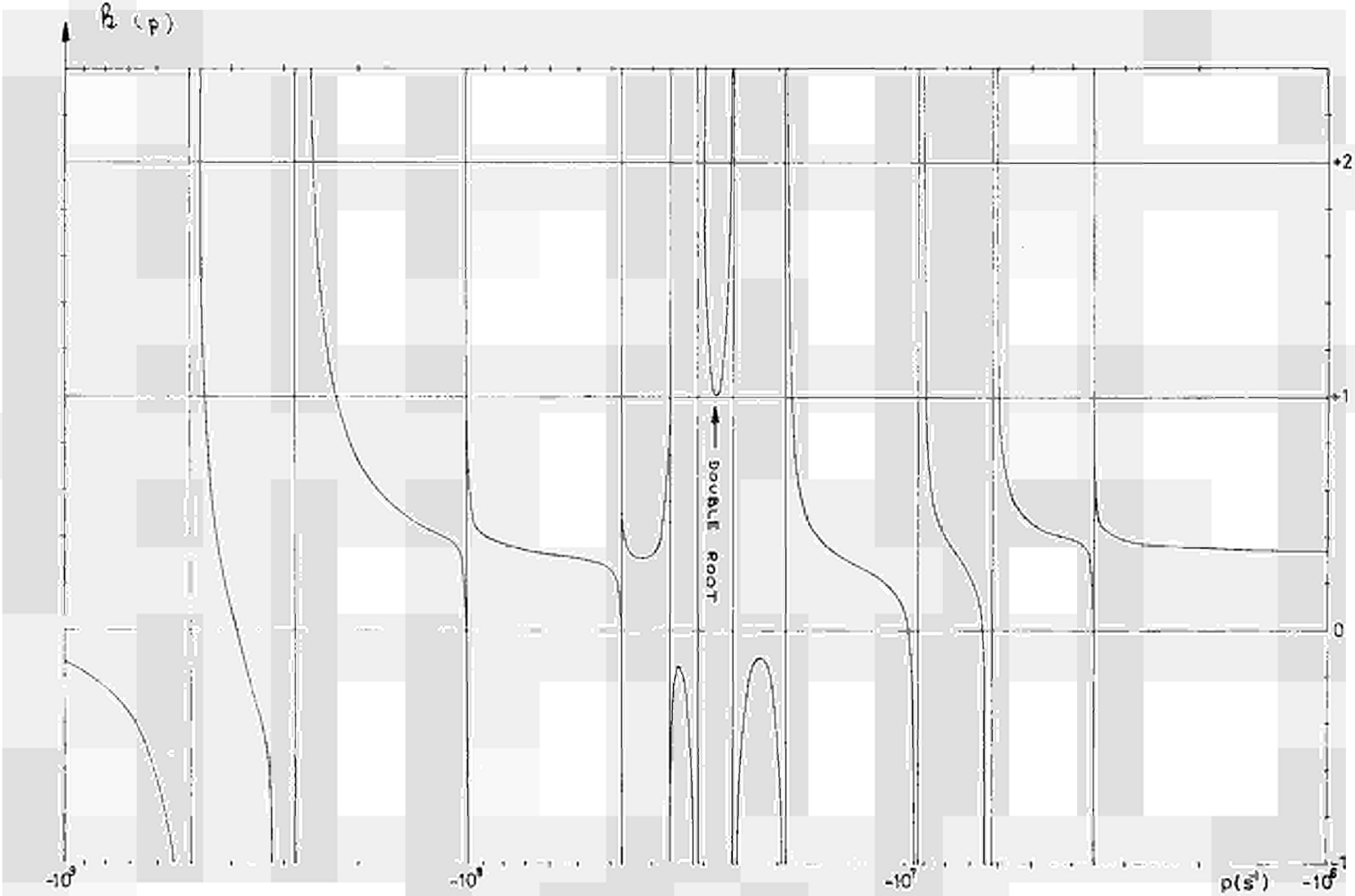


Fig. 10 — NAT-U sphere (D = 71.0 cm) ANL-11 group set

Fig. 11 — NAT U sphere (D = 112,2 cm) ANL 11 group set

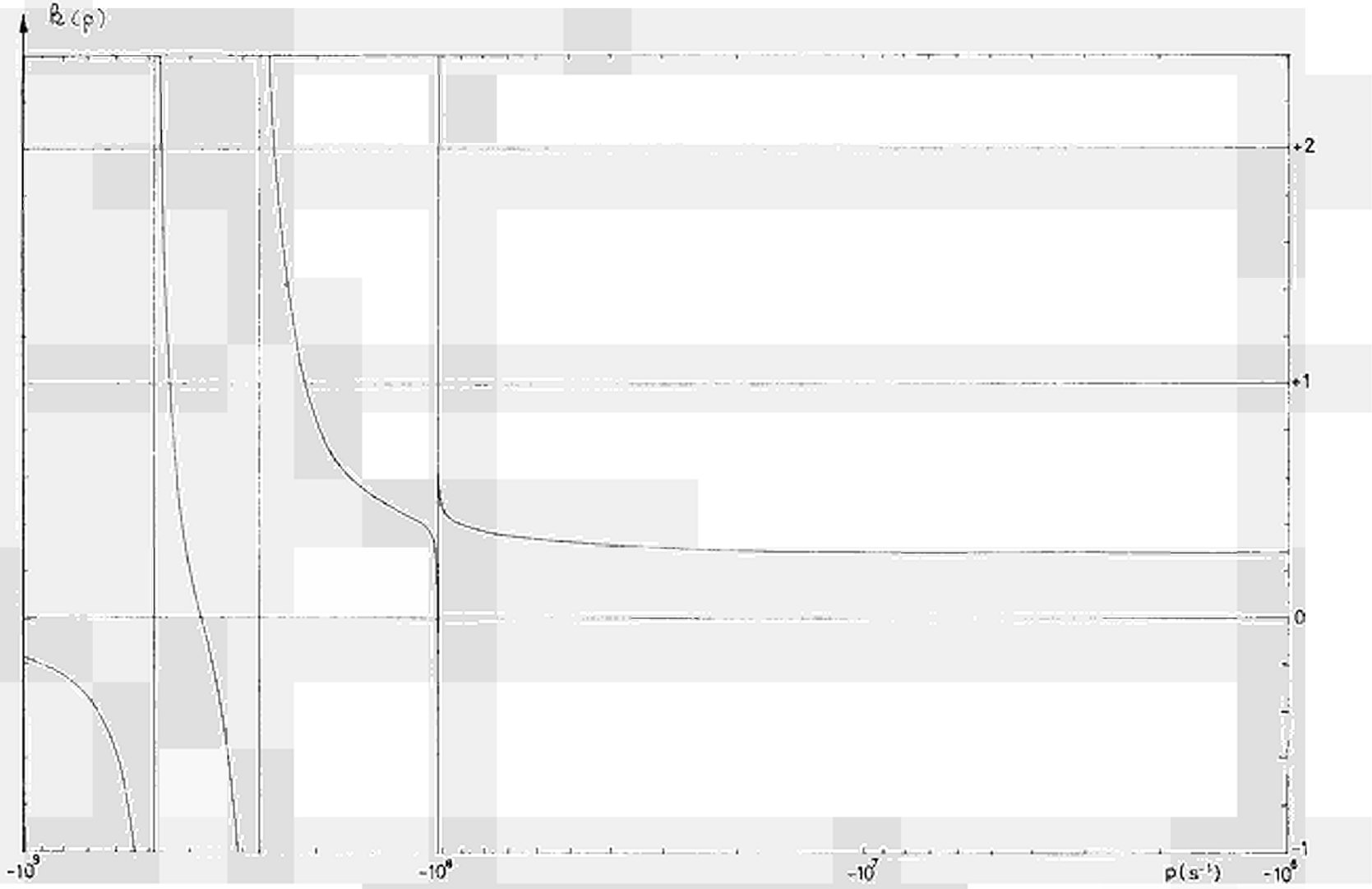


Fig. 12 — Pure U-238 sphere (D = 112,2 cm) ANL H group set

Obviously α_1 corresponds to a parasitic mode (*), but we think that α_2 (about -2.10^8 sec.$^{-1}$), which is accompanied by a large residue probably corresponds to a measurable and discrete pseudo-persistent mode for neutrons above the fission threshold and for the fissions themselves. The neutrons below the fission threshold will decay slower and not according to discrete modes, just as in a non-multiplying medium. Thus we have here a very interesting case where neutrons above some energy may decay according to a discrete mode (plus transient « continuum » modes) and where neutrons below that energy have no discrete modes at all.

When we compare the case of pure U-238 to the case of natural uranium which was just discussed, we see that the former, at least above the fission threshold, probably allows adiscrete mode while the latter does not, even though it is slightly more reactive. The reason can be found qualitatively by looking at inequality (15). Even though $(1-k_p)$ is slightly larger for pure U-238, the ratio $\dfrac{\bar{\nu}\,(\Sigma_a + DB^2)}{(\nu\Sigma)_{min}}$ is much smaller, because of the much narrower range of energies involved in the fission process. However even for pure U-238 the $(\nu\Sigma)_{min}$ is small enough to give rise to a parasitic mode. A better calculation of the pure U-238 case should be performed with a finer group structure just above the fission threshold.

7 — CONCLUSION

The theory of the pulsed technique applied to fast neutron multiplying assemblies is difficult, primarily because it is impossible to describe simply the variation of cross-sections with energy. We have used the multigroup approach which can to some extent describe this variation but which is only approximate at best. We think that the method introduced in this paper, besides yielding a rather easy calculational procedure to determine the eigenvalues, gives a better insight than the conventional method into the localization of these eigenvalues with respect to known quantities, and hence a better understanding of the problem. More work should certainly be done with the multigroup model, and an attempt should be made at solving some simplified continuous energy models. It should be stressed that the basis of our method could still be used with a continuous energy model, so that the results obtained for a non-multiplying system could be used as an essential step towards solving the case of a multiplying system.

As was briefly mentioned in the summary we are presently using a similar method for the analysis of exponential experiments on fast neutron multiplying systems. Here again one has transient modes of two kinds : spatial and spectral. In the analysis of the latter we also develop the theory of multiplying systems from that of non-multiplying systems. It turns out that Σ_{min} rather than $(\nu\Sigma)_{min}$ is the important quantity for this experiment. It will then be possible to determine the appropriate experiment for each case, considering the existence of asymptotic modes and other important factors such as the required inventory of fissionable material, instrumentation problems and hardness of the asymptotic spectrum, which for a same medium is much softer in a pulsed experiment.

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(*) The small residue for this pole follows from the fact that the average fission cross-section is small for the corresponding group.