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ELECTRIC QUADRUPOLE TRANSITIONS BETWEEN SOME EXCITED STATES IN SPHERICAL EVEN-EVEN NUCLEI

by

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1963

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Electric Quadrupole Transitions between Some Excited States in Spherical Even-Even Nuclei (*).

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(ricevuto il 21 Maggio 1962)

Summary. — The branching ratios from the higher excited states in spherical even-even nuclei are calculated using a simplified model, which includes only a single j shell, and a pairing plus quadrupole residual interaction between nucleons. The method is applied to the branching from the 4+ levels: it is found that the $E2$ transition to the second 2+ level can be enhanced against the corresponding one to the first 2+ level for more than one 4+ level.

1. – Introduction.

It has been shown recently by VAN LIESHOUT, RICCI and GIRGIS (1) that several spherical even-even nuclei exhibit an interesting regularity in the decay of some higher excited states. If we consider, for instance, the $E2$ decay of a given 4+ level and introduce the branching ratio:

$$F_x = \frac{B(E2)_{x_1}}{B(E2)_{x_2}},$$

(*) This research was sponsored in part by the Fondazione Beneduce, Napoli. It is also a part of the research programme of the Nuclear Spectroscopy Group of Naples, working under a contract between EURATOM and C.N.E.N.

(1) R. VAN LIESHOUT, R. A. RICCI and R. K. GIRGIS: *Nuovo Cimento*, 21, 379 (1961).

of the decays into the two lower $2+$ excited states, ϖ labelling the $4+$ level of interest (Fig. 1), it appears that F_ϖ can be $\gg 1$ for more than one $4+$ level.

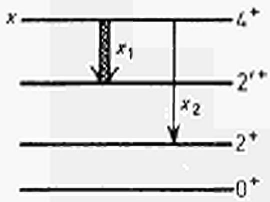

Fig. 1. – Transitions from a 4^+ level.

A qualitative explanation of these enhancements has been suggested by RICCI, JEAN and VAN LIESHOUT [2], based on a simple model of « pairing plus quadrupole force » coupling [3,4], in the case of a single j and only one kind of nucleons [5].

Our purpose, in the present note, is to provide support to that explanation by detailed calculations of the $E2$ branching ratios. We are aware that the model is too drastic and cannot be applied without further refinement to actual nuclei. The more realistic case of various j shells and two kinds of nucleons [6] will be investigated later.

2. – The model.

We introduce the following well-known approximations [7] (we do not want to discuss here their validity):

i) The quasi-particle approximation. The creation operator of a quasi-particle β_m^+ is defined by the Bogoljubov-Valatin [8] transformation:

$$\beta_m^+ = u b_m^+ - (-)^{l+j-m} v b_{-m}\,,$$
$$u^2 + v^2 = 1\,,$$

where b_m^+ is the creation operator of a particle in the state j, m. In our single-j case, $v^2 = N/2\Omega$ and $E = \frac{1}{2}G\Omega$, where N is the number of particles in the shell, $\Omega = j + \frac{1}{2}$ the pair capacity of the shell, G the strength of the pairing force and E the excitation energy of a quasi-particle.

ii) Quasi-boson approximation. We define:

$$B_\mu^{(s)+} = \frac{1}{\sqrt{2}} \sum_{mm'} \langle jj\,mm' | s\mu \rangle \beta_m^+ \beta_{m'}^+\,, \qquad s = 2, 4 \ldots 2j-1\,,$$

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(2) R. A. RICCI, M. JEAN and R. VAN LIESHOUT: *Nuovo Cimento*, **25**, 1389 (1962).
(3) L. S. KISSLINGER and R. A. SØRENSEN: *Mat. Fys. Medd. Dan. Vid. Selsk.*, **32**, no. 9 (1960).
(4) R. A. SØRENSEN: *Nucl. Phys.*, **25**, 674 (1961).
(5) B. R. MOTTELSON: *The Many Body Problem* (Paris, 1959).
(6) T. TAMURA and T. UDAGAWA: *Prog. Theor. Phys.*, **26**, 947 (1961).
(7) B. F. BAYMAN: *Lectures on Seniority, Quasi-particles and Collective Vibrations* (1960).
(8) S. T. BELYAEV: *Mat. Fys. Medd. Dan. Vid. Selsk.*, **31**, no. 11 (1959).

and suppose

$$[B_\mu^{(s)}, B_{\mu'}^{(s')+}] = \delta_{ss'}\delta_{\mu\mu'} .$$

$B_\mu^{(s)+}$ acts then like the creation operator for a boson of spin s; the approximation seems reliable only when the number of quasi-particles is small with respect to the capacity Ω of the shell.

iii) The quadrupole force is introduced according to the linearization method [9-11], *i.e.* only quadratic terms like B^+B and BB (and Hermitian conjugates) are taken into account. With the approximations i), ii) and iii) the Hamiltonian is written as:

$$H \quad = H_{s\neq 2} + H_2 ,$$

$$H_{s\neq 2} = 2E \sum_{s\neq 2,\,\mu} B_\mu^{(s)+} B_\mu^{(s)} ,$$

$$H_2 = 2E \sum_\mu B_\mu^{(2)+} B_\mu^{(2)} - \tfrac{1}{2}\chi Q^2 \sum_\mu (-)^\mu [B_\mu^{(2)+} + (-)^\mu B_{-\mu}^{(2)}][B_{-\mu}^{(2)+} + (-)^\mu B_\mu^{(2)}] ,$$

with

$$Q = (-)^l uv \sqrt{\tfrac{2}{5}}\, \langle j \| r^2 Y^{(2)} \| j \rangle .$$

The second term in H_2 comes from the quadrupole force between two nucleons:

$$V_{ab} = -\chi r_a^2 Y_a^{(2)} \cdot r_b^2 Y_b^{(2)} .$$

If we define a vector operator $\boldsymbol{J}$ by:

$$J_x = \tfrac{1}{4}(S_2^+ - S_2) = -J_x^+ ,$$

$$J_y = -\frac{i}{4}(S_2^+ + S_2) = -J_y^+ ,$$

$$J_z = \tfrac{1}{2}(S_1 + \tfrac{5}{2}) = J_z^+ ,$$

with

$$S_1 = \sum_\mu B_\mu^{(2)+} B_\mu^{(2)} \quad \text{and} \quad S_2 = \sum_\mu (-)^\mu B_\mu^{(2)} B_{-\mu}^{(2)}$$

then

$$H_2 = -5E + 4(E - \tfrac{1}{2}\chi Q^2) J_z - 2i\chi Q^2 J_y .$$

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[9] M. BARANGER: *Phys. Rev.*, **120**, 957 (1960).
[10] R. ARVIEU and M. VENERONI: *Compt. Rend.*, **280**, 992 (1960).
[11] M. KOBAYASI and T. MARUMORI: *Prog. Theor. Phys.*, **23**, 387 (1960).

In order to find the normal modes of the system, we perform a canonical transformation characterized by the operator $U = \exp[-2\alpha J_x]$. Expanding U into a power series, the transformed Hamiltonian can be calculated with the help of the commutation rule $\boldsymbol{J}\wedge\boldsymbol{J} = i\boldsymbol{J}$, and is found to be diagonal under the condition:

$$\operatorname{ctgh} 2\alpha = 1 - (2E/\chi Q^2)\,.$$

We can then write the new Hamiltonian as:

$$U^+H_2U = -\tfrac{5}{2}(2E - \hbar\omega) + \hbar\omega S_1$$

with

$$\hbar\omega = 2E\sqrt{1 - \frac{\chi Q^2}{E}}\,.$$

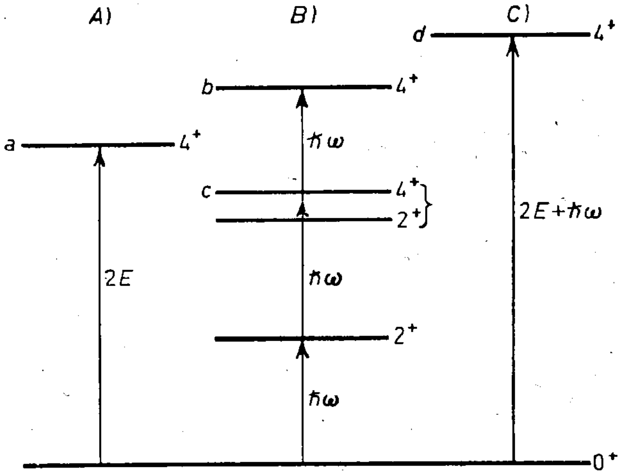

Fig. 2. – Energy spectrum resulting from a « pairing plus quadrupole » force coupling scheme.

The following schematic energy spectrum describes therefore our system of N interacting particles in the j shell (Fig. 2):

A) quasi-boson type excitations with energy $2E$; operators $B_\mu^{(s\neq 1)+}$;

B) phonon type excitations with energy $\hbar\omega$; operators $B_\mu^{(2)+}$;

C) excitations due to the coupling of phonons and quasi-bosons.

It is clear that for large j several $4+$ levels are obtained which lie above an energy of order $2\hbar\omega$. In order to simplify the analysis of the various possible cases, we make the assumption that the quadrupole force is strong enough to depress the $2'+$ two phonon level below the one quasi-boson level ($\hbar\omega < E$).

We have then four types of $4+$ levels labelled by $x = a$, b, c and d:

a) One $4+$ quasi-boson (*A* band).

b) Three phonons coupled to $I = 4+$ (*B* band).

c) Two phonons coupled to $I = 4+$ (*B* band).

d) A phonon and a quasi-boson coupled to $I = 4+$ (*C* band).

We can then calculate the branching ratio F_x for each type x.

3. – Electromagnetic transitions.

The $E2$ transition operator $\mathscr{M}^{(2)}_\mu = \sum_a r_a^2 Y^{(2)}_\mu(a)$ can be written, in second quantized form, after having performed the quasi-particle transformation as:

$$\mathscr{M}^{(2)}_\mu = Q[B^{(2)+}_\mu + (-)^\mu B^{(2)}_{-\mu}] + q[\beta^+\gamma]^{(2)}_\mu ,$$

with

$$q = (v^2 - u^2)\frac{1}{\sqrt{5}}\langle j\|r^2 Y^{(2)}\|j\rangle ,$$

$$[\beta^+\gamma]^{(2)}_\mu = \sum_{mm'}\langle jjmm'|2\mu\rangle\,\beta^+_m\gamma_{m'} ,$$

$$\gamma_m = (-)^{j-m}\beta_{-m} .$$

By the canonical transformation U, the operator $B^{(2)+}_\mu$ becomes

$$U^+ B^{(2)+}_\mu U = \cosh\alpha B^{(2)+}_\mu - (-)^\mu \sinh\alpha B^{(2)}_{-\mu} .$$

One gets therefore:

$$U^+\mathscr{M}^{(2)}_\mu U = Q(\cosh\alpha - \sinh\alpha)[B^{(2)+}_\mu + C^{(2)}_\mu] - q\mathscr{N}^{(2)}_\mu ,$$

$$\mathscr{N}^{(2)}_\mu = -[\beta^+\gamma]^{(2)}_\mu + 2(1-\cosh 2\alpha)M^{(2)}_\mu + \sinh 2\alpha N^{(2)}_\mu + \sinh 2\alpha O^{(2)}_\mu +$$
$$+ 2(1-\cosh\alpha)P^{(2)}_\mu + 2(1-\cosh\alpha)Q^{(2)}_\mu + 2\sinh\alpha R^{(2)}_\mu + 2\sinh\alpha T^{(2)}_\mu ,$$

$$M^{(2)}_\mu = 5\begin{Bmatrix}2&2&2\\ j&j&j\end{Bmatrix}[B^{(2)+}C^{(2)}]^{(2)}_\mu \qquad P^{(2)}_\mu = \sum_{s\neq 2}\sqrt{5(2s+1)}\begin{Bmatrix}2&2&s\\ j&j&j\end{Bmatrix}[B^{(s)+}C^{(2)}]^{(2)}_\mu ,$$

$$N^{(2)}_\mu = 5\begin{Bmatrix}2&2&2\\ j&j&j\end{Bmatrix}[B^{(2)+}B^{(2)+}]^{(2)}_\mu \qquad Q^{(2)}_\mu = \sum_{s\neq 2}\sqrt{5(2s+1)}\begin{Bmatrix}2&2&s\\ j&j&j\end{Bmatrix}[C^{(s)}B^{(2)+}]^{(2)}_\mu ,$$

$$O^{(2)}_\mu = 5\begin{Bmatrix}2&2&2\\ j&j&j\end{Bmatrix}[C^{(2)}C^{(2)}]^{(2)}_\mu \qquad R^{(2)}_\mu = \sum_{s\neq 2}\sqrt{5(2s+1)}\begin{Bmatrix}2&2&s\\ j&j&j\end{Bmatrix}[B^{(s)+}B^{(2)+}]^{(2)}_\mu ,$$

$$T^{(2)}_\mu = \sum_{s\neq 2}\sqrt{5(2s+1)}\begin{Bmatrix}2&2&s\\ j&j&j\end{Bmatrix}[C^{(s)}C^{(2)}]^{(2)}_\mu ,$$

with

$$C^{(s)}_\mu = (-)^\mu B^{(s)}_{-\mu} .$$

The expressions for the operators M, N ... T have been obtained forming commutators of J_x with $[\beta^+\gamma]^{(2)}_\mu$, and making use of the angular momentum recoupling rules ([12]).

One sees that the transitions a_1 and d_2 are strictly forbidden, *i.e.* $F_a = 0$ and $F_d = \infty$. For the B band transitions (b and c) one must evaluate (*):

$$B(E2) = \tfrac{1}{9}\,|\langle I=2\|\mathcal{M}^{(2)}\|I=4\rangle|^2 .$$

Using known results for matrix elements of product of tensor operators and the selection rules for matrix elements of the $B^{(s)}$ operator, one finds

$$F_b = \frac{1}{f}\operatorname{tg}^2\varphi\,\frac{(\cosh\alpha - \sinh\alpha)^2}{\sinh^2 2\alpha},$$

$$F_c = \frac{16}{49}\,f\operatorname{ctg}^2\varphi\,\frac{\cosh^2 2\alpha}{(\cosh\alpha-\sinh\alpha)^2},$$

with

$$f = 100\begin{Bmatrix}2 & 2 & 2\\ j & j & j\end{Bmatrix}^2,$$

$$u = \cos(\varphi/2),$$

$$v = -(-)^{\nu}\sin(\varphi/2).$$

It is also interesting to evaluate the usual branching ratio

$$F_0 = \frac{B(E2)_{2'_+\to 2+}}{B(E2)_{2'_+\to 0+}},$$

which is found to be exactly $2F_b$, together with the value $\mathcal{F}$ of $B(E2)_{2+\to 0+}$ in units of a single particle estimate $B(E2)_{sp}$ for $(j)^2_{J=2}\to(j)^2_{J=0}$ transition, which can be expressed as

$$\frac{\mathcal{F}}{\Omega} = \frac{1}{4}\sin^2\varphi(\cosh\alpha-\sinh\alpha)^2 .$$

4. – Discussion.

Our assumption $\hbar\omega/E<1$, or equivalently $\chi Q^2/G\Omega>\frac{3}{8}$, leads to χ values much too large as compared with the actual values used for instance by KISS-

(12) A. R. EDMONDS: *Angular Momentum in Quantum Mechanics* (Princeton, 1957).

(*) Strictly speaking, the $2'+$ and the $4+$ two-phonon levels are degenerate, and therefore no transition between them can occur. However, we may always evaluate $B(E2)$ for the transition C_1, and suppose that the degeneracy is slightly removed as is the case in actual nuclei (see Fig. 2).

LINGER and SØRENSEN [4]. But it is well known that a single j model is not able to depress strongly the 2+ level. Thus our assumption must be considered only as simulating such a strong effect. We choose as a tentative value $\hbar\omega = 0.8\,E$. On the other hand, the quantity f varies quite smoothly when j increases, and one can take a mean value $f = 0.4$ without affecting significantly the order of magnitude of the branching ratios F_x.

The single parameter left is then:

$$\xi = \frac{1-(\hbar\omega/2E)^2}{(2N/\Omega)(1-N/2\Omega)}\,,$$

which describes the occupancy of the shell (a large ξ occurs at the beginning or at the end of the shell, and $\xi_{\min} = 0.84$ at the middle of the shell).

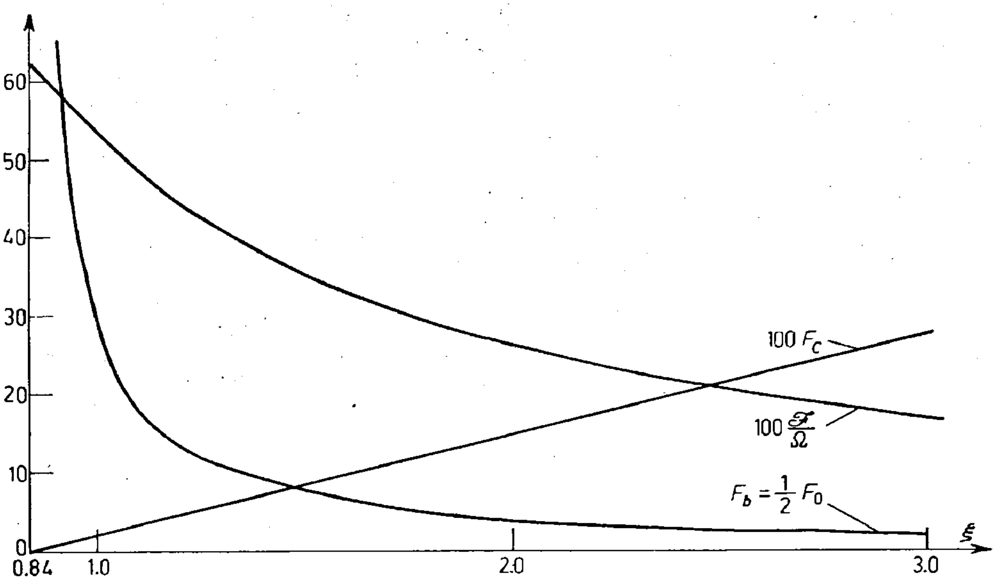

Fig. 3. – Branching ratios F_x and enhancement factor $\mathscr{F}$ *vs.* shell occupancy.

In Fig. 3, we have plotted the quantities of interest against the parameter ξ. If we exclude the region of large ξ where the model is inadequate, we can point out the following trends:

i) The branching ratio F_b is of the order of some units (and even larger when approaching the middle of the shell), whereas F_c remains always $\ll 1$.

ii) The quantity $\mathscr{F}/\Omega$ has a value 0.2 to 0.6, and the factor $\mathscr{F}$ cannot be large if j is not large.

In the middle of the shell, $\mathscr{F}$ has its maximum value (*), $F_c = 0$ and F_b as F_0 becomes infinite. We approach then the situation of the vibrational model.

5. – Conclusion.

It is thus verified that even within the limit of validity of such a simple model there exists among the possible ones a group of 4+ higher excited levels, for which the probability of $E2$ transition to the $2'+$ level is enhanced with respect to that to the first 2+ level (type *a* and *b*). We also find other 4+ levels for which the opposite situation occurs (type *a* and *c*).

As has been mentioned above, the model discussed here can only provide qualitative indications. A more realistic treatment should include several particle levels $j_1, j_2, j_3, \ldots$ with reasonable values of the various parameters. Further calculations in this sense are in progress.

* * *

We wish to thank Prof. M. JEAN and Prof. R. A. RICCI for suggesting the subject of this note and for many stimulating discussions.

(*) It can be remarked that for a pure pairing force, $\mathscr{F}/\Omega$ has its maximum value 0.25 at the middle of the shell, and that our quadrupole force gives an additional enhancement by a factor 2.5.

RIASSUNTO (*)

Si calcolano i rapporti di branching dai più alti livelli eccitati nei nuclei sferici pari-pari, facendo uso di un modello semplificato, che comprende un solo strato j, ed una interazione residua di coppie e di quadrupolo fra i nucleoni. Si applica il metodo al branching dai livelli 4^+; si trova che la transizione $E2$ al secondo livello 2^+ può essere esaltata rispetto a quella corrispondente verso il primo livello 2^+ da più di un livello 4^+.

(*) *Traduzione a cura della Redazione.*